These problems are provided simply for practice with blocking times and the UB and Exact tests. Brief solutions are included. Please post questions to the newsgroup (`uiuc.class.cs331`), or ask during office hours (5:00–8:00pm Thursday in the 331 lab).

1. There are 4 periodic tasks, τ_1 through τ_4, and 2 data structures shared among the tasks, DS_1 and DS_2. τ_1, τ_2, and τ_4 share DS_1 where τ_1's critical section is a ms, τ_2's critical section is b ms, and τ_4's critical section is c ms. τ_2 and τ_3 share DS_2 where τ_2's critical section is d ms and τ_3's critical section is e ms.

   a, b, c, d, and e are all positive integers. Do not make any assumptions about their relative sizes. The *max* function may be used in your answers. It accepts any number of arguments and returns the largest one. For example, $max(5, 20) = 20$ and $max(100, 54, 8) = 100$.

   Assuming that semaphores can be nested, what are the worst case blocking times of each task under the Basic Priority Inheritance Protocol (BIP) and the Priority Ceiling Protocol (PCP)?

   Solution:

Task	**BIP**	**PCP**
τ_1	$max(c, b + d + e)$	$max(c, b + d)$
τ_2	$c + e$	$max(c, e)$
τ_3	c	c
τ_4	0	0

2. There are four periodic tasks with the following computation times (C), periods (T), blocking times (B), and pre-period deadlines (D). The context switching time is $S = \frac{1}{2}$. Use the Utilization Bound and Exact tests as needed to determine if the task set is schedulable.

Task	C	T	B	D
τ_1	15	100	6	30
τ_2	70	150	3	20
τ_3	20	200	4	10
τ_4	26	275	0	10

$S = 0.5$

   Solution:
   The following tests show that the task set is schedulable.

   (a) The UB test on τ_1 passes:

$$\frac{15 + 1 + 6 + 30}{100} = \frac{52}{100} \quad \leq \quad U(1) = 1$$

   (b) The UB test on τ_2 passes:

$$\frac{15 + 1}{100} + \frac{70 + 1 + 3 + 20}{150} = \frac{59}{75} \approx 0.7867 \quad \leq \quad U(2) \approx 0.8284$$

(c) The UB test on τ_3 is inconclusive:

$$\frac{15+1}{100} + \frac{70+1}{150} + \frac{20+1+4+10}{200} = \frac{97}{120} \approx 0.8083 \quad \not\leq \quad U(3) \approx 0.7797$$

(d) The Exact test on τ_3 passes:

$$a_0 = (15+1) + (70+1) + (20+1) = 108$$
$$a_1 = 21 + \left\lceil \frac{108}{100} \right\rceil 16 + \left\lceil \frac{108}{150} \right\rceil 71 = 124$$
$$a_2 = 21 + \left\lceil \frac{124}{100} \right\rceil 16 + \left\lceil \frac{124}{150} \right\rceil 71 = 124$$
$$a_1 = a_2 = 124 \quad \leq \quad (T_3 - B_3 - D_3) = (200 - 4 - 10) = 186$$

(e) The Exact test on τ_4 passes:

$$a_0 = (15+1) + (70+1) + (20+1) + (26+1) = 135$$
$$a_1 = 27 + \left\lceil \frac{135}{100} \right\rceil 16 + \left\lceil \frac{135}{150} \right\rceil 71 + \left\lceil \frac{135}{200} \right\rceil 27 = 151$$
$$a_2 = 27 + \left\lceil \frac{151}{100} \right\rceil 16 + \left\lceil \frac{151}{150} \right\rceil 71 + \left\lceil \frac{151}{200} \right\rceil 27 = 222$$
$$a_3 = 27 + \left\lceil \frac{222}{100} \right\rceil 16 + \left\lceil \frac{222}{150} \right\rceil 71 + \left\lceil \frac{222}{200} \right\rceil 27 = 265$$
$$a_4 = 27 + \left\lceil \frac{265}{100} \right\rceil 16 + \left\lceil \frac{265}{150} \right\rceil 71 + \left\lceil \frac{265}{200} \right\rceil 27 = 265$$
$$a_3 = a_4 = 265 \quad \leq \quad (T_4 - D_4) = (275 - 10) = 265$$

3. There are four periodic tasks with the following computation times (C), periods (T), blocking times (B), and pre-period deadlines (D). The context switching time is $S = 1.5$.

Task	C	T	B	D
τ_1	37	100	0	0
τ_2	60	200	0	0
τ_3	55	250	0	0
τ_4	1	850	0	0

$S = 1.5$

Note: As it is written, this problem requires that the exact test be carried out quite far. Letting T_4 be 800 instead of 850 will make things simpler at the end. Why?

(a) Use the Utilization Bound and Exact tests as needed to show that the task set is *not* schedulable.

