Homework 4, due April 11th 5pm CST

Handin at 1102 DCL. Slide under door if TA not present.

Important: Please type or *neatly* write your solutions. Anything we can't read will receive no credit. You must show appropriate work to receive full credit.

1. (20 points) Suppose we have a PD control system with K_p equal to 3.2, K_d equal to 2.4, and the mass of the object we're trying control equal to 0.4.

   (a) What are the eigenvalues for this system? Is it stable? (5 points)

   (b) Assuming the object is initially at rest, what is the 95% settle time of the system? (The time after which the position errors will be less than 5% of the initial position error). Show your work. (Recall that if s_1 and s_2 are distinct eigenvalues, then the error is $x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$ where c_1 and c_2 are constants.) (15 points)

Solution:

(a) The Laplace transform gives us $s^2 + \frac{2.4}{.4}s + \frac{3.2}{.4} = 0$. Solve for s and we get:

$s_1 = -2$

$s_2 = -4$

The eigenvalues are real and negative. Thus the system is stable.

(b) Using eigenvalues s_1 and s_2, we can set the error $x(t)$ to the following:

$$x(t) = c_1 e^{-2t} + c_2 e^{-4t} \tag{1}$$

We want to solve for 95% settle time, which translates into the following equation:

$$0.05x(0) = c_1 e^{-2t} + c_2 e^{-4t} \tag{2}$$

Substituting 0 in for t in equation (1), we see that:

$$x(0) = c_1 + c_2$$

Differentiating equation (1) and using the initial condition that $x'(0) = 0$, we also get:

$$0 = -2c_1 - 4c_2$$

Using the above two equations, we get that $c_1 = 2x(0)$ and $c_2 = -x(0)$. Substituting these values into equation (2), we get:

$$0.05x(0) = 2x(0)e^{-2t} - x(0)e^{-4t} \tag{3}$$

$$0.05x(0) = x(0)(2e^{-2t} - e^{-4t}) \tag{4}$$

$$0.05 = 2e^{-2t} - e^{-4t} \tag{5}$$

$$0 = -e^{-4t} + 2e^{-2t} - 0.05 \tag{6}$$

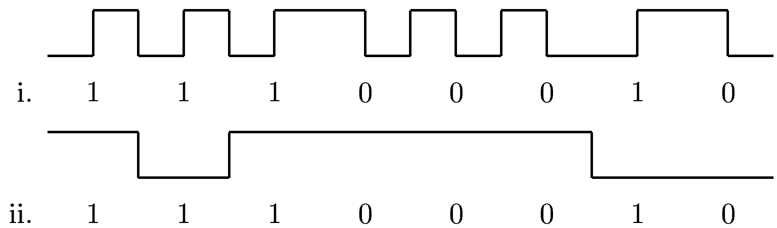
$$0 = -(e^{-2t})^2 + 2e^{-2t} - 0.05 \tag{7}$$

If we view e^{-2t} as a variable, the last equation turns into a quadratic equation with solutions $(1.975, 0.025)$. Solving for t in each of the numbers, we get $t = (-0.340, 1.838)$. Obviously, the negative number doesn't make sense. Hence, 1.838 seconds is the answer.

2. Serial I/O (20 points)

   (a) Show the encoding of the signal `11100010` as a waveform using (i) Manchester Encoding and (ii) NRZI. For NRZI, assume the line is at `0` before transmission starts. For Manchester Encoding, use the method discussed in the class lecture notes. (15 points)

   Solution:



   (b) What are some advantages and disadvantages of using Manchester Encoding? NRZI? (5 points)

   Solution:

   Manchester Encoding is clock-friendly because it doesn't rely on a clock. But it is not bit-friendly because it needs one transition (2 transmissions) per bit.

   NRZI is bit-friendly because sending 0's do not require transitions. It is not clock-friendly because the receiver and sender must have synchronized clocks to calculate boundaries between bits.

3. Schedulability Utilization Bound Test (60 points)

   For the following items, calculate if the tasks listed are schedulable based on the Schedulability Utilization Bound Test and, if necessary, the Exact Schedulability Test.

   (a) τ_1 has execution time $C_1 = 2$ and $T_1 = 7$.
   τ_2 has execution time $C_2 = 6$ and $T_2 = 11$.
   τ_3 has execution time $C_3 = 4$ and $T_3 = 35$.

