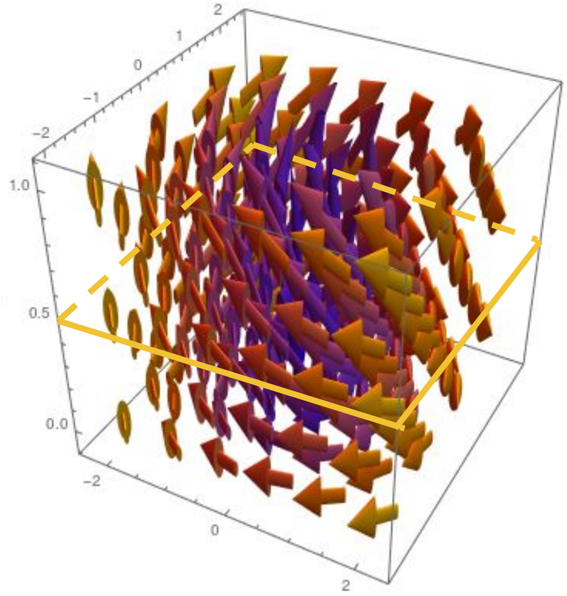
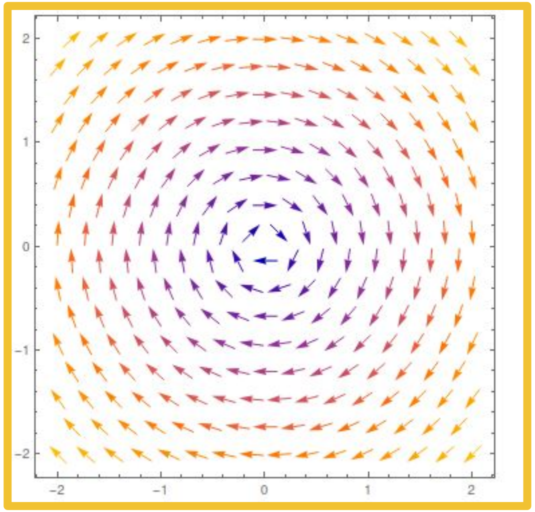
Streamlines:Example

$$\vec{v}(\vec{x}) = \begin{bmatrix} y \\ -x \\ 0.5z \end{bmatrix}$$



$$\vec{v}(\vec{x}) = \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix}$$



$$\frac{dy}{dx} = \frac{v}{u}$$

$$u\,dy = v\,dx$$

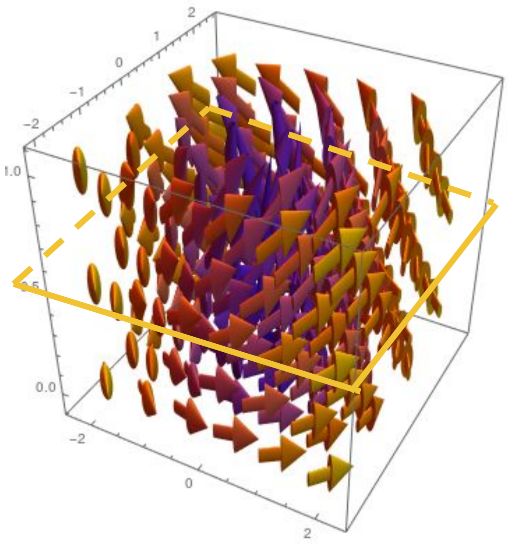
$$ydy = -xdx$$

$$y^2 + x^2 = c$$

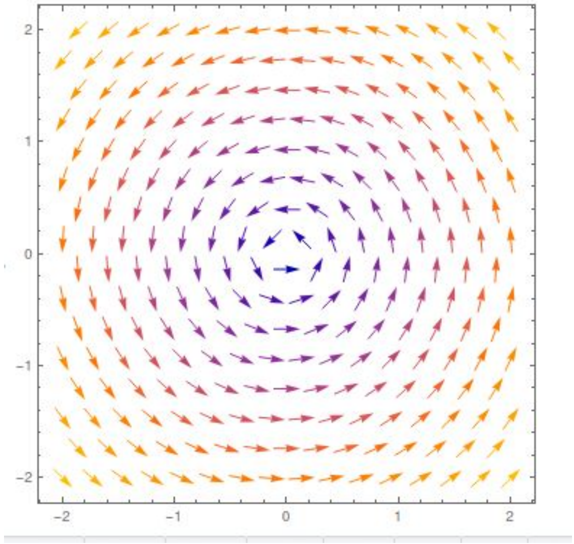
Ref: Chapter 2

Streamlines:Example

$$\vec{v}(\vec{x}) = \begin{bmatrix} -y \\ +x \\ 0.5z \end{bmatrix}$$



$$\vec{v}(\vec{x}) = \begin{bmatrix} -y \\ +x \\ 0 \end{bmatrix}$$



$$\frac{dy}{dx} = \frac{v}{u}$$

$$u\,dy = v\,dx$$

$$ydy = -xdx$$

$$y^2 + x^2 = c$$

Ref: Chapter 2

Streamlines: Example

$$\frac{dy}{dx} = \frac{v}{u}$$

$$u\,dy = v\,dx$$

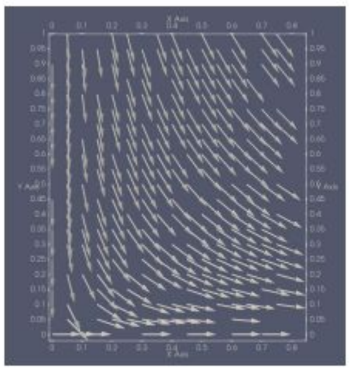
$$\boxed{\vec{v} = a\,x\hat{e}_x - a\,y\hat{e}_y}$$

