Solutions

Name:______________________________

Student ID #:______________________________

Signature:______________________________

Physics 1C Final Exam

- By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
- This exam contains eight workout problems, each 10 points, for a total of 80 points. Remember to write down each step of your calculation, and explain your answers. You have 2:50 to complete this exam.
- Close your exam when time is up, and show your student ID when handing it in.
- Detailed exam rules:
    - By signing above, you agree to the statement below "Academic Integrity – A Bruin's Code of Conduct".
    - You can use any type of calculator that does not have internet capability. Silence and put away your cell phones, tablets, and laptops.
    - Quote numerical answers with 3 significant figures, e.g. 0.262 or 3.72×10^3. Always specify the units, and quote final answers in SI units unless otherwise directed.
    - The last page of the exam is an equation sheet that may be torn off.
    - Try to fit all relevant calculations on the front of the pages, but if you run out of room on the page, use the back of the page and indicate "Problem <n> continued:" to help us in grading.
- If you have questions during the exam, raise your hand. If you are not seated near the end of a row, you may need to come to the aisle or down to the front of the room to ask them.

Academic Integrity - A Bruin's Code of Conduct:

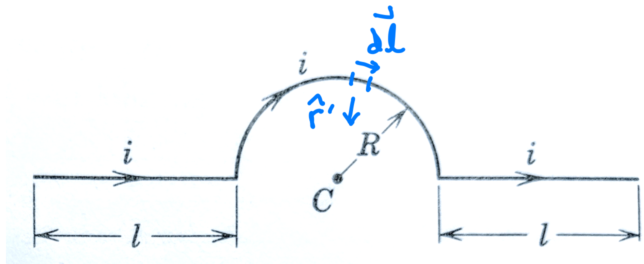
As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behavior includes cheating, fabrication or falsification, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, facilitating academic misconduct, coercion regarding grading or evaluation of coursework, or collaboration not authorized by the instructor. Please review our campus' policy on academic integrity in the UCLA Student Conduct Code: https://deanofstudents.ucla.edu/individual-student-code .

If you engage in these types of unacceptable behaviors in our course, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam. These allegations will be referred to the Office of the Dean of Students and can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

By submitting my assignments and exams for grading in this course, I acknowledge the above-mentioned terms of the UCLA Student Code of Conduct, declare that my work will be solely my own, and that I will not communicate with anyone other than the instructor and proctors in any way during the exams.

Problem 1 (10 pts): The wire shown below carries current *I*. What is the magnetic field ***B*** (magnitude and direction) at the center C of the semicircle arising from:

a) (3 pts) each straight segment of length *l*
b) (5 pts) the semicircular segment of radius *R*, and
c) (2 pts) the entire wire.



Biot - Savart:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl}\times\hat{r}'}{r'^2}$$

where $\vec{r}'$ is the vector from current segment $I\vec{dl}$ to the point $\vec{r}$ where $\vec{B}$ is calculated.

a) On the left, $\vec{dl} \rightarrow$ and $\vec{r}' \rightarrow$ are parallel.
On the right, $\vec{dl} \rightarrow$ and $\vec{r}' \leftarrow$ are antiparallel.
In both cases, $\vec{dl}\times\vec{r}' = 0$, so the integrals $= 0$, and $\boxed{\vec{B} = 0 \text{ from the straight wires.}}$

b) In the semicircle, $\vec{dl}\times\hat{r}' = (R\,d\theta)(-\hat{k})$ (into page)

$$\vec{B} = -\hat{k}\cdot\frac{\mu_0}{4\pi} I\int\frac{R\,d\theta}{R^2} = -\hat{k}\cdot\frac{\mu_0 I}{4\pi R}\int_0^{\pi} d\theta = \boxed{-\hat{k}\,\frac{\mu_0 I}{4R}\text{ (into page)}}$$

Alternatively, the answer for a whole circle is known as $B = \frac{\mu_0 I}{2R}$, and since the integrand is constant, half a circle gives half of that value, $B = \frac{\mu_0 I}{4R}$, and the direction into the page can be deduced from a r.h. rule with fingers curling in direction of I, thumb gives B direction.

c) Just add the contributions, so

$$\boxed{B = \frac{\mu_0 I}{4R}\text{ into the page}}$$

Problem 2 (10 pts): A rectangular loop of wire with resistance R is to the right of a long, straight wire carrying a constant current I, and is pulled at a constant velocity v away from it, as shown below.

