

This document contains the program traces and explanations of the execution of the INDUCENFA procedure presented in the paper entitled *A Comparison of Selected Variable Ordering Methods for NFA Induction*. The paper was written by Tomasz Jastrz b and submitted to the International Conference on Computational Science (ICCS 2019).

Let  $S = (\{a, abb\}, \{aaa, ab, ba\})$  be the input sample over the alphabet  $\Sigma = \{a, b\}$ . Let the number of states  $k = 2$ , and let  $y_1 = 0$  and  $y_2 = 1$ . Let the set of constraints  $C = \{c_1, c_2, c_3, c_4, c_5\}$  be as follows, with constraints  $c_1$  and  $c_2$  corresponding to examples, and constraints  $c_3$ – $c_5$  corresponding to counterexamples:

$$\begin{aligned} \mathbf{c_1} : & \quad x_1y_1 + x_2y_2 & = 1 \\ \mathbf{c_2} : & \quad (x_1x_5^2 + x_1x_6x_7 + x_2x_5x_7 + x_2x_7x_8)y_1 + (x_1x_5x_6 + x_1x_6x_8 + x_2x_6x_7 + x_2x_8^2)y_2 & = 1 \\ \mathbf{c_3} : & \quad (x_1^3 + 2x_1x_2x_3 + x_2x_3x_4)y_1 + (x_1^2x_2 + x_1x_2x_4 + x_2^2x_3 + x_2x_4^2)y_2 & = 0 \\ \mathbf{c_4} : & \quad (x_1x_5 + x_2x_7)y_1 + (x_1x_6 + x_2x_8)y_2 & = 0 \\ \mathbf{c_5} : & \quad (x_1x_5 + x_3x_6)y_1 + (x_2x_5 + x_4x_6)y_2 & = 0 \end{aligned}$$

Since it holds that  $y_1 = 0$  and  $y_2 = 1$  the constraints can be simplified as follows:

$$\begin{aligned} \mathbf{c_1} : & \quad x_2 & = 1 \\ \mathbf{c_2} : & \quad x_1x_5x_6 + x_1x_6x_8 + x_2x_6x_7 + x_2x_8^2 & = 1 \\ \mathbf{c_3} : & \quad x_1^2x_2 + x_1x_2x_4 + x_2^2x_3 + x_2x_4^2 & = 0 \\ \mathbf{c_4} : & \quad x_1x_6 + x_2x_8 & = 0 \\ \mathbf{c_5} : & \quad x_2x_5 + x_4x_6 & = 0 \end{aligned}$$

Given the above constraints the numbers of active products in constraints  $c_1$ – $c_5$  are  $|c_1| = 1$ ,  $|c_2| = 4$ ,  $|c_3| = 4$ ,  $|c_4| = 2$ ,  $|c_5| = 2$ . The number of active products  $d(c, x_i)$  involving variables  $x_i$ , for  $i = 1, 2, \dots, 8$ , are as follows:

1. For  $c_1$ :  $d(c_1, x_1) = 0$ ,  $d(c_1, x_2) = 1$ ,  $d(c_1, x_3) = 0$ ,  $d(c_1, x_4) = 0$ ,  $d(c_1, x_5) = 0$ ,  $d(c_1, x_6) = 0$ ,  $d(c_1, x_7) = 0$ ,  $d(c_1, x_8) = 0$ ,
2. For  $c_2$ :  $d(c_2, x_1) = 2$ ,  $d(c_2, x_2) = 2$ ,  $d(c_2, x_3) = 0$ ,  $d(c_2, x_4) = 0$ ,  $d(c_2, x_5) = 1$ ,  $d(c_2, x_6) = 3$ ,  $d(c_2, x_7) = 1$ ,  $d(c_2, x_8) = 2$ ,
3. For  $c_3$ :  $d(c_3, x_1) = 2$ ,  $d(c_3, x_2) = 4$ ,  $d(c_3, x_3) = 1$ ,  $d(c_3, x_4) = 2$ ,  $d(c_3, x_5) = 0$ ,  $d(c_3, x_6) = 0$ ,  $d(c_3, x_7) = 0$ ,  $d(c_3, x_8) = 0$ ,
4. For  $c_4$ :  $d(c_4, x_1) = 1$ ,  $d(c_4, x_2) = 1$ ,  $d(c_4, x_3) = 0$ ,  $d(c_4, x_4) = 0$ ,  $d(c_4, x_5) = 0$ ,  $d(c_4, x_6) = 1$ ,  $d(c_4, x_7) = 0$ ,  $d(c_4, x_8) = 1$ ,
5. For  $c_5$ :  $d(c_5, x_1) = 0$ ,  $d(c_5, x_2) = 1$ ,  $d(c_5, x_3) = 0$ ,  $d(c_5, x_4) = 1$ ,  $d(c_5, x_5) = 1$ ,  $d(c_5, x_6) = 1$ ,  $d(c_5, x_7) = 0$ ,  $d(c_5, x_8) = 0$ .