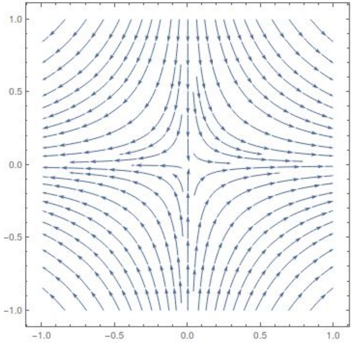
$$\frac{dy}{dx} = \frac{v}{u} = -\frac{y}{x}$$

$$\boxed{\ln y = -\ln x + \ln k}$$

Streamlines are tangent to the velocity at every point in the flow-field

$$\boxed{xy = k}$$





Ref: Chapter 2

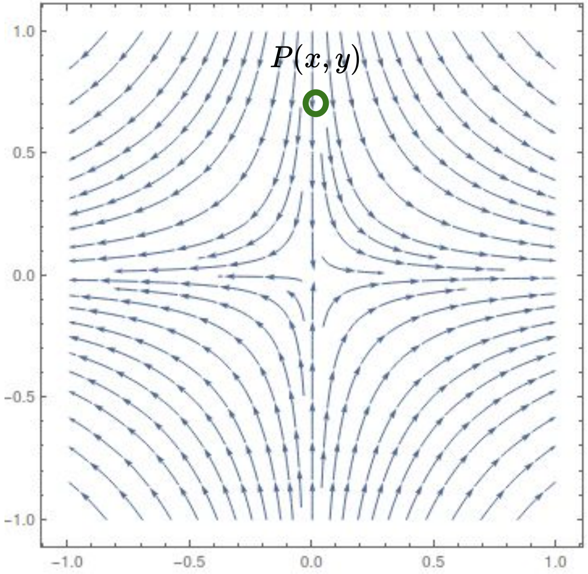
Streamlines:Example

$$\frac{dy}{dx} = \frac{v}{u}$$

$$u\,dy = v\,dx$$

$$\boxed{\vec{v} = a\,x\hat{e}_x - a\,y\hat{e}_y}$$

Streamlines are tangent to the velocity at every point in the flow-field



Note:

1. Along the y-axis the fluid particle decelerate as it approaches the origin
2. The origin is the stagnation point
3. Track fluid-particles (a dyed fluid parcel)-Lagrangian approach
4. Measure velocities at a point (x,y,z;t)-Eulerian approach

Ref: Chapter 2

Pathline & Streamline

$$\boxed{\vec{v} = a\,x\hat{e}_x - a\,y\hat{e}_y}$$

$$\boxed{a = 0.3\text{s}^{-1}}$$

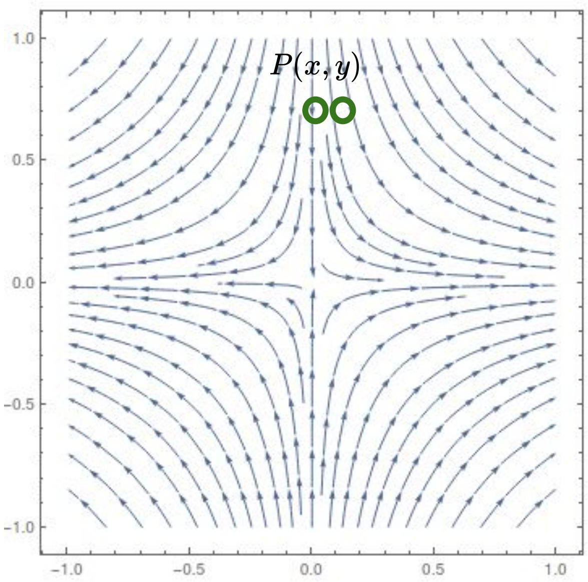
Determine the velocity of fluid particle at point (2,8)

$$\vec{v} = 0.6\hat{e}_x - 2.4\hat{e}_y$$

Streamlines

$$\frac{dy}{dx} = \frac{v}{u}$$

$$u\,dy = v\,dx$$



If the particle is passing through point (2,8) at t = 0, determine the location of the particle at t = 6 s.

$$v_x = ax \qquad v_y = -ay$$

$$dx/dt = ax \qquad dy/dt = -ay$$

$$x = x_o \exp(a\,t) \qquad y = y_o \exp(-a\,t)$$

$$x = 12.1\text{m} \qquad y = 1.32\text{m}$$

If the particle is passing through point (0,8) at t = 0, determine the location of the particle at t = 6 s.

$$x = 0 \qquad y = 1.32\text{m} \qquad v_y = -0.3 \times 1.32 \approx -0.4\text{m/s}$$

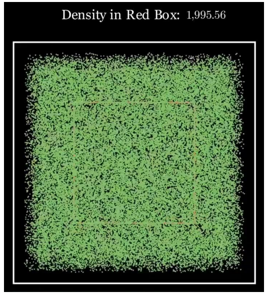
If the particle is passing through point (0,8) at t = 0, determine the location of the particle at t = 60 s.

$$x = 0 \qquad y = 1.2 \times 10^{-7}\text{m} \qquad v_y = -0.3 \times 1.2 \times 10^{-7}\text{m/s}$$

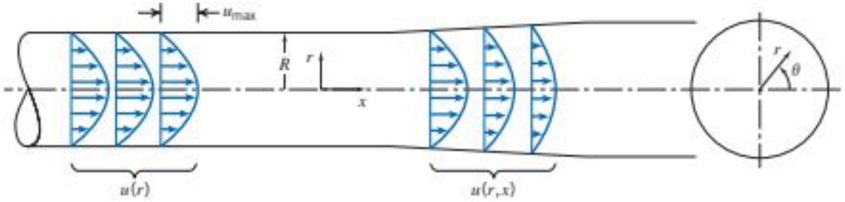
Ref: Chapter 2

Point Properties

Density $\rho = \dfrac{\Delta m}{\Delta V}$



Velocity



Pressure (Stress)

Ref: Chapters 1 - 3

Point Properties: Example

$$\bar{\bar{\sigma}} = \lim_{\delta A->0} \frac{\delta F}{\delta A} \hat{e}_F \hat{e}_A$$

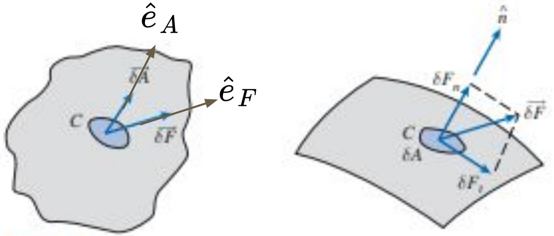


Fig. 2.6 The concept of stress in a continuum.

