RINGS

Cosets

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1 RINGS

Units in $\mathbb{Z}/6\mathbb{Z}$ are [1], [5] so $\mathbb{Z}/6\mathbb{Z}^*$

Definition 1.1. Let R Be a ring. IF $R^* = R \backslash \{0\}$ Then R is a field. In other words, R is a field if every non zero element in R has a multiplicative inverse.

Definition 1.2. IOf $K \subseteq L$ are fields and K is a subring of L, then K is a subfield of L and L is an extension field of K.

example: $\mathbb{Q}$ is a subfield of $\mathbb{R}$ and $\mathbb{R}$ is an extensino field of $\mathbb{Q}$
Ex. $\mathbb{R}$ is a subfield of $\mathbb{C}$ and $\mathbb{C}$ is an extension field of $\mathbb{R}$

Definition 1.3. A Domain is a ring R with no zero divisors.

non ex: $\mathbb{Z}/6\mathbb{Z}$ is not a domain.

Proposition 1.1 (3.1.3)**.** Let R be a domain and $a, x, y \in R$ if $a \neq 0$ and ax = ay then x = y

Proof. if $ax = ay$ then $ax + (-ay) = 0$ and $a(x + (-y)) = 0$ since R is a domain and $a \neq 0$ x = y ☐

Proposition 1.2 (3.1.4)**.** Let F be a field then F is a domain

Proof. suppose $x, y \in F, x \neq 0$ and $xy = 0$ want to show y =0
Since $x \neq 0$, $\exists x^{-1} \in F$ since F is a field. Then $0 = x^{-1} \times 0 = x^{\{} - 1\}(xy) = (x^{\{} - 1\}x)y = y$

☐

Ex: define $\mathbb{Q}(i) = \{a + bi | a, b \in \mathbb{Q}\} \subseteq \mathbb{C}$
Q: Is Q(i) a field ? Need to show that every nonzero element has a multiplicative inverse.