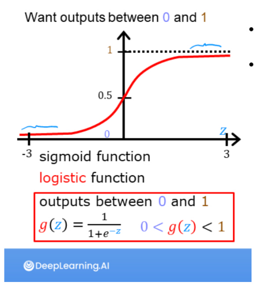
Optional Lab: Logistic Regression

In this ungraded lab, you will

- explore the sigmoid function (also known as the logistic function)
- explore logistic regression; which uses the sigmoid function

In [1]:
```
import numpy as np
%matplotlib widget
import matplotlib.pyplot as plt
from plt_one_addpt_onclick import plt_one_addpt_onclick
from lab_utils_common import draw_vthresh
plt.style.use('./deeplearning.mplstyle')
```

Sigmoid or Logistic Function



As discussed in the lecture videos, for a classification task, we can start by using our linear regression model, $f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = \mathbf{w} \cdot \mathbf{x}^{(i)} + b$, to predict y given x.

- However, we would like the predictions of our classification model to be between 0 and 1 since our output variable y is either 0 or 1.
- This can be accomplished by using a "sigmoid function" which maps all input values to values between 0 and 1.

Let's implement the sigmoid function and see this for ourselves.

Formula for Sigmoid function

The formula for a sigmoid function is as follows -

$$g(z) = \frac{1}{1+e^{-z}} \tag{1}$$

In the case of logistic regression, z (the input to the sigmoid function), is the output of a linear regression model.

- In the case of a single example, z is scalar.
- in the case of multiple examples, z may be a vector consisting of m values, one for each example.
- The implementation of the sigmoid function should cover both of these potential input formats. Let's implement this in Python.

NumPy has a function called `exp()`, which offers a convenient way to calculate the exponential (e^{z}) of all elements in the input array (`z`).

It also works with a single number as an input, as shown below.

In [2]:
```
# Input is an array.
input_array = np.array([1,2,3])
exp_array = np.exp(input_array)

print("Input to exp:", input_array)
print("Output of exp:", exp_array)

# Input is a single number
input_val = 1
exp_val = np.exp(input_val)

print("Input to exp:", input_val)
print("Output of exp:", exp_val)
```

```
Input to exp: [1 2 3]
Output of exp: [ 2.72  7.39 20.09]
Input to exp: 1
Output of exp: 2.718281828459045
```

The `sigmoid` function is implemented in python as shown in the cell below.

In [3]:
```
def sigmoid(z):
    """
    Compute the sigmoid of z

    Args:
        z (ndarray): A scalar, numpy array of any size.

    Returns:
        g (ndarray): sigmoid(z), with the same shape as z

    """

    g = 1/(1+np.exp(-z))

    return g
```

Let's see what the output of this function is for various value of `z`

In [4]:
```
# Generate an array of evenly spaced values between -10 and 10
z_tmp = np.arange(-10,11)

# Use the function implemented above to get the sigmoid values
y = sigmoid(z_tmp)

# Code for pretty printing the two arrays next to each other
np.set_printoptions(precision=3)
print("Input (z), Output (sigmoid(z))")
print(np.c_[z_tmp, y])
```

```
Input (z), Output (sigmoid(z))
[[-1.000e+01  4.540e-05]
 [-9.000e+00  1.234e-04]
 [-8.000e+00  3.354e-04]
 [-7.000e+00  9.111e-04]
 [-6.000e+00  2.473e-03]
 [-5.000e+00  6.693e-03]
 [-4.000e+00  1.799e-02]
 [-3.000e+00  4.743e-02]
 [-2.000e+00  1.192e-01]
 [-1.000e+00  2.689e-01]
 [ 0.000e+00  5.000e-01]
 [ 1.000e+00  7.311e-01]
 [ 2.000e+00  8.808e-01]
 [ 3.000e+00  9.526e-01]
 [ 4.000e+00  9.820e-01]
 [ 5.000e+00  9.933e-01]
 [ 6.000e+00  9.975e-01]
```

