Detecting and Modeling Changes in Stream Nutrient Dynamics

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Joint work with Hannah Waler

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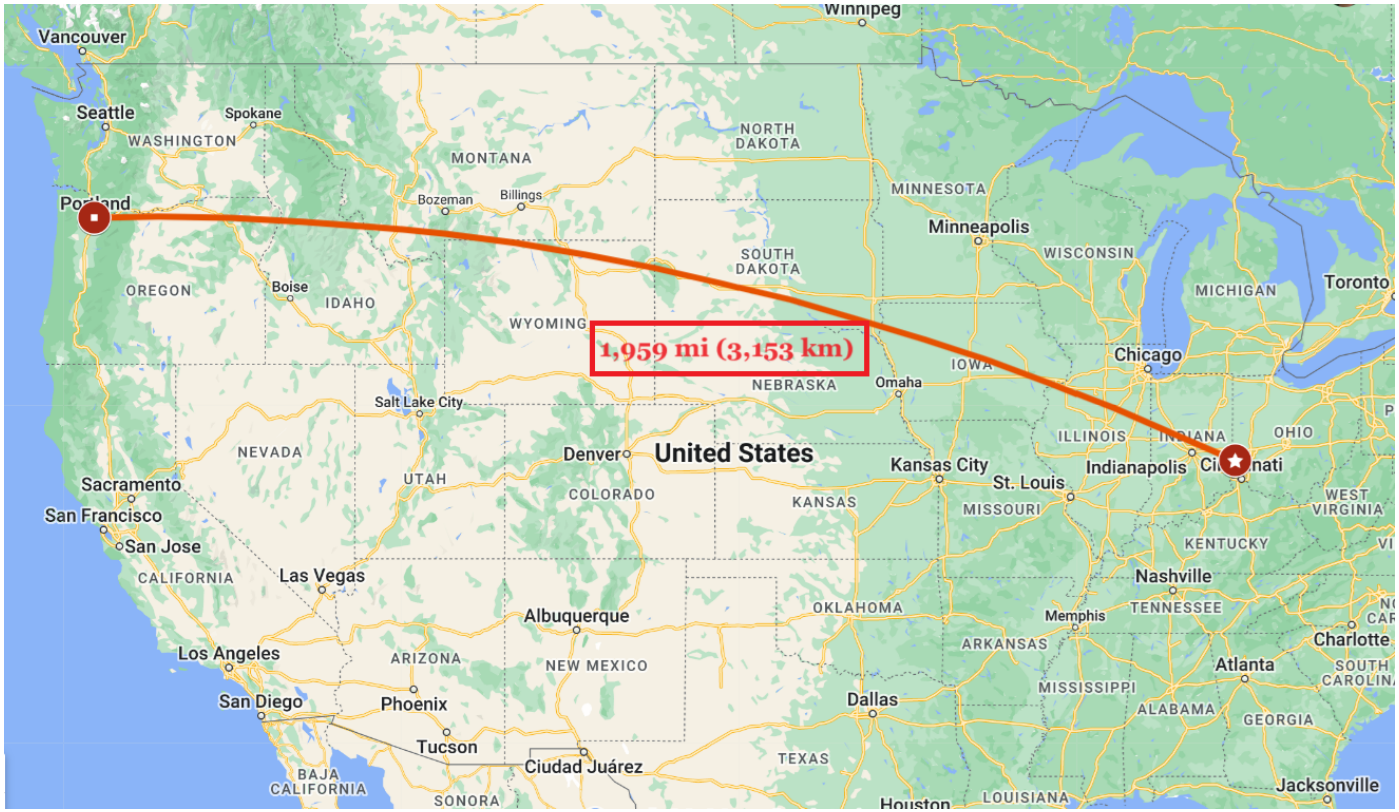
Acton Lake

Where is this?



