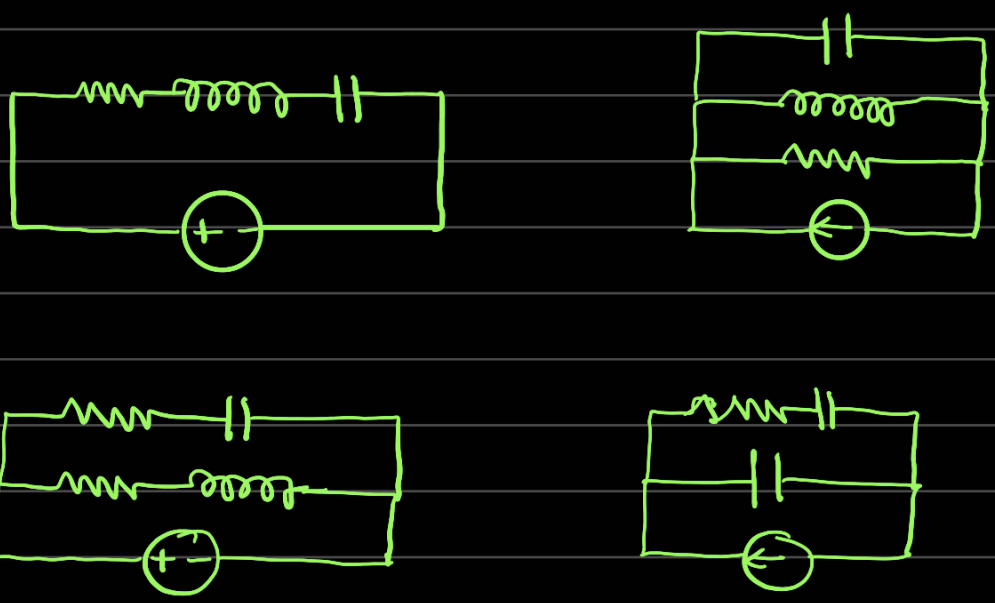
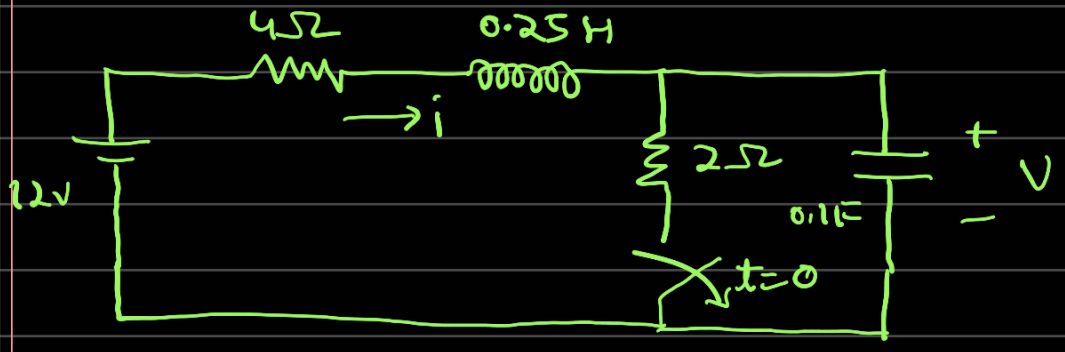
Day - 12

→ 2nd order circuits (DC) –



1)



Find $i(0^+)$, $v(0^+)$, $\frac{di}{dt}(0^+)$, $\frac{dv}{dt}(0^+)$

$i(\infty)$, $v(\infty)$

Ans:) $i(0^+) = \frac{12}{4+2} = \frac{12}{6} = 2A$

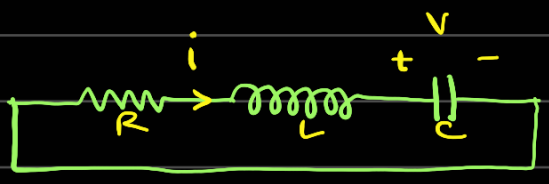
$v(0^+) = 4\ V$

$\frac{di}{dt}(0^+) = 0$, $\frac{dv}{dt}(0^+) = \frac{i}{C} = \frac{2}{0.1}$

$= 20\ V$

$i(\infty) = 0$, $v(\infty) = 12V$

2)



Source-free RLC circuit.

write differential eqⁿ.