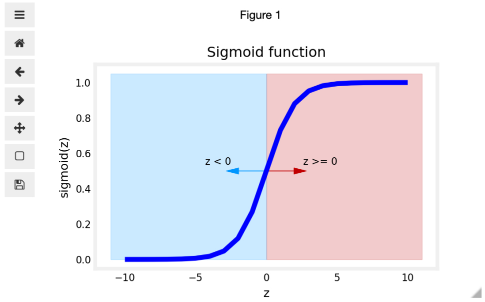
```
[ 7.000e+00  9.991e-01]
[ 8.000e+00  9.997e-01]
[ 9.000e+00  9.999e-01]
[ 1.000e+01  1.000e+00]]
```

The values in the left column are `z`, and the values in the right column are `sigmoid(z)`. As you can see, the input values to the sigmoid range from -10 to 10, and the output values range from 0 to 1.

Now, let's try to plot this function using the `matplotlib` library.

In [5]:
```
# Plot z vs sigmoid(z)
fig,ax = plt.subplots(1,1,figsize=(5,3))
ax.plot(z_tmp, y, c="b")

ax.set_title("Sigmoid function")
ax.set_ylabel('sigmoid(z)')
ax.set_xlabel('z')
draw_vthresh(ax,0)
```



As you can see, the sigmoid function approaches `0` as `z` goes to large negative values and approaches `1` as `z` goes to large positive values.

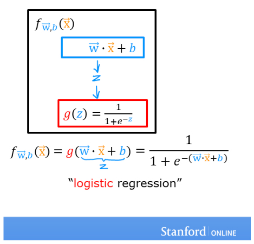
Logistic Regression

A logistic regression model applies the sigmoid to the familiar linear regression model as shown below:

$$f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = g(\mathbf{w} \cdot \mathbf{x}^{(i)} + b) \tag{2}$$

where

$$g(z) = \frac{1}{1+e^{-z}} \tag{3}$$



Let's apply logistic regression to the categorical data example of tumor classification.
First, load the examples and initial values for the parameters.

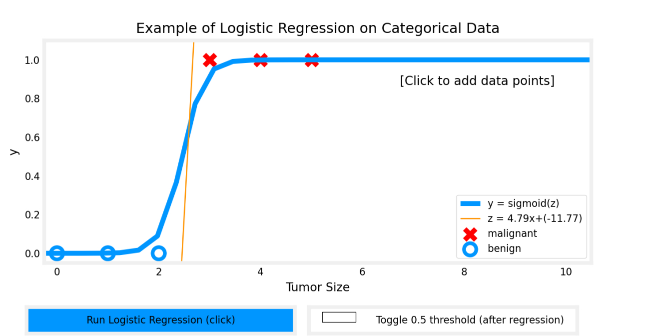
In [6]:
```
x_train = np.array([0., 1, 2, 3, 4, 5])
y_train = np.array([0,  0, 0, 1, 1, 1])

w_in = np.zeros((1))
b_in = 0
```

Try the following steps:

- Click on 'Run Logistic Regression' to find the best logistic regression model for the given training data
  - Note the resulting model fits the data quite well.
  - Note, the orange line is 'z' or $\mathbf{w} \cdot \mathbf{x}^{(i)} + b$ above. It does not match the line in a linear regression model. Further improve these results by applying a *threshold*.
- Tick the box on the 'Toggle 0.5 threshold' to show the predictions if a threshold is applied.
  - These predictions look good. The predictions match the data
  - Now, add further data points in the large tumor size range (near 10), and re-run logistic regression.
  - unlike the linear regression model, this model continues to make correct predictions

In [7]:
```
plt.close('all')
addpt = plt_one_addpt_onclick( x_train,y_train, w_in, b_in, logistic=True)
```



Congratulations!

You have explored the use of the sigmoid function in logistic regression.

In [ ]: