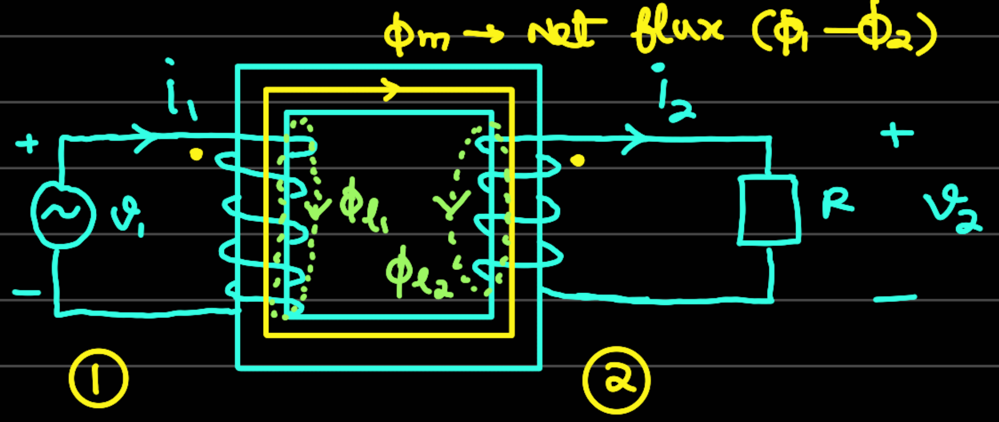
Day-18

→ **Transformer:**



$$v_1 = i_1 R_1 + \frac{d\psi_1}{dt}$$

($R_1 \to$ Resistance of primary winding)

Assumption → lump model

$$\psi_1 = N_1 \phi_1 = N_1(\phi_1 + \phi_{l_1})$$

$\phi_{l_1} \to$ leakage flux (winding 1)

$\phi_{l_2} \to$ leakage flux (winding 2)

$N_1 \phi_{l_1} \to$ Assuming ϕ_{l_1} passes all the windings, individual flux leakage for particular turns is neglected

$$\frac{d\psi_2}{dt} = i_2 R_2 + v_2$$

$$\psi_2 = N_2 \phi_2 = N_2(\phi_m - \phi_{l_2})$$

$$v_1 = i_1 R_1 + N_1 \frac{d\phi_{l_1}}{dt} + N_1 \frac{d\phi_m}{dt}$$

Applying Ampere's circuital law (along path of ϕ_{l_1})

$$\oint \vec{H}\cdot\vec{dl} = N_1 i_1$$ (LHS can't be simplified due to non-uniform path)

$$\Rightarrow \phi_{l_1}\cdot \mathcal{R}_{l_1} = N_1 i_1$$

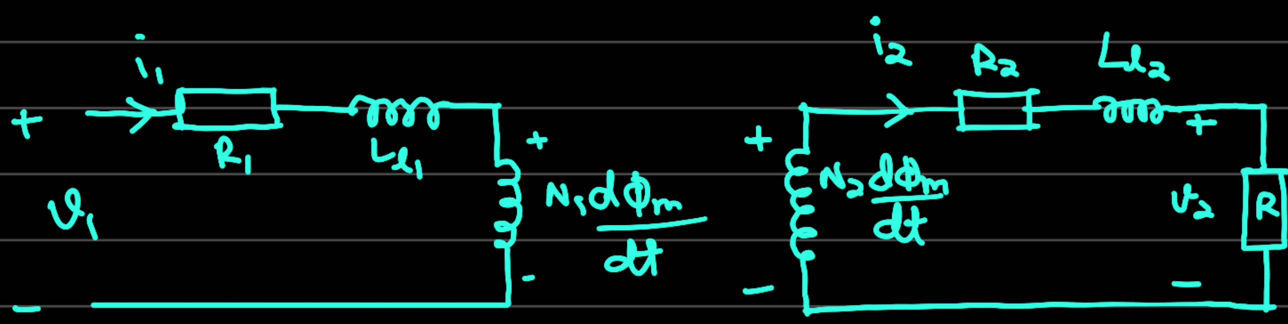
$\hookrightarrow$ equivalent reluctance for leakage flux ϕ_{l_1}

