(6) 4.9 6.2 7.5

$7.5-5=2.5$

$2.5/1.3=1.92$

(7) Coservative Confidence Interval

$$\hat{p} \pm \frac{z^*}{2\sqrt{n}}$$

$\hat{p} = \frac{159}{1400} = 0.1136$

$n = 1400$

$x = 159$

$\alpha = 0.05$

$z^* = 1.96$

$$0.1136 + \frac{1.96}{2\sqrt{1400}} =$$

$$0.1136 + \frac{1.96}{74.833} = 0.1398 = 13.98\%$$

$$0.1136 - \frac{1.96}{74.833} = 0.0874 = 8.74\%$$

(10) binomial probability formula

$n=15$

$p=0.07$

$x=2$

$$P(X=x) = (nCx) * p^x * (1-p)^{n-x}$$

$$P(X=2) = (15C2) * 0.07^2 * (0.93)^{13}$$

$$105 * 0.07^2 * (0.93)^{13} = 0.2003$$

$$\frac{n!}{(n-x)!\,x!} = \frac{15!}{(15-2)!\,2!} = 105$$

$15C2 = 105$

(13) $P(X=10) / (P(x=7) + P(x=8) + P(x=9) + P(x=10))$

$= 0.05 / (0.24 + 0.14 + 0.11 + 0.05)$

$= 0.09$

---

(14) Given: $\hat{p}1 = 0.52$

$\hat{p}2 = 0.35$

$SE_{\hat{p}_1 - \hat{p}_2} = 0.0338$

$CI = c = 90\%$

Formula:

$(\hat{p}_1 - \hat{p}_2 - E, \hat{p}_1 - \hat{p}_2 + E)$

where

$E = Z_c \times SE_{\hat{p}_1 - \hat{p}_2}$

$Z_c$ is z critical value for $c = 90\%$ CI

Find area $= (1+c)/2 = (1+0.90)/2 = 1.90/2 = 0.9500$

Look in z-table for Area $= 0.9500$ or its closest value and find corresponding z-value.

Area 0.9500 is in between 0.9495 and 0.9505 and both the areas are the same distance from 0.9500.

Thus we look for both areas and find both z-values.

Thus Area 0.9495 corresponds to 1.64, and 0.9505 to 1.65.

The Avg of both z-values = $(1.64+1.65)/2 = 1.645$

Thus $Z_C = 1.645$

Thus

$E = Z_C \times SE_{\hat{p}_1-\hat{p}_2}$

$E = 1.645 \times 0.0338$

$E = 0.055601 = 0.0556$

Thus

$(\hat{p}_1 - \hat{p}_2 - E,\ \hat{p}_1 - \hat{p}_2 + E)$

$(0.52 - 0.35 - 0.0556,\ 0.52 - 0.35 + 0.0556)$

$(0.1144,\ 0.2256)$

16 $\hat{p}1 = 0.52$ $N_1 = 501$

$\hat{p}2 = 0.35$ $N_2 = 352$

$SE_{\hat{p}_1-\hat{p}_2} = 0.0338$

First check conditions of normality by observing if n1*P1 and n2*P2 both are greater than 5...

$N1 * P1 = 261$

$N2 * P2 = 123$

Since both are greater than 5, we use standard normal z-table

$Z = (p1 - p2)/SE$ $Z = 5.03$

From z-table, $P(z > 5.03) = 0$

P-value = 0

Since P-value is less than 0.05, we reject Ho

(19) $\hat{X}_1 = 156$ $n_1 = 20$

$\hat{X}_2 = 162$ $n_2 = 22$

$s_1 = 36$ $df = 39.958$

$s_2 = 41$ $t_c = 2.021$

(Rejection Region) $R = \{t : |t| > 2.021\}$

Since it is assumed the population variances are unequal, the t-statistic is computed with:

$$t = \frac{\hat{X}_1 - \hat{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{156-162}{\sqrt{\frac{36^2}{20} + \frac{41^2}{22}}} = -0.505$$

