

# ASPECT

Advanced Solver for Problems in Earth's ConvecTion

Preview release, version 0.8

Wolfgang Bangerth  
Thomas Geenen  
Timo Heister  
Martin Kronbichler

# Contents

<b>1</b>	<b>Introduction</b>	<b>4</b>
1.1	Referencing ASPECT	5
1.2	Acknowledgments	5
<b>2</b>	<b>Equations, models, coefficients</b>	<b>5</b>
2.1	Basic equations	5
2.2	Coefficients	7
2.3	Pressure normalization	8
2.4	Initial conditions and the adiabatic pressure/temperature	9
2.5	Numerical methods	9
2.6	Simplifications of the basic equations	10
2.6.1	The Boussinesq approximation: Incompressibility	10
2.6.2	Almost linear models	11
<b>3</b>	<b>Installation</b>	<b>12</b>
3.1	Prerequisites	12
3.2	Obtaining ASPECT and initial configuration	13
3.3	Compiling ASPECT and generating documentation	14
<b>4</b>	<b>Running Aspect</b>	<b>14</b>
4.1	Overview	14
4.2	Selecting between 2d and 3d runs	18
4.3	Debug or optimized mode	18
4.4	Visualizing results	18
4.4.1	Visualization the graphical output using <i>Visit</i>	19
4.4.2	Visualizing statistical data	21
4.4.3	Large data issues for parallel computations	21
4.5	Checkpoint/restart support	22
<b>5</b>	<b>Run-time input parameters</b>	<b>22</b>
5.1	Overview	22
5.1.1	The structure of parameter files	23
5.1.2	Categories of parameters	24
5.2	Global parameters	24
5.3	Parameters in section <code>Boundary temperature model</code>	26
5.4	Parameters in section <code>Boundary temperature model/Spherical constant</code>	26
5.5	Parameters in section <code>Discretization</code>	27
5.6	Parameters in section <code>Discretization/Stabilization parameters</code>	27
5.7	Parameters in section <code>Geometry model</code>	28
5.8	Parameters in section <code>Geometry model/Box</code>	28
5.9	Parameters in section <code>Geometry model/Spherical shell</code>	28
5.10	Parameters in section <code>Gravity model</code>	29
5.11	Parameters in section <code>Gravity model/Radial constant</code>	29
5.12	Parameters in section <code>Initial conditions</code>	29
5.13	Parameters in section <code>Initial conditions/Spherical gaussian perturbation</code>	30
5.14	Parameters in section <code>Material model</code>	31
5.15	Parameters in section <code>Material model/Simple model</code>	31
5.16	Parameters in section <code>Material model/Table model</code>	32
5.17	Parameters in section <code>Material model/Table model/Viscosity</code>	33
5.18	Parameters in section <code>Mesh refinement</code>	34

5.19	Parameters in section <code>Model settings</code>	35
5.20	Parameters in section <code>Postprocess</code>	36
5.21	Parameters in section <code>Postprocess/Depth average</code>	37
5.22	Parameters in section <code>Postprocess/Visualization</code>	37
<b>6</b>	<b>Cookbooks</b>	<b>38</b>
6.1	Geophysical setups	38
6.2	Benchmarks	38
6.2.1	The SolCx Stokes benchmark	38
6.2.2	The SolKz Stokes benchmark	42
6.2.3	The “inclusion” Stokes benchmark	43
<b>7</b>	<b>Extending Aspect</b>	<b>45</b>
7.1	Materials, geometries, gravitation and other aspects of the model: <code>plugins</code>	46
7.1.1	Material models	48
7.1.2	Geometry models	50
7.1.3	Gravity models	51
7.1.4	Initial conditions	51
7.1.5	Temperature boundary conditions	52
7.1.6	Postprocessors: Evaluating the solution after each time step	53
7.2	Extending the basic solver	55
<b>8</b>	<b>Future plans for Aspect</b>	<b>56</b>
<b>9</b>	<b>Finding answers to more questions</b>	<b>57</b>
	<b>References</b>	<b>58</b>
	<b>Index of run-time parameter entries</b>	<b>59</b>
	<b>Index of run-time parameters with section names</b>	<b>60</b>

# 1 Introduction

ASPECT — short for Advanced Solver for Problems in Earth’s ConvecTion — is a code intended to solve the equations that describe thermally driven convection with a focus on doing so in the context of convection in the earth mantle. It is primarily developed by computational scientists at Texas A&M University based on the following principles:

- *Usability and extensibility:* Simulating mantle convection is a difficult problem characterized not only by complicated and nonlinear material models but, more generally, by a lack of understanding which parts of a much more complicated model are really necessary to simulate the defining features of the problem. To name just a few examples:
  - Mantle convection is often solved in a spherical shell geometry, but the earth is not a sphere – its true shape on the longest lengthscales is dominated by polar oblateness, but deviations from spherical shape relevant to convection patterns may go down to the lengthscales of mountain belts, mid-ocean ridges or subduction trenches. Furthermore, processes outside the mantle like crustal depression during glaciations can change the geometry as well.
  - Rocks in the mantle flow on long time scales, but on shorter time scales they behave more like a visco-elasto-plastic material as the break and as their crystalline structure heals again. The mathematical models discussed in Section 2 can therefore only be approximations.
  - If pressures are low and temperatures high enough, rocks melt, leading to all sorts of new and interesting behavior.

This uncertainty in what problem one actually wants to solve requires a code that is easy to extend by users to support the community in determining what the essential features of convection in the earth mantle are. Achieving this goal also opens up possibilities outside the original scope, such as the simulation of convection in exoplanets or the icy satellites of the gas giant planets in our solar system.

- *Modern numerical methods:* We build ASPECT on numerical methods that are at the forefront of research in all areas – adaptive mesh refinement, linear and nonlinear solvers, stabilization of transport-dominated processes. This implies complexity in our algorithms, but also guarantees highly accurate solutions while remaining efficient in the number of unknowns and with CPU and memory resources.
- *Parallelism:* Many convection processes of interest are characterized by small features in large domains – for example, mantle plumes of a few tens of kilometers diameter in a mantle almost 3,000 km deep. Such problems can not be solved on a single computer but require dozens or hundreds of processors to work together. ASPECT is designed from the start to support this level of parallelism.
- *Building on others’ work:* Building a code that satisfies above criteria from scratch would likely require several 100,000 lines of code. This is outside what any one group can achieve on academic time scales. Fortunately, most of the functionality we need is already available in the form of widely used, actively maintained, and well tested and documented libraries, and we leverage these to make ASPECT a much smaller and easier to understand system. Specifically, ASPECT builds immediately on top of the DEAL.II library (see <http://www.dealii.org/>) for everything that has to do with finite elements, geometries, meshes, etc.; and, through DEAL.II on Trilinos (see <http://trilinos.sandia.gov/>) for parallel linear algebra and on P4EST (see <http://www.p4est.org/>) for parallel mesh handling.

Combining all of these aspects into one code makes for an interesting challenge. We hope to have achieved our goal of providing a useful tool to the geodynamics community and beyond!

**Note:** ASPECT is a community project. As such, we encourage contributions from the community to improve this code over time. Natural candidates for such contributions are implementations of new plugins as discussed in Section 7.1 since they are typically self-contained and do not require much knowledge of the details of the remaining code. Obviously, however, we also encourage contributions to the core functionality in any form!

**Note:** ASPECT will only solve problems relevant to the community if we get feedback from the community on things that are missing or necessary for what you want to do. Let us know by personal email to the developers, or the mantle convection or `aspect-devel` mailing lists hosted at CIG!

## 1.1 Referencing Aspect

As with all scientific work, funding agencies have a reasonable expectation that if we ask for continued funding for this work, we need to demonstrate relevance. To this end, we ask that if you publish results that were obtained to some part using ASPECT, you cite the following, canonical reference for this software:

```
@Article{KHB12,
  author = {M. Kronbichler and T. Heister and W. Bangerth},
  title = {High Accuracy Mantle Convection Simulation through
    Modern Numerical Methods},
  journal = {submitted},
  year = {2011}
}
```

## 1.2 Acknowledgments

The development of ASPECT has been funded through a variety of grants to the authors. Most immediately, it has been supported through the Computational Infrastructure in Geodynamics (CIG-II) grant (National Science Foundation Award No. EAR-0949446, via The University of California – Davis) but the initial portions have also been supported by the original CIG grant (National Science Foundation Award No. EAR-0426271, via The California Institute of Technology). In addition, the libraries upon which ASPECT builds heavily have been supported through many other grants that are equally gratefully acknowledged.

# 2 Equations, models, coefficients

## 2.1 Basic equations

ASPECT solves the a system of equations in a  $d = 2$ - or  $d = 3$ -dimensional domain  $\Omega$  that describes the motion of a highly viscous fluid driven by differences in the gravitational force due to a density that depends on the temperature. In the following, we largely follow the exposition of this material in Schubert, Turcotte and Olson [STO01].

Specifically, we consider the following set of equations for velocity  $\mathbf{u}$ , pressure  $p$  and temperature  $T$ :

$$-\nabla \cdot \left[ 2\eta \left( \varepsilon(\mathbf{u}) - \frac{1}{3}(\nabla \cdot \mathbf{u})\mathbf{1} \right) \right] + \nabla p = \rho \mathbf{g} \quad \text{in } \Omega, \quad (1)$$

$$\nabla \cdot (\rho \mathbf{u}) = 0 \quad \text{in } \Omega, \quad (2)$$

$$\begin{aligned} \rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) - \nabla \cdot k \nabla T = \rho H \\ + 2\eta \left( \varepsilon(\mathbf{u}) - \frac{1}{3}(\nabla \cdot \mathbf{u})\mathbf{1} \right) : \left( \varepsilon(\mathbf{u}) - \frac{1}{3}(\nabla \cdot \mathbf{u})\mathbf{1} \right) \\ + \frac{\partial \rho}{\partial T} T \mathbf{u} \cdot \mathbf{g} \quad \text{in } \Omega, \end{aligned} \quad (3)$$

where  $\varepsilon(\mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$  is the symmetric gradient of the velocity (often called the *strain rate*).

To be finished  
Wouldn't the  
last term need  
to have a  
minus sign  
drho/dT i  
negative...

In this set of equations, (1) and (2) represent the compressible Stokes equations in which  $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$  is the velocity field and  $p = p(\mathbf{x}, t)$  the pressure field. Both fields depend on space  $\mathbf{x}$  and time  $t$ . Fluid flow is driven by the gravity force that acts on the fluid and that is proportional to both the density of the fluid and the strength of the gravitational pull.

Coupled to this Stokes system is equation (3) for the temperature field  $T = T(\mathbf{x}, t)$  that contains heat conduction terms as well as advection with the flow velocity  $\mathbf{u}$ . The right hand side terms of this equation correspond to

- internal heat production for example due to radioactive decay;
- friction heating;
- adiabatic compression of material; as written, this term assumes that the the overall pressure is dominated by the hydrostatic pressure, in which case the variation of the total pressure can be expressed by gravity and density.

The equations we ASPECT currently solves do not include phase change terms, see Section 8.

These equations are augmented by boundary conditions that can either be of Dirichlet-, Neumann, or tangential type on subsets of the boundary  $\Gamma = \partial\Omega$ :

$$\mathbf{u} = 0 \quad \text{on } \Gamma_{0,\mathbf{u}}, \quad (4)$$

$$\mathbf{n} \cdot \mathbf{u} = 0 \quad \text{on } \Gamma_{\parallel,\mathbf{u}}, \quad (5)$$

$$T = T_{\text{prescribed}} \quad \text{on } \Gamma_{D,T}, \quad (6)$$

$$\mathbf{n} \cdot k \nabla T = 0 \quad \text{on } \Gamma_{N,T}. \quad (7)$$

Here,  $\Gamma_{0,\mathbf{u}}$  corresponds to parts of the boundary on which the velocity is fixed to be zero,  $\Gamma_{\parallel,\mathbf{u}}$  to parts of the boundary on which the velocity may be nonzero but must be parallel to the boundary,  $\Gamma_{D,T}$  to places where the temperature is prescribed (for example at the inner and outer boundaries of the earth mantle), and finally  $\Gamma_{N,T}$  to places where the temperature is unknown but the heat flux across the boundary is zero (for example on symmetry surfaces if only a part of the shell that constitutes the domain the Earth mantle occupies is simulated). We require that one of these boundary conditions hold at each point for both velocity and temperature, i.e.,  $\Gamma_{0,\mathbf{u}} \cup \Gamma_{\parallel,\mathbf{u}} = \Gamma$  and  $\Gamma_{D,T} \cup \Gamma_{N,T} = \Gamma$ .

ASPECT solves these equations in essentially the form stated. In particular, the form given in (1) implies that the pressure  $p$  we compute is in fact the *total pressure*, i.e., the sum of hydrostatic pressure and dynamic pressure.<sup>1</sup> Consequently, it allows the direct use of this pressure when looking up pressure dependent material parameters.

<sup>1</sup>Other codes often replace this equation by  $-\nabla \cdot 2\eta \nabla \mathbf{u} + \nabla p_d = (\rho - \rho_0)\mathbf{g}$  where  $p_d = (p + \rho_0\phi)$ , and  $\phi$  is the gravitational potential so that  $\mathbf{g} = -\nabla\phi$  and chosen in such a way that  $\phi = 0$  at that part of the boundary where we want the pressure to be zero (e.g., on the earth surface). Furthermore,  $\rho_0$  is a reference density. In this formulation, it is clear that the quantity that drives the fluid flow is in fact the *buoyancy* caused by the *variation* of densities, not the density itself.  $p_d = p + \rho_0\phi$  is then the *dynamic* pressure, i.e., the difference between total pressure and hydrostatic pressure  $p_s = -\rho_0\phi$ .

