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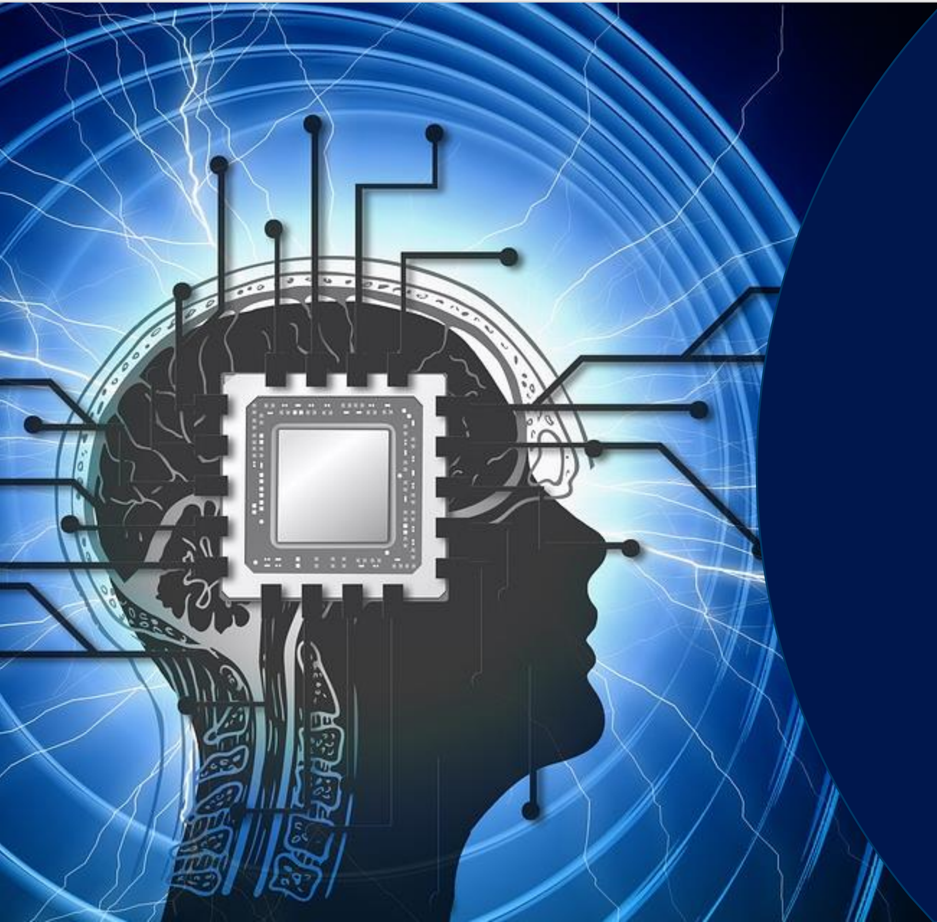


Halbleitertechnik IV-
Nano Electronics

Friedrich-Alexander-Universität Erlangen-Nürnberg | Lehrstuhl für Elektronische Bauelemente | Cauerstraße 6 | 91058 Erlangen | www.leb.tf.fau.de | Michael Jank

Repetition

Charge Carriers in Semiconductors

Objectives of the lecture

Mission and goals?

Charge Carriers in Semiconductors
Objectives

- This lecture repeats in a very compact manner the basic concepts of generation and transport of charge carriers in semiconductors
- Together with the lecture on p-n junctions and metal-oxide-semiconductor devices it lays the groundwork for understanding the basic concepts behind scaled and nanometer size devices
- You should take away a solid understanding of the reasoning and relations behind the formations of different distributions of free carriers as well as their movement caused by concentration gradients or external stimuli

Charge Carriers in Semiconductors
Outline

- **Band Structure Models**
- **Carrier Concentrations in Solids**
- **Transport of Carriers in Electric Field**
- **Impact Ionization**
- **Carrier Diffusion**
- **Balancing Carrier Related Phenomena – Equation of Continuity**

Band Structure Models

Definition of Internal Vigors for Establishment of Carrier Ensembles

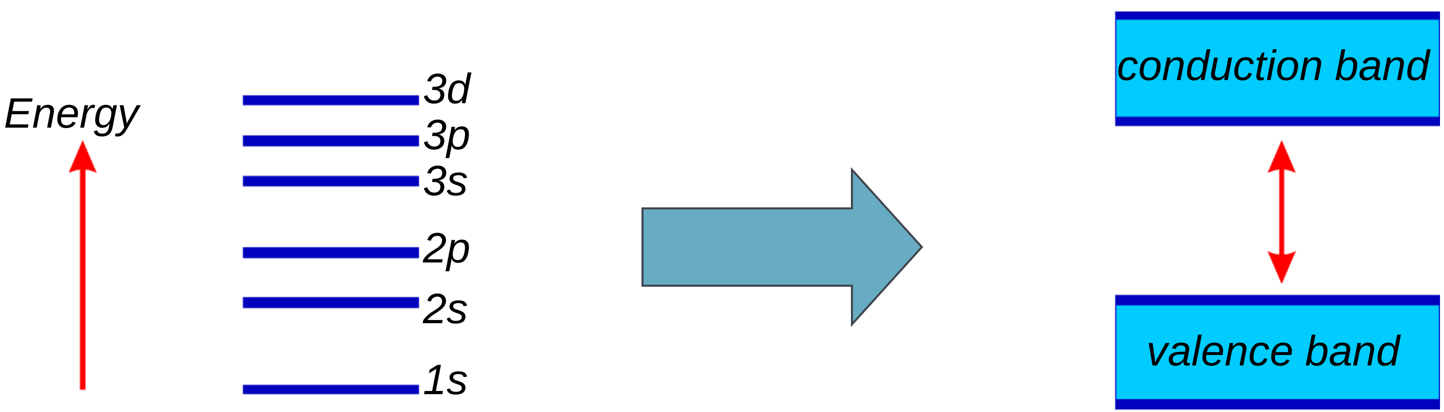
Band Structure Models

Background

- In Semiconductors, Energy Band Structure defines both the generation/distribution as well as transport of free carriers
  - *Simplified band models help to describe most devices in electronics*

