

```
[]: msk = np.random.rand(len(df)) < 0.8
train = cdf[msk]
test = cdf[~msk]</pre>
```

# Polynomial regression

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Sometimes, the trend of data is not really linear, and looks curvy. In this case we can use Polynomial regression methods. In fact, many different regressions exist that can be used to fit whatever the dataset looks like, such as quadratic, cubic, and so on, and it can go on and on to infinite degrees.

In essence, we can call all of these, polynomial regression, where the relationship between the independent variable x and the dependent variable y is modeled as an nth degree polynomial in x. Lets say you want to have a polynomial regression (let's make 2 degree polynomial):

$$y = b + \theta_1 x + \theta_2 x^2$$

Now, the question is: how we can fit our data on this equation while we have only x values, such as **Engine Size**? Well, we can create a few additional features: 1, x, and  $x^2$ .

**PolynomialFeatures()** function in Scikit-learn library, drives a new feature sets from the original feature set. That is, a matrix will be generated consisting of all polynomial combinations of the features with degree less than or equal to the specified degree. For example, lets say the original feature set has only one feature, *ENGINESIZE*. Now, if we select the degree of the polynomial to be 2, then it generates 3 features, degree=0, degree=1 and degree=2:

```
[]: from_sklearn_preprocessing_import_PolynomialEeatures
    from_sklearn_import_linear_model
    train_x = np.asanyarray(train[['ENGINESIZE']])
    train_y = np.asanyarray(train[['COZEMISSIONS']])

test_x = np.asanyarray(test[['ENGINESIZE']])
    test_y = np.asanyarray(test[['COZEMISSIONS']])

poly = PolynomialFeatures(degree=2)
    train_x_poly = poly.fit_transform(train_x)
    train_x_poly
```

fit\_transform takes our x values, and output a list of our data raised from power of 0 to power of 2 (since we set the degree of our polynomial to 2).

The equation and the sample example is displayed below.

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \longrightarrow \begin{bmatrix} [1 & v_1 & v_1^2] \\ [1 & v_2 & v_2^2] \\ \vdots & \vdots & \vdots \\ [1 & v_n & v_n^2] \end{bmatrix}$$

$$\begin{bmatrix} 2. \\ 2.4 \\ 1.5 \\ \vdots \end{bmatrix} \longrightarrow \begin{bmatrix} [1 & 2. & 4.] \\ [1 & 2.4 & 5.76] \\ [1 & 1.5 & 2.25] \\ \vdots & \vdots & \vdots \end{bmatrix}$$

It looks like feature sets for multiple linear regression analysis, right? Yes. It Does. Indeed, Polynomial regression is a special case of linear regression, with the main idea of how do you select your features. Just consider replacing the x with  $x_1$ ,  $x_1^2$  with  $x_2$ , and so on. Then the degree 2 equation would be turn into:

$$y = b + \theta_1 x_1 + \theta_2 x_2$$

Now, we can deal with it as 'linear regression' problem. Therefore, this polynomial regression is considered to be a special case of traditional multiple linear regression. So, you can use the same mechanism as linear regression to solve such a problems.

so we can use LinearRegression() function to solve it:

```
[]: clf = linear_model.LinearRegression()
  train_y_ = clf.fit(train_x_poly, train_y)
  # The coefficients
  print_('Coefficients:..._clf.coef_)
  print_('Intercept.',clf.intercept_)
```

As mentioned before, **Coefficient** and **Intercept**, are the parameters of the fit curvy line. Given that it is a typical multiple linear regression, with 3 parameters, and knowing that the parameters are the intercept and coefficients of hyperplane, sklearn has estimated them from our new set of feature sets. Lets plot it:

```
[]: plt.scatter(train.ENGINESIZE, train.CO2EMISSIONS, color='blue')

XX = np.arange(0.0, 10.0, 0.1)

yy = clf.intercept_[0]+ clf.coef_[0][1]*XX+ clf.coef_[0][2]*np.power(XX, 2)

plt.plot(XX, yy, '-r'_)

plt.xlabel("Engine size")

plt.ylabel("Emission")
```

## Evaluation

```
[]: from_sklearn.metrics_import_r2_score

test_x_poly = poly.fit_transform(test_x)
test_y_ = clf.predict(test_x_poly)

print("Mean absolute error: %.2f" % np.mean(np.absolute(test_y_ - test_y)))
print("Residual sum of squares (MSE): %.2f" % np.mean((test_y_ - test_y) *** 2))
print("R2-score: %.2f" % r2_score(test_y_ _ test_y_ ).
```

#### Practice

Try to use a polynomial regression with the dataset but this time with degree three (cubic). Does it result in better accuracy?

```
    # write your code here

▼ Click here for the solution
    poly3 = PolynomialFeatures(degree=3)
    train_x_poly3 = poly3.fit_transform(train_x)
    clf3 = linean_model.LineanRegression()
```

```
cir3 = linear_model.linearkegression()
train_y3 = clf3.fit(train_x_poly3, train_y)
# The coefficients
print ('Coefficients: ', clf3.coef_)
```

```
print ('Intercept: ',clf3.intercept_)
plt.scatter(train.ENGINESIZE, train.CO2EMISSIONS, color='blue')
XX = np.arange(0.0, 10.0, 0.1)
yy = clf3.intercept_[0]+ clf3.coef_[0][1]*XX + clf3.coef_[0][2]*np.power(XX, 2) + clf3.coef_[0][3]*np.power(XX, 3)
plt.plot(XX, yy, '-r' )
plt.xlabel("Engine size")
plt.xlabel("Engine size")
plt.ylabel("Emission")
test x_poly3 = poly3.fit_transform(test_x)
test_x_poly3 = clf3.predict(test_x_poly3)
print("Mean absolute error: %.2f" % np.mean(np.absolute(test_y3_ - test_y)))
print("Residual sum of squares (MSE): %.2f" % np.mean((test_y3_ - test_y) ** 2))
print("R2-score: %.2f" % r2_score(test_y3_ , test_y) )
```

## Want to learn more?

IBM SPSS Modeler is a comprehensive analytics platform that has many machine learning algorithms. It has been designed to bring predictive intelligence to decisions made by individuals, by groups, by systems – by your enterprise as a whole. A free trial is available through this course, available here: SPSS Modeler

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Thank you for completing this lab!

## Author

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#### Other Contributors

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# **Change Log**

Version	Changed By	Change Description
2.2	Lakshmi	Made changes in markdown of equations
2.1	Lakshmi	Made changes in URL
2.0	Lavanya	Moved lab to course repo in GitLab
	2.2	2.1 Lakshmi

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