Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	6.0	*Other Types of Functions*
Lesson Code	6.1	*Transformation of Functions (Translation)*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

By the end of this learning guide, the students will have been able to:

1. Understand the rule for vertical and horizontal translation of functions.
2. Demonstrate vertical and horizontal functions using graphing tools and software.
3. Illustrate the vertical and horizontal translation of functions on the coordinate plane.

HOOK

Time Allocation: 3 minutes
Actual Time Allocation: ______ minutes

All parabolas are related to the graph of $y = x^2$. Some parabolas may appear in a different location, may have a different opening, or maybe narrower or wider – but they will always resemble $y = x^2$. This makes $y = x^2$ the *parent* graph of the family of parabolas.

A parent function is a basic function that is transformed (recall translation, reflection, rotation, and dilation) to create other members in its family. Some examples of parent functions are shown below. Notice that except for the constant function, the coefficient of x in each equation is 1.

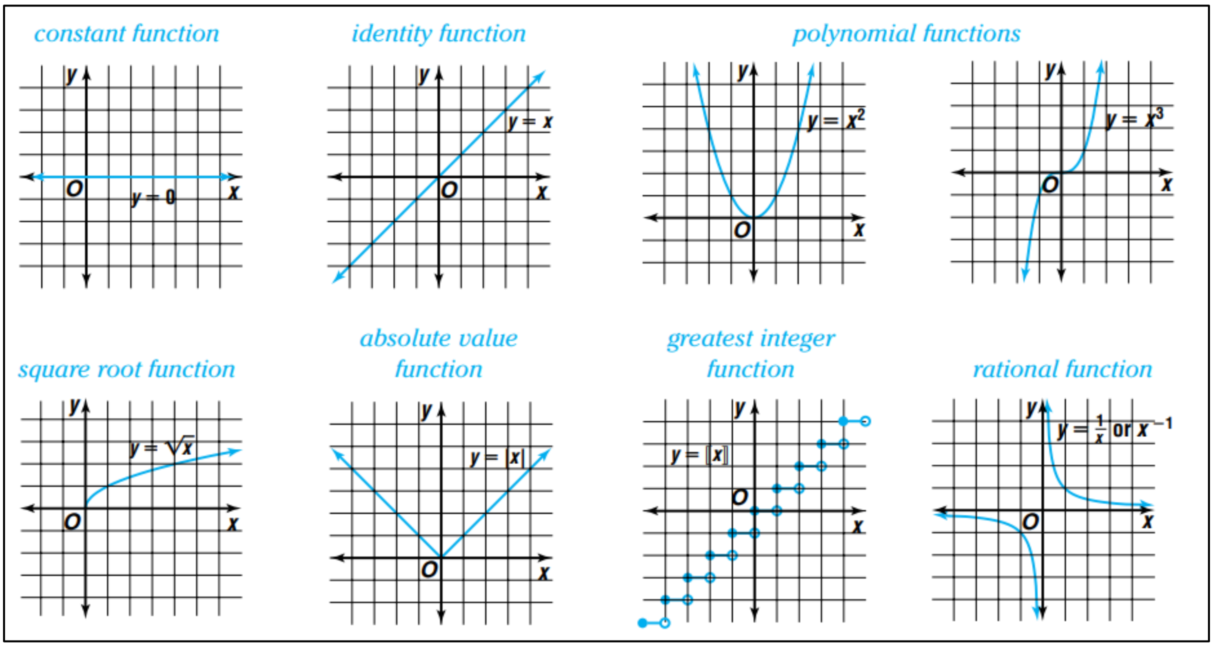


Figure 1: Parent functions Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

From learning guides 2.1 to 2.5, we learned how to transform geometric objects on the coordinate plane. In particular, we learned how to translate, reflect, rotate, and dilate. In this learning guide, we are going to learn how to transform <u>functions</u>. Let's start with translation.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

One simple kind of transformation is shifting the entire graph up or down, this is formally called as vertical translation. Let us explore and visualize vertical translation using GeoGebra.

