Solutions

Name:____________________________

Student ID #:____________________________

Signature:____________________________

Physics 1C Midterm #1 - Version A

(white paper)

- By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
- This exam contains four workout problems, each 10 points, for a total of 40 points. Remember to write down each step of your calculation, and explain your answers. You have 50 minutes to complete this exam.
- Close your exam when time is up, and show your student ID when handing it in.
- Detailed exam rules:
  - By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
  - You can use any type of calculator that does not have internet capability. Silence and put away your cell phones, tablets, and laptops.
  - Quote numerical answers with 3 significant figures, e.g. 0.262 or 3.72×10^3. Always specify the units, and quote final answers in SI units unless otherwise directed.
  - The last page of the exam is an equation sheet that may be torn off.
  - Fit all relevant calculations on the front of the pages. If you run out of room, use the front of the blank page before the equation sheet, and indicate "Problem <n> continued:" to help us in grading.
- If you have questions during the exam, raise your hand. If you are not seated near the end of a row, you may need to come to the aisle or down to the front of the room to ask them.

Academic Integrity - A Bruin's Code of Conduct:

As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behavior includes cheating, fabrication or falsification, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, facilitating academic misconduct, coercion regarding grading or evaluation of coursework, or collaboration not authorized by the instructor. Please review our campus' policy on academic integrity in the UCLA Student Conduct Code: https://deanofstudents.ucla.edu/individual-student-code .

If you engage in these types of unacceptable behaviors in our course, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam. These allegations will be referred to the Office of the Dean of Students and can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

By submitting my assignments and exams for grading in this course, I acknowledge the above-mentioned terms of the UCLA Student Code of Conduct, declare that my work will be solely my own, and that I will not communicate with anyone other than the instructor and proctors in any way during the exams.

B version

Problem 1 (10 pts): An electron moving with a velocity $\vec{v} = (1.0\hat{\imath} + 3.0\hat{\jmath} + 5.0\hat{k}) \times 10^6$ m/s enters a region where there is a uniform electric field and a uniform magnetic field. The magnetic field is given by $\vec{B} = (3.0\hat{\imath} + 3.0\hat{\jmath} + 2.0\hat{k}) \times 10^{-3}$ T.

a) (7 pts) What is the magnetic force vector on the electron at the instant that it enters the magnetic field region?
b) (3 pts) If the electron travels through the region at constant velocity, what is the electric field vector?

$$\vec{F} = q\vec{v} \times \vec{B}, \quad q^- = 1.6 \times 10^{-19}\,C$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.0 & 3.0 & 5.0 \\ 3.0 & 3.0 & 2.0 \end{vmatrix} \times 10^6 \tfrac{m}{s} \times 10^{-3}\,T$$

$$\vec{v} \times \vec{B} = \left[\hat{i}(6-15) - \hat{j}(2-15) + \hat{k}(3-9)\right] \times 10^3 \tfrac{mT}{s}$$

$$\vec{v} \times \vec{B} = (-9\hat{i} + 13\hat{j} - 6\hat{k}) \times 10^3 \tfrac{mT}{s}$$

a) $\boxed{\vec{F} = 1.44 \times 10^{-15}\hat{i} - 2.08 \times 10^{-15}\hat{j} + 9.6 \times 10^{-16}\hat{k}}$ N

b) Constant velocity: $\vec{a} = 0$ so $\vec{F}_{tot} = m\vec{a} = 0$,

so $\vec{F}_{tot} = q(\underbrace{\vec{E} + \vec{v} \times \vec{B}}) = 0$

$\therefore \vec{E} = -\vec{v} \times \vec{B}$

From above,

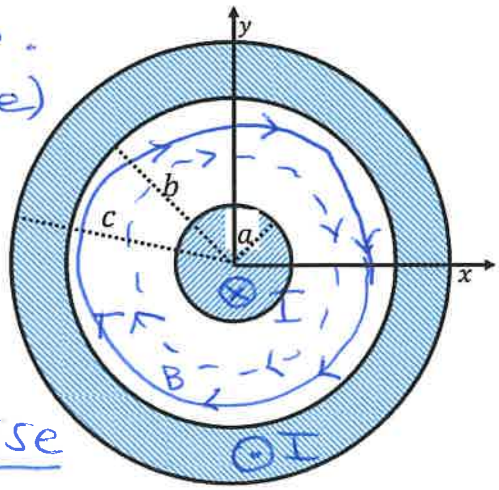
$$\boxed{\vec{E} = -\vec{v} \times \vec{B} = 9.0 \times 10^3\hat{i} - 13.0 \times 10^3\hat{j} + 6.0 \times 10^3\hat{k} \text{ V/m}}$$

B version

Problem 2 (10 pts): Coaxial Cable. A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c in the figure below. The central conductor carries current I in the direction into the page, and the tube carries equal current in the opposite direction (out of the page). The currents are distributed uniformly over the cross sections of each conductor. The conductors are not ferromagnetic.

a) (7 pts) Derive an expression for the magnitude of the magnetic field at points outside the central, solid conductor but inside the tube ($a < r < b$). Indicate the direction of this field on the diagram below if it is not zero.

b) (3 pts) Derive an expression for the magnitude of the magnetic field outside the tube ($r > c$). Indicate the direction of this field on the diagram below if it is not zero.



a) Use clockwise Amperian loop.

