

Assignment - 1

Tejash Agrahari
EP20BTECH11024

Abstract—The following is solution to Exercise 2.32 of Discrete-Time Signal Processing textbook by Oppenheim.

Problem 1. Consider an LTI system with frequency response:

$$H(e^{j\omega}) = e^{-j(\omega-\pi/4)} \left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}} \right), \quad -\pi < \omega < \pi,$$

Determine output $y[n]$ for all n if the input for all n is :

$$x[n] = \cos\left(\frac{\pi n}{2}\right)$$

Solution:

$$\begin{aligned} H(e^{j\omega}) &= e^{j\pi/4} \cdot e^{-j\omega} \left(\frac{1 + e^{-j2\omega} + 4e^{-j4\omega}}{1 + \frac{1}{2}e^{-j2\omega}} \right) \\ &= e^{j\pi/4} G(e^{j\omega}) \end{aligned}$$

Let $y_1[n] = x[n] * g[n]$, then

$$\begin{aligned} x[n] &= \cos\left(\frac{n\pi}{2}\right) = \frac{e^{jn\pi/2} + e^{-jn\pi/2}}{2} \\ y_1[n] &= \frac{G(e^{j\pi/2})e^{jn\pi/2} + G(e^{-j\pi/2})e^{-jn\pi/2}}{2} \end{aligned}$$

Evaluating the frequency response at $\omega = \pm\pi/2$:

$$G(e^{j\pi/2}) = e^{-j\pi/2} \frac{1 + e^{-j\pi} + 4e^{-j2\pi}}{1 + \frac{1}{2}e^{-j\pi}} = 8e^{-j\pi/2}$$

$$G(e^{-j\pi/2}) = 8e^{j\pi/2}$$

Therefore,

$$y_1[n] = (8e^{j(\pi n/2 - \pi/2)} + 8e^{j(-\pi n/2 + \pi/2)})/2 = 8 \cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$$

and

$$y[n] = e^{j\pi/4} y_1[n] = 8e^{j\pi/4} \cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$$