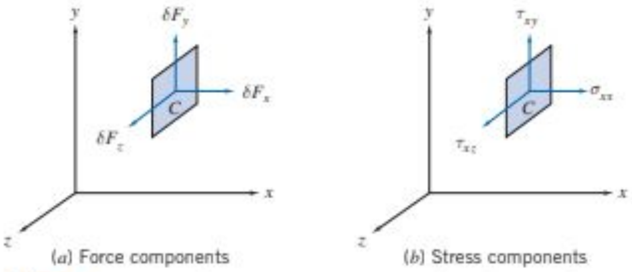


Fig. 2.7 Force and stress components on the element of area δA_x.

1. Surface forces on a fluid particle leads to stress
2. The concept of stress is useful for describing how the forces acting on the boundaries of a control volume is transmitted through the volume
3. Stress is developed if there is a relative motion between fluid particles
4. Outward normal to the surface is considered to be positive (convention followed)

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

1. 9 components
2. Symmetric
3. 6 independent components

Ref: Chapter 2

Point Properties: Example

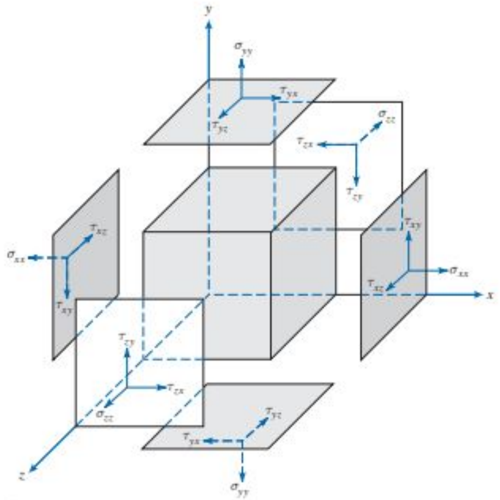


Fig. 2.8 Notation for stress.

$$\bar{\bar{\sigma}} = \lim_{\delta A->0} \frac{\delta F}{\delta A} \hat{e}_F \hat{e}_A$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

1. Outward normal to the surface is considered to be positive (convention followed)
2. Shear stress is *zero* when fluid is at rest

$$\sigma_{xx} = \lim_{(\delta A_x->0)} \frac{\delta F_x}{\delta A_x}$$

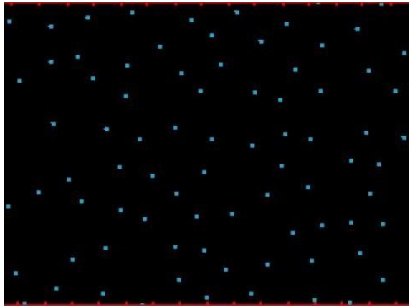
$$\tau_{xy} = \lim_{(\delta A_x->0)} \frac{\delta F_y}{\delta A_x}$$

$$\tau_{xz} = \lim_{(\delta A_x->0)} \frac{\delta F_z}{\delta A_x}$$

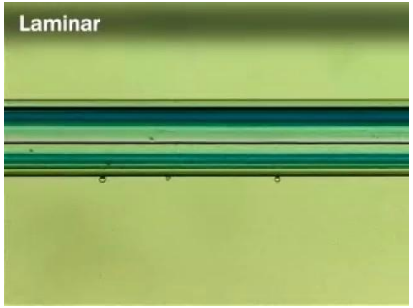
1. 9 components
2. Symmetric
3. 6 independent components

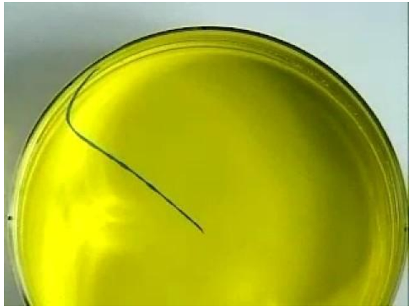
Ref: Chapters 1 - 3

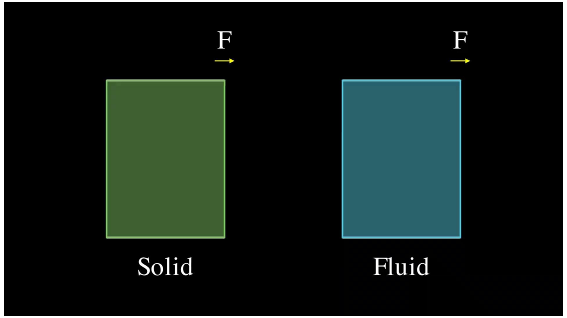
Stress and Strain rate



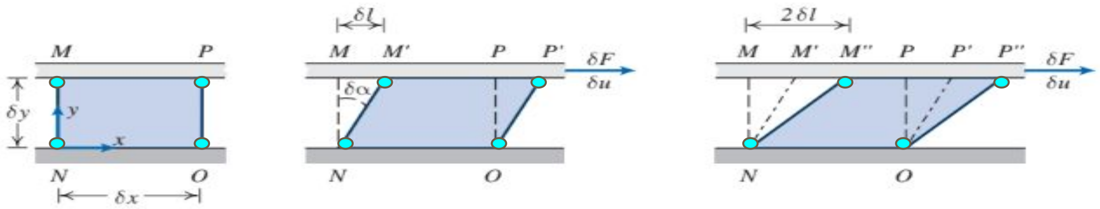
Stress in fluid is generated if there is relative motion between fluid particles







Rate of deformation - One dimensional flow



$$\text{Rate of Deformation} = \frac{d\alpha}{dt} = \lim_{\Delta t -> 0} \frac{\Delta\alpha}{\Delta t}$$

$$\text{Rate of Deformation} = \frac{d\alpha}{dt} = \lim_{\Delta t -> 0} \frac{\frac{\Delta l}{\Delta y}}{\Delta t}$$

$$\text{Rate of Deformation} = \frac{d\alpha}{dt} = \lim_{\Delta t -> 0} \frac{\frac{\Delta u \cancel{\Delta t}}{\Delta y}}{\cancel{\Delta t}}$$

