SOLUTIONS

Name:______________________________

Student ID #:______________________________

Signature:______________________________

Physics 1C PRACTICE Final Exam

- By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
- This exam contains four workout problems, each 10 points, for a total of 40 points. Remember to write down each step of your calculation, and explain your answers. You have 50 minutes to complete this exam.
- Close your exam when time is up, and show your student ID when handing it in.
- Detailed exam rules:
    - By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
    - You can use any type of calculator that does not have internet capability. Silence and put away your cell phones, tablets, and laptops.
    - Quote numerical answers with 3 significant figures, e.g. 0.262 or 3.72×10^3. Always specify the units, and quote final answers in SI units unless otherwise directed.
    - The last page of the exam is an equation sheet that may be torn off.
    - Fit all relevant calculations on the front of the pages. If you run out of room, use the front of the blank page before the equation sheet, and indicate "Problem <n> continued:" to help us in grading.
- If you have questions during the exam, raise your hand. If you are not seated near the end of a row, you may need to come to the aisle or down to the front of the room to ask them.

Academic Integrity - A Bruin's Code of Conduct:

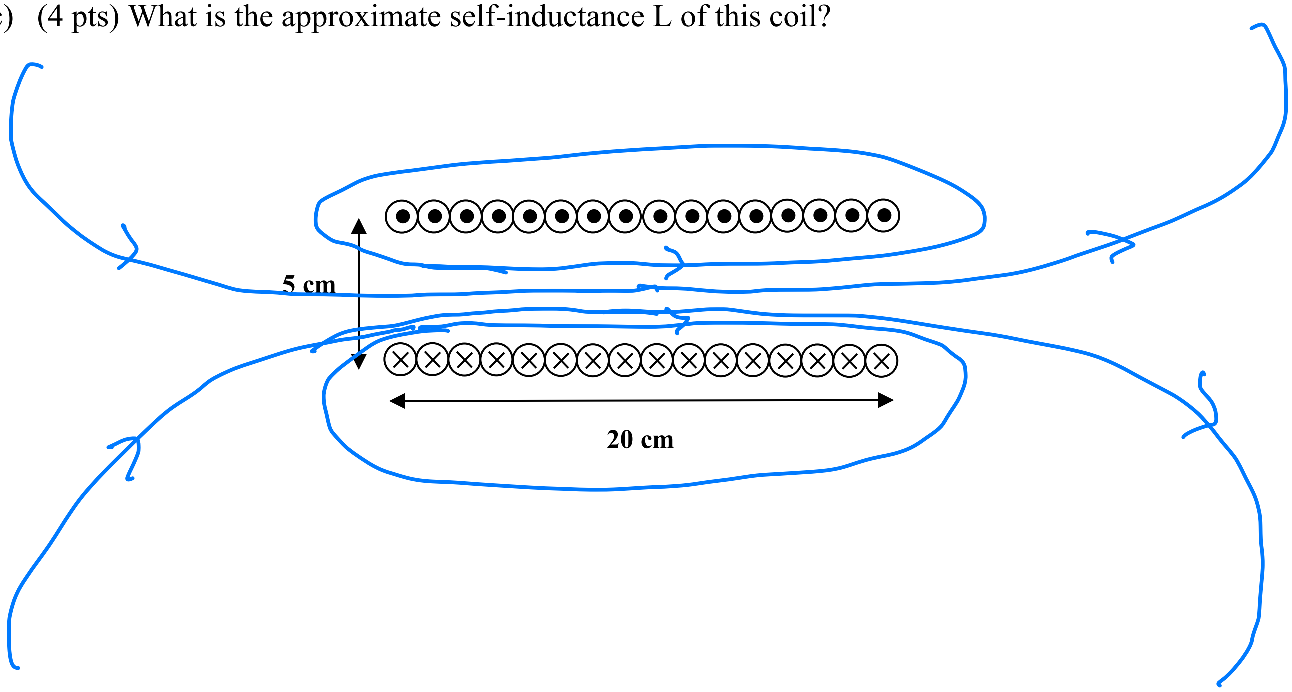
As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behavior includes cheating, fabrication or falsification, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, facilitating academic misconduct, coercion regarding grading or evaluation of coursework, or collaboration not authorized by the instructor. Please review our campus' policy on academic integrity in the UCLA Student Conduct Code: https://deanofstudents.ucla.edu/individual-student-code .