Single atom:
discrete energy levels

Solid:
"blurred" energy bands

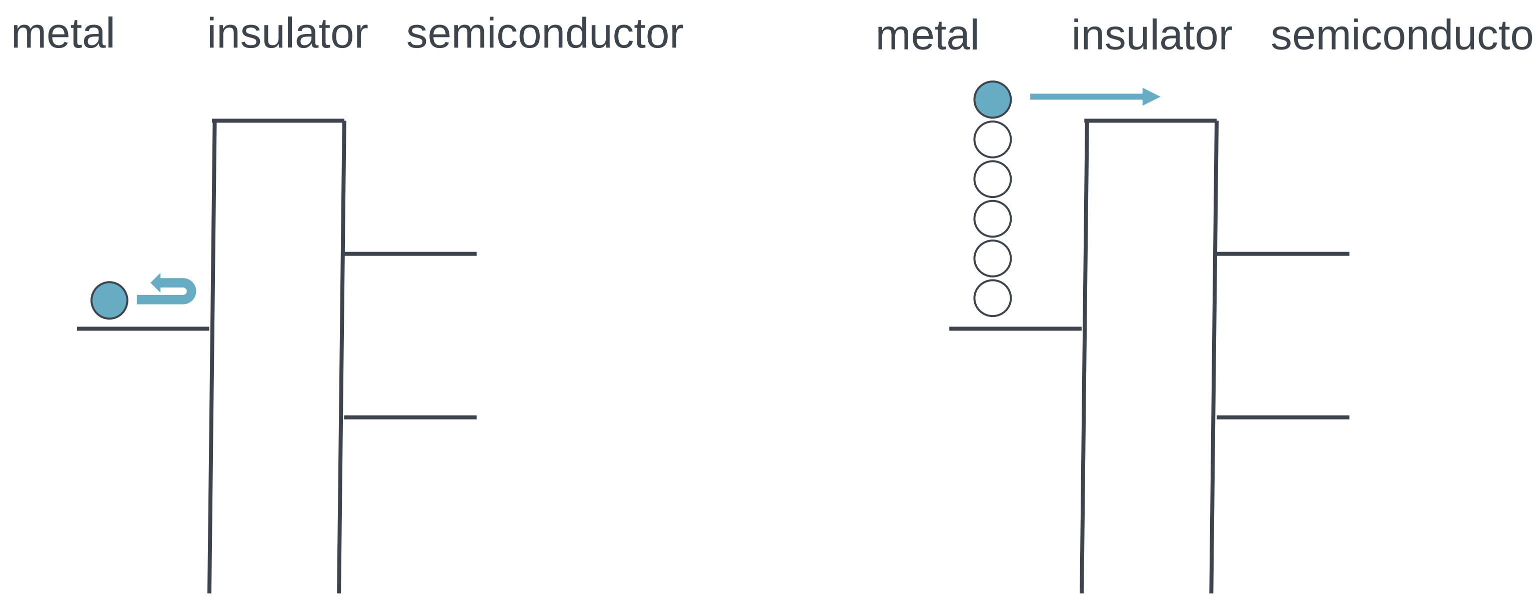


approx. 10^{22} atoms/cm^3

Band Structure Models

Applications – thermionic emission over a barrier

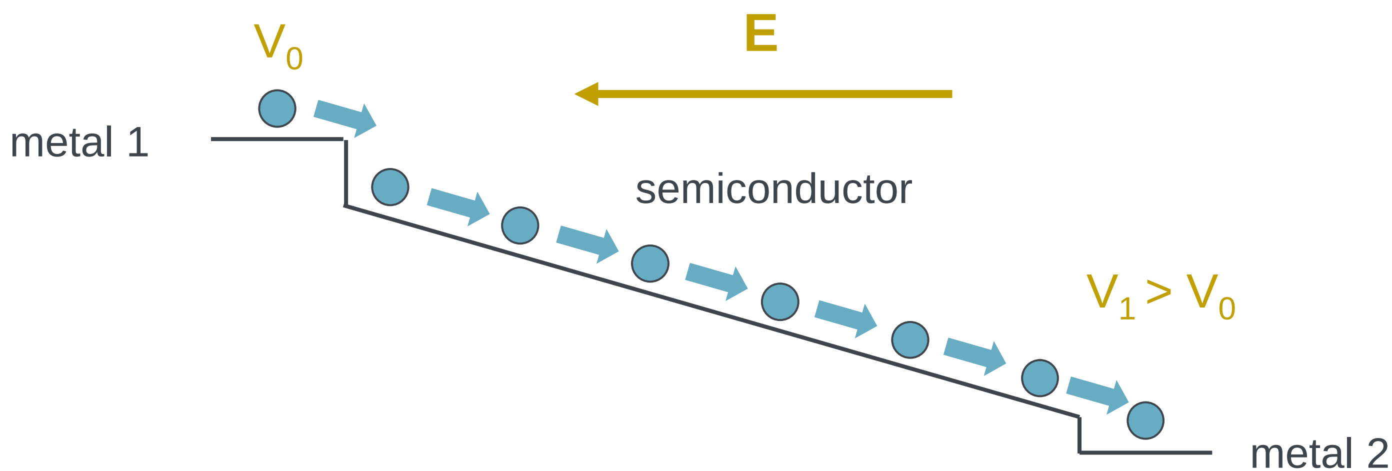
- The energetic level of a carrier defines its transport (options)
- Example: barrier in a MOS capacitor (MOSFET channel)
    - *The carrier will need kinetic energy to overcome the energy barrier*



Band Structure Models

Applications - transport through a semiconductor (conduction band)