In what follows we present the operations performed in the evaluation and ordering phases of the INDUCENFA procedure at the successive levels of recursion. While specifying each recursion level we also provide the values of the procedure arguments, namely variables  $i$  and  $v$  representing the index and value of the most recently set variable  $x$ .

## 1 The *min-max-ex* variable ordering method

**Level 0**,  $i = \emptyset$ ,  $v = \emptyset$ . At first the REORDER procedure is called. The constraint related to examples having the fewest active products is  $c_1$ . The constraint contains only a single variable  $x_2$ , so this variable is selected as the next variable. Therefore,  $i \leftarrow 2$  and  $v \leftarrow 0$ , as the REORDER procedure always assigns zero first (see row 1 in Tab. 1).

**Level 1,  $i = 2, v = 0$ .** Since  $v = 0$ , the EVALUATE procedure updates the values  $|c|$  and  $d(c, x_i)$  in the following way:  $|c_1| \leftarrow 0, |c_2| \leftarrow 2, d(c_1, x_2) \leftarrow 0, d(c_2, x_2) \leftarrow 0, d(c_2, x_6) \leftarrow 2, d(c_2, x_7) \leftarrow 0, d(c_2, x_8) \leftarrow 1$  (products  $x_2, x_2x_6x_7$ , and  $x_2x_8^2$  become inactive). Next the conditions  $|c_1| = 0$  and  $|c_2| = 0$  are checked and since the first condition is satisfied, we set  $p \leftarrow \mathbf{false}$  and return. As a result, we backtrack to level 0, change the value of variable  $x_2 \leftarrow 1$  and invoke INDUCENFA again.

**Level 1,  $i = 2, v = 1$ .** The EVALUATE procedure now checks whether any of the constraints  $c_3$ – $c_5$  contains a product equal to one. As there is no such product, we continue to check whether constraints  $c_1$  or  $c_2$  contain a product equal to one. We find that  $c_1$  becomes satisfied, but since  $c_2$  is not yet satisfied we set  $p \leftarrow \mathbf{true}$  and return (see row 2 in Tab. 1). The REORDER procedure chooses from  $c_2$  (the only unsatisfied constraint related to examples), the most frequent variable  $x_6$ .

**Level 2,  $i = 6, v = 0$ .** The EVALUATE procedure updates the values  $|c_2|$  and  $d(c_2, x_i)$  as follows:  $|c_2| \leftarrow 1, d(c_2, x_1) \leftarrow 0, d(c_2, x_5) \leftarrow 0, d(c_2, x_6) \leftarrow 0, d(c_2, x_7) \leftarrow 0, d(c_2, x_8) \leftarrow 1$  (products  $x_1x_5x_6, x_1x_6x_8$ , and  $x_2x_6x_7$  become inactive). Since  $|c_2| \neq 0$  we set  $p \leftarrow \mathbf{true}$  and proceed (see row 3 in Tab. 1). The REORDER procedure selects now the only remaining variable in  $c_2$ , namely  $x_8$  and sets it to zero.

**Level 3,  $i = 8, v = 0$ .** The EVALUATE procedure updates the values  $|c_2| \leftarrow 0$  and  $d(c_2, x_8) \leftarrow 0$  (product  $x_2x_8^2$  becomes inactive). As a result  $|c_2| = 0$  is satisfied and so we set  $p \leftarrow \mathbf{false}$  and backtrack to level 2 (see row 4 in Tab. 1). There we change the assignment  $x_8 \leftarrow 1$  and proceed.