While this formulation has a number of numerical advantages, it also has significant disadvantages: (i) The pressure we compute is not immediately comparable to quantities that we need to look up pressure-dependent quantities such as the density. (ii) The definition of a reference density is only simple if we have incompressible models for which the density only depends on the temperature; for more complicated models, it is not a priori clear which density  $\rho_0$  to chose so that  $p + \rho_0\phi$  really only contains the dynamic part of the pressure. (iii) To compute the total pressure  $p$  from  $p_d$  one needs to know a gravitational potential  $\phi$  that is consistent with the gravity vector  $\mathbf{g}$ . This is not always trivial because many simple models just prescribe a  $\mathbf{g}$  for which no such potential needs to exist; for example, this is the case when using a radially inward gravitational vector of constant magnitude.

**Note:** In reality, ASPECT has no preferred system of units as long as every material constant, geometry, time, etc., is all expressed in the same system. However, all existing implementations of models uses the SI system, i.e., they express everything in meters, kilograms and seconds – the MKS system –, as well as degrees Kelvin. It is therefore convenient to think that ASPECT actually requires the use of these units throughout and that all material parameters, geometries, etc., must also be expressed in these units, although nothing prevents a user from using a domain that has unit length, a viscosity value of one, a density with reference value of zero that can be either positive or negative depending on whether the temperature is positive or negative, etc, to achieve a model description that is exactly equal to what one would do when posing the model in non-dimensionalized form.

All this said, for convenience, output quantities are sometimes provided in units meters per *year* instead of meters per *second* (velocities) or in *years* instead of *seconds* (the current time, the time step size); however, this conversion happens at the time output is generated, and is not part of the solution process. Whether this conversion should happen is determined by the flag “Use years in output instead of seconds” in the input file, see Section 5.2.

## 2.2 Coefficients

The equations above contain a significant number of coefficients that we will discuss in the following. In the most general form, many of these coefficients depend nonlinearly on the solution variables pressure  $p$ , temperature  $T$  and, in the case of the viscosity, on the strain rate  $\varepsilon(\mathbf{u})$ . Alternatively, they may be parameterized as a function of the spatial variable  $\mathbf{x}$ . ASPECT allows both kinds of parameterizations.

**Note:** One of the next versions of ASPECT will actually iterate out nonlinearities in the material description. However, in the current version, we simply evaluate all nonlinear dependence of coefficients at the solution variables from the previous time step or a solution suitably extrapolated from the previous time steps.

Note that below we will discuss examples of the dependence of coefficients on other quantities; which dependence is actually implemented in the code is a different matter. As we will discuss in Section 5 and 7, some versions of these models are already implemented and can be selected from the input parameter file; others are easy to add to ASPECT by providing self-contained descriptions of a set of coefficients that the rest of the code can then use without a need for further modifications.

Concretely, we consider the following coefficients and dependencies:

- *The viscosity*  $\eta = \eta(p, T, \varepsilon(\mathbf{u}), \mathbf{x})$ : Units  $\text{Pa} \cdot \text{s} = \text{kg} \frac{1}{\text{m} \cdot \text{s}}$ .

The viscosity is the proportionality factor that relates total forces (external gravity minus pressure gradients) and fluid velocities  $\mathbf{u}$ . The simplest models assume that  $\eta$  is constant, with the constant often chosen to be on the order of  $10^{21} \text{Pa s}$ .

More complex (and more realistic) models assume that the viscosity depends on pressure, temperature and strain rate. Since this dependence is often difficult to quantify, one modeling approach is to make  $\eta$  spatially dependent.

- *The density*  $\rho = \rho(p, T, \mathbf{x})$ : Units  $\frac{\text{kg}}{\text{m}^3}$ .

In general, the density depends on pressure and temperature, both through pressure compression, thermal expansion, and phase changes the material may undergo as it moves through the pressure-temperature phase diagram.

The simplest parameterization for the density is to assume a linear dependence on temperature, yielding the form  $\rho(T) = \rho_{\text{ref}}[1 - \beta(T - T_{\text{ref}})]$  where  $\rho_{\text{ref}}$  is the reference density at temperature  $T_{\text{ref}}$  and  $\beta$  is the linear thermal expansion coefficient. For the earth mantle, typical values for this parameterization would be  $\rho_{\text{ref}} = 3300 \frac{\text{kg}}{\text{m}^3}$ ,  $T_{\text{ref}} = 293\text{K}$ ,  $\beta = 2 \cdot 10^{-5} \frac{1}{\text{K}}$ .



- *The gravity vector  $\mathbf{g} = \mathbf{g}(\mathbf{x})$* : Units  $\frac{\text{m}}{\text{s}^2}$ .

Simple models assume a radially inward gravity vector of constant magnitude (e.g., the surface gravity of Earth,  $9.81 \frac{\text{m}}{\text{s}^2}$ ), or one that can be computed analytically assuming a homogenous mantle density.

A physically self-consistent model would compute the gravity vector as  $\mathbf{g} = -\nabla\varphi$  with a gravity potential  $\varphi$  that satisfies  $-\Delta\varphi = 4\pi G\rho$  with the density  $\rho$  from above and  $G$  the universal constant of gravity. This would provide a gravity vector that changes as a function of time. Such a model is not currently implemented.

- *The specific heat capacity  $C_p = C_p(p, T, \mathbf{x})$* : Units  $\frac{\text{J}}{\text{kg}\cdot\text{K}} = \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}$ .

The specific heat capacity denotes the amount of energy needed to increase the temperature of one kilogram of material by one degree. Wikipedia lists a value of  $790 \frac{\text{J}}{\text{kg}\cdot\text{K}}$  for granite<sup>2</sup> For the earth mantle, a value of  $1250 \frac{\text{J}}{\text{kg}\cdot\text{K}}$  is within the range suggested by the literature.

- *The thermal conductivity  $k = k(p, T, \mathbf{x})$* : Units  $\frac{\text{W}}{\text{m}\cdot\text{K}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^3\cdot\text{K}}$ .

The thermal conductivity denotes the amount of thermal energy flowing through a unit area for a given temperature gradient. It depends on the material and as such will from a physical perspective depend on pressure and temperature due to phase changes of the material as well as through different mechanisms for heat transport (see, for example, the partial transparency of perovskite, the most abundant material in the earth mantle, at pressures above around 120 GPa [BRV<sup>+</sup>04]).

As a rule of thumb for its order of magnitude, wikipedia quotes values of  $1.83\text{--}2.90 \frac{\text{W}}{\text{m}\cdot\text{K}}$  for sandstone and  $1.73\text{--}3.98 \frac{\text{W}}{\text{m}\cdot\text{K}}$  for granite.<sup>3</sup> The values in the mantle are almost certainly higher than this though probably not by much. The exact exact value is not really all that important: heat transport through convection is several orders of magnitude more important than through thermal conduction.

- *The intrinsic specific heat production  $H = H(\mathbf{x})$* : Units  $\frac{\text{W}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^3}$ .

This term denotes the intrinsic heating of the material, for example due to the decay of radioactive material. As such, it depends not on pressure or temperature, but may depend on the location due to different chemical composition of material in the earth mantle. The literature suggests a value of  $\gamma = 7.4 \cdot 10^{-12} \frac{\text{W}}{\text{kg}}$ .

## 2.3 Pressure normalization

The equations described above, (1)–(3), only determine the pressure  $p$  up to an additive constant. On the other hand, since the pressure appears in the definition of many of the coefficients, we need a pressure that has some sort of *absolute* definition. A physically useful definition would be to normalize the pressure in such a way that the average pressure along the “surface” has a prescribed value where the geometry description (see Section 7.1.2) has to determine which part of the boundary of the domain is the “surface” (we call a part of the boundary the “surface” if its depth is “close to zero”).

Typically, one will choose this average pressure to be zero, but there is a parameter “**Surface pressure**” in the input file (see Section 5.2) to set it to a different value. One may want to do that, for example, if one wants to simulate the earth mantle without the overlying lithosphere. In that case, the “surface” would be the interface between mantle and lithosphere, and the average pressure at the surface to which the solution of the equations will be normalized should in this case be the hydrostatic pressure at the bottom of the lithosphere.

An alternative is to normalize the pressure in such a way that the *average* pressure throughout the domain is zero or some constant value. This is not a useful approach for most geodynamics applications but is common in benchmarks for which analytic solutions are available. Which kind of normalization is chosen is determined by the “**Pressure normalization**” flag in the input file, see Section 5.2.

<sup>2</sup>See [http://en.wikipedia.org/wiki/Specific\\_heat](http://en.wikipedia.org/wiki/Specific_heat).

<sup>3</sup>See [http://en.wikipedia.org/wiki/Thermal\\_conductivity](http://en.wikipedia.org/wiki/Thermal_conductivity) and [http://en.wikipedia.org/wiki/List\\_of\\_thermal\\_conductivities](http://en.wikipedia.org/wiki/List_of_thermal_conductivities).



## 2.4 Initial conditions and the adiabatic pressure/temperature

Equations (1)–(3) require us to pose initial conditions for the temperature, and this is done by selecting one of the existing models for initial conditions in the input parameter file, see Section 5.12. The equations themselves do not require that initial conditions are specified for the velocity and pressure variables (since there are no time derivatives on these variables in the model).

Nevertheless, a nonlinear solver will have difficulty converging to the correct solution if we start with a completely unphysical pressure for models in which coefficients such as density  $\rho$  and viscosity  $\eta$  depend on the pressure and temperature. To this end, ASPECT computes pressure and temperature fields  $p_{\text{ad}}(z), T_{\text{ad}}(z)$  that satisfy adiabatic conditions:

$$\rho C_p \frac{d}{dz} T_{\text{ad}}(z) = \frac{\partial \rho}{\partial T} T_{\text{ad}}(z) g_z, \quad (8)$$

$$\frac{d}{dz} p_{\text{ad}}(z) = \rho g_z, \quad (9)$$

where strictly speaking  $g_z$  is the magnitude of the vertical component of the gravity vector field, but in practice we take the magnitude of the entire gravity vector.

These equations can be integrated numerically starting at  $z = 0$ , using the depth dependent gravity field and values of the coefficients  $\rho = \rho(p, T, z), C_p = C_p(p, T, z)$ . As starting conditions at  $z = 0$  we choose a pressure  $p_{\text{ad}}(0)$  equal to the average surface pressure (often chosen to be zero, see Section 2.3), and an adiabatic surface temperature  $T_{\text{ad}}(0)$  that is also selected in the input parameter file.

**Note:** The adiabatic surface temperature is often chosen significantly higher than the actual surface temperature. For example, on earth, the actual surface temperature is on the order of 290 K, whereas a reasonable adiabatic surface temperature is maybe 1200 K. The reason is that the bulk of the mantle is more or less in thermal equilibrium with a thermal profile that corresponds to the latter temperature, whereas the very low actual surface temperature and the very high bottom temperature at the core-mantle boundary simply induce a thermal boundary layer. Since the temperature and pressure profile we compute using the equations above are simply meant to be good starting points for nonlinear solvers, it is important to choose this profile in such a way that it covers most of the mantle well; choosing an adiabatic surface temperature of 290 K would yield a temperature and pressure profile that is wrong almost throughout the entire mantle.

## 2.5 Numerical methods

There is no shortage in the literature for methods to solve the equations outlined above. The methods used by ASPECT use the following, interconnected set of strategies in the implementation of numerical algorithms:

- *Mesh adaptation:* Mantle convection problems are characterized by widely disparate length scales (from plate boundaries on the order of kilometers or even smaller, to the scale of the entire earth). Uniform meshes can not resolve the smallest length scale without an intractable number of unknowns. Fully adaptive meshes allow resolving local features of the flow field without the need to refine the mesh globally. Since the location of plumes that require high resolution change and move with time, meshes also need to be adapted every few time steps.
- *Accurate discretizations:* The Boussinesq problem upon which most models for the earth mantle are based has a number of intricacies that make the choice of discretization non-trivial. In particular, the finite elements chosen for velocity and pressure need to satisfy the usual compatibility condition for saddle point problems. This can be worked around using pressure stabilization schemes for low-order discretizations, but high-order methods can yield better accuracy with fewer unknowns and offer more reliability. Equally important is the choice of a stabilization method for the highly advection-dominated temperature equation. ASPECT uses a nonlinear artificial diffusion method for the latter.

- *Efficient linear solvers:* The major obstacle in solving the Boussinesq system is the saddle-point nature of the Stokes equations. Simple linear solvers and preconditioners can not efficiently solve this system in the presence of strong heterogeneities or when the size of the system becomes very large. ASPECT uses an efficient solution strategy based on a block triangular preconditioner utilizing an algebraic multigrid that provides optimal complexity even up to problems with hundreds of millions of unknowns.
- *Parallelization of all of the steps above:* Global mantle convection problems frequently require extremely large numbers of unknowns for adequate resolution in three dimensional simulations. The only realistic way to solve such problems lies in parallelizing computations over hundreds or thousands of processors. This is made more complicated by the use of dynamically changing meshes, and it needs to take into account that we want to retain the optimal complexity of linear solvers and all other operations in the program.
- *Modularity of the code:* A code that implements all of these methods from *scratch* will be unwieldy, unreadable and unusable as a community resource. To avoid this, we build our implementation on widely used and well tested libraries that can provide researchers interested in extending it with the support of a large user community. Specifically, we use the DEAL.II library [BHK07, BHK12] for meshes, finite elements and everything discretization related; the TRILINOS library [HBH<sup>+</sup>05, H<sup>+</sup>11] for scalable and parallel linear algebra; and P4EST [BWG11] for distributed, adaptive meshes. As a consequence, our code is freed of the mundane tasks of defining finite element shape functions or dealing with the data structures of linear algebra, can focus on the high-level description of what is supposed to happen, and remains relatively compact. The code will also automatically benefit from improvements to the underlying libraries with their much larger development communities. ASPECT is extensively documented to enable other researchers to understand, test, use, and extend it.

