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Norwegian University of Science and Technology

Electronic Voting Using Lattice-Based Commitments and Verifiable Encryption

Aarhus Crypto Seminar

Carsten Baum, Kristian Gjøsteen, **Tjerand Silde** and Thor Tunge

Department of Mathematical Sciences,
NTNU Trondheim

Takeaway I

We propose a protocol for remote electronic voting using lattice-based commitments and verifiable encryption. It is the first practical construction for electronic voting that supports complex ballots and is built from post-quantum assumptions. Our scheme is also the first to defend against compromise of the voter's computer using return codes.

Takeaway II

The core of our protocol is a verifiable shuffle of known values inspired by Neff's construction (ACM CCS 2001). Our shuffle uses the lattice- based commitments from Baum et al. (SCN 2018).

Takeaway III

To prevent malformed encryptions, we use the verifiable encryption scheme of Lyubashevsky and Neven (EUROCRYPT 2017). We reuse this trick of verifiable encryption of commitment openings to create a practical return code mechanism.

Overview

– Security Definitions
– The Protocol Architecture
– Lattice-Based Commitments
– The Shuffle Protocol
– Proof of Encrypted Opening
– Return Codes
– The Voting Scheme
– Parameters and Efficiency
– Improvements and Future Work

Security Definitions I

If a voting system gives the correct answer, relative to some ideally determined collection of ballots and some counting function, we have *integrity*.

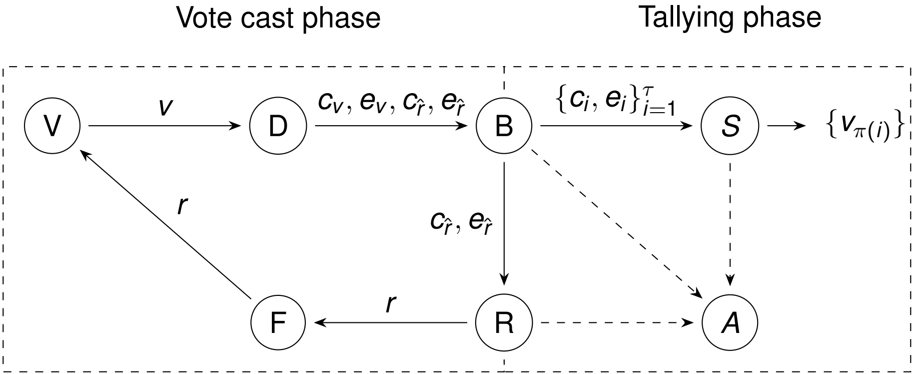
If it is hard to determine what ballot a given voter cast, up to what can be deduced from the election outcome, we have *privacy*.

Security Definitions II

If voters can ensure that the ballot of their choice is counted, even when an adversary can control the voter during ballot casting, we have *coercion resistance*.

If the ballot casting and the talley phase produce transcripts that allow voters to verify that the count included their ballots, we have *verifiability*.

The Protocol Architecture



Lattice-Based Commitments from Baum et al.

$\texttt{Com}$ commits to messages $m \in R_p$ by sampling an $\boldsymbol{r}_m \xleftarrow{\$} S_{\beta_\infty}^k$ and computing

$$\texttt{Com}(m; \boldsymbol{r}_m) = \boldsymbol{B} \cdot \boldsymbol{r}_m + \begin{bmatrix} \mathbf{0}^n \\ m \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_1 \\ c_2 \end{bmatrix} = [m] \, .$$

$\texttt{Com}$ outputs $c = [m]$ and $d = (m; \boldsymbol{r}_m, 1)$.

$\texttt{Open}$ verifies whether an opening $(m, \boldsymbol{r}_m, f)$ with $f \in \bar{\mathcal{C}}$ is a valid opening by checking if

$$f \cdot \begin{bmatrix} \boldsymbol{c}_1 \\ c_2 \end{bmatrix} \stackrel{?}{=} \boldsymbol{B} \cdot \boldsymbol{r}_m + f \cdot \begin{bmatrix} \mathbf{0}^n \\ m \end{bmatrix},$$

and that $\|r_i\| \leq 4\sigma\sqrt{N}$ for $\boldsymbol{r}_m = (r_0, \ldots r_{k-1})$ with $\sigma = 11 \cdot \nu \cdot \beta_\infty \cdot \sqrt{kN}$.
$\texttt{Open}$ outputs 1 if all these conditions holds, and 0 otherwise.