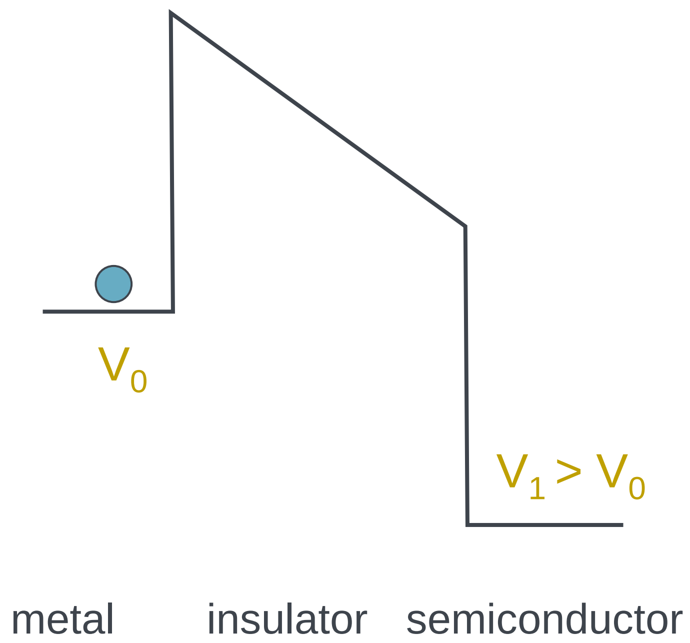
- The energetic level of a carrier defines its transport (options)
- Example: potential gradient in a (ohmic) contacted semiconductor (conduction band)

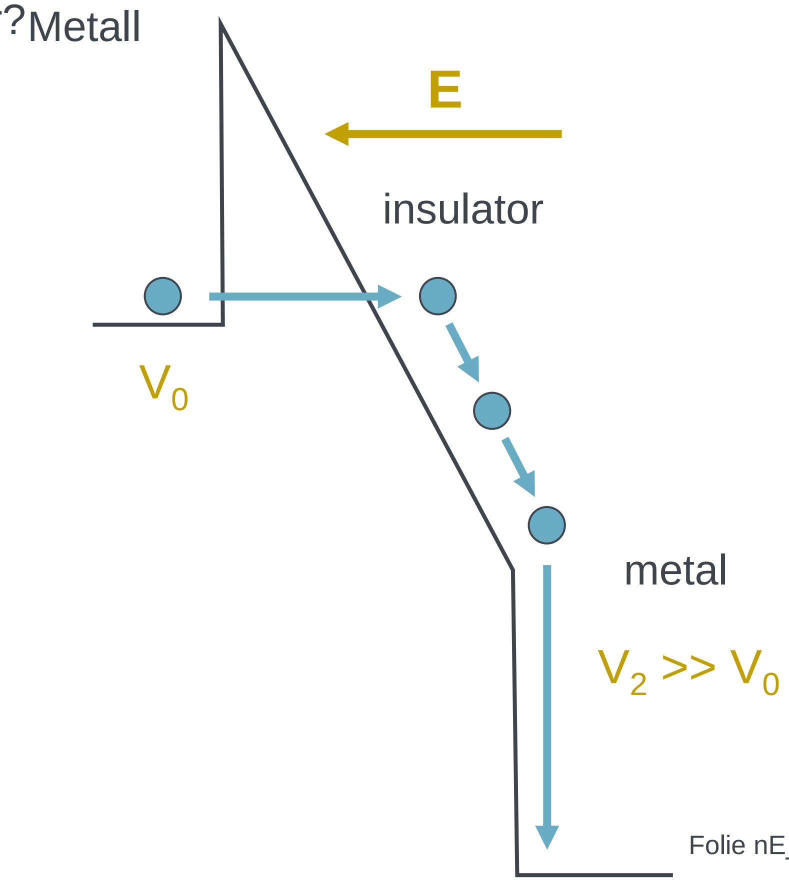


Band Structure Models

Applications - tunneling through a barrier

- The energetic level of a carrier defines its transport (options)
- Example: carrier transport through an insulator?





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Carrier Concentration in Solids

Density of States (DoS)

- The Density of States ***D(E)*** describes the number of states („places“ to be occupied by carriers) at energy level ***E*** that are available per volume and energy interval
- ***N*** atoms contribute to y (number of energy levels in the interval forming the band) x 2 (spins) states that are overlapping
- Densities of States for electrons in conduction band D_C and holes in valence band D_V can be derived from the effective masses of the respective carriers (m_e*, m_h*). In a 3d ideal crystal thay can be calculated as

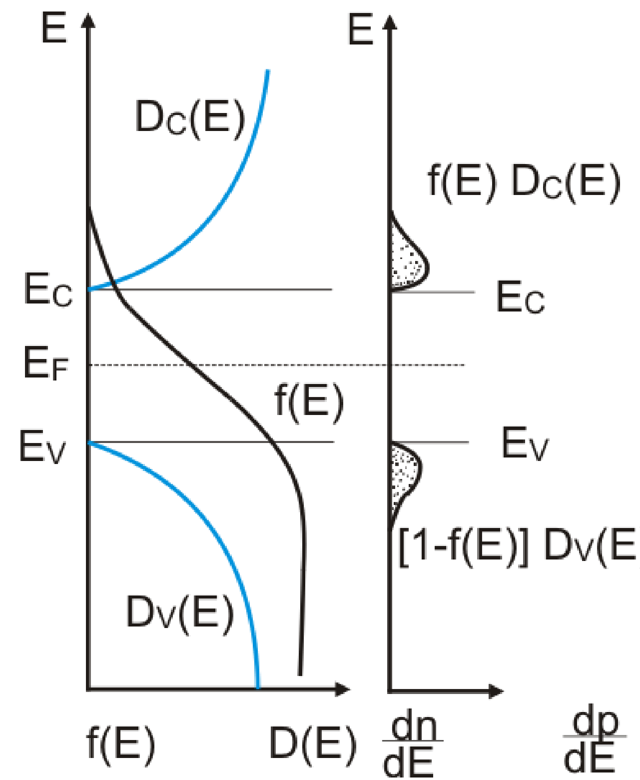
$$D_C(E) = \frac{4\pi(2m_e^*)^{\frac{3}{2}}}{h^3}\sqrt{E - E_C}$$

