UCLA Statistics

STATISTICS 10
Introduction to Statistical Reasoning

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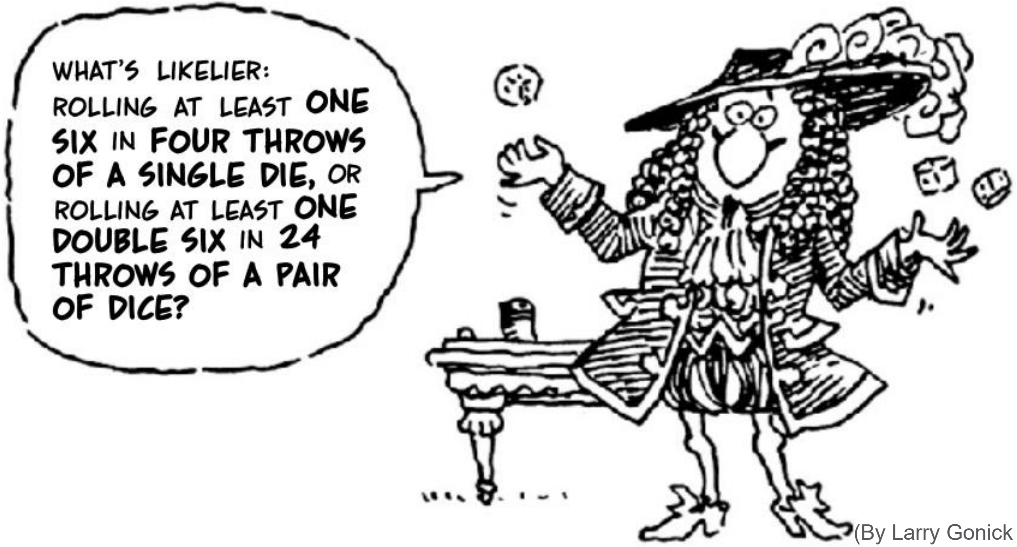
RANDOMNESS AND PROBABILITIES

A Little History on Probability Theory

Antoine Gombaud, Chevalier de Méré (1607-1684) – **The Dice Problem**

Solved by Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665)



(By Larry Gonick & Woollcott Smith)

Randomness

No predictable pattern occurs, and no outcome is more likely to appear than any other

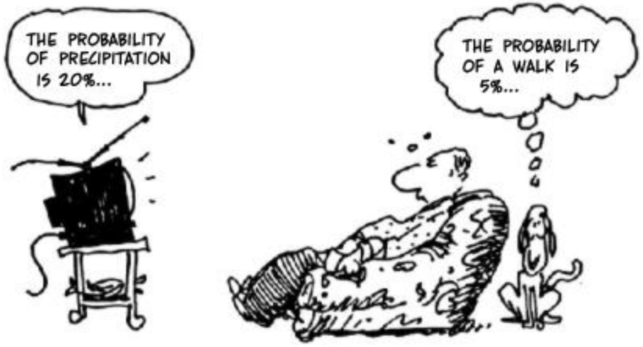
Randomness can be achieved with help from computer or some other randomizing device.

[Random numbers]

- Pseudo-random numbers are generated by algorithm that we do not know.
  - The algorithm starts with an initial number (seed). In R: $set.seed()$
  - Specifying the same seed will give us the same set of pseudo-random numbers.
- For most practical purposes, these pseudo-random numbers are as random as we need.

Probability

- What is the chance that it will rain tomorrow?
- What is the chance that a dog will live longer than 10 years?
- What is the probability that I will win the lottery?
- What are the chances that at least two of the students in class share the same birthday?



Probability is a measure of how likely an event is to occur.

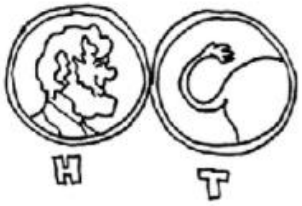
Notation:

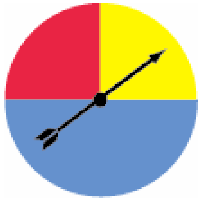
- **P(rain)**: the probability that it will rain tomorrow
- **P(A)**: the probability that event A will occur
- **P(B)**: the probability that event B will occur

Two Types of Probabilities

Theoretical Probability

Long-run relative frequency at which an event occurs after infinitely many repetitions





Empirical Probability

Relative frequencies based on experiment or observations of a real-life process

Example: In one experiment, we flip a fair coin 20 times and get 8 heads.
The empirical probability of getting heads is $\frac{8}{20} = 0.4 = 40\%$.

Theoretical vs. Empirical

Difference

- Theoretical probabilities are always the same value.
- Empirical probabilities could change from experiment to experiment.

Connection

We can use empirical probabilities to estimate and test theoretical probabilities.

- **Estimate**
  - Theoretical probabilities may be too difficult to compute.
  - Empirical probabilities can help us estimate the theoretical probabilities base on what we observe.
- **Test**
  - The assumptions we make to compute theoretical probabilities could be incorrect.
  - The Empirical probabilities can help us verify the correctness of a theoretical value

FINDING THEORETICAL PROBABILITIES

Terminologies

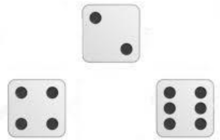
- Experiment or Trial: any repeatable procedure with well-defined set of possible outcomes.

