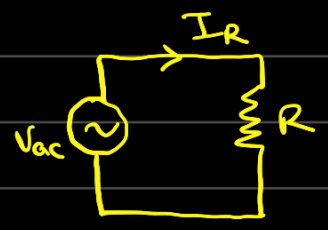
Day-7

→ AC circuits:

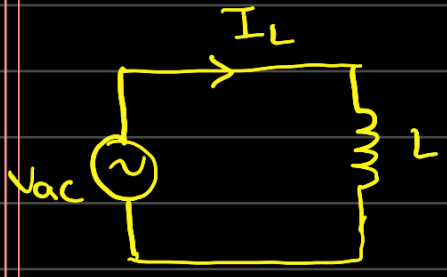
$$V_{ac} = V_m \cos(\omega t + \phi)$$



$$I_R = \frac{V_m}{R}\cos(\omega t+\phi)$$

$$= I_m \cos(\omega t+\phi)$$

I_R, V_{ac} in phase



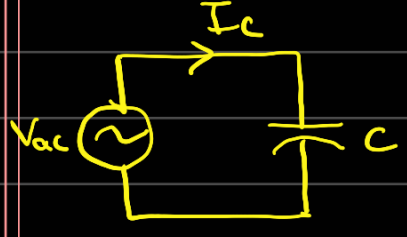
$$V_{ac} = L\frac{dI_L}{dt}$$

$$\Rightarrow \frac{V_m}{L}\cos(\omega t+\phi) = \frac{dI_L}{dt}$$

$$\Rightarrow I_L = \frac{V_m}{\omega L}\sin(\omega t+\phi)$$

$$= \frac{V_m}{\omega L}\cos\left(\omega t+\phi-\frac{\pi}{2}\right)$$

I_L lags V_{ac} by $\pi/2$



$$C\frac{dV_{ac}}{dt} = I_C$$

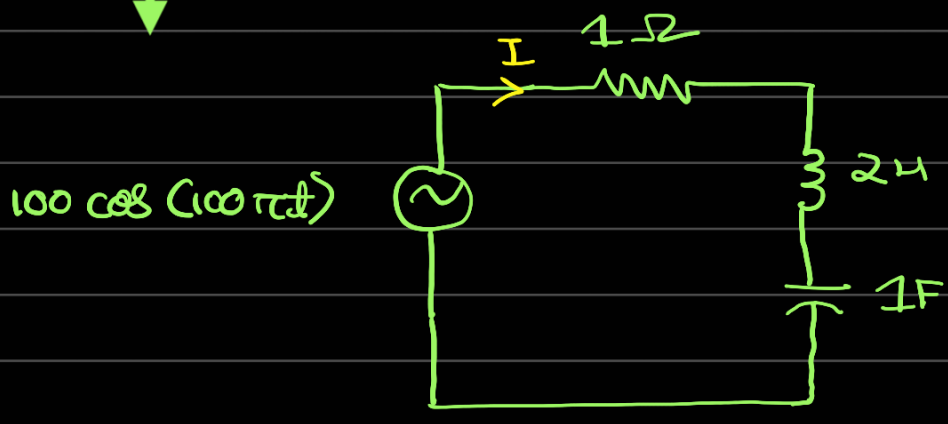
$$\Rightarrow -CV_m\omega\sin(\omega t+\phi) = I_C$$

$$\Rightarrow I_C = \frac{V_m}{1/\omega C}\cos\left(\omega t+\phi+\frac{\pi}{2}\right)$$

I_C leads V_{ac} by $\pi/2$

→ Phasor diagram :

1.)



Ans) $100\cos 100\pi t = I + 2\frac{dI}{dt} + \int I\,dt$

Too much time taking. Use phasor.

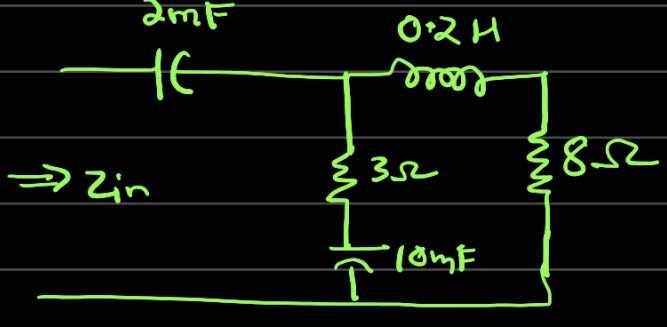
$X_L = j\omega L$ (Inductive reactance)

$X_C = \frac{1}{j\omega C}$ (Capacitive reactance)

So $100\cos 100\pi t = I\left(1 + 200j - \frac{j}{100}\right)$

$\Rightarrow I = \frac{100\cos 100\pi t}{1 + 199.99j}$

2.)



$\omega = 50\ rads^{-1}$

Find Zin.

