LECTURE X (JUNE 5)

Comparison Theorem, Triangle Inequalities, Bolzao Weirstrass Thoerem

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1 q2

question: $a_{n+1} = \sin(a_n)$, $a_1 = \frac{1}{100}$ a. show (a_n) is converging (decreasing) b. show $a_n \to 0$ u can use taylor series

Proof 1.1.

$$b_{n+1} = b_n - \frac{b_n^3}{6} + b_n^4$$
$$sin(x) = x - \frac{x^3}{6} + \frac{\sin^{(4)}(c)}{4!};; \text{ taylor series with remainder and } 0 \le c \le x$$

Now show $a_n \le b_n$ (squeeze theorem)

$$a_1 = b_1$$
$$a_n \le b_n;; \text{ induction assume this and show}$$
$$a_{n+1} \le b_{n+1}$$
$$\sin(a_n) \le b_n - \frac{b_n^3}{6} + b_n^4$$
$$\sin(a_n) = a_n - \frac{a_n^3}{6} + \frac{\sin(a_n)}{4!}a_n^4 \le b_n - \frac{b_n^3}{6} + b_n^4$$
$$(b_n - a_n) - \frac{b_n^3}{6} + \frac{a_n^3}{6} + b_n^4 - a_n^4 \ge 0?$$

let $l = \lim a_n$ show that l satisfies $l = \sin(l)$

$$|l - \sin(l)| = |l - a_{n+1} + a_{n+1} - \sin l| \le |a_{n+1} - l| + |\sin(a_n) - \sin(l)| \le \epsilon$$

2 Q4

every convergence sequence goes to 1. but (a_n) does not converge. is it possible for (a_n) to be bounded? tge answer is no (a_n) will have limsup, lim inf but a_n doesn't converge so limsup $\neq$ lim inf (for not convergence part) for goeing to 1 $liminf \le 1 \le limsup$ then we have convergence seequence $a > 1$ which is a contradiction

3 Q5

(a_n) and $C = \{\text{cluster pts of } (a_n)\}$ (b_m) such that $b_m \in C$ $\forall M, \exists (n_k)$ such that $a_{n_k} \to b_m$ b is a cluster pt of b_m meaning b is a cluster pt of (a_n)

$\exists (b_{m_j}) \le (b_m) b_{m_j} \to b$

let $\epsilon > 0$ $\exists J$ s.t. $\forall j \geq J$ $|b_{m_j} - b| < \epsilon$

$$\exists K \text{ st } \forall k \geq K \exists a_{n_k}(m_j) \to b_{m_j}$$

$$|a_{n_k(m_j)} - b_{m_j}| < \epsilon$$

$$|a_{n_k(m_j)} - b| \leq |a_{n_k(m_j)} - b_{m_j}| + |b_{m_j} - b| < 2\epsilon$$