Unstructured Moving Mesh

Department of Physics, National Taiwan University

計算天文物理（ASPHYS7020）

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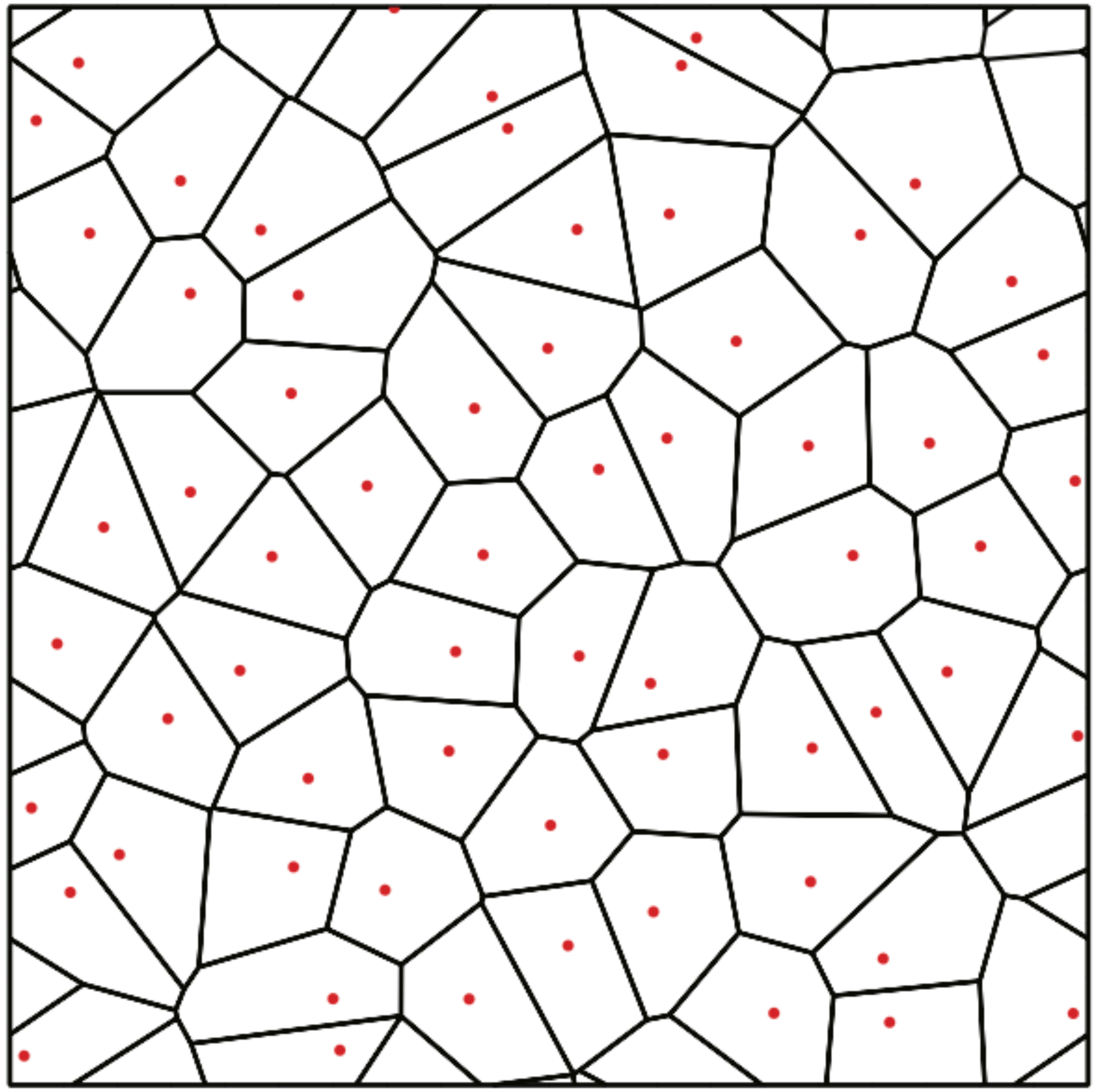
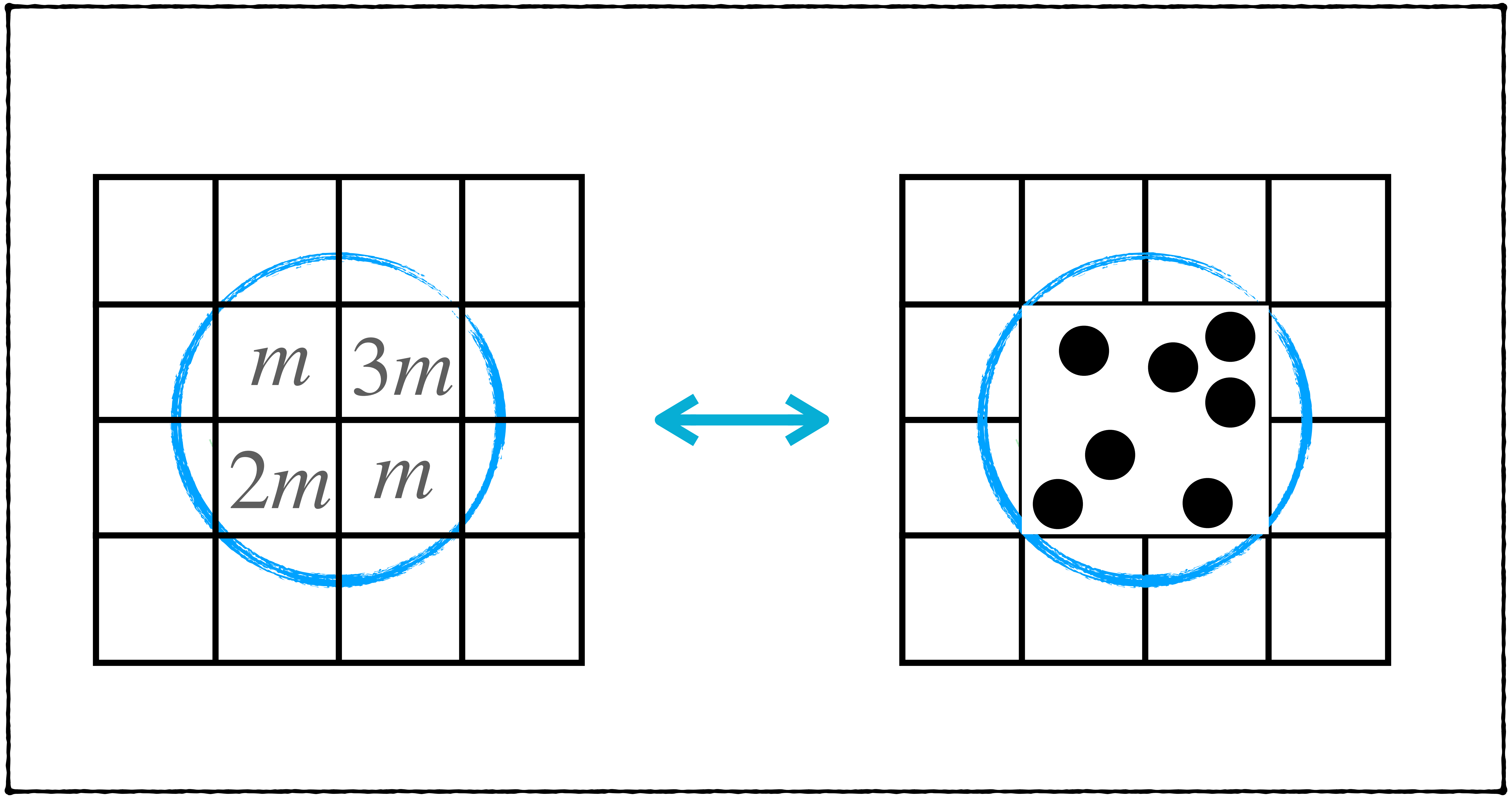
Outline

