Complex Network Stability: Exploring Comprehensive Anchored Edge s-Core

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Introduction. Graph is widely used for modelling complex data, especially in social network analysis. A well-known metric to measuring network cohesiveness is the k-core [1], which has recently been applied to maximise user engagement. Previous papers have addressed this by anchoring edges or nodes [2, 3, 4], but inconsistent budget allocation has limited their practicality when designing real-world solutions. In this paper, we focus on weighted graphs and propose an edge anchoring problem with efficient budget allocation based on edge weights. This proposed problem can be formulated as a comprehensive version of several existing problems, allowing us to overcome the previously mentioned limitations. We demonstrate that the proposed problem is NP-hard and APX-hard, and design a heuristic algorithm to solve it effectively.

Problem Statement. We utilise the s-core [5] as a fundamental metric to measure the cohesiveness in weighted non-simple graphs. Additionally, to model more complex networks, we define the Comprehensive Increased Edge s-core (CEsC) problem, which considers both lower and upper bounds on edge weights.

Problem definition 1. *(Comprehensive increased edge s-core (**CEsC**)). Given an undirected weighted non-simple graph $G = (V, E)$, a positive integer s and b, and a user-defined bound function $T(\cdot) : V \times V \rightarrow \mathcal{I}$, where $\mathcal{I}$ represent the set of all intervals, the CEsC problem is to find a set of pairs $A = \{\langle(u_1, v_1), \delta_1\rangle, \langle(u_2, v_2), \delta_2\rangle, \ldots, \langle(u_n, v_n), \delta_n\rangle\}$, where (u_i, v_i) is an edge with weight increased by a positive integer δ_i such that (1) Maximise the size of the comprehensive s-core limited on T, while anchoring A; and (2) The sum of all δ in A must be less than or equal to b (i.e., $\sum_{i=1}^{n} \delta_i \leq b$).*

We prove that existing methods [2, 3, 4] which operate with mutually inconsistent budget criteria, can be reduced to the CEsC problem, demonstrating that the CEsC problem serves as a broad and comprehensive framework that covers existing methods.

Example 1. *As illustrated in Figure 1, a budget of 1 is applied to increase the weight of (x_3, x_4) from 6 to 7 in order to enlarge the s-core ($s = 9$). This mechanism demonstrates how the size of the s-core can be increased through iterative budget allocations.*

Algorithm. Given that the CEsC problem is both NP-hard and APX-hard, we propose two heuristic algorithms that iteratively select the node pair for weight increase. We define *followers* as the set of nodes that are iteratively added to the s-core after an edge e is anchored. The first algorithm, referred to as the naïve algorithm, iteratively selects a set of edges to maximise the size of the s-core. To efficiently choose the edges, we employ the follower ratio (i.e., $|\text{followers}|/\delta$) to avoid dominated assignments. The primary bottleneck of the naive approach is the large number of candidate edges. In the advanced algorithm, we utilise the property that edges between followers contribute fewer additional followers than the candidate edge itself. Instead of directly calculating the s-core to determine the number of followers, we optimise this process by comparing the presence or absence of potential followers.

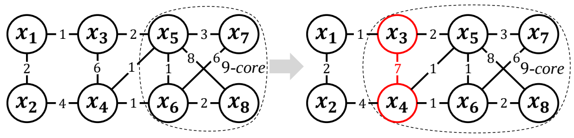

Figure 1: Toy example

Conclusion and Future work. In this work, we propose CEsC problem, which address efficient budget allocation for edge anchoring in weighted graphs. Our work provides a more comprehensive approach to previous methods aimed at maximising network engagement through edge and node anchoring. We propose two Greedy algorithms: naïve algorithm that considers all edges and advanced algorithm that reduces the candidate edge set by leveraging follower properties. The experimental results are expected to demonstrate the efficiency of the advanced algorithm. As future work, we aim to extend our approach to various networks such as heterogeneous information networks, hypergraphs, and uncertain graphs and diverse environment like dynamic networks.

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