Announcement

- Last Time:
  - Stability and Proportional Control
- Today
  - Proportional + Derivative Control
  - Proportional + Derivative + Integral Control
  - Modern State Space Design

- Next lecture, we will talk about some really simple but very useful MATLAB tools that can help you with filtering and analyze control. This concludes our review on signals and control, a very common application of embedded systems.

Marble under Proportional Control

- Stability (does the error grow, shrink to zero or be bounded?)
  - Stability: all eigenvalues' real parts are negative
  - Instability: at least one eigenvalue's real part is greater than zero.
  - Marginal stability: none greater than zero but some eigenvalues equal to zero.

For example, suppose that s = 5 + 6j, is the system unstable? The error y(t):

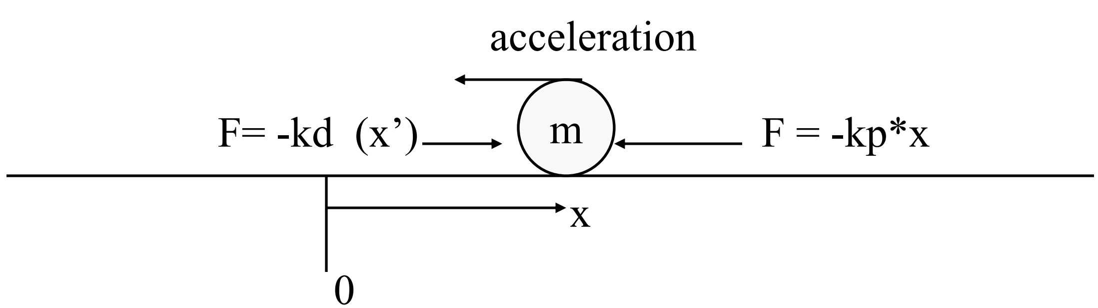
x(t) =exp(5+6j)
=exp(5 t)* exp(6 j t)

- exp(5 t) grows larger and larger
- exp(6j t) = cost(6 t) + j sin(6 t) oscillates with a constant magnitude

- The combined effect is that the oscillation grows larger and larger

Intuition

- When the position error, x, is positive, proportional control force is negative. It pushes the marble back to the origin (setpoint) at 0. What will be the force and the speed when marble reaches origin if we use only proportional control?
- When the marble is moving towards setpoint, the velocity is negative. So the force due to derivative control -kd*x’ is positive. This counters the proportional force and slows down the marble’s motion towards the origin.
- In summary, proportional control is like your car’s gas pedal that moves the car towards the setpoint (where you want it to be) while derivation control is like your brake.



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Marble Under P + D control

- F = m x'', //where m is the mass of the marble
- F = -kp*x // proportional control
- F = -kp*x - kd*x' //proportional + derivative control

- m*x'' + kd*x' + kp*x = 0 //putting it together
- m*s^2 +kd*s + kp = 0
- s = (-kd ± SQRT(kd^2 - 4m*kp))/2m

- m > 0, kp > 0 and kd > 0

- Is this system stable? Can you prove it?

How to Tune a Simple PD Controller

- Experimental tuning procedure
  - first to set derivative gain to zero.
  - Slowly increase the proportional gain until the device oscillates around the set point with a speed that is satisfactory. At this point,
    - the real part of the eigenvalue is _________
    - the imaginary part of the eigenvalue determines the __________ of oscillation
  - Slowly increase the derivative gain until the device settles down at the setpoint. At this point, the real part of the eigenvalue is ____________

- Fine tuning
  - If the motion move towards setpoint is too slow, we can __________ proportional gain or ___________ derivative gain

  - If the motion overshoots the setpoint too much, we can __________ the derivative gains or __________ proportional gain

- Avoid using too large a proportional gain and too large a derivative gain at the same time. This will saturate the actuator. (Like slam gas and brake).

Integral Control

- A system often has friction or changing workloads that may not be able modeled in advance
  – In auto-cruise control, we cannot know how many passengers will be in the car
  – Frictions may be change due to machine conditions
- Unmodeled heavy load often results in steady state error, the system will settle near rather than at the setpoint.

What Does Integral Control Do?

- Integral control adds up (integrates) the past errors and then give a force that is proportional to the cumulative errors
  - So if the marble gets stuck near a set-point due to some friction, the position error adds-up over time, eventually generate a force large enough to help get the marble going
  - So if the car has a heavier load and the velocity settles on a speed lower than the set-point for a while, the error adds up and the integral control leads to increased throttle.

The Dark Side of Integral Control

- Integral control acts on cumulative errors. It takes a while to reach a large sum and it will take time to reduce the sum. Consider the following case:
  - the marble stuck on the left side of the set-point
  - After 10 sec, the integral control is large enough to help get the marble moving
  - The integral will keep increasing until the marble cross the origin
  - It will take a while to “wash out” the cumulative error.
- Overdose of integral control is a common source of overshoot, oscillation and even instability.