- Outcome: a result of an experiment.

- Sample space: the collection/set of all possible outcomes of an experiment, denoted as S or Ω.

- Event: any collection of outcomes in the sample space.



Probability Basics

Let A be some event, S be the sample space.

- The probability of an event A happening is denoted as $P(A)$.
- The probability of the set of all possible outcomes is $P(S)$.
- The probability can be expressed as decimals, fractions, or percentages

  Example: $P(A) = 0.5$, or $\frac{1}{2}$, or 50%

RULE 1

The probability that the event occurs is always between 0 and 1 (including 0 and 1)

- $0 \leq P(A) \leq 1$
- If an event can never happen, the probability of the event is 0.
- If an event is certain to happen, the probability of the event is 1.

The Complement Rule

The **complement** refers to the event NOT happening.

- It is the set of outcomes where the event does not happen.
- The complement of event A is denoted as A^c.

RULE 2

The probability that an event will not occur is 1 minus the probability that it occurs.

$$P(A^c) = P(\text{NOT } A) = 1 - P(A)$$

Example:

A = "It will rain tomorrow". Suppose $P(A) = 0.3$

A^c = "It will NOT rain tomorrow", $P(A^c) = 1 - 0.3 = 0.7$

B = "Left the wallet at home". Suppose $P(B) = 0.4$

B^c = "Left the wallet somewhere else". $P(B^c) = 1 - 0.4 = 0.6$

Equally Likely Outcomes

Definition: Situations where all the possible outcomes of a random experiment occur with the same frequency.

E.g., flip a fair coin, roll fair die

RULE 3 If all possible outcomes are equally likely, then for any event A in S.

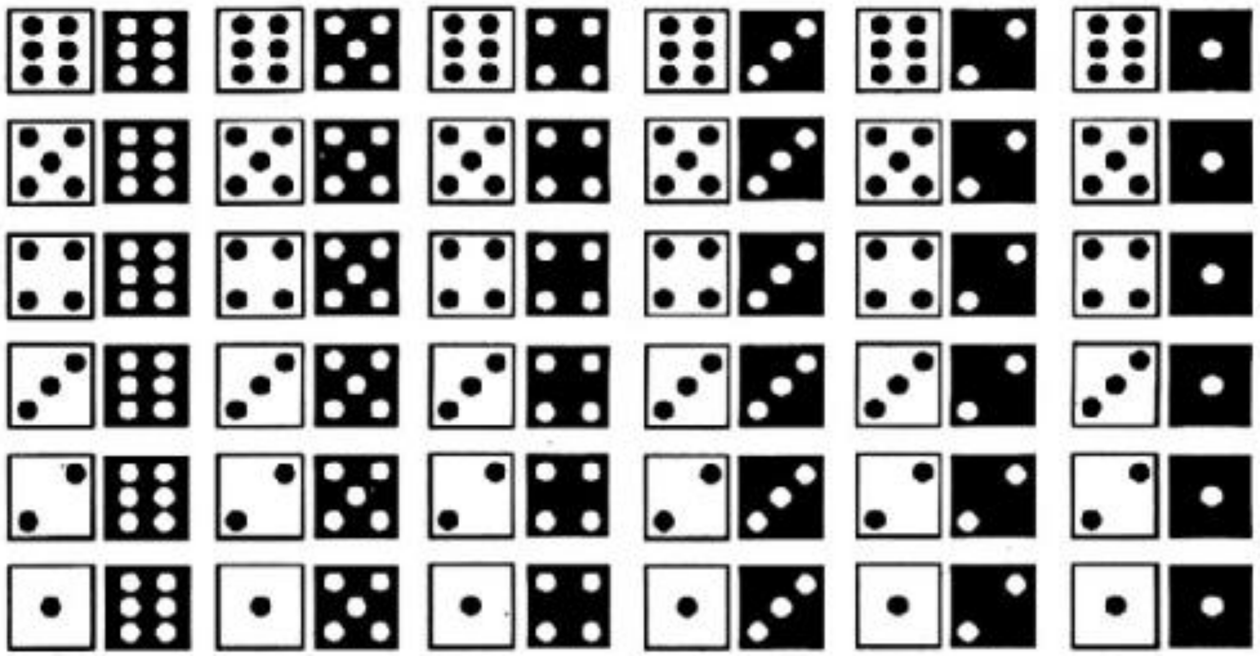
$$P(A) = \frac{\text{Number of outcomes in } A}{\text{Number of all possible outcomes in } S}$$

Example:

What is the probability of rolling a six on a fair die?

What is the probability of rolling a double six on a pair of fair dice?

Sample space for rolling a pair of dice.



Exercise

Reach into a bowl that contains 5 red dice, 3 green dice, and 2 white dice. Assume that the dice have been well mixed.

What is the probability of picking

(a) A red die?

(b) A green die?

(c) A white die?