$$D_V(E) = \frac{4\pi(2m_h^*)^{\frac{3}{2}}}{h^3}\sqrt{E_V - E}$$

Carrier Concentration in Solids

Carrier concentrations in conduction and valence band

- Example of silicon
- On the left side you see both
  - *the densities of states for conduction and valence band as well as*
  - *the Fermi function f(E) giving the probability of an available state at a certain energy level being occupied by a carrier*
- The DoS distributions are different in the conduction and valence bands due to different effective masses m_e^* and m_h^*
- Carrier concentrations per energy are calculated by multiplying the DoS with the occupation probability (right)



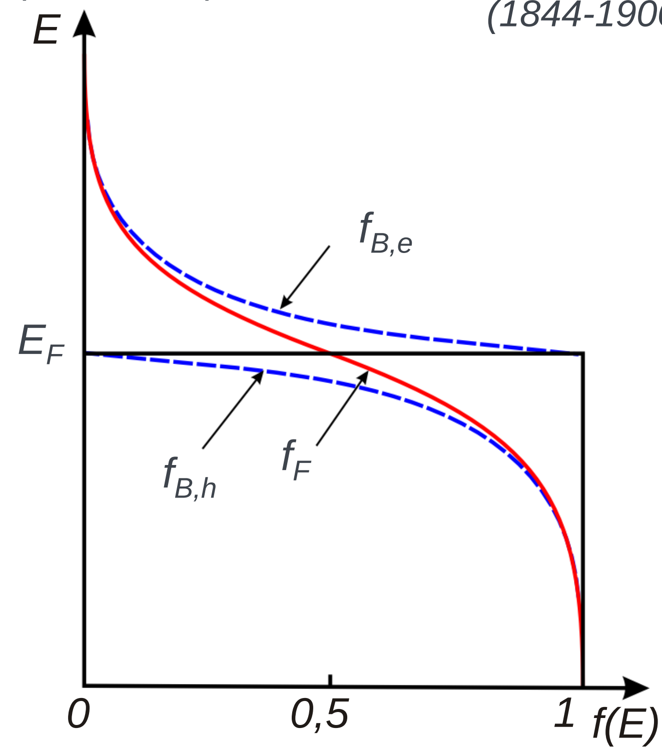
Carrier Concentration in Solids

FERMI-DIRAC statistics

- In conduction band, *f(E)* is directly describing the probability for presence of a free electron
- The concept of holes is based on the absence of electrons, i.e. unoccupied states
  - *consequently 1-f(E) describes the appearence of holes in valence band*
- The FERMI Distribution is given by

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Enrico Fermi (1901-1954) *Paul Dirac (1902 - 1984)* *Ludwig Boltzmann (1844-1906)*



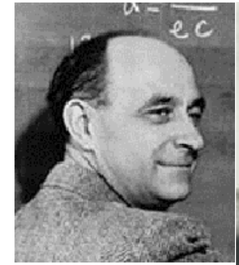
Carrier Concentration in Solids

BOLTZMANN distribution

- The FERMI distribution function always refers to the so called FERMI level E_F at which $f(E)$ is equal to 0.5
- The position of E_F is in first order describing the doping level and as such defining the electrical behavior of the semiconductor
- Fo $|E-E_F| >> kT$, the FERMI distribution may be approximated by the MAXWELL-BOLTZMAN distribution

$$f_{B,e}(E) = \exp\left(-\frac{E - E_F}{kT}\right)$$

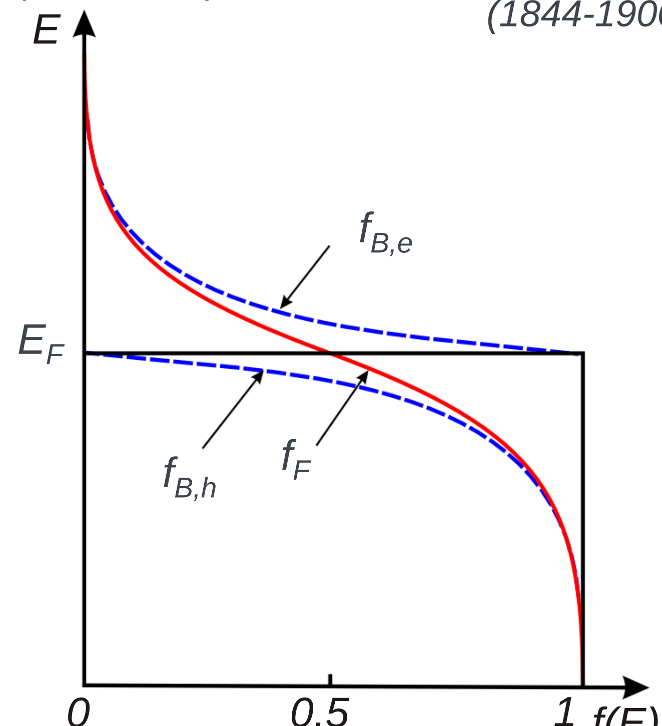
$$f_{B,h}(E) = 1 - \exp\left(\frac{E - E_F}{kT}\right)$$



Enrico Fermi (1901-1954)

Paul Dirac (1902 - 1984)

