Integral Technique

1 Gaussian Integral

1.1 Gauss-Hermite quadrature

$$\int_{-\infty}^{\infty} e^{-x^2/2} f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} w_i f(x_i) \tag{.1}$$

where n is the number of sampled points used. The x_i are the roots of the physicists' version of the Hermite polynomial $H_n(x)(i = 1, 2, \dots, n)$ and the associated weights w_i are given by

$$w_i = \frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}(x_i)]^2} \tag{.2}$$

1.2 Stein's lemma

For a differentiable function $\phi : \mathbb{R} \to \mathbb{R}$ with $\mathbb{E}_{x\sim\mathcal{N}(0,1)}[|\phi'(x)|] < \infty$

$$\mathop{\mathbb{E}}_{x\sim\mathcal{N}(0,1)}[\phi(x)x] = \mathop{\mathbb{E}}_{x\sim\mathcal{N}(0,1)}[\phi'(x)] \tag{.3}$$

1.3 Change of measure

$$\mathcal{N}(\boldsymbol{w}|\boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n)\,\mathrm{d}\boldsymbol{w} = \mathcal{N}(\boldsymbol{x}^\top \boldsymbol{w}|\boldsymbol{x}^\top \mu_n, \boldsymbol{x}^\top \boldsymbol{\Sigma}_n \boldsymbol{x})\,\mathrm{d}(\boldsymbol{x}^\top \boldsymbol{w}) \tag{.4}$$