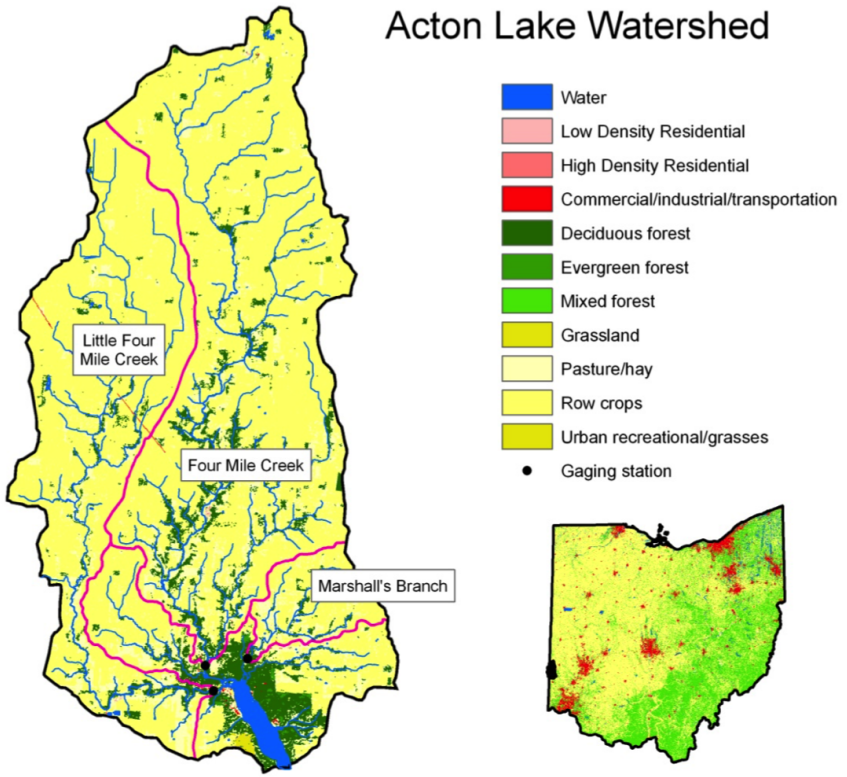
Acton Lake Watershed & Nutrients

Three different streams

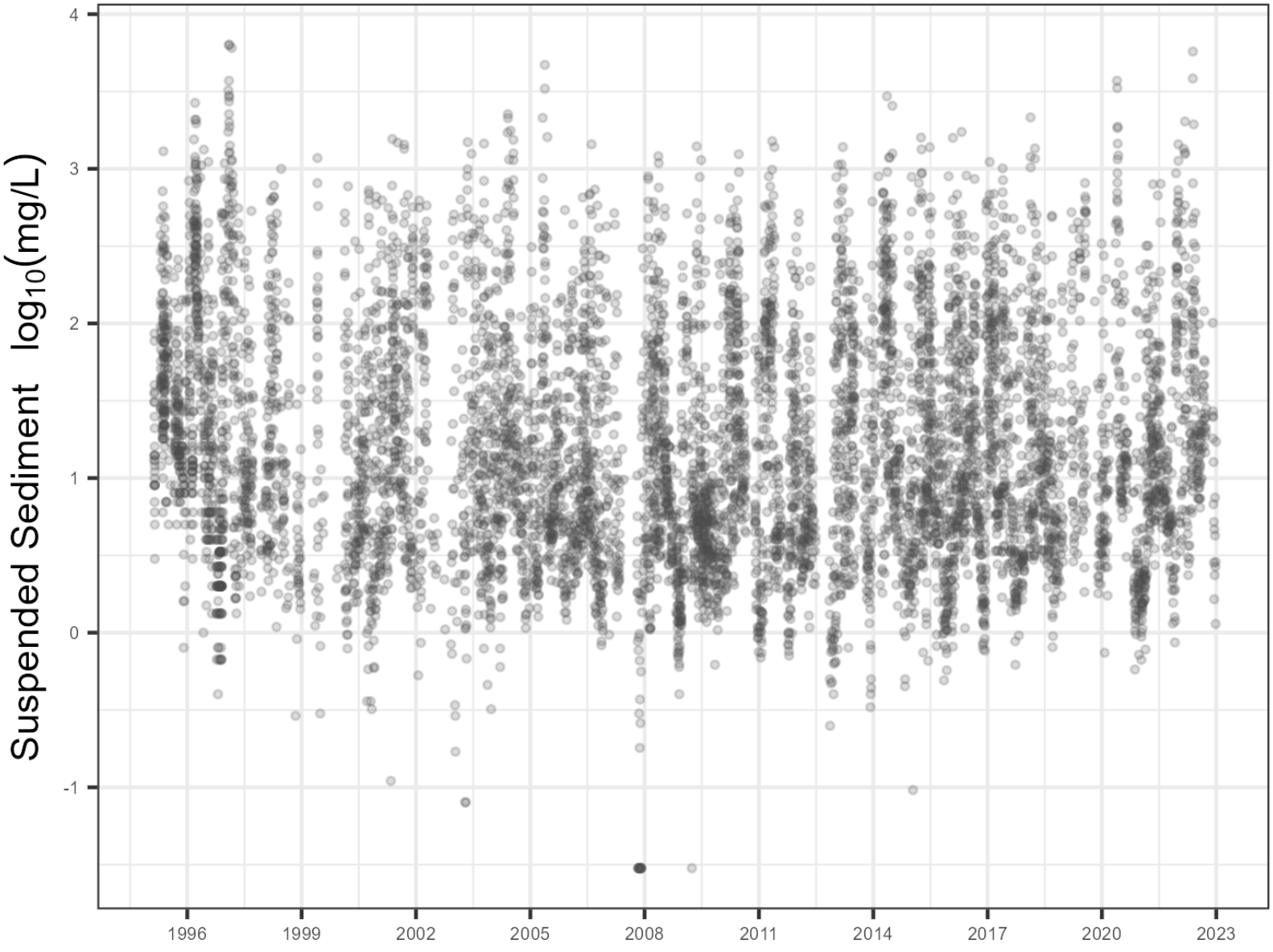
- Four Mile Creek
- Little Four Mile
- Marshall's Branch