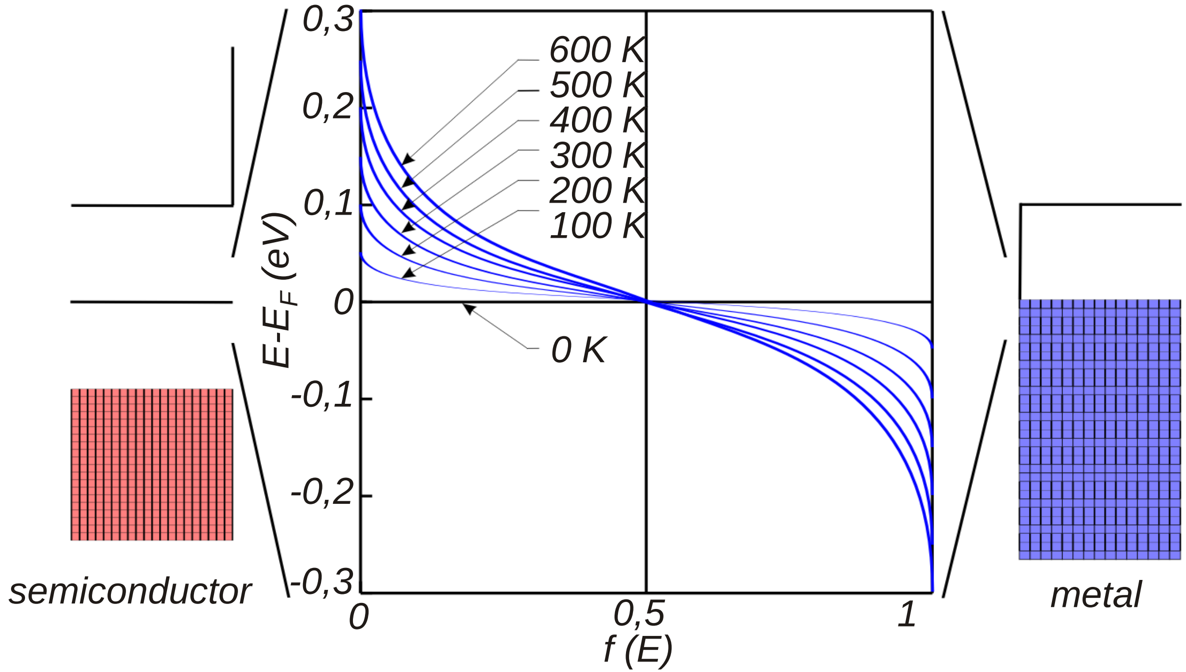
Ludwig Boltzmann (1844-1906)



Carrier Concentration in Solids

Summary and implications of carrier densities

- The FERMI-DIRAC statistics is strongly dependent on temperature
- The position of the FERMI level describes the behavior of materials



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Carrier Transport in an Electric Field

Charge carriers in vacuum vs. solids

- Particle kinetics according to NEWTON's equation of motion

$$m\frac{\partial \mathbf{v_n}}{\partial t} + \frac{m}{\tau}\mathbf{v_n} = -q\mathbf{E}$$

τ : averaged time between two impacts

in vacuum:
($\tau \rightarrow \infty$)

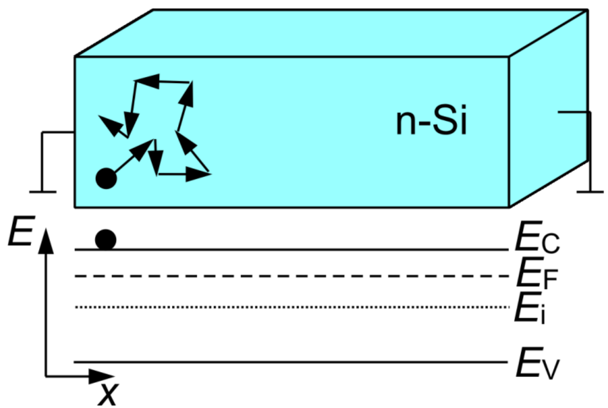
$$\frac{\partial \mathbf{v_n}}{\partial t} = -\frac{q}{m}\mathbf{E}$$

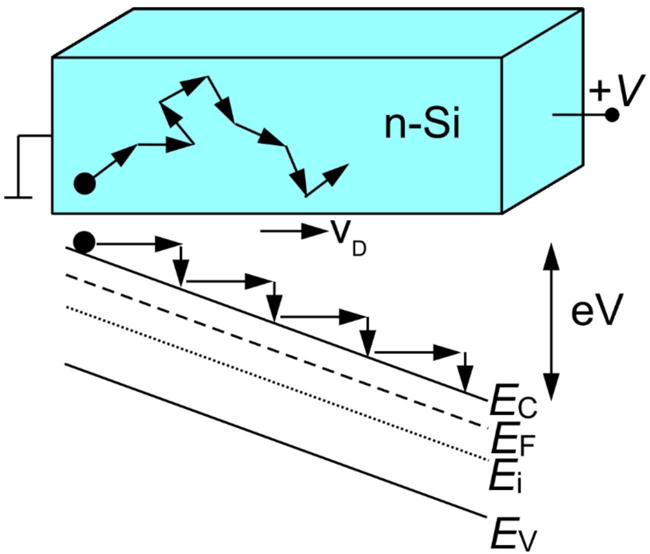
in solid:
($\boldsymbol{v_n}$=const.)

$$\mathbf{v_n} = \frac{q\tau}{m}\mathbf{E} = \mu_n\mathbf{E}$$

constant velocity!