**Level 3,  $i = 8, v = 1$ .** The EVALUATE procedure finds now that the product  $x_2x_8 \in c_4$  is equal to one. So we set  $p \leftarrow \mathbf{false}$  and backtrack to level 2 again (see row 5 in Tab. 1). We unset variable  $x_8$ , backtrack to level 1 and change the value  $x_6 \leftarrow 1$ .

**Level 2,  $i = 6, v = 1$ .** The EVALUATE procedure does not find any contradictions nor  $c_2$  satisfied, so we set  $p \leftarrow \mathbf{true}$  and continue (see row 6 in Tab. 1). In the REORDER procedure we find that there are two most frequent variables  $x_1$  and  $x_8$ . In case of tie we pick the variable with lower index, so we pick  $x_1$  and proceed.

**Level 3,  $i = 1, v = 0$ .** The EVALUATE procedure updates the values  $|c_2|$  and  $d(c_2, x_i)$  as follows:  $|c_2| \leftarrow 2, d(c_2, x_1) \leftarrow 0, d(c_2, x_5) \leftarrow 0, d(c_2, x_8) \leftarrow 1$  (products  $x_1x_5x_6$  and  $x_1x_6x_8$  become inactive). We proceed with  $p \leftarrow \mathbf{true}$  as  $|c_2| \neq 0$  (see row 7 in Tab. 1). In the REORDER procedure we choose variable  $x_7$  (as  $d(c_2, x_7) = d(c_2, x_8) = 1$  and  $7 < 8$ ) and make another recursive call.

**Level 4,  $i = 7, v = 0$ .** The EVALUATE procedure updates the values  $|c_2| \leftarrow 1$  and  $d(c_2, x_7) \leftarrow 0$  (product  $x_2x_6x_7$  becomes inactive), and proceed with  $p \leftarrow \mathbf{true}$  as  $|c_2| \neq 0$  (see row 8 in Tab. 1). In the REORDER procedure we choose variable  $x_8$  as the only one remaining and set it to zero.

**Level 5,  $i = 8, v = 0$ .** As before we find a contradiction in  $c_2$ , as the number of active products  $|c_2| = 0$  and we backtrack (see row 9 in Tab. 1). Then we set  $x_8 \leftarrow 1$ .

**Level 5,  $i = 8, v = 1$ .** As before we find a contradiction in  $c_4$ , since the product  $x_2x_8 = 1$  so we backtrack (see row 10 in Tab. 1), unset variable  $x_8$  and change the value of  $x_7 \leftarrow 1$ .

**Level 4**,  $i = 7$ ,  $v = 1$ . We find no contradiction in constraints  $c_3$ – $c_5$ , but we find that the product  $x_2x_6x_7 \in c_2$  is equal to one, and so constraint  $c_2$  is satisfied. Since both  $c_1$  and  $c_2$  are satisfied we set  $f \leftarrow \mathbf{true}$  and terminate the procedure (see row 11 in Tab. 1).

The final results of the execution of the INDUCENFA procedure using the *min-max-ex* variable ordering method are gathered in Tab. 1. The resulting automaton is shown in Fig. 1.

Table 1: INDUCENFA algorithm execution for the *min-max-ex* variable ordering method and  $y_1 = 0$ ,  $y_2 = 1$ . No. – step number,  $c_+$  – selected constraint,  $x_i$ ,  $v$  – next variable and value to set, Result – result of the EVALUATE procedure.

No.	$c_+$	$x_i$	$v$	Result
1.	$c_1$	$x_2$	0	$p \leftarrow \mathbf{false}$ due to $c_1$
2.	-	$x_2$	1	$c_1$ satisfied
3.	$c_2$	$x_6$	0	$p \leftarrow \mathbf{true}$
4.	$c_2$	$x_8$	0	$p \leftarrow \mathbf{false}$ due to $c_2$
5.	-	$x_8$	1	$p \leftarrow \mathbf{false}$ due to $c_4$
6.	-	$x_6$	1	$p \leftarrow \mathbf{true}$
7.	$c_2$	$x_1$	0	$p \leftarrow \mathbf{true}$
8.	$c_2$	$x_7$	0	$p \leftarrow \mathbf{true}$
9.	$c_2$	$x_8$	0	$p \leftarrow \mathbf{false}$ due to $c_2$
10.	-	$x_8$	1	$p \leftarrow \mathbf{false}$ due to $c_4$
11.	-	$x_7$	1	$c_2$ satisfied, $x_1 \leftarrow 0$ , $x_3 \leftarrow 0$ , $x_4 \leftarrow 0$ , $x_5 \leftarrow 0$ , $x_8 \leftarrow 0$ , $f \leftarrow \mathbf{true}$

