SOLUTIONS

Name:______________________________

Student ID #:______________________________

Signature:______________________________

Physics 1C Midterm #2 - Version A

- By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
- This exam contains four workout problems, each 10 points, for a total of 40 points. Remember to write down each step of your calculation, and explain your answers. You have 50 minutes to complete this exam.
- Close your exam when time is up, and show your student ID when handing it in.
- Detailed exam rules:
    - By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
    - You can use any type of calculator that does not have internet capability. Silence and put away your cell phones, tablets, and laptops.
    - Quote numerical answers with 3 significant figures, e.g. 0.262 or 3.72×10^3. Always specify the units, and quote final answers in SI units unless otherwise directed.
    - The last page of the exam is an equation sheet that may be torn off.
    - Fit all relevant calculations on the front of the pages. If you run out of room, use the front of the blank page before the equation sheet, and indicate "Problem <n> continued:" to help us in grading.
- If you have questions during the exam, raise your hand. If you are not seated near the end of a row, you may need to come to the aisle or down to the front of the room to ask them.

Academic Integrity - A Bruin's Code of Conduct:

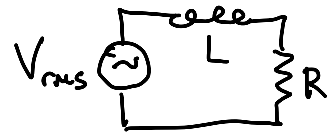
As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behavior includes cheating, fabrication or falsification, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, facilitating academic misconduct, coercion regarding grading or evaluation of coursework, or collaboration not authorized by the instructor. Please review our campus' policy on academic integrity in the UCLA Student Conduct Code: https://deanofstudents.ucla.edu/individual-student-code .

If you engage in these types of unacceptable behaviors in our course, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam. These allegations will be referred to the Office of the Dean of Students and can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

By submitting my assignments and exams for grading in this course, I acknowledge the above-mentioned terms of the UCLA Student Conduct Code, declare that my work will be solely my own, and that I will not communicate with anyone other than the instructor and proctors in any way during the exams.

Problem 1 (10 pts): A resistor with $R = 40\ \Omega$ and an inductor are connected in series across a $V_{rms} = 230\ V$ AC power source with frequency $f = 50\ Hz$. The average electrical power to the resistor is 160 W.

a) (2 pts) What is the RMS current through the resistor?
b) (2 pts) What is the RMS voltage across the resistor?
c) (3 pts) What is the impedance Z of the circuit?
d) (3 pts) What is the RMS voltage across the inductor?



a) $P = I_{rms}^2 \cdot R$ so $I_{rms} = \sqrt{\frac{P}{R}} = \sqrt{\frac{160}{40}}\ A = \boxed{2.0\ A}$

b) $V_{rms} = R \cdot I_{rms}$ so $V_{rms} = 40 \cdot 2.0\ v = \boxed{80\ v}$ for resistor

c) $Z = \frac{V_{rms}}{I_{rms}}$ where $V_{rms} = 230\ v$ (the whole circuit)

$$\boxed{Z = \frac{230}{2.0}\ \Omega = 115\ \Omega}$$

d) $V_{rms}(L) = I_{rms} \cdot X_L$, get X_L from

$Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L = \sqrt{Z^2 - R^2}$

$V_{rms}(L) = I_{rms}\sqrt{Z^2 - R^2}$

$V_{rms}(L) = 2.0 \cdot \sqrt{115^2 - 40^2}\ v$

$\boxed{V_{rms}(L) = 215.6\ v}$ (215 v or 216 v OK)

Problem 2 (10 pts): A source of sinusoidal electromagnetic waves radiates uniformly in all directions. At a distance of 2.0 m from this source, the amplitude of the electric field is measured to be 100 N/C. E_{max}

a) (3 pts) What is the intensity of the source at distance 2.0 m?
b) (4 pts) What is the total power radiated by the source?
c) (3 pts) What is the electric field amplitude at a distance of 10.0 m from the source?

a) Use $I = \frac{1}{2}\epsilon_0 c E_{max}^2$

$$I = \frac{1}{2} \cdot 8.85\times10^{-12} \cdot 3\times10^{8} \cdot (100)^2 \ W/m^2$$

$$\boxed{I = 13.3 \ W/m^2}$$

b) Power in all directions: multiply intensity I by area A. The area is of a sphere with radius R = 2.0 m, $A = 4\pi R^2$

$$P = I \cdot 4\pi R^2$$

$$P = 13.3 \cdot 4\pi \, (2.0)^2 \ \text{Watts}$$

$$\boxed{P = 667 \ W}$$

c) The power travels outward without absorption, so P = 667 W regardless of R. Ratio:

$$\frac{P_{10}}{P_{2.0}} = 1 = \frac{I_{10} \cdot \cancel{4\pi} \cdot (10)^2}{I_{2.0} \cdot \cancel{4\pi} \, (2.0)^2} \Rightarrow \frac{I_{10}}{I_{2.0}} = \left(\frac{2.0}{10}\right)^2 = (0.2)^2$$

Also, $I \propto E^2$.