Rather than detailing the various techniques upon which ASPECT is built, we refer to the paper by Kronbichler, Heister and Bangerth [KHB12] that gives a detailed description and rationale for the various building blocks.

## 2.6 Simplifications of the basic equations

There are two common variations to equations (1)–(3) that are frequently used and that make the system much simpler to solve and analyze: assuming that the fluid is incompressible (the Boussinesq approximation) and a linear dependence of the density on the temperature with constants that are otherwise independent of the solution variables. These are discussed in the following; ASPECT has run-time parameters that allow both of these simpler models to be used.

### 2.6.1 The Boussinesq approximation: Incompressibility

The original Boussinesq approximation assumes that the density can be considered constant in all occurrences in the equations with the exception of the buoyancy term on the right hand side of (1). The primary result of this assumption is that the continuity equation (2) will now read

$$\nabla \cdot \mathbf{u} = 0.$$

This makes the equations *much* simpler to solve: First, because the divergence operation in this equation is the transpose of the gradient of the pressure in the momentum equation (1), making the system of these two equations symmetric. And secondly, because the two equations are now linear in pressure and velocity (assuming that the viscosity  $\eta$  and the density  $\rho$  are considered fixed). In addition, one can drop all terms involving  $\nabla \cdot \mathbf{u}$  from the left hand side of the momentum equation (1) as well as from the shear heating term on the right hand side of (3); while dropping these terms does not affect the solution of the equations, it makes assembly of linear systems faster. In addition, in the incompressible case, one needs to neglect the adiabatic heating term  $\frac{\partial \rho}{\partial T} T \mathbf{u} \cdot \mathbf{g}$  on the right hand side of (3).

From a physical perspective, the assumption that the density is constant in the continuity equation but variable in the momentum equation is of course inconsistent. However, it is justified if the variation is small since the momentum equation can be rewritten to read

$$-\nabla \cdot 2\eta\varepsilon(\mathbf{u}) + \nabla p_d = (\rho - \rho_0)\mathbf{g},$$

where  $p_d$  is the *dynamic* pressure and  $\rho_0$  is the constant reference density. This makes it clear that the true driver of motion is in fact the *deviation* of the density from its background value, however small this value is: the resulting velocities are simply proportional to the density variation, not to the absolute magnitude of the density.

As such, the Boussinesq approximation can be justified. On the other hand, given the real pressures and temperatures at the bottom of the earth mantle, it is arguable whether the density can be considered to be almost constant. Most realistic models predict that the density of mantle rocks increases from somewhere around 3300 at the surface to over 5000 kilogram per cubic meters at the core mantle boundary, due to the increasing lithostatic pressure. While this appears to be a large variability, if the density changes slowly with depth, this is not in itself an indication that the Boussinesq approximation will be wrong. To this end, consider that the continuity equation can be rewritten as  $\frac{1}{\rho}\nabla \cdot (\rho\mathbf{u}) = 0$ , which we can multiply out to obtain

$$\nabla \cdot \mathbf{u} + \frac{1}{\rho}\mathbf{u} \cdot \nabla \rho = 0.$$

The question whether the Boussinesq approximation is valid is then whether the second term (the one omitted in the Boussinesq model) is small compared to the first. To this end, consider that the velocity can change completely over length scales of maybe 10 km, so that  $\nabla \cdot \mathbf{u} \approx \|u\|/10\text{km}$ . On the other hand, given a smooth dependence of density on pressure, the length scale for variation of the density is the entire earth mantle, i.e.,  $\frac{1}{\rho}\nabla \mathbf{u} \cdot \rho \approx \|u\|0.5/3000\text{km}$  (given a variation between minimal and maximal density of 0.5 times the density itself). In other words, for a smooth variation, the contribution of the compressibility to the continuity equation is very small. This may be different, however, for models in which the density changes rather abruptly, for example due to phase changes at mantle discontinuities.

In summary, models that use the approximation of incompressibility solve the following set of equations instead of (1)–(3):

$$-\nabla \cdot [2\eta\varepsilon(\mathbf{u})] + \nabla p = \rho\mathbf{g} \quad \text{in } \Omega, \quad (10)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega, \quad (11)$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) - \nabla \cdot k \nabla T = \rho H + 2\eta\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) \quad \text{in } \Omega, \quad (12)$$

where the coefficients  $\eta, \rho, \mathbf{g}, C_p$  may possibly depend on the solution variables.

**Note:** As we will see in Section 7, it is easy to add new material models to ASPECT. Each model can decide whether it wants to use the Boussinesq approximation or not. The description of the models in Section 5.14 also gives an answer which of the models already implemented uses the approximation or considers the material sufficiently compressible to go with the fully compressible continuity equation.

### 2.6.2 Almost linear models

A further simplification can be obtained if one assumes that all coefficients with the exception of the density do not depend on the solution variables but are, in fact, constant. In such models, one typically assumes that the density satisfies a relationship of the form  $\rho = \rho(T) = \rho_0(1 - \beta(T - T_0))$  with a small thermal expansion

coefficient  $\beta$  and a reference density  $\rho_0$  that is attained at temperature  $T_0$ . Since the thermal expansion is considered small, this naturally leads to the following variant of the Boussinesq model discussed above:

$$\begin{aligned} -\nabla \cdot [2\eta\varepsilon(\mathbf{u})] + \nabla p &= \rho_0(1 - \beta(T - T_0))\mathbf{g} && \text{in } \Omega, \\ \nabla \cdot \mathbf{u} &= 0 && \text{in } \Omega, \\ \rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) - \nabla \cdot k \nabla T &= \rho H + 2\eta\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) && \text{in } \Omega, \end{aligned}$$

If the gravitational acceleration  $\mathbf{g}$  results from a gravity potential  $\varphi$  via  $\mathbf{g} = -\nabla\varphi$ , then one can rewrite the equations above in the following, commonly used form:<sup>4</sup>

$$-\nabla \cdot [2\eta\varepsilon(\mathbf{u})] + \nabla p_d = -\beta\rho_0 T \mathbf{g} \quad \text{in } \Omega, \quad (13)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega, \quad (14)$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) - \nabla \cdot k \nabla T = \rho H + 2\eta\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) \quad \text{in } \Omega, \quad (15)$$

where  $p_d = p + \rho_0(1 + \beta T_0)\varphi$  is the dynamic pressure, as opposed to the total pressure  $p = p_d + p_s$  that also includes the hydrastatic pressure  $p_s = -\rho_0(1 + \beta T_0)\varphi$ . Note that the right hand side forcing term in (13) is now only the deviation of the gravitational force from the force that would act if the material were at temperature  $T_0$ .

Under the assumption that all other coefficients are constant, one then arrives at equations in which the only nonlinear terms are the advection term,  $\mathbf{u} \cdot \nabla T$ , and the shear friction,  $2\eta\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u})$ , in the temperature equation (15). This facilitates the use of a particular class of time stepping scheme in which one does not solve the whole set of equations at once, iterating out nonlinearities as necessary, but instead in each time step solves first the Stokes system with the previous time step's temperature, and then uses the so-computed velocity to solve the temperature equation. These kind of time stepping schemes are often referred to as *IMPES* methods (they originate in the porous media flow community, where the acronym stands for *Implicit Pressure, Explicit Saturation*). For details see [KHB12].

**Note:** In ASPECT 0.1, using the IMPES scheme is the only available option. However, in later versions we will implement a fully nonlinear scheme that treats the equations as coupled, and one will be able to choose between the two variants using a run-time parameter.

## 3 Installation

This is a brief explanation of how to install all the required software and ASPECT itself.

### 3.1 Prerequisites

ASPECT builds on a few other libraries that are widely used in the computational science area and that provide most of the lower-level functionality such as finite element descriptions or parallel linear algebra. Specifically, it builds on DEAL.II which in turn uses Trilinos and P4EST. These need to be installed first before you can compile and run ASPECT. All of these libraries can readily be installed in a user's home directory, without the need to modify the overall system directories.

The following steps should guide you through the installation of these prerequisites:

<sup>4</sup>Note, however, that ASPECT does not solve the equations in the form given in (13)–(15). Rather, it takes the original form with the real density, not the variation of the density. That said, you can use the formulation (13)–(15) by implementing a material model (see Section 7.1.1) in which the density in fact has the form  $\rho(T) = \beta\rho_0 T$  even though this is not physical.

1. *Trilinos*: Trilinos can be downloaded from <http://trilinos.sandia.gov/>. At the current time we recommend Trilinos Version 10.4.x.<sup>5</sup> For installation instructions see [the deal.II README file on installing Trilinos and PETSc](#). Note that you have to configure with MPI by using

```
TPL_ENABLE_MPI:BOOL=ON
```

in the call to `cmake`. After that, run `make install`.

2. *P4EST*: Download and install P4EST as described in the [deal.II p4est installation instructions](#). This is done using the `p4est-setup.sh`; do not use the P4EST stand-alone installation instructions.
3. *DEAL.II*: The current version of ASPECT requires features of DEAL.II that have been developed after the 7.1 release. Since at the time of writing, DEAL.II 7.2 has not yet appeared, we currently require the development version of DEAL.II, which can be obtained by running

```
svn checkout http://www.dealii.org/svn/dealii/trunk/deal.II
```

Once DEAL.II 7.2 is available, this will suffice as well.

You may want to set the environment variable<sup>6</sup> `DEAL_DIR` to the directory where you checked out DEAL.II.

4. *Configuring and compiling DEAL.II*: Now it is time to configure DEAL.II. To this end, follow the [DEAL.II installation instructions](#). Remember to point the `./configure` script to the paths where you installed p4est and Trilinos and make sure you use MPI compilers. A typical command line would look like this:

```
./configure CXX=mpicxx --enable-mpi --disable-threads \  
--with-trilinos=/u/username/bin/trilinos-10.4.2 \  
--with-p4est=/u/username/bin/p4est-0.3.3.8
```

if the Trilinos and P4EST packages have been installed in the subdirectory `/u/username/bin`. Make sure the configuration succeeds and detects the MPI compilers correctly. For more information see the documentation of DEAL.II.

Now you are ready to compile DEAL.II by running `make all`. If you have multiple processor cores, feel free to do `make all -jN` where `N` is the number of processors in your machine to accelerate the process.

5. *Testing your installation*: Test that your installation works by running the `step-32` example that you can find in `$DEAL_DIR/examples/step-32`. Compile by running `make` and run with `mpirun -n 2 ./step-32`.

## 3.2 Obtaining Aspect and initial configuration

The development version of ASPECT can be downloaded by executing the command

```
svn checkout http://dealii.org/svn/aspect/trunk/aspect
```

If `$DEAL_DIR` points to your DEAL.II installation, there is no further configuration that needs to be done, otherwise you need to edit the `Makefile` accordingly.

---

<sup>5</sup>There are newer versions of Trilinos, but at least Trilinos 10.6.x and 10.8.x have bugs that make these versions unusable for our purpose.

<sup>6</sup>For bash this would be adding the line `export DEAL_DIR=/path/to/dealii/` to the file `~/.bashrc`.

### 3.3 Compiling Aspect and generating documentation

After downloading ASPECT and having built the libraries it builds on, you can compile it by typing

```
make
```

on the command line (or `make -jN` if you have multiple processors in your machine, where `N` is the number of processors). This builds the ASPECT executable which will reside in the `lib/` subdirectory and will be named `lib/aspect-2d` or `lib/aspect-3d`, depending on the space dimension you compile for (see Section 4.2 for more information on this). If you intend to modify ASPECT for your own experiments, you may want to also generate documentation about the source code. This can be done using the command

```
make doc
```

which assumes that you have the `doxygen` documentation generation tool installed. Most linux distributions have packages for `doxygen`. The result will be the file `doc/doxygen/index.html` that is the starting point for exploring the documentation.

## 4 Running Aspect

### 4.1 Overview

After compiling ASPECT as described above, you should have an executable file in the `lib/` subdirectory. It can be called as follows:

```
./lib/aspect parameter-file.prm
```

or, if you want to run the program in parallel, using something like

```
mpirun -np 32 ./lib/aspect parameter-file.prm
```

to run with 32 processors. In either case, the argument denotes the (path and) name of a file that contains input parameters. When you download ASPECT, you should already have a sample input file in the top-level directory that gives you an idea of the parameters that can be set. A full description of all parameters is given in Section 5.