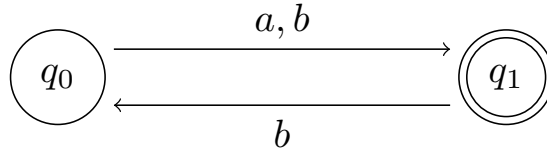


Figure 1: An NFA consistent with the sample  $S = (\{a, abb\}, \{aaa, ab, ba\})$

## 2 The *min-max-cex* variable ordering method

**Level 0**,  $i = \emptyset$ ,  $v = \emptyset$ . At first the REORDER procedure is called. The constraints related to counterexamples having the fewest active products are  $c_4$  and  $c_5$ , but we choose the one with the lower index, i.e.,  $c_4$ . The constraint contains four equally frequent variables, and we choose  $x_1$  as the next variable. Therefore,  $i \leftarrow 1$  and  $v \leftarrow 1$ , as the REORDER procedure now always assigns one first.

**Level 1**,  $i = 1$ ,  $v = 1$ . Since  $v = 1$ , the EVALUATE procedure checks whether any of the constraints  $c_3$ – $c_5$  contains a product equal to one. Since there is no such product we proceed with  $p \leftarrow \mathbf{true}$  (see row 1 in Tab. 2). In the REORDER procedure we again choose constraint  $c_4$  as the one with the fewest products, and we choose  $x_2$  as the next variable.

**Level 2**,  $i = 2$ ,  $v = 1$ . The EVALUATE procedure now checks whether any of the constraints  $c_3$ – $c_5$  contains a product equal to one. As the product  $x_1^2x_2 \in c_3$  becomes equal to one we backtrack and change the value of  $x_2$  (see row 2 in Tab. 2).

**Level 2**,  $i = 2$ ,  $v = 0$ . The EVALUATE procedure checks now that with  $x_2 = 0$  the number of active products in  $c_1$  becomes zero, so we need to backtrack once again. We unset the variable  $x_2$  and change the value of  $x_1 \leftarrow 0$  (see row 3 in Tab. 2).

**Level 1,  $i = 1, v = 0$ .** The EVALUATE procedure does not find any contradiction in either  $c_1$  or  $c_2$ . Hence, it updates the values  $|c_3| \leftarrow 2, |c_4| \leftarrow 1, d(c_3, x_1) \leftarrow 0, d(c_3, x_2) \leftarrow 2, d(c_3, x_4) \leftarrow 1$ , and  $d(c_4, x_6) \leftarrow 0$  (products  $x_1^2 x_2, x_1 x_2 x_4$ , and  $x_1 x_6$  become inactive). We set  $p \leftarrow \mathbf{true}$  (as none of the constraints  $c_3$ – $c_5$  became satisfied) and continue (see row 4 in Tab. 2). The REORDER procedure chooses constraint  $c_4$  and variable  $x_2$ .

**Level 2,  $i = 2, v = 1$ .** The EVALUATE procedure finds no contradiction in  $c_3$ – $c_5$ , so we proceed with  $p \leftarrow \mathbf{true}$  (see row 5 in Tab. 2). In the REORDER procedure we choose constraint  $c_4$  again and variable  $x_8$ , the only remaining variable of this constraint.

**Level 3,  $i = 8, v = 1$ .** The EVALUATE procedure finds that the product  $x_2 x_8 \in c_4$  is equal to one, so we return  $p \leftarrow \mathbf{false}$  and backtrack (see row 6 in Tab. 2).

**Level 3,  $i = 8, v = 0$ .** The EVALUATE procedure finds no contradiction in  $c_1$  and  $c_2$  and updates the values  $|c_4| \leftarrow 0$  and  $d(c_4, x_8) \leftarrow 0$  (product  $x_2 x_8$  becomes inactive). This way  $c_4$  becomes satisfied (see row 7 in Tab. 2) and we continue. In the REORDER procedure we choose constraint  $c_3$  (as  $|c_3| = |c_5|$ , but  $3 < 5$ ) and we set  $x_3 \leftarrow 1$ .