mobility μ_n





Carrier Transport in an Electric Field

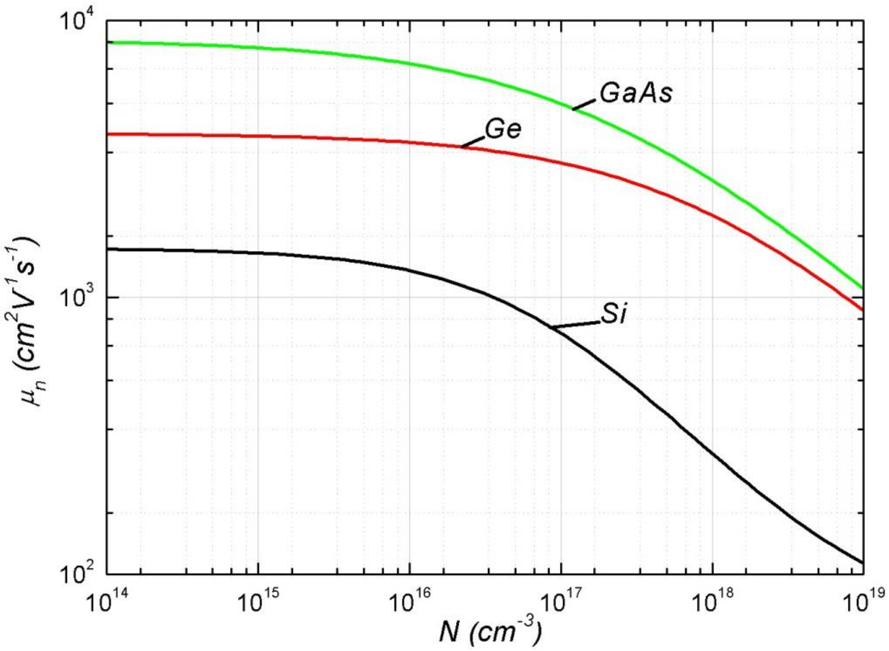
Mobility μ

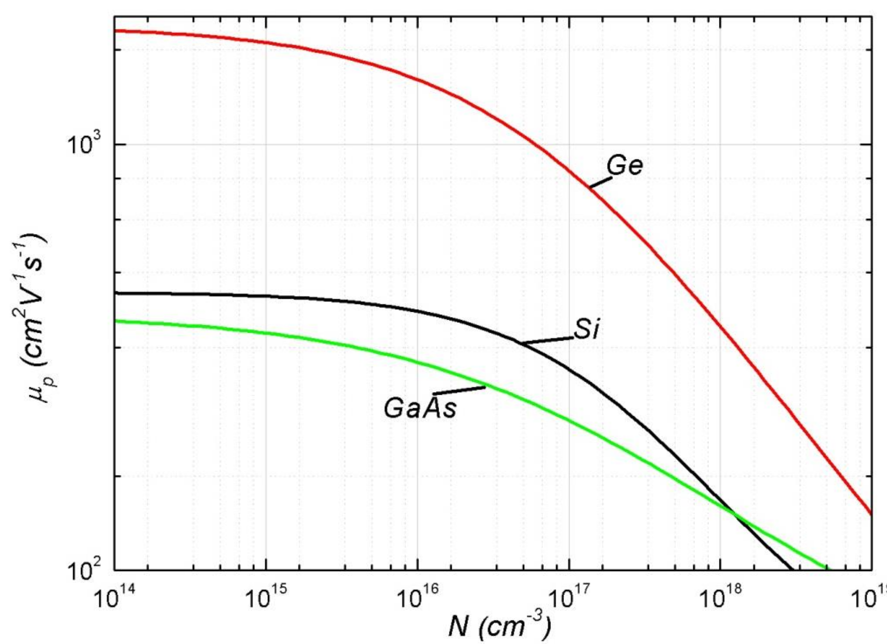
- The mobility is defined by scattering of carriers in the crystal lattice and at defects
- It is the most important measure for describing the movement of carriers in solids
- Scattering is observed due to
  - *Lattice vibrations (phonon scattering)*
  - *Latice defects (dislocations, interstitials, vacancies)*
  - *Grain boundaries*
  - *Extrinsic atoms (dopants, impurities)*
  - *Other carriers*
- Different effects are sumed up by reciprocal addition: $1/\mu_{total} = 1/\mu_1 + 1/\mu_2 + \ldots$
- Other effects on transport by
  - *External electric fields*
  - *External magnetic fields*

Carrier Transport in an Electric Field

Characteristic mobilities

- Example: scattering at ionized donors and acceptors (drift mobilities)
  - *Mobility is decreasing with increasing doping level N*

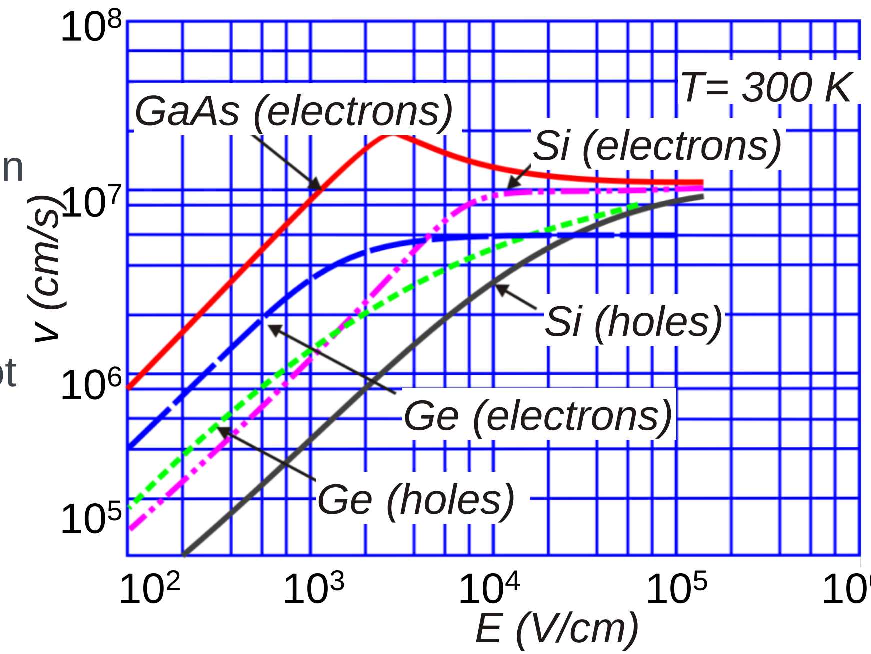




Carrier Transport in an Electric Field

Saturation velocity

- Dependence of velocity/mobility on electric field
- At small fields (< 10kV/cm) the drift velocity is much smaller than the thermal velocity of the carriers ($v_{th} \approx 10^7$ cm/s). The mean time between two impacts and the mobility are constant, vD increases linearly with the electric field.
- For higher electric fields, the drift velocity cannot be neglected anymore. Thus scattering is enhanced, the mean time between impacts and the mobility decrease. Drift velocities approach saturation values of ($v_D \approx 10^7$ cm/s).



