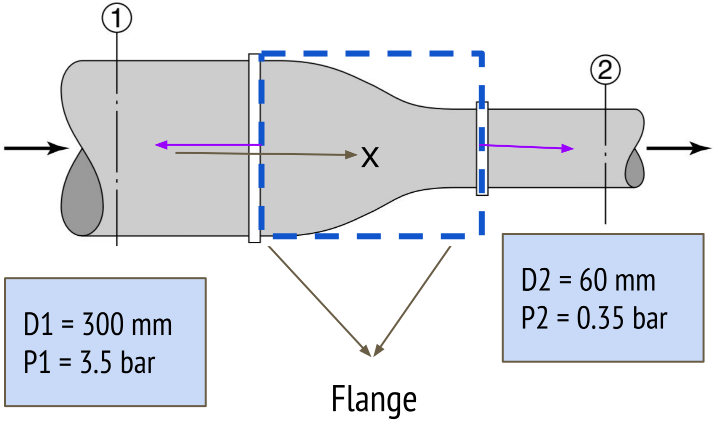
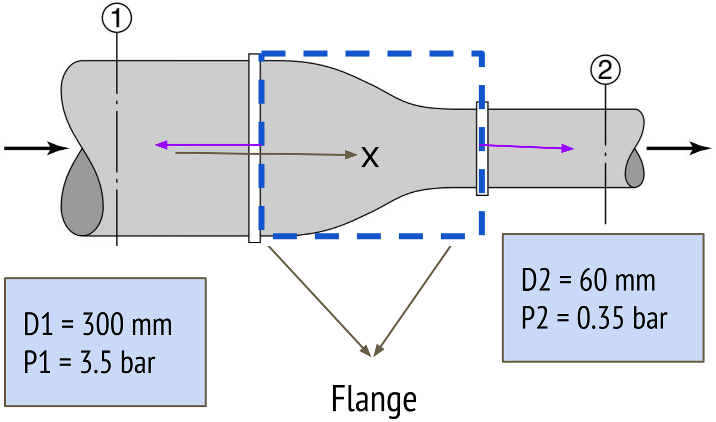
Momentum Conservation



Oil (sp gr. 0.8) flows through the reducer at a rate of 0.1 m^3/s. Assume that the velocity is uniform across the entry and exit cross-section of the reducer. Determine the force needed to hold the reducer in place.

Momentum Conservation

$$\boxed{\int_{\mathrm{CS}} \rho\vec{v}\,(\vec{n}\cdot\vec{v})\,dA = \Sigma\left(\vec{F}_s + \vec{F}_B\right)}$$



Oil (sp gr. 0.8) flows through the reducer at a rate of 0.1 m^3/s. Assume that the velocity is uniform across the entry and exit cross-section of the reducer. Determine the force needed to hold the reducer in place.

Assumptions:

1. Steady State
2. Shear stress is neglected

Selection of Control Volume:

1. Flange to Flange

Net rate of "x" momentum in = rate out - rate in

$$\Sigma F_x = \rho v_2^2 A_2 - \rho\, v_1^2\, A_1$$

Net "x" component of force acting on the CV:

$$\Sigma F_x = P_1\, A_1 - P_2\, A_2 - P_{\mathrm{atm}}\,(A_1 - A_2) + R_x$$

$$\Sigma F_x = P_{1,g}\, A_1 - P_{2,g}\, A_2 + R_x$$

Force acting on CV:

$$\boxed{Rx = -\left(P_{1,g} A_a - P_{2,g} A_2\right) + \left(\rho v_2^2 A_2 - \rho\, v_1^2\, A_1\right)}$$

Ref: Chapter 4

Momentum Conservation

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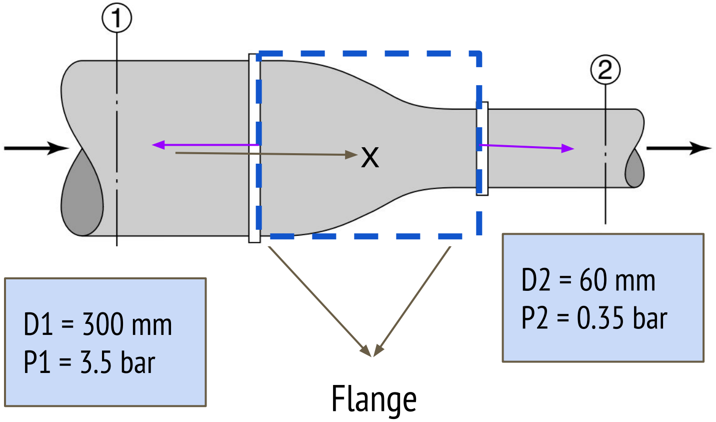