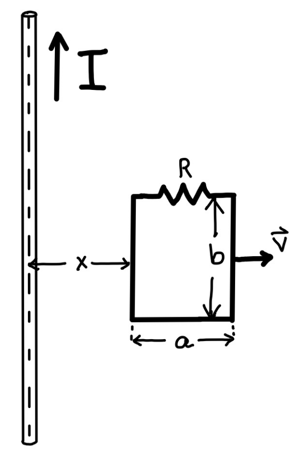
b) (1 pts) What is the magnitude of the magnetic field at an arbitrary distance r from the wire? (If you remember the formula, you don't have to re-derive this.)

a) (1 pts) What is the direction of the magnetic field inside the loop of wire?

c) (3 pts) What is the magnetic flux Φ_B in the loop of wire due to the current in the straight wire, as a function of the distance x between the near side of the circuit and the straight wire (as shown below)?

d) (1 pts) What is the direction of current I' induced in the loop of wire, clockwise, or counterclockwise?

e) (4 pts) What is the magnitude of the current I' induced in the loop of wire? Express your final answer, *not* as a function of time, but as a function of position $x = vt$?



a) Into the page, by r.h. rule (thumb in direction of I, fingers curl in direction of B)

b) $\boxed{B = \frac{\mu_0 I}{2\pi r}}$

c) $$\Phi_B = \iint \vec{B}\cdot\hat{n}\, dA = \int_x^{x+a} dr \int_{z_0}^{z_0+b} dz\, \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \frac{\mu_0 I}{2\pi}\int_x^{x+a} dr\, \frac{1}{r}\underbrace{\int_{z_0}^{z_0+b} dz}_{b} = \frac{\mu_0 I b}{2\pi}\int_x^{x+a}\frac{dr}{r}$$

$$\boxed{\Phi_B = \frac{\mu_0 I b}{2\pi}\left[\ln(x+a) - \ln x\right]}$$

d) Lenz's Law: Φ_B decreasing, so I' in direction that tends to oppose this, i.e. increase B into page, which is $\boxed{\text{clockwise}}$ by r.h. rule.

e) $$|I'| = \frac{|\mathcal{E}|}{R} \quad \text{where} \quad \mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\mu_0 I b}{2\pi}\cdot\frac{\partial}{\partial t}\left[\ln(vt+a) - \ln(vt)\right]$$

$$\mathcal{E} = -\frac{\mu_0 I b}{2\pi}\cdot\left[\frac{v}{vt+a} - \frac{v}{vt}\right] = \frac{\mu_0 I b v}{2\pi}\left[\frac{1}{x} - \frac{1}{x+a}\right]$$

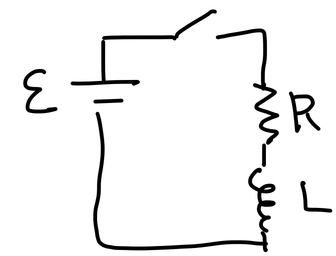
$$\boxed{|I'| = \frac{\mu_0 I b v}{2\pi R}\left[\frac{1}{x} - \frac{1}{x+a}\right]}$$

(This just happens to be positive as is.)

Problem 3 (10 pts): An inductor with an inductance of 2.50 H and a resistance of 5.4 Ω is connected by a switch at time $t = 0$ to the terminals of a car battery with a DC emf of 12.5 V and negligible internal resistance.

a) (3 pts) What is the initial rate of increase of current in the circuit?
b) (3 pts) What is the rate of increase of current at the instant when the current is 0.500 A ?
order d) (2 pts) What is the current 0.463 s after the circuit is closed?
c) (2 pts) What is the steady-state current a long time after the circuit is closed?

Sketch:



a) At $t=0$ current has not built up, due to inductance, so zero volts across resistor, all EMF across inductor.

$$|\mathcal{E}| = L\frac{dI}{dt} \quad \text{so} \quad \boxed{\frac{dI}{dt} = \frac{|\mathcal{E}|}{L} = \frac{12.5\,V}{2.50\,H} = 5.0\ A/s}$$

b) Then $V_R = IR = 0.500\,A \cdot 5.4\,\Omega = 2.70\,V$, leaving

$V_L = \mathcal{E} - V_R = 12.5 - 2.7\,V = 9.8\,V$, and

$$\frac{dI}{dt} = \frac{V_L}{L} = \frac{9.8\,V}{2.50\,H} = \boxed{3.92\ A/s}$$

c) Steady-state current $\Rightarrow$ no voltage $V_L = 0$.
So all $\mathcal{E}$ across resistor

