Day-12

1) $E = f(t, R, P, \rho)$

	E	t	R	P	ρ
M	1	0	0	1	1
L	2	0	1	-1	-3
T	-2	1	0	-2	0

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$

1	0	0	1	1
0	0	1	-3	-5
0	1	0	0	2

$R_2 \leftrightarrow R_3$

1	0	0	1	1
0	1	0	0	2
0	0	1	-3	-5

Rank = 3 , Nullity = 2.

Choose t, R, ρ

$$E \propto \rho^x R^y t^z$$

$$\Rightarrow [ML^2T^{-2}] = [ML^{-3}]^x [L]^y [T]^z$$

$\Rightarrow x = 1$

$-3x + y = 2 \Rightarrow y = 5$

$z = -2$

So $E \propto \frac{R^5 \rho}{t^2}$

2.) $d = f(\rho, \mu, \sigma, v, D)$

	d	ρ	μ	σ	v	D
M	0	1	1	1	0	0
L	1	−3	−1	0	1	1
T	0	0	−1	−2	−1	0

$R_2 \leftrightarrow R_1$

1	−3	−1	0	1	1
0	1	1	1	0	0
0	0	−1	−2	−1	0

Rank = 3, Nullity = 3

Choose ρ, D, μ

$d \propto \rho^x D^y \mu^z$

$L = (ML^{-3})^x (L)^y (ML^{-1}T^{-1})^z$

$x + z = 0$

$-3x + y - z = 1$

$-z = 0$

So $z = 0 = x$, $y = 1$

So $\pi_1 : d/D$

$v \propto \rho^x D^y \mu^z$

$\Rightarrow LT^{-1} = (ML^{-3})^x (L)^y (ML^{-1}T^{-1})^z$

$\Rightarrow z = 1, x = -1, y = -1$

So π_2: $\dfrac{\vartheta \rho D}{\mu}$

$\sigma \propto \rho^x D^y \mu^z$

$\Rightarrow m T^{-2} = (m L^{-3})^x (L)^y (m L^{-1} T^{-1})^z$

$\Rightarrow x + z = 1$

$-3x + y - z = 0$

$-2 = -2$

$\Rightarrow z = 2$, $x = -1 = y$

So π_3: $\dfrac{\sigma \rho D}{\mu^2}$

$\dfrac{\mu \vartheta}{\sigma}$ → capillary no.

$\dfrac{\rho \vartheta^2 D}{\sigma}$ → weber no.

4.) $\dfrac{dT}{dt} = f(C_s, k, L, C_{air}, \rho, \mu, \vartheta)$

	dT/dt	C_s ✓	k	L	C_{air}	ρ	μ ✓	ϑ
m	0	0	1	0	0	1	1	0
L	0	2	1	1	2	−3	−1	1
T	−1	−2	−3	0	−2	0	−1	−1
K	1	−1	−1	0	−1	0	0	0

1 1 −1 0 1 0 0 0

−1 −2 −3 0 −2 0 −1 −1

0 2 1 1 2 −3 −1 1

0 0 1 0 0 1 1 0

$R_2 \to R_2 + R_1$

$$\begin{matrix} 1 & -1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -4 & 0 & -1 & 0 & -1 & -1 \\ 0 & 2 & 1 & 1 & 2 & -3 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{matrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{matrix} 1 & -1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -4 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & -7 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{matrix}$$

$R_4 \to R_4 + \frac{1}{7} R_3$

$\Rightarrow$ Rank $= 4$, Nullity $= 4$

$\Pi_1 : \quad \frac{C_R}{C_{air}}$

$$\mu \propto k^w L^x C_{air}^y \rho^z$$

$$\Rightarrow mL^{-1}T^{-1} = (mLT^{-3}K^{-1})^w L^x (L^2T^{-2}K^{-1})^y \times (mL^{-3})^z$$

