Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	4.0	*Graphs of Polynomial and Rational Functions*
Lesson Code	4.5	*Sketching the graph of a rational function*
Time Frame		30 minutes

TARGET

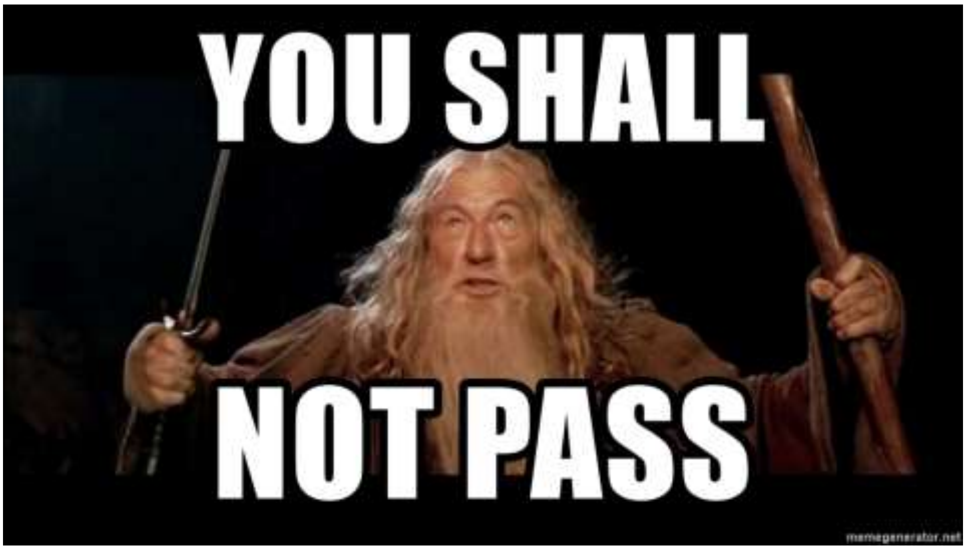
Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, the students should be able to sketch the graph of a rational function.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

What can you say about the meme below? To which aspect of your life does this apply? For one thing, we may recall our parents stating a long list of do's and don'ts and with those don'ts, we may hear them saying a lot of times, 'this is your limit, you shall not pass!' In school, our teachers may say, 'if you do not study hard, you shall not pass.'



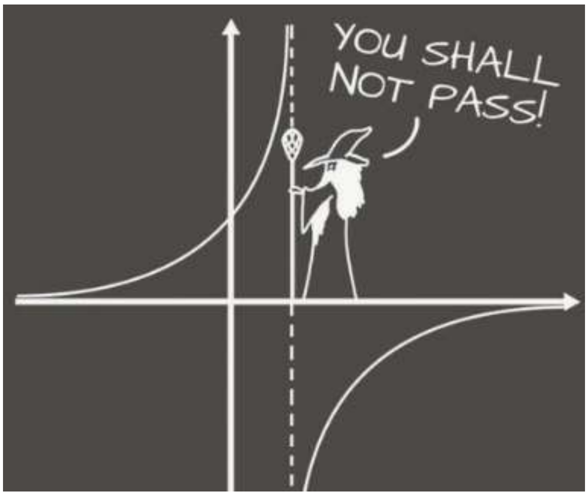
Source: https://memegenerator.net/instance/67496522/you-shall-not-pass-hd-you-shall-not-pass

And what does this have to do with our topic in this learning guide? Let's find out!

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

"You shall not pass" has its applications in sketching the graph of rational functions as well. Recall the concepts you have learned earlier. The curve shall not pass through the asymptotes. It will only get closer and closer to the line, but it will not touch it. This is the first thing that you have to put in mind when graphing rational functions. This is just a portion of the real picture though.



Source:https://www.pinterest.ph/pin/430938258069679756/

To apply the concepts that you have previously mastered, let us consider the following examples on sketching the graph of rational functions. And again, knowing which lines you should not pass through is IMPORTANT!

Note: Graphs of rational functions can cross horizontal or oblique asymptotes.