Since $|t| = 0.505$ which is not $> 2.021$, we do not reject the null hypothesis

(24)
$$\frac{\text{Total \# Not Satisfied} \times \text{Total \# with salary \$20k-\$35k}}{\text{Total \# in sample}}$$

$$\frac{72 \times 76}{230} = 23.79$$

(25)
$$x^2 = \frac{(13-19.17)^2}{19.17} + \frac{(11-15.63)^2}{15.63} + \frac{(19-14.37)^2}{14.37} + \frac{(15-8.83)^2}{8.83}$$

$$+ \frac{(29-33.04)^2}{33.04} + \frac{(31-26.96)^2}{26.96} + \frac{(28-24.78)^2}{24.78} + \frac{(12-15.22)^2}{15.22}$$

$$+ \frac{(34-23.79)^2}{23.79} + \frac{(20-19.41)^2}{19.41} + \frac{(10-17.84)^2}{17.84} + \frac{(8-10.96)^2}{10.96}$$

$$\chi^2 = 1.986 + 1.372 + 1.492 + 4.311$$
$$+ 0.494 + 0.605 + 0.418 + 0.681$$
$$+ 4.382 + 0.018 + 3.445 + 0.799$$

$$\chi^2 = 20.003 \approx 20.0043$$

p-value = CHISQ.DIST.RT($\chi^2$, df)

= CHISQ.DIST.RT(20.0043, 6)

= 0.0028

$0.0028 < 0.05$ reject the null

(30) Type I error = investigator rejects a null hypothesis that is actually true (false-positive)

Type II error = investigator fails to reject a null hypothesis that is actually false (false-negative)

## Statistics Practice Test 2

(1) Non Participation Bias: When a representative sample is chosen but a subset cannot be contacted or does not respond.

(2) $n = 1368$ $\quad C = 95\%$

$x = 460$ $\quad \alpha = 0.05$

$\hat{p} = \frac{x}{n} = \frac{460}{1368} = 0.3363$ $\quad \frac{\alpha}{2} = 0.025$

$\hat{q} = 1 - \hat{p} = 0.66$

Requirement for constructing meaningful confidence interval about the population

$= n\hat{p}(1-\hat{p}) \geq 10$

$= 1368(0.33)(0.66) \geq 10$

$= 297.9504 > 10$

So our requirement is fulfilled.

We use the normal z table

Standard normal deviate for $z_{\frac{\alpha}{2}} = 0.025 = 1.96$

Formula for confidence interval for proportion

$$\hat{p} \pm \alpha/2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.33 \pm 1.96\sqrt{\frac{0.33 * 0.66}{1368}}$$

$$0.33 - 1.96\sqrt{\frac{0.33*0.66}{1368}} \quad , \quad 0.33 + 1.96\sqrt{\frac{0.33*0.66}{1368}}$$

$$(0.311 \quad , \quad 0.361)$$

$$(31.1\% \quad , \quad 36.1\%) = (31.07\%, 36.19\%)$$

5 $$(nC_x) * p^x * (1-p)^{n-x}$$

$n = 15$
$p = 0.1$
$x = 14$

$$({}_{15}C_{14}) * (0.1)^{14} * (1-0.1)^{15-14}$$

$$15 * (0.1)^{14} * (0.9)^1 = 1.35 \times 10^{-13} \approx 0.0000$$

$$\frac{n!}{(n-x)!\,x!} = \frac{15!}{(15-14)!\,14!} = \frac{15!}{14!} = 15$$

6 $$0.58 + 0.18 + 0.10 + 0.07 + 0.05 + x = 1$$

$$x = 0.02$$

7 1

0.58 0.76 0.86 0.93 0.98 1.00

8 $$P(6) / (P(2) + P(3) + P(4) + P(5) + P(6))$$

$$0.02 / (0.18 + 0.10 + 0.07 + 0.05 + 0.02)$$

$$0.02 / 0.42 = 0.0476 = 0.05$$

(9) The 90% CI is

$$\hat{p} +/- Z_{0.05} * \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.53 +/- 1.645 \sqrt{\frac{0.53(1-0.53)}{861}}$$