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$$\Sigma F_x = P_{1,g}\, A_1 - P_{2,g}\, A_2 + R_x$$

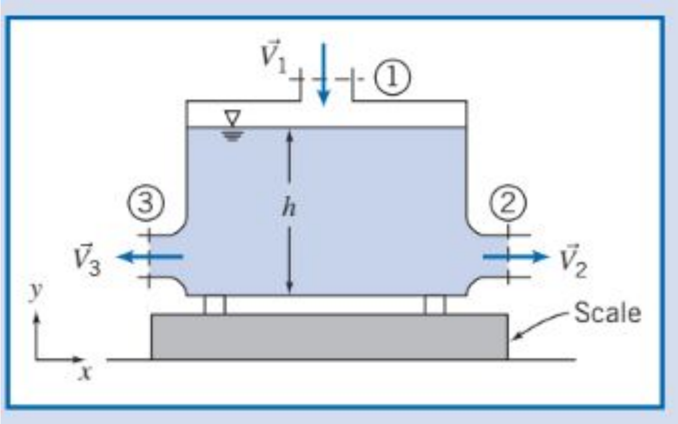
$$\boxed{Rx = -\left(P_{1,g} A_a - P_{2,g} A_2\right) + \left(\rho v_2^2 A_2 - \rho\, v_1^2\, A_1\right)}$$

Momentum Conservation

In a laboratory experiment, the water flow rate is to be measured catching the water as it vertically exits a pipe into an empty open tank that is on a zeroed balance. The tank is 10 m directly below the pipe exit, and the pipe diameter is 50 mm. One student obtains a flow rate by noting that after 60 s the volume of water (density 1000 kg/m^3) in the tank was 2 m^3. Another student obtains a flow rate by reading the instantaneous weight accumulated of 3150 kg indicated at the 60-s point. Find the mass flow rate each student computes. Why do they disagree? Which one is more accurate? Show that the magnitude of the discrepancy can be explained by any concept you may have.

Momentum Conservation

A metal container 0.6m high, with an inside cross-sectional area of 0.1 m^2 weighs 2.5 kg when empty. The container is placed on a scale and water flows in through an opening in the top and out through the two equal-area openings in the sides. Under steady flow conditions, the height of the water in the tank is h 0.57 m. Your teacher claims that the scale will read the weight of the volume of water in the tank plus the tank weight, i.e., that we can treat this as a simple statics problem. You disagree, claiming that a fluid flow analysis is required. Who is right, and what does the scale indicate?



Data :
$A_1 = A_2 = A_3 = 9 \times 10^{-3}\mathrm{m}^2$
$V_1 = 3(-\hat{j})\mathrm{m/s}$

Assumptions:
Steady Flow (quasi-steady)
Incompressible fluid
Uniform velocity (average)

$$\mathrm{y-momentum\ balance:}$$
$$F_{BY} + F_{SY} = \rho v_1^2 A_1$$

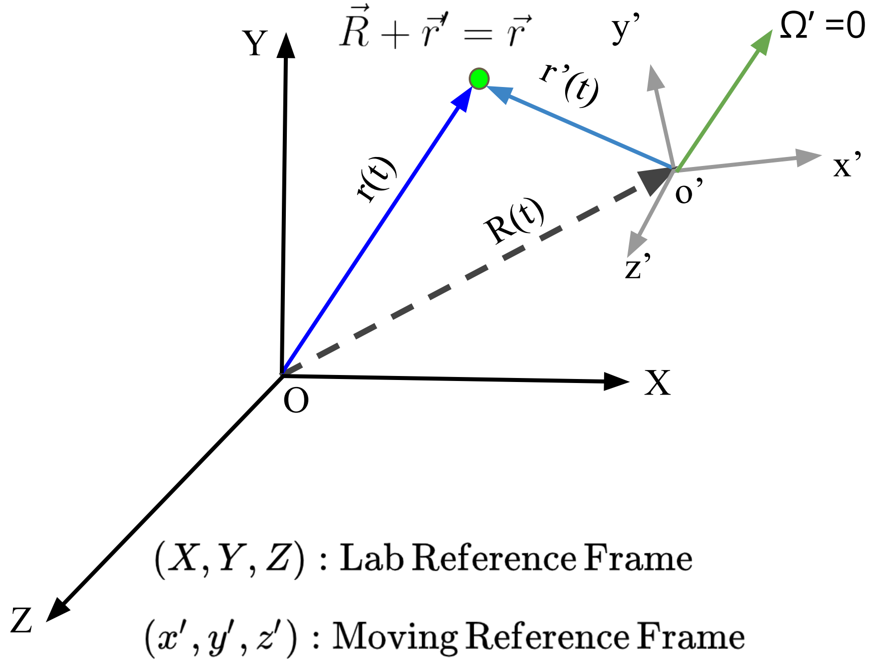
$$R_y - M_t - \rho_W A\, h_w = \rho v_1^2 A_1 / g$$