- Initial condition:
  - Voronoi diagram
  - Density distribution
  - Other settings
- Hydrodynamical scheme
  - Euler equation
  - Update step
- Result
- Future

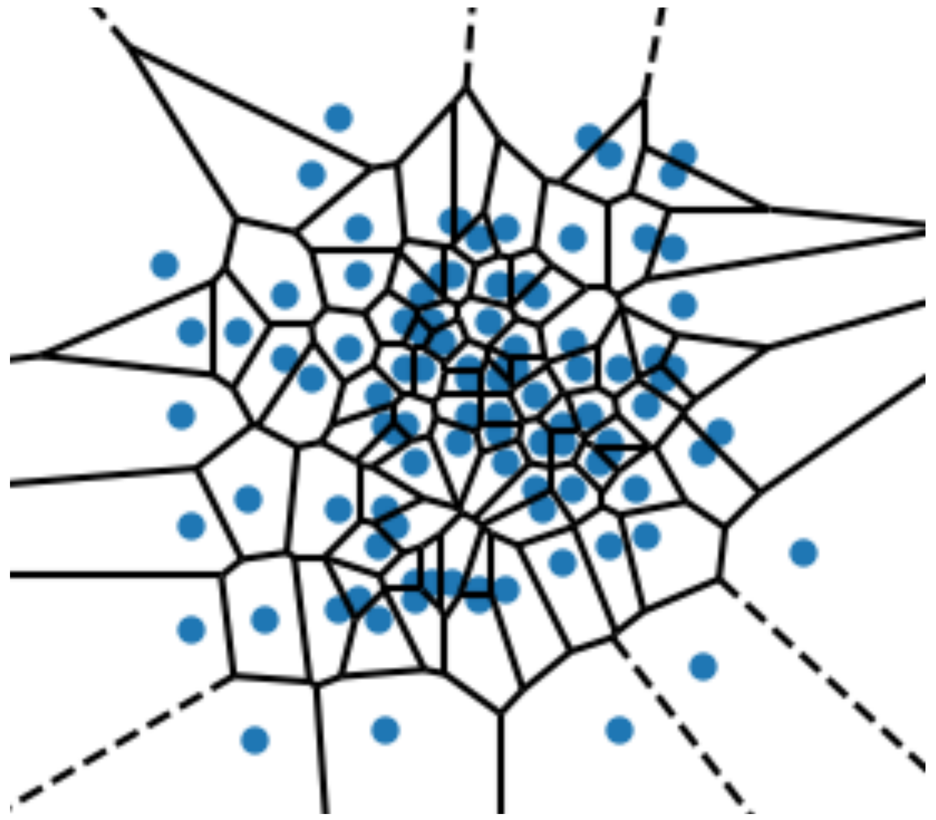
* **Voronoi diagram**



* Density distribution



$$if(random < Ne^{-(x^2+y^2)}) : \ set \ a \ point \ at \ (x, y)$$



Other definitions

- State vector:

$$U = \begin{pmatrix} \rho \\ \rho \overrightarrow{w} \\ \rho e \end{pmatrix}_{4\times 1} = \begin{pmatrix} \rho \\ \rho \overrightarrow{w} \\ \rho u + \frac{1}{2}\rho(\overrightarrow{w})^2 \end{pmatrix} \qquad \begin{pmatrix} \rho \\ \overrightarrow{w} \\ e \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 50 \end{pmatrix}$$

ρ : the mass density. $\overrightarrow{w}$: the velocity field. e : total energy per unit mass. u : the thermal energy per unit mass

- Flux function:

$$F(U) = \begin{pmatrix} \rho \overrightarrow{w} \\ \rho \overrightarrow{w}\,\overrightarrow{w}^T + P \\ (\rho e + P)\overrightarrow{w} \end{pmatrix}_{4\times 2}$$

$P = (\gamma - 1)\rho u$: the pressure of the fluid.

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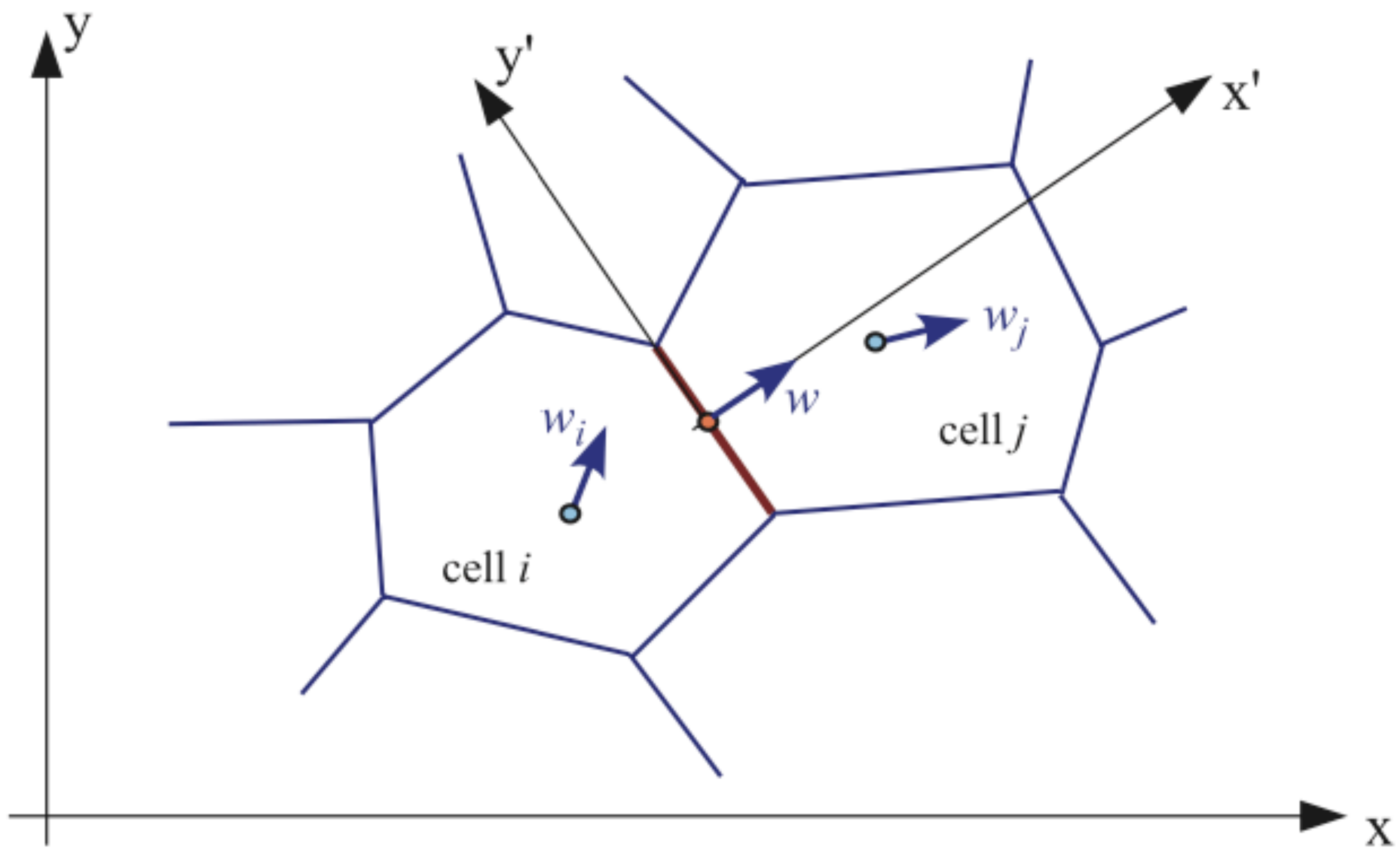
Euler equation