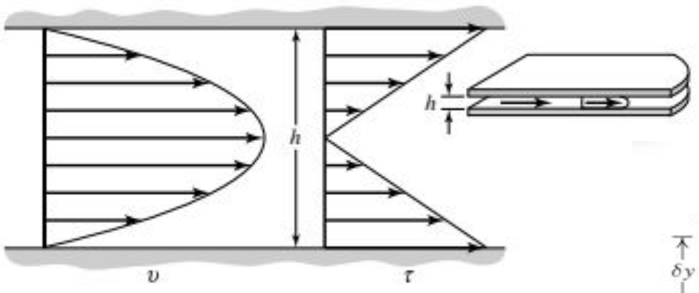
$$\text{Rate of Deformation} = \frac{d\alpha}{dt} = \frac{du}{dy} = \lim_{\Delta y -> 0} \frac{\Delta u}{\Delta y}$$

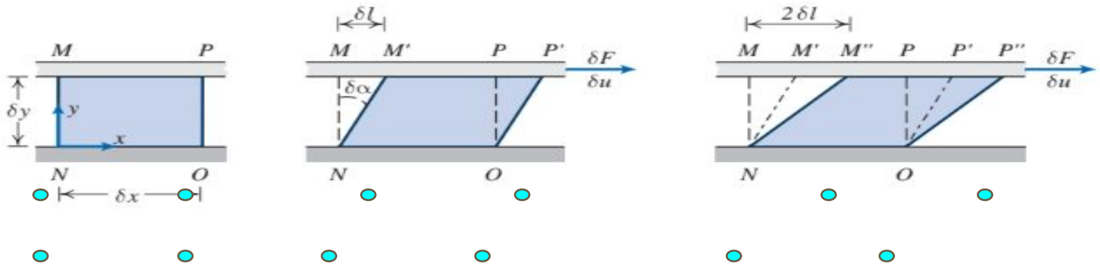
$$\Delta\alpha = \frac{\Delta l}{\Delta y}$$

$$\Delta l = \Delta u \, \Delta t$$

$$\boxed{\text{Rate of Deformation} = \frac{d\alpha}{dt} = \frac{du}{dy}}$$

Shear Stress and Strain rate: Newtonian Fluid

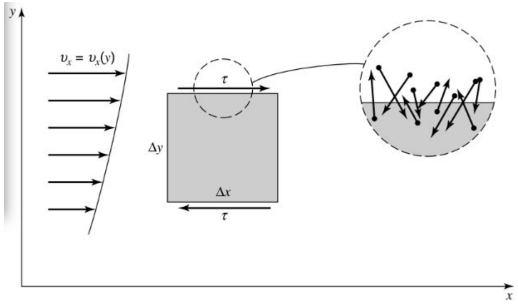




$$\tau_{xy} = \mu \frac{du}{dy}$$

Newtonian Fluid, One dimensional flow

Coefficient of Viscosity - Some points



1. Dependence of coefficient of viscosity on temperature is determined by the nature of intermolecular interaction of the fluid
2. In case of gas, the viscosity increases with increase in temperature; for ideal gas the dependence is the square root of the temperature (for real gas power may vary between 0.6 - 1)
3. Viscosity does not depend on pressure
4. In case of liquid, viscosity decreases with increase in temperature

Units of viscosity

$$\tau_{xy} = \mu \frac{du}{dy}$$

$[M][L]^{-1}[T]^{-1}$ → kg/m-s

kg/m-s

Pa-s

N-s/m^2

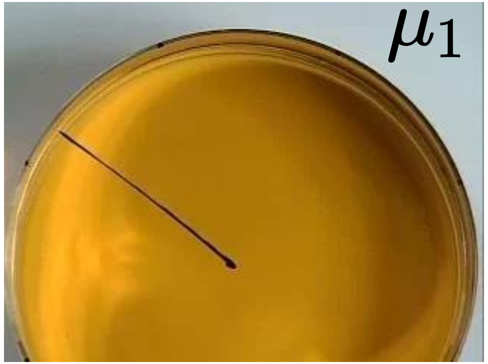
→ Poise (g/cm-s)

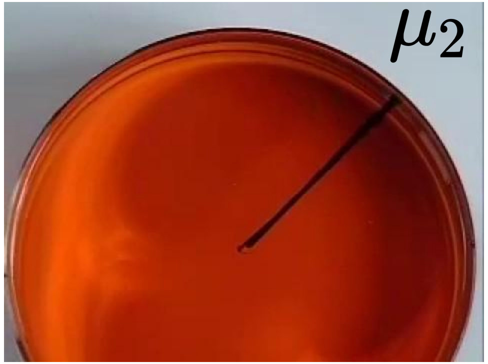
1 cP = 0.01 Poise

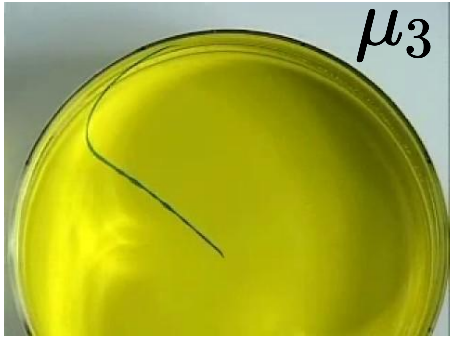
Viscosity of water (commonly used for ambient temperature is 1 cP)

Test Yourself

$$\tau_{xy} = \mu \frac{du}{dy}$$





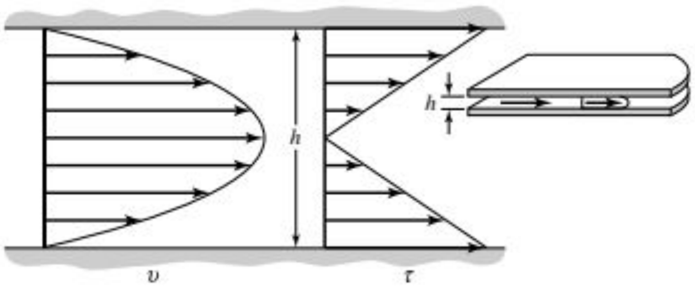


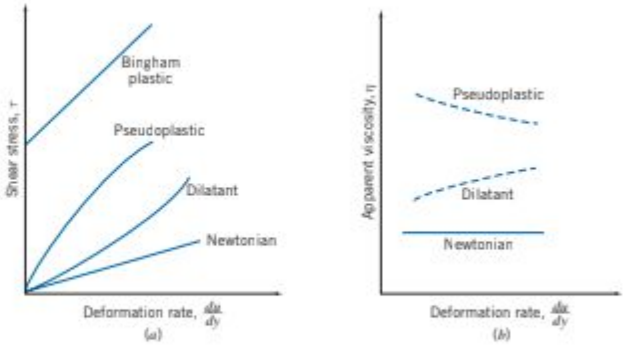
$$\mu_3 > \mu_2 > \mu_1$$

$$\mu_3 < \mu_1 < \mu_2$$

$$\mu_3 < \mu_2 < \mu_1$$

Shear Stress and Strain rate





Type	Description	Example
Real (Bingham) Plastic	Deforms linearly after the yield stress $\tau_{xy} = \tau_o + \mu \frac{du}{dy}$	Toothpaste
Pseudoplastic (shear-thinning)	"Viscosity" decreases as strain rate increases (n<1)	Paint, cream, hair gel
Dialatant (shear thickening)	"Viscosity" increases as strain rate increases (n>1)	Sand, silly putty