If you engage in these types of unacceptable behaviors in our course, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam. These allegations will be referred to the Office of the Dean of Students and can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

By submitting my assignments and exams for grading in this course, I acknowledge the above-mentioned terms of the UCLA Student Code of Conduct, declare that my work will be solely my own, and that I will not communicate with anyone other than the instructor and proctors in any way during the exams.

Problem 1 (10 pts): A cylindrical solenoid coil is wound exactly as shown below in cross-section.

a) (3 pts) Sketch the magnetic field lines created by current that passes through the wires in the directions indicated.
b) (3 pts) What is the <u>approximate</u> magnetic field near the center of the coil that is produced by a current of 10.0 A?
c) (4 pts) What is the approximate self-inductance L of this coil?



a) See above: (1pt) for ~straight, ~equally spaced inside
(1pt) for loops (coming around) with correct direction
(1pt) for up/down, left/right symmetry

b) $B=\mu_0 n I$ (1pt)

$n = \frac{16 \text{ loops}}{20 \text{ cm}} = 0.8/\text{cm} = 80/\text{m}$ (1pt)

$B = 4\pi\times10^{-7}\,\frac{T\cdot m}{A}\cdot 80/m \cdot 10.0\,A = \boxed{1.00\times10^{-3}\,T}$ (1pt)

c) $L = N\frac{\Phi}{I}$ (1pt)

$L = N\frac{BA}{I} = N\frac{\mu_0 n \cancel{I}}{\cancel{I}}\cdot\frac{\pi}{4}d^2 = \frac{\mu_0 N^2}{L}\cdot\frac{\pi}{4}d^2$ (1pt)

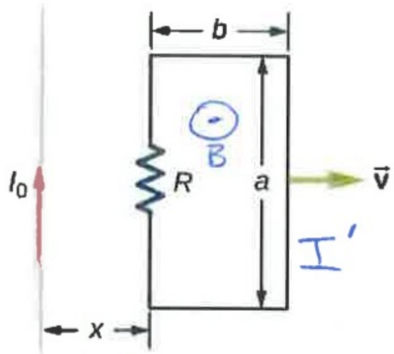
$L = \frac{\cancel{4}\pi\times10^{-7}\cdot(16)^2}{0.2\,m}\cdot\frac{\pi}{\cancel{4}}(0.05)^2\,H = \boxed{3.16\times10^{-6}\,H}$ (2pts)

Problem 2 (10 pts): A rectangular circuit containing a resistance R is pulled at a constant velocity v away from a long, straight wire carrying a constant current I (see below).

a) (1 pt) What is the magnetic field at distance r from the wire?
b) (4 pts) What is the magnetic flux Φ_B through the circuit as a function of the distance x between the near side of the circuit and the wire?
c) (5 pts) What is the current I' induced in the circuit as a function of $x=vt$? (of course it is on the right, so t is positive)

a) $B = \boxed{\dfrac{\mu_0 I}{2\pi r}}$ (1 pt)

b) $\Phi_B = \iint \vec{B} \cdot \hat{n} \, dA = \int_0^a dz \int_x^{x+b} dr \, \dfrac{\mu_0 I}{2\pi r}$ (1 pt) (1 pt)



$\Phi_B = \dfrac{\mu_0 I}{2\pi} \cdot a \cdot \int_x^{x+b} \dfrac{dr}{r}$ (1 pt)

$\Phi_B = \boxed{\dfrac{\mu_0 I a}{2\pi} \left[\ln(x+b) - \ln(x) \right]}$ (1 pt)

(Φ_B decreases as loop pulled away. By Lenz Law, I' will be clockwise to add a small B' to oppose this, B' also ⊙)

c) $x = vt$, so

$\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 I a}{2\pi} \dfrac{d}{dt}\left[\ln(vt+b) - \ln(vt) \right]$ (1 pt) (1 pt)

$\mathcal{E} = \dfrac{\mu_0 I a}{2\pi} \cdot \left[\underbrace{\dfrac{1}{vt+b}}_{x+b} \cdot v - \underbrace{\dfrac{1}{vt}}_{x} \cdot v \right]$ (1 pt)

$\mathcal{E} = -\dfrac{\mu_0 I a v}{2\pi} \left(\dfrac{1}{x+b} - \dfrac{1}{x} \right)$ (1 pt)

$I' = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 I a v}{2\pi R} \left[\dfrac{1}{x} - \dfrac{1}{x+b} \right]}$ (1 pt)