Hands-on Activity using GeoGebra

1. Open your GeoGebra application or go to the link: https://www.geogebra.org/calculator
2. Graph the parent function $y = x^2$. Let us name this graph as f.
3. Graph the parabola $y = x^2 + 1$. Let us name this graph as g and set its color to red. Your graphs should now look like this:

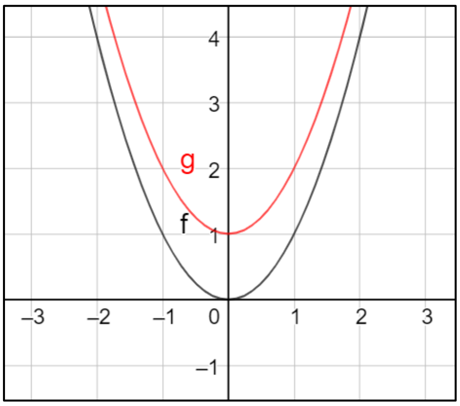


Figure 2: Functions f and g in hands-on activity.

Notice that graph g is the result when graph f is shifted one unit upwards. Let us make use of the *slider* function in GeoGebra.

4. Delete graph g and enter a new function $y = x^2 + a$. Let's name this graph as g and set its color to red. Once you enter this function, you will notice that there is a slider above it.

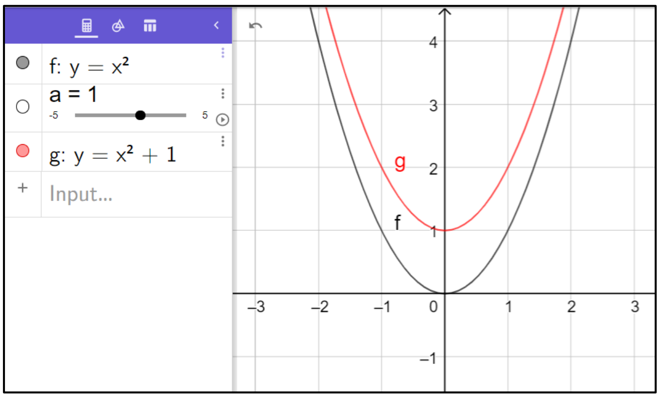


Figure 3: Slider in GeoGebra.

5. Explore the slider by dragging its point to the left or to the right. Notice that the current value of the slider becomes the value of a in the function g. As you drag the slider to the right, the value of a increases and as you drag it to the left, the value of a decreases. What do you notice as you change the value of a? You might observe the following:
   a. When $a = 0$, the function g is equal to the function f.
   b. When $a > 0$, the function g is shifted upwards.
   c. When $a < 0$, the function g is shifted downwards.

Certainly enough, these are the rules for vertical translation. Let's generalize these observations by formally defining vertical translation of functions.

In **Vertical Translation of Graphs**, suppose $c > 0$.

- To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upwards c units.
- To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downwards c units.

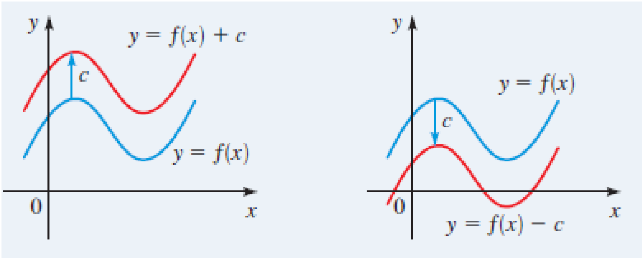


Figure 4: Vertical Translation
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

6. Delete the slider and graph g. Graph a new function $y = (x + a)^2$. Let us name this graph as h and set its color to red. Your graph should now look like this.

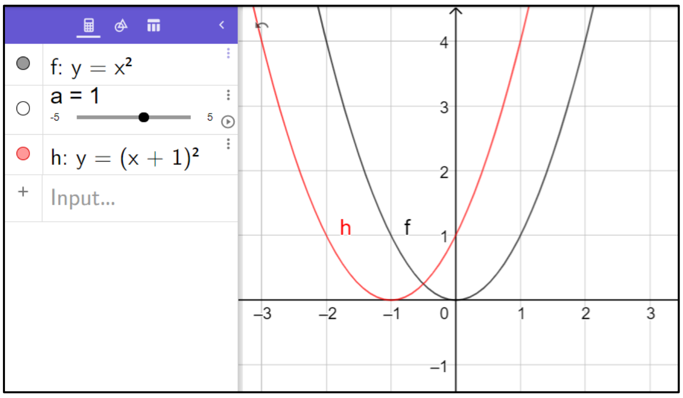


Figure 5: Functions f and h in hands-on activity.

Notice that at $a = 1$, graph h is the result when graph f is shifted one unit to the left.

7. Drag the slider to the left or to the right. What do you notice as you change the value of a? You might observe the following:
   a. When $a = 0$, the function g is equal to the function f.
   b. When $a > 0$, the function g is shifted to the right.
   c. When $a < 0$, the function g is shifted to the left.

Likewise, these are the rules for horizontal translation. Let's generalize these observations by formally defining the horizontal translation of functions.

In **Horizontal Translation of Graphs**, suppose $c > 0$.

- To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.
- To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c units.

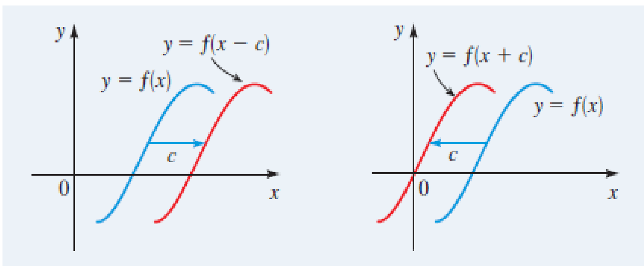


Figure 6: Horizontal Translation
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

Example 1: Translation of a Parent Function. Use the parent graph $y = |x|$ to sketch the graph of each function.

a. $y = |x| + 3$
b. $y = |x + 3|$

Answer: The function in (a) is the graph of $y = |x|$ shifted 3 units upwards. On the other hand, the function in (b) is the graph of $y = |x|$ shifted 3 units to the left.

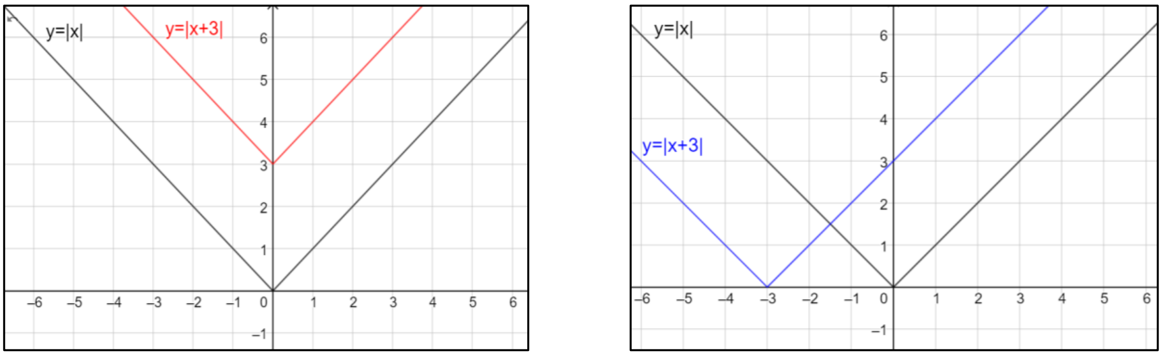


Figure 7: Functions $y = |x| + 3$ and $y = |x + 3|$

Example 2: Changes in Domain and Range. Consider the function $f(x) = \sqrt{9 - x^2}$. Consider as well the semicircles $g(x) = \sqrt{9 - x^2} - 2$ and $h(x) = \sqrt{9 - (x - 2)^2}$. Do the following:

a. Describe in words how the functions $g(x)$ and $h(x)$ are related to the function $f(x)$.
b. Graph functions $g(x)$ and $f(x)$ in one coordinate plane and functions $h(x)$ and $f(x)$ in another coordinate plane
c. Using the graphs in (b), complete the table below.
d. What do you observe on the changes in domain and range after vertical and horizontal transformation?

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-3,3]$		
Range	$[0,3]$		

Answer:

a. The function $g(x)$ is the graph of $f(x)$ shifted 2 units downwards. The graph $h(x)$ is the graph od $f(x)$ shifted 2 units to the right.
b. The graphs are shown below.