Example 1: **Function with Horizontal and Vertical Asymptotes**

Sketch the graph of $f(x) = \frac{5}{x^2-1}$.

For Example 1, start with those that you shall not pass. The vertical asymptotes are $x = 1$ and $x = -1$ and the horizontal asymptote is the line $y = 0$ or the $x -$ axis. To denote the asymptotes, we make use of dashed lines. Another important part is identifying whether the given function is odd, even, or neither. Since it is even, then we can say that $f(-x) = f(x)$. Therefore, the graph is symmetric with respect to the y −axis. The y −intercept is $(0, -5)$. Since $f(x) = 0$ has no solution, there is no x −intercept. And considering other points to see a clearer behavior of the curve gives us the table below.

x	0.9	1.1	2	3
y	-26.32	23.81	1.67	0.625

From the table, we can see that from the left the curve goes downward toward $x = 1$ and from the right, the curve goes upward toward $x = 1$.

1. Vertical Asymptotes:

   $x = 1$ and $x = -1$
2. Horizontal Asymptote:

   $y = 0$ or the $x-$ axis
3. Symmetric about the $y-$axis.
4. Intercept: $(0, -5)$.

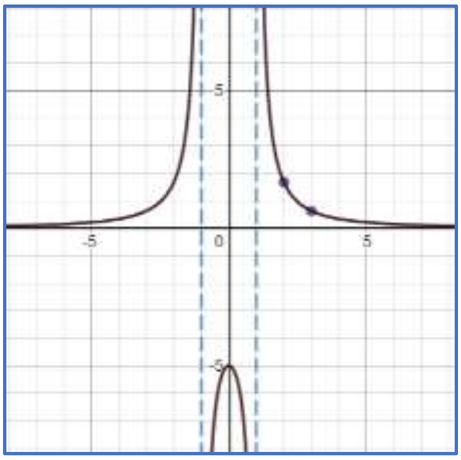


Figure 1: The graph of $f(x) = \frac{5}{x^2-1}$

Example 2: **A Rational Function That Crosses an Asymptote**

Sketch the graph of $f(x) = \frac{2x}{(x+4)^2}$.

For Example 2, the vertical asymptote is $x = -4$. Since the degree of the numerator is less than the degree of the denominator, then the $x-$axis is a horizontal asymptote. The intercept is at $(0,0)$. There is no symmetry. Considering other points gives the table below.

x	1	2	-2	-3
y	0.08	0.11	-1	-6

1. Vertical Asymptote:

   $x = -4$
2. Horizontal Asymptote:

   $y = 0$ or the $x-$ axis
3. Intercept : $(0,0)$.

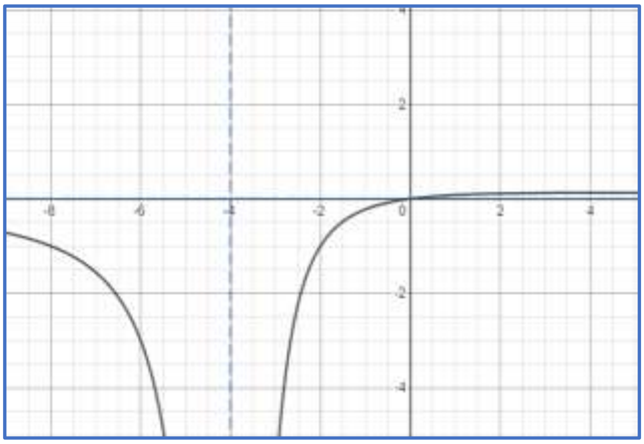


Figure 2: The graph of $f(x) = \frac{2x}{(x+4)^2}$

And from Figure 2, we can see that the graph crosses the horizontal asymptote.

Example 3: **Graphing a Function With an Oblique Asymptote**

Sketch the graph of $f(x) = \frac{2x^2+3x-5}{x-2}$.

