



NTNU

Norwegian University of
Science and Technology

PROTOCOL APIS

TTM4205 – Lecture 11

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12.10.2023

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Protocol APIs

Distributed Schnorr Signatures

BLS Multisignatures

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Protocol APIs

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- ▶ holds secrets where clients can make queries
- ▶ holds secrets that clients can interact with
- ▶ combine inputs to verify batches at once

Protocol APIs

We will look at examples where a client can:

- ▶ extract secret signing keys
- ▶ forge signatures
- ▶ trick a verifier

Several of which are similar to the weekly problems.

We will also look at some mitigations to these issues.

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Recap: Schnorr Signatures

Let \mathbb{G} be a group of prime order p and let g be a generator for \mathbb{G} . Denote by pp the public parameters (\mathbb{G}, g, p) .

Let H be a cryptographic hash function that outputs uniformly random elements in \mathbb{Z}_p .

Let the secret key $\text{sk} \leftarrow \$ \mathbb{Z}_p$ be sampled uniformly at random, and let the public key be $\text{pk} = g^{\text{sk}}$, where pk is made public.

Recap: Schnorr Signatures

The Schnorr signature of message m is computed as:

1. Sample random $r \leftarrow \mathbb{Z}_p$ and compute $R = g^r$.
2. Compute the output challenge as $c = H(\text{pp}, \text{pk}, m, R)$.
3. Compute the response $z = r - c \cdot \text{sk}$. Output $\sigma = (c, z)$.

To verify the signature, compute $R' = g^z \cdot \text{pk}^c$ and check if $c \stackrel{?}{=} H(\text{pp}, \text{pk}, m, R')$. If correct, accept, and otherwise reject.

Distributed Schnorr Signatures

Assume that two parties P_0 and P_1 wants to compute a joint Schnorr signature. Then P_i does the following:

KGen :

- ▶ Sample random $sk_i \leftarrow \mathbb{Z}_p$ and compute $pk_i = g^{sk_i}$.
- ▶ Send pk_i to party P_{1-i} . Set $pk = pk_0 \cdot pk_1 = g^{sk_0 + sk_1}$.

This is called an additive secret sharing of the signing key.

Distributed Schnorr Signatures

Sign:

- ▶ Sample random $r_i \leftarrow \mathbb{Z}_p$ and compute $R_i = g^{r_i}$.
- ▶ Send R_i to party P_{1-i} . Set $c = H(\text{pp}, \text{pk}, m, R_0 \cdot R_1)$.
- ▶ Send the response $z_i = r_i - c \cdot \text{sk}_i$ to party P_{1-i} .

The signature $\sigma = (c, z_0 + z_1)$ can be verified as usual.

Question: How can a malicious client P_0 interacting with an honest (protocol API) P_1 break this signature scheme?

Potential Attacks

- ▶ The adversary can control the nonce values
- ▶ Repeated nonces for different m 's leak sk_1
- ▶ (The adversary can bias the secret-public keys)
- ▶ (The adversary can abort to deny signatures)
- ▶ (All parties need to be online to sign together)

Mitigations in Practice

- ▶ Send hashes in an extra round in KGen and Sign
- ▶ Send $h_i = H(pk_i)$ then pk_i and $h'_i = H(R_i)$ then R_i
- ▶ (If signatures are deterministic we need other solutions)
- ▶ Make it a t -out-of- n signature instead of 2-out-of-2

Proactive Two-Party Signatures for User Authentication

Antonio Nicolosi, Maxwell Krohn, Yevgeniy Dodis, and David Mazières
NYU Department of Computer Science
`{nicolosi,max,dodis,dm}@cs.nyu.edu`

Figure:

<https://www.scs.stanford.edu/~dm/home/papers/nicolosi:2schnorr.pdf>

Two-Round Stateless Deterministic Two-Party Schnorr Signatures From Pseudorandom Correlation Functions

Yashvanth Kondi, Claudio Orlandi, and Lawrence Roy

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Figure: <https://eprint.iacr.org/2023/216.pdf>

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BLS Signatures

Let $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ be groups of prime order p with generators g_1, g_2, g_T . Let $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ be a bilinear pairing such that $\hat{e}(g_1^a, g_2^b) = g_T^{a \cdot b}$ for all $a, b \in \mathbb{Z}_p$ and H be a cryptographic hash function that outputs uniformly random elements in \mathbb{G}_2 .

Let the secret key $sk \leftarrow \mathbb{Z}_p$ be sampled uniformly at random, and let the public key be $pk = g_1^{sk}$, where pk is made public.

A signature is computed as $\sigma = H(m)^{sk}$. The verifier checks $\hat{e}(g_1, \sigma) = \hat{e}(pk, H(m))$. If correct; accept, otherwise reject.

BLS Multisignatures

We can efficiently verify many signatures at once:

- ▶ Given many triples (pk_i, m_i, σ_i) , compute: $\sigma = \Pi_i \sigma_i$
- ▶ Verify all signatures as: $\hat{e}(g_1, \sigma) = \Pi_i \hat{e}(pk_i, H(m_i))$
- ▶ If all messages are identical: $\hat{e}(g_1, \sigma) = \hat{e}(\Pi_i pk_i, H(m))$
- ▶ If the same signers we can aggregate keys: $apk = \Pi_i pk_i$

Question: Fix m and pk_0 , how can an adversary forge a signature for pk_0 that verifies in the aggregated setting?

Potential Attacks

- ▶ Set $pk_1 = g_1^\alpha \cdot (pk_0)^{-1}$ and signature $\sigma = H(m)^\alpha$
- ▶ Then $\hat{e}(g_1, \sigma) = \hat{e}(g_1^\alpha, H(m)) = \hat{e}(pk_0 \cdot pk_1, H(m))$

Mitigations in Practice

- ▶ Require a proof for secret key knowledge
- ▶ Only aggregate distinct messages each time
- ▶ Verify a random linear combination of keys/signatures

Compact Multi-Signatures for Smaller Blockchains

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³ ETH Zurich

Figure: <https://eprint.iacr.org/2018/483.pdf>

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DL Parameters

For security of (EC)DH and (EC)DSA, we need to work in prime order (sub-) groups for the discrete logarithm problem to be hard. What happens if this is not the case?

DL Attacks

Recall from earlier that:

- ▶ Hardness of DL depends on the divisors p of the order
- ▶ We have generic attacks that runs in \sqrt{p} time
- ▶ We have sub-exponential attacks for finite field groups

Faulty parameters

What information might leak if:

- ▶ The order of the (sub-) group is not prime?
- ▶ The element is not in the correct (sub-) group?

Use $g^{\text{sk}} \bmod p$ as an example (EC in weekly problems).

Question: How might this happen in practice?

Mitigations in Practice

Always verify:

- ▶ given parameters
- ▶ input elements
- ▶ output elements

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- ▶ verify protocol parameters
- ▶ verify API inputs
- ▶ check API outputs
- ▶ enforce honest interaction
- ▶ avoid corner case leakage
- ▶ hinder replay attacks

Questions?