$$\tau_{xy} = \mu \frac{du}{dy} \left| \frac{du}{dy} \right|^{(n-1)}$$

Ref: Chapter 2

Point to point variation

Consider a function $f(x,y)$

$$df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$$

$$\frac{df}{ds} = \left(\frac{\partial f}{\partial x}\right)_y \frac{dx}{ds} + \left(\frac{\partial f}{\partial y}\right)_x \frac{dy}{ds}$$

$$\frac{df}{ds} = \left(\frac{\partial f}{\partial x}\right)_y \cos\alpha + \left(\frac{\partial f}{\partial y}\right)_x \sin\alpha$$

along an isoline $\frac{df}{ds} = 0;$

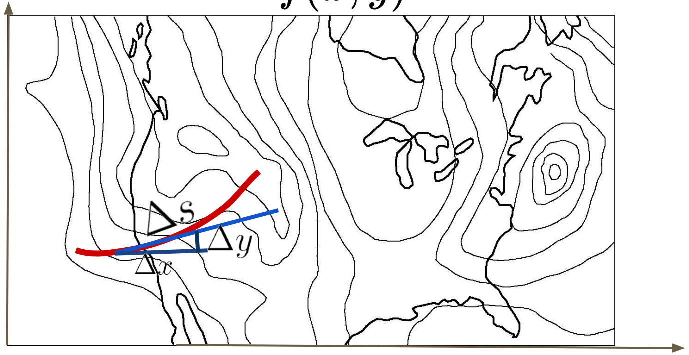
$$\tan\alpha^0 = -\frac{\left(\frac{\partial f}{\partial x}\right)_y}{\left(\frac{\partial f}{\partial y}\right)_x}$$

along a direction of maximum change

$$\frac{d}{d\alpha}\frac{df}{ds} = 0;$$

$$\tan\alpha^m = +\frac{\left(\frac{\partial f}{\partial y}\right)_x}{\left(\frac{\partial f}{\partial x}\right)_y}$$

$f(x,y)$



$$\tan\alpha^0 = -\cot\alpha^m$$

$$\tan\alpha^0 = -\tan\left(90^\circ - \alpha^m\right)$$

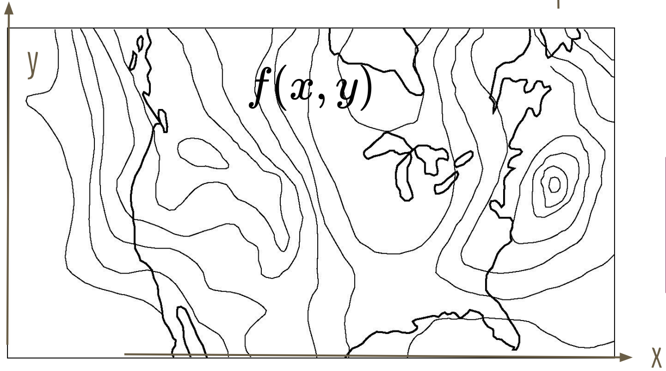
$$\boxed{\alpha^o - \alpha^m = 90^\circ}$$

Isolines are normal to the direction of the maximum changes

Point to point variation

Consider a function $f(x,y)$

$$\frac{df}{ds} = \left(\frac{\partial f}{\partial x}\right)_y \cos\alpha + \left(\frac{\partial f}{\partial y}\right)_x \sin\alpha$$



direction of maximum change

$$\tan\alpha^m = +\frac{\left(\frac{\partial f}{\partial y}\right)_x}{\left(\frac{\partial f}{\partial x}\right)_y}$$

$$\frac{df}{ds}\Big|_{\max} = \left(\frac{\partial f}{\partial x}\right)_y \cos\alpha^m + \left(\frac{\partial f}{\partial y}\right)_x \sin\alpha^m$$

$$\boxed{\frac{df}{ds}\Big|_{\max} = \sqrt{\left(\frac{\partial f}{\partial x}\right)_y^2 + \left(\frac{\partial f}{\partial y}\right)_x^2}}$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{e}_x + \frac{\partial f}{\partial y}\hat{e}_y$$

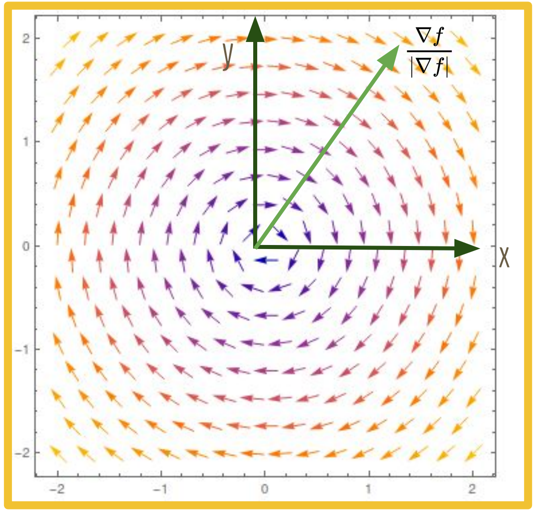
$$\boxed{|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)_y^2 + \left(\frac{\partial f}{\partial y}\right)_x^2}}$$

$$\sin\alpha^m = \frac{\tan\alpha^m}{\sqrt{1+\tan^2\alpha^m}} \qquad \sin\alpha^m = \frac{\left(\frac{\partial f}{\partial y}\right)_x}{\sqrt{\left(\frac{\partial f}{\partial x}\right)_y + \left(\frac{\partial f}{\partial y}\right)_x}}$$

$$\cos\alpha^m = \frac{1}{\sqrt{1+\tan^2\alpha^m}} \qquad \cos\alpha^m = \frac{\left(\frac{\partial f}{\partial x}\right)_y}{\sqrt{\left(\frac{\partial f}{\partial x}\right)_y + \left(\frac{\partial f}{\partial y}\right)_x}}$$