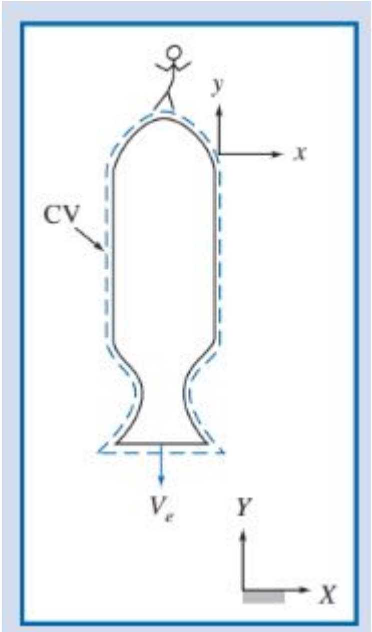
$$R_y = M_t + \rho_W A\, h_w + \rho v_1^2 A_1 / g$$

$$R_y = 67.7kg$$

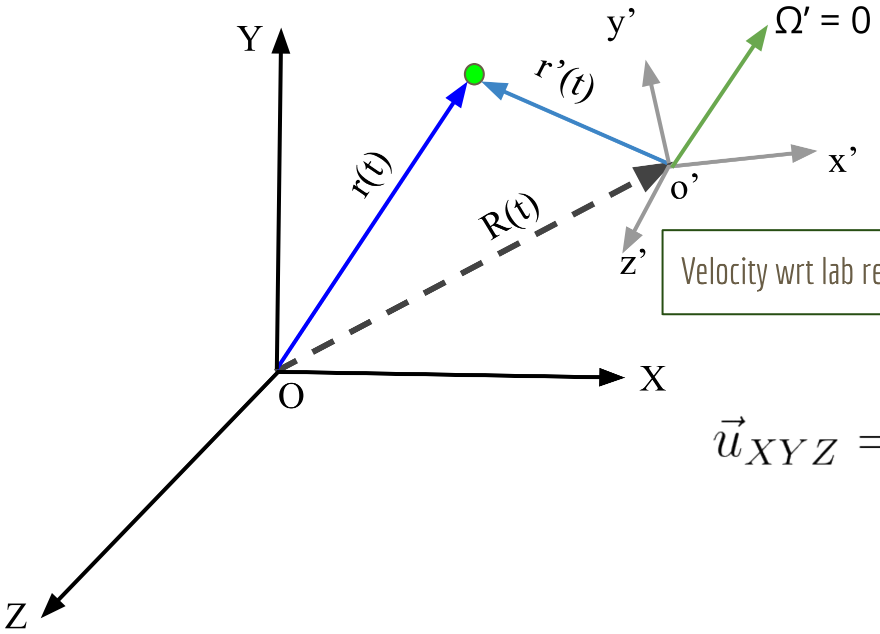
Example 4.5 from Fox-McDonald, 8th Edition

Non-inertial frame of reference (Control Volume with Rectilinear Acceleration)





Non-inertial frame of reference: derivation



$$\boxed{\vec{r} = \vec{R} + \overrightarrow{r'}}$$

Velocity wrt lab reference frame:

$$\vec{u}_{XYZ} = \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} + \frac{d\vec{r'}}{dt}$$

$$\vec{u}_{XYZ} = \vec{U}_{\mathrm{of\,x'y'z'\,rel\,to\,XYZ}} + \frac{d}{dt}\left[x'\hat{e}_{x'} + y'\,\hat{e}_{y'} + z'\,\hat{e}_{z'}\right]$$