$$\frac{I_{10}}{I_{2.0}} = \frac{E_{10}^2}{E_{2.0}^2} = (0.2)^2 \quad \text{so} \quad \frac{E_{10}}{E_{2.0}} = 0.2$$

$$E_{10} = 0.2 \cdot E_{2.0} = 0.2 \cdot 100 \ N/c$$

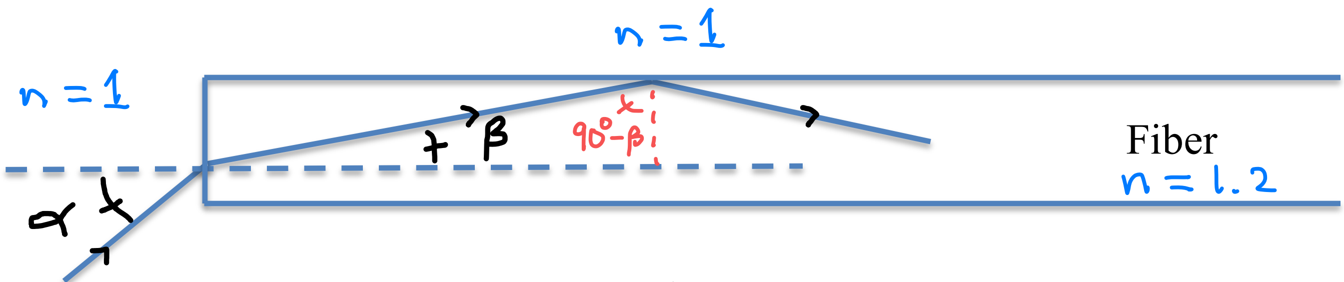
$$\boxed{E_{10} = 20 \ N/c}$$

(Can also do by direct calculation from P, but that takes more work.)

Problem 3 (10 pts): Light enters the end of a long optical fiber from air as shown below. [Take the fiber as extending to the right to infinity.] In order to avoid complications, this optical fiber has no cladding, that is, it is surrounded by air, and we take the fiber as a 2-dimensional object. The refractive index of air is $n = 1.0$, and the refractive index of the fiber is $n = 1.2$.

a) (4 pts) What range of entering angles α between $0^{\circ} - 90^{\circ}$ allow the light to enter the end of the fiber? Explain your reasoning.

b) (6 pts) What range of entering angles α allow the light to be totally internally reflected at subsequent encounters with the walls of the fiber?



a) $n_a \sin\theta_a = n_b \sin\theta_b$, here $1.0 \cdot \sin\alpha = 1.2 \sin\beta$

$\sin\beta = \frac{1.0}{1.2}\sin\alpha = 0.8\overline{3}\sin\alpha$

Since $\sin\alpha \le 1.0$, we have $\sin\beta \le 0.8\overline{3}$

which always has a solution.

$\therefore$ Any angle $\boxed{0^{\circ} \le \alpha \le 90^{\circ}}$ will work.

b) Small α leads to small β and total internal reflection. Angle of incidence $90^{\circ}-\beta$ as shown.

Reflection if no solution for refraction at wall:

$1.2\sin(90^{\circ}-\beta) \ge 1.0 \cdot \underbrace{1.0}_{\text{max of sin}}$

$\sin(90^{\circ}-\beta) \ge \frac{1}{1.2} = 0.8\overline{3} \Rightarrow 90^{\circ}-\beta \ge 56.4^{\circ}$

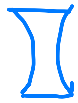
$\beta < 33.5^{\circ}$

Then $\sin\alpha = 1.2\sin\beta < 0.66 \Rightarrow \boxed{\alpha < 41.5^{\circ}}$

($<41^{\circ}$ or $<42^{\circ}$ okay)

R₁ negative, R₂ positive

Problem 4 (10 pts): A thin lens made of glass with $n = 1.40$ has concave surfaces on both sides, each with radius of curvature 80.0 cm. An object 30.0 cm high is placed 150.0 cm to the left of the lens.



a) (5 pts) How far from the lens is the image located? Is this image upright or inverted? What is the size of the image?

b) (5 pts) Make a sketch on the line below, with the lens placed at $x = 0$, and using an arrow with its tail on the optical axis to denote the object. Draw two or three well-chosen rays to verify that the image is consistent with your answers in part a). Explain the choice of each ray in a sentence.

a) First, calculate f from $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