$$\frac{\partial U}{\partial t} + \nabla \cdot F = 0$$

Emphasize : their character as conservation laws for mass, momentum and energy.

$$\int (\nabla \cdot F)\, dV = \oint_{\partial V} (F \cdot \hat{n})\, dA$$

$$Q_i = \begin{pmatrix} m_i \\ \overrightarrow{p}_i \\ E_i \end{pmatrix} = \int_{V_i} U\, dV \qquad \frac{dQ_i}{dt} = -\oint_{\partial V_i} [F(U) - U \overrightarrow{w}^T]\, d\hat{n}$$



- Lax-Friedrichs Scheme for Hydro:

$$F(U_{ij}) = \frac{1}{2}[F(u_i^n) + F(u_j^n) - \frac{\Delta x}{\Delta t}(u_i^n - u_j^n)]$$

$$\Delta x = |\vec{r}_j - \vec{r}_{j-1}|$$

$$\Delta t \le \frac{\Delta x}{|\overrightarrow{w}| + C_s}$$

- The averaged flux across the face i-j:

$$F_{ij} = \frac{1}{A_{ij}} \int_{A_{ij}} [F(U_{ij}) - U_{ij} \overrightarrow{w}^T]\, d\overrightarrow{A}_{ij}$$

Let $\overrightarrow{A}_{ij}$ describe the oriented area of the face between cells i and j (pointing from i to j).