Running the program should produce output that will look something like this (numbers will all be different, of course):

```
Number of active cells: 1,536 (on 5 levels)
Number of degrees of freedom: 20,756 (12,738+1,649+6,369)

*** Timestep 0: t=0 years

    Rebuilding Stokes preconditioner...
    Solving Stokes system... 30+3 iterations.
    Solving temperature system... 8 iterations.

Number of active cells: 2,379 (on 6 levels)
Number of degrees of freedom: 33,859 (20,786+2,680+10,393)

*** Timestep 0: t=0 years

    Rebuilding Stokes preconditioner...
    Solving Stokes system... 30+4 iterations.
    Solving temperature system... 8 iterations.

Postprocessing:
  Writing graphical output: output/solution-00000
  RMS, max velocity:      0.0946 cm/year, 0.183 cm/year
  Temperature min/avg/max: 300 K, 3007 K, 6300 K
```

```

    Inner/outer heat fluxes: 1.076e+05 W, 1.967e+05 W

*** Timestep 1: t=1.99135e+07 years

Solving Stokes system... 30+3 iterations.
Solving temperature system... 8 iterations.

Postprocessing:
  Writing graphical output: output/solution-00001
  RMS, max velocity:      0.104 cm/year, 0.217 cm/year
  Temperature min/avg/max: 300 K, 3008 K, 6300 K
  Inner/outer heat fluxes: 1.079e+05 W, 1.988e+05 W

*** Timestep 2: t=3.98271e+07 years

Solving Stokes system... 30+3 iterations.
Solving temperature system... 8 iterations.

Postprocessing:
  RMS, max velocity:      0.111 cm/year, 0.231 cm/year
  Temperature min/avg/max: 300 K, 3008 K, 6300 K
  Inner/outer heat fluxes: 1.083e+05 W, 2.01e+05 W

*** Timestep 3: t=5.97406e+07 years
...

```

This output was produced by a parameter file that, among other settings, contained the following values:

```

set Dimension           = 2
set End time            = 2e9
set Output directory    = output

subsection Geometry model
  set Model name        = spherical shell
end

subsection Mesh refinement
  set Initial global refinement = 4
  set Initial adaptive refinement = 1
end

subsection Postprocess
  set List of postprocessors = all
end

```

In other words, these run-time parameters specify that we should start with a geometry that represents a spherical shell (see Sections 5.7 and 5.9 for details). The coarsest mesh is refined 4 times globally, i.e., every cell is refined into four children (or eight, in 3d) 4 times. This yields the initial number of 1,536 cells on a mesh hierarchy that is 5 levels deep. We then solve the problem there once and, based on the number of adaptive refinement steps at the initial time set in the parameter file, use the solution so computed to refine the mesh once adaptively (yielding 2,379 cells on 6 levels) on which we start the computation over at time  $t = 0$ .

Within each time step, the output indicates the number of iterations performed by the linear solvers, and we generate a number of lines of output by the postprocessors that were selected (see Section 5.20). Here, we have selected to run all postprocessors that are currently implemented in ASPECT which includes the ones that evaluate properties of the velocity, temperature, and heat flux as well as a postprocessor that generates graphical output for visualization.

While the screen output is useful to monitor the progress of a simulation, it's lack of a structured output makes it not useful for later plotting things like the evolution of heat flux through the core-mantle boundary. To this end, ASPECT creates additional files in the output directory selected in the input parameter file



(here, the `output/` directory relative to the directory in which ASPECT runs). In a simple case, this will look as follows:

```
aspect> ls -l output/
total 780
-rw-rw-rw- 1 b   9863 Dec  1 15:13 parameters.prm
-rw-rw-rw- 1 b 306562 Dec  1 15:13 solution-00000.0000.vtu
-rw-rw-rw- 1 b  97057 Nov 30 05:58 solution-00000.0001.vtu
...
-rw-rw-rw- 1 b   1061 Dec  1 15:13 solution-00000.pvtu
-rw-rw-rw- 1 b    35 Dec  1 15:13 solution-00000.visit
-rw-rw-rw- 1 b 306530 Dec  1 15:13 solution-00001.0000.vtu
-rw-rw-rw- 1 b   1061 Dec  1 15:13 solution-00001.pvtu
-rw-rw-rw- 1 b    35 Dec  1 15:13 solution-00001.visit
...
-rw-rw-rw- 1 b   924 Dec  1 15:13 statistics
```

The purpose of these files is as follows:

- *A listing of all run-time parameters:* The `output/parameters.prm` file contains a complete listing of all run-time parameters. In particular, this includes the one that have been specified in the input parameter file passed on the command line, but it also includes those parameters for which defaults have been used. It is often useful to save this file together with simulation data to allow for the easy reproduction of computations later on.
- *Graphical output files:* One of the postprocessors you select when you say “all” in the parameter files is the one that generates output files that represent the solution at certain time steps. The screen output indicates that it has run at time step 0, producing output files of the form `output/solution-00000`. At the current time, the default is that ASPECT generates this output in VTK format<sup>7</sup> as that is widely used by a number of excellent visualization packages and also supports parallel visualization.<sup>8</sup> If the program has been run with multiple MPI processes, then the list of output files will look as shown above, with the base `solution-x.y` denoting that this the *x*th time we create output files and that the file was generated by the *y*th processor.

VTK files can be visualized by many of the large visualization packages. In particular, the [Visit](#) and [ParaView](#) programs, both widely used, can read the files so created. However, while VTK has become a de-facto standard for data visualization in scientific computing, there doesn’t appear to be an agreed upon way to describe which files jointly make up for the simulation data of a single time step (i.e., all files with the same *x* but different *y* in the example above). Visit and Paraview both have their method of doing things, through `.pvtu` and `.visit` files. To make it easy for you to view data, ASPECT simply creates both kinds of files in each time step in which graphical data is produced.

For more on visualization, see Section 4.4.

- *A statistics file:* The `output/statistics` file contains statistics collected during each time step, both from within the simulator (e.g., the current time for a time step, the time step length, etc.) as well as from the postprocessors that run at the end of each time step. The file is essentially a table that allows for the simple production of time trends. In the example above, it looks like this:

```
# 1: Time step number
# 2: Time (years)
# 3: Iterations for Stokes solver
# 4: Time step size (year)
```

<sup>7</sup>The output is in fact in the VTU version of the VTK file format. This is the XML-based version of this file format in which contents are compressed. Given that typical file sizes for 3d simulation are substantial, the compression saves a significant amount of disk space.

<sup>8</sup>The underlying DEAL.II package actually supports output in around a dozen different formats, but most of them are not very useful for large-scale, 3d, parallel simulations. If you need a different format than VTK, you can select this using the run-time parameters discussed in Section 5.22.

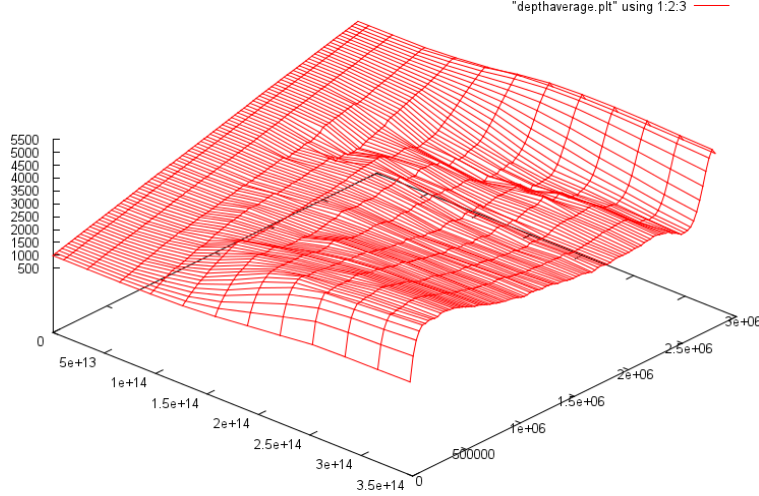


Figure 1: Example output for depth average statistics. On the left axis are 13 time steps, on the right is the depth (from the top at 0 to the bottom of the mantle on the far right), and the upwards pointing axis is the average temperature. This plot is created by calling `splot "depthaverage.plt" using 1:2:3 with lines` in `gnuplot`.

```
# 5: Iterations for temperature solver
# 6: Visualization file name
# 7: RMS velocity (m/year)
# 8: Max. velocity (m/year)
# 9: Minimal temperature (K)
# 10: Average temperature (K)
# 11: Maximal temperature (K)
# 12: Average nondimensional temperature (K)
# 13: Core-mantle heat flux (W)
# 14: Surface heat flux (W)
0 0.0000e+00 33 2.9543e+07 8      "" 0.0000 0.0000 0.0000 0.0000 ...
0 0.0000e+00 34 1.9914e+07 8 output/solution-00000 0.0946 0.1829 300.0000 3007.2519 ...
1 1.9914e+07 33 1.9914e+07 8 output/solution-00001 0.1040 0.2172 300.0000 3007.8406 ...
2 3.9827e+07 33 1.9914e+07 8      "" 0.1114 0.2306 300.0000 3008.3939 ...
```

The actual columns you have in your statistics file may differ from the ones above, but the format of this file should be obvious. Since the hash mark is a comment marker in many programs (for example, `gnuplot` ignores lines in text files that start with a hash mark), it is simple to plot these columns as time series. Alternatively, the data can be imported into a spreadsheet and plotted there.

**Note:** As noted in Section 2.1, ASPECT can be thought to compute in the meter-kilogram-second (MKS, or SI) system. Unless otherwise noted, the quantities in the output file are therefore also in MKS units.

- *Depth average statistics:* Similar to the `output/statistics` file, Aspect can generate depth-average statistics into `output/depthaverage.plt`. This is done by the “depth average” postprocessor and the user can control how often this file is updated.

The data is written in text format that can be easily displayed by e.g. `gnuplot`. For an example, see Figure 1. The plot shows how an initially linear temperature profile forms upper and lower boundary layers.

## 4.2 Selecting between 2d and 3d runs

ASPECT can solve both two- and three-dimensional problems. You select which one you want by putting a line like the following into the parameter file (see Section 5):

```
set Dimension = 2
```

Internally, dealing with the dimension builds on a feature in DEAL.II, upon which ASPECT is based, that is called *dimension-independent programming*. In essence, what this does is that you write your code only once in a way so that the space dimension is a variable (or, in fact, a template parameter) and you can compile the code for either 2d or 3d. The advantage is that codes can be tested and debugged in 2d where simulations are relatively cheap, and the same code can then be re-compiled and executed in 3d where simulations would otherwise be prohibitively expensive for finding bugs; it is also a useful feature when scoping out whether certain parameter settings will have the desired effect by testing them in 2d first, before running them in 3d. This feature is discussed in detail in the [DEAL.II tutorial program step-4](#). Like there, all the functions and classes in ASPECT are compiled for both 2d and 3d. Which dimension is actually called internally depends on what you have set in the input file, but in either case, the machine code generated for 2d and 3d results from the same source code and should, thus, contain the same set of features and bugs. Running in 2d and 3d should therefore yield comparable results. Be prepared to wait much longer for computations to finish in the latter case, however.

## 4.3 Debug or optimized mode

ASPECT utilizes a DEAL.II feature called *debug mode*. By default, ASPECT uses debug mode, i.e., it calls a version of the DEAL.II library that contain lots of checks for the correctness of function arguments, the consistency of the internal state of data structure, etc. If you program with DEAL.II, for example to extend ASPECT, it has been our experience over the years that, by number, most programming errors are of the kind where one forgets to initialize a vector, one accesses data that has not been updated, one tries to write into a vector that has ghost elements, etc. If not caught, the result of these bugs is that parts of the program use invalid data (data written into ghost elements is not communicated to other processors), that operations simply make no sense (adding vectors of different length), that memory is corrupted (writing past the end of an array) or, in rare and fortunate cases, that the program simply crashes.

Debug mode is designed to catch most of these errors: It enables some 7,300 assertions (as of late 2011) in DEAL.II where we check for errors like the above and, if the condition is violated, abort the program with a detailed message that shows the failed check, the location in the source code, and a stacktrace how the program got there. The downside of debug mode is, of course, that it makes the program much slower – depending on application by a factor of 4–10.

ASPECT by default uses debug mode because most users will want to play with the source code, and because it is also a way to verify that the compilation process worked correctly. If you have verified that the program runs correctly with your input parameters, for example by letting it run for the first 10 time steps, then you can switch to optimized mode by editing the top of the [Makefile](#) and following the steps to build and run in Section 3.3; alternatively, you can just build the entire application using the command `make debug-mode=off`.

**Note:** It goes without saying that if you make significant modifications to the program, you should do the first runs in debug mode to verify that your program still works as expected.

## 4.4 Visualizing results

Among the postprocessors that can be selected in the input parameter file (see Sections 4.1 and 5.22) are some that can produce files in a format that can later be used to generate a graphical visualization of the solution variables  $\mathbf{u}$ ,  $p$  and  $T$  at select time steps, or of quantities derived from these variables.

By default, the files that are generated are in VTU format, i.e., the XML-based, compressed format defined by the VTK library, see <http://public.kitware.com/VTK/>. This file format has become a broadly accepted pseudo-standard that many visualization program support, including two of the visualization programs used most widely in computational science: Visit (see <http://www.llnl.gov/visit/>) and ParaView (see <http://www.paraview.org/HTML/Index.html>). The VTU format has a number of advantages beyond being widely distributed:

- It allows for compression, keeping files relatively small even for sizeable computations.
- It is a structured XML format, allowing other programs to read it without too much trouble.
- It has a degree of support for parallel computations where every processor would only write that part of the data to a file that this processor in fact owns, avoiding the need to communicate all data to a single processor that then generates a single file. This requires a master file for each time step that then contains a reference to the individual files that together make up the output of a single time step. Unfortunately, there doesn't appear to be a standard for these master records; however, both ParaView and Visit have defined a format that each of these programs understand and that requires placing a file with ending `.pvtu` or `.visit` into the same directory as the output files from each processor. Section 4.1 gives an example of what can be found in the output directory.

**Note:** You can select other formats for output than VTU, see the run-time parameters in Section 5.22. However, none of the numerous formats currently implemented in DEAL.II other than the VTK/VTU formats allows for splitting up data over multiple files in case of parallel computations, thus making subsequent visualization of the entire volume impossible. Furthermore, given the amount of data ASPECT can produce, the compression that is part of the VTU format is an important part of keeping data manageable.