So $$v_1 = i_1 R_1 + \frac{d}{dt}\left(\frac{N_1 i_1}{\mathcal{R}_{l_1}}\right) + N_1 \frac{d\phi_m}{dt}$$

$\mathcal{R}_{l_1} \rightarrow$ constant (most of it is air)

$$v_1 = i_1 R_1 + \underbrace{\frac{N_1^2}{\mathcal{R}_{l_1}}}_{L_{l_1}} \frac{di_1}{dt} + N_1 \frac{d\phi_m}{dt}$$

$$\Rightarrow v_1 = i_1 R_1 + L_{l_1}\frac{di_1}{dt} + N_1\frac{d\phi_m}{dt}$$



If 2ndary winding is wound on primary winding, leakage flux $\rightarrow 0$

Apply ACL along path of ϕ_m.

$$\oint \vec{H}\cdot\vec{dl} = N_1 i_1 - N_2 i_2$$

$$\Rightarrow H_c l_c = N_1 i_1 - N_2 i_2$$

$\hookrightarrow$ Assumed uniform — all confined to core

$$\Rightarrow \frac{B_c l_c}{\mu_0 \mu_r} = N_1 i_1 - N_2 i_2$$

Assumed $\mu_r \rightarrow$ constant
B vs H $\rightarrow$ linear $\rightarrow$ No core loss
No saturation

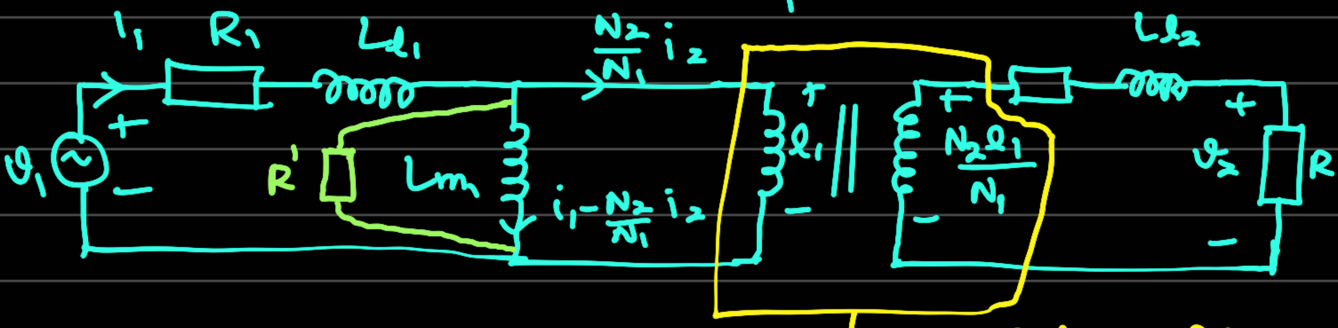
$$\Rightarrow \frac{\phi_m l_c}{\mu_0 \mu_r A_c} = N_1 i_1 - N_2 i_2$$

$$\Rightarrow \phi_m \mathcal{R}_c = N_1 i_1 - N_2 i_2$$

[For ideal transformer, $\mu_r \rightarrow \infty$
$\Rightarrow \mathcal{R}_c \rightarrow 0$]

$$v_1 = i_1 R_1 + L_{l_1} \frac{di_1}{dt} + N_1 \frac{d}{dt}\left(\frac{N_1 i_1 - N_2 i_2}{\mathcal{R}_c}\right)$$

$$= i_1 R_1 + L_{l_1} \frac{di_1}{dt} + \underbrace{\frac{N_1^2}{\mathcal{R}_c}}_{L_{m_1}} \frac{d}{dt} \underbrace{\left(i_1 - \frac{N_2}{N_1} i_2\right)}_{\text{magnetising current}}$$



2ndary winding

$$-N_2 \frac{d\phi_{l_2}}{dt} + N_2 \frac{d\phi_m}{dt} = i_2 R_2 + v_2$$

$$\Rightarrow N_2 \frac{d\phi_m}{dt} = i_2 R_2 + L_{l_2} \frac{di_2}{dt} + v_2$$

$R' \rightarrow$ for core loss