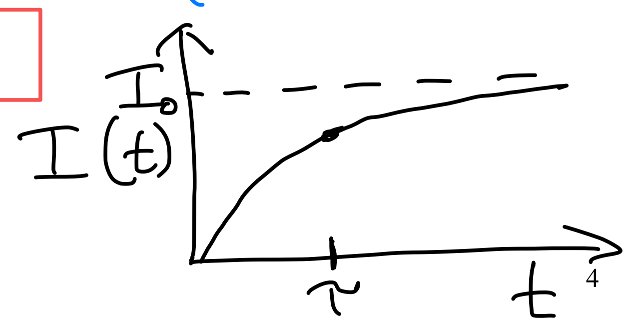
$$I(t=\infty) = \frac{V}{R} = \frac{12.5\,V}{5.40\,\Omega} = 2.31\,A$$

d) Note eqn. $\tau = L/R$ time constant of LR circuit.

$$\tau = \frac{2.50\,H}{5.4\,\Omega} = 0.463\,s$$

Current builds up as $I = I_0\left(1 - e^{-t/\tau}\right)$
so here $t = \tau$ and $I = I_0(1 - e^{-1}) = 0.632\,I_0$

$$I = 0.632 \cdot 2.31\,A = \boxed{1.46\,A}$$



Problem 4 (10 pts): A source of electromagnetic waves radiates uniformly in all directions. At a distance of 80 m from this source, the RMS amplitude of the electric field is measured to be 2.0 N/C.

a) (3 pts) What is the intensity of the source at distance 80 m?
b) (3 pts) What is the total power radiated by the source?
c) (4 pts) What is the RMS electric field amplitude at a distance 10.0 m from the source?

a) $I = \frac{1}{2}\epsilon_0 c E_{max}^2$ but $E_{rms} = \frac{E_{max}}{\sqrt{2}}$ so

** $I = \epsilon_0 c E_{rms}^2 = 8.85\times10^{-12}\cdot 3\cdot10^8\cdot(2.0)^2 = \boxed{1.06\times10^{-2}\ W/m^2}$

b) $P = I\cdot A$, here A is area of a sphere of radius 80 m, $A = 4\pi r^2$, so * $P = I\cdot 4\pi r^2$

$P = 1.06\times10^{-2}\ W/m^2 \cdot \underbrace{4\pi(80m)^2}_{8.04\times10^4 m^2} = \boxed{852\ W}$

c) Method I, brute force: invert eqn. * to get I at 10.0 m, then invert eqn. ** to get E_{rms}:

$*:\ I = \frac{P}{4\pi r^2} = \frac{852\ W}{4\pi(10.0)^2} = 0.678\ W/m^2$

$**:\ E_{rms} = \sqrt{\frac{I}{\epsilon_0 c}} = \sqrt{\frac{0.678}{8.85\times10^{-12}\cdot 3\cdot10^8}}\ N/C = \boxed{16.0\ N/C}$

Method II, ratios:

$E \propto \sqrt{I} \propto \sqrt{\frac{1}{Area}} \propto \sqrt{\frac{1}{r^2}} = \frac{1}{r}$ so $E \propto \frac{1}{r}$

"proportional to"

So $E_{10} = E_{80}\cdot\frac{80m}{10m} = 2.0\frac{N}{C}\times 8 = \boxed{16.0\ N/C}$

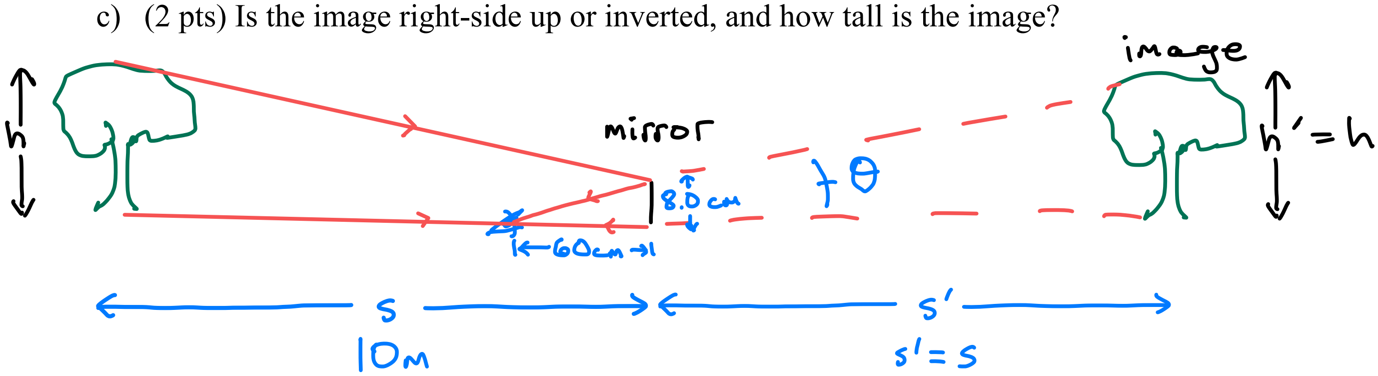
A little more written-out:

$$\frac{E(r=10)}{E(r=80)} = \sqrt{\frac{I(r=10)}{I(r=80)}} = \sqrt{\frac{1/Area(10)}{1/Area(80)}} = \sqrt{\frac{\cancel{4\pi}(80m)^2}{\cancel{4\pi}(10m)^2}} = 8$$

$E(r=10) = 8\cdot E(r=80) = 8\cdot 2.0\ N/C = \boxed{16.0\ N/C}$

Problem 5 (10 pts): The image of a tree just covers the length of a plane mirror 8.0 cm tall when the mirror is held upright at arm's length 60 cm from the eye. The tree is 10 m from the mirror.

a) (2 pts) Where is the image located?
b) (6 pts) What is the tree's height?
c) (2 pts) Is the image right-side up or inverted, and how tall is the image?



a) Plane mirrors: image behind mirror equal distance, i.e. $\boxed{\text{10 m behind the mirror}}$.

Formally, $f = \frac{R}{2}$ but $R = \infty$ so $f = \infty$, $\frac{1}{f} = 0 = \frac{1}{s} + \frac{1}{s'}$ so $s' = -s$.

b) For similar triangles, ratios of side lengths are the same. So comparing

$$\tan\theta = \frac{8.0\text{ cm}}{60\text{ cm}} = \frac{h'}{s' + 0.6\text{ m}} = \frac{h}{10.6\text{ m}}$$

$$h = 10.6\text{ m} \times \frac{8}{60} = \boxed{1.41\text{ m}}$$

c) $\boxed{\text{Right-side up, } h' = h = 1.41\text{ m}}$

Formally,

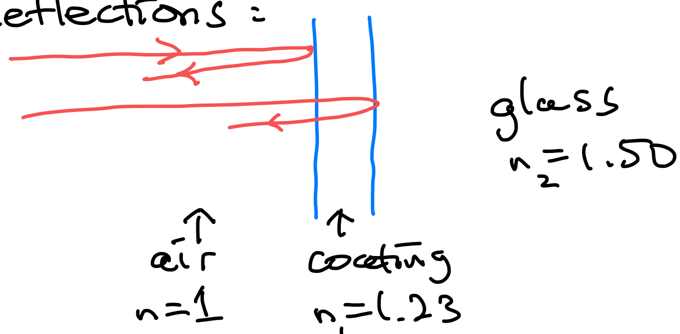
$M = \frac{-s'}{s} = \frac{-(-s)}{s} = 1$, which means the image height h' is the same as h, and being positive, right-side up.

Problem 6 (10 pts): An anti-reflection coating with refractive index $n_1 = 1.23$ is deposited on glass having refractive index $n_2 = 1.50$. The coating is designed to minimize reflections at a wavelength $\lambda_0 = 500\ nm$ in the middle of the visible spectrum.

a) (2 pts) What is the wavelength of this light λ, in nm units, inside the anti-reflection coating?
b) (5 pts) To minimize reflections at 500 nm, what is the minimum thickness t (in nm) of the coating that will do the job?
c) (3 pts) There are various thicknesses of anti-reflection coating that can do the job. In words, describe a physics reason why the thinnest antireflection coating is best.

a) $\lambda = \frac{\lambda_0}{n_1} = \frac{500\text{nm}}{1.23} = \boxed{406.5\text{nm}}$ ← in a.r. coating

b) Reflections:



At both surfaces, interface to higher n.
∴ Phase change just due to reflection is the same, and phase difference just due to path length in coating, 2t. For destructive reflection interference,

$2t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots \left(\frac{1}{2}+m\right)\lambda$

The thinnest is $m=0$ where

$2t = \frac{\lambda}{2}$ or $t = \frac{\lambda}{4} = \frac{406.5\text{nm}}{4} = \boxed{101.4\text{nm}}$

c) Thinnest is best because thicker ones will have more variation of phase between 2 reflections
i) for angles not head-on ($\theta \neq 0$)
ii) for different λ (different colors)
... and then won't cancel reflections as well.

