Solutions

☆ Vacancies in metals –

1.) $$\frac{(n_v/N)_{T_1}}{(n_v/N)_{T_2}} = \frac{e^{-\Delta H_v/RT_1}}{e^{-\Delta H_v/RT_2}}$$

$$= e^{\frac{\Delta H_v}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$$

$$\Rightarrow \Delta H_v = \frac{RT_1T_2}{T_1-T_2} \ln\left[\frac{(n_v/N)_{T_1}}{(n_v/N)_{T_2}}\right]$$

Take $T_1 = 2410^\circ C = 2683 K$

$T_2 = 1234^\circ C = 1507 K$

$$\therefore \Delta H_v = \frac{8.314 \times 2683 \times 1507}{1176 \times 1000} \ln\left(\frac{5.26\times10^{-3}}{3.091\times10^{-5}}\right) \text{ kJ mol}^{-1}$$

$$\approx 146.835 \text{ kJ mol}^{-1}$$

$$= \frac{146835}{1.6\times10^{-19}\times6.022\times10^{23}} \text{ eV/atom}$$

$$\approx \boxed{1.524 \text{ eV/atom}}$$

2.) $$\frac{(n_v/N)_{T_2}}{(n_v/N)_{T_1}} = e^{\frac{\Delta H_v}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

$$\Rightarrow \frac{1}{T_2} = \frac{1}{T_1} - \frac{R}{\Delta H_v} \ln\left[\frac{(n_v/N)_{T_2}}{(n_v/N)_{T_1}}\right]$$

$$= \frac{1}{1161} - \frac{8.617}{1.5} \ln\left[\frac{10^{-3}}{10^{-6}}\right]$$

$$= \frac{1}{1161} - \frac{8.617}{1.5} \times 3 \ln 10 < 0$$

$$\Rightarrow T_2 < 0 \quad \text{(That's not possible!)}$$

So it is not possible to achieve a vacancy fraction of one vacancy for every thousand atomic sites by simply raising the temperature.

3.) $$\frac{n_v}{N_A} = \frac{26.98}{26.2} \times \frac{7.57 \times 10^{23}}{10^6 \times 6.022 \times 10^{23}}$$

$$\approx 1.294 \times 10^{-6}$$

Now, $$1.294 \times 10^{-6} = e^{\frac{-E_a}{RT}}$$

$$\Rightarrow E_a = -RT \ln(1.294 \times 10^{-6})$$

$$= \frac{8.617}{10^5} \times 873 \ln\left(\frac{10^6}{1.294}\right) \text{ eV/atom}$$

$$\approx \boxed{1.02 \text{ eV/atom}}$$

4.) $$n_v \,(\text{in } m^{-3}) = \exp\left(\frac{-1.08}{8.617 \times 10^{-5} \times 1123}\right) \times 6.022 \times 10^{23} \times \frac{7.65}{55.85}$$

$$\approx 1.173 \times 10^{18} \; m^{-3}$$

☆ Dislocations –

1.) $$|\vec{b}_{Fe}| = 2 \times 0.124 \text{ nm} = 0.248 \text{ nm}$$
$$|\vec{b}_V| = 2 \times 0.205 \text{ nm} = 0.41 \text{ nm}$$

2.) Clearly, the burgers' vectors will be

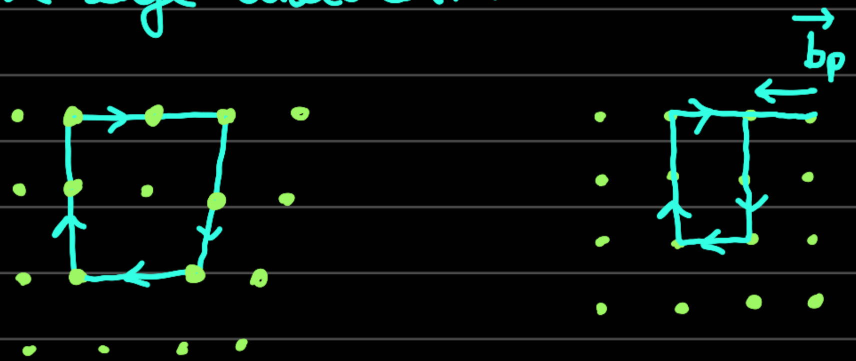
$$\vec{b}_1 = \frac{1}{4}[1\,1\,2] \quad \text{and} \quad \vec{b}_2 = \frac{1}{4}[1\,1\,\bar{2}]$$

