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set stuff

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1 Connected Set

A set E is connected if $\nexists$ a separation of E into two open componenets. $E = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$ and $V_1 \neq \emptyset \neq V_2$ and $\bar{V_1} \cap V_2 = V_1 \cap \bar{V_2} = \emptyset$ and $V_1 \neq E \neq V_2$.

Theorem 1.1. *$E \subseteq \mathbb{R}$, E connected is equivalent to E is on interval I.*

Proof. suppose $E \subseteq \mathbb{R}$ is connected. solution implying $H_1 \subseteq H_2 \subseteq H_3 \dots$ (sequences in $\mathbb{R}^n$) E is empty or, if E contains only c, $E = (c, c)$ or $[c, c]$. suppose E contains at least 2 points.

$$a = \inf(E)$$

$$b = \sup(E)$$

and $a < b$ two casese we can take: $a \in E$ or $a \not{i}nE$, $b \in E$ or $b \not{i}nE$. property of supremums and infimums.

consider $E_k = [a_k, b_k]$ Q: is $E_k \subseteq E$? E is one of the following: $E = (a, b)$, $E = [a, b]$, $E = (a, b]$, $E = [a, b)$ how to prove $E_k \subseteq E$? suppose not then, $\exists x \in E_k$ such that $x \notin E$ $a < a_k \leq x \leq b_k < b$ consider $E \setminus \{x\} = E$ $E_R = \{y \in E : y > x\}$

Definition 1.1. U is erlatively open wrt E if $\exists$ open set A such that $U = E \cap A$

by $E = [a, b]$ and $U = E \cap A$ then $A = (\frac{a+b}{2}, b + \epsilon)$ and $U = [\frac{a+b}{2}, b]$ $\square$

back to E_R, show $\exists V$ open s.t. $V \cap E = E_2$

$\bigcup_{k=1}^{\infty} E_k \subseteq E$ $x \in E_k$ for some k which meanns $x \in E$

case 1: $a_k = a \in E$ true

case 2: $a_k > a \notin E : x \in \tilde{E} \to x \in E_k$ for some $k \to a < a_k \leq x$

Definition 1.2. A set E is perfect if $E =$ limit pots of E
limit points of $E : \{x \in X : \forall B \text{ open}, x \in B, \exists y (\neq x) \in E \cap N\}$

Definition 1.3. A set E is nowhere dense if $\bar{A}$ (closure of A) has empty interior

$\bar{E} = \bigcap V$, $\{V \text{ closed } : V \supseteq E\}$ $E^0 = \{x \in E : \exists V \text{ open } x \in V \subseteq E\}$

Cantor Set: $C = \bigcap_{k=1}^{\infty} I_k$ where $I_1 = [0, 1]$, $I_2 = [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$, $I_3 = [0, \frac{1}{9}] \cup [\frac{2}{9}, \frac{3}{9}] \cup [\frac{6}{9}, \frac{7}{9}] \cup [\frac{8}{9}, 1]$ Q: is C closed? yes intersection of closed I_k

Q: is C compact, yes, closed and bounded.

Q: is perfect?

Q: is C nowhere dense?

$$C = \{\sum_{k=1}^{\infty} a_k (\frac{1}{3})^k : a_k \in \{0, 2\}\}$$