Carrier Transport in an Electric Field

Specific resistance

- The overall current density is made up of the movement (drift) of both electrons and holes in conduction and valence bands

$$\boldsymbol{j}_{\boldsymbol{Drift}} = q(p\boldsymbol{v}_{\boldsymbol{p}} - n\boldsymbol{v}_{\boldsymbol{n}}) = q(p\mu_p + n\mu_n)\boldsymbol{E}$$

- The specific resistance of the crystal is given by

$$\rho = \frac{1}{\sigma} = \frac{1}{q(p\mu_p + n\mu_n)}$$

- For moderate electric fields the mobility is constant and OHM's law can be applied

$$\boldsymbol{j}_{\boldsymbol{Drift}} = \sigma \boldsymbol{E}$$

Georg Simon Ohm (1789 – 1854)

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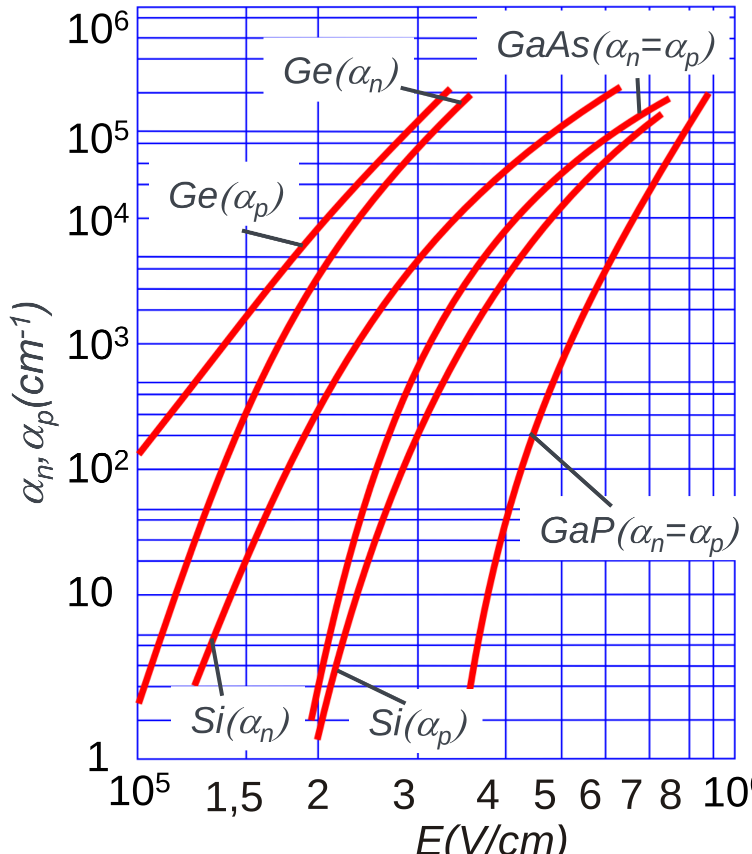
Impact Ionization

Ionization rates

- At high fields, carriers can gain sufficient energy to ionize lattice atoms upon scattering (**impact ionization**)
  - *Mean free path has to be sufficient to yield the critical energy before interacting with a bound carrier!*
- Ionization rate α is the medium number of carriers generated (=ionization events) per particle and distance
- Ionisation rates for electrons and holes differ (i.e.: α_n, α_p) but often simplified according to $\alpha_p \approx \alpha_n \approx \alpha$ for calculations

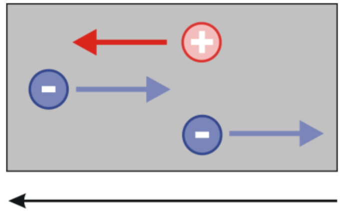


ionization rate α: approximate values

Impact Ionization

Avalanche breakdown

- If the generated carriers (electrons and holes) can themselves gain sufficient energy to ionize further atoms, the effect may grow exponentially



- The respective effect is named **Avalanche Breakdown** and is used for example in solid-state secondary electron multipliers for single-photon detection
- The condition for avalanche breakdown (for $\alpha \approx \alpha_n \approx \alpha_p$)

$$\int_0^d \alpha \, dx = 1$$

is typically fulfilled at an critical ionization rate α of 10^4 cm^{-1} meaning that 1 ionization per µm is needed for the carrier multiplication

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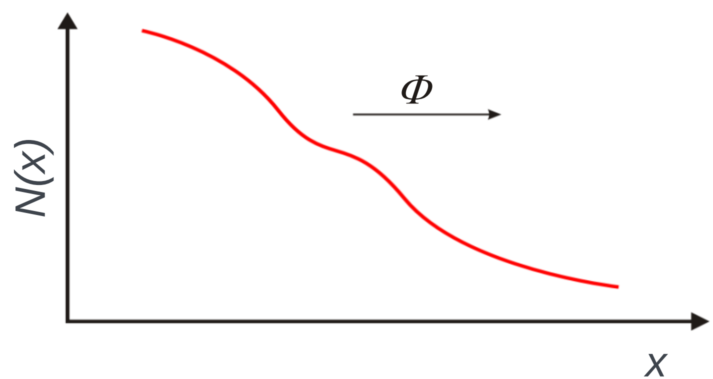
Diffusion Current
Diffusion of Carriers