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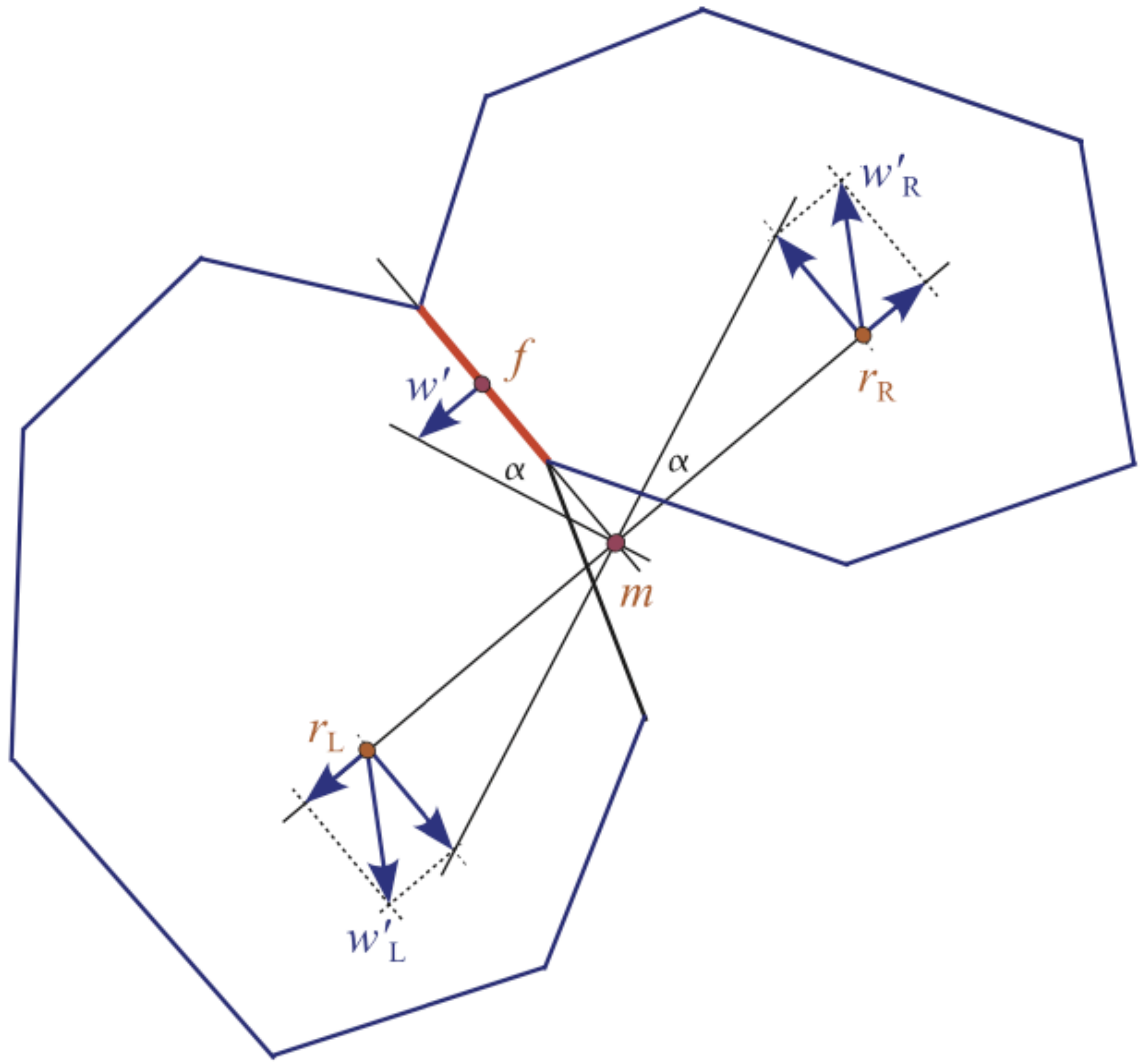
- The full velocity $\vec{w}$ of the face :

$$\vec{w}' = \frac{(\vec{w}_L - \vec{w}_R)\cdot[\vec{f} - \frac{(\vec{r}_R + \vec{r}_L)}{2}]}{|\vec{r}_R - \vec{r}_L|}\frac{(\vec{r}_R - \vec{r}_L)}{|\vec{r}_R - \vec{r}_L|}$$

$$\vec{w} = \frac{\vec{w}_R + \vec{w}_L}{2} + \vec{w}'$$



- The Euler equation in finite-volume form become

$$\frac{dQ_i}{dt} = -\sum_j A_{ij}F_{ij}$$

$$Q_i^{(n+1)} = Q_i^{(n)} - \Delta t \sum_j A_{ij}\hat{F}_{ij}^{(n+1/2)}$$

the $\hat{F}_{ij}$ are now an appropriately time-averaged approximation to the true flux F_{ij} across the cell face.

$$\frac{dQ_i}{dt} = -\oint_{\partial V_i} [F(U) - U\vec{w}^T]\, d\hat{n}$$

$$F(U_{ij}) = \frac{1}{2}[F(u_i^n) + F(u_j^n) - \frac{\Delta x}{\Delta t}(u_i^n - u_j^n)]$$

$$F_{ij} = \frac{1}{A_{ij}}\int_{A_{ij}} [F(U_{ij}) - U_{ij}\vec{w}^T]\, d\vec{A}_{ij}$$

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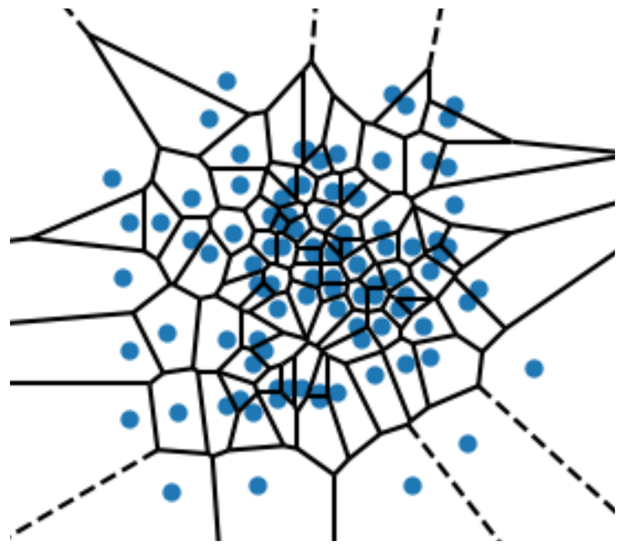
* Update steps



