Finite-alphabet and Decision-feedback based channel estimation for space-time coded OFDM systems

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Abstract

A novel blind channel estimation scheme is presented for space-time coded OFDM (STC-OFDM) systems. This scheme is composed of coarse channel estimation by exploiting the finite-alphabet property of information signals and fine channel estimation by using decision-directed method, which employs the priori knowledge of the transmitted signals other than the finite-alphabet property of mapping constellation. At the cost of a little more computational complexity, it achieves better performance than the competitive one. The scheme is tested with simulations and also compared with the subspace-based channel estimation.

1 Introductions

In recent years, space-time coding technique has gained much attention due to its ability to greatly increase the system capacity [1]. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) technique has received increasingly wider applications for its advantages such as multipath mitigation and simple frequency domain equalization. As the combination of these two techniques, the STC-OFDM systems enjoy the advantages of both these two techniques, and has become a new research front [2].

Like the conventional OFDM systems, STC-OFDM systems also require channel state information to perform the coherent demodulation. However, the channel estimation algorithm for the conventional OFDM systems can not be extended to the STC-OFDM systems, since independent fading signals from multiple antennae overlaps on the received signals in STC-OFDM systems. Much research work [2-6] has been done on this issue, and the existing estimators can be roughly divided into two classes, namely, non-blind estimation and blind estimation. In existing non-blind estimation methods, channel estimation is usually performed by employing training sequence [2] or pilot tones [3], as a result, these methods often suffer significant loss of bandwidth efficiency. Recently numerous blind estimation methods have been developed because they can save bandwidth. A subspace-based blind estimator has been introduced in [4], but it requires additional precoding for signals to be modulated and this additional precoding consumes additional bandwidth. Constant modulus (CM) based channel estimation resting on the CM property of signals and the special structure of orthogonal ST coding has been presented in [5]. However, the requirement of the orthogonal structure ST coding limits its application, since orthogonal ST coding implies great loss of transmission rate for more than two transmit antennae. Another subspace-based estimator is presented in [6], while it requires the transmitted symbol is real or symmetric, which means great loss of bandwidth efficiency, thus this requirement limits its application.

In this paper, we proposed a new blind channel estimation algorithm for STC-OFDM systems by exploiting the finite-alphabet property of the transmitted symbols. Unlike the existing blind estimators in [4-6], it can be applied to STC-OFDM systems with non-orthogonal ST coding and no-real or non-symmetric transmitted symbols, and needs no additional precoding as well. At the cost of a little computation complexity, it provides better performance than the competitive one.

This paper is organized as follows. Section 2 introduces the STC-OFDM system model and section 3 describes the proposed algorithm. Some simulation examples to demonstrate the performance of the proposed estimator are provided in section 4 and the conclusions are drawn in section 5.

2 STC-OFDM System model

Consider a STC-OFDM system with two transmit antennas and one receive antenna as in [5]. Like in the conventional OFDM systems, the information symbol sequence are parsed into blocks $\mathbf{s}_k = [s_k(0), \cdots, s_k(N-1)]^T$, where k, N are the block index and subcarrier number, respectively. The ST encoder maps every two consecutive blocks $\mathbf{s}_{2k}$ and $\mathbf{s}_{2k+1}$ into two blocks $\bar{\mathbf{s}}_k$ and $\tilde{\mathbf{s}}_k$, where

$$\bar{\mathbf{s}}_{2k} = \mathbf{s}_{2k},\ \bar{\mathbf{s}}_{2k+1} = -\mathbf{s}_{2k+1}^*,\ \tilde{\mathbf{s}}_{2k} = \mathbf{s}_{2k+1},\ \tilde{\mathbf{s}}_{2k+1} = \mathbf{s}_{2k}^*. \quad (1)$$

And the blocks $\bar{\mathbf{s}}_k$ and $\tilde{\mathbf{s}}_k$ are firstly IFFT modulated by OFDM and then transmitted at time interval k through transmit antenna 1 and 2, respectively.

Provided that the frequency selective channel between the ith transmit antenna and the receive antenna is expressed as $\mathbf{h}_i = [h_i(0), \cdots, h_i(L)]^T$, i = 1,2, where L is the upper bound for the channel orders of $\mathbf{h}_1$ and $\mathbf{h}_2$. The inter-symbol interference (ISI) is not taken into account since the ISI can be removed by cyclic-prefix or zero padding removal. After being propagated through these channels and FFT demodulated in the receiver, the signals can be expressed as [2]

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$$r_k(n) = H_1(n)\bar{s}_k(n) + H_2(n)\tilde{s}_k(n) + w_k(n),\quad n = 0,1,\cdots,N-1 \tag{2}$$

where $H_i(n)$ is the channel frequency response for the nth tone, corresponding to channel $\mathbf{h}_i$, and $w_k(n)$ denotes the additive complex Gaussian noise, with zero mean and variance σ_n^2.

3 Blind Channel Estimation

Based on the received data model (2), we will next derive our blind channel estimation by two steps. In the first step the coarse channel estimation can be obtained by using the finite-alphabet property of the signals, and in the second step the fine channel estimation can be further obtained on the basis of the coarse estimation by using decision-directed method.

a. The step of coarse channel estimation

First give the following assumptions, which hold true in our STC-OFDM systems.

a1) Symbols $s_k(n)$ are drawn equiprobably from a finite alphabet set $\{\varsigma_q\}_{q=1}^Q$ of size Q;

a2) The finite alphabet set is symmetric, i.e., $-\varsigma_q,\ \varsigma_q^* \in \{\varsigma_q\}_{q=1}^Q$

The assumption a1) implies $\prod_{q=1}^{Q}[s_k(n)-\varsigma_q]=0$, i.e., $s_k^Q(n)+\alpha_1 s_k^{Q-1}(n)+\cdots+\alpha_Q=0$, where $\alpha_1,\cdots,\alpha_Q$ are determined by the constellation points $\{\varsigma_q\}_{q=1}^Q$. Define J satisfying $\alpha_J \neq 0$; $\alpha_n = 0,\ \forall n < J$.

It has been proven in [7-8] that the following result holds

$$E\{s_k^m(n)\} = \begin{cases} -(J/Q)\alpha_J \neq 0, & m = J; \\ 0 & ,\ 0 < m < J \end{cases} \tag{3}$$

for any constellation under the condition a1). Fortunately J is not very big for most constellations, such as $J=4$ for QAM constellations with $Q = 16,32,64,128,256$, and $J = Q$ for PSK constellations, which makes the following estimation more practicable.

Since $w_k(n)$ is zero-mean complex circular Gaussian noise and independent of $s_k(n)$, we have $E\{w_k^m(n)\}=0,\ \forall m>0$. Starting from the received data model (2) and the circularity of $w_k(n)$, we derive that

$$\begin{aligned} E\{r_{2k}^J(n)\} &= E\{[H_1(n)\bar{s}_{2k}(n) + H_2(n)\tilde{s}_{2k}(n) + w_{2k}(n)]^J\} \\ &= H_1^J(n)E\{s_{2k}^J(n)\} + H_2^J(n)E\{s_{2k+1}^J(n)\} \end{aligned} \tag{4}$$

similarly, from the condition a2) we can obtain that

$$\begin{aligned} E\{r_{2k+1}^J(n)\} &= E\{[H_1(n)\bar{s}_{2k+1}(n) + H_2(n)\tilde{s}_{2k+1}(n) + w_{2k+1}(n)]^J\} \\ &= (-1)^J H_1^J(n)E\{s_{2k+1}^J(n)\} + H_2^J(n)E\{s_{2k}^J(n)\} \end{aligned} \tag{5}$$

. If $\mathbf{s}_{2k}(n)$ and $\mathbf{s}_{2k+1}(n)$ satisfy

$$E^2\{s_{2k}^J(n)\} \neq (-1)^J E^2\{s_{2k+1}^J(n)\} \tag{6}$$

$H_1^J(n)$ and $H_2^J(n)$ can be uniquely resolved from (4-5) according to the matrix theorem. To satisfy (6), different mapping constellations can be used for $\mathbf{s}_{2k}(n)$ and $\mathbf{s}_{2k+1}(n)$. In our scheme the constellations $\{\varsigma_q\}_{q=1}^Q$ $\{\bar{\varsigma}_q\}_{q=1}^Q$ for $\mathbf{s}_{2k}(n)$ and $\mathbf{s}_{2k+1}(n)$ are simply chosen as

$$\varsigma_p \in \{\alpha\bar{\varsigma}_q\}_{q=1}^Q,\ \forall p \in [1,Q] \tag{7}$$

with real boosting factor α satisfying $\alpha^{2J} \neq (-1)^J$, i.e., $\mathbf{s}_{2k}(n)$ and $\mathbf{s}_{2k+1}(n)$ are mapped from the same constellation and then $\mathbf{s}_{2k}(n)$ is multiplied by the boosting factor α. And the boosting factor α has to be well chosen since too large boosting factor means that too small power is distributed to the second transmit antenna.