**Level 4,  $i = 3, v = 1$ .** Setting  $x_3 \leftarrow 1$  makes the EVALUATE procedure detect a contradiction in constraint  $c_3$ , which sets  $p \leftarrow \mathbf{false}$  and forces backtracking (see row 8 in Tab. 2).

**Level 4,  $i = 3, v = 0$ .** As the current setting does not affect constraints  $c_1$  and  $c_2$ , we update the values  $|c_3| \leftarrow 1$  and  $d(c_3, x_3) \leftarrow 0$  (product  $x_2^2 x_3$  becomes inactive). We set  $p \leftarrow \mathbf{true}$  (see row 9 in Tab. 2) and call REORDER procedure. In this procedure we choose  $c_3$  and variable  $x_4$ .

**Level 5,  $i = 4, v = 1$ .** Setting  $x_4 \leftarrow 1$  makes the EVALUATE procedure detect a contradiction in constraint  $c_3$ , which sets  $p \leftarrow \mathbf{false}$  and forces backtracking (see row 10 in Tab. 2).

**Level 5,  $i = 4, v = 0$ .** Variable  $x_4$  does not appear in constraints  $c_1$  and  $c_2$ , so we set  $|c_3| \leftarrow 0, |c_5| \leftarrow 1$ , and  $d(c_3, x_4) \leftarrow 0$  and we mark constraint  $c_3$  as satisfied (see row 11 in Tab. 2). In the REORDER procedure we select constraint  $c_5$  as the only not-yet-satisfied constraint related to counterexamples and we pick variable  $x_5$ .

**Level 6,  $i = 5, v = 1$ .** The assignment  $x_5 \leftarrow 1$  causes a contradiction in  $c_5$  so we set  $p \leftarrow \mathbf{false}$  and backtrack (see row 12 in Tab. 2).

**Level 6,  $i = 5, v = 0$ .** The assignment  $x_5 \leftarrow 0$  causes the constraint  $c_5$  to become satisfied (again  $c_1$  and  $c_2$  are not affected). Since all constraints  $c_3$ – $c_5$  are satisfied, we set  $x_6 \leftarrow 1, x_7 \leftarrow 1$  and  $f \leftarrow \mathbf{true}$  and terminate the procedure (see row 13 in Tab. 2).

The final results of the execution of the INDUCENFA procedure using the *min-max-cex* variable ordering method are gathered in Tab. 2. The resulting automaton is the same as the NFA in Fig. 1.

### 3 The *deg* variable ordering method

When using the *deg* method we first establish the order of variables based on the value  $D(x_j) = \sum_{i=1}^5 d(c_i, x_j)$ , for  $j \in [1, 8]$ . These values are as follows:  $D(x_1) = 5, D(x_2) = 9, D(x_3) = 1, D(x_4) = 3, D(x_5) = 2, D(x_6) = 5, D(x_7) = 1, D(x_8) = 3$ . Sorting by  $D(x_j)$  and in case of ties picking the variable with smaller index, we get the following order of variables:  $x_2, x_1, x_6, x_4, x_8, x_5, x_3$ , and  $x_7$ .

Table 2: INDUCENFA algorithm execution for *min-max-cex* variable ordering method and  $y_1 = 0$ ,  $y_2 = 1$ . No. – step number,  $c_-$  – selected constraint,  $x_i$ ,  $v$  – next variable and value to set, Result – result of EVALUATE procedure.

No.	$c_-$	$x_i$	$v$	Result
1.	$c_4$	$x_1$	1	$p \leftarrow \mathbf{true}$
2.	$c_4$	$x_2$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
3.	-	$x_2$	0	$p \leftarrow \mathbf{false}$ due to $c_1$
4.	-	$x_1$	0	$p \leftarrow \mathbf{true}$
5.	$c_4$	$x_2$	1	$p \leftarrow \mathbf{true}$
6.	$c_4$	$x_8$	1	$p \leftarrow \mathbf{false}$ due to $c_4$
7.	-	$x_8$	0	$c_4$ satisfied
8.	$c_3$	$x_3$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
9.	-	$x_3$	0	$ c_3  \leftarrow 1$
10.	$c_3$	$x_4$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
11.	-	$x_4$	0	$c_3$ satisfied, $ c_5  \leftarrow 1$
12.	$c_5$	$x_5$	1	$p \leftarrow \mathbf{false}$ due to $c_5$
13.	-	$x_5$	0	$c_5$ satisfied, $x_6 \leftarrow 1$ , $x_7 \leftarrow 1$ , $f \leftarrow \mathbf{true}$

**Level 0**,  $i = \emptyset$ ,  $v = \emptyset$ . The REORDER procedure simply return  $x_2$  with the initial assignment  $x_2 \leftarrow 0$ .

