MM 225 AI and Data Science

2024-25-1

Problems for practice 1

1. Let random variable X ~ Exp(λ) then show the following:

    a. $P(X < t_1+t_2 | X > t_2) = P(X < t_1)$

    b. $E(X) = 1/\lambda$

    c. $Var(X) = 1/\lambda^2$

2. Jones figures that the total number of thousands of miles that a used auto can be driven before it would need to be junked is an exponential random variable with parameter 1/20. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed but rather is (in thousands of miles) uniformly distributed over (0, 40).

3. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1$.

    a. What is the probability that a repair time exceeds 2 hours?

    b. What is the conditional probability that a repair takes at least 3 hours, given that its duration exceeds 2 hours?

4. An IQ test produces scores that are normally distributed with mean value 100 and standard deviation 14.2. The top 1 percent of all scores are in what range?

5. A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

6. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes? If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

7. Consider independent Bernoulli trials with probability p. Random variable X is a number of trials to get the first success. So if X =k then there are (k-1) failures and the kth trial is a success.

    a. Write the sample space for X.

    b. Determine pmf for X

    c. $E(X) = ?$

    d. $Var(X) = ?$

    e. Show that X is memoryless.