CMPT 225-D100 Midterm Exam 1
Summer 2023, Burnaby

Sample Solutions

This is a **50 minute closed book exam**: notes, books, computers, calculators, electronic devices, etc. are **not** permitted. Do not speak to any other students during their exam or look at their work. Please remain seated and **raise your hand** if you have a question.

Linked Lists

(10 marks) Suppose you have a C++ program with a **singly-linked list** based on this:

```
struct Node {
   int data;
   Node* next;
};

Node* head = nullptr;   // head is a global variable
```

`head` is a global variable that points to the first element of the list. If `head` is `nullptr`, then the list is empty. Note that there is *no* class, and the `negate_biggest()` function requested below should directly use `head`.

Question

Using **detailed C++-like pseudocode**, write a function called `negate_biggest()` that:

- Returns a copy of the biggest data value in the list.
- Negates (i.e. multiplies by -1) the biggest value in the list. No other changes are made to the list.
- Traverses the list at most one time.
- **Doesn't** use recursion.

Assume the list is non-empty, and that there are *no* duplicates (i.e. every data value is unique).

For example, suppose your program creates a list like this:

```
head = new Node{0, nullptr};
head = new Node{2, head};
head = new Node{1, head};
```

When you call `negate_biggest()` it returns 2, and the 2 in the list is negated:

Your `negate_biggest()` should work for *any* valid singly-linked list, not just this example. Your answer should be efficient and not use any unnecessary memory. **Don't** use arrays or vectors or other such data structures in your answer.

Please write your answer on the next page.

Sample Solution

```
int negate_biggest() {
    Node *p = head;
    Node *biggest_node = p;

    // find the node with the biggest value
    while (p != nullptr) {
        if (p->data > biggest_node->data) {
            biggest_node = p;
        }
        p = p->next;
    }

    // negate the biggest value and return it
    biggest_node->data *= -1;
    return biggest_node->data * -1;
}
```

The idea of this code is that `p` points to each node of the list, one at a time, and `biggest_node` points to the node with the currently biggest value that `p` has seen.

Stacks

(10 marks) Suppose the `Stack` class contains `int`s, and has these methods:

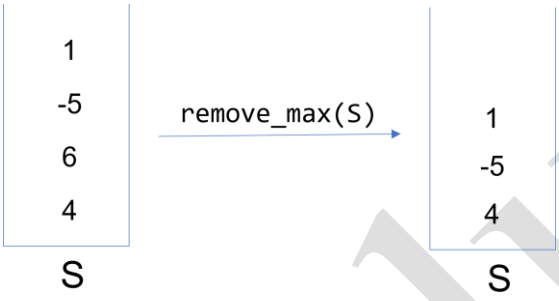
- `S.push(x)` inserts `x` on the top of `S`.
- `S.pop()` removes, but does **not** return, the top `int` of `S`; if `S` is empty, an error occurs.
- `S.peek()` returns, but does **not** remove, a copy of the top `int` of `S`; if `S` is empty, an error occurs.
- `S.empty()` returns `true` if `S` is empty, and `false` otherwise.
- `S.size()` returns the number of `int`s in `S`.
- When you define a new stack it starts empty, e.g.:

```
Stack S;   // S is empty
Stack T;   // T is empty
```

Question

Using **detailed C++-like pseudocode**, write a function (not a method!) called `remove_max(Stack& S)` that removes the *biggest* element in `S`, and leaves the rest of the elements in the same order. To make things easier, you can assume `S` is non-empty, and that all the `int`s on `S` are different (i.e. `S` has no duplicates).

For example:



Important Only use `Stack`s in your answers: **don't** use arrays, vectors, lists, or other such data structures. You can use as many stacks as you need (but try to use as few as possible).

Important `remove_max(Stack& s)` is a *function*, **not** a method in a class. So it can only use the `Stack` methods described above, and should not make any assumptions about how `Stack` is implemented.

Write your answer on the next page.

Please use brief comments to explain the main sections of your answer.

```
// Pre-conditions:
//    S is not empty
//    S has no duplicates
// Post-condition:
//    modifies S so that its max element is removed,
//    and other elements are still there in the same order

void remove_max(Stack& S)
{
    //
    // max is always the max element seen
    // so far
    //
    int max = S.top();
    S.pop();

    //
    // move all elements from S to temp,
    // updating max as we go
    //
    Stack temp;
    while (!S.empty()) {
        if (S.top() > max) {
            max = S.top();
        }
        temp.push(S.top());
        S.pop();

    }

    //
    // move all elements from temp back to S,
    // except for max
    //
    while (!temp.empty()) {
        if (temp.top() != max) {
            S.push(temp.top());
        }
        temp.pop();
    }
}
```

O-notation and Analysis

a) (5 mark) State the precise mathematical definition of "$f(n)$ is $O(g(n))$".

Solution

$f(n)$ is $O(g(n))$ if there is a real constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \leq cg(n)$ for all $n \geq n_0$.

b) (5 marks) Using the definition of O-notation, mathematically prove that $20n - 50$ is $O(n)$.

Solution

Consider $20n - 50 \leq cn$. Re-arranging this gives $20n \leq cn + 50$. Setting, say, $c = 20$ we get $20n \leq 20n + 50$, which simplifies to $0 \leq 50$ which is true for all integers $n \geq 1$. So setting $n_0 = 1$ works.

Trees

Let p be a node in a tree T, as defined in the textbook and lectures. The **depth** of p is defined to be the number of ancestors of p, *excluding* p itself.

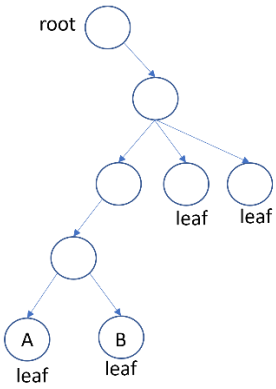
a) (4 marks) Draw a tree:

- that is **not** binary.
- where the root is labelled "root".
- with **exactly** 4 **leaf** nodes, each labelled "leaf".
- with **exactly** 2 different nodes of **depth** 4, labelled A and B.

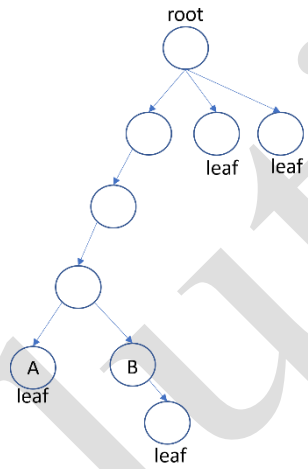
Solution

Many trees are possible. Here is are two examples:

Example 1



Example 2



b) (2 marks) Give a **recursive definition** of the depth of a node p in a tree T.

Solution

- If p is the root, then the **depth** of p is 0.
- Otherwise, the **depth** of p is one plus the depth of the parent of p.