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ positive (r.h. rule)

$2\pi r \cdot B = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r}}$ for $a < r < b$

Since positive, B is in clockwise direction as shown.

b) Outside, $I_{enc} = 0$ for $r > c$ (currents cancel)

$\oint \vec{B} \cdot d\vec{l} = 0 = B \cdot 2\pi r$

$\vec{B} = 0$

B version

Problem 3 (10 pts): Suppose we want to make our own AC electrical generator that produces EMF that varies sinusoidally at 60 Hz. We have a permanent magnet that makes a uniform field of 0.1 Tesla, and we use a circular coil of diameter 10 cm that fits inside the magnet.

a) (4 pts) What is the maximum magnetic flux through a single loop of this coil?
b) (4 pts) What is the maximum EMF that would be made by rotating a single loop of this coil?
c) (2 pts) How many loops of wire are then needed to produce an EMF that varies between −170 V and +170 V (the usual U.S. AC "120 volt" wall voltage)?

a) Area $A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi (0.05\,m)^2 = 7.85 \times 10^{-3}\,m^2$

$\Phi_B(max) = BA \underbrace{\cancel{\cos} \omega t}_{1 \text{ at max}} = BA = 0.1\,T \cdot 7.85 \times 10^{-3}\,m^2$

$\Phi_B(max) = \boxed{7.85 \times 10^{-4}\,T \cdot m^2}$ (Wb or Weber)

b) $\omega = 2\pi f = 2\pi \cdot 60/s = 377/s$

$\mathcal{E} = BA\omega \sin \omega t$ for $N = 1$

$\mathcal{E}_{max} = BA\omega = \Phi_B(max) \cdot \omega$

$\mathcal{E}_{max} = 7.85 \times 10^{-4}\,T \cdot m^2 \cdot 377/s$

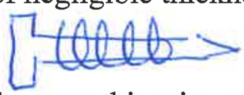
$\boxed{\mathcal{E}_{max} = 0.296\,V}$

c) $\mathcal{E}_{max} = 170\,V = N \cdot 0.296\,V$

$N = \frac{170\,V}{0.296\,V} = \boxed{574}$ loops

B version

Problem 4 (10 pts): Assume you have a spool of 22-gauge copper wire, which has a diameter of 0.6426 mm, and a thick "20d" nail, which has a diameter of 5.15 mm. The copper wire is coated with a thin insulator, usually made of plastic but called "enamel", of negligible thickness. The nail is made out of steel with a relative permeability $K_m = 1.3 \times 10^3$.



a) (3 pts) How many times can you tightly wrap this wire around the nail without overlapping to form a solenoid with a length of 8 cm?

b) (5 pts) Estimate the inductance of this solenoid, assuming the magnetic field inside is constant.

c) (2 pts) If a current of 5.0 A flows through the wire wound around the nail, how much magnetic energy will be stored inside?

a) Wires spaced $\Delta L = 0.6426$ mm from the next wire

$$N = \frac{L}{\Delta L} = \frac{80\text{ mm}}{0.6426\text{ mm}} = \boxed{124} \text{ (nearest integer)}$$

b) $L = N\Phi_B / I$ where

$$\Phi_B = B \cdot A = (\mu n I) \cdot \left(\frac{\pi}{4}\delta^2\right) = (K_m \mu_0 n)\left(\frac{\pi}{4}\delta^2\right) \cdot I$$

$$L = N K_m \mu_0 \frac{N}{L} \cdot \frac{\pi}{4}\delta^2$$

$$= 124 \cdot 1.3 \times 10^3 \cdot 4\pi \times 10^{-7} \cdot \frac{124}{0.08} \cdot \frac{\pi}{4}(5.79 \times 10^{-3})^2$$

5.15 mm + 0.6426 mm

$\doteq$ 5.79 mm

$$L = \cancel{0.0083}\ 8.27 \times 10^{-3}\text{ H} = \boxed{8.27\text{ mH}}$$

c) $U = \frac{1}{2}LI^2 = \frac{1}{2}(8.27 \times 10^{-3})(5.0)^2$ J

$$\boxed{U = 0.103\text{ J}}$$

Midterm 1 possibly useful constants:

$\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2}$

$\mu_0 = 4\pi \times 10^{-7}\ Wb/A \cdot m$

$e = 1.6 \times 10^{-19}\ C$

Midterm 1 possibly useful equations:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad , \quad \overrightarrow{dF} = I\overrightarrow{dl} \times \vec{B}$$

$$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$$

$$R = {}^{mv}\!/_{|q|B}$$

$$\vec{\mu} \equiv I\vec{A} \ , \ \vec{\tau} = \vec{\mu} \times \vec{B} \ , \ U = -\vec{\mu} \cdot \vec{B}$$

$$\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2} \quad , \quad \overrightarrow{dB} = \frac{\mu_0 I \overrightarrow{dl} \times \hat{r}}{4\pi r^2}$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot \overrightarrow{dA}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad , \quad \frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

$$B = \frac{\mu_0 N I}{2\pi r} \quad , \quad B = \mu_0 n I \ , \ n \equiv N/L$$

$$\mu = K_m \cdot \mu_0 \quad , \quad \chi_m = K_m - 1$$

$$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B} \cdot \overrightarrow{dA}$$

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \overrightarrow{dl}$$

$$\mathcal{E} = NBA\omega \cdot sin\omega t$$

$$\mathcal{E} = -L\frac{di}{dt} \quad , \quad L = \frac{N\Phi_B}{i}$$

$$U = \frac{1}{2}LI^2 \quad , \quad u \equiv \frac{U}{Vol.} = \frac{B^2}{2\mu_0}$$

$$\tau = \frac{L}{R}$$

$$\omega = \sqrt{\frac{1}{LC}} \quad , \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ and } \tau = \frac{2L}{R}$$