



NTNU

Norwegian University of  
Science and Technology

# LEGACY CRYPTO 2

TTM4205 – Lecture 6

Tjerand Silde

07.09.2023

# Contents

**Announcements**

**Legacy Crypto**

**Legacy PKC**

**Attacks on TLS**

**Backdoors**

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## Announcements

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# Reference Group

I am looking for (at least) three students to form a reference group in this course, preferably students from different programs. We will meet three times during the semester, and your feedback is extremely valuable.

Send me an email and/or talk to me in the break :)

# Deadlines

- ▶ Topic/scope/group approval: **November 1st**
- ▶ Short oral presentations: **November 23rd** (TBC)
- ▶ Draft submission for feedback: **November 23rd**
- ▶ "Weekly Problems": **December 1st at 23:59.**
- ▶ "Special Topics Project": **December 22nd at 23:59**

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# Legacy Crypto is...

- ▶ Old and outdated crypto
- ▶ Insecure, weakened, or flawed crypto
- ▶ Crypto regulated by export control
- ▶ Potentially backdoored crypto
- ▶ Key escrow and surveillance
- ▶ Downgradable crypto protocols

## Two Categories

Secret Key Crypto

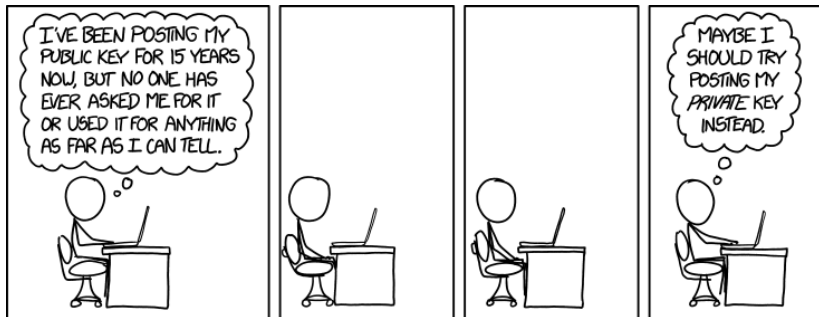
Public Key Crypto



Secret Key Crypto

**Public Key Crypto**

# Public Key Crypto



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# Legacy Ciphers

While we have many attacks against symmetric key ciphers that made them obsolete, we do not have groundbreaking attacks against the legacy public key ciphers.

However, we need to be careful when setting parameters, and (as with CBC) be careful when using padding schemes.

Here are some examples...

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  - ▶ computing DL depends on largest prime factor  $p|(q-1)$
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- ▶ Supersingular curves  $\rightarrow$  Can break Decisional DH
- ▶ Choose safe curves?  $\rightarrow$  Standardized P-256, X25519, ...

## Elliptic Curve Cryptography in Practice

Joppe W. Bos<sup>1</sup>, J. Alex Halderman<sup>2</sup>, Nadia Heninger<sup>3</sup>, Jonathan Moore, Michael Naehrig<sup>1</sup>,  
and Eric Wustrow<sup>2</sup>

<sup>1</sup> Microsoft Research

<sup>2</sup> University of Michigan

<sup>3</sup> University of Pennsylvania

**Figure:** <https://eprint.iacr.org/2013/734.pdf>

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- ▶ PKCS 1 padding  $\rightarrow$  Bleichenbacher's padding attack

## Twenty Years of Attacks on the RSA Cryptosystem

Dan Boneh  
dabo@cs.stanford.edu

**Figure:**

<https://crypto.stanford.edu/~dabo/papers/RSA-survey.pdf>

# RSA Challenges

Challenge Name	Digits	Bits	Date Factored	Factored by
RSA-100	100	330	Apr 1, 1991	A. K. Lenstra
RSA-110	110	364	Apr 14, 1992	A. K. Lenstra and M.S. Manasse
RSA-120	120	397	Jul 9, 1993	T. Denny et al.
RSA-130	130	430	Apr 10, 1996	A. K. Lenstra et al.
RSA-140	140	463	Feb 2, 1999	H. te Riele et al.
RSA-150	150	496	Apr 16, 2004	K. Aoki et al.
RSA-155	155	512	Aug 22, 1999	H. te Riele et al.
RSA-160	160	530	Apr 1, 2003	J. Franke et al.
RSA-170	170	563	Dec 29, 2009	D. Bonenberger and M. Krone
RSA-576	174	576	Dec 3, 2003	J. Franke et al.
RSA-180	180	596	May 8, 2010	S. A. Danilov and I. A. Popovyan
RSA-190	190	629	Nov 8, 2010	A. Timofeev and I. A. Popovyan
RSA-640	193	640	Nov 2, 2005	J. Franke et al.
RSA-200	200	663	May 9, 2005	J. Franke et al.
RSA-210	210	696	Sep 26, 2013	R. Propper
RSA-704	212	704	Jul 2, 2012	S. Bai, E. Thomé and P. Zimmermann
RSA-220	220	729	May 13, 2016	S. Bai, P. Gaudry, A. Kruppa, E. Thomé and P. Zimmermann
RSA-230	230	762	Aug 15, 2018	S. S. Gross
RSA-768	232	768	Dec 12, 2009	T. Kleinjung et al.
RSA-240	240	795	Nov 24, 2019	F. Boudot, P. Gaudry, A. Guille- vic, N. Heninger, E. Thomé and P. Zimmermann
RSA-250	250	829	Feb 28, 2020	F. Boudot, P. Gaudry, A. Guille- vic, N. Heninger, E. Thomé and P. Zimmermann

