

Module 2 - The LP Model

Problem 1: Back Savers Backpack Production

a) Decision Variables

Symbol	Definition
x1	Collegiate backpacks produced per week
x2	Mini backpacks produced per week

b) Objective Function

Maximize weekly profit:

$$Z = 32x_1 + 24x_2$$

c) Constraints

Type	Constraint per week
Material -> Nylon	$3x_1 + 2x_2 \leq 5000$ sq ft
Labor	$45x_1 + 40x_2 \leq 84000$ minutes
Sales limits	$x_1 \leq 1000$; $x_2 \leq 1200$
Non-negativity	$x_1, x_2 \geq 0$

d) Full Mathematical Formulation

Maximize $Z = 32x_1 + 24x_2$

Subject to:

$$3x_1 + 2x_2 \leq 5000 \quad 45x_1 + 40x_2 \leq 84000 \quad x_1 \leq 1000$$

$$x_2 \leq 1200$$

$$x_1, x_2 \geq 0$$

Problem 2: Weigelt Corporation Plant Allocation

a) Decision Variables

Symbol	Meaning
x_{1L}	Large units at Plant 1
x_{2L}	Large units at Plant 2
x_{3L}	Large units at Plant 3
x_{1M}	Medium units at Plant 1
x_{2M}	Medium units at Plant 2
x_{3M}	Medium units at Plant 3

x1S	Small units at Plant 1
x2S	Small units at Plant 2
x3S	Small units at Plant 3

Product Data:

Size	Profit/ unit \$	Storage / Unit (sq ft)	Daily Demand Cap (units)
Large (L)	420	20	900
Medium (M)	360	15	1200
Small (S)	300	12	750

Plant data:

Plant	Excess Capacity (units/day)	Storage Available (sq ft/day)
1	750	13000
2	900	12000
3	450	5000

b) Objective Function

Maximize daily profit:

$$Z = 420(x_{1L} + x_{2L} + x_{3L}) + 360(x_{1M} + x_{2M} + x_{3M}) + 300(x_{1S} + x_{2S} + x_{3S})$$

c) Constraints

Constraint Group	Constraint
Plant capacity (P1)	$x_{1L} + x_{1M} + x_{1S} \leq 750$
Plant capacity (P2)	$x_{2L} + x_{2M} + x_{2S} \leq 900$
Plant capacity (P3)	$x_{3L} + x_{3M} + x_{3S} \leq 450$

Constraint Group	Constraint
Storage (P1)	$20x1L + 15x1M + 12x1S \leq 13000$
Storage (P2)	$20x2L + 15x2M + 12x2S \leq 12000$
Storage (P3)	$20x3L + 15x3M + 12x3S \leq 5000$

Constraint Group	Constraint
Demand (L)	$x1L + x2L + x3L \leq 900$
Demand (M)	$x1M + x2M + x3M \leq 1200$
Demand (S)	$x1S + x2S + x3S \leq 750$

Constraint Group	Constraint
Equal % capacity use	$(x_{1L}+x_{1M}+x_{1S})/750 = (x_{2L}+x_{2M}+x_{2S})/900 = (x_{3L}+x_{3M}+x_{3S})/450$

Constraint Group	Constraint
Non-negativity	$x_{ij} \geq 0$ for all $i \in \{1,2,3\}, j \in \{L,M,S\}$

d) Full Mathematical Formulation

Maximize $Z = 420(x_{1L}+x_{2L}+x_{3L}) + 360(x_{1M}+x_{2M}+x_{3M}) + 300(x_{1S}+x_{2S}+x_{3S})$

with taking into consideration all the constraints.