Problem 3 (10 pts): A circuit contains two elements in series, but it is not known which of L, R, or C they are. The current in the circuit, when connected to AC power that is 120 volt (RMS) and 60 Hz, has RMS magnitude 2.7 A and leads the voltage by 20.0°.

a) (4 pts) What are the two elements of the circuit? Explain why.
b) (6 pts) What values of inductance, resistance, and/or capacitance do they have?

$$\tan\phi = \tan 20^\circ = 0.364 = \frac{|X_L - X_c|}{R}$$

a) Must have $R \neq 0$, otherwise $\tan\phi \to \infty$ i.e. $\phi = \pm\frac{\pi}{2}$. (2pts)

From "ELI the ICE man" mnemonic, here current leads, so capacitive case. (2pts)

So the elements are $\boxed{\text{resistor and capacitor}}$.

b) $$Z = \frac{V_{RMS}}{I_{RMS}} = \frac{120\,v}{2.7\,A} = 44.4\,\Omega = \sqrt{R^2 + X_c^2},$$ (2pts)

$$Z = \sqrt{R^2\left(1+\left(\frac{X_c}{R}\right)^2\right)} = R\sqrt{1+\tan^2\phi}$$

$$R = \frac{Z}{\sqrt{1+\tan^2\phi}} = \frac{44.4\,\Omega}{\sqrt{1+0.364^2}} = \boxed{41.7\,\Omega}$$ (2pts)

$$X_c = R\tan\phi = 41.7\,\Omega \cdot 0.364 = \boxed{15.2\,\Omega}$$ (2pts)

and $X_c = \frac{1}{\omega C}$ so $C = \frac{1}{\omega X_c} = \frac{1}{2\pi f X_c}$

$$C = \frac{1}{2\pi \cdot 60 \cdot 15.2}\,F = \boxed{1.75 \times 10^{-4}\,F}$$

Problem 4 (10 pts): A laser produces a beam of light along the *z*-axis that is polarized so that the electric field is only parallel to the *y*-axis. The average power in the laser beam is 20 mW. The wavelength of the light is 632.8 nm. The beam is round and its diameter is 5 mm.

a) (2 pts) What is the frequency of the light?
b) (3 pts) What is the intensity of the beam coming out of the laser?
c) (5 pts) Write down a possible equation for the *magnetic* field in the electromagnetic wave produced by the laser, including as many specific numbers as possible. [Hint: your answer should be a vector.]

a) $f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\ m/s}{6.328\times10^{-7}\ m} = \boxed{4.74\times10^{14}\ Hz}$ (1pt)

(1pt)

b) $I = S_{avg} = \frac{Power}{Area} = \frac{P}{\frac{\pi}{4}d^2} = \frac{20\times10^{-3}\ W}{\frac{\pi}{4}\cdot(5\times10^{-3}\ m)^2}$ ← (1pt area)

(1pt)

$I = \frac{20\times10^{-3}\ W}{1.96\times10^{-5}\ m^2} = \boxed{1{,}018\ W/m^2}$ (1pt)

c) Light travels in direction $\vec{E}\times\vec{B}$. If $\vec{E} \parallel \hat{j}$ and it travels along z $(\hat{k})$, $\vec{B}$ must be along the remaining axis $(\hat{i})$.

Magnitude of $\vec{B}$: from $I = \frac{cB_0^2}{2\mu_0}$ (1pt), solve for

$$B_0 = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2\cdot4\pi\times10^{-7}\cdot1018}{3\times10^{8}}}\ T$$

$B_0 = 2.92\times10^{-6}\ T$

Generally $\vec{B} = \hat{i}\,B_0 \sin(kz - \omega t + \phi)$, where

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6.328\times10^{-7}\ m} = 9.93\times10^{6}/m$

$\omega = 2\pi f = 2.98\times10^{15}/s$

(1pt)

So $\boxed{\vec{B} = \hat{i}\cdot(2.92\times10^{-6}\ T)\sin(9.93\times10^{6}/m\cdot z - 2.98\times10^{15}/s\cdot t + \phi)}$

(1pt) (1pt)

sin( ) or cos( ) are fine

(1pt)

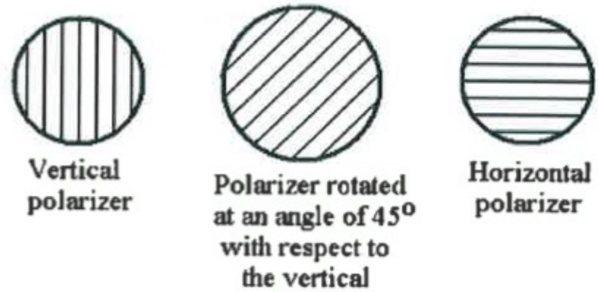
ϕ cannot be determined from the info. given, so is arbitrary number.