The Shuffle Protocol I

1. The prover $\mathcal{P}$ and verifier $\mathcal{V}$ receives a set of commitments $\{[m_i]\}_{i=1}^{\tau}$,
2. $\mathcal{P}$ also receives the set of openings $\{(m_i, r_m)\}_{i=1}^{\tau}$ of the commitments,
3. $\mathcal{V}$ picks a random $\rho \xleftarrow{\$} \Sigma_{\beta_\infty}$ and sends ρ to $\mathcal{P}$,
4. $\mathcal{P}$ and $\mathcal{V}$ shifts the commitments to get $M_i = m_i - \rho$,
5. $\mathcal{P}$ picks a random permutation $\pi \in S_\tau$,
6. $\mathcal{P}$ shuffles the messages by defining $\hat{M}_i := M_{\pi^{-1}(i)}$ for all $i \in \{1, \ldots, \tau\}$,
7. $\mathcal{P}$ sends the set of shuffled messages $\{\hat{M}_i\}_{i=1}^{\tau}$ to the verifier $\mathcal{V}$,
8. $\mathcal{P}$ proves to the verifier $\mathcal{V}$ in a public coin 6-move protocol Π_S that the set of shuffled messages is indeed the openings of the received commitments.

The Shuffle Protocol II

$\mathcal{P}$ picks a random permutation $\pi \in S_\tau$ and defines $\hat{M}_i = M_{\pi^{-1}(i)}$ for all i.

$\mathcal{P}$ draws $\theta_i \xleftarrow{\$} R_p$ uniformly at random for each i and computes

$$
\begin{aligned}
D_1 &= \left[\theta_1 \hat{M}_1\right] \\
D_2 &= \left[\theta_1 M_2 + \theta_2 \hat{M}_2\right] \\
&\vdots \\
D_{\tau-1} &= \left[\theta_{\tau-2} M_{\tau-1} + \theta_{\tau-1} \hat{M}_{\tau-1}\right] \\
D_\tau &= \left[\theta_{\tau-1} M_\tau\right].
\end{aligned}
$$

The Shuffle Protocol III

$\mathcal{V}$ chooses a challenge $\beta \in R_p$.

$\mathcal{P}$ computes $s_i \in R_q$ such that the following equations are satisfied:

$$
\begin{aligned}
\beta M_1 + s_1 \hat{M}_1 &= \theta_1 \hat{M}_1 \\
s_1 M_2 + s_2 \hat{M}_2 &= \theta_1 M_2 + \theta_2 \hat{M}_2 \\
&\vdots \\
s_{\tau-2} M_{\tau-1} + s_{\tau-1} \hat{M}_{\tau-1} &= \theta_{\tau-2} M_{\tau-2} + \theta_{\tau-1} \hat{M}_{\tau-1} \\
(-1)^{\tau} \beta \hat{M}_{\tau} + s_{\tau-1} M_{\tau-1} &= \theta_{\tau-1} M_{\tau}.
\end{aligned}
$$

The Shuffle Protocol IV

This can be written as a matrix-equation:

$$\underbrace{\begin{bmatrix} \hat{M}_1 & 0 & \dots & 0 & 0 \\ M_2 & \hat{M}_2 & \dots & 0 & 0 \\ 0 & M_3 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & M_{\tau-1} & \hat{M}_{\tau-1} \\ 0 & 0 & \dots & 0 & M_\tau \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} s_1 - \theta_1 \\ s_2 - \theta_2 \\ s_3 - \theta_3 \\ \vdots \\ s_{\tau-2} - \theta_{\tau-2} \\ s_{\tau-1} - \theta_{\tau-1} \end{bmatrix}}_{s-\theta} = \underbrace{\begin{bmatrix} -\beta M_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ (-1)^{\tau-1}\beta\hat{M}_\tau \end{bmatrix}}_{b}.$$

The Shuffle Protocol V

Theorem

The shuffle protocol is complete.

Theorem

The shuffle protocol in is sound for any PPT $\mathcal{P}^*$ that wins with probability $> \frac{\tau^\delta}{p^N}$.

The Shuffle Protocol VI

Simulation to prove HVZK:

<u>Real</u>

$$\begin{aligned}
1:&\quad \theta_i \xleftarrow{\$} R_p\\
2:&\quad D_i \leftarrow (2)\\
3:&\quad \text{Send } D_i\\
4:&\quad s_i \leftarrow (3)\\
5:&\quad \text{Send } s_i\\
6:&\quad \text{SHVZK}
\end{aligned}$$

<u>Game 1</u>

$$\begin{aligned}
1:&\quad \theta_i \xleftarrow{\$} R_p\\
2:&\quad D_i \leftarrow (2)\\
3:&\quad \text{Send } D_i\\
4:&\quad s_i \leftarrow (3)\\
5:&\quad \text{Send } s_i\\
6:&\quad \boxed{\textbf{Sim}}
\end{aligned}$$

<u>Game 2</u>

$$\begin{aligned}
1:&\quad \boxed{s_i \xleftarrow{\$} R_p}\\
2:&\quad \theta_i \xleftarrow{\$} R_p\\
3:&\quad \boxed{D_i \leftarrow (6)}\\
4:&\quad \text{Send } D_i\\
5:&\quad \text{Send } s_i\\
6:&\quad \text{Sim}
\end{aligned}$$

<u>Simulator</u>

$$\begin{aligned}
1:&\quad \boxed{D_i \leftarrow [0]}\\
2:&\quad s_i \xleftarrow{\$} R_p\\
3:&\quad \text{Send } D_i\\
4:&\quad \text{Send } s_i\\
5:&\quad \text{Sim}
\end{aligned}$$

Proof of Encrypted Opening I

We want to prove that we know μ such that $T\mu = u$ for public T and u.

