

## Day - 15

→ Alternative crystal structure description —

Atoms tend to arrange in close-packed configurations

→ hard spheres

close-packed structures :  $\geq 6$  nearest neighbours

↓  
Simplest eg: Simple Cubic (6)

BCC  $\rightarrow 8$

FCC  $\rightarrow 12$

HCP  $\rightarrow 12$

→ Atomic Packing Fraction (APF) =  $\frac{V_{\text{atoms}}}{V_{\text{unit cell}}}$

↓  
only for hard sphere model.

$$\text{for BCC : } V_{\text{atoms}} = 2 \times \frac{4}{3} \pi r^3 = \frac{8\pi r^3}{3}$$

$$V_{\text{uc}} = a^3$$

$$\text{so APF} = \frac{8\pi}{3} \left(\frac{r}{a}\right)^3$$

$$\text{But } 4r = \sqrt{3}a \Rightarrow \frac{r}{a} = \frac{\sqrt{3}}{4}$$

$$\text{so APF} = \frac{8\pi}{3} \times \frac{3\sqrt{3}}{64}$$

$$= \frac{\sqrt{3}\pi}{8}$$

$$\approx 68\%$$

for FCC:  $V_{atoms} = 4 \times \frac{4}{3} \pi r^3$

$$= \frac{16 \pi r^3}{3}$$

$$APF = \frac{16 \pi}{3} \left( \frac{r}{a} \right)^3$$

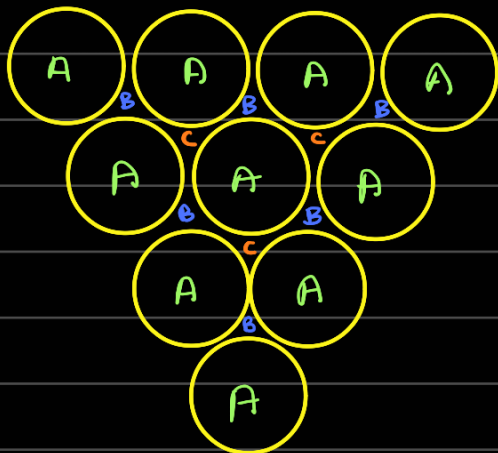
But  $4r = \sqrt{2}a \Rightarrow \frac{r}{a} = \frac{1}{2\sqrt{2}}$

$$= \frac{16 \pi}{3} \times \frac{1}{16\sqrt{2}} = \frac{\pi}{3\sqrt{2}} \approx 74\%$$



Highest achievable  
APF  
(Kepler conjecture)

→ Closed Packed plane



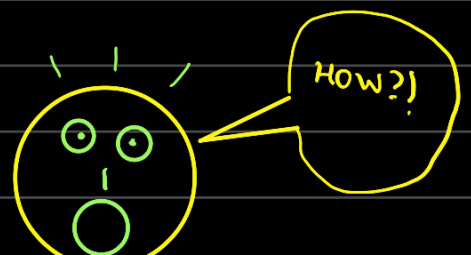
A-plane  
(arbitrarily labelled)

We can choose either B or C

Choose B, then stack on C. It can be repeated again (ABCABC....)

We can have ABAB....

OR ABC ABCABC also!



But nature chooses — ABC — (CCP)  
— AB — (HCP)

(001) plane (HP) with 2 atoms/u.c.

(111) plane of FCC

☆ Even though you can close-pack spheres in 3D, a max. of 74% of u.c. can be occupied.

26% lost space  $\rightarrow$  putting different-sized atoms

Locations where the centres of these different sized atoms can be arranged in a regular fashion  $\rightarrow$  interstitial positions

$\downarrow$   
2 types: I finds itself

no. of lattice atoms (L)  
coordinated to it

4  $\rightarrow$  tetrahedral interstice

6  $\rightarrow$  octahedral interstice

No. of interstitial position in one u.c. or per lattice atom.

Octahedral:

1 Body centre  $\rightarrow$  1

12 edge centres  $\rightarrow 12 \times 1/4 = 3$

$1+3 = 4$  octahedral sites  $\rightarrow 1:1$   
(4 lattice sites)

8 tetrahedral sites  $\rightarrow 2:1$

hcp: no. of lattice atoms = 2

Octahedral  $\rightarrow 2$

Tetrahedral  $\rightarrow 4$

BCC  $\rightarrow$  no regular tetrahedral sites.

