EN 313: Power Electronics

dc-dc Converters

Suryanarayana Doolla

Department of Energy Science and Engineering
Indian Institute of Technology Bombay
email: suryad@iitb.ac.in

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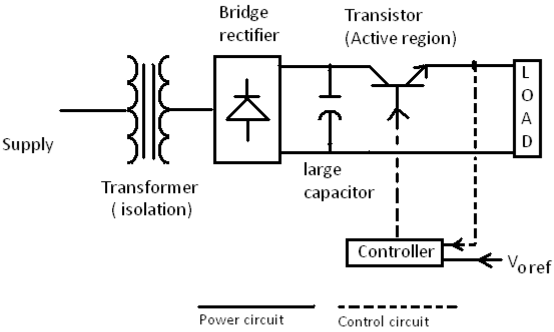
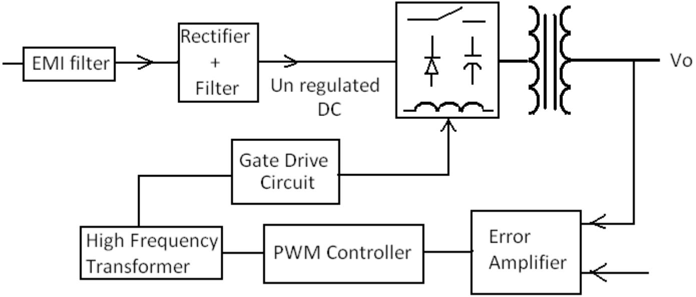
Linear Power Supply



- Require bulky transformer
- Efficiency is very low (30-60%), and preferred for power supply rating $<$ 25W

Switched Mode Power Supply (SMPS)

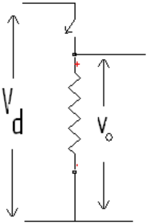


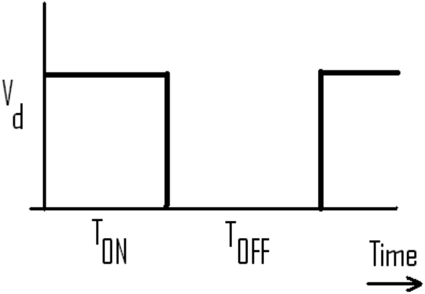
- Reduced size of transformer
- High efficiency (70-90%)
- Transistor operated in on/off mode has large power handling capability compared to one in linear mode

dc-dc Converter - Classification

- Direct Converters (Non-Isolated)
  - Buck, Boost
- Derived Converters (Non-Isolated)
  - Buck-Boost, Cuk
- Full Bridge dc-dc Converters (Non-Isolated)
  - Bi Polar voltage switching, Uni Polar voltage switching
- Isolated
  - Unidirectional core excitation (Flyback, Forward)
  - Bidirectional core excitation (Push-Pull, Half Bridge, Full bridge)

Pulse Width Modulation (PWM)





$$V_0 = \frac{1}{T_s}\int_0^{t_{ON}} V_d dt + \frac{1}{T_s}\int_0^{t_{ON}} 0.dt$$

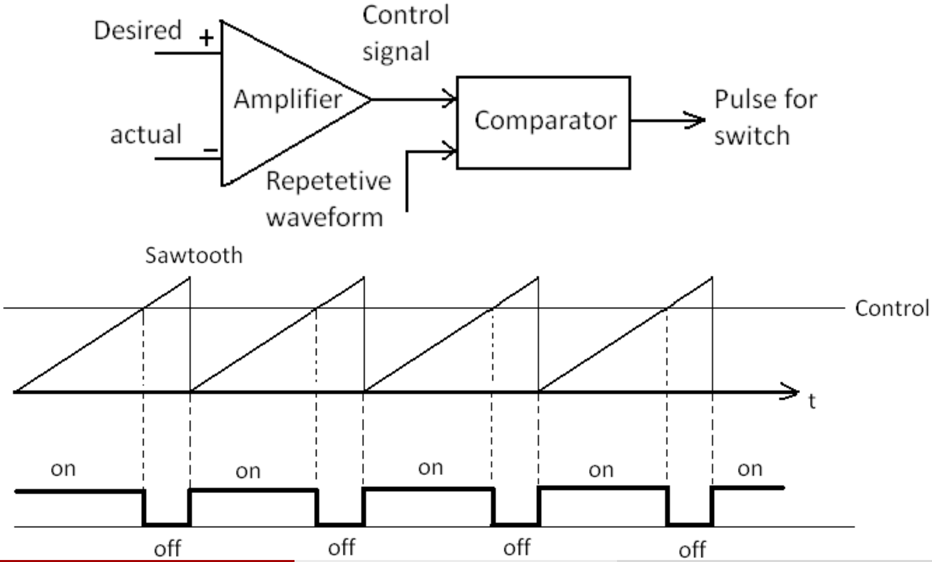
$$V_0 = DV_s$$

D is defined as the ratio of on time to total time and is given by $D = \frac{T_{on}}{T_s}$