**Level 1**,  $i = 2$ ,  $v = 0$ . Since  $v = 0$ , the EVALUATE procedure checks first whether there is any contradiction in the constraints  $c_1$  and  $c_2$ . Since  $|c_1| = 0$  holds, we need to backtrack (see row 1 in Tab. 3).

**Level 1**,  $i = 2$ ,  $v = 1$ . The EVALUATE procedure now checks whether any of the constraints  $c_3$ – $c_5$  contains a product equal to one. As no such product is found, we check whether constraint  $c_1$  or  $c_2$  is satisfied. We mark constraint  $c_1$  as satisfied and proceed (see row 2 in Tab. 3). The REORDER procedure returns variable  $x_1$ .

**Level 2**,  $i = 1$ ,  $v = 0$ . The EVALUATE procedure checks that there is no contradiction with respect to  $c_2$  and updates the values  $|c|$  and  $d(c, x_i)$  in the following way:  $|c_2| \leftarrow 2$ ,  $|c_3| \leftarrow 2$ ,  $|c_4| \leftarrow 1$ ,  $d(c_2, x_1) \leftarrow 0$ ,  $d(c_2, x_5) \leftarrow 0$ ,  $d(c_2, x_6) \leftarrow 1$ ,  $d(c_2, x_8) \leftarrow 1$ ,  $d(c_3, x_1) \leftarrow 0$ ,  $d(c_3, x_4) \leftarrow 1$ ,  $d(c_4, x_1) \leftarrow 0$  and  $d(c_4, x_6) \leftarrow 0$  (products  $x_1x_5x_6$ ,  $x_1x_6x_8$ ,  $x_1^2x_2$ ,  $x_1x_2x_4$ , and  $x_1x_6$  become inactive). We proceed with  $p \leftarrow \mathbf{true}$  (see row 3 in Tab. 3). The REORDER procedure returns variable  $x_6$ .

**Level 3**,  $i = 6$ ,  $v = 0$ . As there is no contradiction, the EVALUATE procedure updates the values  $|c_2| \leftarrow 1$ ,  $|c_5| \leftarrow 1$ ,  $d(c_2, x_6) \leftarrow 0$ ,  $d(c_2, x_7) \leftarrow 0$ , and  $d(c_5, x_4) \leftarrow 0$  (products  $x_2x_6x_7$  and  $x_4x_6$  become inactive). We set  $p \leftarrow \mathbf{true}$  and continue (see row 4 in Tab. 3). The REORDER procedure chooses returns variable  $x_4$ .

**Level 4**,  $i = 4$ ,  $v = 0$ . Since  $x_4$  does not appear in  $c_2$ , the EVALUATE procedure updates the values  $|c_3| \leftarrow 1$  and  $d(c_3, x_4) \leftarrow 0$  (product  $x_2x_4^2$  becomes inactive). We set  $p \leftarrow \mathbf{true}$  and continue (see row 5 in Tab. 3). The REORDER procedure returns variable  $x_8$ .

**Level 5**,  $i = 8$ ,  $v = 0$ . Setting  $x_8 \leftarrow 0$  causes a contradiction in  $c_2$ , so we need to backtrack (see row 6 in Tab. 3).

**Level 5**,  $i = 8$ ,  $v = 1$ . Setting  $x_8 \leftarrow 1$  causes a contradiction in  $c_4$ , so we need to backtrack again (see row 7 in Tab. 3). We unset variable  $x_8$  and change the value  $x_4 \leftarrow 1$ .

**Level 4**,  $i = 4$ ,  $v = 1$ . Setting  $x_4 \leftarrow 1$  causes a contradiction in  $c_3$ , so we need to backtrack again (see row 8 in Tab. 3). We unset variable  $x_4$  and change the value  $x_6 \leftarrow 1$ .

