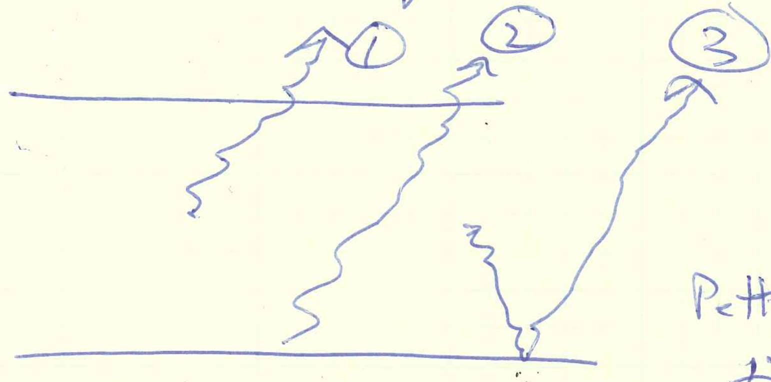
Day 10:

Show that the brightness temperature received by a satellite is

$$T_B \sim T_S - R_p T_S T_r^2$$

where T_S is the surface temperature R_p is the polarized reflectivity (either V or H) and T_r is the transmittance. Assume that the atmosphere and the surface are at the same temperature, and that the atmosphere is isothermal



Petty Stephens
$t^* = T_r$

From Petty 8.46 we have

$$I^{\uparrow}(\infty) = \left(\varepsilon_p B(T_S) + (1-\varepsilon_p) I_{\downarrow}(0) \right) t^*$$

where ε_p is the polarized emissivity of the surface, so $(1-\varepsilon_p)$ is the reflectivity, since ε_p = absorptivity. t^* is the transmissivity between the surface and TOA

From 8.34 and 8.35 we know that, if there is no downward radiance at the top of the atmosphere $I\downarrow(\infty) = 0$ and

$$I_\downarrow(0) = B_\downarrow(1 - t^*)$$ where t^*

is the transmissivity. So for the 3 arrows in the sketch, from 8.35

$\cancel{1 + 2 =}$

$\cancel{1 + 2}$

$$\overset{2}{}\qquad\qquad\qquad\overset{3}{}$$

$$I_\uparrow(\infty) = \left(\varepsilon_p B(T_s) + (1-\varepsilon_p) I_\downarrow(0)\right) t^*$$

$$+ \underset{1}{B(T_s)(1 - t^*)}$$

Now use the Rayleigh-Jean approximation

$$B_\lambda(T) = \frac{2 c k_B T}{\lambda^4} \quad (6.7) \text{ sec. C.1.4}$$

$$B(T_s) = \frac{2 c k_B T_s}{\lambda^4}$$

$$I = \frac{2 c k_B T_B}{\lambda^4}$$

$$I_{\uparrow}(\infty) = \left(\varepsilon_p B(T_S) + (1-\varepsilon_p) B_{\downarrow} (1-t^*)\right) t^*$$
$$+ B(T_S)(1-t^*)$$

Use the R-J approx: and divide by $\frac{2cK_B}{\lambda^4}$

$$T_B = \left(\varepsilon_p T_S + (1-\varepsilon_p) T_S (1-t^*)\right) t^*$$
$$+ T_S(1-t^*)$$

write $(1-\varepsilon_p) = R_p$

$$T_B = (1-R_p) t^* T_S + R_p T_S t^*$$
$$- R_p T_S t^{*2} + T_S - t^* T_S$$

$$T_B = T_S - R_p T_S t_*^2$$

That means if we define

$$\Delta T_{B19} = T_{B19H} - T_{B19V}$$

$$\Delta T_{B37} = T_{B37H} - T_{B37V}$$

Then

$$\Delta T_{B19} = (T_S - R_{19H} T_S t_*^2) - (T_S - R_{19V} T_S t_*^2)$$

$$\Delta T_{B19} = (R_{19V} - R_{19H}) T_S t_*^2$$

$$\Delta T_{B37} = (R_{37V} - R_{37H}) T_S t_*^2$$

Also $t^{*2} = \cancel{T_{r\ell}^2 \, T_{rw}} \; T_{r\ell}^2 \cdot T_{rw}^2 \cdot T_{rox}^2$

where ℓ, w and ox refer to liquid, vapor and oxygen absorber paths.

$$T_{rox} = \exp(-K_{ox}\ell/\mu)$$

$$T_{r\ell} = \exp(-K_\ell \ell/\mu)$$

$$T_{rw} = \exp(-K_w w/\mu)$$

$$t_*^2 = \exp(-2K_\ell \ell/\mu)\exp(-2K_w w/\mu) T_{rox}^2$$

$$\log \Delta T_{B19} = \log\left[T_S(R_v - R_H) T_{rox}^2\right] + \log T_{r_\ell}^2 T_{rw}^2$$

$$= \log\left[T_S(R_v - R_H) T_{rox}^2\right] + 2\left(-k_\ell\, l/\mu\right) + 2\left(-k_w\, w/\mu\right)$$

$$k_\ell l + k_w w = -\mu/2 \log\left(\frac{\Delta T_{B19}}{T_S(R_{v19} - R_{H19}) T_{rox}^2}\right)$$

$$k_\ell l + k_w w = -\mu/2 \log\left(\frac{\Delta T_{B37}}{T_S(R_{v37} - R_{H37}) T_{rox}^2}\right)$$

Final step: Solve for l (liquid water path) and w (water vapor path)

Rewrite as matrix equation

$$\begin{bmatrix} k_{w19} & k_{\ell 19} \\ k_{w37} & k_{\ell 37} \end{bmatrix} \begin{bmatrix} w \\ l \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

where $R_1 = -\frac{\mu}{2} \log\left(\frac{\Delta T_{B19}}{T_S(R_{v19} - R_{H19}) T_{rox}^2}\right)$

$$R2 = -\frac{\mu}{2} \log\left(\frac{\Delta T_{B37}}{T_S(R_{v37} - R_{H37}) T_{rox}^2}\right)$$

In general, if

$$Ax = b$$

then

$$x = A^{-1}b$$

where $A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \times \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

so for our problem:

$$\begin{bmatrix} w \\ l \end{bmatrix} = \frac{1}{K_{w19}K_{l37} - K_{l19}K_{w37}} \begin{bmatrix} K_{l37} & -K_{l19} \\ -K_{w37} & K_{w19} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

write $\delta = K_{w19}K_{l37} - K_{l19}K_{w37}$

$$\boxed{\begin{aligned} w &= \frac{1}{\delta}\left(R_1 K_{l37} - R_2 K_{l19}\right) \\ l &= \frac{1}{\delta}\left(R_2 K_{w19} - R_1 K_{w37}\right) \end{aligned}}$$