Problem 5 (10 pts): Unpolarized light having intensity I_0 is passed through three polarizing filters, with the first oriented vertically, the second at an angle of 45° to the first, and the third at an angle of 90° to the first, as shown below. Neglect reflections and any additional sources of inefficiency of the filters.

a) (3 pts) What is the intensity of light I coming through the first filter?
b) (2 pts) What is the intensity of light I coming through the combination of the first two filters?
c) (2 pts) What is the intensity of light I coming through the combination of all three filters?
d) (3 pts) When we remove the second filter, what is the intensity of light I coming through the combination of filters one and three?

through the combination of filters one and three?

a) $I_1 = I_0 \cos^2\phi$ (1pt)
but ϕ is random, so use average.



$\langle I_1 \rangle = I_0 \underbrace{\langle \cos^2\phi \rangle}_{\frac{1}{2}}$ (1 pt)

$\langle I_1 \rangle = \boxed{\frac{I_0}{2}}$ (1 pt)

b) Here, $I_2 = \langle I_1 \rangle \cdot \underbrace{\cos^2\phi}_{1/2}$ with $\phi = 45°$ (1pt)

$I_2 = \frac{I_0}{2} \cdot \frac{1}{2} = \boxed{\frac{I_0}{4}}$ (1pt)

c) ~~After~~ 2 filters, the light is at 45° polarization, 3rd filter apply same formula with $\phi = 45°$ representing angle of 3rd filter with respect to the 45° of the light: (1pt)

$I_3 = \frac{I_2}{2} = \boxed{\frac{I_0}{8}}$ (1pt)

d) Between light from 1st filter ↕ vertically polarized and 3rd filter, $\phi = 90°$, (1pt)

$I = \frac{I_0}{2} \cdot \underbrace{\cos^2(90°)}_{0} = 0$ (1pt)

6/14

Problem 6 (10 pts): A setup with double thin slits is illuminated by a source of coherent, plane light waves such as a laser, just as was done during several lecture demonstrations.

Later, a thin, 100% transparent film is put in front of one of the slits. The thickness of the film is $\lambda_0/(n-1)$, where λ_0 is the wavelength of the incident light and n is the index of refraction of the film.

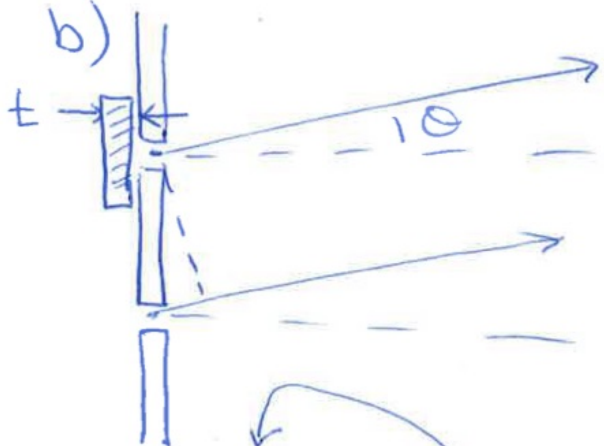
a) (3 pts) Describe in one or two coherent sentences the interference pattern observed on a screen far away <u>before</u> the film is put in front, in as much detail as possible given the limited information.
b) (5 pts) After the film is put in front, what is the difference in phase $\Delta\phi$ between the light reaching the two slits?
c) (2 pts) Describe what happens to the interference pattern seen on the screen as a result of putting the film in front, and explain why.

result of putting the film in front, and explain why.

a) There are alternating bright and dark spots, equally spaced, with a fall-off in brightness that depends on the width of each slit (not given).

