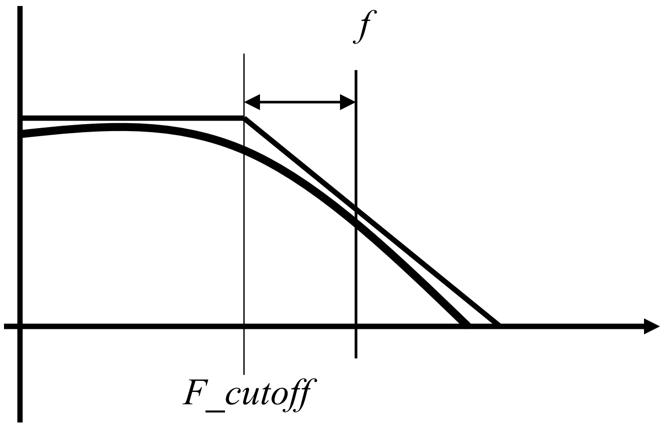
Announcement

- In the next 3 lectures, we will study the basics about signal processing and control.
- Although we will spend only a very short time on this subject, you can still do quite impressive things with what is taught here, for example, controlling the highly unstable water-saw in the lab.

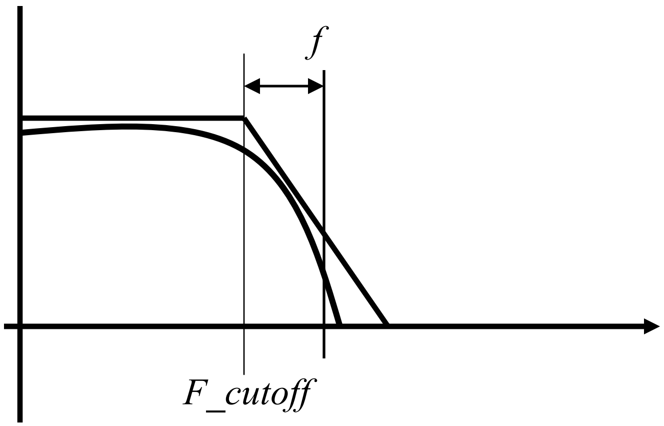
How to specify Low Pass filter parameters:
Effect of cutoff frequency and order (stage)

•The further away from the cutoff frequency, the more is the attenuation (reduction)

•The higher the order of a filter, the sharper is the slope of attenuation



filter 1



filter 2

Which filter has a higher order?

How To Specify Filter Parameters

For a given filter design, you need to pick the cutoff frequency and the order to meet the design requirements.

$$|a(\omega)| = |a_0| \left(\frac{1}{\sqrt{1 + (\frac{\omega}{\omega\_cutoff})^2}} \right)^N$$

$$\theta(\omega) = -N(\arctan \frac{\omega}{\omega\_cutoff})$$

The formula for the commonly used Butterworth filter is SQRT(1 /(1 + (w/w_cutoff)^{2N}))

Design an Anti-aliasing Filter

- The frequencies of interest in the signal are from 10 - 60 Hz. There are serious noises with frequencies in the range 500 - 1000 Hz seen on the scope. Pick the cutoff frequency and the order so that the signal will be reduced no more that 30% while the noise will be reduced at least 96%.

- You need to try various different cutoff frequencies and orders. The rule of thumb is
  - if the attenuation to signal is too large, increase the cutoff frequency
  - if the attenuation to noise is not enough, reduce the cutoff freq. or increase the order

- Max magnitude reduction in signal.

$$\left(\frac{1}{\sqrt{1+\left(\frac{60}{100}\right)^2}}\right)^2 = 0.74$$

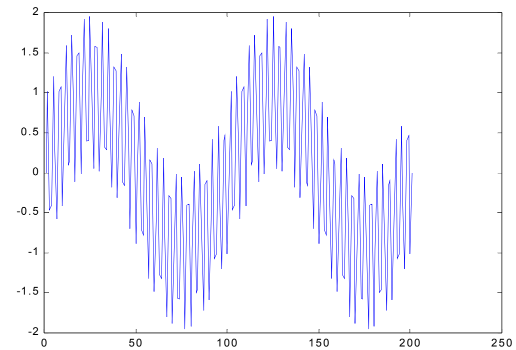
- Min magnitude reduction in noise.

$$\left(\frac{1}{\sqrt{1+\left(\frac{500}{100}\right)^2}}\right)^2 = 0.038$$

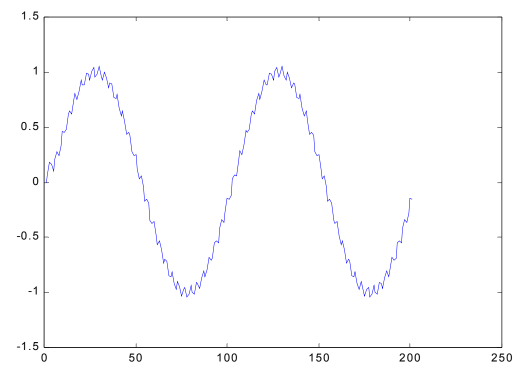
Why not using 10 Hz and 1000 Hz in the formulae?

Example

- t = 0:0.005:2; %sample at 200 Hz for 2 sec.
- x = sin(2*pi*t); % 1 Hz signal
- y = sin(2*pi*30*t); % 30 Hz noise
- plot(x+y)



- wn = 20/(200*0.5)=0.2 % cut off at 20 Hz
- Nyquist
- [b, a] = butter(2, 0.2) % 2^{nd} order filter
  % cutoff 20 Hz
- z = filter(b, a, (x+y));
- plot(z)



- "%" comment. ";" suppress output.

