# Lecture Notes, Week 5

## Textbook Definition of a Tree

A **tree T** is a set of nodes, using storing values, in a parent-child relationship with the following properties:

* T could be empty, i.e. it has no nodes. This is called the **empty tree**.
* If T is non-empty, it has a special node called the **root of T** that has no parent.
* Each node v of T *different* from the root has a *unique parent* node w. We say v is a **child** of w, and w is the **parent** of v.

Notes and terminology:

* Every non-empty tree has **exactly one root**.
* When drawing a tree, we typically put the root at the top.
* The **root is the only node with no parents**.
* All nodes in a tree can have 0 or more child nodes.
* A node with **0 children** is called an **external node**, or a **leaf node**.
* A node with **1 or more children** is called an **internal node**.

If x is a node in a tree, then:

* the **ancestors** of x are x itself, plus the ancestors of x’s parent
* the **descendants** of x are x itself, plus all the nodes that have x as an ancestor

**Fact**: The root is an ancestor of all nodes in a tree.

**Fact**: All nodes in a tree, including the root, are descendants of the root.

An **edge** in a tree is a pair of nodes x and y such that either x is the parent of y, or y is the parent of x.

A **path** in a tree is a sequence of nodes such that there is an edge between each adjacent node.

**Fact**: If R is the root node of a tree, and x is some other node in the tree, then there is exactly one path from R to x. Intuitively, you can see this is true by pretending there were two different paths from R to x. If there were two paths, then there must be some node that has two or more parents, which is impossible in a tree.