2017년 3월 8일

$$r_{xy} = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y}) / n}{\sqrt{\frac{\sum_i^n (x_i - \bar{x})^2}{n}} \sqrt{\frac{\sum_i^n (y_i - \bar{y})^2}{n}} / n}$$

분자 $\sum_i^n (x_i - \bar{x})(y_i - \bar{y}) = \sum (x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x}\bar{y})$

$= \sum x_i y_i - \bar{x}\sum y_i - \bar{y}\sum x_i + n\bar{x}\bar{y}$ $\quad \bar{x} = \frac{1}{n}\sum x_i$ 이고 $\bar{y} = \frac{1}{n}\sum y_i$

$= \sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i - \frac{1}{n}\sum y_i \sum x_i + \frac{1}{n}\sum x_i \sum y_i$

$= \sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i$

분모도 같은 방법으로 $= \sqrt{\left(\sum x_i^2 - \frac{1}{n}(\sum x_i)^2\right)\left(\sum y_i^2 - \frac{1}{n}(\sum y_i)^2\right)}$

$= \sqrt{\sum x_i^2 \sum y_i^2 - \frac{1}{n}\sum x_i^2 (\sum y_i)^2 - \frac{1}{n}(\sum x_i)^2 \sum y_i^2 + \frac{(\sum x_i)^2 (\sum y_i)^2}{n^2}}$

$= \frac{1}{n}\sqrt{n^2 \sum x_i^2 \sum y_i^2 - n\sum x_i^2 (\sum y_i)^2 - n(\sum x_i)^2 \sum y_i^2 + (\sum x_i)^2 (\sum y_i)^2}$

$= \frac{1}{n}\sqrt{\left[n\sum x_i^2 - (\sum x_i)^2\right]\left[n\sum y_i^2 - (\sum y_i)^2\right]}$

이것을 확인해 보자.

$$\Rightarrow \frac{\sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i \times n}{\frac{1}{n}\sqrt{\quad} \times n}$$

$$= \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{\left[n\sum x_i^2 - (\sum x_i)^2\right]\left[n\sum y_i^2 - (\sum y_i)^2\right]}}$$

이므로, 위 두 식이 동일함을 보였다.