$$= 0.53 +/- 0.0280$$

$$= 0.5020, 0.5580$$

(11) The test statistic $Z = (\hat{p} - p) / \sqrt{\frac{p(1-p)}{n}}$

$$= (0.53 - 0.81) / \sqrt{\frac{0.81(1-0.81)}{861}}$$

$$= -20.94$$

p-value $= P(Z < -20.94) = 0.0000$

Since the p-value is less than 0.05, we reject $H_0$

(16) $1332/7 = 190.29$ (calculate the avg)

(17)

$$X^2 = \frac{(218-190.2857)^2}{190.2857} + \frac{(191-190.2857)^2}{190.2857} + \frac{(239-190.2857)^2}{190.2857}$$

$$+ \frac{(189-190.2857)^2}{190.2857} + \frac{(178-190.2857)^2}{190.2857} + \frac{(168-190.2857)^2}{190.2857}$$

$$+ \frac{(149-190.2857)^2}{190.2857}$$

$$= 4.0365 + 0.0027 + 12.4711 + 0.0087 + 0.7932 + 2.61 + 8.9576$$

$= 28.88$

Excel code

p-value = $\overbrace{\text{CHISQ.DIST.RT}(28.88, 6)}$ $\chi^2$ (variance) df

$= 6.410 \times 10^{-5} = 0.00006410 = 0.0000$

p-value $< 0.05$ reject null

(23) $(75 + 163)/2 = 119$

(24) $2 \times 1.96\,\sigma = 163 - 75$ (z-score for 95% and $n > 100 = \pm 1.96$)

$$\sigma = \frac{88}{2 \times 1.96} = 22.45$$

(25) $$z = \frac{x - \mu}{\sigma} = \frac{182 - 119}{22.45} = 2.81$$

(26) $n_1 = 87$ $\mu_1 = 0.451$ $\sigma_1 = 0.139$ $c = 90\%$

$n_2 = 94$ $\mu_2 = 0.581$ $\sigma_2 = 0.167$

We have to find 90% CI for $\mu_1 - \mu_2$

$[(\mu_1 - \mu_2) - \text{margin of error}, (\mu_1 - \mu_2) + \text{margin of error}]$

Where,

$$\text{Margin of error} = t_c \sqrt{\frac{(n_1 - 1)\sigma_1^2 + (n_2 - 1)\sigma_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

for $\alpha = 0.10$

$dof = 87+94-2$
$= 179$

$t_c = 1.653$

$$\text{Margin of error} = 1.653\sqrt{\frac{(87-1)(0.139)^2+(94-1)(0.167)^2}{87+94-2}\left(\frac{1}{87}+\frac{1}{94}\right)}$$

$= 0.03792$

$\text{Lower Limit} = (\mu_1-\mu_2) - \text{margin of error}$

$= 0.451-0.581-0.03792$

$= -0.16792$

$\text{Upper Limit} = (\mu_1-\mu_2) + \text{margin of error}$

$= (0.451-0.581) + 0.03792$

$= -0.09208$

The 90% CI for $\mu_1-\mu_2$ is:

$-0.1679 < \mu_1-\mu_2 < -0.0921$

28 test statistic $$t = \frac{\mu_1-\mu_2}{\sqrt{\frac{(n_1-1)(\sigma_1^2)+(n_2-1)(\sigma_2^2)}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$= \frac{0.451 - 0.581}{\sqrt{\frac{(87-1)(0.139)^2 + (94-1)(0.167)^2}{87+94-2}\left(\frac{1}{87} + \frac{1}{94}\right)}}$$

$$t = \frac{-0.13}{0.022937888} = -5.6675$$

P-value = 0

$\alpha = 0.10$

$0 < 0.10$

We reject the null hypothesis Ho