Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	4.0	*Graphs of Polynomial and Rational Functions*
Lesson Code	4.4	*Sketching the graph of a polynomial function*
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, the students should be able to sketch the graph of a polynomial function.

HOOK

Time Allocation: 7 minutes
Actual Time Allocation: _____ minutes

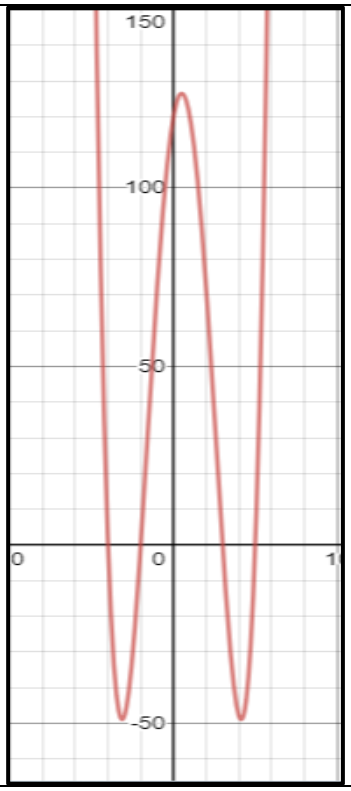
Having discussed several concepts about the graph of polynomial functions in the previous lessons, you are now ready to sketch the graph of polynomial functions using the important properties of polynomial functions that you have learned.

But before we proceed, allow yourself to play the role of a "matchmaker" in the following exercises. Your goal is to find a perfectly matched function to the graph given to you. You will work on few challenges crafted to enhance your understanding on roots, multiplicity, vertical dilation, and end behavior. You may still be aided by your graphing app for ease of going through the activity.

Let the challenge begin!

1. Replace the △ symbols in the row function with the correct number to match the graph on the right
   $f(x) = (x + 4)(x + \triangle)(x - 3)(x - \triangle)$.

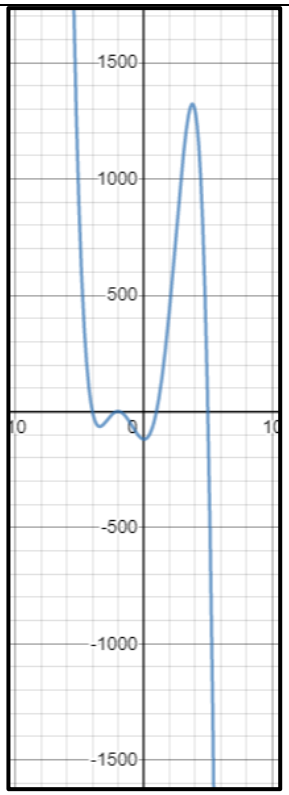
Final Answer:



2. Replace △ symbols in the row function with the correct number to match the graph on the right
$f(x) =\triangle\ (x+4)(x+2)^2(x-1)(x-\triangle)$.

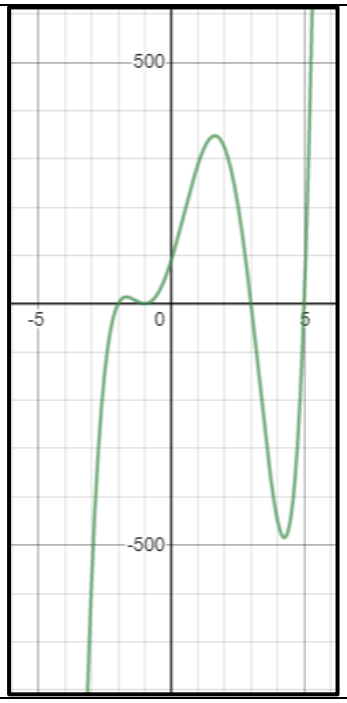
Hint: Since it rises to the left and falls to the right, try something negative. Try a value between -1 and -2.

Final Answer:



3. Write a polynomial function in factored form that matches the graph on the right. Calculate the value of a given that $(0,100)$ is a point on the graph.

Final Answer:



So how did you go through the activity? Do you think it helped that you already know concepts such as zeros/roots, end behavior, multiplicity, intercepts?

*Note:*This activity is not graded.

IGNITE

Time Allocation: 10 minutes
Actual Time Allocation: _____ minutes

Now we are ready to set aside our graphing app and get engaged with a method for getting a rough sketch of a general polynomial function. The most important information we need is a complete list of all the zeros and their corresponding multiplicity, and the value of a that dictates its vertical dilation. In the succeeding discussion, either the list of zeros will be given, or the zeros of the function will be easy to find. Let us consider the following examples.

Example 1:

Consider the Legendre Polynomial defined by $P(x) = \frac{1}{2}(5x^3 - 3x)$ which we have mentioned earlier during the first quarter.

First, we need to apply the leading coefficient test that you have mastered by this time. Since the leading coefficient is positive and the degree is odd, then we know that the graph eventually falls to the left and rises to the right.

Second, we find the real zeros of the function by factoring.

$$P(x) = \frac{1}{2}(5x^3 - 3x)$$
$$= \frac{1}{2}x(5x^2 - 3)$$

Equating the factors to zero shows that the real zeros of P are $x = 0$ and $x = \pm\frac{\sqrt{15}}{5}$, all zeroes of *odd multiplicity.*

Third, let us find a few additional points as shown on the table.

x	– 1	0	1	2
$f(x)$	– 1	0	1	17

Fourth, we plot the points and draw the graph by sketching a continuous curve through the points that we have identified. From the determined multiplicity, we know that the graph would simply cross the $x-$axis at $x = 0$ and $x = \pm\frac{\sqrt{15}}{5}$.

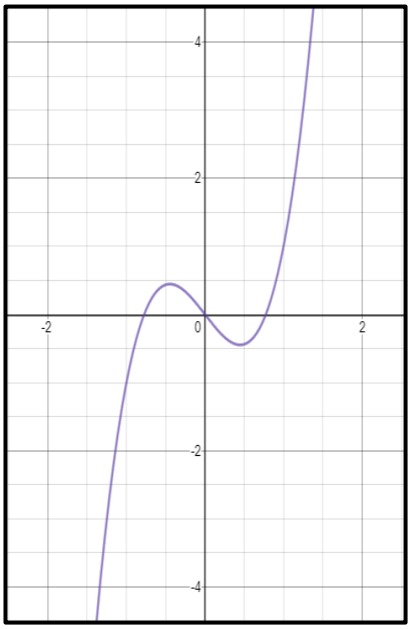


Figure 1: Graph of $P(x) = \frac{1}{2}(5x^3 - 3x)$

Example 2:

This time we consider $f(x) = 2x^4 - 8x^3 + 2x^2 + 12x$.
Just as what we did in the first example, let us apply the leading coefficient test. The leading coefficient is positive, and the degree of f is even, then we know that the graph eventually rises both to the left and to the right.

Second, we find the real zeros of the function by factoring.

$$f(x) = 2x^4 - 8x^3 + 2x^2 + 12x$$
$$f(x) = 2x(x^3 - 4x^2 + x + 6)$$
$$f(x) = 2x(x-2)(x^2 - 2x - 3)$$
$$f(x) = 2x(x-2)(x-3)(x+1)$$

Equating the factors to zero shows that the real zeros of f are $x = 0, 2, 3, -1,$ all of *odd multiplicity.*

Third, let us find a few additional points as shown on the table.

x	-0.581	0	1	2.581
$f(x)$	-4.5	0	8	-4.5

Fourth, we plot the points and draw the graph by sketching a continuous curve through the points that we have identified. From the determined multiplicity, we know that the graph would simply cross the x −axis at $x = 0, 2, 3,$ and -1 .