1pt - any interference
1pt - equal spacing
1pt - fall-off or equal intensity (very narrow slits)

b)



Phase change:

$\lambda' = \frac{\lambda_0}{n}$ in film (shorter) (1pt)

In thickness t, phase changes by $\frac{2\pi}{\lambda} \cdot t = \phi_1$ (1pt)

versus air $\frac{2\pi}{\lambda_0} t = \phi_2$ (1pt)

$\Delta\phi = \phi_1 - \phi_2$

$\Delta\phi = \left(\frac{2\pi}{\lambda} - \frac{2\pi}{\lambda_0}\right) t$

$\Delta\phi = 2\pi t \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 2\pi \left(\frac{\lambda_0}{n-1}\right) \underbrace{\left(\frac{1}{\lambda_0/n} - \frac{1}{\lambda_0}\right)}_{\frac{n-1}{\lambda_0}}$ (1pt)

$\boxed{\Delta\phi = 2\pi}$ (1pt)

8/14

c) This 2π is equivalent to no change of phase as ~~far~~ as the interference pattern goes, so $\boxed{\text{not any change}}$. (2pts)

Problem 7 (10 pts): Visible light wavelengths range from 400 to 700 nm. Suppose that we want to design a diffraction grating that produces a full spectrum of visible light in the first order (m=1) without any m=2 or higher order diffraction of any visible light wavelength.

To do this, we need to find out the acceptable range of values for d, the separation between grooves in the diffraction grating:

a) (5 pts) What is the minimum value of d so that we can have m=1 diffraction over the full range of the visible light spectrum?
b) (5 pts) What is the maximum value of d so that we completely exclude m=2 diffraction as mentioned?

diffraction as mentioned?

a) We get strong diffraction at angles θ according to $d\sin\theta = m\lambda$! (1pt) For $m=1$, $d\sin\theta = \lambda$, so $\sin\theta = \frac{\lambda}{d}$. (1pt) Obviously $d \geq \lambda$ (1pt) is required, for all $\lambda \in [400, 700]$ nm

So $\boxed{d > 700\,nm}$ is required (2 pts)

b) For $m=2$, $d\sin\theta = 2\lambda$, or $\sin\theta = 2\frac{\lambda}{d}$. (1pt)
If we want to exclude this, we need $2\frac{\lambda}{d} > 1$ (2pts) so no value of θ works.
$2\lambda > d$ for all λ between 400-700 nm
So $\boxed{d < 800\,nm}$ works for all visible light.
(2pts)

Bottom line: $700\,nm < d < 800\,nm$, a narrow range!

Problem 8 (10 pts): The Higgs particle, discovered in 2012, has a rest mass equivalent energy of 125 GeV (a GeV is a billion electron-volts). The electron, discovered in 1897, has a rest mass equivalent energy of 511 KeV.

a) (2 pts) Express the rest mass equivalent energy of the Higgs particle in joules.
b) (2 pts) Suppose that a Higgs particle at rest decays into two electrons (in reality, this is exceedingly rare). How much total relativistic energy does each electron have?
c) (3 pts) What is the relativistic γ factor of each electron?
d) (3 pts) What is the difference $\Delta v = (c\text{-}v)$ between the speed of light and the speed of each electron? [Hint: the answer is NOT zero.]

a) $e = 1.6\times10^{-19}\,C$ so $1\,eV = e\cdot 1\,V = 1.6\times10^{-19}\,J$ (1pt)

$E = 125\times10^{9}\,eV \cdot 1.6\times10^{-19}\,\frac{J}{eV} = \boxed{2.0\times10^{-8}\,J}$ (1pt)

b) Must conserve momentum so symmetric decay

e ⟵ • ⟶ e
H

Each electron has half the energy (1pt) of the Higgs (conserve energy). This is the rest mass energy, so $E_e = \frac{1}{2}M_H c^2 = \frac{125\text{ GeV}}{2}$

$E_e = \boxed{62.5\text{ GeV} = 1.0\times10^{-8}\,J}$ either OK. (1pt)

c) $\gamma = \frac{E_e}{m_e c^2}$ (1pt) $= \frac{62.5\times10^{9}\,eV}{511\times10^{3}\,eV} = \boxed{1.22\times10^{5}}$ (1pt)

(1pt) number

d) $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ solve ~~for~~ β^2: $\beta^2 = 1 - \frac{1}{\gamma^2}$ (1pt)

$\beta^2 = 1 - \frac{1}{(1.22\times10^5)^2} = 1 - 6.72\times10^{-11}$ } (1pt

$1-\beta^2 = 6.72\times10^{-11}$

$(1-\beta)(1+\beta) = 6.72\times10^{-11}$ but $1+\beta \doteq 2$ to high accuracy

$1-\beta \doteq \frac{6.72\times10^{-11}}{2} = 3.36\times10^{-11}$

(1pt)

$c - v = c(1-\beta) = 3.0\times10^{8}\,\frac{m}{s}\cdot 3.36\times10^{-11} = \boxed{0.010\,\frac{m}{s}}$

