LECTURE 7

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January 29, 2025

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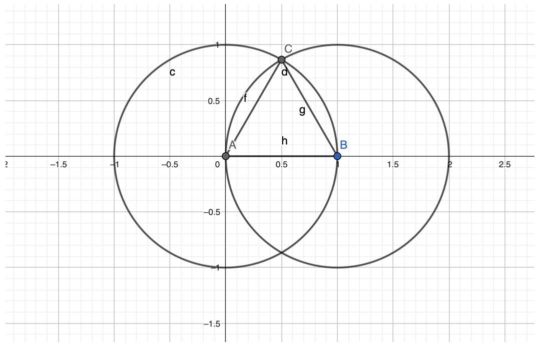
1 continuing from last lecture



a field generated by 0, 1 is $\mathbb{Q}$. with this equilateral trinalge we generated $\mathbb{Q}(\sqrt{3})$

1.1 intersecting lines

$$\alpha_1 x + \beta_1 y = \gamma_1$$
$$\alpha_2 x + \beta_2 y = \gamma_2$$
$$\alpha_i, \beta_i, \gamma_i \in K$$
$$x, y \in K$$

1.2 intersecting line with circle

$$\alpha x + \beta y = \gamma$$
$$(x - c_1)^2 + (y - c_2)^2 = r^2$$
$$\text{quadratic in x}$$

We may need to add square roots to the field. The degree of extension = 1 or 2. This is because the degree of the extension is the degree of the minimal polynomial.
Degree $[K : \mathbb{Q}]$ either stays the same or doubles.
$[K_{i+1} : \mathbb{Q}] = [K_{i+1} : K_i] \cdot [K_i : \mathbb{Q}]$

1.3 Intersecting two cricles

$$(x - c_1)^2 + (y - c_2)^2 = r_1^2$$
$$(x - d_1)^2 + (y - d_2)^2 = r_2^2$$
$$c_i, d_i, r_i \in K_s$$

Solving this system of equations will give us the intersection points.

$$x^2 - 2c_1x + c_1^2 + y^2 - 2c_2y + c_2^2 = r_1^2$$
$$x^2 - 2d_1x + d_1^2 + y^2 - 2d_2y + d_2^2 = r_2^2$$
$$(x - c_1)^2 + (y - c_2)^2 = r_1^2$$
linear equation x of y
$$\rightarrow K_o = \mathbb{Q}$$
$$[K_s : \mathbb{Q}] = 2^j \quad j \in \mathbb{Z}, j \geq 0$$

We must construct $\sqrt[3]{2}$ Using Eisenstein criteria $[\mathbb{Q}(\sqrt[3]{2}) : \mathbb{Q}] = 3$ This is not possible because:

$$[K_s : \mathbb{Q}](= 2^j) = [K_s : \mathbb{Q}(\sqrt[3]{2})] \cdot [\mathbb{Q}(\sqrt[3]{2}) : \mathbb{Q}](= 3)$$

2 Trisecting an angle