



NTNU

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# SIDE-CHANNEL ATTACKS 2

TTM4205 – Lecture 8

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# Contents

**Announcements**

**Previous Lecture**

**SCA on RSA**

**CT Arithmetic**

**SCA on ECC**

# Contents

**Announcements**

Previous Lecture

SCA on RSA

CT Arithmetic

SCA on ECC

# Reference Group Meeting

The reference group will meet Thursday morning. Get in touch with the reference group members if you have any feedback about the course. You can also provide feedback (anonymously) on the Piazza forum.

# Contents

Announcements

**Previous Lecture**

SCA on RSA

CT Arithmetic

SCA on ECC

# Black Box Crypto

We design the security of a cryptographic scheme to follow Kerckhoff's principle: if everything about the scheme, except for the key, is known, then the scheme should be secure.

We then analyze the scheme mathematically as black-box algorithms that take some (public or secret) input and give some (public or secret) output, and prove that it is secure concerning the algorithm description and the public data.

However, security depends on your model. In practice, it matters how these algorithms are implemented and what kind of information the *physical* system leaks about the inner workings of the algorithm computing on secret data.

# Leakage

- ▶ The time it takes to compute
- ▶ The power usage while computing
- ▶ The electromagnetic radiation...
- ▶ The temperature increase...
- ▶ The memory pattern accessed...
- ▶ The sounds your laptop makes...

# Attack Categories

- ▶ Remote vs physical attacks
- ▶ Software and hardware attacks
- ▶ Passive vs active attacks
- ▶ Invasive vs non-invasive attacks



# Contents

Announcements

Previous Lecture

**SCA on RSA**

CT Arithmetic

SCA on ECC

# RSA Exponentiation

In the RSA cryptosystem (encryption, decryption, signing and verification), we need to compute an exponentiation.

If the exponent is a secret (decryption or signing) key, we must protect this value against side-channel attacks.

# Assumptions

In this example we assume a few things:

- ▶ the RSA primes are generated securely
- ▶ order  $\phi$  is computed as  $\text{lcm}(p - 1, q - 2)$
- ▶ we have a way of representing larger integers

# Weaknesses and Defenses

In the following slides we will look at the common ways to compute modular exponentiation. For each algorithm, try to come up with attacks and defenses for the algorithm.

# Square and Multiply

```
1  # compute m = c**d mod n
2  def squareAndMultiply(c, d, n):
3      m = c
4
5      for i in range(len(d)):
6          m = m * m % n
7
8          if ( d[i] == 1 ):
9              m = m * c % n
10
11     return m
```

# Potential Weaknesses

The following might trivially leak the key:

- ▶ timing or power traces might leak the 1's in  $d$
- ▶ multiplication might not be constant time
- ▶ modular reduction might not be constant time

# Potential Defenses

We must at least ensure the following:

- ▶ algorithm must be independent of the 1's in  $d$
- ▶ bit int multiplication must be constant time
- ▶ modular reduction must be constant time

Assume that the two latter operations are constant time.

# Square and Always Multiply

```
1  # compute m = c**d mod n
2  def squareAndAlwaysMultiply(c, d, n):
3      m, x = c, c
4
5      for i in range(len(d)):
6          m = m * m % n
7
8          if ( d[i] == 1 ):
9              m = m * c % n
10
11         else:
12             x = m * c % n
13
14     return m
```



# Potential Weaknesses

- ▶ dummy operations might leak memory information
- ▶ "smart" compilers might skip dummy operations
- ▶ fault injections might expose dummy operations

# Potential Defenses

- ▶ make the result dependent on every operation
- ▶ perform the same operations independent of  $d$

# Montgomery Ladder

```
1      # compute m = c**d mod n
2      def MontgomeryLadder(c, d, n):
3          m1, m2 = c, c * c % n
4
5          for i in range(len(d)):
6
7              if ( d[i] == 1 ):
8                  m1 = m1 * m2 % n
9                  m2 = m2 * m2 % n
10
11             else:
12                 m2 = m1 * m2 % n
13                 m1 = m1 * m1 % n
14
15         return m1
```

# Potential Weaknesses

There might still be issues:

- ▶ if  $c$  is chosen adaptively, many power traces might leak  $d$

# Potential Defenses

Randomization to the rescue:

- ▶ randomize the computation to make it independent of  $c$

# Randomized Montgomery Ladder

```
1  # compute m = c**d mod n
2  # we have e*d = 1 mod phi
3  def randMontgomeryLadder(c, e, d, phi, n):
4
5      r1 = secrets.randbelow(n)
6      r2 = squareAndMultiply(r1, e, n)
7      r1Inv = MontgomeryLadder(r1, phi-1, n)
8
9      m1 = c * r2 % n
10     m2 = m1 * m1 % n
11
12     for i in range(len(d)):
13
14         if ( d[i] == 1 ):
15             m1 = m1 * m2 % n
16             m2 = m2 * m2 % n
17
18         else:
19             m2 = m1 * m2 % n
20             m1 = m1 * m1 % n
21
22     m1 = m1*r1Inv % n
23     return m1
```

# Potential Weaknesses

There might still be issues:

- ▶ if key is fixed, many power traces might leak  $d$

# Potential Defenses

Randomization to the rescue (again):

- ▶ randomize the exponent to mask the key  $d$



# Doubly randomized Montgomery Ladder

```
1  # compute m = c**d mod n
2  # we have e*d = 1 mod phi
3  def randRandMontgomeryLadder(c, e, d, phi, n, t):
4
5      r1 = secrets.randbelow(n)
6      r2 = squareAndMultiply(r1, e, n)
7      r1Inv = MontgomeryLadder(r1, phi-1, n)
8
9      r = secrets.randbelow(t)
10     # get dNew = d + r * phi
11     dNew = convert(d, r, phi)
12
13     m1 = c * r2 % n
14     m2 = m1 * m1 % n
15
16     for i in range(len(d)):
17
18         if ( d[i] == 1 ):
19             m1 = m1 * m2 % n
20             m2 = m2 * m2 % n
21
22         else:
23             m2 = m1 * m2 % n
24             m1 = m1 * m1 % n
25
26     m1 = m1*r1Inv % n
27     return m1
```

# Summary

Protecting secret key computations are difficult. We need:

- ▶ all binary operations to be constant time
- ▶ the algorithmic operations to be constant time
- ▶ correctness of output to depend on all operations
- ▶ the base element to be randomized (masked)
- ▶ the exponent to be randomized (masked)

# Contents

Announcements

Previous Lecture

SCA on RSA

**CT Arithmetic**

SCA on ECC

# Representing Large Integers

This is usually done by representing them as a list of integers of 32 or 64 bits. Binary operations is then done over the list of integers and must remember the carry when it overflows.

For example, a RSA-4096 moduli can be represented using a list of 128 integers of 32 bits or 64 integers of 64 bits.

Takes in two 32 bit integers to be multiplied and outputs two 32 bit integers representing the upper and lower 32 bits of the product. This operation is constant time.

Disclaimer 1: this depends on the machine your are using.

Disclaimer 2: this depends on the compiler your are using.

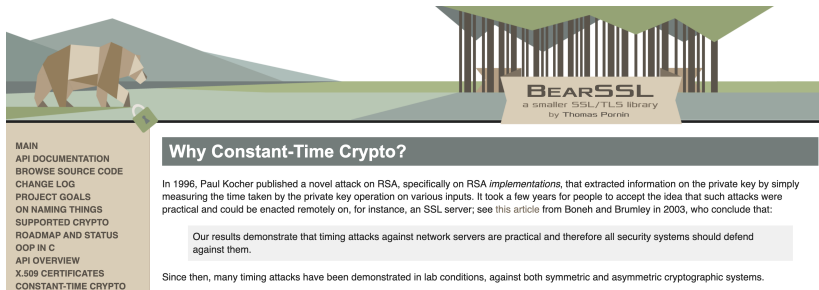
# Arm MUL

CPU type	32→32	32→64	MUL31	64→64
ARM7T (A32)	N	N	Y	S-
ARM7T (Thumb)	N	S-	S-	S-
ARM9T (A32)	N	N	Y	S-
ARM9T (Thumb)	N	S-	S-	S-
ARM9E	Y	Y	Y	S+
ARM10E	Y	Y	Y	S+
ARM11	Y	Y	Y	S+
Cortex-A (A32)	Y	Y	Y	S+
Cortex-A (A64)	?	?	?	?
Cortex-R (A32)	Y	Y	Y	S+
Cortex-M0/M0+/M1	Y	S-	S-	S-
Cortex-M3	Y	N	N	S-
Cortex-M4	Y	Y	Y	S+