both terms negative for concave

$$\frac{1}{f} = (1.40-1)\left(\frac{-1}{80\,cm} - \frac{1}{80\,cm}\right)$$

$$f = -100\,cm$$

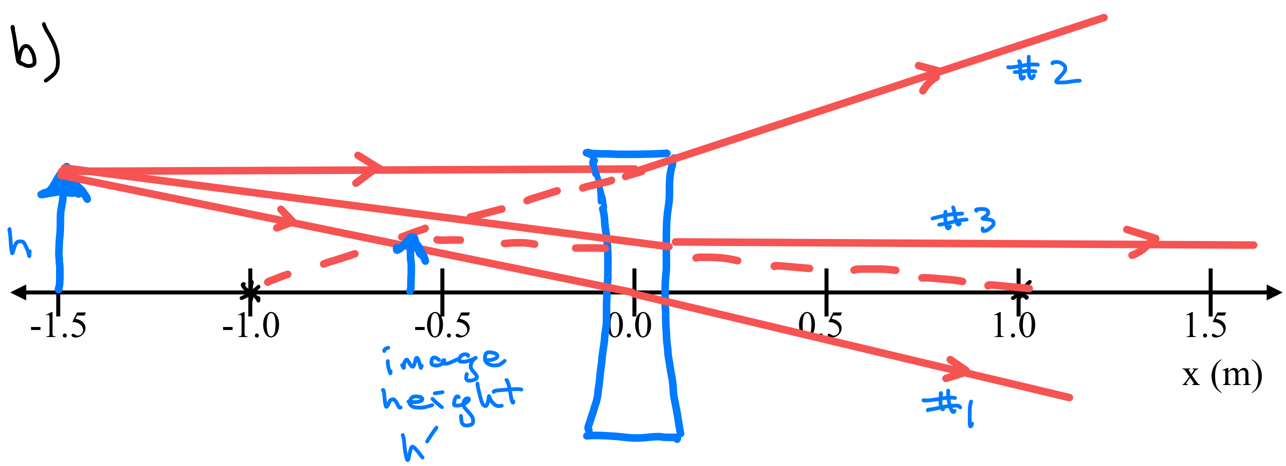
Then, image distance from $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$,

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{-1}{100\,cm} - \frac{1}{150\,cm} \Rightarrow \boxed{s' = -60\,cm}$$

Since $M = -\frac{s'}{s} = \frac{+60\,cm}{150\,cm} = 0.4$ is positive, $\boxed{\text{image is upright.}}$

Size $h' = Mh = 0.4 \cdot 30cm = \boxed{12\,cm}$

b)



any 2 OK

Ray #1 is easiest: thru center of lens ⇒ undeflected

Ray #2 parallel to optic axis, bent to pass thru focal point - on near side since f negative.

Ray #3 more difficult: pointed at far focal point, will emerge from lens parallel to optic axis.

Rays appear to diverge from location of image.

Midterm 2 constants:

$\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2}$

$\mu_0 = 4\pi \times 10^{-7}\ Wb/A \cdot m$

$e = 1.6 \times 10^{-19}\ C$

$c = 3.00 \times 10^{8}\ m/s$

Midterm 2 equations:

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, $\overrightarrow{dF} = I\overrightarrow{dl} \times \vec{B}$

$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$

$R = {mv}/{|q|B}$

$\vec{\mu} \equiv I\vec{A}$, $\vec{\tau} = \vec{\mu} \times \vec{B}$, $U = -\vec{\mu} \cdot \vec{B}$

$\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$, $\overrightarrow{dB} = \frac{\mu_0 I \overrightarrow{dl} \times \hat{r}}{4\pi r^2}$

$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot \overrightarrow{dA}$

$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$, $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

$B = \frac{\mu_0 N I}{2\pi r}$, $B = \mu_0 n I$, $n \equiv N/L$

$\mu = K_m \cdot \mu_0$, $\chi_m = K_m - 1$

$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B} \cdot \overrightarrow{dA}$

$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \overrightarrow{dl}$

$\mathcal{E} = NBA\omega \cdot sin\omega t$

$\mathcal{E} = -L\frac{di}{dt}$, $L = \frac{N\Phi_B}{i}$

$U = \frac{1}{2}LI^2$, $u \equiv \frac{U}{Vol.} = \frac{B^2}{2\mu_0}$

$\tau = \frac{L}{R}$

$\omega = \sqrt{\frac{1}{LC}}$, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and $\tau = \frac{2L}{R}$

$V_{rms} = \frac{V_0}{\sqrt{2}}$, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\langle P \rangle = \frac{V_{rms}^2}{R} = I_{rms}^2 R = I_{rms} V_{rms}$ (resistive)

$V_R = IR, V_L = IX_L, V_C = IX_C$, $X_L = \omega L$, $X_C = {1}/{\omega C}$

$V = IZ$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R}$, $cos\phi = \frac{R}{Z}$

$\langle P \rangle = I_{rms} V_{rms} cos\phi$

$\frac{I_1}{I_2} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

$k\lambda = 2\pi$, $v = f\lambda = \omega/k$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, $B = E/c$

$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$, $I = S_{av} = \frac{1}{2}\epsilon_0 c E_{max}^2$

$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c}$, $p_{rad} = \frac{(1-2)I}{c}$

$\frac{c}{v} \equiv n = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0 \mu_0}}$, $\lambda = \frac{\lambda_0}{n}$

$\theta_r = \theta_a$ or $n_a sin\theta_a = n_b sin\theta_b$

$I = I_{max}\cos^2\phi$

$tan\theta_p = \frac{n_b}{n_a}$

$I_{scat} \propto \lambda^{-4}$

$P \equiv \frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$M \equiv \frac{y'}{y} = -\frac{s'}{s}$

Or $M \equiv \frac{\theta'}{\theta} = \frac{25\ cm}{f}$ or $M = \frac{-f_1}{f_2}$ or $M = \frac{-25\ cm \cdot s_1'}{f_1 f_2}$

f-number $\equiv \frac{f}{D}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$