Non-inertial frame of reference: derivation

$$\vec{u}_{XYZ} = \vec{U}_{\mathrm{of\,x'y'z'\,rel\,to\,XYZ}} + \frac{d}{dt}\left[x'\hat{e}_{x'} + y'\,\hat{e}_{y'} + z'\,\hat{e}_{z'}\right]$$

$$\vec{u}_{XYZ} = \vec{U}_{\mathrm{of\,x'y'z'\,rel\,to\,XYZ}} + \left[\frac{dx'}{dt}\hat{e}_{x'} + \frac{dy'}{dt}\,\hat{e}_{y'} + \frac{dz'}{dt}\,\hat{e}_{z'}\right] + \left[x'\frac{d\hat{e}_{x'}}{dt} + y'\frac{d\hat{e}_{y'}}{dt} + z'\frac{d\hat{e}_{z'}}{dt}\right]$$

Only rectilinear acceleration

Rotating frame of reference: Acceleration

$$\vec{u}_{XYZ} = \vec{U}_{\mathrm{of\,x'y'z'\,rel\,to\,XYZ}} + \vec{u}'_{x'y'z'}$$

Velocity of a point wrt the lab reference frame

Velocity of the moving reference frame wrt the lab reference frame

Velocity of the point wrt the moving reference frame

Rotating frame of reference: Acceleration

$$\vec{u}_{XYZ} = \vec{U}_{\text{of x'y'z' rel to XYZ}} + \vec{u'}_{x'y'z'}$$

$$\vec{a}_{XYZ} = \frac{d}{dt}\left(\vec{u}_{XYZ}\right)$$

$$\frac{d}{dt}\left(\vec{u}_{XYZ}\right) = \frac{d}{dt}\left[\vec{U}_{\text{of x'y'z' rel to XYZ}} + \vec{u'}_{x'y'z'}\right]$$

$$\boxed{\frac{d}{dt}\left(\vec{u}_{XYZ}\right) = \frac{d\vec{U}}{dt} + \vec{a'}_{x'y'z'}}$$

$$\vec{a}_{XYZ} = \vec{a}_{rf} + \vec{a}_{x'y'z'}$$

Non-inertial frame of reference: Acceleration

$$\vec{a}_{XYZ} = \vec{a}_{rf} + \vec{a}_{x'y'z'}$$

$\vec{a}_{rf}$: Acceleration of
the particle
wrt lab reference frame

$\vec{a}_{rf}$: Acceleration of
the moving reference frame
wrt lab reference frame

$\vec{a}_{rf}$: Acceleration of
the particle
wrt moving reference frame

Non-inertial frame of reference: Acceleration

$$\vec{a}_{XYZ} = \vec{a}_{rf} + \vec{a}_{x'y'z'}$$

Newton's second law of motion (inertial frame of reference)

$$\boxed{\left.\frac{d\vec{P}_{XYZ}}{dt}\right|_{\text{system}} = \Sigma\vec{F}}$$

where, $\boxed{\vec{P}_{XYZ} = \int_{CV} \rho \vec{v}_{XYZ} dV}$

$$\boxed{\Sigma\vec{F} = \frac{d}{dt}\int_{M_{\text{sys}}} \vec{v}_{XYZ}\, dm}$$

$$\boxed{\Sigma\vec{F} = \int_{M_{\text{sys}}} \frac{d\vec{v}_{XYZ}}{dt} dm}$$

$$\vec{P}_{x'y'z'} = \int_{CV} \rho \vec{v}_{x'y'z'} dV$$

$$\frac{d\vec{P}_{XYZ}}{dt} = \frac{d\vec{P}_{x'y'z'}}{dt} + \int_{M\text{sys}} \vec{a}_{rf}\, dm$$

$$\Sigma\,\vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v}_{x'y'z'} dV + \int_{CS} \left(\rho\, \vec{v}_{x'y'z'}\right)\, \left(\vec{n}\cdot\vec{v}_{x'y'z'}\right) + \int_{CV} \vec{a}_{rf}\, \rho dV$$

Non-inertial frame of reference: Acceleration

$$\vec{a}_{XYZ} = \vec{a}_{rf} + \vec{a}_{x'y'z'}$$

Newton's second law of motion (inertial frame of reference) $\boxed{\frac{d\vec{P}_{XYZ}}{dt}\Big|_{\text{system}} = \Sigma\vec{F}}$ where, $\boxed{\vec{P}_{XYZ} = \int_{CV} \rho\vec{v}_{XYZ} dV}$