Notice that the degree of the numerator is greater than the degree of the denominator. This implies the presence of an oblique asymptote. Earlier you have learned that by dividing the numerator with the denominator, you will obtain the equation for the oblique asymptote. Thus, the vertical asymptote is $x = 2$ and the oblique asymptote is $y = 2x + 7$. The intercepts are $(0, 2.5), (-2.5,0)$ and $(1,0)$. Considering other points to see a clearer behavior of the curve gives us the table below.

x	-3	-1	3	4
y	-0.8	2	22	19.5

1. Vertical Asymptote:

$$x = 2$$

2. Oblique asymptote:

$$y = 2x + 7$$

3. Intercepts:

$$(0,2.5), (-2.5,0), (1,0)$$

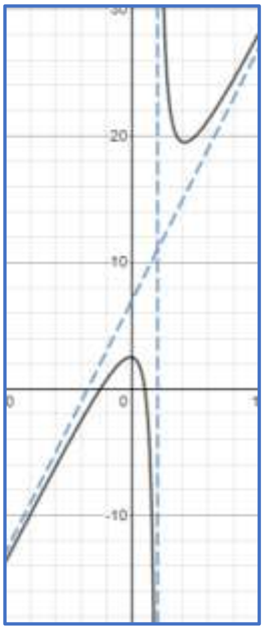


Figure 3: The graph of $f(x) = \frac{2x^2+3x-5}{x-2}$

Example 4: **Discontinuity (Hole) in Graphs**

Sketch the graph of $f(x) = \frac{x+2}{x^2-4}$.

Since $x^2 - 4 = (x-2)(x+2)$, then the domain of f is the set of all real numbers except 2 and -2. Notice that the rational expression can be simplified. As such, the function can also be defined as $f(x) = \frac{1}{x-2}$ for $x \neq 2$ and $x \neq -2$. In this case, the domain is determined even before the function is simplified. The graph of $f(x) = \frac{1}{x-2}$ has a vertical asymptote $x = 2$. Since the degree of the numerator is less than the degree of the denominator, then the y –axis is a horizontal asymptote. Also, $f(2)$ is undefined, thus the point $(-2, -0.25)$ will not be part of the graph and it will be indicated on the graph as an open circle.

1. Vertical Asymptote:

   $$x = 2$$
2. Horizontal asymptote:

   $$y = 0$$
3. Intercept: $\left(0, -\frac{1}{2}\right)$

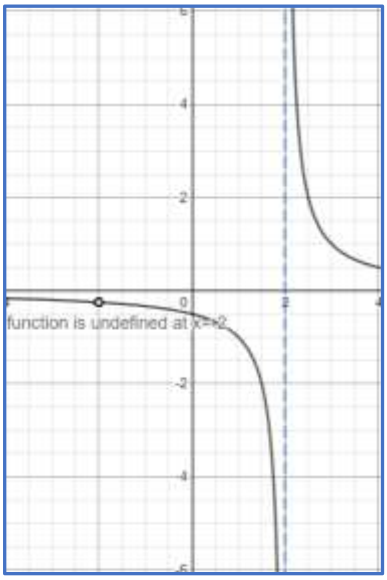


Figure 4: The graph of $f(x) = \frac{x+2}{x^2-4}$

NAVIGATE

Time Allocation: 8 minutes
Actual Time Allocation: _____ minutes

Sketch the graph of the following functions and complete the requirements on the table. Items marked with an asterisk (*) are graded.

Function	Asymptotes (Horizontal/Vertical/ Oblique) Symmetry (if any) Intercepts (if any)	Graph
1. $f(x) = \frac{x+1}{x^2-9}$		
*2. $f(x) = \frac{x}{(x+2)^2}$		
3. $f(x) = \frac{3x^2+4}{x-1}$		
*4. $f(x) = \frac{x+1}{x^2-1}$		

