

Day-10

→ Class Test 1 Solⁿ

1.) $n_1 = n_2 = n_3 = \frac{1}{\sqrt{3}}$

$$D = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & 1/2 \\ 1/2 & 1 & -1/2 \\ -1/2 & 1/2 & 1 \end{bmatrix}$$

2.) a_1, a_2 and $\frac{a_1 + a_2}{2}, \gamma = 120^\circ$

6-fold rotation: $\vec{a}_1 \times \vec{a}_2$

$$\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \times \text{l.p.}$$

$$\vec{a}_1 = \frac{\sqrt{3}}{2} a \hat{i} - \frac{a}{2} \hat{j}$$

$$\vec{a}_2 = a \hat{j}$$

60° rotation:

$$\begin{aligned} \vec{a}_{1 \text{ rot}} &= D_{ij} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -1/2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \\ &= \vec{a}_1 + \vec{a}_2 \end{aligned}$$

$$\text{Now, } D_{ij} \left(\frac{\vec{a}_1 + \vec{a}_2}{2} \right) = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

↓
not a lattice

point!

\therefore There is no 6-fold.

Aliter:

If $\vec{a}_1, \vec{a}_2, \frac{\vec{a}_1 + \vec{a}_2}{2}$ are lattice points

$\frac{\vec{a}_1 + \vec{a}_2}{2} - \vec{a}_1 = \frac{-\vec{a}_1 + \vec{a}_2}{2}$ is also a lattice points

$$\vec{b}_1 = \frac{\vec{a}_1 + \vec{a}_2}{2}$$

$$\vec{b}_2 = \frac{-\vec{a}_1 + \vec{a}_2}{2}$$

$$g_{ij} = \frac{a^2}{2} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\vec{b}_1 \neq \vec{b}_2$$

$$\angle b_1, b_2 = 90^\circ$$