   Solution:

   $U_1 = 2 \div 7 = 0.286$
   $U_2 = 6 \div 11 = 0.546$
   $U_3 = 4 \div 35 = 0.115$
   $U = U_1 + U_2 + U_3 = 0.946 > U(3) = 0.779$. Thus the 3 tasks are *not* guaranteed to be schedulable by the UB test alone. Therefore, the exact test is needed. We see that $U_1 + U_2 = 0.832 > U(2)$, so we apply the exact test on these 2 tasks first.

   $a_0 = 2 + 6 = 8$
   $a_1 = 6 + \lceil \frac{8}{7} \rceil 2 = 10$
   $a_2 = 6 + \lceil \frac{10}{7} \rceil 2 = 10 < T_2$

   The first two tasks are schedulable. Now, for the entire set.

   $a_0 = 2 + 6 + 4 = 12$
   $a_1 = 4 + \lceil \frac{12}{7} \rceil 2 + \lceil \frac{12}{11} \rceil 6 = 20$
   $a_2 = 4 + \lceil \frac{20}{7} \rceil 2 + \lceil \frac{20}{11} \rceil 6 = 22$

$a_3 = 4 + \lceil \frac{22}{7} \rceil 2 + \lceil \frac{22}{11} \rceil 6 = 24$
$a_4 = 4 + \lceil \frac{24}{7} \rceil 2 + \lceil \frac{24}{11} \rceil 6 = 30$
$a_5 = 4 + \lceil \frac{30}{7} \rceil 2 + \lceil \frac{30}{11} \rceil 6 = 32$
$a_6 = 4 + \lceil \frac{32}{7} \rceil 2 + \lceil \frac{32}{11} \rceil 6 = 32 < T_3$
The entire set of 3 tasks is schedulable.

(b) τ_1 has execution time $C_1 = 2$ and $T_1 = 9$.
τ_2 has execution time $C_2 = 6$ and $T_2 = 21$.
τ_3 has execution time $C_3 = 4$ and $T_3 = 25$.
τ_4 has execution time $C_4 = 3$ and $T_4 = 35$.
Solution:
$U_1 = 2 \div 9 = 0.223$
$U_2 = 6 \div 21 = 0.286$
$U_3 = 4 \div 25 = 0.16$
$U_4 = 3 \div 35 = 0.086$
$U = U_1 + U_2 + U_3 + U_4 = 0.754 \leq U(4) = 0.756$. Thus the 4 tasks are schedulable.

(c) τ_1 has execution time $C_1 = 2$ and $T_1 = 9$.
τ_2 has execution time $C_2 = 6$ and $T_2 = 21$.
τ_3 has execution time $C_3 = 4$ and $T_3 = 25$.
τ_4 has execution time $C_4 = 4$ and $T_4 = 35$.
Solution:
$U_1 = 2 \div 9 = 0.223$
$U_2 = 6 \div 21 = 0.286$
$U_3 = 4 \div 25 = 0.16$
$U_4 = 4 \div 35 = 0.115$
$U = U_1 + U_2 + U_3 + U_4 = 0.782 > U(4) = 0.756$. Thus the 4 tasks are not schedulable by the UB test alone. Need to use the exact test.
$a_0 = 2 + 6 + 4 + 4 = 16$
$a_1 = 4 + \lceil \frac{16}{9} \rceil 2 + \lceil \frac{16}{21} \rceil 6 + \lceil \frac{16}{25} \rceil 4 = 18$
$a_2 = 4 + \lceil \frac{18}{9} \rceil 2 + \lceil \frac{18}{21} \rceil 6 + \lceil \frac{18}{25} \rceil 4 = 18 < T_4$
Hence, the entire set is schedulable.

(d) τ_1 has execution time $C_1 = 6$ and $T_1 = 10$.
τ_2 has execution time $C_2 = 3$ and $T_2 = 20$.
τ_3 has execution time $C_3 = 8$ and $T_3 = 40$.
Solution:
$U_1 = 6 \div 10 = 0.6$
$U_2 = 3 \div 20 = 0.15$
$U_3 = 8 \div 40 = 0.2$
Since the 3 tasks are harmonic, $U(3) = 1.0$. $U = U_1 + U_2 + U_3 = 0.95 \leq U(3) = 1.0$. Thus the 3 tasks are schedulable.