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

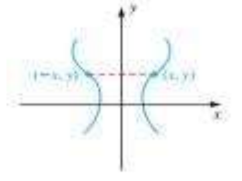
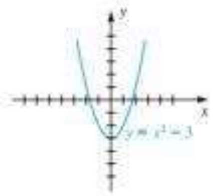
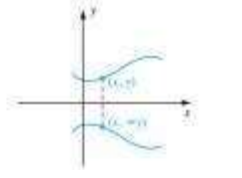
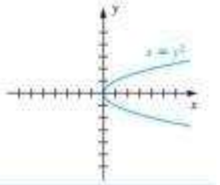
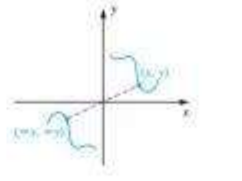
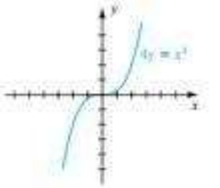
From the previous examples, we can say that sketching the graph of a rational function is not that difficult if we know very well how to get the ASIS, that is:

1. A-Asymptote

At this point, it should already be clear how to determine the asymptotes whether it is horizontal, vertical or oblique. Remember that asymptotes are drawn as dashed lines.

2. S-Symmetry

Always check for symmetry. This will help you have a rough idea of where the graphs are located on the Cartesian plane. Recall the table below which was provided to you during the first quarter.

Terminology	**Graphical Interpretation**	**Test for Symmetry**	**Illustration**
The graph is symmetric with respect to the y-axis		Substitution of $-x$ for x leads to the same equation.	
The graph is symmetric with respect to the x-axis.		Substitution of $-y$ for y leads to the same equation.	
The graph is symmetric with respect to the Origin.		Simultaneous substitution of $-x$ for x and $-y$ for y leads to the same equation.	

Source: Algebra and Trigonometry with Analytic Geometry/ Swokowski & Cole/ p.134

3. I-Intercepts

Finding the intercepts gives one an idea as to whether the curves intersect the axes or not.

4. S-Several Selected Points

Consider several selected points to determine how the graph approaches the asymptotes and plot them. Draw the graph through the points you selected.

References:

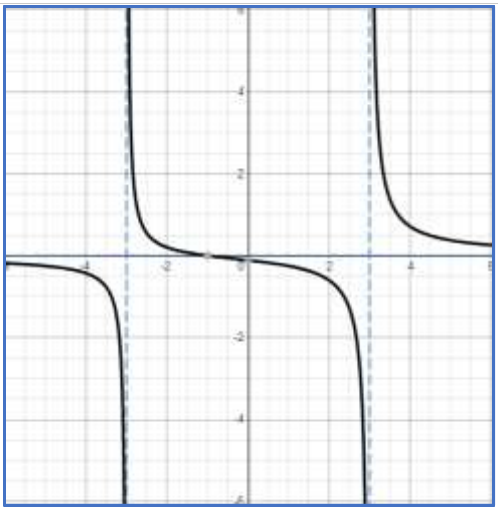
1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Quezon City: Rex Bookstore Inc
2. Larson, Ron. Hostetler, Robert. Edwards, Bruce (2005). *College Algebra: A graphing Approach, 4th Edition*. Boston, New York: Houghton Mifflin Company.
3. Swokowski, Earl. Cole, Jeffrey (2010*). Algebra and Trigonometry with Analytic Geometry. Classic 12th Edition.* Belmont, CA: Cengage Learning
4. PSHS System. (2020). *Math 1 Chapter 1 Module Version 2* [PDF]. Philippines: PSHS System.
5. PSHS CBZRC. (2020). *Template-Editable-1* [DOC]. Batangas: PSHS CBZRC.

Prepared by: Maria Cecilia C. Bastian
Position: Special Science Teacher (SST) IV
Campus: PSHS- CAR

Reviewed by: Arlene C. Agosto
Position: Special Science Teacher (SST) V
Campus: PSHS-CVisC

Answer Key:

NAVIGATE:

1. $f(x) = \frac{x+1}{x^2-9}$ Vertical asymptotes: $x = 3$ and $x = -3$ Horizontal asymptote: $y = 0$ Symmetry is about the origin Intercept: $(-1,0)$	
3. $f(x) = \frac{3x^2+4}{x-1}$ Vertical asymptote: $x = 1$ Oblique asymptote: $y = 3x + 3$ Intercept: $(0, -4)$