COMPUTER ORGANIZATION

Homework Four

1. Consider a Cache with nominal size 1MByte, and 4 words per block. The main memory address is 32 bits wide. How many blocks does it hold? How many bits would the *offset need*? Consider these cases: Direct Mapped, 2-way set associative and 4 ways associative. Calculate the size of the index and the tag for each case. Calculate de real space in bytes that the cache uses. How does the number of sets affect the size of the cache?

- $1MB = 2^{20}B$
- 4 words/block $= 4 \times 4$ B/block $= 16$ B/block $= 2^4$ B/blocks
- $\#\text{blocks} = \frac{2^{20}B}{2^4 B/\text{blocks}} = 2^{16}$ blocks

offset is 4 bits

Direct Mapped: 2^{16} blocks → 16 bit index

tag $= 32 - (16+4) = 12$ bits

Real size $= 1MB + 2^{16}\left[\frac{12+1}{23 \text{ bits/Byte}}\right] = 1MB + (13 \times 8)2^{10}B$ (valid bit)

$\simeq \boxed{1,128 KB}$

2-way SA: $\frac{2^{16}\text{ blocks}}{2} = 2^{15}$ sets → 15 bit index

tag $= 32 - (15+4) = 13$ bits

Real size $= 1MB + 2^{16}\left[\frac{13+2}{23 b/B}\right] = \boxed{1,144 KB}$ (V bit + LRU bit)

4 way SA; $\frac{2^{16}\text{ blocks}}{4} = 2^{14}$ sets → 14 bit index

tag $= 32 - (14+4) = 14$ bits

Real Size $= 1MB + 2^{16}\left[\frac{14+3}{23 b/B}\right] = \boxed{1,160 KB}$ (V bits + 2 LRU bits)

2. Consider a 4-way Set associative cache with nominal size 1MByte. The main memory address is 32 bits wide. Consider these cases: block holds 1 word, block holds 2 words, block holds 4 words, and block holds 8 words. For each case: How many blocks does the whole cache hold? How many bits would the *offset need*? What is the number of sets? Calculate the size of the index and the tag. Calculate de real space in bytes that the cache uses. How does the number of sets affect the size of the cache?

4 way → $1MB = 2^{20}B$

① 1 word/block = 4 bytes/block → # blocks = $\frac{2^{20}B}{2^2 B/block} = 2^{18}$ blocks

↳ 2^2 B/block → offset → 2 bits

→ tag = 32 − (16+2) = 14 bits

(1 valid bit, 2 LRU bits)

Real Size = $1MB + 2^{18}\left[\frac{14+3}{8 b/B}\right] = 1568$ KB

② 2 words/block → 8 B/block = 2^3 B/block → offset = 3 bits

blocks = $\frac{2^{20}}{2^3} = 2^{17}$ blocks → # sets = $\frac{2^{17}\text{ blocks}}{4\text{ way}} = 2^{15}$ sets → index 15 bits

tag = 32 − [15+3] = 14 bits → Real Size = $1MB + 2^{17} \cdot \left[\frac{14+3}{8}\right] = 1{,}296$ Bytes

④ 4 words/block → 16 B/block = 2^4 B/blocks → offset = 4 bits

blocks = $\frac{2^{20}}{2^4} = 2^{16}$ blocks → # sets = $\frac{2^{16}\text{ blocks}}{4\text{ ways}} = 2^{14}$ sets → index 14 bits

tag = 32 − [14+4] = 14 bits → Real Size = $1MB + 2^{16}\left[\frac{14+3}{8}\right] = 1{,}160$ Bytes

If the solution includes only one status bit instead of 3, thats fine.

block addresses: 20 3 3 11 20 6 12 2 10 11 9 9 2 14 11 20 21 6

3. [41, 7, 6, 22, 40, 13, 24, 5, 21, 23, 18, 19, 5, 28, 22, 40, 42, 13] represent these accesses to a memory cache, size 256Bytes, 2 words per block, in these two cases: direct mapped and 2-way set associative. Show the status of the cache after the accesses. Show the number of hits, and calculate the hit and miss rate.