#### 4.4.1 Visualization the graphical output using *Visit*

In the following, let us discuss the process of visualizing a 2d computation using Visit. The steps necessary for other visualization programs will obviously differ but are, in principle, similar.

To this end, let us consider a simulation of convection in a box-shaped, 2d region (see the “cookbooks” section, Section 6, for suitable input files). We can run the program with 4 processors using

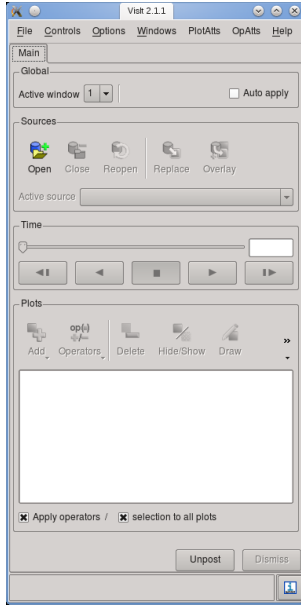
```
mpirun -np 4 ./lib/aspect-2d box.prm
```

Letting the program run for a while will result in several output files as discussed in Section 4.1 above.

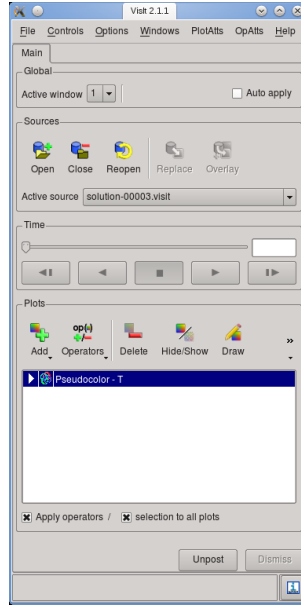
In order to visualize one time step, follow these steps:<sup>9</sup>

- *Selecting input files:* As mentioned above, in parallel computations we usually generate one output file per processor in each time step for which visualization data is produced (see, however, Section 4.4.3). To tell Visit which files together make up one time step, ASPECT creates a `solution-NNNNN.visit` file in the output directory. To open it, start Visit, click on the “Open” button in the “Sources” area of its main window (see Fig. 2(a)) and select the file you want. Alternatively, you can also select files using the “File > Open” menu item, or hit the corresponding keyboard short-cut. After adding an input source, the “Sources” area of the main window should list the selected file name.
- *Selecting what to plot:* ASPECT outputs all sorts of quantities that characterize the solution, such as temperature, pressure, velocity, and many others on demand (see Section 5.22). Once an input file has been opened, you will want to add graphical representations of some of this data to the still empty canvas. To this end, click on the “Add” button of the “Plots” area. The resulting menu provides a

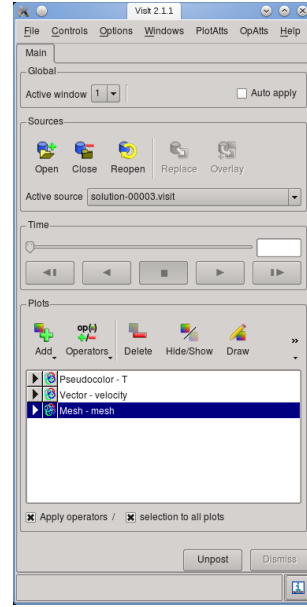
<sup>9</sup>The instructions and screenshots were generated with Visit 2.1. Later versions of Visit differ slightly in the arrangement of components of the graphical user interface, but the workflow and general idea remains unchanged.



(a)

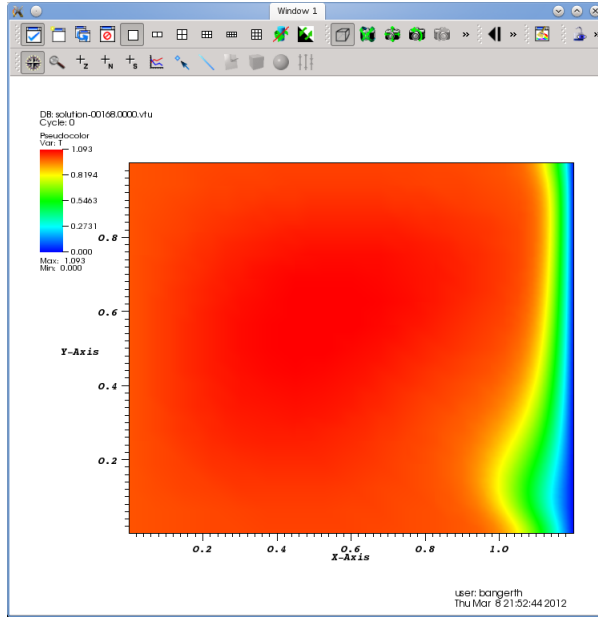


(b)

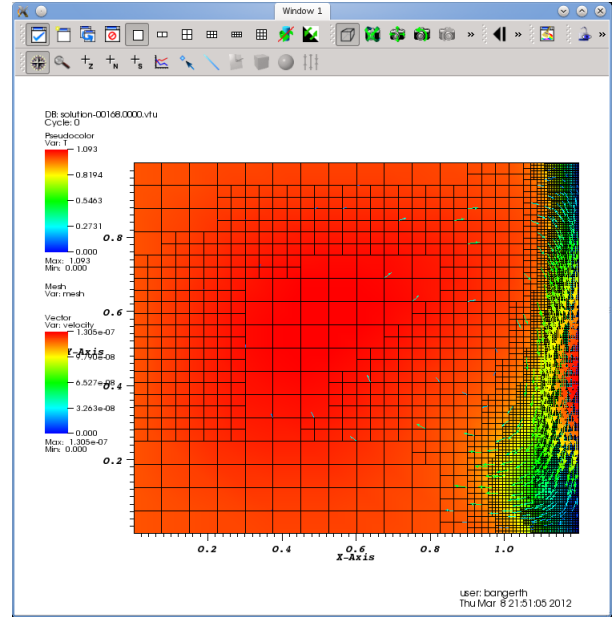


(c)

Figure 2: Main window of Visit, illustrating the different steps of adding content to a visualization.



(a)



(b)

Figure 3: Display window of Visit, showing a single plot and one where different data is overlaid.

number of different kinds of plots. The most important for our purpose are: (i) “Pseudocolor” allows the visualization of a scalar field (e.g., temperature, pressure, density) by using a color field. (ii) “Vector” displays a vector-valued field (e.g., velocity) using arrows. (iii) “Mesh” displays the mesh. The “Contour”, “Streamline” and “Volume” options are also frequently useful, in particular in 3d.

Let us choose the “Pseudocolor” item and select the temperature field as the quantity to plot. Your main window should now look as shown in Fig. 2(b). Then hit the “Draw” button to make Visit generate data for the selected plots. This will yield a picture such as shown in Fig. 3(a) in the display window of Visit.

- *Overlaying data:* Visit can overlay multiple plots in the same view. To this end, add another plot to the view using again the “Add” button to obtain the menu of possible plots, then the “Draw” button to actually draw things. For example, if we add velocity vectors and the mesh, the main window looks as in Fig. 2(c) and the main view as in Fig. 3(b).
- *Adjusting how data is displayed:* Without going into too much detail, if you double click onto the name of a plot in the “Plots” window, you get a dialog in which many of the properties of this plot can be adjusted. Further details can be changed by using “Operators” on a plot.
- *Making the output prettier:* As can be seen in Fig. 3, Visit by default puts a lot of clutter around the figure – the name of the user, the name of the input file, color bars, axes labels and ticks, etc. This may be useful to explore data in the beginning but does not yield good pictures for presentations or publications. To reduce the amount of information displayed, go to the “Controls > Annotations” menu item to get a dialog in which all of these displays can be selectively switched on and off.
- *Saving figures:* To save a visualization into a file that can then be included into presentations and publications, go to the menu item “File > Save window”. This will create successively numbered files in the directory from which Visit was started each time a view is saved. Things like the format used for these files can be chosen using the “File > Set save options” menu item. We have found that one can often get better looking pictures by selecting the “Screenshot” method in this dialog.

More information on all of these topics can be found in the Visit documentation, see <http://www.llnl.gov/visit/>.

#### 4.4.2 Visualizing statistical data

#### 4.4.3 Large data issues for parallel computations

Among the challenges in visualizing the results of parallel computations is dealing with the large amount of data. The first bottleneck this presents is during run-time when ASPECT wants to write the visualization data of a time step to disk. Using the compressed VTU format, ASPECT generates on the order of 10 bytes of output for each degree of freedom in 2d and more in 3d; thus, output of a single time step can run into the range of gigabytes that somehow have to get from compute nodes to disk. This stresses both the cluster interconnect as well as the data storage array.

There are essentially two strategies supported by ASPECT for this scenario:

- If your cluster has a fast interconnect, for example Infiniband, and if your cluster has a fast, distributed file system, then ASPECT can produce output files that are already located in the correct output directory (see the options in Section 5.2) on the global file system. ASPECT uses MPI I/O calls to this end, ensuring that the local machines do not have to access these files using slow NFS-mounted global file systems.
- If your cluster has a slow interconnect, e.g., if it is simply a collection of machines connected via ethernet, then writing data to a central file server may block the rest of the program for a while. On the other hand, if your machines have fast local storage for temporary file systems, then ASPECT can

Screenshot  
would be nice

Create screen  
shot

To be written

write data first into such a file and then move it in the background to its final destination while already continuing computations. To select this mode, set the appropriate variables discussed in Section 5.22. Note, however, that this scheme only makes sense if every machine on which MPI processes run has fast local disk space for temporary storage.

**Note:** An alternative would be if every processor directly writes its own files into the global output directory (possibly in the background), without the intermediate step of the temporary file. In our experience, file servers are quickly overwhelmed when encountering a few hundred machines wanting to open, fill, flush and close their own file via NFS mounted file system calls, sometimes completely blocking the entire cluster environment for extended periods of time.

## 4.5 Checkpoint/restart support

If you do long runs, especially when using parallel computations, there are a number of reasons to periodically save the state of the program:

- If the program crashes for whatever reason, the entire computation may be lost. A typical reason is that a program has exceeded the requested wallclock time allocated by a batch scheduler on a cluster.
- Most of the time, no realistic initial conditions for strongly convecting flow are available. Consequently, one typically starts with a somewhat artificial state and simply waits for a long while till the convective state enters the phase where it shows its long-term behavior. However, getting there may take a good amount of CPU time and it would be silly to always start from scratch for each different parameter setting. Rather, one would like to start such parameter studies with a saved state that has already passed this initial, unphysical, transient stage.

To this end, ASPECT creates a set of files in the output directory (selected in the parameter file) every 50 time steps in which the entire state of the program is saved so that a simulation can later be continued at this point. The previous checkpoint files will then be deleted. To resume operations from the last saved state, you need to set the `Resume computation` flag in the input parameter file to `true`, see Section 5.2.

**Note:** It is not imperative that the parameters selected in the input file are exactly the same when resuming a program from a saved state than what they were at the time when this state was saved. For example, one may want to choose a different parametrization of the material law, or add or remove postprocessors that should be run at the end of each time step. Likewise, the end time, the times at which some additional mesh refinement steps should happen, etc., can be different.

Yet, it is clear that some other things can't be changed: For example, the geometry model that was used to generate the coarse mesh and describe the boundary must be the same before and after resuming a computation. Likewise, you can not currently restart a computation with a different number of processors than initially used to checkpoint the simulation. Not all invalid combinations are easy to detect, and ASPECT may not always realize immediate what is going on if you change a setting that can't be changed. However, you will almost invariably get non-sensical results after some time.

## 5 Run-time input parameters

### 5.1 Overview

What ASPECT computes is driven by two things:



- The models implemented in ASPECT. This includes the geometries, the material laws, or the initial conditions currently supported. Which of these models are currently implemented is discussed below; Section 7 discusses in great detail the process of implementing additional models.
- Which of the implemented models is selected, and what their run-time parameters are. For example, you could select a model that prescribes constant coefficients throughout the domain from all the material models currently implemented; you could then select appropriate values for all of these constants. Both of these selections happen from a parameter file that is read at run time and whose name is specified on the command line. (See also Section 4.1.)

In this section, let us give an overview of what can be selected in the parameter file. Specific parameters, their default values, and allowed values for these parameters are documented in the following subsections. An index with page numbers for all run-time parameters can be found on page 59.

### 5.1.1 The structure of parameter files

Most of the run-time behavior of ASPECT is driven by a parameter file that looks in essence like this:

```
set Dimension                = 2
set Resume computation       = false
set End time                 = 1e10
set CFL number               = 1.0
set Output directory         = bin

subsection Mesh refinement
  set Initial adaptive refinement = 1
  set Initial global refinement   = 4
end

subsection Material model
  set Model name                 = simple

  subsection Simple model
    set Reference density        = 3300
    set Reference temperature    = 293
    set Viscosity                 = 5e24
  end
end
...
```

Some parameters live at the top level, but most parameters are grouped into subsections. An input parameter file is therefore much like a file system: a few files live in the root directory; others are in a nested hierarchy of sub-directories. And just as with files, parameters have both a name (the thing to the left of the equals sign) and a content (what's to the right).

All parameters you can list in this input file have been *declared* in ASPECT. What this means is that you can't just list anything in the input file with entries that are unknown simply being ignored. Rather, if your input file contains a line setting a parameter that is unknown to something, you will get an error message. Likewise, all declared parameters have a description of possible values associated with them – for example, some parameters must be non-negative integers (the number of initial refinement steps), can either be true or false (whether the computation should be resumed from a saved state), or can only be a single element from a selection (the name of the material model). If an entry in your input file doesn't satisfy these constraints, it will be rejected at the time of reading the file (and not when a part of the program actually accesses the value and the programmer has taken the time to also implement some error checking at this location). Finally, because parameters have been declared, you do not *need* to specify a parameter in the input file: if a parameter isn't listed, then the program will simply use the default provided when declaring the parameter.