11/14

Final exam constants:

$\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{Nm^2}$

$\mu_0 = 4\pi \times 10^{-7}\ Wb/A \cdot m$

$e = 1.6 \times 10^{-19}\ C$

$c = 3.00 \times 10^8\ m/s$

Final exam equations:

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, $\overrightarrow{dF} = I\overrightarrow{dl} \times \vec{B}$

$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$

$R = {mv}/{|q|B}$

$\vec{\mu} \equiv I\vec{A}$, $\vec{\tau} = \vec{\mu} \times \vec{B}$, $U = -\vec{\mu} \cdot \vec{B}$

$\vec{B} = \frac{\mu_0 q\vec{v} \times \hat{r}}{4\pi r^2}$, $\overrightarrow{dB} = \frac{\mu_0 I\overrightarrow{dl} \times \hat{r}}{4\pi r^2}$

$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot \overrightarrow{dA}$

$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$, $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

$B = \frac{\mu_0 NI}{2\pi r}$, $B = \mu_0 nI$, $n \equiv N/L$

$\mu = K_m \cdot \mu_0$, $\chi_m = K_m - 1$

$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B} \cdot \overrightarrow{dA}$

$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \overrightarrow{dl}$

$\mathcal{E} = NBA\omega \cdot sin\omega t$

$\mathcal{E} = -L\frac{di}{dt}$, $L = \frac{N\Phi_B}{i}$

$U = \frac{1}{2}LI^2$, $u \equiv \frac{U}{Vol.} = \frac{B^2}{2\mu_0}$

$\tau = \frac{L}{R}$

$\omega = \sqrt{\frac{1}{LC}}$, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and $\tau = \frac{2L}{R}$

$V_{rms} = \frac{V_0}{\sqrt{2}}$, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\langle P \rangle = \frac{V_{rms}^2}{R} = I_{rms}^2 R = I_{rms}V_{rms}$ (resistive)

$V_R = IR, V_L = IX_L, V_C = IX_C$, $X_L = \omega L$, $X_C = {1}/{\omega C}$

$V = IZ$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R}$, $cos\phi = \frac{R}{Z}$

$\langle P \rangle = I_{rms}V_{rms}cos\phi$

$\frac{I_1}{I_2} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

$k\lambda = 2\pi$, $v = f\lambda = \omega/k$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, $B = E/c$

$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$, $I = S_{av} = \frac{1}{2}\epsilon_0 c E_{max}^2$

$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c}$, $p_{rad} = \frac{(1-2)I}{c}$

$\frac{c}{v} \equiv n = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}}$, $\lambda = \frac{\lambda_0}{n}$

$\theta_r = \theta_a$ or $n_a sin\theta_a = n_b sin\theta_b$

$I = I_{max}\cos^2\phi$

$tan\theta_p = \frac{n_b}{n_a}$

$I_{scat} \propto \lambda^{-4}$

$P \equiv \frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$M \equiv \frac{y'}{y} = -\frac{s'}{s}$

Or $M \equiv \frac{\theta'}{\theta} = \frac{25\ cm}{f}$ or $M = \frac{-f_1}{f_2}$ or $M = \frac{-25\ cm \cdot s_1'}{f_1 f_2}$

f-number $\equiv \frac{f}{D}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$

$d\sin\theta = \begin{Bmatrix} m \\ m + \frac{1}{2} \end{Bmatrix}\lambda$

$I = I_0\cos^2(\phi/2)$

$2t = \begin{Bmatrix} m \\ m + \frac{1}{2} \end{Bmatrix}\lambda$, $\lambda = \frac{\lambda_0}{n}$

$\sin\theta = m \cdot \frac{\lambda}{a}$ or $\sin\theta = m \cdot \frac{\lambda}{d}$ or $\sin\theta = 1.22 \cdot \frac{\lambda}{a}$

$\beta \equiv \frac{v}{c}$, $\gamma \equiv (1 - \beta^2)^{-1/2}$

$x' = \gamma(x - ut)$, $y' = y$, $z' = z$, $t' = \gamma\left(t - \frac{ux}{c^2}\right)$

$v_x = \frac{v_x' + u}{1 + uv_x'/c^2}$

$f = \sqrt{\frac{c+u}{c-u}}\, f_0$

$\vec{p} = \gamma m\vec{v}$

$E = K + mc^2 = \gamma mc^2$, $E^2 = (mc^2)^2 + (pc)^2$