Using Integral Control

- As a rule of thumb, start from zero and use it lightly
- Check the eigenvalue of the system make sure all of them are sufficiently negative.

- $X'' = F/m$, where $F = -Kp\, x - Kd\, x' - \int Ki * x$

- $s^2 + Kd/m\, s + Kp/m + Ki/m\, 1/s = 0$, Laplace transform of an integral is 1/s

- $s^3 + kd/m\, s^2 + kp/m\, s + Ki/m = 0$

- As we can see, the effect of integral is to add an order to the system. Large value of Ki could lead to positive eigenvalue.

- We can solve the equation to see if the real part of the eigenvalues are still negative. This is best done by using tools such as Matlab. A subject that we will overview next lecture

How to Tune a Simple PID Controller

- Experimental tuning procedure
  - first to set derivative gain and integral gains to zero.
  - Slowly increase the proportional gain until the device oscillates around the set point with a speed that is satisfactory. At this point,
    - the real part of the eigenvalue is _________
    - the imaginary part of the eigenvalue determines the __________ of oscillation
  - Slowly increase the derivative gain until the device settles down at the setpoint. At this point, the real part of the eigenvalue is ____________
  - If there is steady state error, slightly increase the integral gain until the steady state error is corrected and yet not causing serious oscillation. This means that
    - the real part of the eigenvalues are still _________.

- Fine tuning
  - If the motion move towards setpoint is too slow, we can __________ proportional gain or ___________ derivative gain, don't play with integral gain.
  - If there is steady state error, we can add a <u>little</u> __________ gain
  - If the motion overshoots the setpoint and oscillate we can __________ the derivative gains or reduce __________ gain and __________ gain

Modern State Space Control

- We have learned the effect of control is to change the eigenvalues, e.g., under proportional control of the marble, the real-part is zero. By adding the derivative term, the eigenvalues turn negative.
- Matrix is far more powerful tool to manipulate a linear system's eigenvalues. The use of the matrix method is called the state space method.
- The state of a dynamic system is its position and velocities along all the axes.
- (Recall that in the absence of external force, an object will keep its velocity. If it is rest, it will stay rest and if it is moving, it will keep moving. This is known as ___________ law)

Summary

- We have come a very long way in these two lectures. We have learned
  - The key concepts of control
  - How to experimentally turn a PID controller
  - How to use modern state space design to control an inverted pendulum.
- Next lecture you will learn how to use Matlab to simulate the control of the water-saw to prepare you to control the real water-seesaw. This concludes one of the most common applications of embedded real time systems.
- We will return to embedded real-time computing issues.

Appendix:Marble on a Table in Matrix Form

The general form of linear control is: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$,

where **x** is state and we let x_1 =position error, and x_2= error derivative

A is the coefficients of the PDE without control

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ without external force, $\dot{x}$(i.e., x_2) unchange and $\ddot{x}$ (i.e. $\dot{x}_2$)$= 0$

$\mathbf{u} = -\mathbf{kx}$, the control is a linear combination error and error derivative

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} \begin{bmatrix} k_p & k_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{\mathbf{x}} \quad = \quad \mathbf{A} \quad \mathbf{x} \quad - \quad \mathbf{B} \quad \mathbf{k} \quad \mathbf{x}$$

where **B** maps external control into velocity and acceleration

Since $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} = (\mathbf{A} - \mathbf{BK})\mathbf{x}$, the effect of **u** is to change the system matrix from **A** to **(A − BK)**

A describes how the states will change when there is no control and **(A - BK)** tells us how the states will change with feedback control.

Appendix: Using Matlab LQR

- One of the powerful and practical tools in control is the Linear Quadratic Regulator (

LQR is designed to minimize the cost function

$$J = \int (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt$$

where $\mathbf{x}^T \mathbf{Q} \mathbf{x}$ is the square of errors weighted by $\mathbf{Q}$

$\mathbf{u}^T \mathbf{R} \mathbf{u}$ is the square of control signal weighted by $\mathbf{R}$

Using LQR to design a Marble controller

```
>>m = 0.01;  A = [0,1;0,0]; B = [0; 1/m];  Q=[1,0;0,1]; R = 1;
>>[K, S, E] = lqr(A, B, Q, R] %where k is the gains,  E are the eigenvalues
% Matlab will give us K = 1.0000, 1.0100 and E = -1.0001,   -99.9950
```

Appendix: A Challenge for Extra-Credit

- Telelab allows you to control a real inverted pendulum across the Web and watching your control via streaming video.
- Go to www-drii.cs.uiuc.edu/download
- Follow the instructions to install Telelab on a machine.
- Read the IP_Control_Overview.pdf and review the LQR design examples for the inverted pendulum.
- Do your own LQR design and try it out! (Tips: the controller has maximal of +- 5 volts to play with and don't push the controller too hard.)
- Actually control the inverted pendulum and show it to Dan that your team is able to control an inverted pendulum and get extra 50 (out of 100) points of a lab assignment! You can do it anytime before the final but make the arrangement early.