### 5.1.2 Categories of parameters

The parameters that can be provided in the input file can roughly be categorized into the following groups:

- Global parameters (see Section 5.2): These parameters determine the overall behavior of the program. Primarily they describe things like the output directory, the end time of the simulation, or whether the computation should be resumed from a previously saved state.
- Parameters for certain aspects of the numerical algorithm: These describe, for example, the specifics of the spatial discretization. In particular, this is the case for parameters concerning the polynomial degree of the finite element approximation (Section 5.5), some details about the stabilization (Section 5.6), and how adaptive mesh refinement is supposed to work (Section 5.18).
- Parameters that describe certain global aspects of the equations to be solved: This includes, for example, a description if certain terms in the model should be omitted or not. See Section 5.19 for the list of parameters in this category.
- Parameters that characterize plugins: Certain behaviors of ASPECT are described by what we call *plugins* – self-contained parts of the code that describe one particular aspect of the simulation. An example would be which of the implemented material models to use, and the specifics of this material model. The sample parameter file above gives an indication of how this works: within a subsection of the file that pertains to the material models, one can select one out of several plugins (or, in the case of the postprocessors, any number, including none, of the available plugins), and one can then specify the specifics of this model in a sub-subsection dedicated to this particular model.

A number of components of ASPECT are implemented via plugins. These are, together with the sections in which their parameters are declared:

- The material model: Sections 5.14, 5.15.
- The geometry: Sections 5.7, 5.9.
- The gravity description: Sections 5.10, 5.11.
- Initial conditions for the temperature: Sections 5.12, 5.13.
- Temperature boundary conditions: Sections 5.3, 5.4.
- Postprocessors: Sections 5.20, 5.22.

The details of parameters in each of these categories can be found in the sections linked to above. Some of them will also be used in the cookbooks in Section 6.

## 5.2 Global parameters

- *Parameter name:* `Adiabatic surface temperature`

*Value:* 0

*Default:* 0

*Description:* In order to make the problem in the first time step easier to solve, we need a reasonable guess for the temperature and pressure. To obtain it, we use an adiabatic pressure and temperature field. This parameter describes what the ‘adiabatic’ temperature would be at the surface of the domain (i.e. at depth zero). Note that this value need not coincide with the boundary condition posed at this point. Rather, the boundary condition may differ significantly from the adiabatic value, and then typically induce a thermal boundary layer. For more information, see the section in the manual that discusses the general mathematical model.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- **Parameter name:** CFL number

*Value:* 1.0

*Default:* 1.0

*Description:* In computations, the time step  $k$  is chosen according to  $k = c \min_K \frac{h_K}{\|u\|_{\infty, K} p_T}$  where  $h_K$  is the diameter of cell  $K$ , and the denominator is the maximal magnitude of the velocity on cell  $K$  times the polynomial degree  $p_T$  of the temperature discretization. The dimensionless constant  $c$  is called the CFL number in this program. For time discretizations that have explicit components,  $c$  must be less than a constant that depends on the details of the time discretization and that is no larger than one. On the other hand, for implicit discretizations such as the one chosen here, one can choose the time step as large as one wants (in particular, one can choose  $c > 1$ ) though a CFL number significantly larger than one will yield rather diffusive solutions. Units: None.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- **Parameter name:** End time

*Value:* 1e10

*Default:* 1e8

*Description:* The end time of the simulation. Units: years if the 'Use years in output instead of seconds' parameter is set; seconds otherwise.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- **Parameter name:** Output directory

*Value:* output

*Default:* output

*Description:* The name of the directory into which all output files should be placed. This may be an absolute or a relative path.

*Possible values:* [DirectoryName]

- **Parameter name:** Resume computation

*Value:* false

*Default:* false

*Description:* A flag indicating whether the computation should be resumed from a previously saved state (if true) or start from scratch (if false).

*Possible values:* [Bool]

- **Parameter name:** Start time

*Value:* 0

*Default:* 0

*Description:* The start time of the simulation. Units: years if the 'Use years in output instead of seconds' parameter is set; seconds otherwise.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- **Parameter name:** Surface pressure

*Value:* 0

*Default:* 0

*Description:* The mathematical equations that describe thermal convection only determine the pressure up to an arbitrary constant. On the other hand, for comparison and for looking up material parameters

it is important that the pressure be normalized somehow. We do this by enforcing a particular average pressure value at the surface of the domain, where the geometry model determines where the surface is. This parameter describes what this average surface pressure value is supposed to be. By default, it is set to zero, but one may want to choose a different value for example for simulating only the volume of the mantle below the lithosphere, in which case the surface pressure should be the lithostatic pressure at the bottom of the lithosphere. For more information, see the section in the manual that discusses the general mathematical model.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- *Parameter name:* `Use years in output instead of seconds`

*Value:* false

*Default:* true

*Description:* When computing results for mantle convection simulations, it is often difficult to judge the order of magnitude of results when they are stated in MKS units involving seconds. Rather, some kinds of results such as velocities are often stated in terms of meters per year (or, sometimes, centimeters per year). On the other hand, for non-dimensional computations, one wants results in their natural unit system as used inside the code. If this flag is set to 'true' conversion to years happens; if it is 'false', no such conversion happens.

*Possible values:* [Bool]

### 5.3 Parameters in section Boundary temperature model

- *Parameter name:* `Model name`

*Value:* box

*Default:*

*Description:* Select one of the following models:

'spherical constant': A model in which the temperature is chosen constant on the inner and outer boundaries of a spherical shell. Parameters are read from subsection 'Spherical constant'.

'box': A model in which the temperature is chosen constant on the left and right sides of a box.

*Possible values:* [Selection spherical constant—box ]

### 5.4 Parameters in section Boundary temperature model/Spherical constant

- *Parameter name:* `Inner temperature`

*Value:* 6300

*Default:* 6000

*Description:* Temperature at the inner boundary (core mantle boundary). Units: K.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- *Parameter name:* `Outer temperature`

*Value:* 300

*Default:* 0

*Description:* Temperature at the outer boundary (lithosphere water/air). Units: K.

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

## 5.5 Parameters in section Discretization

- *Parameter name:* Stokes velocity polynomial degree  
*Value:* 2  
*Default:* 2  
*Description:* The polynomial degree to use for the velocity variables in the Stokes system. Units: None.  
*Possible values:* [Integer range 1...2147483647 (inclusive)]
- *Parameter name:* Temperature polynomial degree  
*Value:* 2  
*Default:* 2  
*Description:* The polynomial degree to use for the temperature variable. Units: None.  
*Possible values:* [Integer range 1...2147483647 (inclusive)]
- *Parameter name:* Use locally conservative discretization  
*Value:* false  
*Default:* false  
*Description:* Whether to use a Stokes discretization that is locally conservative at the expense of a larger number of degrees of freedom (true), or to go with a cheaper discretization that does not locally conserve mass, although it is globally conservative (false).  
*Possible values:* [Bool]

## 5.6 Parameters in section Discretization/Stabilization parameters

- *Parameter name:* alpha  
*Value:* 2  
*Default:* 2  
*Description:* The exponent  $\alpha$  in the entropy viscosity stabilization. Units: None.  
*Possible values:* [Double 1...2 (inclusive)]
- *Parameter name:* beta  
*Value:* 0.078  
*Default:* 0.078  
*Description:* The  $\beta$  factor in the artificial viscosity stabilization. An appropriate value for 2d is 0.052 and 0.078 for 3d. Units: None.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* cR  
*Value:* 0.5  
*Default:* 0.11  
*Description:* The  $c_R$  factor in the entropy viscosity stabilization. Units: None.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.7 Parameters in section Geometry model

- *Parameter name:* **Model name**

*Value:* box

*Default:*

*Description:* Select one of the following models:

‘spherical shell’: A geometry representing a spherical shell or a pice of it. Inner and outer radii are read from the parameter file in subsection ‘Spherical shell’.

‘box’: A box geometry parallel to the coordinate directions. The extent of the box in each coordinate direction is set in the parameter file.

*Possible values:* [Selection spherical shell—box ]

## 5.8 Parameters in section Geometry model/Box

- *Parameter name:* **X extent**

*Value:* 1.2

*Default:* 1

*Description:* Extent of the box in x-direction. Units: m.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* **Y extent**

*Value:* 1

*Default:* 1

*Description:* Extent of the box in y-direction. Units: m.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* **Z extent**

*Value:* 1

*Default:* 1

*Description:* Extent of the box in z-direction. This value is ignored if the simulation is in 2d Units: m.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.9 Parameters in section Geometry model/Spherical shell

- *Parameter name:* **Inner radius**

*Value:* 5698e3

*Default:* 3481000

*Description:* Inner radius of the spherical shell. Units: m.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* **Opening angle**

*Value:* 180

*Default:* 360

*Description:* Opening angle in degrees of the section of the shell that we want to build. Units: degrees.

*Possible values:* [Double 0...360 (inclusive)]

- *Parameter name:* `Outer radius`

*Value:* 10415e3

*Default:* 6336000

*Description:* Outer radius of the spherical shell. Units: m.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.10 Parameters in section Gravity model

- *Parameter name:* `Model name`

*Value:* vertical

*Default:*

*Description:* Select one of the following models:

‘vertical’: A gravity model in which the gravity direction is vertically downward and at constant magnitude.

‘radial constant’: A gravity model in which the gravity direction is radially inward and at constant magnitude. The magnitude is read from the parameter file in subsection ‘Radial constant’.

‘radial earth-like’: A gravity model in which the gravity direction is radially inward and with a magnitude that matches that of the earth at the core-mantle boundary as well as at the surface and in between is physically correct under the assumption of a constant density.

*Possible values:* [Selection vertical—radial constant—radial earth-like ]

## 5.11 Parameters in section Gravity model/Radial constant

- *Parameter name:* `Magnitude`

*Value:* 30

*Default:* 30

*Description:* Magnitude of the gravity vector in  $m/s^2$ . The direction is always radially outward from the center of the earth.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.12 Parameters in section Initial conditions

- *Parameter name:* `Model name`

*Value:* perturbed box

*Default:*

*Description:* Select one of the following models:

‘spherical hexagonal perturbation’: An initial temperature field in which the temperature is perturbed following a six-fold pattern in angular direction from an otherwise spherically symmetric state.

‘spherical gaussian perturbation’: An initial temperature field in which the temperature is perturbed by a single Gaussian added to an otherwise spherically symmetric state. Additional parameters are read from the parameter file in subsection ‘Spherical gaussian perturbation’.

‘perturbed box’: An initial temperature field in which the temperature is perturbed slightly from an otherwise constant value equal to one. The perturbation is chosen in such a way that the initial temperature is constant to one along the entire boundary.

*Possible values:* [Selection spherical hexagonal perturbation—spherical gaussian perturbation—perturbed box ]



### 5.13 Parameters in section Initial conditions/Spherical gaussian perturbation

- *Parameter name:* **Amplitude**  
*Value:* 0.01  
*Default:* 0.01  
*Description:* The amplitude of the perturbation.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* **Angle**  
*Value:* 4.71238898038468985769  
*Default:* 0e0  
*Description:* The angle where the center of the perturbation is placed.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* **Filename for initial geotherm table**  
*Value:* initial\_geotherm\_table  
*Default:* initial\_geotherm\_table  
*Description:* The file from which the initial geotherm table is to be read. The format of the file is defined by what is read in source/initial\_conditions/spherical\_shell.cc.  
*Possible values:* [FileName (Type: input)]
- *Parameter name:* **Non-dimensional depth**  
*Value:* 0.7  
*Default:* 0.7  
*Description:* The non-dimensional radial distance where the center of the perturbation is placed.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* **Sigma**  
*Value:* 0.2  
*Default:* 0.2  
*Description:* The standard deviation of the Gaussian perturbation.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* **Sign**  
*Value:* 1  
*Default:* 1  
*Description:* The sign of the perturbation.  
*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

## 5.14 Parameters in section Material model

- *Parameter name:* `Model name`

*Value:* simple

*Default:*

*Description:* Select one of the following models:

‘table’: A material model that reads tables of pressure and temperature dependent material coefficients from files.

‘Steinberger’: lookup from the paper of Steinberger/Calderwood

‘simple’: A simple material model that has constant values for all coefficients but the density. This model uses the formulation that assumes an incompressible medium despite the fact that the density follows the law  $\rho(T) = \rho_0(1 - \beta(T - T_{\text{ref}}))$ . The value for the components of this formula and additional parameters are read from the parameter file in subsection ‘Simple model’.

*Possible values:* [Selection table—Steinberger—simple ]

## 5.15 Parameters in section Material model/Simple model

- *Parameter name:* `Reference density`

*Value:* 1

*Default:* 3300

*Description:* Reference density  $\rho_0$ . Units:  $kg/m^3$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `Reference specific heat`

*Value:* 1250

*Default:* 1250

*Description:* The value of the specific heat  $cp$ . Units:  $JG/kgK$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `Reference temperature`

*Value:* 1

*Default:* 293

*Description:* The reference temperature  $T_0$ . Units:  $K$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `Thermal conductivity`

*Value:* 1e-6

*Default:* 4.7

*Description:* The value of the thermal conductivity  $k$ . Units:  $W/m/K$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `Thermal expansion coefficient`

*Value:* 2e-5

*Default:* 2e-5

*Description:* The value of the thermal expansion coefficient  $\beta$ . Units:  $1/K$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* **Viscosity**

*Value:* 1

*Default:* 5e24

*Description:* The value of the constant viscosity. Units:  $kg/m/s$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.16 Parameters in section **Material model/Table model**

- *Parameter name:* **Composition**

*Value:* olixene

*Default:* olixene

*Description:* The Composition of the model.

