Practice Sheet-4

1.) $\bar{x} = \frac{1}{10}(0.5 + 0.55 + 0.53 + 0.56 + 0.54 + 0.57 + 0.52 + 0.62 + 0.55 + 0.58)$

$= 0.55$

$$s = \sqrt{\frac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{9}}$$

$\simeq 0.029$

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_9$$

Now, $\frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{0.55 - \mu}{0.029}\sqrt{10}$

For 95% confidence interval, $\frac{\alpha}{2} = 0.025$

$t_{0.025,9} = 2.262$

So $-2.262 < \sqrt{10}\,\frac{(0.55 - \mu)}{0.029} < 2.262$

$\Rightarrow 0.55 - 2.262 \times \frac{0.029}{\sqrt{10}} < \mu < 0.55 + 2.262 \times \frac{0.029}{\sqrt{10}}$

$\Rightarrow \boxed{0.53 < \mu < 0.57}$

2.) $\mu \in \left(\bar{x} - t_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}}\right)$

$\Rightarrow \mu \in \left(320 - 2.262 \times \frac{16}{\sqrt{100}},\ 320 + 2.262 \times \frac{16}{\sqrt{100}}\right)$

$$\Rightarrow \boxed{\mu \in (316.38,\ 323.62)}$$

3.) $$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{1.2-\mu}{0.2/\sqrt{20}}$$

For 99% confidence interval, $\frac{\alpha}{2} = 0.005$

$$Z_{0.995} = 2.576$$

So $$\mu \in \left(1.2 - 2.576 \times \frac{0.2}{\sqrt{20}},\ 1.2 + 2.576 \times \frac{0.2}{\sqrt{20}}\right)$$

$$\Rightarrow \boxed{\mu \in (1.085,\ 1.315)}$$

4.) $n = 30$, $\bar{x} = 2.5$, $s = 2.12$

So $$\frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$$

$$P\left\{ \frac{\bar{x}-\mu}{s/\sqrt{n}} > -t_{\alpha, n-1} \right\} = 1-\alpha$$

Here, $1-\alpha = 0.9 \Rightarrow \alpha = 0.1$

Now, $t_{0.1, 29} = 1.311$

So $$\frac{2.5-\mu}{2.12/\sqrt{30}} > -1.311$$

$$\Rightarrow \mu < 2.5 + 1.311 \times \frac{2.12}{\sqrt{30}}$$

$$\Rightarrow \boxed{\mu < 3.007}$$

5.) a.) s^2 is the point estimator of σ^2

So $\boxed{s^2 = \dfrac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{9} = 32.233}$

$(\bar{x} = 144.3)$

b.) $\alpha = 0.01$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\Rightarrow \chi^2_{1-\frac{\alpha}{2}, n-1} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{\frac{\alpha}{2}, n-1}$$

$$\Rightarrow \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}$$

$$\Rightarrow \frac{9 \times 32.233}{\chi^2_{0.05, 9}} < \sigma^2 < \frac{9 \times 32.233}{\chi^2_{0.95, 9}}$$

$$\Rightarrow \frac{290.1}{16.919} < \sigma^2 < \frac{290.1}{3.325}$$

$$\Rightarrow \boxed{17.146 < \sigma^2 < 87.248}$$

c.) $\dfrac{(n-1)s^2}{\sigma^2} > \chi^2_{1-\alpha, n-1}$ $\quad (1-\alpha = 0.9)$

$$\Rightarrow \sigma^2 < \frac{290.1}{\chi^2_{0.9, 9}} = \frac{290.1}{4.168}$$

$$\Rightarrow \boxed{\sigma^2 < 69.6}$$

6.) we calculate $\bar{x}_1 = 18$, $s_1^2 = 2.5$

$\bar{x}_2 = 5$, $s_2^2 = 7$

so $s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

$= \frac{10+14}{6}$

$= 4$

$\Rightarrow \boxed{s = 2}$ (estimate of σ)

7.) a.) s (estimate of σ) $= \frac{\sum_{i=1}^{8}(x_i-\bar{x})^2}{} = 0.008$

b.) $\sigma \in \left(s\sqrt{\frac{(n-1)}{\chi^2_{\frac{\alpha}{2},n-1}}}, s\sqrt{\frac{n-1}{\chi^2_{1-\frac{\alpha}{2},n-1}}} \right)$ $(\alpha = 0.1)$

$\Rightarrow \sigma \in \left(0.008\sqrt{\frac{7}{\chi^2_{0.05,7}}}, 0.008\sqrt{\frac{7}{\chi^2_{0.95,7}}} \right)$

$\Rightarrow \sigma \in \left(0.008\sqrt{\frac{7}{14.067}}, 0.008\sqrt{\frac{7}{2.167}} \right)$

$\Rightarrow \boxed{\sigma \in (0.0056, 0.0144)}$

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