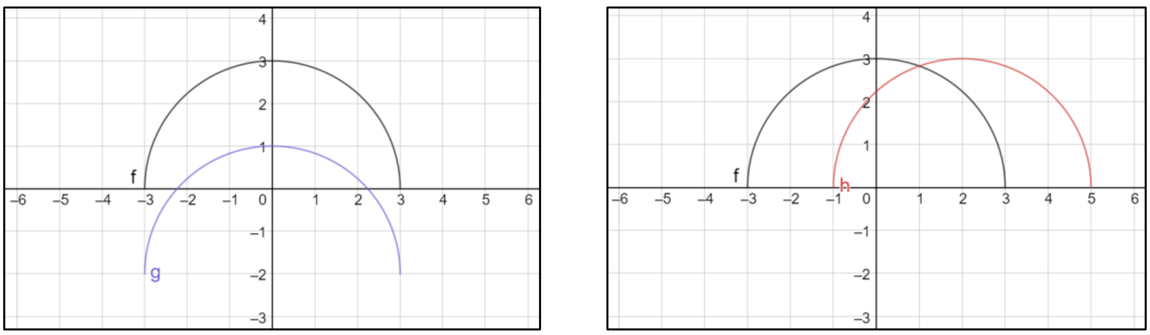


Figure 8: Functions $g(x) = \sqrt{9 - x^2} - 2$ and $h(x) = \sqrt{9 - (x - 2)^2}$

c. The completed table is shown below.

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-3,3]$	$[-3,3]$	$[-1,5]$
Range	$[0,3]$	$[-2,1]$	$[0,3]$

d. For function $g(x)$, <u>the domain is unchanged while the range is changed</u>. The resulting range is shifted two units downward as shown below.

Range of $g(x)$: $[0,3] \rightarrow [0-2, 3-2] \rightarrow [-2,1]$

For function $h(x)$, <u>the domain is changed while the range is unchanged</u>. The resulting domain is shifted two units to the right as shown below.

Domain of $h(x)$: $[-3,3] \rightarrow [-3+2, 3+2] \rightarrow [-1,5]$

Note: Any horizontal translation will affect the domain and leave the range unchanged. On the other hand, any vertical translation will affect the range and leave the domain unchanged.

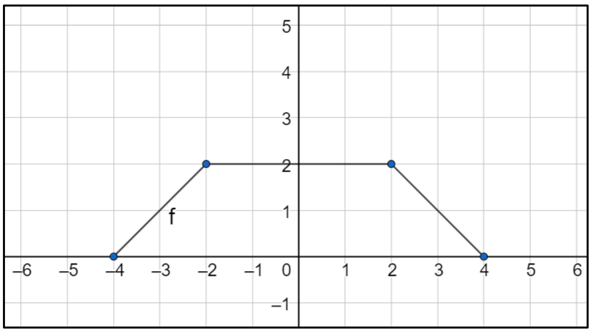
NAVIGATE

Time Allocation: 10 minutes
Actual Time Allocation: ______ minutes

Answer the following questions. Items marked with an asterisk (*) will be graded.

1. Describe how the graph of g is obtained from the graph of f.
   a. $f(x) = x^2$, $g(x) = x^2 + \sqrt{2}$
   b. $*f(x) = x^3$, $g(x) = (x - \sqrt{2})^3$

2. The graph of $f(x)$ is given below. Consider the function $g(x) = f(x + 3)$ and do the following:



   a. Describe in words how the $g(x)$ is related to the function $f(x)$.
   b. Graph g(x) in the same coordinate plane as $f(x)$.
   c. Complete the table below.

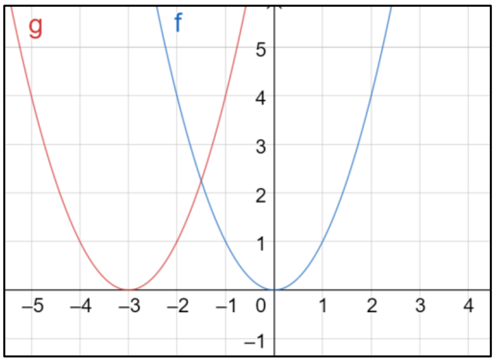
	$f(x)$	$g(x)$
Interval at which the function is strictly increasing		
Interval at which the function is strictly decreasing		
Interval at which the function is constant		

   d. What are your observations?