$w + z = 1$

$w + x + 2y - 3z = -1$

$-2y = -1 \Rightarrow y = 1/2$

$-w - y = 0 \Rightarrow w = -1/2$

$z = 3/2$

$x = 3$

So $u \propto k^{-1/2} L^{3} C_{air}^{1/2} \rho^{3/2}$

π_2: $\dfrac{u}{L^3}\sqrt{\dfrac{k}{\rho^3 C_{air}^2}}$

$v \propto k^{w} L^{x} C_{air}^{y} \rho^{z}$

$w + z = 0$

$w + x + 2y - 3z = 1$

$-2y = -1 \Rightarrow y = 1/2$

$-w - y = 0 \Rightarrow w = -1/2$

$z = 1/2$

$-1/2 + x + 1 - 3/2 = 1$

$\Rightarrow x = 2$

So $v \propto k^{-1/2} L^{2} C_{air}^{1/2} \rho^{1/2}$

π_3: $\dfrac{v}{L^2}\sqrt{\dfrac{k}{\rho C_{air}}}$

$\dfrac{dT}{dt} \propto k^{w} L^{x} C_{air}^{y} \rho^{z}$

$\Rightarrow w + z = 0$

$w + x + 2y - 3z = 0$

$-2y = -1 \Rightarrow y = 1/2$

$-w - y = 1 \Rightarrow w = -3/2$

$z = 3/2$

$-3/2 + x + 1 - 9/2 = 0 \Rightarrow x = 5$

So $\dfrac{dT}{dt} \propto k^{-3/2} L^{5} C_{air}^{1/2} \rho^{3/2}$

$$\Rightarrow \Pi_4: \quad \frac{dT/dt}{L^5}\sqrt{\frac{k^3}{\rho^3 C_{air}}}$$

3.) $h = f(D, d, V, \rho, \mu, W)$

	h	D	d	V	ρ	μ	W
m	0	0	0	0	1	1	1
L	1	1	1	1	-3	-1	1
T	0	0	0	-1	0	-1	-2

Rank = 3, Nullity = 4
Choose d, ρ, W

$\Pi_1: h/d$ $\quad \Pi_2: D/d$ is obvious

$$V \propto d^x \rho^y W^z$$

$$\Rightarrow LT^{-1} = L^x (mL^{-3})^y (mLT^{-2})^z$$

$$\Rightarrow z = 1/2$$
$$y = -1/2$$
$$x = -1$$

So $\Pi_3: \quad \frac{Vd}{\sqrt{\frac{P}{W}}}$

$$\mu \propto d^x \rho^y W^z$$

$$\Rightarrow mL^{-1}T^{-1} = L^x (mL^{-3})^y (mLT^{-2})^z$$

$$\Rightarrow z = 1/2 = y$$
$$x = 0$$

So $\Pi_4: \quad \frac{\mu}{\sqrt{\rho W}}$

Replace sphere by plate.
Replace 'D' with 'A', the area of the plate (F depends on A is the assumption)

	F	A	d	v	ρ	μ	W
m	1	0	0	0	1	1	1
L	1	2	1	1	-3	-1	1
T	-2	0	0	-1	0	-1	-2

Rank = 3, Nullity = 4
Choose d, ρ, v

$$\pi_1 : \frac{F}{\rho v^2 d^2}$$

$$\pi_2 : \frac{A}{d^2}$$

$$\pi_3 : \frac{\mu}{\rho v d}$$

$$\pi_4 : \frac{W}{\rho v^2 d^2}$$

5) $P = f\left(d, \omega, \frac{dV}{dt}, \Delta p, \rho\right)$ (Δp → pressure diff.)

P → power; ω → angular speed; $\frac{dV}{dt}$ → volume flow rate

	P	d	ω	dV/dt	ΔP	ρ
m	1	0	0	0	1	1
L	2	1	0	3	-1	-3
T	-3	0	-1	-1	-2	0

Rank = 3, Nullity = 3
Choose d, ω, ρ

$$P \propto d^x \omega^y \rho^z$$

$$\Rightarrow mL^2T^{-3} = L^x (T^{-1})^y (mL^{-3})^z$$

$$\Rightarrow z = 1,\ y = 3,\ x = 5$$

So $\Pi_1 : \dfrac{P}{d^5 \omega^3 \rho}$

$$\frac{dV}{dt} \propto d^x \omega^y \rho^z$$

$$\Rightarrow L^3 T^{-1} = L^x (T^{-1})^y (mL^{-3})^z$$

$$\Rightarrow z = 0,\ y = 1,\ x = 3$$

So $\Pi_2 : \dfrac{dV/dt}{d^3 \omega}$

$$\Delta P \propto d^x \omega^y \rho^z$$

$$\Rightarrow mL^{-1}T^{-2} = L^x (T^{-1})^y (mL^{-3})^z$$

$$\Rightarrow z = 1,\ y = 2,\ x = 2$$

So $\Pi_3 : \dfrac{\Delta P}{d^2 \omega^2 \rho}$

—x—