MM 225 – 2024-25 – I

In class Tutorial 3 (22 October 2024)

Consider following logistic regression for i = 1, 2, 3, …, n:

$$P(y_i|w) = \psi_i^{y_i}(1-\psi_i)^{(1-y_i)}$$

where,

$$\psi_i = \frac{1}{1+e^{-wx_i}}$$

The steps to be taken to determine the formula to maximum likelihood estimator of w are as follows:

1. Deriving the likelihood function for the w

$$L(w) = \prod_{i=1}^{n} P(y_i|w) = \prod_{i}^{n} \psi_i^{y_i}(1-\psi_i)^{(1-y_i)}$$

2. Taking log of likelihood function:

$$\log(L(w)) = \sum_{i=1}^{n} y_i \log(\psi_i) + (1-y_i)\log(1-\psi_i)$$

3. Determine w that would maximize log(L(w)) by solving $\frac{dlog(L(w))}{dw} = 0$.

Question 1: Step iii find $\frac{dlog(L(w))}{dw}$.

Question 2:

The steps for Gradient Descent Algorithm for estimating the weights of the logistic regression, the cost function is given by

$$\varepsilon = -\sum_{i=1}^{n} y_i \log(\psi_i) + (1-y_i)\log(1-\psi_i)$$

The algorithm has following steps. Fill up the step # 3 and 4 below:

1. Choose 0.0001
2. Initial value for w be 0.02
3. $w_{k+1} = ----------$
4. Carry out two iterations for the data:

Time	78	72	70	56	66
Failure	0	0	1	1	0