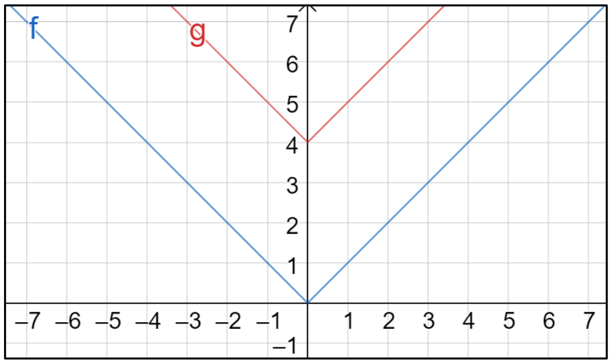
3. *Using the same function $f(x)$ in item (3), consider the function $g(x) = f(x) + 3$ and repeat steps a to d.

4. Write a formula for the function g that results when the graph of $f(x)$ is transformed as described (simplify your answer as necessary). Without solving, what is the domain and range of g?
   a. The graph of $f(x) = \sqrt{x}$ is shifted 2 units up.
   b. *The graph of $f(x) = x^2$ is shifted 5 units left.

5. For the following exercises, the graphs of f and g are given. Find a formula for the function g from the function of f. Simplify your answer. Note that the graph in blue is the function of f. You may verify your answer by graphing it using GeoGebra.

a.


$f(x) = x^2$

*b.


$f(x) = |x|$

KNOT

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

- A parent function is a basic function that is transformed (recall translation, reflection, rotation, and dilation) to create other members in its family.
- In **Vertical Translation of Graphs**, suppose $c > 0$.
  - To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.
  - To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downwards c units.
- In **Horizontal Translation of Graphs**, suppose $c > 0$.
  - To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.
  - To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c units.
- The table below summarizes the properties of the functions affected by vertical and horizontal translation.

	Vertical Translation	Horizontal Translation
Domain	Not affected	Affected
Range	Affected	Not affected
Interval at which the function is strictly increasing	Not affected	Affected
Interval at which the function is strictly decreasing	Not affected	Affected
Interval at which the function is constant	Not affected	Affected

References:

Albarico, J.M. (2013). THINK Framework. Based on *Science LINKS* by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Carter, J., Cuevas, G., Day, R., and Malloy, C., (2012). *Glencoe Geometry*. USA: The McGraw-Hill Companies, Inc.

International Geogebra Institute. (2020). *GeoGebra*. www.geogebra.org

Stewart, J., Redlin, L., Watson, S (2012). *Precalculus: Mathematics for Calculus*. Brooks/Cole, Cengage Learning.

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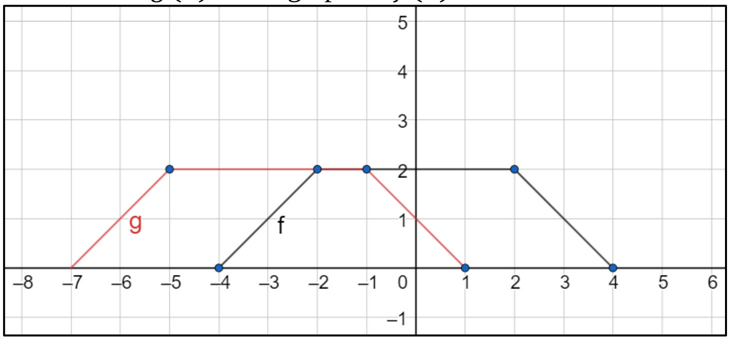
Answer Key:

Navigate

1. a. The graph of $g(x)$ is the graph of $f(x)$ shifted up $\sqrt{2}$ units.

2. a. The function $g(x)$ is the graph of $f(x)$ shifted left 3 units.
   b . 

   c.

	$f(x)$	$g(x)$
Interval at which the function is strictly increasing	$(-4-2)$	$(-7,-5)$
Interval at which the function is strictly decreasing	$(2,4)$	$(-1,1)$
Interval at which the function is constant	$(-2,2)$	$(-5,-1)$

   d. The intervals at which $g(x)$ is strictly increasing, strictly decreasing, and constant are the corresponding intervals of $f(x)$ shifted 3 units to the left.

4. a. $g(x) = \sqrt{x} + 2$
   domain: $[0, \infty)$
   range: $[2, \infty)$

5. a. $f(x) = x^2 + 6x + 9$