Point to point variation

$$y^2 + x^2 = f(x, y)$$



$$\text{Streamline Equation} : y^2 + x^2 = c$$

$$\nabla f = 2x\hat{e}_x + 2y\hat{e}_y$$ Radial Direction

Classification of Fluid Motion

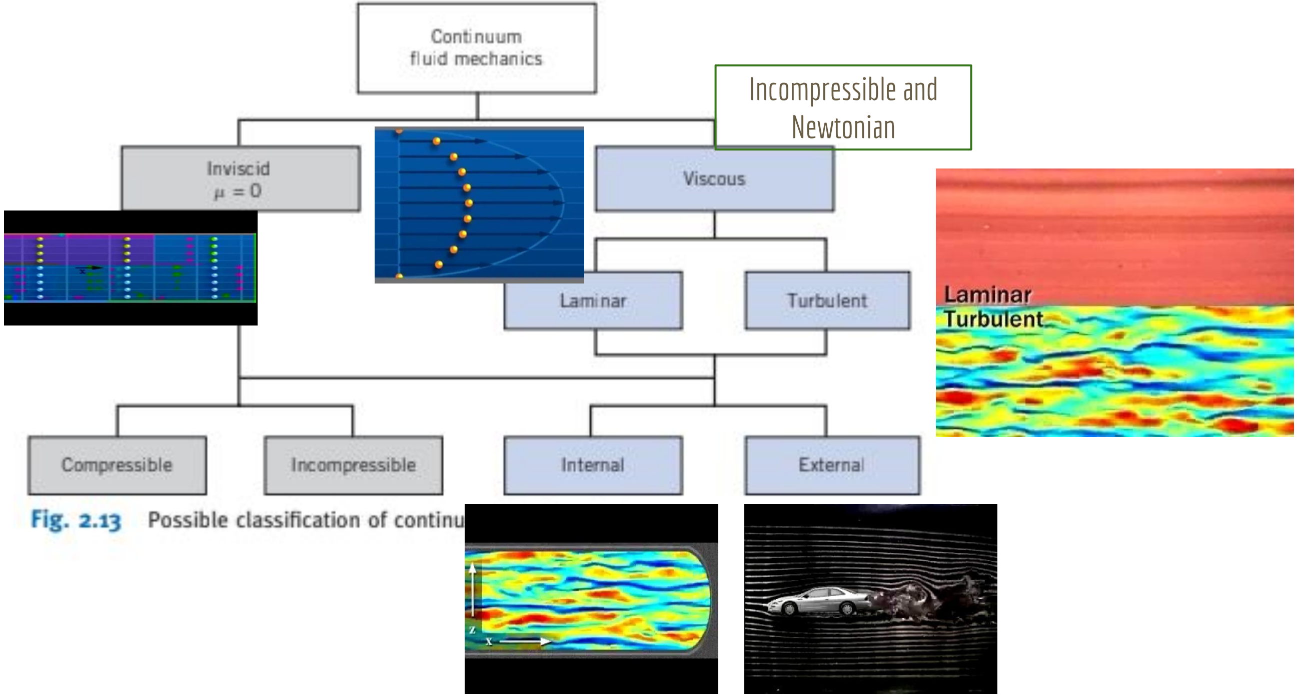


Fig. 2.13 Possible classification of continu

Summary and Way Forward

1. Fluids and their Properties:
   a. What is a fluid?
   b. Difference between Solid and Fluid
   c. System and Control Volume
   d. Units
   e. Fluid and Continuum
   f. (In)compressibility
   g. Properties at a point
   h. Classification
   i. Point to point variation
   j. Static Pressure
2. Fluid in motion

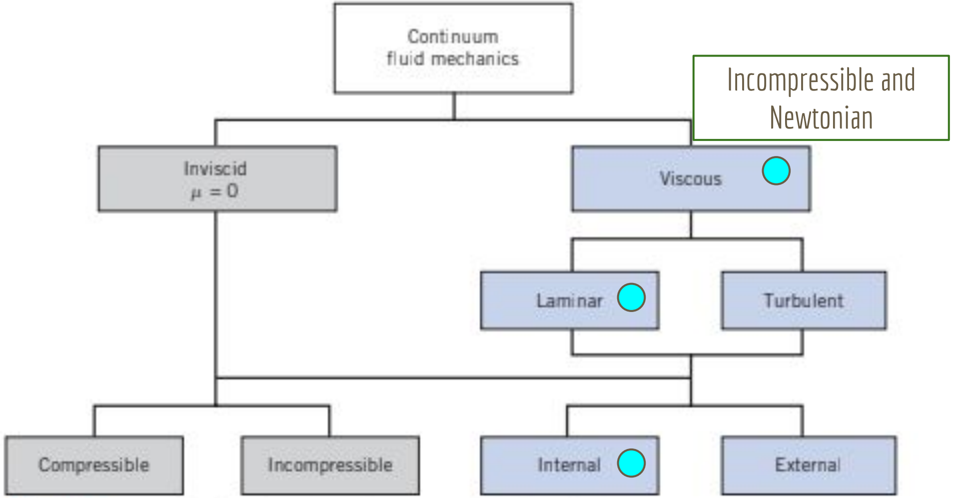
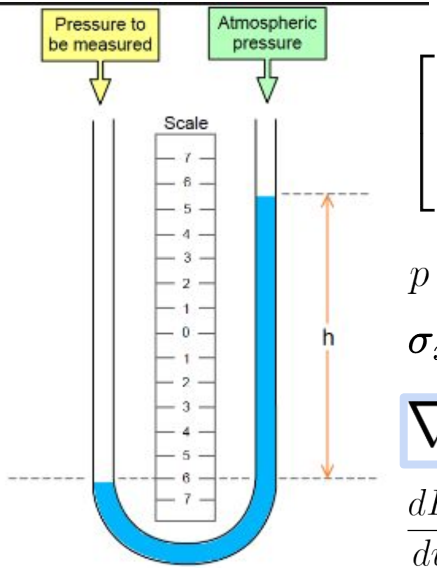


Fig. 2.13 Possible classification of continuum fluid mechanics.