Solution:
The Exact test on τ_3 fails:

$$a_0 = (37+3) + (60+3) + (55+3) = 161$$
$$a_1 = 58 + \left\lceil \frac{161}{100} \right\rceil 40 + \left\lceil \frac{161}{200} \right\rceil 63 = 201$$
$$a_2 = 58 + \left\lceil \frac{201}{100} \right\rceil 40 + \left\lceil \frac{201}{200} \right\rceil 63 = 304$$
$$a_2 = 304 \quad \not\leq \quad T_3 = 250$$

(b) Devise a period transformation that makes the task set schedulable. Show that your transformation works.

Solution:
Make τ_3 into τ_3' by dividing its computation time and period by 5. This makes the three highest priority tasks harmonic. The following tests show that the modified task set is schedulable.

Task	C	T	B	D
τ_3'	11	50	0	0
τ_1	37	100	0	0
τ_2	60	200	0	0
τ_4	1	850	0	0

$S = 1.5$

i. The UB test on τ_3' passes:

$$\frac{11+3}{50} = \frac{14}{50} \quad \leq \quad U(1) = 1$$

ii. The UB test on τ_1 passes (harmonic):

$$\frac{11+3}{50} + \frac{37+3}{100} = \frac{68}{100} \quad \leq \quad U(2) = 1$$

iii. The UB test on τ_2 passes (harmonic):

$$\frac{11+3}{50} + \frac{37+3}{100} + \frac{60+3}{200} = \frac{199}{200} \quad \leq \quad U(3) = 1$$

iv. The UB test on τ_4 is inconclusive:

$$\frac{11+3}{50} + \frac{37+3}{100} + \frac{60+3}{200} + \frac{1+3}{850} \approx 0.9997 \quad \not\leq \quad U(4) \approx 0.7568$$

v. The Exact test on τ_4 passes:

$$a_0 = (11+3)+(37+3)+(60+3)+(1+3) = 121$$

$$a_1 = 4 + \left\lceil \frac{121}{50} \right\rceil 14 + \left\lceil \frac{121}{100} \right\rceil 40 + \left\lceil \frac{121}{200} \right\rceil 63 = 189$$

$$a_2 = 4 + \left\lceil \frac{189}{50} \right\rceil 14 + \left\lceil \frac{189}{100} \right\rceil 40 + \left\lceil \frac{189}{200} \right\rceil 63 = 203$$

$$a_3 = 4 + \left\lceil \frac{203}{50} \right\rceil 14 + \left\lceil \frac{203}{100} \right\rceil 40 + \left\lceil \frac{203}{200} \right\rceil 63 = 320$$

$$a_4 = 4 + \left\lceil \frac{320}{50} \right\rceil 14 + \left\lceil \frac{320}{100} \right\rceil 40 + \left\lceil \frac{320}{200} \right\rceil 63 = 388$$

$$a_5 = 4 + \left\lceil \frac{388}{50} \right\rceil 14 + \left\lceil \frac{388}{100} \right\rceil 40 + \left\lceil \frac{388}{200} \right\rceil 63 = 402$$

$$a_6 = 4 + \left\lceil \frac{402}{50} \right\rceil 14 + \left\lceil \frac{402}{100} \right\rceil 40 + \left\lceil \frac{402}{200} \right\rceil 63 = 519$$

$$a_7 = 4 + \left\lceil \frac{519}{50} \right\rceil 14 + \left\lceil \frac{519}{100} \right\rceil 40 + \left\lceil \frac{519}{200} \right\rceil 63 = 587$$

$$a_8 = 4 + \left\lceil \frac{587}{50} \right\rceil 14 + \left\lceil \frac{587}{100} \right\rceil 40 + \left\lceil \frac{587}{200} \right\rceil 63 = 601$$

$$a_9 = 4 + \left\lceil \frac{601}{50} \right\rceil 14 + \left\lceil \frac{601}{100} \right\rceil 40 + \left\lceil \frac{601}{200} \right\rceil 63 = 718$$

$$a_{10} = 4 + \left\lceil \frac{718}{50} \right\rceil 14 + \left\lceil \frac{718}{100} \right\rceil 40 + \left\lceil \frac{718}{200} \right\rceil 63 = 786$$

$$a_{11} = 4 + \left\lceil \frac{786}{50} \right\rceil 14 + \left\lceil \frac{786}{100} \right\rceil 40 + \left\lceil \frac{786}{200} \right\rceil 63 = 800$$

$$a_{12} = 4 + \left\lceil \frac{800}{50} \right\rceil 14 + \left\lceil \frac{800}{100} \right\rceil 40 + \left\lceil \frac{800}{200} \right\rceil 63 = 800$$

$$a_{11} = a_{12} = 800 \quad \leq \quad T_4 = 850$$