Four different nutrients measured

- Ammonium (NH_4)
- Nitrate (NO_3)
- Soluble Reactive Phosphorus (SRP)
- Suspended Sediment (SS)



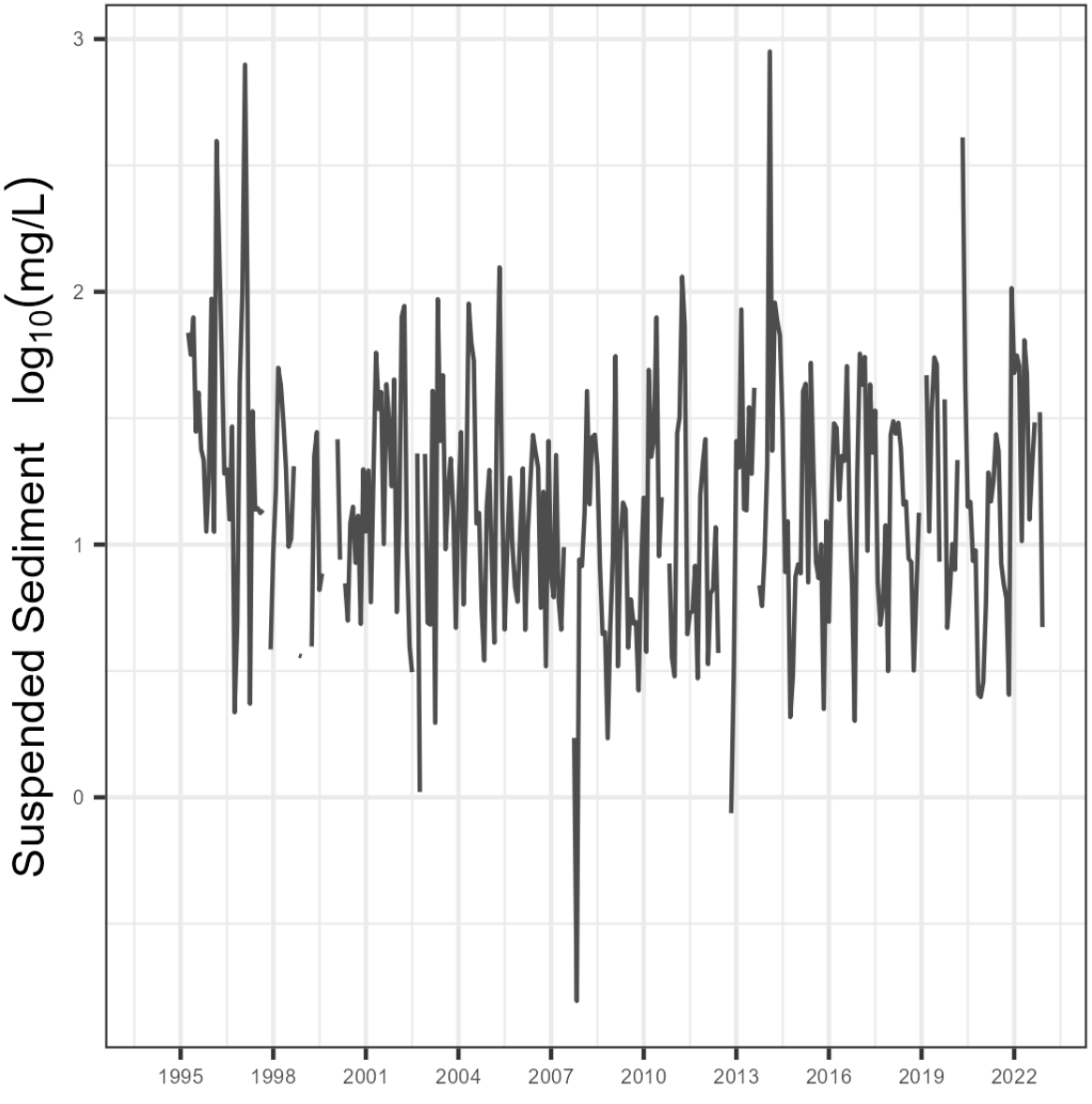
Example Data Set - Four Mile Creek Suspended Sediment