**Level 3**,  $i = 6$ ,  $v = 1$ . The EVALUATE procedure does not detect any contradiction, so we proceed with  $p \leftarrow \mathbf{true}$  (see row 9 in Tab. 3). The REORDER procedure returns variable  $x_4$ .

**Level 4**,  $i = 4$ ,  $v = 0$ . The EVALUATE procedure does not detect any contradiction, so we proceed with  $p \leftarrow \mathbf{true}$  (see row 10 in Tab. 3). The REORDER procedure returns variable  $x_8$ .

**Level 5**,  $i = 8$ ,  $v = 0$ . There is no contradiction in  $c_2$ , but the EVALUATE procedure detects that  $c_4$  is satisfied. So we continue with  $p \leftarrow \mathbf{true}$  (see row 11 in Tab. 3). The REORDER procedure returns variable  $x_5$ .

**Level 6**,  $i = 5$ ,  $v = 0$ . The assignment  $x_5 \leftarrow 0$  makes constraint  $c_5$  satisfied (product  $x_2x_5$  becomes inactive, and  $|c_5| \leftarrow 0$ ), so we set  $p \leftarrow \mathbf{true}$  (see row 12 in Tab. 3). The REORDER procedure returns variable  $x_3$ .

**Level 7**,  $i = 3$ ,  $v = 0$ . The assignment  $x_3 \leftarrow 0$  makes constraint  $c_3$  satisfied (product  $x_2^2x_3$  becomes inactive and  $|c_3| \leftarrow 0$ ), so we continue (see row 13 in Tab. 3). The REORDER procedure returns variable  $x_7$ .

**Level 8**,  $i = 7$ ,  $v = 0$ . The assignment  $x_7 \leftarrow 0$  causes a contradiction in constraint  $c_2$  (as product  $x_2x_6x_7$  becomes inactive and so  $|c_2| \leftarrow 0$ ). We set  $p \leftarrow \mathbf{false}$  and backtrack (see row 14 in Tab. 3).

**Level 8**,  $i = 7$ ,  $v = 1$ . All the constraints become satisfied as the product  $x_2x_6x_7 \in c_2$  becomes equal to one. We set  $f \leftarrow \mathbf{true}$  and terminate the procedure without setting any other variable (see row 15 in Tab. 3).

The final results of the execution of the INDUCENFA procedure using the *deg* variable ordering method are gathered in Tab. 3. The resulting automaton is the same as the NFA in Fig. 1.

Table 3: INDUCENFA algorithm execution for *deg* variable ordering method and  $y_1 = 0$ ,  $y_2 = 1$ . No. – step number,  $c_-$  – selected constraint,  $x_i$ ,  $v$  – next variable and value to set, Result – result of EVALUATE procedure.

No.	$c_-$	$x_i$	$v$	Result
1.	$c_4$	$x_1$	1	$p \leftarrow \mathbf{true}$
2.	$c_4$	$x_2$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
3.	-	$x_2$	0	$p \leftarrow \mathbf{false}$ due to $c_1$
4.	-	$x_1$	0	$p \leftarrow \mathbf{true}$
5.	$c_4$	$x_2$	1	$p \leftarrow \mathbf{true}$
6.	$c_4$	$x_8$	1	$p \leftarrow \mathbf{false}$ due to $c_4$
7.	-	$x_8$	0	$c_4$ satisfied
8.	$c_3$	$x_3$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
9.	-	$x_3$	0	$ c_3  \leftarrow 1$
10.	$c_3$	$x_4$	1	$p \leftarrow \mathbf{false}$ due to $c_3$
11.	-	$x_4$	0	$c_3$ satisfied, $ c_5  \leftarrow 1$
12.	$c_5$	$x_5$	1	$p \leftarrow \mathbf{false}$ due to $c_5$
13.	-	$x_5$	0	$c_5$ satisfied, $x_6 \leftarrow 1$ , $x_7 \leftarrow 1$ , $f \leftarrow \mathbf{true}$
14.	-	$x_5$	0	$c_5$ satisfied, $x_6 \leftarrow 1$ , $x_7 \leftarrow 1$ , $f \leftarrow \mathbf{true}$
15.	-	$x_5$	0	$c_5$ satisfied, $x_6 \leftarrow 1$ , $x_7 \leftarrow 1$ , $f \leftarrow \mathbf{true}$