- Besides the drift transport due to electric fields, also carrier redistribution by diffusion has to be taken into account
- In case of a gradient in the carrier concentration (electrons, holes or both), there is a tendency to reduce the gradient
- Background is the thermal motion of the carriers. Consider a plane with different carrier concentrations on either sides: Thermal motion will lead to a net movement from the side with higher concentration to the side with lower concentration

$$\Phi(x,t) = -D\frac{\partial N(x,t)}{\partial x}$$

with **Φ: carrier flux**
N: carrier concentration
D: diffusion coefficient



Adolf Fick (1829 - 1901)

Diffusion Current

Diffusion currents for electrons and holes

- The particle flux Φ is transformed into a current by multiplying it with the respective charge (-q for electrons, +q for holes) and accounting for the individual concentration gradients as well as diffusion coefficients

$$j_{p,Diff} = -qD_p \frac{\partial p(x,t)}{\partial x} \qquad j_{n,Diff} = qD_n \frac{\partial n(x,t)}{\partial x}$$

- The total diffusion current is calculated by summing up the components

$$j_{Diff}(x,t) = q\left(D_n \frac{\partial n(x,t)}{\partial x} - D_p \frac{\partial p(x,t)}{\partial x}\right)$$

or more in general (in 3-d)

$$\boldsymbol{j}_{Diff} = q(D_n \,\mathbf{grad}\, n - D_p \,\mathbf{grad}\, p)$$

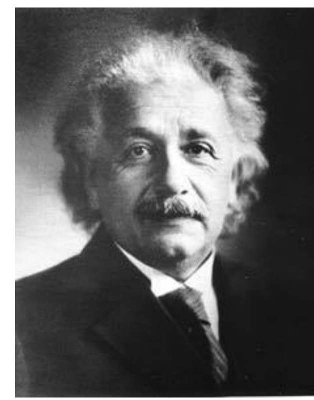
Diffusion Current

EINSTEIN relation

- Diffusion constant is dependent on the average velocity v_{TH} (thermal) and also on the median time between two scattering events
- This also holds for the mobility (transport in electric field)
- Diffusion constant and mobility are linked by the EINSTEIN relation

$$D_{n,p} = \frac{kT}{q} \mu_{n,p} = V_T \mu_{n,p}$$

where k: BOLTZMANN constant
V_T: thermal voltage



Albert Einstein
(1879 - 1955)

Diffusion Current

Selected materials properties

- Diffusion constants and mobilities at room temperature and low doping concentration (cf. slide 20)

Semi-conductor	D_n (cm²/s)	D_p (cm²/s)	μ_n (cm²/Vs)	μ_p (cm²/Vs)
Ge	100	49	3900	1900
Si	39	12	1500	450
GaAs	220	10	8500	400

- The Einstein relation is only valid for low electric fields ($E < 10^3$ V/cm), where also the mobility $\mu(E)$ is constant

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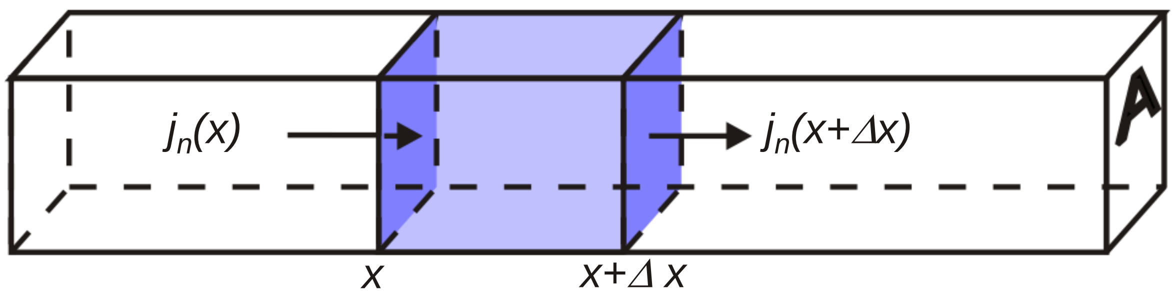
Equation of Continuity
Motivation

- The equation of continuity integrates (=superimposes) the transport mechnisms of **drift** and **diffusion**
- Furthermore, it includes **generation*** and **recombination***, thus allowing for a full balancing of a specific volume $A \cdot \Delta x$ of the semiconductor



- e.g. for electrons:

$$\frac{\partial n}{\partial t} = \left[\frac{j_n(x) - j_n(x+\Delta x)}{-q\Delta x}\right] + G_n - R_n$$

where
G_n: generation rate of electrons
R_n: recombination rate

* not relevant for Nano Electronics

Equation of Continuity
Full representation

- Inserting drift as well as diffusion currents and generation as well as recombination rates and letting Δx approach 0 yields the EoCs for both electrons and holes

$$\frac{\partial n(x,t)}{\partial t} = n(x,t)\,\mu_n \frac{\partial E(x,t)}{\partial x} + \mu_n E(x,t)\frac{\partial n(x,t)}{\partial x} + D_n \frac{\partial^2 n(x,t)}{\partial x^2} + G_{n,ext} - \frac{\Delta n(x,t)}{\tau_n}$$

$$\frac{\partial p(x,t)}{\partial t} = -p(x,t)\,\mu_p \frac{\partial E(x,t)}{\partial x} - \mu_p E(x,t)\frac{\partial p(x,t)}{\partial x} + D_p \frac{\partial^2 p(x,t)}{\partial x^2} + G_{p,ext} - \frac{\Delta p(x,t)}{\tau_p}$$

- In combination with Poisson equation, relating the electric field to charge concentrations

$$\frac{\partial E(x,t)}{\partial x} = \frac{\rho(x,t)}{\varepsilon_0 \varepsilon_{HL}} \quad \text{mit} \quad \rho(x,t) = q(p + N_D^+ - n - N_A^-)$$

where ρ(x,t) is space-charge density

the EoCs provide the solution to the most relevant questions in semiconductor physics

Thanks for your attention!