Day-3

→ Streamlines -

$$\frac{dy}{dx} = \frac{v}{u} \qquad \vec{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Here, $\vec{v} = \begin{bmatrix} y \\ -0.5x \\ 0 \end{bmatrix}$ (2D flow)

So $\frac{dy}{dx} = \frac{-0.5x}{y}$

$$\Rightarrow \int y\,dy = \int -0.5x\,dx$$

$$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{4} + \frac{c^2}{2}$$

$$\Rightarrow \frac{x^2}{2} + y^2 = c^2$$

→ 3D flow :

$$\vec{v} = \begin{bmatrix} y \\ -x \\ 0.5z \end{bmatrix}$$

Take cross-section (arbitrarily)

$$\frac{dy}{dx} = \frac{-x}{y} \Rightarrow x^2 + y^2 = c^2$$

→ $\vec{v} = ax\hat{i} - ay\hat{j}$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow x\,dy + y\,dx = 0$$

$$\Rightarrow xy = c$$

$$\rightarrow \quad \vec{v} = 0.3\,(x\hat{i} - y\hat{j})$$

$$\vec{v}\,(2,8) = 0.6\,\hat{i} - 2.4\,\hat{j}$$

$$\vec{r}\,(0) = (2, 8)$$
$$\vec{r}\,(6) = ?$$

$$\frac{dx}{dt} = 0.3x \quad , \quad \frac{dy}{dt} = -0.3y$$

$$\Rightarrow x = 2e^{0.3t} \quad , \quad y = 8e^{-0.3t}$$

Put $t = 6$

$$\rightarrow \quad \text{Pressure} = \ominus \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

(-ve sign shows it's compressive)

(trace is rotation invariant)

$$\text{Stress matrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\overline{\overline{\sigma}} = \lim_{\delta\vec{A} \to \vec{0}} \frac{\delta\vec{F}}{\delta\vec{A}}$$

↓ symmetric func^n

(angular momentum conservation)

$$\sigma_{xx} = \lim_{\delta A_x \to 0} \frac{\delta F_x}{\delta A_x}$$

$$\tau_{xy} = \lim_{\delta A_x \to 0} \frac{\delta F_y}{\delta A_x}$$

$$\tau_{xz} = \lim_{\delta A_x \to 0} \frac{\delta F_z}{\delta A_x}$$

τ_{xy} → y-area

x-force