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$$\boxed{\Sigma\vec{F} = \int_{M_{\text{sys}}} \frac{d\vec{v}_{XYZ}}{dt} dm}$$

$$\boxed{\Sigma\vec{F} = \int_{M_{\text{sys}}} \frac{d\vec{v}_{x'y'z'}}{dt} dm + \int_{M_{\text{sys}}} \vec{a}_{rf}\, dm}$$

Note that: $\boxed{\vec{P}_{x'y'z'} = \int_{CV} \rho\vec{v}_{x'y'z'} dV}$

$$\frac{d\vec{P}_{XYZ}}{dt} = \frac{d\vec{P}_{x'y'z'}}{dt} + \int_{M\text{sys}} \vec{a}_{rf}\, dm$$

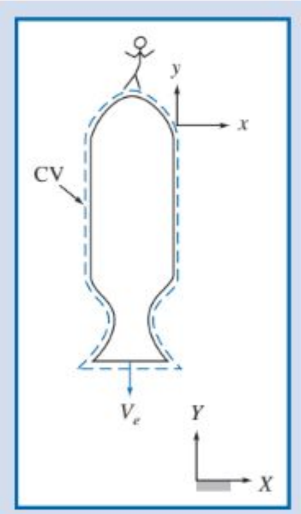
$$\boxed{\Sigma\,\vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho\vec{v}_{x'y'z'} dV + \int_{CS} (\rho\,\vec{v}_{x'y'z'})\,(\vec{n}\cdot\vec{v}_{x'y'z'}) + \int_{CV} \vec{a}_{rf}\,\rho dV}$$

Non-inertial frame of reference: Acceleration

$$\Sigma\,\vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v}_{x'y'z'} dV + \int_{CS} \left(\rho\,\vec{v}_{x'y'z'}\right)\left(\vec{n}\cdot\vec{v}_{x'y'z'}\right) + \int_{CV} \vec{a}_{rf}\,\rho dV$$

$$\Sigma\,\vec{F} - \int_{CV} \vec{a}_{rf}\,\rho dV = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v}_{x'y'z'} dV + \int_{CS} \left(\rho\,\vec{v}_{x'y'z'}\right)\left(\vec{n}\cdot\vec{v}_{x'y'z'}\right)$$

Non-inertial frame of reference: Acceleration



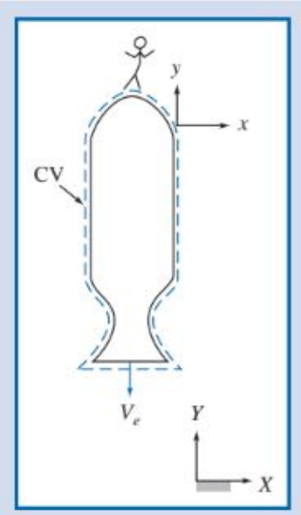
A rocket with an initial mass of 400 kg is to be launched vertically. The fuel consumption rate is 5 kg/s. The rocket ejects gas to the atmosphere at 3500 m/s relative to the rocket. Determine the initial acceleration of the rocket and the speed of the rocket after 10s.

Given:

1. Small rocket accelerates vertically from rest
2. Initial mass: 400 kg
3. Air resistance may be neglected
4. Rate of fuel consumption: 5 kg/s
5. Exhaust velocity: 3500 m/s, relative to rocket,leaving at atmospheric pressure

$$\Sigma \vec{F} - \int_{CV} \vec{a}_{rf}\, \rho dV = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v}_{x'y'z'} dV + \int_{CS} \left(\rho\, \vec{v}_{x'y'z'}\right)\left(\vec{n}\cdot \vec{v}_{x'y'z'}\right)$$

Non-inertial frame of reference: Acceleration



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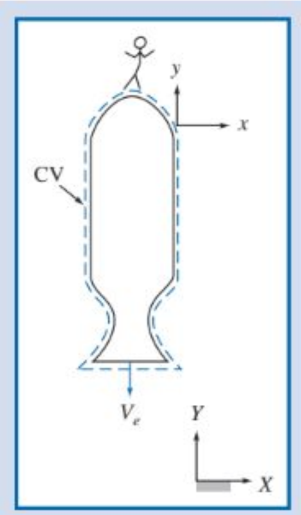
$$\cancel{F_{sy}} + F_{BY} - \int_{CV} a_y \rho dV =$$

$$\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv + \int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA$$

Non-inertial frame of reference: Acceleration



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$$F_{BY} - \int_{CV} a_y \rho dV =$$
$$\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv + \int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA$$