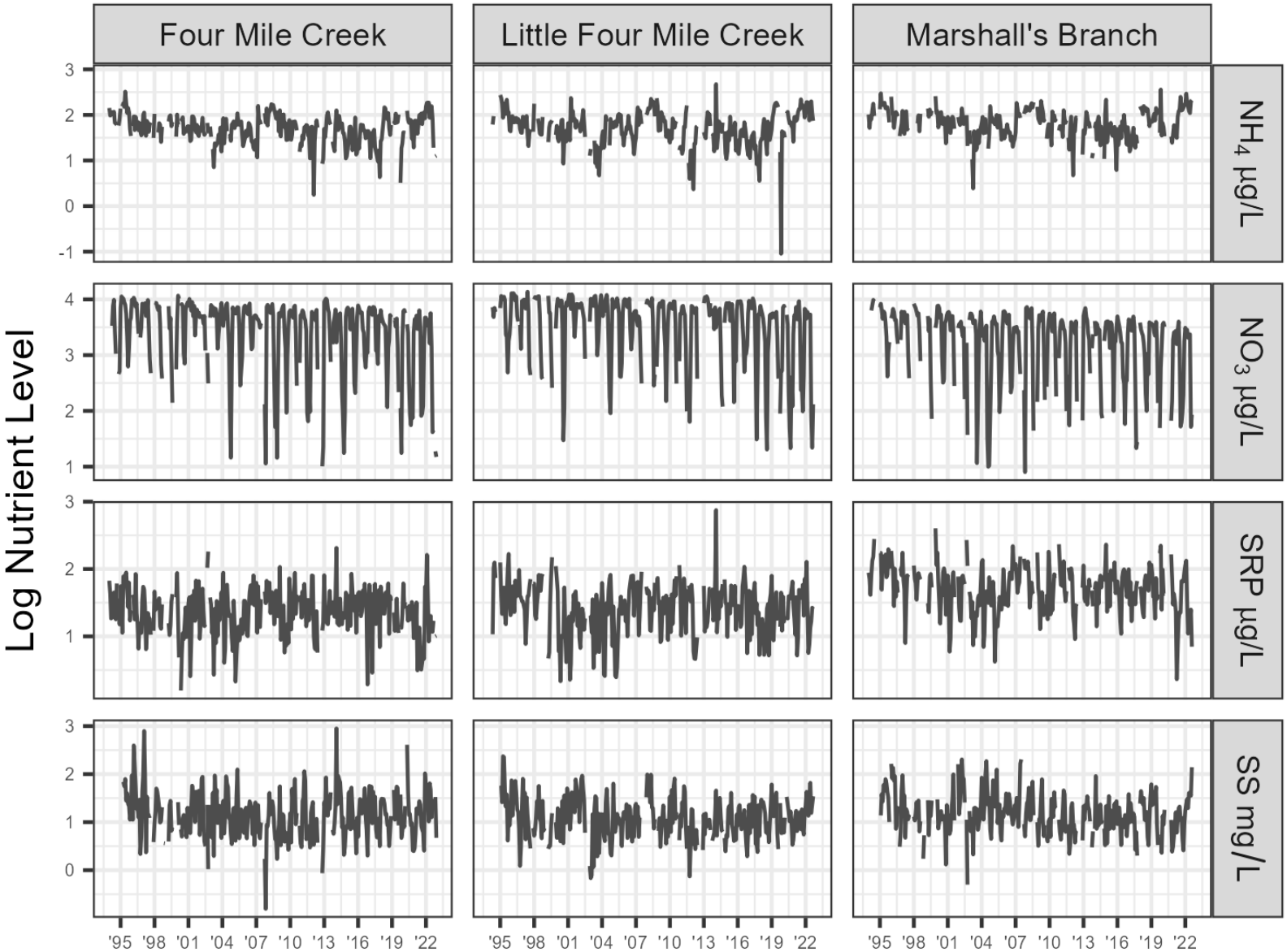
Example Aggregated Data Set

Monthly averages on a log scale

- $\log_{10}$ on each observation
- For each month-year, compute the mean of all observations



All Series


Four Mile Creek
Little Four Mile Creek
Marshall's Branch
NH4 µg/L
NO3 µg/L
SRP µg/L
SS mg/L
Log Nutrient Level
'95 '98 '01 '04 '07 '10 '13 '16 '19 '22

Features of the Data

Influence by stream flow (discharge)

- More water flow - more nutrients
- Known to be nonlinear relationship

Seasonal

- Weather patterns
- Seasonal Agricultural practices

Temporal correlation

- Underlying AutoRegressive type process

Possible long-term trends

Model Form

$$Y_t = \mu + \alpha t + \gamma_1 Q_t + \gamma_2 Q_t^2 + \sum_{j=1}^{5} \beta_j \sin\left(\frac{2\pi jt}{12}\right) + \sum_{j=1}^{6} \beta_j^* \cos\left(\frac{2\pi jt}{12}\right) + \varepsilon_t$$

- The term ε_t follows an AR(p) process
- The term Q_t models the effects of discharge
- The sin and cos terms model seasonality

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Previous work looked at long term trends (importance of α); see Renwick et al. [2008] or Renwick et al. [2018].

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We are looking for potential changes in μ and α

Model form (with change points)

$$Y_t = \mu + \delta_t + (\alpha + \Delta_t)t \; + \; \gamma_1 Q_t + \gamma_2 Q_t^2 + \sum_{j=1}^{5} \beta_j \sin\left(\frac{2\pi jt}{12}\right) + \sum_{j=1}^{6} \beta_j^* \cos\left(\frac{2\pi jt}{12}\right) + \varepsilon_t$$

where

$$\delta_t = \begin{cases} 0 & t < \tau_1 \\ \mu_1 & \tau_1 \le t < \tau_2 \\ \mu_2 & \tau_2 \le t < \tau_3 \\ \vdots & \vdots \\ \mu_m & \tau_m \le t \le N \end{cases}, \quad \Delta_t = \begin{cases} 0 & t < \tau_1 \\ \alpha_1 & \tau_1 \le t < \tau_2 \\ \alpha_2 & \tau_2 \le t < \tau_3 \\ \vdots & \vdots \\ \alpha_m & \tau_m \le t \le N \end{cases}$$