*Possible values:* [Anything]

- *Parameter name:* **Compressible**

*Value:* true

*Default:* true

*Description:* whether the model is compressible.

*Possible values:* [Bool]

- *Parameter name:* **ComputePhases**

*Value:* false

*Default:* false

*Description:* whether to compute phases.

*Possible values:* [Bool]

- *Parameter name:* **Gravity**

*Value:* 30

*Default:* 30

*Description:* The value of the gravity constant. Units:  $m/s^2$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* **Path to model data**

*Value:* datadir

*Default:* datadir

*Description:* The path to the model data.

*Possible values:* [DirectoryName]

- *Parameter name:* **Reference density**

*Value:* 3300

*Default:* 3300

*Description:* Reference density  $\rho_0$ . Units:  $kg/m^3$ .

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `Reference specific heat`  
*Value:* 1250  
*Default:* 1250  
*Description:* The value of the specific heat  $cp$ . Units:  $JG/kgK$ .  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* `Reference temperature`  
*Value:* 293  
*Default:* 293  
*Description:* The reference temperature  $T_0$ . Units:  $K$ .  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* `Thermal conductivity`  
*Value:* 4.7  
*Default:* 4.7  
*Description:* The value of the thermal conductivity  $k$ . Units:  $W/m/K$ .  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* `Thermal expansion coefficient`  
*Value:* 2e-5  
*Default:* 2e-5  
*Description:* The value of the thermal expansion coefficient  $\beta$ . Units:  $1/K$ .  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.17 Parameters in section Material model/Table model/Viscosity

- *Parameter name:* `ReferenceViscosity`  
*Value:* 5e24  
*Default:* 5e24  
*Description:* The value of the constant viscosity. Units:  $kg/m/s$ .  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* `ViscosityModel`  
*Value:* Exponential  
*Default:* Exponential  
*Description:* Viscosity Model  
*Possible values:* [Anything]
- *Parameter name:* `Viscosity increase lower mantle`  
*Value:* 4e0  
*Default:* 4e0  
*Description:* The Viscosity increase (jump) in the lower mantle.  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]

- *Parameter name:* `exponential_P`  
*Value:* 1  
*Default:* 1  
*Description:* multiplication factor or Pressure exponent  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]
- *Parameter name:* `exponential_T`  
*Value:* 1  
*Default:* 1  
*Description:* multiplication factor or Temperature exponent  
*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.18 Parameters in section Mesh refinement

- *Parameter name:* `Additional refinement times`  
*Value:*  
*Default:*  
*Description:* A list of times so that if the end time of a time step is beyond this time, an additional round of mesh refinement is triggered. This is mostly useful to make sure we can get through the initial transient phase of a simulation on a relatively coarse mesh, and then refine again when we are in a time range that we are interested in and where we would like to use a finer mesh. Units: each element of the list has units years if the 'Use years in output instead of seconds' parameter is set; seconds otherwise.  
*Possible values:* [List list of  $;$ [Double 0...1.79769e+308 (inclusive)] $;$  of length 0...4294967295 (inclusive)]
- *Parameter name:* `Coarsening fraction`  
*Value:* 0.05  
*Default:* 0.05  
*Description:* The fraction of cells with the smallest error that should be flagged for coarsening.  
*Possible values:* [Double 0...1 (inclusive)]
- *Parameter name:* `Initial adaptive refinement`  
*Value:* 1  
*Default:* 2  
*Description:* The number of adaptive refinement steps performed after initial global refinement but while still within the first time step.  
*Possible values:* [Integer range 0...2147483647 (inclusive)]
- *Parameter name:* `Initial global refinement`  
*Value:* 3  
*Default:* 2  
*Description:* The number of global refinement steps performed on the initial coarse mesh, before the problem is first solved there.  
*Possible values:* [Integer range 0...2147483647 (inclusive)]

- *Parameter name:* **Refinement fraction**  
*Value:* 0.3  
*Default:* 0.3  
*Description:* The fraction of cells with the largest error that should be flagged for refinement.  
*Possible values:* [Double 0...1 (inclusive)]
- *Parameter name:* **Strategy**  
*Value:* Density c\_p temperature  
*Default:* Density c\_p temperature  
*Description:* The method used to determine which cells to refine and which to coarsen.  
*Possible values:* [Selection Temperature—Normalized density and temperature—Weighted density and temperature—Density c\_p temperature ]
- *Parameter name:* **Time steps between mesh refinement**  
*Value:* 10  
*Default:* 10  
*Description:* The number of time steps after which the mesh is to be adapted again based on computed error indicators.  
*Possible values:* [Integer range 1...2147483647 (inclusive)]

## 5.19 Parameters in section **Model settings**

- *Parameter name:* **Fixed temperature boundary indicators**  
*Value:* 0, 1  
*Default:*  
*Description:* A comma separated list of integers denoting those boundaries on which the temperature is fixed and described by the boundary temperature object selected in its own section of this input file. All boundary indicators used by the geometry but not explicitly listed here will end up with no-flux (insulating) boundary conditions.  
*Possible values:* [List list of ;[Integer range 0...2147483647 (inclusive)]<sub>i</sub> of length 0...4294967295 (inclusive)]
- *Parameter name:* **Include adiabatic heating**  
*Value:* false  
*Default:* false  
*Description:* Whether to include adiabatic heating into the model or not. From a physical viewpoint, adiabatic heating should always be used but may be undesirable when comparing results with known benchmarks that do not include this term in the temperature equation.  
*Possible values:* [Bool]
- *Parameter name:* **Include shear heating**  
*Value:* false  
*Default:* true  
*Description:* Whether to include shear heating into the model or not. From a physical viewpoint, shear heating should always be used but may be undesirable when comparing results with known benchmarks that do not include this term in the temperature equation.  
*Possible values:* [Bool]

- *Parameter name:* Prescribed velocity boundary indicators

*Value:* 0

*Default:*

*Description:* A comma separated list of integers denoting those boundaries on which the velocity is tangential but prescribed, i.e., where external forces act to prescribe a particular velocity. This is often used to prescribe a velocity that equals that of overlying plates.

*Possible values:* [List list of  $i$  [Integer range 0...255 (inclusive)] $_i$  of length 0...4294967295 (inclusive)]

- *Parameter name:* Radiogenic heating rate

*Value:* 0

*Default:* 0e0

*Description:* H0

*Possible values:* [Double -1.79769e+308...1.79769e+308 (inclusive)]

- *Parameter name:* Tangential velocity boundary indicators

*Value:* 1

*Default:*

*Description:* A comma separated list of integers denoting those boundaries on which the velocity is tangential and unrestrained, i.e., where no external forces act to prescribe a particular tangential velocity (although there is a force that requires the flow to be tangential).

*Possible values:* [List list of  $i$  [Integer range 0...255 (inclusive)] $_i$  of length 0...4294967295 (inclusive)]

- *Parameter name:* Zero velocity boundary indicators

*Value:* 2, 3

*Default:*

*Description:* A comma separated list of integers denoting those boundaries on which the velocity is zero.

*Possible values:* [List list of  $i$  [Integer range 0...255 (inclusive)] $_i$  of length 0...4294967295 (inclusive)]

## 5.20 Parameters in section Postprocess

- *Parameter name:* List of postprocessors

*Value:* visualization, velocity statistics, temperature statistics, heat flux statistics

*Default:* all

*Description:* A comma separated list of postprocessor objects that should be run at the end of each time step. Some of these postprocessors will declare their own parameters which may, for example, include that they will actually do something only every so many time steps or years. Alternatively, the text 'all' indicates that all available postprocessors should be run after each time step.

The following postprocessors are available:

'visualization': A postprocessor that takes the solution and writes it into files that can be read by a graphical visualization program. Additional run time parameters are read from the parameter subsection 'Visualization'.

'velocity statistics': A postprocessor that computes some statistics about the velocity field.

'temperature statistics': A postprocessor that computes some statistics about the temperature field.



‘velocity statistics for the table model’: A postprocessor that computes some statistics about the velocity field.

‘heat flux statistics for the table model’: A postprocessor that computes some statistics about the heat flux across boundaries.

‘heat flux statistics’: A postprocessor that computes some statistics about the heat flux across boundaries.

‘depth average’: A postprocessor that computes depth averaged quantities and writes them out.

*Possible values:* [MultipleSelection visualization—velocity statistics—temperature statistics—velocity statistics for the table model—heat flux statistics for the table model—heat flux statistics—depth average—all ]

## 5.21 Parameters in section Postprocess/Depth average

- *Parameter name:* Time between graphical output

*Value:* 1e8

*Default:* 1e8

*Description:* The time interval between each generation of graphical output files. A value of zero indicates that output should be generated in each time step. Units: years if the ‘Use years in output instead of seconds’ parameter is set; seconds otherwise.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 5.22 Parameters in section Postprocess/Visualization

- *Parameter name:* Number of grouped files

*Value:* 0

*Default:* 0

*Description:* VTU file output supports grouping files from several CPUs into one file using MPI I/O when writing on a parallel filesystem. Select 0 for no grouping. This will disable parallel file output and instead write one file per processor in a background thread. A value of 1 will generate one big file containing the whole solution.

*Possible values:* [Integer range 0...2147483647 (inclusive)]

- *Parameter name:* Output format

*Value:* vtu

*Default:* vtu

*Description:* The file format to be used for graphical output.

*Possible values:* [Selection none—dx—ucd—gnuplot—povray—eps—gmw—tecplot—tecplot\_binary—vtk—vtu—deal.II intermediate ]

- *Parameter name:* Time between graphical output

*Value:* 1e5

*Default:* 1e8

*Description:* The time interval between each generation of graphical output files. A value of zero indicates that output should be generated in each time step. Units: years if the ‘Use years in output instead of seconds’ parameter is set; seconds otherwise.

*Possible values:* [Double 0...1.79769e+308 (inclusive)]

## 6 Cookbooks

In this section, let us present a number of “cookbooks” – examples of how to use ASPECT in typical or less typical ways. As discussed in Sections 4 and 5, ASPECT is driven by run-time parameter files, and so setting up a particular situation primarily comes down to creating a parameter file that has the right entries. Thus, the subsections below will discuss in detail what parameters to set and to what values. Note that parameter files need not specify *all* parameters – of which there is a bewildering number – but only those that are relevant to the particular situation we would like to model. All parameters not listed explicitly in the input file are simply left at their default value (the default values are also documented in Section 5).

Of course, there are situations where what you want to do is not covered by the models already implemented. Specifically, you may want to try a different geometry, a different material or gravity model, or different boundary conditions. In such cases, you will need to implement these extensions in the actual source code. Section 7 provides information on how to do that.

The remainder of this section shows a number of applications of ASPECT. They are grouped into two categories: Setups that try to model geophysical situations (Section 6.1) and setups that are used to benchmark ASPECT to ensure correctness or to test accuracy of our solvers (Section 6.2).

**Note:** The input files discussed in the following sections can generally be found in the `cookbooks/` directory of your ASPECT installation.

### 6.1 Geophysical setups

### 6.2 Benchmarks

To be written

Benchmarks are used to verify that a solver solves the problem correctly, i.e., to *verify* correctness of a code.<sup>10</sup> Over the past decades, the geodynamics community has come up with a large number of benchmarks. Depending on the goals of their original inventors, they describe stationary problems in which only the solution of the flow problem is of interest (but the flow may be compressible or incompressible, with constant or variable viscosity, etc), or they may actually model time-dependent processes. Some of them have solutions that are analytically known and can be compared with, while for others, there are only sets of numbers that are approximately known. We have implemented a number of them in ASPECT to convince ourselves (and our users) that ASPECT indeed works as intended and advertised. Some of these benchmarks are discussed below.

#### 6.2.1 The SolCx Stokes benchmark

The SolCx benchmark is intended to test the accuracy of the solution to a problem that has a large jump in the viscosity along a line through the domain. Such situations are common in geophysics: for example, the viscosity in a cold, subducting slab is much larger than in the surrounding, relatively hot mantle material.

The SolCx benchmark computes the Stokes flow field of a fluid driven by spatial density variations, subject to a spatially variable viscosity. Specifically, the domain is  $\Omega = [0, 1]^2$ , gravity is  $\mathbf{g} = (0, -1)^T$  and the density is given by  $\rho(\mathbf{x}) = \sin(\pi x_1) \cos(\pi x_2)$ ; this can be considered a density perturbation to a constant background density. The viscosity is

$$\eta(\mathbf{x}) = \begin{cases} 1 & \text{for } x_1 \leq 0.5, \\ 10^6 & \text{for } x_1 > 0.5. \end{cases}$$

This strongly discontinuous viscosity field yields an almost stagnant flow in the right half of the domain and consequently a singularity in the pressure along the interface. Boundary conditions are free slip on all of  $\partial\Omega$ .

<sup>10</sup>Verification is the first half of the *verification and validation* (V&V) procedure: *verification* intends to ensure that the mathematical model is solved correctly, while *validation* intends to ensure that the mathematical model is correct. Obviously, much of the aim of computational geodynamics is to validate the models that we have.

