Assignment-2

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1.) Net heat power provided to water $= (1 - 0.16)$ kW
$= 0.84$ kW

Consider density of water $= 1\ kg\ L^{-1}$

As $V_{water} = 2L$

$\Rightarrow m_{water} = 2L \times 1\ kg\ L^{-1} = 2\ kg$

We have, $\frac{dH}{dt} = ms\frac{dT}{dt}$

$\Rightarrow \frac{dT}{dt} = \frac{0.84\ kJ\ s^{-1}}{2kg \times 4.2 kJ\ kg^{-1} K^{-1}}$

$= 0.1\ K\ s^{-1}$

$\Rightarrow \int_0^t dt = \int_{27+273}^{77+273} 10\, dT$

$\Rightarrow \boxed{t = 10 \times 50\ s = 500\ s}$

2.) Heat transfer = Energy consumed during charging − Energy lost during discharging

$\Rightarrow \Delta H = (0.53 \times 3600 - 250 \times 4.184)$ kJ

$\Rightarrow \boxed{\Delta H = 862\ kJ}$ [This is the heat lost by battery during charging]

3.) mass of water, $m = 9\ kg$

Specific heat capacity of water, $S = 4.184\ kJ\ kg^{-1}\ °C^{-1}$

From 1st law of thermodynamics,

$Q = \Delta U + W$

$$\Rightarrow \frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

$$\Rightarrow -K_1 T = mS\frac{dT}{dt} - K_2 T$$

$$\Rightarrow \frac{dT}{dt} = \frac{(K_2 - K_1)}{mS} T$$

$$\Rightarrow \int_{T_0}^{T} \frac{dT}{T} = \int_{0}^{t} \frac{K_2 - K_1}{mS}\, dt$$

$$\Rightarrow \ln\left(\frac{T}{T_0}\right) = \frac{(K_2 - K_1)}{mS} t$$

$$\Rightarrow T = T_0\, e^{\frac{(K_2 - K_1)}{mS} t}$$

$$\Rightarrow \frac{dT}{dt} = \frac{(K_2 - K_1) T_0}{mS} e^{\frac{(K_2 - K_1) t}{mS}}$$

$$mS = 9 \times 4.184 \times 1000 \text{ J } ^\circ C^{-1}$$
$$= 37656 \text{ J } ^\circ C^{-1}$$

(considering K_1, K_2 in W $^\circ C^{-1}$)

So $$\boxed{\frac{dT}{dt} = \frac{(K_2 - K_1)}{37656} T_0\, e^{\frac{(K_2 - K_1) t}{37656}} \quad ^\circ C\, s^{-1}}$$

4.) mass flow rate of gas $= \dot{m}$ (kg s^{-1})

We have, $\dot{m} C_p \Delta T =$ Power produced + Heat loss rate

$$\Rightarrow \dot{m} \times 1 \times (1100 - 400) = 4600 - 300$$

$$\Rightarrow \boxed{\dot{m} = \frac{4300}{700} \text{ kg } s^{-1} \approx 6.143 \text{ kg } s^{-1}}$$

5.)a.) $\Delta H = \int_{t_1}^{t_2} C_p \, dt$ ($t \rightarrow$ temperature in °C)

$$= \int_0^{100} \left(2.093 + \frac{41.87}{t+100}\right) dt$$

$$= \left[2.093t + 41.87 \ln(t+100)\right]_0^{100}$$

$$= 209.3 + 41.87 \ln 2 \quad J$$

$$\Rightarrow \Delta H = 238.322 \text{ J}$$

b.) $\Delta U = \Delta H - W$

$$= \left[238.322 - 101325 \left(\frac{2400-2000}{10^6}\right)\right] J$$

$$\Rightarrow \boxed{\Delta U = 197.792 \text{ J}}$$

6.) a.) $1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$ $\left(\eta = 1 - \frac{T_{rej}}{T_{abs.}}\right)$

$$\Rightarrow \boxed{T_2 = \sqrt{T_1 T_3}}$$

b.) $W_{E_1} = W_{E_2}$

$$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow Q_2 = \frac{Q_1 + Q_3}{2}$$

Now, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ and $\frac{Q_3}{Q_2} = \frac{T_3}{T_2}$

So $\frac{Q_3}{Q_1} = \frac{Q_2}{Q_1} \times \frac{Q_3}{Q_2} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} = \frac{T_3}{T_1}$

In the equation $Q_2 = \frac{Q_1 + Q_3}{2}$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{1 + Q_3/Q_1}{2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{1 + T_3/T_1}{2}$$

$$\Rightarrow \boxed{T_2 = \frac{T_1 + T_3}{2}}$$

7.) $\eta_{max} = 1 - \frac{27+273}{627+273} = 1 - \frac{300}{900} = \frac{2}{3} \simeq 0.67$

$\eta_{actual} = \frac{50}{3 \times 75000/3600} = 0.8$

As $\eta_{actual} > \eta_{max}$ so such an engine is **not possible**

Since such an engine cannot exist, it is unnecessary to check whether it is economical or not.

8.) $\eta_{carnot} = 1 - \frac{T_{low}}{T_{high}}$

Clearly, η_{carnot} is independent of the working substance used.

$\therefore$ **All engines have the same efficiency**

9.) 1st law of thermodynamics : $Q = \Delta U + W$

Here the insulated storage tank is our system.

$\dot{Q} = \dot{\Delta U} + \dot{W}$

$\Rightarrow 0 = \dot{m}(u_f - u_i) - \dot{m} h_i$

$\Rightarrow \boxed{h_i = u_f}$ (as $u_i = 0$; tank was empty)

10.) Let the area of plate $= A\ m^2$

So $\dfrac{1}{0.5A} = 0.2$

$\Rightarrow \boxed{A = 10\ m^2}$

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