First Term: CV is a function of time

$$F_{BY} = -gM_{CV}$$
$$M_{CV} = m_i - \dot{m}_c \, t$$

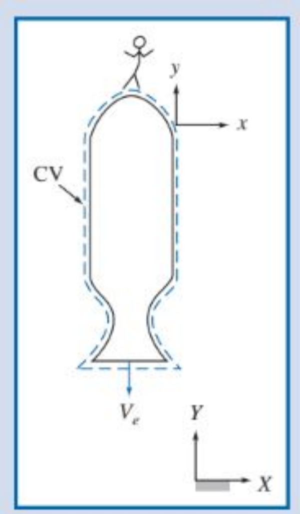
$$F_{BY} = -g(m_i - \dot{m}_c t)$$

$\dot{m}_c$: fuel consumption rate

Second Term: Like a solid body acceleration

$$\int_{CV} a_y \rho dV = a_y (m_i - \dot{m}_c t)$$

Non-inertial frame of reference: Acceleration



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$$F_{BY} - \int_{CV} a_y \rho dV =$$

$$\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv + \int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA$$

Third Term:

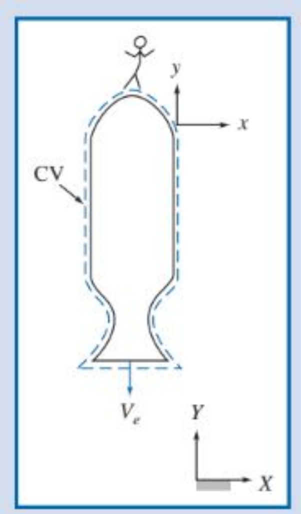
$$\boxed{\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv = 0}$$

Since the exit velocity does not change with time

Fourth Term:

$$\int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA = -V_e \dot{m}_e$$

Non-inertial frame of reference: Acceleration



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$$\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv + \int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA$$

$$F_{BY} = -g(m_i - \dot{m}_c t)$$

$$\int_{CV} a_y \rho dV = a_y (m_i - \dot{m}_c t)$$

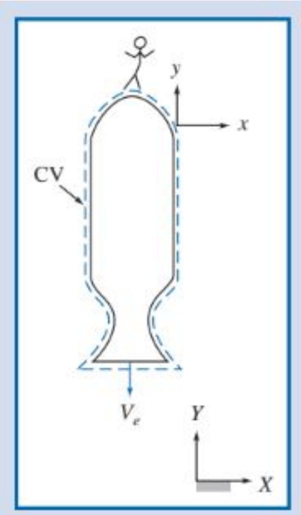
$$\boxed{\frac{\partial}{\partial t} \int_{CV} \rho v_{y,xyz} dv = 0}$$

$$\int_{CS} \rho V_{xyz} (n \cdot v_{xyz}) dA = -V_e \dot{m}_e$$

Instantaneous Acceleration :

$$a_y = \frac{V_e \dot{m}_e}{m_i} - g$$
$$= 43.75 \text{ms}^{-2}$$

Non-inertial frame of reference: Acceleration



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$$F_{BY} - \int_{CV} a_y \rho dV = \frac{\partial}{\partial t}\int_{CV} \rho v_{y,xyz} dv + \int_{CS} \rho V_{xyz}(n \cdot v_{xyz}) dA$$

$$F_{BY} = -g(m_i - \dot{m}_c t)$$

$$\int_{CV} a_y \rho dV = a_y(m_i - \dot{m}_c t)$$

$$\boxed{\frac{\partial}{\partial t}\int_{CV} \rho v_{y,xyz} dv = 0}$$

$$\int_{CS} \rho V_{xyz}(n \cdot v_{xyz}) dA = -V_e \dot{m}_e$$

After 10s :