Either of these answers is OK for full credit.

Problem 7 (10 pts): When laser light of wavelength 632.8 nm passes through a diffraction grating, the first bright spots occur at $\pm 22.0°$ from the central maximum.

a) (3 pts) What is the spacing between the slits (also called rulings or lines) of the diffraction grating?
b) (1 pts) What is the number of lines per mm of the diffraction grating?
c) (3 pts) How many bright spots are there in total?
d) (3 pts) At what angles do all of the bright spots occur?

a) Use $d \sin\theta = m\lambda$ for bright spots

$$d = \frac{m\lambda}{\sin\theta} = \frac{1 \cdot 632.8\,\text{nm}}{\sin(22°)} = \boxed{1690\,\text{nm} = 1.69\times10^{-3}\,\text{mm} = 1.69\times10^{-6}\,\text{m}}$$

all OK answers

b) In a certain length L, say 1 mm, we can write $L = N \cdot d$ since they are closely spaced, so

$$n = \frac{N}{L} = \frac{1}{d} = \frac{1}{1.69\times10^{-3}\,\text{mm}} = \boxed{592/\text{mm}}$$

c) Consider other m:

$$\sin\theta = \frac{m\lambda}{d} = m \cdot \frac{632.8\,\text{nm}}{1690\,\text{nm}} = m \cdot 0.374$$

OK choices are $m = 0$, $m = \pm 1$, $m = \pm 2$.

For $m = \pm 3$ or greater, $|\sin\theta| > 1$ so impossible.

There are $\boxed{\text{5 bright spots}}$ (3 if mistake of not considering negative m).

d) $m = 0 \Rightarrow \boxed{\theta = 0}$

$m = \pm 1 \Rightarrow \boxed{\theta = \pm 22.0°}$ as stated

$m = \pm 2 \Rightarrow \sin\theta = \pm 2 \cdot 0.374 = \pm 0.749$

$$\boxed{\theta = \pm 48.5°}$$

Problem 8 (10 pts): All K^0 particles have mass of $m_{K^0} = 8.85 \times 10^{-28}$ kg. The K^0 particle is unstable and decays to a pair of π^0 particles (this reaction is denoted $K^0 \to \pi^0\pi^0$). If the K^0 is at rest when it decays, the kinetic energy of each π^0 particle is the same $K = 1.82 \times 10^{-11}$ Joules. [Hint: use only relativistic equations in this problem.]

a) (2 pts) What is the rest energy of the K^0 particle?
b) (3 pts) What is the total energy of each π^0 particle?
c) (3 pts) What is the mass of the π^0 particle?
d) (2 pts) What are the rest energies of the K^0 and π^0 particles, expressed in MeV (millions of electron-volts)?

a) $E_0 = mc^2 = (8.85 \times 10^{-28}\,\text{kg})(3 \times 10^8\,\text{m/s})^2 = \boxed{7.96 \times 10^{-11}\,\text{J}}$

b) Energy conservation in $K^0 \to \pi^0\pi^0$:

$E_0 = 2E_{\pi^0} \Rightarrow E_{\pi^0} = \frac{E_0}{2} = \boxed{3.98 \times 10^{-11}\,\text{J}}$

c) Use $E_{\pi^0} = E_0(\pi^0) + K$ so

$E_0(\pi^0) = E_{\pi^0} - K = (3.98 - 1.82) \times 10^{-11}\,\text{J} = 2.16 \times 10^{-11}\,\text{J}$

$m_{\pi^0} = \frac{E_0(\pi^0)}{c^2} = \frac{2.16 \times 10^{-11}\,\text{J}}{(3 \times 10^8\,\text{m/s})^2} = \boxed{2.40 \times 10^{-28}\,\text{kg}}$

d) Conversion:

$1\,\text{MeV} = 1.0 \times 10^6\,\text{eV} = (1.0 \times 10^6\,\text{eV})(1.6 \times 10^{-19}\,\text{J/eV}) = 1.6 \times 10^{-13}\,\text{J/MeV}$

$E_0(K^0) = 7.96 \times 10^{-11}\,\text{J} \div 1.6 \times 10^{-13}\,\text{J/MeV} = \boxed{497\,\text{MeV}}$

$E_0(\pi^0) = 2.16 \times 10^{-11}\,\text{J} \div 1.6 \times 10^{-13}\,\text{J/MeV} = \boxed{135\,\text{MeV}}$

(additional calculation paper)

Final exam constants:

$\epsilon_0 = 8.854 \times 10^{-12}\, \frac{C^2}{Nm^2}$

$\mu_0 = 4\pi \times 10^{-7}\ Wb/A \cdot m$

$e = 1.6 \times 10^{-19}\ C$

$c = 3.00 \times 10^8\ m/s$

Final exam equations:

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, $\overrightarrow{dF} = I\overrightarrow{dl} \times \vec{B}$

$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$

$R = {mv}/{|q|B}$

$\vec{\mu} \equiv I\vec{A}$, $\vec{\tau} = \vec{\mu} \times \vec{B}$, $U = -\vec{\mu} \cdot \vec{B}$

$\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$, $\overrightarrow{dB} = \frac{\mu_0 I \overrightarrow{dl} \times \hat{r}}{4\pi r^2}$

$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot \overrightarrow{dA}$

$\vec{B} = \frac{\mu_0 I}{2\pi r}\, \hat{\phi}$, $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

$B = \frac{\mu_0 N I}{2\pi r}$, $B = \mu_0 n I$, $n \equiv N/L$

$\mu = K_m \cdot \mu_0$, $\chi_m = K_m - 1$

$\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\iint \vec{B} \cdot \overrightarrow{dA}$

$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \overrightarrow{dl}$

$\mathcal{E} = NBA\omega \cdot \sin\omega t$

$\mathcal{E} = -L\frac{di}{dt}$, $L = \frac{N\Phi_B}{i}$

$U = \frac{1}{2}LI^2$, $u \equiv \frac{U}{Vol.} = \frac{B^2}{2\mu_0}$

$\tau = \frac{L}{R}$

$\omega = \sqrt{\frac{1}{LC}}$, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and $\tau = \frac{2L}{R}$

$V_{rms} = \frac{V_0}{\sqrt{2}}$, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\langle P \rangle = \frac{V_{rms}^2}{R} = I_{rms}^2 R = I_{rms} V_{rms}$ (resistive)

$V_R = IR, V_L = IX_L, V_C = IX_C$, $X_L = \omega L$, $X_C = {1}/{\omega C}$

$V = IZ$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R}$, $\cos\phi = \frac{R}{Z}$

$\langle P \rangle = I_{rms} V_{rms} \cos\phi$

$\frac{I_1}{I_2} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

$k\lambda = 2\pi$, $v = f\lambda = \omega/k$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, $B = E/c$

$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$, $I = S_{av} = \frac{1}{2}\epsilon_0 c E_{max}^2$

$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c}$, $p_{rad} = \frac{(1-2)I}{c}$

$\frac{c}{v} \equiv n = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0 \mu_0}}$, $\lambda = \frac{\lambda_0}{n}$

$\theta_r = \theta_a$ or $n_a \sin\theta_a = n_b \sin\theta_b$

$I = I_{max} \cos^2 \phi$

$\tan\theta_p = \frac{n_b}{n_a}$

$I_{scat} \propto \lambda^{-4}$

$P \equiv \frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$M \equiv \frac{y'}{y} = -\frac{s'}{s}$

Or $M \equiv \frac{\theta'}{\theta} = \frac{25\ cm}{f}$ or $M = \frac{-f_1}{f_2}$ or $M = \frac{-25\ cm \cdot s_1'}{f_1 f_2}$

f-number $\equiv \frac{f}{D}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$

$d\sin\theta = \begin{Bmatrix} m \\ m + \frac{1}{2} \end{Bmatrix} \lambda$

$I = I_0 \cos^2(\phi/2)$

$2t = \begin{Bmatrix} m \\ m + \frac{1}{2} \end{Bmatrix} \lambda$, $\lambda = \frac{\lambda_0}{n}$

$\sin\theta = m \cdot \frac{\lambda}{a}$ or $\sin\theta = m \cdot \frac{\lambda}{d}$ or $\sin\theta = 1.22 \cdot \frac{\lambda}{a}$

$\beta \equiv \frac{v}{c}$, $\gamma \equiv (1 - \beta^2)^{-1/2}$

$x' = \gamma(x - ut)$, $y' = y$, $z' = z$, $t' = \gamma\left(t - \frac{ux}{c^2}\right)$

$v_x = \frac{v_x' + u}{1 + u v_x'/c^2}$

$f = \sqrt{\frac{c+u}{c-u}}\, f_0$

$\vec{p} = \gamma m \vec{v}$

$E = K + mc^2 = \gamma mc^2$, $E^2 = (mc^2)^2 + (pc)^2$