Now, $\frac{\mu}{2}\left[2 \times \frac{1}{16}(1^2+1^2+2^2) - \frac{1}{4}(1^2+1^2+0^2)\right]$

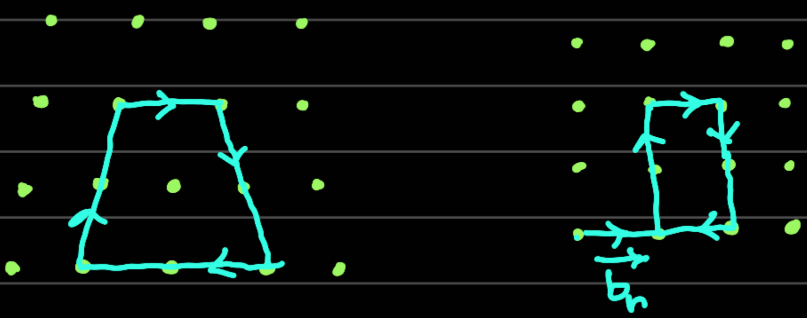
$$= \frac{\mu}{16}(6-4) = \frac{\mu}{8} > 0$$

$\therefore$ $\boxed{\text{Non-spontaneous}}$

3.) Positive edge dislocation:



Negative edge dislocation:



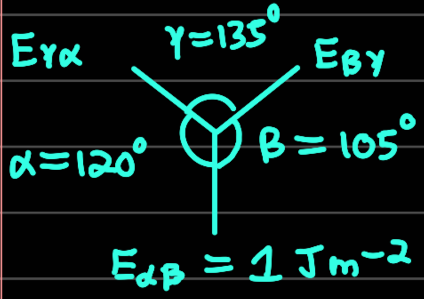
Burgers' vectors are shown in the figures.
We find $\underline{\vec{b}_p + \vec{b}_n = 0}$

4.) Dislocation energy density $= \frac{1}{2} G b^2 \rho$

$$= \frac{1}{2} \times 45 \times 10^9 \times \left(\frac{3.61}{\sqrt{2}} \times 10^{-10}\right)^2 \times 10^{10} \ \text{J m}^{-3}$$

$$\approx \boxed{14.66 \ \text{J m}^{-3}}$$

☆ Surfaces and Grain Boundaries –

1.) 

From Lami's Theorem,

$$\frac{E_{\alpha\beta}}{\sin\gamma} = \frac{E_{\beta\gamma}}{\sin\alpha} = \frac{E_{\gamma\alpha}}{\sin\beta}$$

So $$E_{\beta\gamma} = \frac{\sin 120^\circ}{\sin 135^\circ} = \frac{\sqrt{3}/2}{1/\sqrt{2}} \approx 1.22\ J m^{-2}$$

$$E_{\gamma\alpha} = \frac{\sin 105^\circ}{\sin 135^\circ} = \frac{(\sqrt{3}+1)/2\sqrt{2}}{1/\sqrt{2}}$$

$$\approx 1.366\ J\ m^{-2}$$

2.) No. of grains per unit volume $= \dfrac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$

$$= \frac{6}{\pi d^3}$$

Grain boundary area per unit volume

$$= 4\pi\left(\frac{d}{2}\right)^2 \times \frac{6}{\pi d^3} = \frac{6}{d}$$

If $\gamma \rightarrow$ Grain boundary energy per unit area ($J\ m^{-2}$)

Energy per unit volume, $E = \dfrac{6\gamma}{d}$

If d increases, E decreases $\Rightarrow \Delta E < 0$

$\Rightarrow$ Spontaneous process.

$$\Delta E = 6\gamma\left(\frac{1}{0.1\times10^{-3}} - \frac{1}{0.01\times10^{-3}}\right)\ J\ m^{-3}$$

$$= -5.47 \times 10^{5} \ J \ m^{-3}$$
$$= -0.547 \ MJ \ m^{-3}$$

3) wrong formula in question. It is actually

$$N = 2^{n-1} \Rightarrow n = 1 + \log_2 N$$

Put $N = 45$

so $n = 1 + \log_2 45$

$= 1 + 5.492$

$= 6.492$

——x——