Pulse Width Modulation

- Constant Frequency
  - Commonly employed
- Variable Frequency
  - Difficulty in filtering out harmonics of output waveform
  - Generally employed using thyristors
- Two Modes of operation
  - Continuous current mode
  - Dis-Continuous current mode

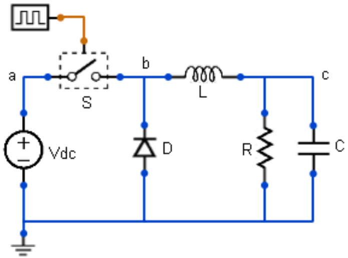
Pulse Generation



dc-dc Converter - Steady state analysis

- The converters are analyzed in steady state.
- Switches are considered as ideal, the losses in inductive and capacitive elements are neglected.
- The dc input voltage to the converters is assumed to have zero/low impedance.
- Switched mode converters utilize one or more switches to convert input voltage from one state to other at the output
- The frequency of repetitive waveform is kept constant and amplitude of control signal is varied.

Buck converter

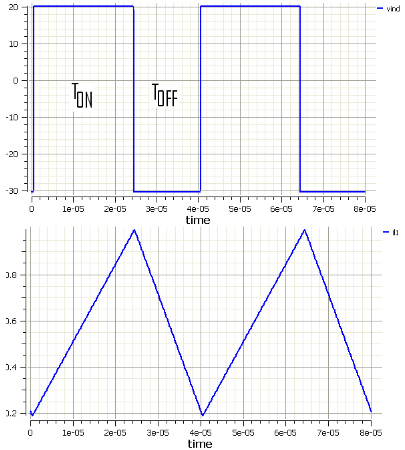


V_s= Supply Voltage,
V_0= Output Voltage,
V_L= Inductor Voltage= $V_s - V_0$

- The average output voltage is less than the input voltage V_d
- The average output voltage varies linearly with control voltage
- The filter capacitor is assumed to be high so that the output voltage is more of less constant

Buck converter-CC Mode

Inductor Voltage and Current



Analysis

When the switch in ON, Inductor current is rising
When the switch in OFF, Inductor current is falling

$$V_L = \frac{1}{T_s}\int_0^{T_{on}} (V_d - V_0).dt$$

$$+\frac{1}{T_s}\int_{T_{on}}^{T_s} -(V_0).dt$$

Buck converter-CC Mode

$$V_L = \frac{1}{T_s}\int_0^{T_{on}} (V_d - V_0).dt + \frac{1}{T_s}\int_{T_{on}}^{T_s} -(V_0).dt$$

$$V_L = \frac{T_{on}}{T_s}(V_d - V_0) - \frac{V_0}{T_s}(T_s - T_{on})$$

The average voltage across inductor in a cycle is zero.

$$V_0 = \frac{T_{on}}{T_s}V_d$$

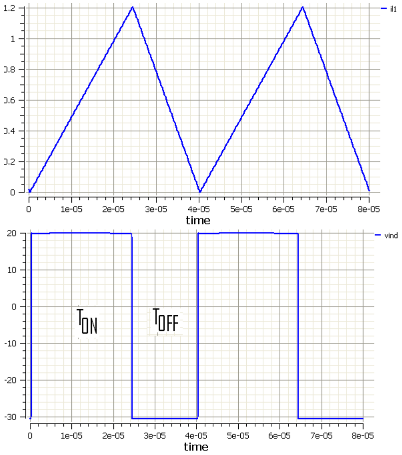
$$V_0 = DV_d$$

Assuming a lossless circuit,

$$\frac{I_d}{I_0} = \frac{V_0}{V_d} = D$$

Boundary Condition -CCM and DCM

Inductor Current and Voltage



Analysis

$$I_{LB} = \frac{1}{2} I_{L,peak}$$

$$\implies I_{LB} = \frac{1}{2}\frac{V_d - V_0}{L} T_{on}$$

$$\implies I_{LB} = \frac{DT_s}{2L}(V_d - V_0)$$

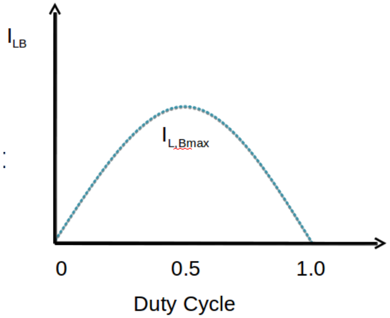
also,

$$V_0 = DV_d$$

$$\implies I_{LB} = \frac{V_d T_s}{2L}[D(1-D)]$$

Boundary Condition

Inductor Current with duty cycle



Analysis

The inductor current is minimum at D=0 and D=1, and is maximum at D=0.5, also $I_{L,Bmax} = \frac{V_d T_s}{8L}$
If we consider that the average current through the capacitor is zero then $I_{OB} = I_{LB}$