$$a_y = \frac{V_e \dot{m}_e}{m_i - \dot{m}_e\, t} - g$$

$$\frac{dv_y}{dt} = \frac{V_e \dot{m}_e}{m_i - \dot{m}_e\, t} - g$$

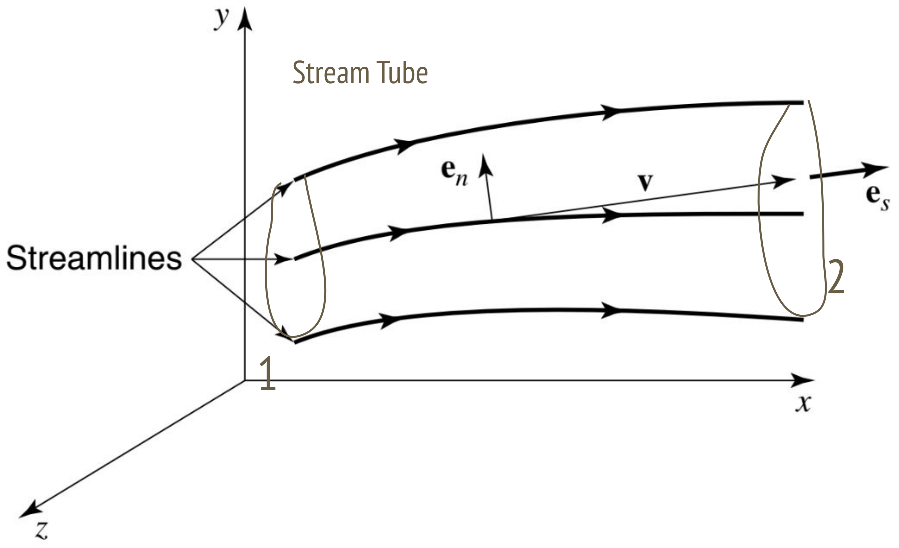
$$v_y = V_e \ln \frac{m_i}{m_i - \dot{m}_e t} - gt$$

$$v_y = 367.4 \text{m/s}$$

Bernoulli Equation

Statement:
For a steady process with an inviscid fluid
Along a streamline the total mechanical energy is conserved

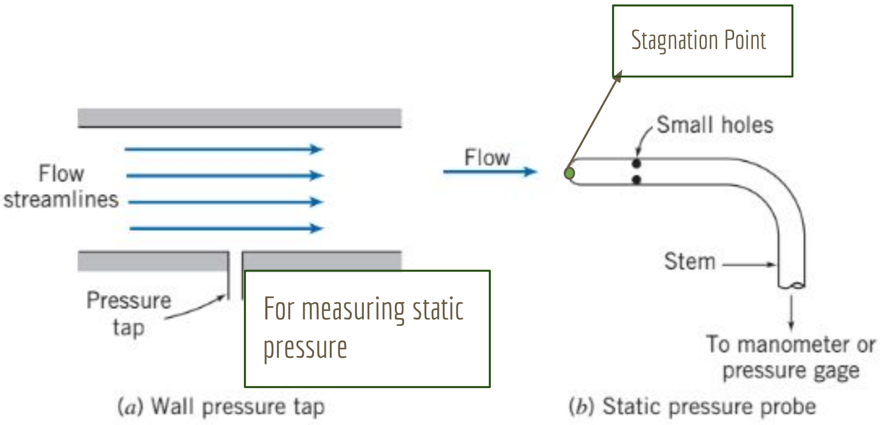
$$\boxed{\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \mathrm{Const}}$$



Static Pressure Stagnation Pressure and Dynamic Pressure

Statement:
For a steady process with an inviscid fluid
Along a streamline the total mechanical energy is conserved

$$\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \text{Const}$$



1. Static Pressure: Measured by the manometer or any pressure gauge
2. Stagnation Pressure: Pressure of the fluid when fluid decelerated to zero velocity
3. Dynamic Pressure: $\frac{1}{2}\rho v^2$

Along a streamline going through the stagnation point

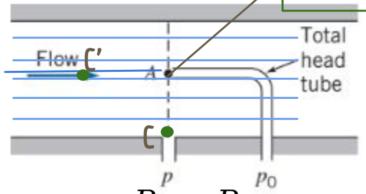
$$\frac{P_o}{\rho} + \frac{1}{2}v_o^2 = \frac{P}{\rho} + \frac{1}{2}v^2$$

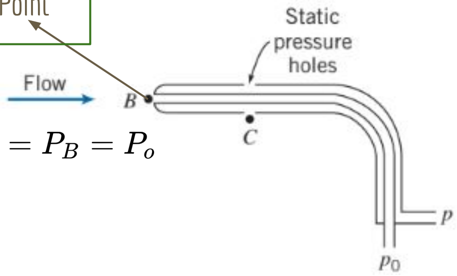
$$v = \sqrt{2\,\frac{P_o - P}{\rho}}$$

Stagnation Pressure Probe (Pitot Tube - Pea toe tube)

$$\boxed{\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \text{Const}}$$

Stagnation Point





$$P_A = P_B = P_o$$

$$P_C = P$$
$$v_C = v$$

Note that since the fluid is inviscid and the cross-section in uniform:

$$\frac{P_{C'}}{\rho} + \frac{1}{2}v_{C'}^2 = \frac{P_C}{\rho} + \frac{1}{2}v_c^2$$