**Table 1.** The solved RSA Challenges

**Figure:** <https://eprint.iacr.org/2021/894.pdf>

# Key Sizes

	Parameter	Legacy	Future System Use	
			Near Term	Long Term
Symmetric Key Size	k	80	128	256
Hash Function Output Size	m	160	256	512
MAC Output Size	m	80	128	256
RSA Problem	$\ell(n) \geq$	1024	3072	15360
Finite Field DLP	$\ell(p^n) \geq$	1024	3072	15360
	$\ell(p), \ell(q) \geq$	160	256	512
ECDLP	$\ell(q) \geq$	160	256	512
Pairing	$\ell(q^n) \geq$	1024	3072	15360
	$\ell(p), \ell(q) \geq$	160	256	512

**Table 2.** Key Size Analysis, where  $\ell(\cdot)$  refers to the bit-length of the parameter.

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- ▶ 2015: two 1024 groups break 18% HTTPS and 26% SSH

# Logjam Attack

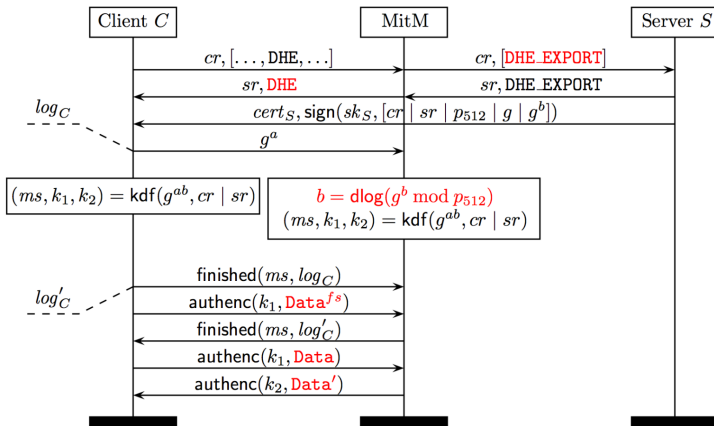


Figure: <https://weakdh.org/imperfect-forward-secrecy.pdf>

# Old Attacks on TLS

## RC4

- Roos's Bias 1995
- Fluhrer, Martin & Shamir 2001
- Klein 2005
- Combinatorial Problem 2001
- Royal Holloway 2013
- Bar-mitzvah 2015
- NOMORE 2015

## RSA-PKCS#1 v1.5 Encryption

- Bleichenbacher 1998
- Jager 2015
- DROWN 2016

## Renegotiation

- Marsh Ray Attack 2009
- Renegotiation DoS 2011
- Triple Handshake 2014

## 3DES

- Sweet32

## AES-CBC

- Vaudenay 2002
- Boneh/Brumley 2003
- BEAST 2011
- Lucky13 2013
- POODLE 2014
- Lucky Microseconds 2015

## Compression

- CRIME 2012

## MD5 & SHA1

- SLOTH 2016
- SHAttered 2017

**Figure:** [https://owasp.org/www-chapter-london/assets/slides/OWASPLondon20180125\\_TLSv1.3\\_Andy\\_Brodie.pdf](https://owasp.org/www-chapter-london/assets/slides/OWASPLondon20180125_TLSv1.3_Andy_Brodie.pdf)

# Downgrade Attacks on TLS

## TLS: a long year of downgrade attacks

- POODLE      TLS 1.2 → SSLv3      [Dec'14]
- FREAK      RSA-2048 → RSA-512      [Mar'15]
- LOGJAM      DH-2048 → DH-512      [May'15]
- BLEICH?      RSA-Sign → RSA-Enc      [Aug'15]
- SLOTH      RSA-SHA256 → RSA-MD5      [Jan'16]

**Figure:** <https://rwc.iacr.org/2016/Slides/Downgrade.pdf>

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- ▶ Removed AES-CBC encryption mode
- ▶ Removed static (EC) Diffie-Hellman
- ▶ Only standardized groups/curves

# New Cipher Suites

TLS 1.3 only allows for 5 different cipher suites:

- ▶ (EC)DHE-AES-128-GCM-SHA256
- ▶ (EC)DHE-AES-256GCM-SHA384
- ▶ (EC)DHE-CHACHA20-POLY1305-SHA256
- ▶ (EC)DHE-AES-128-CCM-SHA256
- ▶ (EC)DHE-AES-128-CCM-8-SHA256

# Matthew Green's Blog

- ▶ Standards: <https://blog.cryptographyengineering.com/2011/10/04/how-standards-go-wrong-constructive>
- ▶ Logjam: <https://blog.cryptographyengineering.com/2015/05/22/attack-of-week-logjam>
- ▶ FREAK: <https://blog.cryptographyengineering.com/2015/03/03/attack-of-week-freak-or-factoring-nsa>

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# Dual EC



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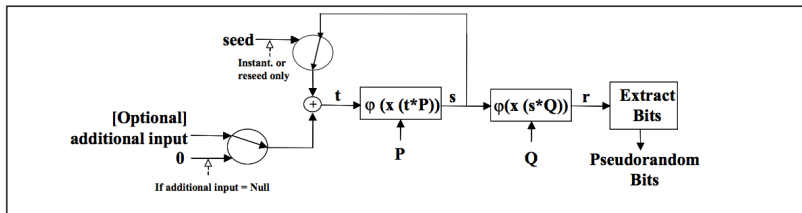
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- ▶ Let  $x(P)$  output the  $x$  coordinate of the point  $P$
- ▶ Let  $\phi$  be a function that truncates  $x(P)$  to bits

# Dual EC



This is provably biased if  
you know  $DLOG \log_P Q$



## DUAL\_EC **Backdoor** (Simplified)

### The user

- Two parameters  
 $(P, Q)$
- Compute next state  
 $s_{i+1} = P^{s_i} \bmod N$
- Compute next output  
 $r_i = Q^{s_i} \bmod N$

### The attacker

- Keep  $x$  such that  
 $P = Q^x \bmod N$
- Observe any output  
 $r_i$
- Compute next state  
 $s_{i+1} = r_i^x \bmod N$
- Predict all future outputs!

$$s_{i+1} = P^{s_i} = (Q^x)^{s_i} = (Q^{s_i})^x = r_i^x \bmod N$$

## A Security Analysis of the NIST SP 800-90 Elliptic Curve Random Number Generator

Daniel R. L. Brown\* and Kristian Gjøsteen†

February 15, 2007

**Figure:** <https://eprint.iacr.org/2007/048.pdf>

# Matthew Green's Blog

- ▶ Dual-EC-DRBG: <https://blog.cryptographyengineering.com/2013/09/18/the-many-flaws-of-dualecdrbg>
- ▶ RSA warning: <https://blog.cryptographyengineering.com/2013/09/20/rsa-warns-developers-against-its-own>
- ▶ NSA random number: <https://blog.cryptographyengineering.com/2013/12/28/a-few-more-notes-on-nsa-random-number>
- ▶ Juniper backdoor: <https://blog.cryptographyengineering.com/2015/12/22/on-juniper-backdoor>

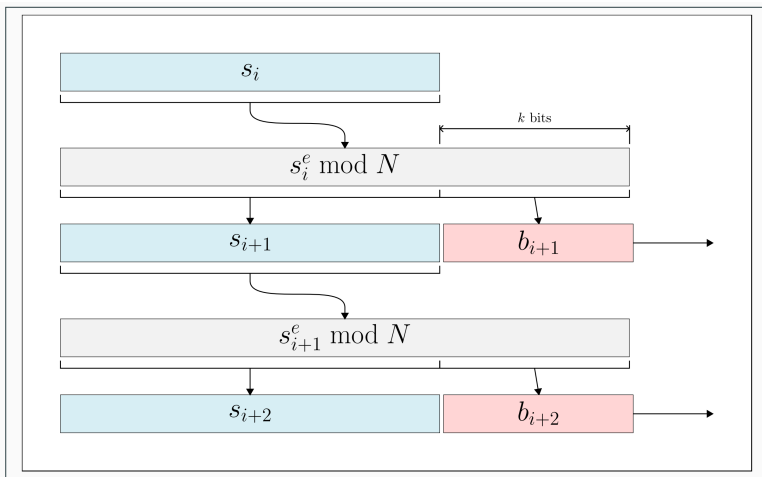
# On the Possibility of a Backdoor in the Micali-Schnorr Generator

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Hannah Davis<sup>1</sup>   Matthew Green<sup>2</sup>   Nadia Heninger<sup>1</sup>  
Keegan Ryan<sup>1</sup>   Adam Suhl<sup>1</sup>

**Figure:** paper: <https://eprint.iacr.org/2023/440.pdf>, talk: <https://www.youtube.com/watch?v=608NQdTn39Q&t=2629s>, slides: <https://iacr.org/submit/files/slides/2023/rwc/rwc2023/119/slides.pdf>

# Micali-Schnorr?



**Unclear how to recover the state using RSA decryption.**

# Questions?