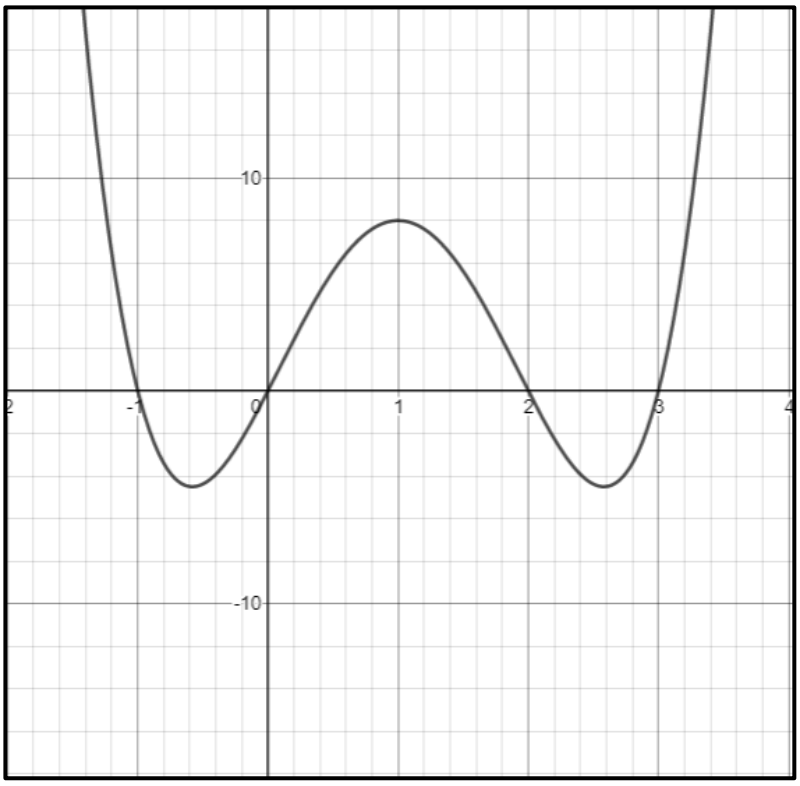


Figure 2: Graph of $P(x) = x^3 - 4x^2 + x + 6$

NAVIGATE

Time Allocation: 10 minutes
Actual Time Allocation: _____ minutes

Sketch the graph of the following functions and complete the requirements on the table. Items marked with an asterisk (*) are graded.

Function	Leading Coefficient	Zeros and Multiplicity	End Behavior	Graph
1. $f(x) = -x^5 + x$				
*2. $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$				
3. $f(x) = -x^4 + 1$				
*4. $f(x) = x^4 - 4x^2 + 1$				

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

During the first quarter, specifically on the lesson about exploring polynomial functions, you were tasked to search for other polynomial functions that are being used in other learning areas (biology, chemistry, computer science, physics, engineering, economics, geology etc.). Choose only one of the two that you have written earlier then sketch the graph of the polynomial using the steps provided in this learning guide. Please accurately identify the zeros and its corresponding multiplicity by writing them below the graph you sketched. This activity is graded.

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.*) Science LINKS*. Quezon City: Rex Bookstore Inc
2. Larson, Ron. Hostetler, Robert. Edwards, Bruce (2005). *College Algebra: A Graphing Approach, 4thEdition*. Boston, New York: Houghton Mifflin Company.
3. Swokowski, Earl. Cole, Jeffrey (2010). *Algebra and Trigonometry with Analytic Geometry.* Classic 12th Edition. Belmont, CA: Cengage Learning
4. PSHS System. (2020). *Math 1 Chapter 1 Module Version 2* [PDF]. Philippines: PSHS System.
5. PSHS CBZRC. (2020). *Template-Editable-1* [DOC]. Batangas: PSHS CBZRC.

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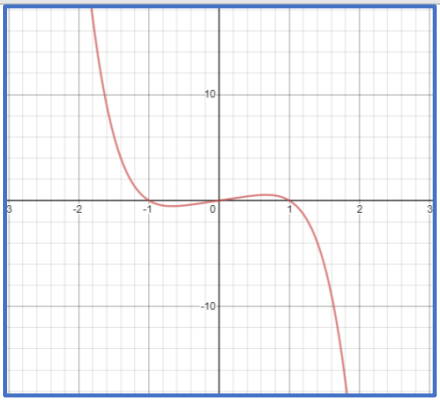
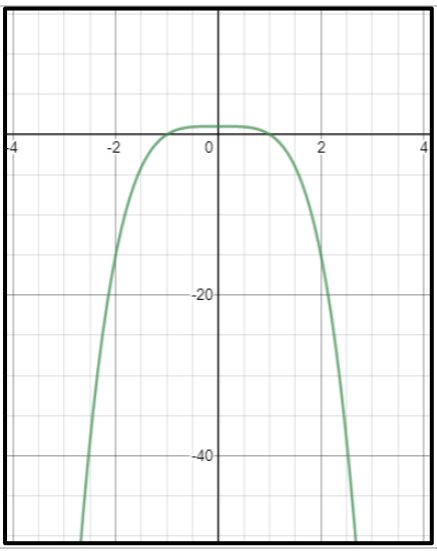
Answer Key:

NAVIGATE:

Function	Leading Coefficient	Zeros and Multiplicity	End Behavior	Graph
1. $f(x) = -x^5 + x$	Negative, odd	$x = -1,0,1$ Odd Multiplicity	Rises to the left, falls to the right	
3. $f(x) = -x^4 + 1$	Negative, even	$x = -1,1$ Odd multiplicity	Falls both to the right and left	

KNOT:
Answers may vary.