Ans 2)

$0.2H \equiv 50 \times 0.2 = 10\ \Omega$

$2mF \equiv \frac{1}{2\times10^{-3}\times 50} = 10\ \Omega$

$10mF \equiv \frac{1}{10\times10^{-3}\times50} = 2\ \Omega$

So $8 + 10j \parallel 3 - 2j$

$= \frac{(8+10j)(3-2j)}{11+8j}$

$= \frac{44 + 14j}{11+8j}$

Add to $-10j$

So $\frac{44 + 14j - 110j + 80}{11+8j}$

$11 - 8j$

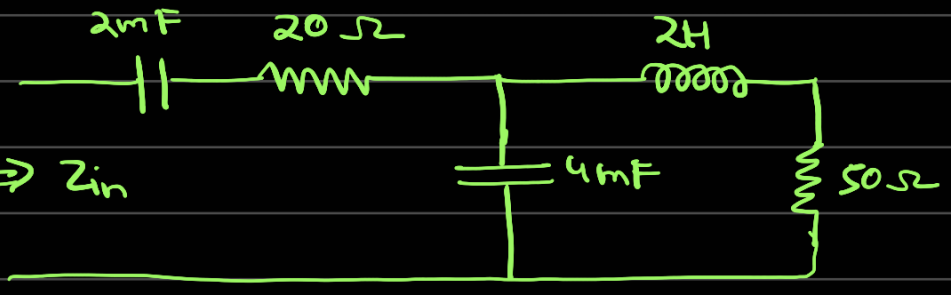
$= \frac{124 - 96j}{11+8j}$

$= \frac{596 - 2048j}{185}$

992
$+1056$

$= 3.22 - 11.07j\ \Omega$

3.)



$\Rightarrow Z_{in}$

$\omega = 10\ rad\ s^{-1}$

Ans.)

$2H \equiv 20j$

$4mF \equiv \frac{1}{4\times10^{-3}\times10} = -25j$

$2mF = \frac{1}{2\times10^{-3}\times10} = -50j$

$$\frac{(50+20j)(-25j)}{50-5j} + 20 - 50j$$

$$= \frac{-5j(50+20j)}{10-j} + 20 - 50j$$

$$= 10\left[\frac{10-25j}{10-j} + 2 - 5j\right]$$

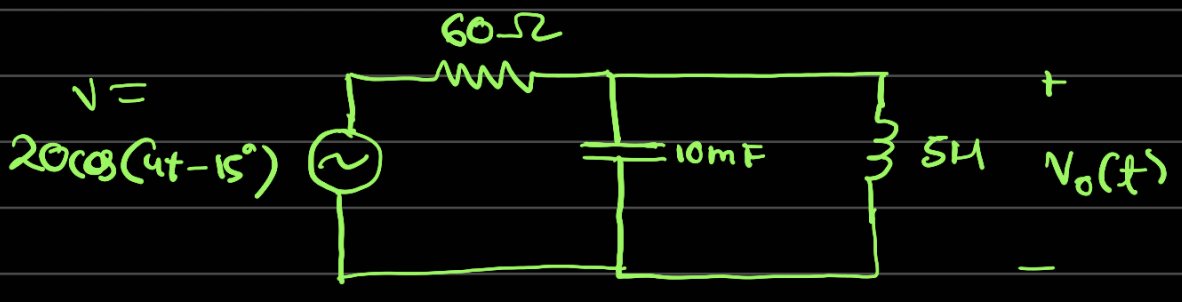
$$= 10\left[\frac{10-25j+20-2j-50j-5}{10-j}\right]$$

$$= 10\left[\frac{25-77j}{10-j}\right]$$

$$= \frac{10}{101}(327-745j)\ \Omega$$

$$= 32.376 - 73.762j\ \Omega$$

4.)



Find $V_o(t)$.

Ans:)

$$5H \equiv 4 \times 5 = 20j$$

$$10mF \equiv \frac{1}{4 \times 10 \times 10^{-3}} = -25j$$

$$Z_{eq} = 60 + \frac{20j \times -25j}{20j - 25j}$$

$$= 60 + \frac{500}{-5j}$$

$$= 60 + 100j^{\circ}$$

$$\text{so} \quad V_o(t) = \frac{V}{60 + 100j^{\circ}} \times 100j^{\circ}$$

$$= \frac{20\cos(4t - 15^{\circ})}{60 + 100j} \times 100j^{\circ}$$

$$= \frac{\cos(4t - 15^{\circ})}{3 + 5j} \times 100j^{\circ}$$

$$= \frac{1}{34} \times \cos(4t - 15^{\circ}) \times (500 + 300j)$$

$$= \frac{50}{17}(5 + 3j)\cos(4t - 15^{\circ})$$

$$= \frac{100}{\sqrt{34}} \angle \tan^{-1}(0.6) \cos(4t - 15^{\circ})$$

$$= 17.15 \cos(4t + 15.96^{\circ}) \ V$$