In practice, $E\{r_k^J(n)\}$ is always replaced by consistent sample averages, i.e.,

$$E\{r_k^J(n)\} \approx aver\_r_k^J(n) = \frac{1}{K}\sum_{k=0}^{K-1} r_k^J(n) \tag{8}$$

where K is the total number of blocks averaged.

Consider that $r_{2k}^J(n)$ is a random variable and let the mean square error of $r_{2k}^J(n)$ be denoted by $\sigma_{2k}(n)$, then $\sigma_{2k}(n)$ can be expressed as

$$\begin{aligned} \sigma_{2k}^2(n) &= E\{[r_{2k}^J(n) - E(r_{2k}^J(n))]^2\} \\ &= \sum_{s_{2k}(n),s_{2k+1}(n)\in\{\varsigma_q\}_{q=1}^Q} \{[H_1(n)s_{2k}(n) + H_2(n)s_{2k}(n)]^J - E(r_{2k}^J(n))\}/Q^2 \end{aligned} \tag{9}$$

It is obvious that $\sigma_{2k}(n)$ only depends on the constellation for a certain channel. According to the central limit theorem, when K is sufficiently large, $aver\_r_k^J(n)$ approximates Gaussian distribution with its mean $E\{aver\_r_{2k}^J(n)\} = E\{r_{2k}(n)\}$ and its variance $\sigma_{2K}(n)$ satisfying $D\{aver\_r_{2k}^J(n)\} = \sigma_{2k}^2(n)/K$. Similarly we have $E\{aver\_r_{2k+1}^J(n)\} = E\{r_{2k+1}(n)\}$ and $D\{aver\_r_{2k+1}^J(n)\} = \sigma_{2k+1}^2(n)/K$. Therefore, the mean square errors of $H_1^J(n)$ and $H_2^J(n)$ only depend on the mapping constellation with fixed channels, and decrease with K increasing. When K is sufficiently large, the estimate error is negligible. However, too large K is not reasonable in even some slowly time-varying channels, hence the further fine channel estimation is needed for certain K in the following step.

b. The step of fine channel estimation

Based on the coarse estimate of the Lth order channel response $\hat{H}_1^J(n)$, $\hat{H}_2^J(n)$, $n=0,1,\cdots,N-1$, the fine estimate of channels can be obtain by using the following decision-directed decoding method since more priori knowledge of finite set constellation other than the finite-alphabet property (3) is employed in this method.

Given $\hat{H}_i^J(n)$, $i=1,2$, we have $\hat{H}_i(n) \in \{e^{j2m\pi/J}[\hat{H}_1^J(n)]^{1/J}\}_{m=0}^{J-1}$ with a scalar ambiguity $e^{j2m\pi/J}$, hence we have J^2 possible vectors $\hat{\mathbf{H}}(n) = [\hat{H}_1(n)\ \ \hat{H}_2(n)]^T$. Exploiting the linear decoding of ST codes in [1]

$$s_{2k}(n) = \frac{2H_1^*(n)r_{2k}(n) + 2H_2(n)r_{2k+1}^*(n)}{|H_1(n)|^2 + |H_2(n)|^2} \tag{10}$$

$$s_{2k+1}(n) = \frac{2H_2^*(n)r_{2k}(n) - 2H_1(n)r_{2k+1}^*(n)}{|H_1(n)|^2 + |H_2(n)|^2} \tag{11}$$

a joint maximum likelihood (ML) channel estimation and

ST decoding could be first developed by searching over all possible symbols and combined channel vectors $\hat{\mathbf{H}}(n)=[\hat{H}_1(n)\ \ \hat{H}_2(n)]^T$. Provided that the decoded information symbol and channel estimate through ML method are denoted by $\hat{\mathbf{S}}(n)=[\hat{s}_{2k}(n)\ \ \hat{s}_{2k+1}(n)]^T$ and $\hat{\mathbf{H}}(n)=[\hat{H}_1(n)\ \ \hat{H}_2(n)]^T$, respectively. From the ST coding structure, the channel can be re-estimated as

$$\widetilde{H}_1(k,n)=\frac{r_{2k}(n)\hat{s}_{2k}^*(n)-r_{2k+1}(n)\hat{s}_{2k+1}(n)}{|\hat{s}_{2k}(n)|^2+|\hat{s}_{2k+1}(n)|^2} \tag{12}$$

$$\widetilde{H}_2(k,n)=\frac{r_{2k}(n)\hat{s}_{2k+1}^*(n)-r_{2k+1}(n)\hat{s}_{2k}(n)}{|\hat{s}_{2k}(n)|^2+|\hat{s}_{2k+1}(n)|^2} \tag{13}$$

where $\widetilde{H}_i(k,n), i=1,2$ denotes the re-estimated channel frequency response from the ith transmit antenna for the nth tone during interval $2k$ and $2k+1$. We call this re-estimation method as decision-directed method. Then we obtain the averaged re-estimated channel frequency response over the K block intervals.

$$\widetilde{H}_i(n)=\frac{2}{K}\sum_{k=0}^{K/2-1}\widetilde{H}_i(k,n) \tag{14}$$

One more thing to be mentioned is that due to the scalar ambiguity of $\hat{H}_i(n)$ and the symmetric property of the constellation, the decoded symbol and channel estimate in the ML method also have scalar ambiguity problem in the ML search method, and the scalar ambiguity α_1 satisfies $\alpha_1^J=1$. Further, this problem results in the same scalar ambiguity problem in the decision–directed re-estimation method above. However, the Jth order of re-estimated channel $\{\widetilde{H}_i^J(n)\}_{n=0}^{N-1}$ can be obtained without ambiguity. Based on the Jth order of re-estimated channel $\{\widetilde{H}_i^J(n)\}_{n=0}^{N-1}$, the same method including MD algorithms and inserting two pilot symbols in every two symbol blocks are employed to compute the channel estimate as in [7].

c. Distinct Features

We next discuss some characteristics of the proposed estimator: error floor phenomena and computational complexity of the estimator.

1) **Channel Estimation Error Floor Problem**

It has been shown in the coarse channel estimation step that when the AWGN is negligible, $\hat{H}_1^J(n)$ and $\hat{H}_2^J(n)$ follow the Gaussian distribution, and their mean square errors depend only on the mapping constellation with fixed channels, and are reciprocal to K.

Although the decision-directed method is not a linear estimation algorithm, it is not difficult to conclude from the results above that the mean square error of the re-estimated channels depends on the mapping constellation sets and the total symbol block number K, decreases with K increasing. Therefore, the channel estimate in our algorithm has error floor phenomenon, i.e., there is the estimate error even without noise, and the error floor decreases with K increasing for a mapping constellation.

2) **computational complexity**

The computational complexity is obviously dominated by the second step, and the computational complexity of the decision-directed method and the MD method in [7] in the second step is proportional to J^2N and J^{L+1} , respectively, hence the computational complexity of our new estimator is approximately proportional to J^2N+J^{L+1} . While the computation complexity of the subspace-based algorithm in [6] is proportional to N^2 since its computation mainly comprises covariance matrix SVD operations.

Therefore, our algorithm has bigger computation load than the subspace-base algorithm for small N, but may have smaller computation load for large N.

4 Simulation Results

In this section Monte Carlo simulations were performed to evaluate the performance of the proposed estimator for an STC-OFDM system with two transmit antenna and single receive antennae, $N=32$ subcarriers. The bit error rate (BER) and normalized mean square error (NMSE) (averaging on all channels) of the channel estimate are applied to measure the performance of the estimator. As usual, the signal-noise-ratio (SNR) is defined as

$$SNR=E\{\sum_{k=0}^{K-1}\|r_k(n)-w_k(n)\|^2\}/E\{\sum_{k=0}^{K-1}\|w_k(n)\|^2\} \tag{15}$$

The NMSE is defined as

$$NMSE=\sum_{i=1}^{2}\sum_{l=0}^{L}\|\bar{h}_i(l)-h_i(l)\|^2/\sum_{i=1}^{2}\sum_{l=0}^{L}\|h_i(l)\|^2 \tag{16}$$

where $\bar{h}_i(l)$ is the estimated channel. The boosting factor α in the proposed estimator satisfies $\alpha^J=2$.

In the simulations, the number of paths of both the two multipath channels is assumed to be 5, i.e., $L1=L2=L=4$, and the complex path gains $h_i(l)$ are generated randomly as per Gaussian distribution for both real and imaginary parts and the maximum path time delay is assumed to be $12\mu s$. Without loss of generality, the energy of multipath channels is normalized: $\|\mathbf{h}_i\|^2=1,\ \mathrm{i}=1,2\cdot$

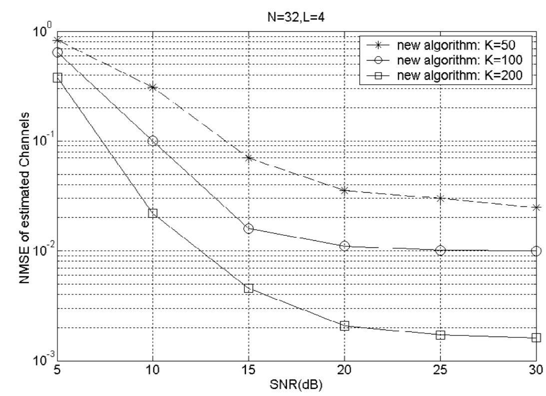

Fig.1 NMSE performance of the proposed estimator with $K=50$, 100 and 200

Fig.1 shows the performance of the proposed estimator with $K=50$, 100 and 200 in multipath channels respectively. Here, QPSK has been used as the mapping constellation, and the results are averaged over 500 channel realizations. In Fig.1, we can see that for a certain mapping constellation, the estimate error decreases with with K increasing, and the channel estimate in our algorithm has error floor phenomenon, which has confirms the analysis of the distinct features in section 3.

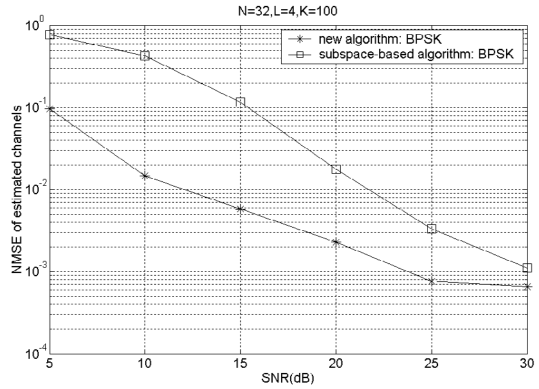

Fig.2 time domain NMSE performance of the proposed estimator and the subspace-based estimator

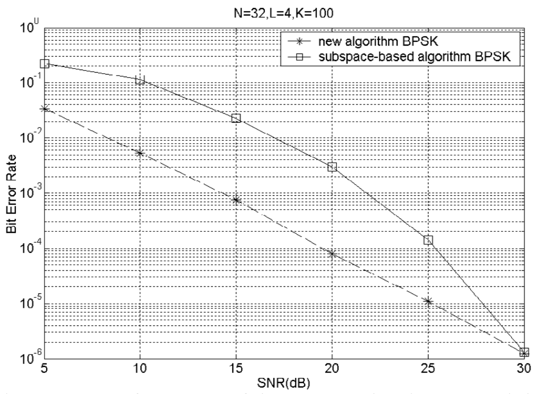

Fig.3 BER performance of the proposed estimator and the subspace-based estimator

Fig.2 and Fig.3 compare the performance of the estimator with that of the subspace-based algorithm in [6]. To maintain fairness, BPSK constellation is used in this simulation since the subspace-based algorithm requires the transmitted signals are real or symmetric. The results in Fig.2 and Fig.3 are respectively averaged over 500 and 1000 Monte Carlo realizations. From Fig.2, the NMSE performance of the proposed estimator is better than that of the subspace-based algorithm, since the new algorithm obtains the channel estimate by making full use of the priori knowledge of all symbols constellations, whereas the subspace-based estimator rests on the redundancy induced only by L ZP every symbol block. Fig.3 compares the BER performance of these two estimators when the system has been equalized by the estimated channels. The NMSE performance translates into the BER performance of Fig.3, which shows the BER obtained using the proposed algorithm outperforms the subspace-based estimator.

5 Conclusions

A new semi-blind finite-alphabet and decision-directed based channel estimator for STC-OFDM systems has been described. Unlike the subspace-based semi-blind channel estimation algorithm in [6], which assumes that the transmitted symbol is real or symmetric, the proposed estimator can avoid great loss of bandwidth efficiency. At the cost of a little more computation load, it achieves better performance than the subspace-based estimator for low SNRs when STC-OFDM systems has number small of subcarriers. With large number of subcarriers, it can achieve much smaller computation load than the subspace-based one. Moreover the proposed estimator can be straightforward extended to STC-OFDM systems with more than two transmit antennae.

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