$$F(U_{ij}) = \frac{1}{2}[F(u_i^n) + F(u_j^n) - \frac{\Delta x}{\Delta t}(u_i^n - u_j^n)]$$

$$\vec{w}' = \frac{(\vec{w}_L - \vec{w}_R) \cdot [\vec{f} - \frac{(\vec{r}_R + \vec{r}_L)}{2}]}{|\vec{r}_R - \vec{r}_L|} \frac{(\vec{r}_R - \vec{r}_L)}{|\vec{r}_R - \vec{r}_L|}$$

$$\vec{w} = \frac{\vec{w}_R + \vec{w}_L}{2} + \vec{w}'$$

$$F_{ij} = \frac{1}{A_{ij}} \int_{A_{ij}} [F(U_{ij}) - U_{ij}\vec{w}^T] \, d\vec{A}_{ij}$$

$$Q_i^{(n+1)} = Q_i^{(n)} - \Delta t \sum_j A_{ij} \hat{F}_{ij}^{(n+1/2)}$$

Calculate a new Voronoi tessellation based on the current coordinates ***ri*** of the mesh-generating points.

Move the mesh-generating points with their assigned velocities for this time-step.

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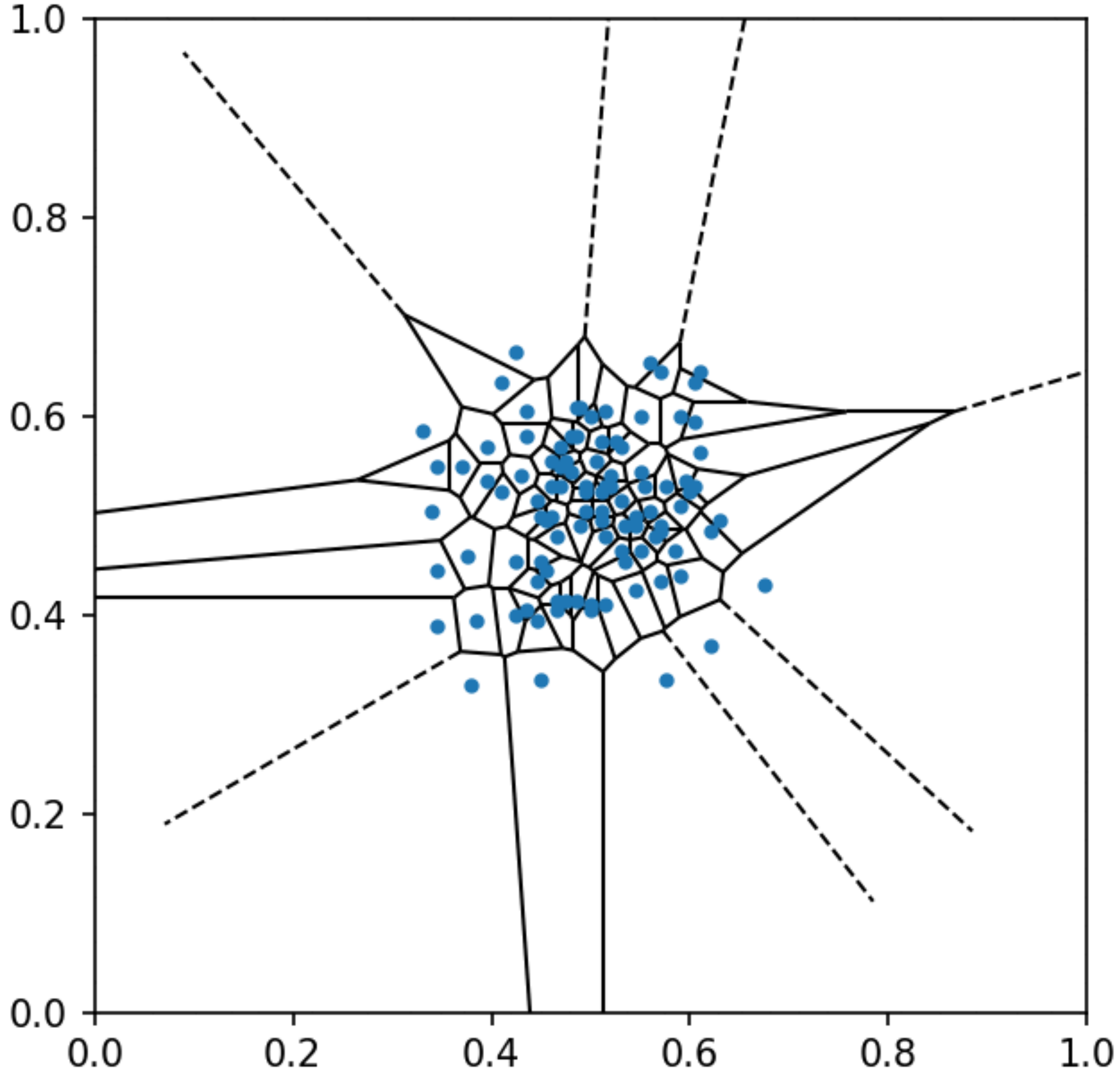
• Result