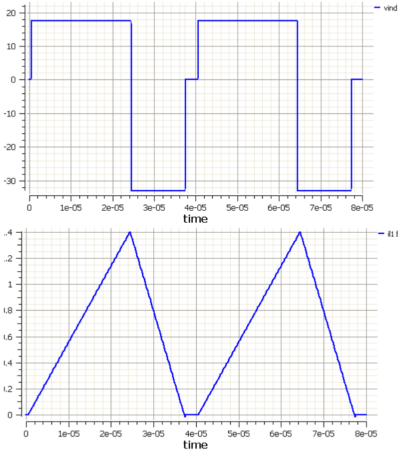
If the current is less than I_{OB} or I_{LB}, it is said to be discontinuous in nature ie., i_L become discontinuous.

Buck Converter - DCM

- Either input voltage or output voltage is constant
- DCM with constant input voltage (V_d)
- Boundary Condition:
  $I_{LB} = \frac{V_d T_s}{2L}[D(1-D)]$, $I_{L,Bmax} = \frac{V_d T_s}{8L}$, $I_{L,B} = 4I_{LBmax}D(1-D)$
- Assume that initially the converter is operated at edge of CCM and the output load power is decreased, i.e., "R" increases and hence "i_L" decreases introducing discontinuity in the current waveform

Buck Converter - DCM

Inductor Current with duty cycle



Analysis

Average voltage across inductor in zero.

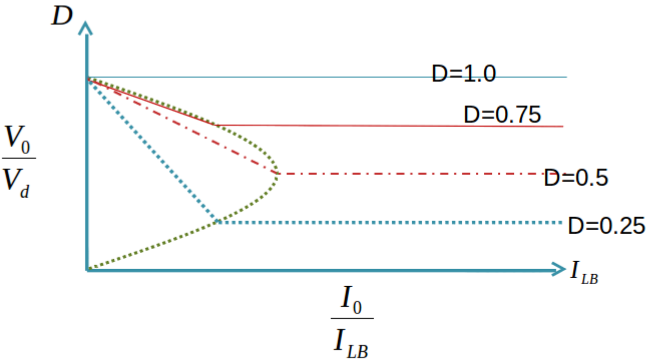
$$(V_d - V_0)DT_s + (-V_0)\Delta_1 T_s = 0$$
$$\implies V_d D.T_S = T_s V_0(D + \Delta_1)$$
$$\implies V_0 = \frac{D}{D+\Delta_1} V_d$$

The unknown parameter (Δ_1) can be derived in terms of known parameters,

$$V_0 = \frac{D^2}{D^2 + \frac{I_0}{I_{LBmax} \times \frac{1}{4}}} \times V_d$$

$$\Delta_1 = \frac{I_0}{I_{LBmax} \times \frac{1}{4D}}$$

Boundary Condition



- Boundary between CCM and DCM is given by dotted line.

Output Ripple -CCM

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{1}{C} \times \frac{1}{2} \times \frac{\Delta I_L}{2} \times \frac{T_s}{2}$$

Also during the turnoff t_{off}

$$\Delta I_L = \frac{V_0}{L}(1-D)T_s; \qquad \Delta V_0 = \frac{T_s V_0}{8LC}(1-D)T_s$$

$$\frac{\Delta V_0}{V_0} = \frac{1}{8} T_s^2 \frac{1}{LC}(1-D); \qquad \frac{\Delta V_0}{V_0} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2$$

Where f_c is the corner frequency given by $f_c = \frac{1}{2\pi\sqrt{LC}}$

Comparison of Converters

Converter	Output Voltage	Boundary Condition
Buck	$V_0 = DV_d$	$I_{LB} = \frac{V_d T_s}{2L}[D(1-D)]$
Boost	$V_0 = \frac{D}{1-D} V_d$	$I_{OB} = \frac{V_o T_s}{2L}[D(1-D)^2]$

Summary

- DC/DC Converters
  - Introduction to DC/DC Converters.
  - Linear and Switched Mode Amplifiers.
  - Buck Converter (CCM and DCM, Boundary Condition).

  Next Class
  - DC/DC Converter - Boost, Buck-Boost Converter

For Further Reading:

- Power Electronics: Converters, Applications, and Design: N. Mohan, T. M. Undeland, W. P. Robbins, John Wiley and Sons.
- Power electronics and motor drives: advances and trends: Bimal K Bose. Pearson Education.