for change points

$$1 < \tau_1 < \tau_2 < \ldots < \tau_m \le N$$

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$$Y_t = \boxed{\mu + \delta_t + (\alpha + \Delta_t)t} + \gamma_1 Q_t + \gamma_2 Q_t^2 + \sum_{j=1}^{5} \beta_j \sin\left(\frac{2\pi jt}{12}\right) + \sum_{j=1}^{6} \beta_j^* \cos\left(\frac{2\pi jt}{12}\right) + \varepsilon_t$$

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Model Selection

Minimum Description Length

- Based on coding and information theory
  - How much storage is required to describe a model
- Better models = models with minimal storage
- Each parameter can uniquely be penalized

Similar to the well-known AIC or BIC, but penalty structure is different.

The MDL approach has been shown to be effective in the change point problem; see Shi et al. [2022].

MDL Objective Function

$$
\begin{aligned}
-2\log L \;+\; 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} \;+\; & \\
\sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + & \\
\frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p) &
\end{aligned}
$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

MDL Objective Function

$$\boxed{-2\log L} \;+\; 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} \;+\; \sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + \frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

Log-Likelihood...

MDL Objective Function

$$-2\log L + \boxed{2\frac{\log(N)}{2} + k\frac{\log(N)}{2}} + \sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + \frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

Cost for estimating two discharge coefficients and k sin and cos terms.

MDL Objective Function

$$-2\log L \;+\; 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} \;+\; \sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + \boxed{\frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)}$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

Cost for determining autoregressive order and modeling AR(p).

MDL Objective Function

$$-2\log L + 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} + \boxed{\sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1})} + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + \frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

Cost for fitting μ_j and α_j in $m+1$ regimes, each of length $\tau_j - \tau_{j-1}$.

MDL Objective Function

$$-2\log L + 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} + \sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \boxed{\log(m) + \sum_{j=2}^{m}\log(\tau_j)} + \frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

Cost for determining the number of change points and their locations.

MDL Objective Function

$$-2\log L + 2\frac{\log(N)}{2} + k\frac{\log(N)}{2} + \sum_{j=1}^{m+1}\log(\tau_j - \tau_{j-1}) + \log(m) + \sum_{j=2}^{m}\log(\tau_j) + \frac{\log(N)}{2} + p\frac{\log(2N)}{2} + \log(p)$$

with $\tau_0 = 0$ and $\tau_{m+1} = N$.

The details can be found in Lu et al. [2010].

Implementation

How to find the optimal MDL objective function?

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Genetic Algorithm

- Binary search on $N - 1$ potential change point locations
  - Max *iterations* 5000, a *run* of 500 for convergence
  - Mutation probability: 0.1; Crossover probability: 0.8
- *Suggestions* for initial populations
  - No change point configuration
  - All single change point configurations
  - Random selection of 2 change point configurations
- Additional regime penalty (minimum regime length of 60 months or 5 years)

See Li and Lund [2012] for more on genetic algorithms and MDL.