**Figure:** <https://www.bearssl.org/ctmul.html>

# Modular Montgomery multiplication

```
28 void
29 br_131_montymul(uint32_t *d, const uint32_t *x, const uint32_t *y,
30                 const uint32_t *m, uint32_t m01)
31 {
32     size_t len, len4, u, v;
33     uint64_t dh;
34
35     len = (n[0] + 31) >> 5;
36     len4 = len & ~(size_t)3;
37     br_131_zero(d, n[0]);
38     dh = 0;
39     for (u = 0; u < len; u++) {
40         uint32_t f, xu;
41         uint64_t r, zh;
42
43         xu = x[u + 1];
44         f = MUL31_lo((d[1] + MUL31_lo(x[u + 1], y[1])), m01);
45
46         r = 0;
47         for (v = 0; v < len4; v += 4) {
48             uint64_t z;
49
50             z = (uint64_t)d[v + 1] + MUL31(xu, y[v + 1])
51               + MUL31(f, m[v + 1]) + r;
52             r = z >> 31;
53             d[v + 0] = (uint32_t)z & 0x7FFFFFFF;
54             z = (uint64_t)d[v + 2] + MUL31(xu, y[v + 2])
55               + MUL31(f, m[v + 2]) + r;
56             r = z >> 31;
57             d[v + 1] = (uint32_t)z & 0x7FFFFFFF;
58             z = (uint64_t)d[v + 3] + MUL31(xu, y[v + 3])
59               + MUL31(f, m[v + 3]) + r;
60             r = z >> 31;
61             d[v + 2] = (uint32_t)z & 0x7FFFFFFF;
62             z = (uint64_t)d[v + 4] + MUL31(xu, y[v + 4])
63               + MUL31(f, m[v + 4]) + r;
64             r = z >> 31;
65             d[v + 3] = (uint32_t)z & 0x7FFFFFFF;
66         }
67         for (; v < len; v++) {
68             uint64_t z;
69
70             z = (uint64_t)d[v + 1] + MUL31(xu, y[v + 1])
71               + MUL31(f, m[v + 1]) + r;
72             r = z >> 31;
73             d[v] = (uint32_t)z & 0x7FFFFFFF;
74         }
75
76         zh = dh + r;
77         d[len] = (uint32_t)zh & 0x7FFFFFFF;
78         dh = zh >> 31;
79     }
80
81     /*
82     * We must write back the bit length because it was overwritten in
83     * the loop (not overwriting it would require a test in the loop,
84     * which would yield bigger and slower code).
85     */
86     d[0] = n[0];
87
88     /*
89     * d[] may still be greater than m[] at that point; notably, the
90     * 'dh' word may be non-zero.
91     */
92     br_131_sub(d, m, NEG(dh, 0) | NOT(br_131_sub(d, m, 0)));
93 }
```



**Figure:** <https://www.bearssl.org/constanttime.html>



# Montgomery Modular Multiplication

```
function REDC is  
  input: Integers  $R$  and  $N$  with  $\gcd(R, N) = 1$ ,  
         Integer  $N'$  in  $[0, R - 1]$  such that  $NN' \equiv -1 \pmod R$ ,  
         Integer  $T$  in the range  $[0, RN - 1]$ .  
  output: Integer  $S$  in the range  $[0, N - 1]$  such that  $S \equiv TR^{-1} \pmod N$   
  
   $m \leftarrow ((T \bmod R)N') \bmod R$   
   $t \leftarrow (T + mN) / R$   
  if  $t \geq N$  then  
    return  $t - N$   
  else  
    return  $t$   
  end if  
end function
```

**Figure:** [https://en.wikipedia.org/wiki/Montgomery\\_modular\\_multiplication](https://en.wikipedia.org/wiki/Montgomery_modular_multiplication)

# Constant Time IF

A possible way to compute an IF in constant time:

$$(t < N) \cdot t + (1 - (t < N)) \cdot (t - N)$$

Disclaimer: "smart" compilers might make it a regular IF.

# Contents

Announcements

Previous Lecture

SCA on RSA

CT Arithmetic

**SCA on ECC**

We can essentially re-use most mechanisms for RSA in ECC.

**Q:** Do you see any immediate differences between the two?

We can essentially re-use most mechanisms for RSA in ECC.

**A:** We need to be a bit careful about the following:

- ▶ scalar multiplication must depend on curve params
- ▶ addition formulas involve inversion of secret elements
- ▶ addition formulas depends on the input points

We can essentially re-use most mechanisms for RSA in ECC.

**Sol:** Some possible solutions to avoid the above:

- ▶ verify points and use curve-dependent formulas
- ▶ use curves and formulas that are universal
- ▶ compute inversion in constant time (Fermat trick)
- ▶ avoid (most) inversions using projective coordinates

# Comparative Study of ECC Libraries for Embedded Devices

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**Figure:** <https://tjerandsilde.no/files/Comparative-Study-of-ECC-Libraries-for-Embedded-Devices.pdf>

# Questions?