$\frac{256B}{8\,B/block} = \frac{2^8 B}{2^3 B/block} = 2^5 = 32$ blocks;

DM

hit rate = $\frac{8}{18}$

0	
1	
2	b2 [W4 (5)]
3	b3 [W (6) 7]
4	
5	
6	b6 [W12 (13)]
7	
8	
9	b9 [W18 (19)]
10	b10 [W20,21]
11	b11 [W (22) (23)]
12	b12 [24,25]
13	
14	b14 [W28,29]
15	
16	
17	
18	
19	
20	b20 [W (40) 41]
21	b21 [W 42,43]
22	
23	
24	
25	
26	
27	
28	
29	
30	
31	

2-way SA → $\frac{32 \text{ blocks}}{2} = 16$ sets

hit rate = $\frac{8}{18}$

set		
0		
1		
2	b2 [W4 (5)]	
3	b3 [W (6) 7]	
4	b20 [W (40) 41]	
5	b21 [W 42,43]	
6	b6 [W12 (13)]	
7		
8		
9	b9 [W18,19]	
10	b10 [20,21]	
11	b11 [W (22) (23)]	
12	b12 [W (24) 25]	
13		
14	b14 [W28,29]	
15		

b.add = $\frac{41}{2} = 20$

20 MOD 32 = 20

$\frac{7}{2} = 3$ MOD 32 = 3

$\frac{6}{2} = 3$ MOD 32 = 3 ✓

$\frac{22}{2} = 11$ MOD 32 = 11

$\frac{40}{2} = 20$ MOD 32 = 20 ✓

$\frac{13}{2} = 6$ MOD 32 = 6

$\frac{24}{2} = 12$ MOD 32 = 12

$\frac{5}{2} = 2$ MOD 32 = 2

Direct Mapped

20 MOD 16 = 4

3 MOD 16 = 3

11 MOD 16 = 11

20 MOD 16 = 4 ✓

6 MOD 16 = 6 ✓

12 MOD 16 = 12 ✓

2 MOD 16 = 2

10 MOD 16 = 10

9 MOD 16 = 9

14 MOD 16 = 14

21 MOD 16 = 5

set associative

block addresses
word address

✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
10	1	15	10	3	6	15	5	4	4	1	7	5	10	10	3	4

4. [41, 7, 6, 22, 40, 13, 24, 5, 21, 23, 18, 19, 5, 28, 22, 40, 42, 13] represent these accesses to a memory cache, size 256Bytes, 4 words per block, in these two cases: direct mapped and 2-way set associative. Show the status of the cache after the accesses. Show the number of hits, and calculate the hit and miss rate.

$\frac{256\,B}{4\times4\ B/block} = \frac{2^8}{2^4} = 2^4 = 16$ blocks

⊛ Direct Mapped:

w41 → $\frac{41}{4}$ = 10r1 → it's the second word in block 10

w7 → $\frac{7}{4}$ = 1r3 → it's the fourth word in block 1

w22 → $\frac{22}{4}$ = 5r2 → third word in block 5

0	
1	b1[w4 5 6 7]
2	
3	b3[w12 13 14,15]
4	b4[w16,17,18 19]
5	b5[w20 21 22 23]
6	b6[w24,25,26,27]
7	b7[w28,29,30,31]
8	
9	
10	b10[w40 41 42 43]
11	
12	
13	
14	
15	

hit rate = $\frac{11}{18}$ miss rate = $\frac{7}{18}$

⊛ Set Associative

sets = $\frac{\#\text{ blocks}}{\#\text{ ways}} = \frac{16}{2} = 8$ sets

w41 → b10 MOD 8 = 2

0		
1	b1[w4 5 6 7]	
2	b10[w40 41 42 43]	
3	b3[12 13 14,15]	
4	b4[16,17,18,19]	
5	b5[w20 21 22 23]	
6	b6[24,25,26,27]	
7	b7[w28,29,30,31]	

hits = $\frac{11}{18}$ # misses = $\frac{7}{18}$