- This simulation can be run several steps.
- However, sometimes will crash.
  - Due to $P < 0$
  - Due to $m < 0$
- Since we did not implement reconstruct the position of the point, we cannot see the density change just by point position.

```
P<0
P<0
P<0
P<0
P<0
/home/vivi/Computational Astrophysics/Final_Project/NTU_asphy_final_project_666/unstructed_mo
  a = ( gamma*P/U[:,0] )**0.5
m<0
P<0
P<0
P<0
P<0
P<0
P<0
Traceback (most recent call last):
  File "/home/vivi/Computational Astrophysics/Final_Project/NTU_asphy_final_project_666/unstr
    vor = Voronoi(points)
  File "qhull.pyx", line 2615, in scipy.spatial.qhull.Voronoi.__init__
  File "qhull.pyx", line 284, in scipy.spatial.qhull._Qhull.__init__
ValueError: Points cannot contain NaN
```

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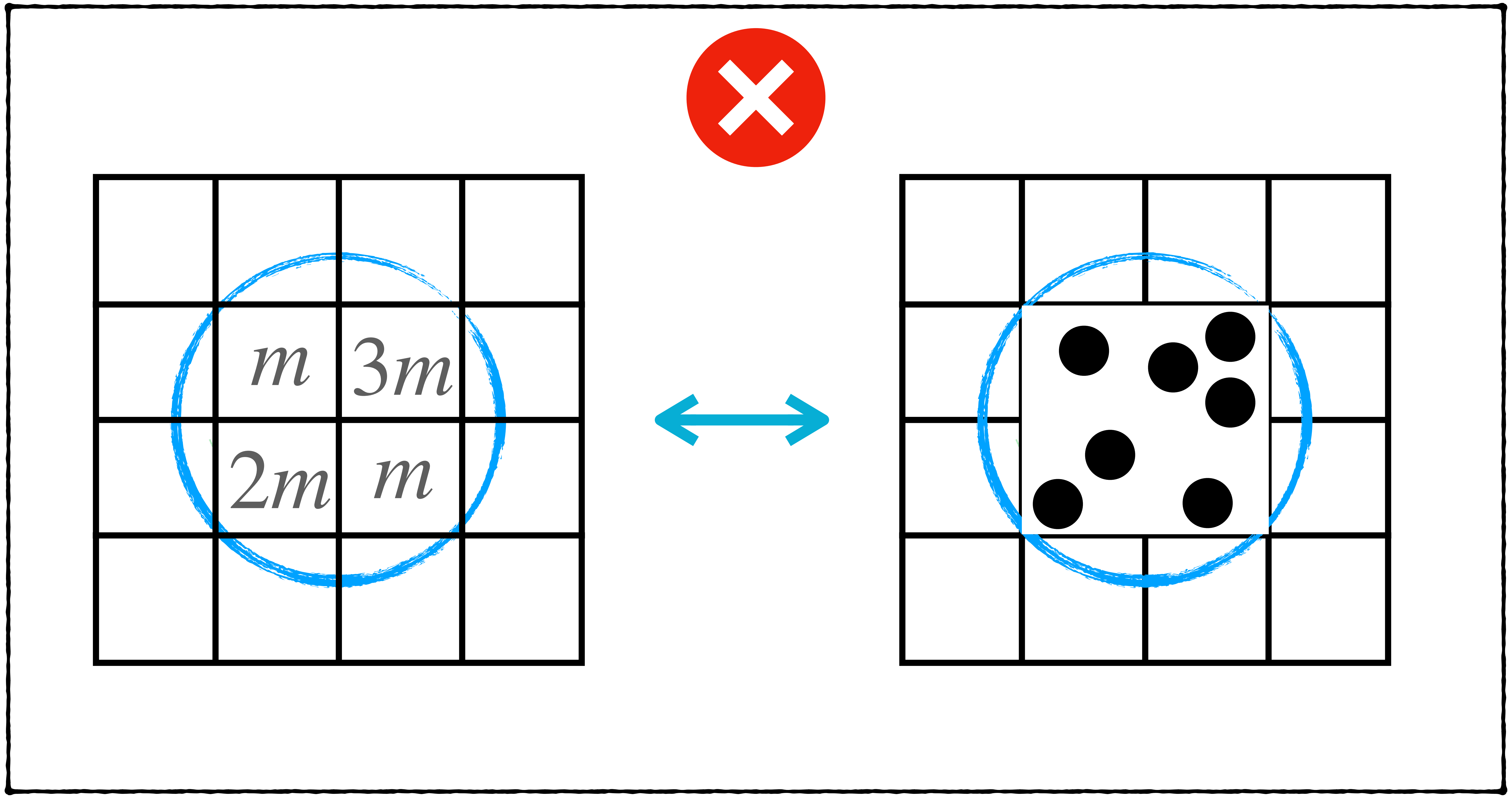
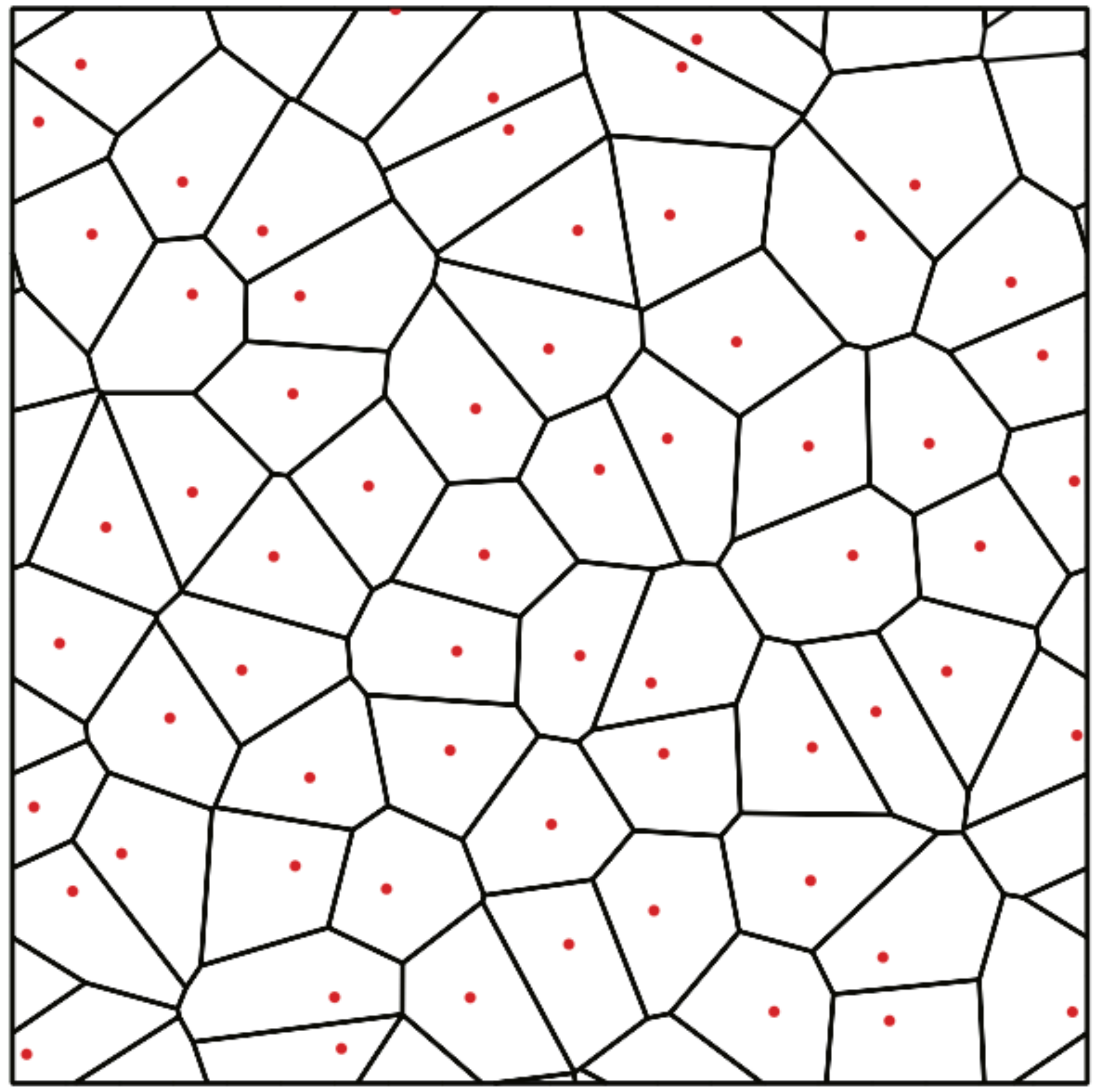
$$Q_i^{(n+1)} = Q_i^{(n)} - \Delta t \sum_j A_{ij} \hat{F}_{ij}^{(n+1/2)}$$





• Future

- The problem:
  - * Boundary condition
  - * Density