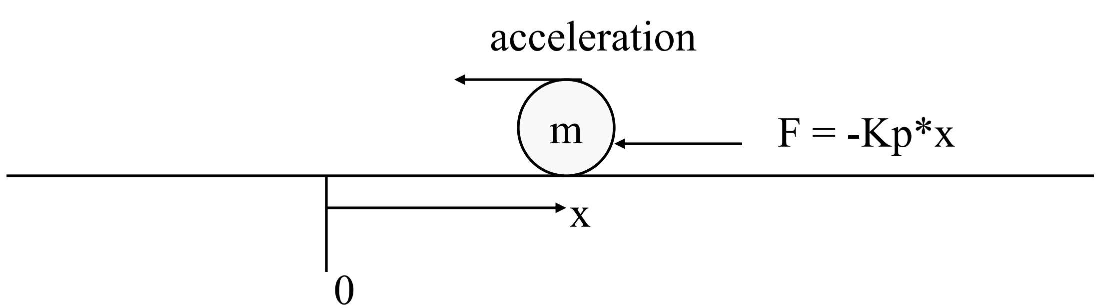
Automatic Control

There are many exciting automatic control applications. The challenging ones are those which are open loop (I.e. without control) unstable.

You will learn the key concepts in 2 lectures and will be able to control an open loop unstable system, the water-seesaw, at the lab by yourselves. For those who want extra credits, you may use Matlab to design a controller to control a real inverted pendulum in our Telelab.

A Simple System with Proportional Control

- The control is proportion to the size of the error
- Consider a marble on a flat and perfectly leveled table.
  - Any point can be an equilibrium point (just pick one)
  - Its motion can be described by Newton's law F = ma, or x'' = F/m
  - suppose that we want to keep the marble at x = 0, by applying proportional control: $F = -k_p x$.
    - The feedback is negative since if the marble *position error* is negative, it pushes with a positive force and vice versa.
    - K_p is a positive number known as proportional control constant.



Concept of Stability

- Can you describe the motion of this marble under proportional control?

- Intuitively, is the controlled motion of marble stable?

- What would be a suitable definition of stability? Imagine that you are the early scientists who need to define it.

- Can you apply your stability definition to explain why we consider the motion of earth going around the sun is a stable motion?

Concepts of Stability

- Error is the difference between the set point (where you want the state to be) and the actual state.

- Set point can follow a prescribed trajectory, e.g., an orbit. Or as a constant, e.g., keep room temperate at 75 F.

- The objective of controlling the marble is to put it at rest at the origin. I.e. x = 0, and x' = 0, from any initial condition. Or for that matter, after the marble was disturbed by some force.
  – This intuition is formalized as the notion of stability. Or more precisely, asymptotic stability, I.e. the ***errors*** will converge to zero over time.

  – The opposite of stability is instability, meaning that the ***errors*** will grow larger and larger without bound. That is, the marble will leave the origin for good.

  – In between is marginal stability, the ***errors*** stay within some bound.

Stability

So

- Is the marble under proportional control
  - Stable
  - Unstable
  - Marginally stable?

- What is the mathematical tool that would allow us to reason about the stability of controlled motion?

Stability and Eigenvalues

- Most systems are modeled by linear systems, or by a collection of linear systems via piecewise linearization. For example, a fighter jet F16, is controlled by six linear models for six different flight conditions.

- From the calculus courses, we know that the solution of Linear Partial Differential equations. e.g., 3*y'' + 4*y' + 5*y = 0, is sum of exponentials:
  - $y(t) = c_1 \exp(\lambda_1 t) + c_2 \exp(\lambda_2 t) + ...$
    - Where the exponents λ_l, λ_2 ... are known as eigenvalues of the system.
    - eigenvalues can be complex, e.g. $\exp(RE(\lambda)\ t)*\exp(IM(\lambda)\ jt)$

Stability Analysis

- What are the relationship between the magnitude of eigenvalues and the notion of (asymptotic) stability, marginal stability and instability?

  – Stability: all eigenvalues' real parts are negative
  – Instability: at least one eigenvalue's real part is greater than zero.
  – Marginal stability: none greater than zero but some eigenvalues equal to zero.

  For example, suppose that s = 5 + 6j
  Is the system unstable? The error y(t):

  y(t) =exp((5+6j)t)
  =exp(5 t)* exp(6 j t)

  – exp(5 t) grows larger and larger
  – exp(6j t) = cost(6 t) + j sin(6 t) oscillates with a constant magnitude

  – The combined effect is that the oscillation grows larger and larger

Solving Simple LPE Using Laplace Transform

- $3*y'' + 4*y' + 5*y = 0$ (replace n^{th} derivative with s^n)
- $3s^2 + 4s + 5 = 0$

- Solve for s and you will get the eigenvalues

- Now, let us analyze the stability of our marble under proportional control.

- From Newton's law, we have $F = m x''$
- From proportional control we have $F = -kp * x$

- Hence, we have $mx'' = -kp*x$, with $m>0$ and $kp > 0$,
- $-kpx = mx''$ -> $x'' = -kpx/m$ -> $s^2 = -kp/m$
- s = SQRT(-Kp/m) = SQRT(Kp/m) j

- The marble under proportional control is therefore:
  - stable?
  - Unstable?
  - marginally stable?

Summary

- In this lecture, we have learned
  - The concept of stability (does the error grow or shrink?)
  - How to analyze the system stability using differential equations.