# Optional Lab: Logistic Regression, Decision Boundary

## Goals

In this lab, you will:

- Plot the decision boundary for a logistic regression model. This will give you a better sense of what the model is predicting.

In [1]:
```
import numpy as np
%matplotlib widget
import matplotlib.pyplot as plt
from lab_utils_common import plot_data, sigmoid, draw_vthresh
plt.style.use('./deeplearning.mplstyle')
```

## Dataset

Let's suppose you have following training dataset

- The input variable `X` is a numpy array which has 6 training examples, each with two features
- The output variable `y` is also a numpy array with 6 examples, and `y` is either `0` or `1`

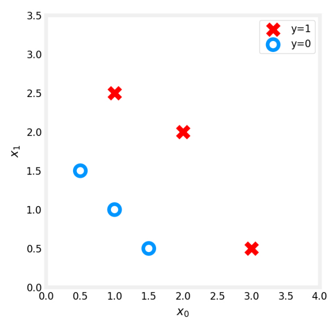
In [2]:
```
X = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y = np.array([0, 0, 0, 1, 1, 1]).reshape(-1,1)
```

### Plot data

Let's use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

In [3]:
```
fig,ax = plt.subplots(1,1,figsize=(4,4))
plot_data(X, y, ax)

ax.axis([0, 4, 0, 3.5])
ax.set_ylabel('$x_1$')
ax.set_xlabel('$x_0$')
plt.show()
```




## Logistic regression model

- Suppose you'd like to train a logistic regression model on this data which has the form

  $f(x) = g(w_0x_0 + w_1x_1 + b)$

  where $g(z) = \frac{1}{1+e^{-z}}$, which is the sigmoid function

- Let's say that you trained the model and get the parameters as $b = -3, w_0 = 1, w_1 = 1$. That is,

  $f(x) = g(x_0 + x_1 - 3)$

  (You'll learn how to fit these parameters to the data further in the course)

Let's try to understand what this trained model is predicting by plotting its decision boundary

### Refresher on logistic regression and decision boundary

- Recall that for logistic regression, the model is represented as

$$f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = g(\mathbf{w} \cdot \mathbf{x}^{(i)} + b) \tag{1}$$

  where $g(z)$ is known as the sigmoid function and it maps all input values to values between 0 and 1:

$$g(z) = \frac{1}{1+e^{-z}} \tag{2}$$

  and $\mathbf{w} \cdot \mathbf{x}$ is the vector dot product:

$$\mathbf{w} \cdot \mathbf{x} = w_0x_0 + w_1x_1$$

- We interpret the output of the model ($f_{\mathbf{w},b}(x)$) as the probability that $y = 1$ given $\mathbf{x}$ and parameterized by $\mathbf{w}$ and $b$.
  - Therefore, to get a final prediction ($y = 0$ or $y = 1$) from the logistic regression model, we can use the following heuristic -

  if $f_{\mathbf{w},b}(x) >= 0.5$, predict $y = 1$

  if $f_{\mathbf{w},b}(x) < 0.5$, predict $y = 0$

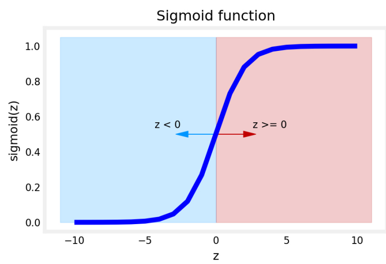
- Let's plot the sigmoid function to see where $g(z) >= 0.5$

In [4]:
```
# Plot sigmoid(z) over a range of values from -10 to 10
z = np.arange(-10,11)

fig,ax = plt.subplots(1,1,figsize=(5,3))
# Plot z vs sigmoid(z)
ax.plot(z, sigmoid(z), c="b")
```

```
ax.set_title('Sigmoid function')
ax.set_ylabel('sigmoid(z)')
ax.set_xlabel('z')
draw_vthresh(ax,0)
```

Figure 2




- As you can see, $g(z) >= 0.5$ for $z >= 0$
- For a logistic regression model, $z = \mathbf{w} \cdot \mathbf{x} + b$. Therefore,

  if $\mathbf{w} \cdot \mathbf{x} + b >= 0$, the model predicts $y = 1$

  if $\mathbf{w} \cdot \mathbf{x} + b < 0$, the model predicts $y = 0$

### Plotting decision boundary

Now, let's go back to our example to understand how the logistic regression model is making predictions.

- Our logistic regression model has the form

  $f(\mathbf{x}) = g(-3 + x_0 + x_1)$

- From what you've learnt above, you can see that this model predicts $y = 1$ if $-3 + x_0 + x_1 >= 0$

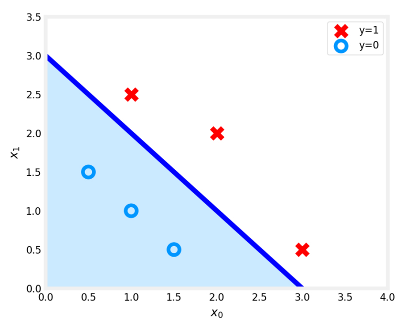
Let's see what this looks like graphically. We'll start by plotting $-3 + x_0 + x_1 = 0$, which is equivalent to $x_1 = 3 - x_0$.

In [5]:
```
# Choose values between 0 and 6
x0 = np.arange(0,6)

x1 = 3 - x0
fig,ax = plt.subplots(1,1,figsize=(5,4))
# Plot the decision boundary
ax.plot(x0,x1, c="b")
ax.axis([0, 4, 0, 3.5])

# Fill the region below the line
ax.fill_between(x0,x1, alpha=0.2)

# Plot the original data
plot_data(X,y,ax)
ax.set_ylabel(r'$x_1$')
ax.set_xlabel(r'$x_0$')
plt.show()
```




- In the plot above, the blue line represents the line $x_0 + x_1 - 3 = 0$ and it should intersect the x1 axis at 3 (if we set $x_1$ = 3, $x_0$ = 0) and the x0 axis at 3 (if we set $x_1$ = 0, $x_0$ = 3).
- The shaded region represents $-3 + x_0 + x_1 < 0$. The region above the line is $-3 + x_0 + x_1 > 0$.
- Any point in the shaded region (under the line) is classified as $y = 0$. Any point on or above the line is classified as $y = 1$. This line is known as the "decision boundary".

As we've seen in the lectures, by using higher order polynomial terms (eg: $f(x) = g(x_0^2 + x_1 - 1)$, we can come up with more complex non-linear boundaries.

## Congratulations!

You have explored the decision boundary in the context of logistic regression.

In [ ]:

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