Using the verifiable encryption scheme by Lyubashevsky and Neven, we can prove that we know a small $\bar{\mu}$ and $\bar{c}$ such that

$$T\bar{\mu} = \bar{c}u.$$

Proof of Encrypted Opening II

We can re-write the commitment as a single matrix-vector multiplication:

$$\mathtt{Com}(m; r_m) = \begin{bmatrix} B_1 & 0^n \\ b_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_m \\ m \end{bmatrix}.$$

Denote this matrix by C.

Proof of Encrypted Opening III

We can use the verifiable encryption scheme to encrypt the opening $\mu = (r_m, m)$, and prove that the voter knows a witness for the relation

$$C\mu = C \cdot \begin{bmatrix} r_m \\ m \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

Return Codes I

Assume that the voters have ω different options in the election.

Further, let

- $v_1, v_2, \ldots, v_\omega \in S_{\beta_\infty}$ be possible ballots,
- $a_0 \in R_p$ be a blinding-key for a voter V,
- $\mathtt{PRF}_k : \{0,1\}^* \times R_p \to \{0,1\}^n$ be a pseudo-random function,

Return Codes II

The *pre-code* $\hat{r}_j$ corresponding to the ballot v_j is $\hat{r}_j = v_j + a_0$.

The *return code* r_j corresponding to the ballot v_j is $r_j = \mathtt{PRF}_k(V, \hat{r}_j)$.

Let c_{a_0}, c_{v_j} and $c_{\hat{r}}$ be commitments to the blinding key, the ballot and the pre-code.

We can prove in zero-knowledge that $\hat{r}_j = a_0 + v_j$ is correct.

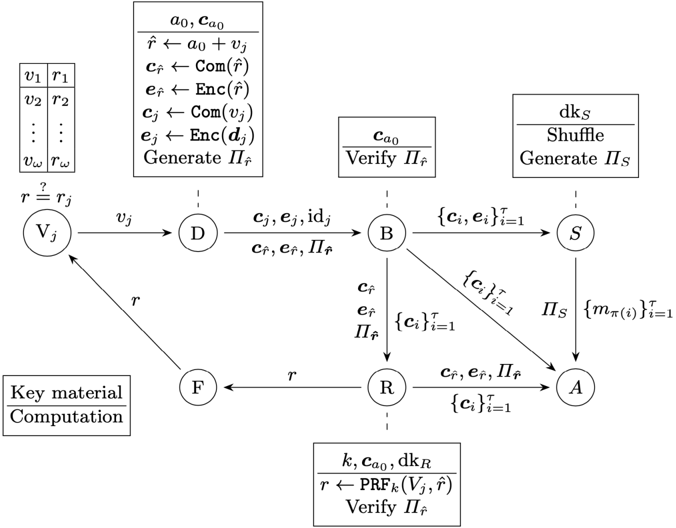
Return Codes III

A commitment to voter V's voter-unique blinding key a_0 is public.

The voter V's *computer* computes the pre-code $\hat{r}_j = v_j + a_0$, a commitment $c_{\hat{r}_j}$ to $\hat{r}_j$ and a proof $\Pi_{\hat{r}_j}$. Finally, it will verifiably encrypt as $e_{\hat{r}_j}$ the opening of $c_{\hat{r}_j}$ with the return code generator's public key.

The *return code generator* receives V, $c_{\hat{r}_j}$, c_{v_j}, $e_{\hat{r}_j}$ and $\Pi_{\hat{r}_j}$. It verifies the proof and the encryption, and then decrypts the ciphertext to get $\hat{r}_j$. It computes the return code as $r_j = \mathtt{PRF}_k(V, \hat{r}_j)$.

The Voting Scheme



Parameters and Efficiency

A vote $(c_j, e_j, c_{\hat{r}}, e_{\hat{r}}, \Pi_{\hat{r}})$ is of total size $\approx$ 400 KB.

For τ voters, the ballot box $\mathcal{B}$ receives $\approx 400\tau$ KB of data.

The shuffle proof is of total size $\approx 21\tau$ KB.

Commitments	Encryption	Verification	Shuffle Proof
0.12ms	60ms	4ms	12τms

Improvements and Future Work

Can we...

- extend the shuffle to handle arbitrary ring elements?
- extend this into a mix-net with more than one shuffle-server?
- extend the return-code mechanism to handle re-voting?
- aggregate zero-knowledge proofs for all equations in the shuffle?

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Thank you! Questions?