Ans:) $iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$

$\Rightarrow L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$

Apply laplace [$\mathcal{L}(i) = s$]

$\Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ ⟶ Characteristic eqⁿ.

(One can substitute e^{st} for this)

$\Rightarrow s = \frac{-R/L \pm \sqrt{(R/L)^2 - 4/LC}}{2}$

$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$\begin{cases} s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{cases}$ ⟶ Natural roots

$\alpha = \frac{R}{2L}$ ⟶ damping factor

$\omega_0 = \frac{1}{\sqrt{LC}}$ ⟶ Natural frequency

Case 1: $\alpha > \omega_0$ ⟶ Real roots
(overdamp)

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case 2: $\alpha = \omega_0$ (critical damp)

$$i(t) = (A_1 + A_2 t) e^{st} \qquad (s = s_1 = s_2)$$

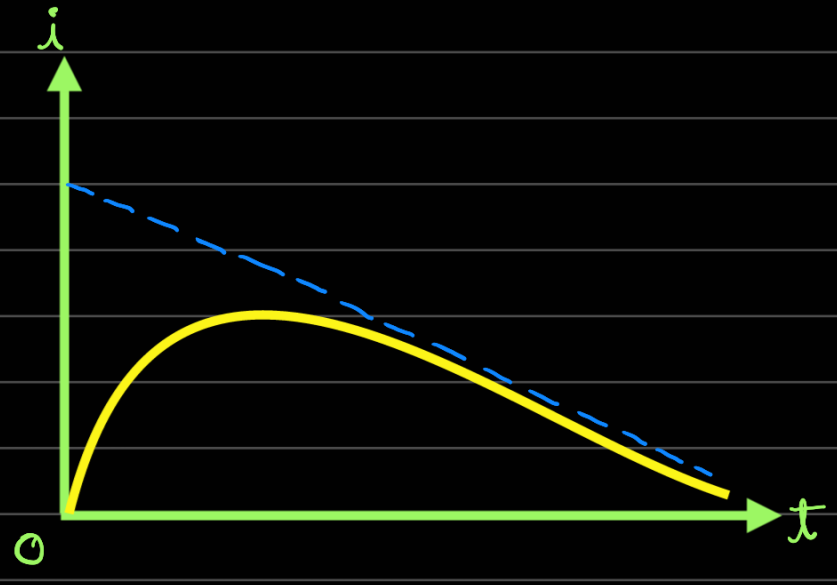
Case 3: $\alpha < \omega_0$ (underdamp)

$$i(t) = e^{-\alpha t}[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

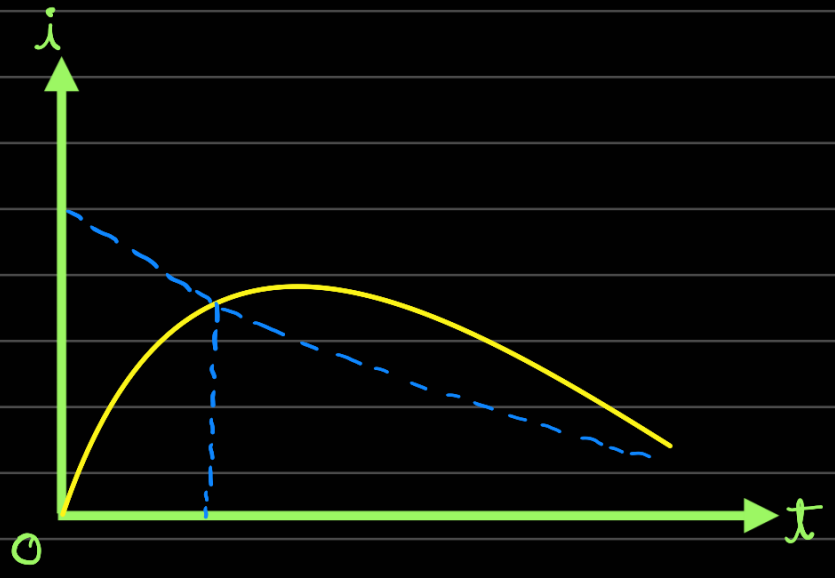
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

→ **waveforms :**

(i) **Overdamp**



(ii) **Critical damp**



iii) Underdamp