Point Properties: Pressure

Fluid at rest



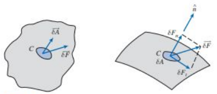


Fig. 2.6 The concept of stress in a continuum.

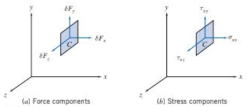


Fig. 2.7 Force and stress components on the element of area δA_x.

$$\bar{\bar{\sigma}} = \lim_{\delta A -> 0} \frac{\delta F}{\delta A} \hat{e}_F \hat{e}_A$$

$$\begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

$$p = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\boldsymbol{\sigma_{xx} = \sigma_{yy} = \sigma_{zz}}$$

$$\boxed{\nabla p = \rho \bar{g}}$$

$$\frac{dP}{dy} = -\rho g$$

$$\boxed{\Delta P = h \rho g}$$

1. Pressure is defined in compressive sense
2. Pressure is isotropic (*Derivation on next slide as additional learning material)*
3. Pressure is a scalar quantity
4. For incompressible fluid density does not depend on pressure
5. Principle of hydrostatics is important in:
    a. Computing forces on submerged bodies
    b. Developing instruments to measure pressure
    c. Determining forces on hydraulic systems

y

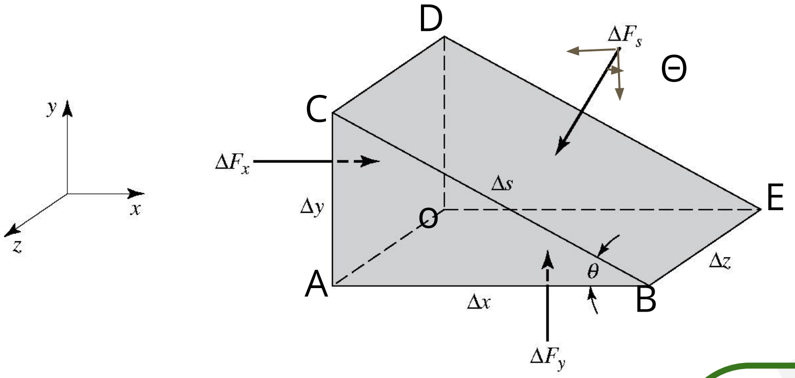
x

z

https://instrumentationtools.com/u-tube-manometer-principle/

Ref: Chapters 1 - 3

Pressure is isotropic



1. Newton's equation is applied to a fluid element as the fluid element approaches to infinitesimally small volume
2. Fluid at rest, so shear force is zero, only normal force acts on a surface
3. Force Balance on the differential fluid element ABCDEO

$$\Sigma F_x = 0;\ \Sigma F_y = 0$$

$$\Delta F_x - \Delta F_s \sin\theta = 0$$

$$\Delta F_x - \Delta F_s \frac{\Delta y}{\Delta s} = 0$$

$$\frac{\Delta F_x}{\Delta y \Delta z} - \frac{\Delta F_s}{\Delta s \Delta z} = 0$$

$$\boxed{\sigma_{xx} = \sigma_{ss}}$$

$$\Delta F_y - \Delta F_s \cos\theta - \rho g_y \frac{1}{2}(\Delta x \Delta y \Delta z) = 0$$

$$\Delta F_y - \Delta F_s \frac{\Delta x}{\Delta s} - \rho g_y \frac{1}{2}(\Delta x \Delta y \Delta z) = 0$$

$$\frac{\Delta F_y}{\Delta x \Delta z} - \frac{\Delta F_s}{\Delta s \Delta z} - \rho g_y \frac{1}{2}(\Delta y) = 0$$

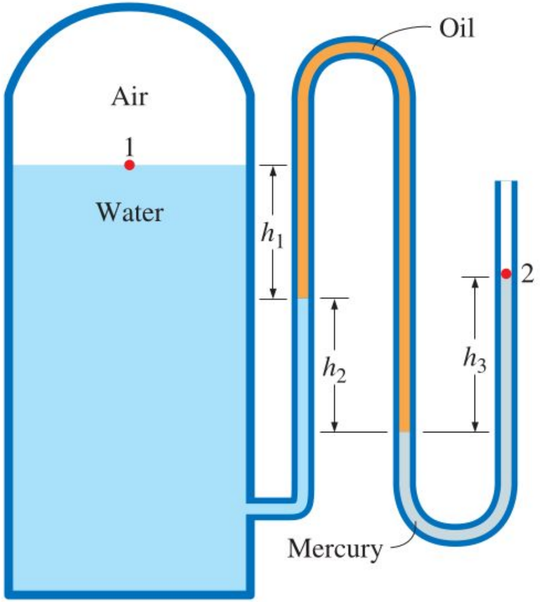
$$\boxed{\sigma_{yy} = \sigma_{ss}}$$

Vanishes as Δy ->0

Ref: Chapter 3

Practice Problem 1: Hydrostatic

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1= 0.1 m, h_2= 0.2 m, and h_3= 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively. (~130 kPa)

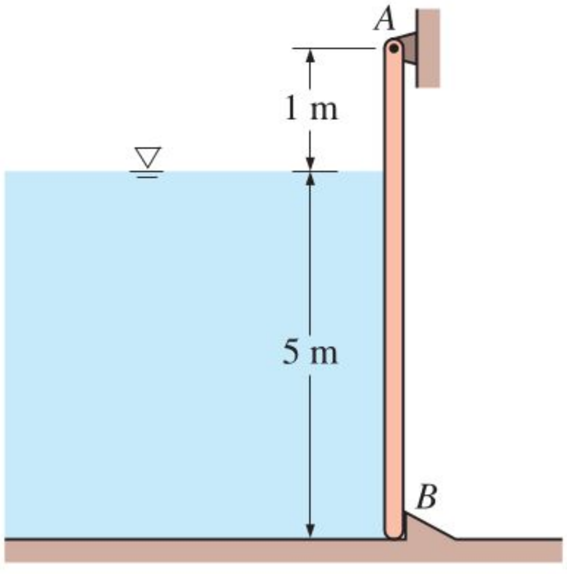


Ref: Cengel, Y., Cimbala, J.: Fluid Mechanics Fundamentals and Applications, McGraw-Hill Education.