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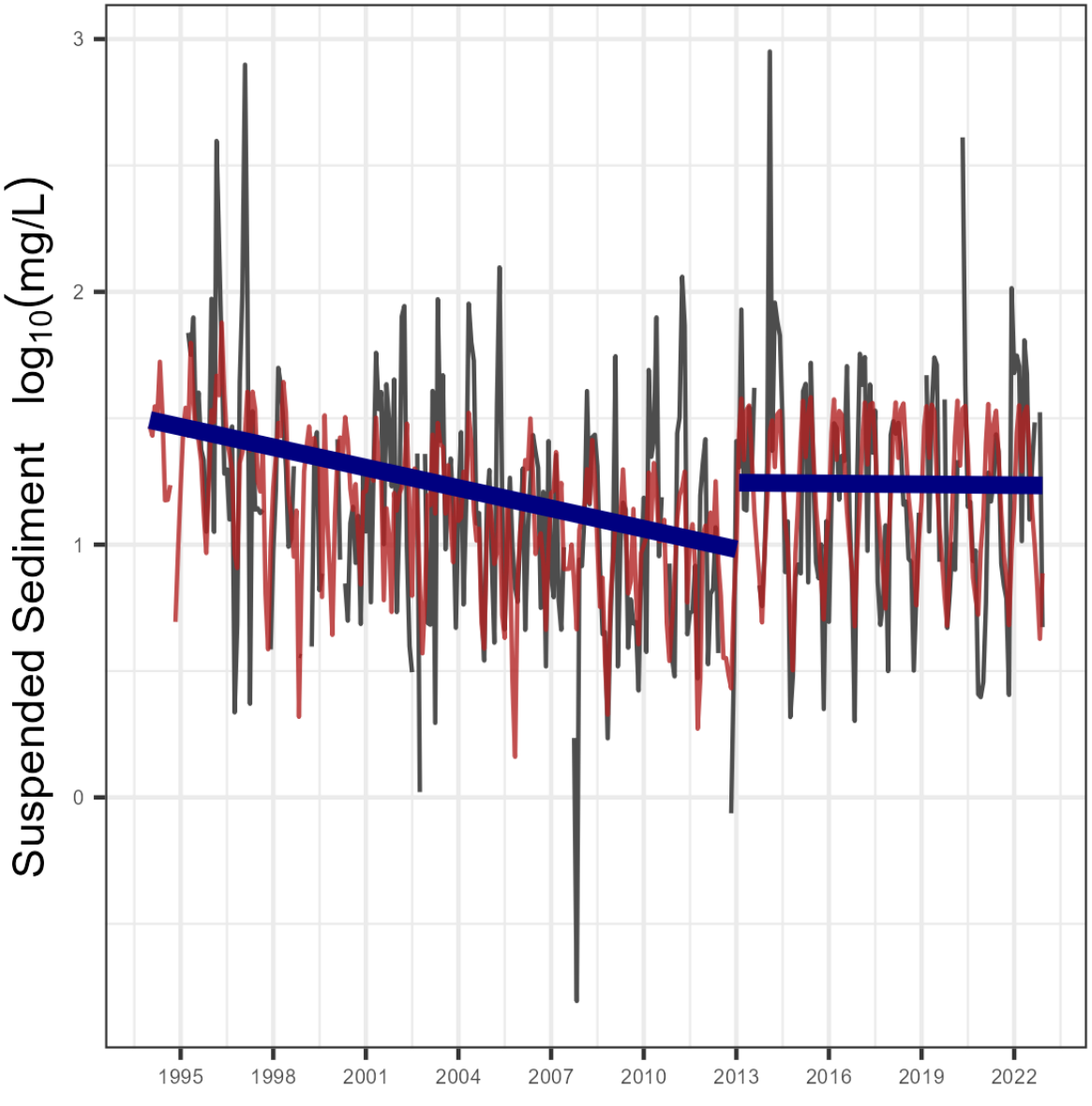
Repeated on all 4×3 combinations of streams & nutrients.