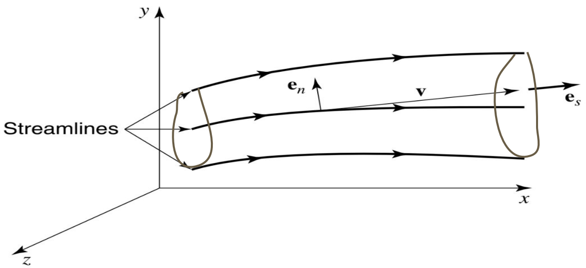
Along a streamline going through the stagnation point

$$\boxed{\frac{P_o}{\rho} + \frac{1}{2}v_o^2 = \frac{P}{\rho} + \frac{1}{2}v^2}$$

$$\boxed{v = \sqrt{2\,\frac{P_o - P}{\rho}}}$$

Discussion on Bernoulli Equation

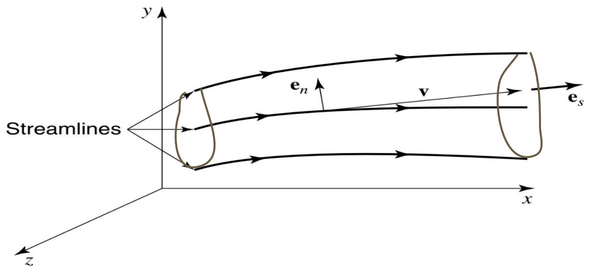
$$\boxed{\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \mathrm{Const}}$$



1. Bernoulli Equation is obtained by (any one of the way):
   a. Integrating the momentum conservation equation along the streamline
   b. Conserving the mechanical energy (derived from the momentum conservation equation)
   c. From the first law of thermodynamics for special cases when the change in internal energy is exactly compensated by the transfer of heat or in other words, the mechanical and the thermal energy are independently balanced
2. For fluids with non-zero viscosity mechanical energy is converted into undesired thermal energy, termed as dissipation (losses)

Discussion on Bernoulli Equation

Statement:
For a steady process with an inviscid fluid
Along a streamline the total mechanical energy is conserved



$$\boxed{\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \text{Const}}$$

For a real fluid

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gz_2 + h_{\text{loss}}$$

Energy Grade Line and Hydraulic Grade Line

For a real fluid

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gz_2 + h_{\text{loss}}$$

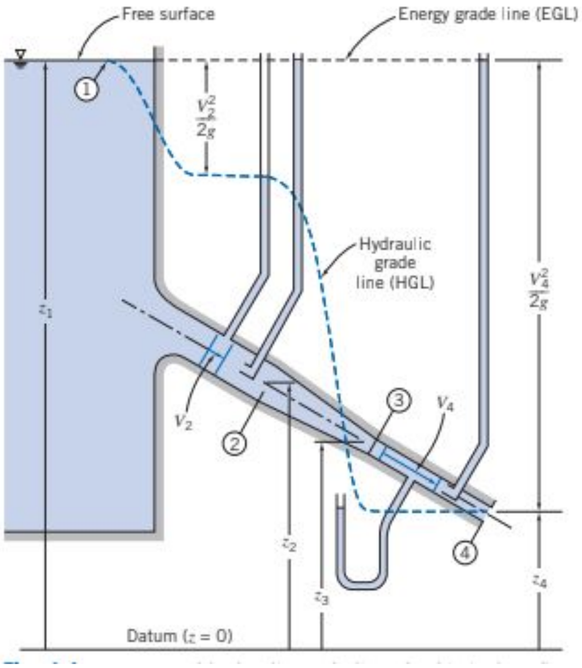
For an ideal fluid

$$\boxed{\left(\frac{1}{2}v^2 + gz + \frac{P}{\rho}\right) = \text{Const}}$$

Divide by g and write in the dimension of Length [L]

$$\frac{P}{\rho\, g} + \frac{1}{2\, g}v^2 + z = H$$

1. Since the terms have dimension of [L], this is also termed as “head”;
2. H is the total head of the flow
3. This is also known as Energy Grade Line (Graphical representation)
4. For incompressible, inviscid fluid, EGL is constant



Energy Grade Line and Hydraulic Grade Line

For a real fluid

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gz_2 + h_{\text{loss}}$$

For an ideal fluid

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