Assignment-2

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1:) Net heat power provided to water $= (1-0.16)$ kW
$= 0.84$ kW

Consider density of water $= 1\ \text{kg}\ L^{-1}$

As $V_{water} = 2L$

$\Rightarrow m_{water} = 2L \times 1\ \text{kg}\ L^{-1} = 2$ kg

We have, $\frac{dH}{dt} = ms\frac{dT}{dt}$

$$\Rightarrow \frac{dT}{dt} = \frac{0.84 \quad \text{kJ s}^{-1}}{2\text{kg} \times 4.2\text{kJ kg}^{-1}\text{K}^{-1}}$$

$$= 0.1\ \text{K s}^{-1}$$

$$\Rightarrow \int_0^t dt = \int_{27+273}^{77+273} 10\, dT$$

$$\Rightarrow \boxed{t = 10 \times 50\text{s} = 500\text{s}}$$

2.) Heat transfer = Energy consumed during charging − Energy lost during discharging

$\Rightarrow \Delta H = (0.53 \times 3600 - 250 \times 4.184)$ kJ

$\Rightarrow \boxed{\Delta H = 862\ \text{kJ}}$ [This is the heat lost by battery during charging]

3.) mass of water, $m = 9$ kg

Specific heat capacity of water, $S = 4.184\ \text{kJ kg}^{-1}\,{}^{\circ}\text{C}^{-1}$

From 1st law of thermodynamics,

$Q = \Delta U + W$

$$\Rightarrow \frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

$$\Rightarrow -K_1 T = ms\frac{dT}{dt} - K_2 T$$

$$\Rightarrow \frac{dT}{dt} = \frac{(K_2-K_1)}{mS}T$$

$$\Rightarrow \int_{T_0}^{T} \frac{dT}{T} = \int_0^t \frac{K_2-K_1}{mS}\,dt$$

$$\Rightarrow \ln\left(\frac{T}{T_0}\right) = \frac{(K_2-K_1)}{mS}t$$

$$\Rightarrow T = T_0\, e^{\frac{(K_2-K_1)}{mS}t}$$

$$\Rightarrow \frac{dT}{dt} = \frac{(K_2-K_1)T_0}{mS}\, e^{\frac{(K_2-K_1)t}{mS}}$$

$$mS = 9 \times 4.184 \times 1000 \text{ J }°C^{-1}$$
$$= 37656 \text{ J }°C^{-1}$$

(considering K_1, K_2 in W °C^{-1})

So $\boxed{\frac{dT}{dt} = \frac{(K_2-K_1)}{37656} T_0\, e^{\frac{(K_2-K_1)t}{37656}} \; °C\,s^{-1}}$

4.) mass flow rate of gas $= \dot{m}$ (kg s^{-1})

We have, $\dot{m} C_p \Delta T$ = Power produced + Heat loss rate

$$\Rightarrow \dot{m} \times 1 \times (1100-400) = 4600 + 300$$

$$\Rightarrow \boxed{\dot{m} = \frac{4900}{700} \text{ kg s}^{-1} \approx 7 \text{ kg s}^{-1}}$$

5.) a.) $\Delta H = \int_{t_1}^{t_2} C_p \, dt$ ($t \to$ temperature in °C)

$$= \int_0^{100} \left(2.093 + \frac{41.87}{t+100}\right) dt$$

$$= \left[2.093t + 41.87 \ln(t+100)\right]_0^{100}$$

$$= 209.3 + 41.87 \ln 2 \quad J$$

$$\Rightarrow \Delta H = 238.322 \text{ J}$$

b.) $\Delta U = \Delta H - W$

$$= \left[238.322 - 101325 \frac{(2400 - 2000)}{10^6}\right] J$$

$$\Rightarrow \boxed{\Delta U = 197.792 \text{ J}}$$

6.) a.) $1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$ $\left(\eta = 1 - \frac{T_{rej}}{T_{abs.}}\right)$

$$\Rightarrow \boxed{T_2 = \sqrt{T_1 T_3}}$$

b.) $W_{E_1} = W_{E_2}$

$$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow Q_2 = \frac{Q_1 + Q_3}{2}$$

Now, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ and $\frac{Q_3}{Q_2} = \frac{T_3}{T_2}$

So $\frac{Q_3}{Q_1} = \frac{Q_2}{Q_1} \times \frac{Q_3}{Q_2} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} = \frac{T_3}{T_1}$

In the equation $Q_2 = \frac{Q_1 + Q_3}{2}$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{1 + Q_3/Q_1}{2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{1 + T_3/T_1}{2}$$

$$\Rightarrow \boxed{T_2 = \frac{T_1 + T_3}{2}}$$

7.) $\eta_{max} = 1 - \frac{27+273}{627+273} = 1 - \frac{300}{900} = \frac{2}{3} \simeq 0.67$

$\eta_{actual} = \frac{50}{3 \times 75000/3600} = 0.8$

As $\eta_{actual} > \eta_{max}$ so such an engine is $\boxed{\text{not possible}}$

Since such an engine cannot exist, it is unnecessary to check whether it is economical or not.

8.) $\eta_{carnot} = 1 - \frac{T_{low}}{T_{high}}$

Clearly, η_{carnot} is independent of the working substance used.

$\therefore$ $\boxed{\text{All engines have the same efficiency}}$

9.) 1st law of thermodynamics: $Q = \Delta U + W$

Here the insulated storage tank is our system.

$\dot{Q} = \Delta\dot{U} + \dot{W}$

$\Rightarrow 0 = \dot{m}(u_f - u_i) - \dot{m}h_i$

$\Rightarrow \boxed{h_i = u_f}$ (as $u_i = 0$; tank was empty)

10.) Let the area of plate $= A\ m^2$

So $\frac{1}{0.5A} = 0.2$

$\Rightarrow \boxed{A = 10\ m^2}$

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