Results - Four Mile Suspended Sediment

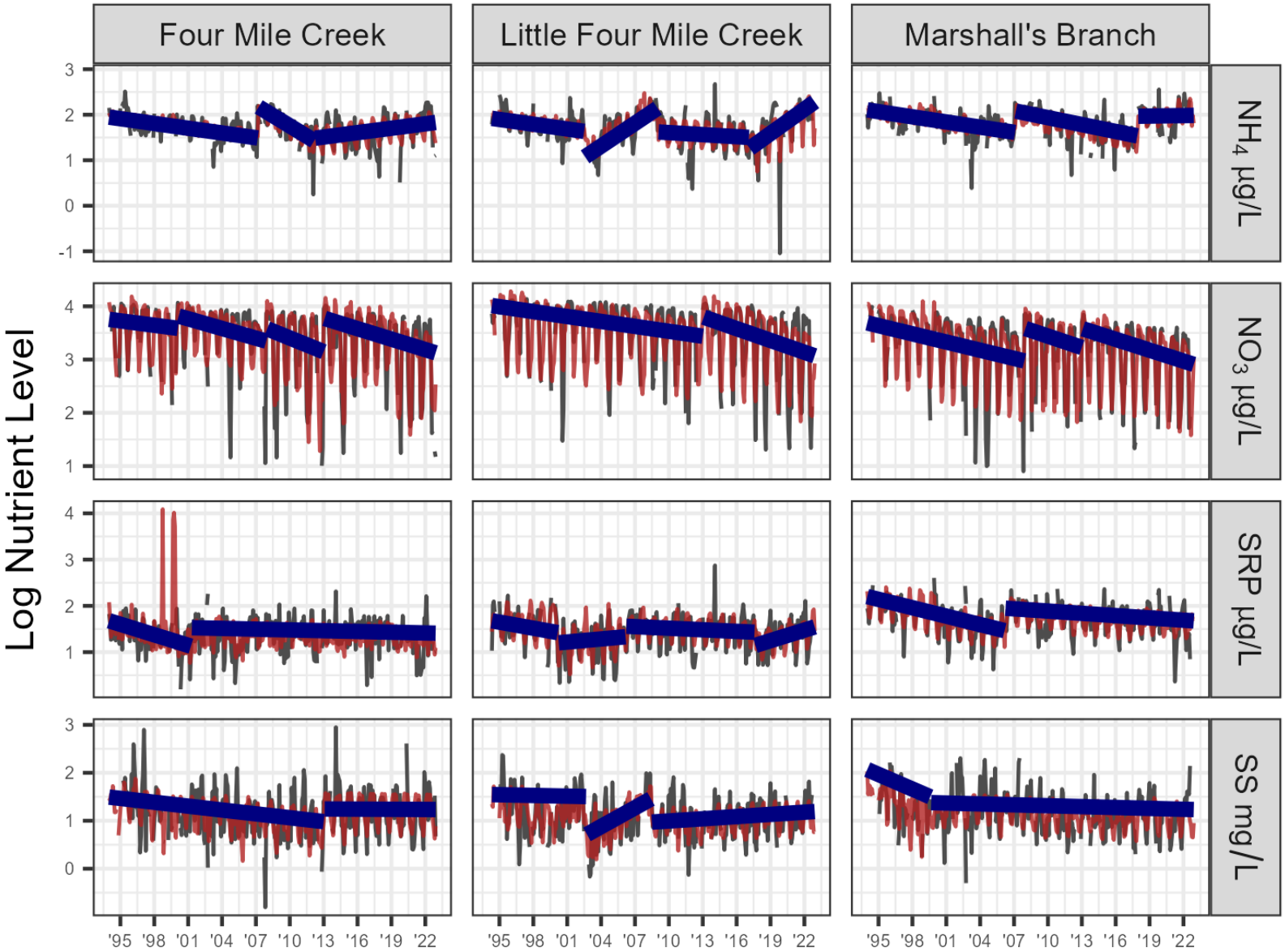
Original series

Fitted Model

Underlying Trend



Results - All stream-nutrient combinations



Conclusions

Some contextual conclusions

Ammonium (NH_4)

- Decreasing until ~2007, increasing since? (*squirrely*)

Nitrate (NO_3)

- Several decreasing regimes, with intermittent jumps

Soluble Reactive Phosphorus (SRP)

- Relatively stable after an initial decrease

Suspended Sediment (SS)

- Early decrease, now pretty stable

References

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Thanks!

Collaborator

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  Data Analyst at Progressive Insurance

Other contributors

- Dr. Mike Vanni - Ecologist
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  Department of Geography - Miami University
- Dr. Bill Renwick - Geographer
  Department of Geography - Miami University
- Dr. Emily Morris - Former Undergraduate Student
  Food & Drug Administration

Questions? Comments? Suggestions?

Shameless Self Promotion

Slides are available on my github site:
`https://tjfisher19.github.io/`

Github handle: **`tjfisher19`**

Code available: `https://github.com/tjfisher19`

Email: **`fishert4@miamioh.edu`**