Ref: Chapters 1 - 3

Practice Problem 2: Hydrostatic

A 6-m-high, 5-m-wide rectangular plate blocks the end of a 5-m-deep freshwater channel.. The plate is hinged about a horizontal axis along its upper edge through a point A and is restrained from opening by a fixed ridge at point B. Determine the force exerted on the plate by the ridge. (~450 kN)



Ref: Cengel, Y., Cimbala, J.: Fluid Mechanics Fundamentals and Applications, McGraw-Hill Education.

Ref: Chapters 1 - 3

Content - Lecture 1

1. Fluids and their Properties:
    a. What is a fluid?
    b. Difference between Solid and Fluid
    c. System and Control Volume
    d. Units
    e. Fluid and Continuum
    f. (In)compressibility
    g. Properties at a point
    h. Point to point variation
    i. Static Pressure
2. Fluid in motion

PHILIP J. PRITCHARD
Fox AND McDonald's
INTRODUCTION TO
Fluid Mechanics

Test Yourself

For the velocity field

$$\vec{v}(\vec{x}) = \begin{bmatrix} Ax^2y \\ Bxy^2 \\ 0 \end{bmatrix}$$

Obtain the equation of streamlines
Plot several streamlines in the first quadrant
$A = 2\ m^{-2}\ s^{-1}$, $B = 1\ m^{-2}\ s^{-1}$

Test Yourself

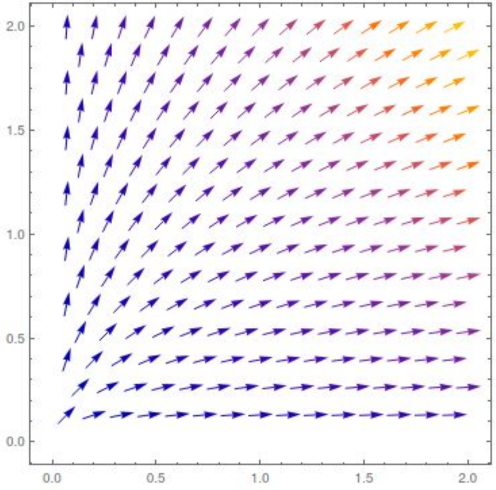
For the velocity field

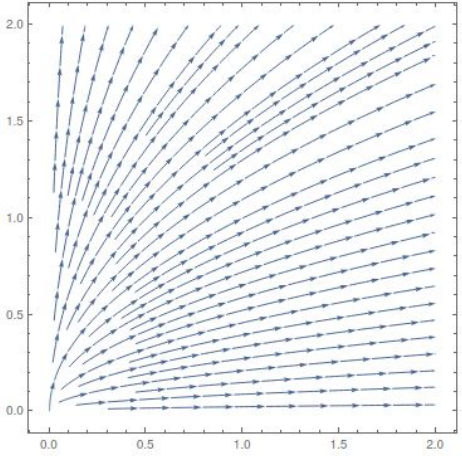
$$\vec{v}(\vec{x}) = \begin{bmatrix} Ax^2y \\ Bxy^2 \\ 0 \end{bmatrix}$$

$$\frac{dy}{dx} = \frac{y}{2x}$$

$$y = cx^{\frac{1}{2}}$$

Obtain the equation of streamlines
Plot several streamlines in the first quadrant
A = 2 m^{-2} s^{-1}, B = 1 m^{-2} s^{-1}





Mass and Momentum Conservation

Mass and Momentum Conservation

$$\boxed{\frac{dM}{dt}|_{\text{system}} = 0}$$

Mass of a system can not be created or destroyed

$$\boxed{\frac{d\vec{P}}{dt}|_{\text{system}} = \Sigma\vec{F}}$$

Rate of change of linear momentum is summation of forces acting on the system

Rate of change of the mass of the system

Rate of change of the amount of mass in the control volume

Net rate of inflow of mass to the control volume through the control surface

$$\frac{dM}{dt}|_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\,(1)\,dV + \int_{\text{CS}} \rho\,(1)\;(\vec{n}\cdot\vec{v})\,dA$$

Rate of change of the momentum of the system

Rate of change of the amount of the linear momentum in the control volume

Net rate of inflow of the linear momentum to the control volume through the control surface

$$\frac{d\vec{P}}{dt}|_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\,(\vec{v})\,dV + \int_{\text{CS}} \rho\,(\vec{v})\;(\vec{n}\cdot\vec{v})\,dA$$

Mass and Momentum Conservation

$$\boxed{\frac{dM}{dt}\Big|_{\text{system}} = 0}$$

$$\boxed{\frac{d\vec{P}}{dt}\Big|_{\text{system}} = \Sigma\vec{F}}$$

System to CV:

$$\frac{dM}{dt}\Big|_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\,(1)\,dV + \int_{\text{CS}} \rho\,(1)\,\left(\vec{n}\cdot\vec{v}\right)dA$$

$$\frac{d\vec{P}}{dt}\Big|_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\,(\vec{v})\,dV + \int_{\text{CS}} \rho\,(\vec{v})\,\left(\vec{n}\cdot\vec{v}\right)dA$$

Body force: Acts throughout the control volume; in the context of fluid mechanics, it is usually gravity

Surface force: Acts on the control surface, e.g., normal and the shear stresses

$$\frac{\partial}{\partial t}\int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho\,\left(\vec{n}\cdot\vec{v}\right)dA = 0$$

$$\frac{\partial}{\partial t}\int_{\text{CV}} \rho\vec{v} dV + \int_{\text{CS}} \rho\vec{v}\,\left(\vec{n}\cdot\vec{v}\right)dA = \Sigma\left(\vec{F}_s + \vec{F}_B\right)$$

$\vec{F_B}$: Body Force

$\vec{F_s}$: Surface Force

Mass Conservation

$$\frac{\partial}{\partial t}\int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho\left(\vec{n}\cdot\vec{v}\right) dA = 0$$

$$\textbf{@ steady state : } \int_{\text{CS}} \boldsymbol{\rho}\left(\vec{n}\cdot\vec{v}\right) d\boldsymbol{A} = \mathbf{0}$$

Convention followed: outward normal: +ve