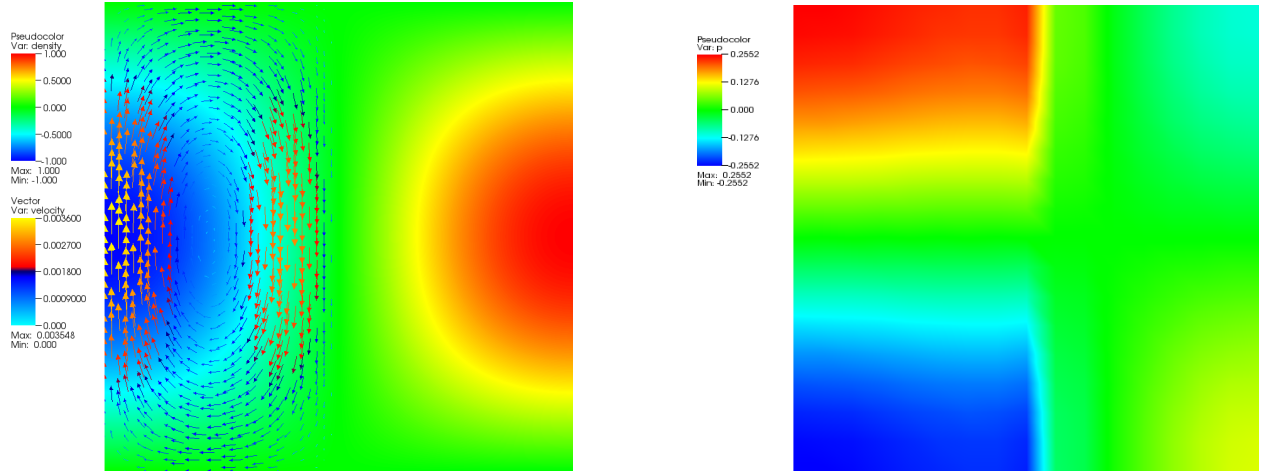


Figure 4: SolCx Stokes benchmark. Left: The density perturbation field and overlaid to it some velocity vectors. The viscosity is very large in the right hand, leading to a stagnant flow in this region. Right: The pressure on a relatively coarse mesh, showing the internal layer along the line where the viscosity jumps.

The temperature plays no role in this benchmark. The prescribed density field and the resulting velocity field are shown in Fig. 4.

The SolCx benchmark was previously used in [DMGT11, Section 4.1.1] (references to earlier uses of the benchmark are available there) and its analytic solution is given in [Zho96]. ASPECT contains an implementation of this analytic solution taken from the Underworld package (see [MQL<sup>+</sup>07] and <http://www.underworldproject.org/>, and correcting for the mismatch in sign between the implementation and the description in [DMGT11]).

To run this benchmark, the following input file will do:

```
##### Global parameters

set Dimension                = 2
set Start time                = 0
set End time                  = 0

set Output directory          = output
set Pressure normalization    = volume

##### Parameters describing the model

subsection Geometry model
  set Model name = box

  subsection Box
    set X extent = 1
    set Y extent = 1
  end
end

subsection Model settings
  set Prescribed velocity boundary indicators =
  set Tangential velocity boundary indicators = 0,1,2,3
  set Zero velocity boundary indicators      =
```

```

end

subsection Material model
  set Model name = SolCx

  subsection SolCx
    set Viscosity jump = 1e6
  end
end

subsection Gravity model
  set Model name = vertical
end

##### Parameters describing the temperature field

subsection Boundary temperature model
  set Model name = box
end

subsection Initial conditions
  set Model name = perturbed box
end

##### Parameters describing the discretization

subsection Discretization
  set Stokes velocity polynomial degree = 2
  set Use locally conservative discretization = false
end

subsection Mesh refinement
  set Initial adaptive refinement = 0
  set Initial global refinement = 4
end

##### Parameters describing the what to do with the solution

subsection Postprocess
  set List of postprocessors = DuretzEtAl error , visualization
end

```

Since this is the first cookbook in the benchmarking section, let us go through the different parts of this file in more detail:

- The first part consists of parameter setting for overall parameters. Specifically, we set the dimension in which this benchmark runs to two and choose an output directory. Since we are not interested in a time dependent solution, we set the end time equal to the start time, which results in only a single time step being computed.

The last parameter of this section, **Pressure normalization**, is set in such a way that the pressure is chosen so that its *domain* average is zero, rather than the pressure along the surface, see Section 2.3.

- The next part of the input file describes the setup of the benchmark. Specifically, we have to say how the geometry should look like (a box of size  $1 \times 1$ ) and what the velocity boundary conditions

shall be (tangential flow all around – the box geometry defines four boundary indicators for the left, right, bottom and top boundaries, see also Section 5.7). This is followed by subsections choosing the material model (where we choose a particular model implemented in ASPECT that describes the spatially variable density and viscosity fields, along with the size of the viscosity jump) and finally the chosen gravity model (a gravity field that is the constant vector  $(0, -1)^T$ , see Section 5.10).

- The part that follows this describes the boundary and initial values for the temperature. While we are not interested in the evolution of the temperature field in this benchmark, we nevertheless need to set something. The values given here are the minimal set of inputs.
- The second-to-last part sets discretization parameters. Specifically, it determines what kind of Stokes element to choose (see Section 5.5 and the extensive discussion in [KHB12]). We do not adaptively refine the mesh but only do four global refinement steps at the very beginning. This is obviously a parameter worth playing with.
- The final section on postprocessors determines what to do with the solution once computed. Here, we do two things: we ask ASPECT to compute the error in the solution using the setup described in the Duretz et al. paper [DMGT11], and we request that output files for later visualization are generated and placed in the output directory. The functions that compute the error automatically query which kind of material model had been chosen, i.e., they can know whether we are solving the SolCx benchmark or one of the other benchmarks discussed in the following subsections.

Upon running ASPECT with this input file, you will get output of the following kind (obviously with different timings, and details of the output may also change as development of the code continues):

```
aspect/cookbooks> ../lib/aspect sol_cx.prm
Number of active cells: 256 (on 5 levels)
Number of degrees of freedom: 3,556 (2,178+289+1,089)

*** Timestep 0: t=0 years
Solving temperature system... 0 iterations.
Rebuilding Stokes preconditioner...
Solving Stokes system... 30+3 iterations.

Postprocessing:
Errors u_L1, p_L1, u_L2, p_L2: 1.125997e-06, 2.994143e-03, 1.670009e-06, 9.778441e-03
Writing graphical output:      output/solution-00000
```

Total wallclock <b>time</b> elapsed since start		1.51s	
Section	no. calls	wall <b>time</b>	% of total
Assemble Stokes system	1	0.114s	7.6%
Assemble temperature system	1	0.284s	19%
Build Stokes preconditioner	1	0.0935s	6.2%
Build temperature preconditioner	1	0.0043s	0.29%
Solve Stokes system	1	0.0717s	4.8%
Solve temperature system	1	0.000753s	0.05%
Postprocessing	1	0.627s	42%
Setup dof systems	1	0.19s	13%

One can then visualize the solution in a number of different ways (see Section 4.4), yielding pictures like those shown in Fig. 4. One can also analyze the error as shown in various different ways, for example as a function of the mesh refinement level, the element chosen, etc.; we have done so extensively in [KHB12].

### 6.2.2 The SolKz Stokes benchmark

The SolKz benchmark is another variation on the same theme as the SolCx benchmark above: it solves a Stokes problem with a spatially variable viscosity but this time the viscosity is not a discontinuous function but grows exponentially with the vertical coordinate so that its overall variation is again  $10^6$ . The forcing is again chosen by imposing a spatially variable density variation. For details, refer again to [DMGT11].

The following input file, only a small variation of the one in the previous section, solves this benchmark:

```
##### Global parameters

set Dimension                = 2

set Start time                = 0
set End time                  = 0

set Output directory          = output

set Pressure normalization    = volume

##### Parameters describing the model

subsection Geometry model
  set Model name = box

  subsection Box
    set X extent = 1
    set Y extent = 1
  end
end

subsection Model settings
  set Prescribed velocity boundary indicators =
  set Tangential velocity boundary indicators = 0,1,2,3
  set Zero velocity boundary indicators      =
end

subsection Material model
  set Model name = SolKz
end

subsection Gravity model
  set Model name = vertical
end

##### Parameters describing the temperature field

subsection Boundary temperature model
  set Model name = box
end

subsection Initial conditions
  set Model name = perturbed box
end

##### Parameters describing the discretization
```

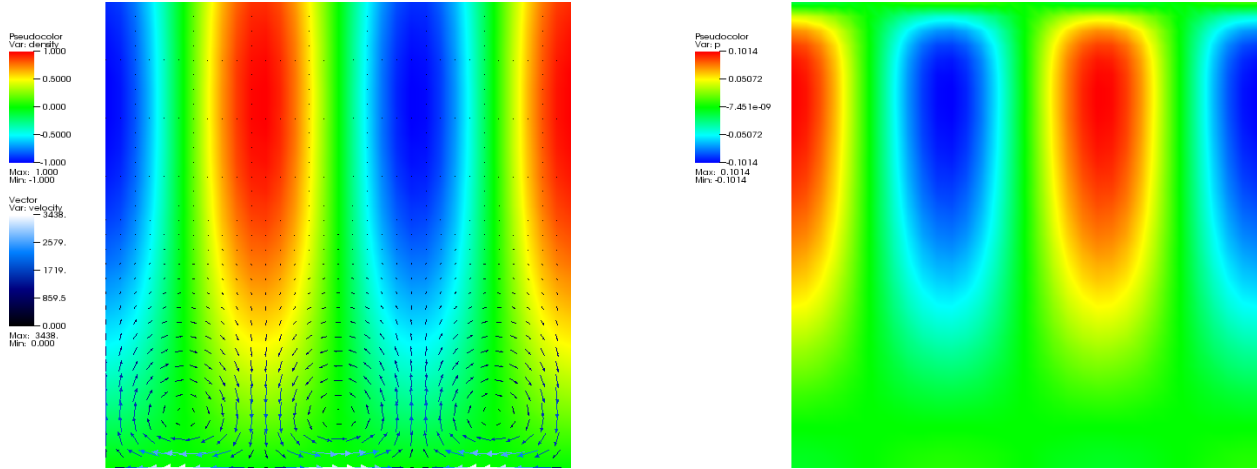


Figure 5: SolKz Stokes benchmark. Left: The density perturbation field and overlaid to it some velocity vectors. The viscosity grows exponentially in the vertical direction, leading to small velocities at the top despite the large density variations. Right: The pressure.

```
subsection Discretization
  set Stokes velocity polynomial degree      = 2
  set Use locally conservative discretization = false
end

subsection Mesh refinement
  set Initial adaptive refinement            = 0
  set Initial global refinement              = 4
end

##### Parameters describing the what to do with the solution

subsection Postprocess
  set List of postprocessors = DuretzEtAl error , visualization
end
```

The output when running ASPECT on this parameter file looks similar to the one shown for the SolCx case. The solution when computed with one more level of global refinement is visualized in Fig. 5.

### 6.2.3 The “inclusion” Stokes benchmark

The “inclusion” benchmark again solves a problem with a discontinuous viscosity, but this time the viscosity is chosen in such a way that the discontinuity is along a circle. This ensures that, unlike in the SolCx benchmark discussed above, the discontinuity in the viscosity never aligns to cell boundaries, leading to much larger difficulties in obtaining an accurate representation of the pressure. Specifically, the almost discontinuous pressure along this interface leads to oscillations in the numerical solution. This can be seen in the visualizations shown in Fig. 6. As before, for details we refer to [DMGT11]. The analytic solution against which we compare is given in [SP03]. An extensive discussion of convergence properties is given in [KHB12].

As before, the benchmark can be run with a small variation of the input files already discussed above:

```
##### Global parameters
```

Revisit this once we have the machinery in place to choose non-zero boundary conditions in a more elegant way.

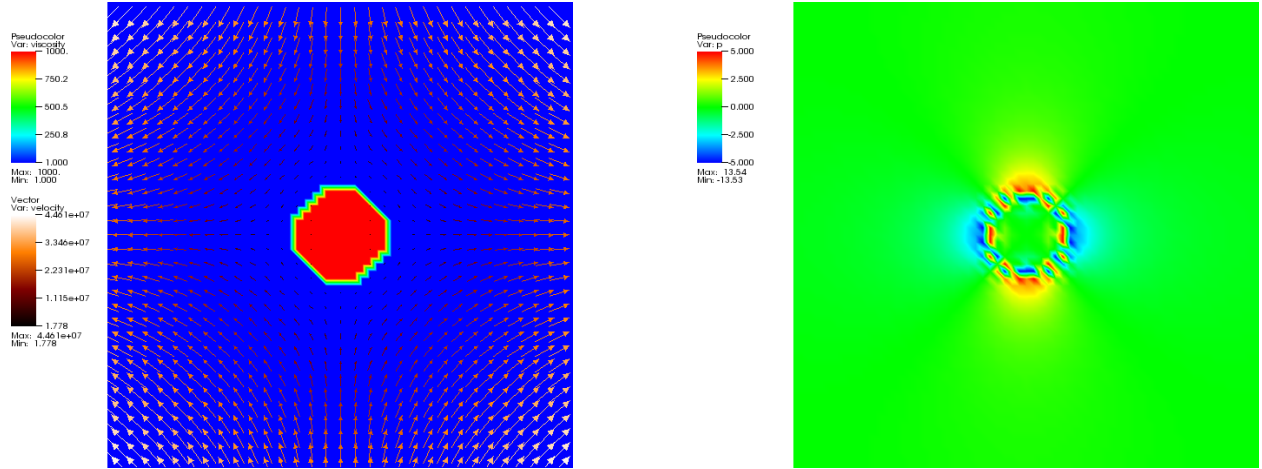


Figure 6: Inclusion Stokes benchmark. Left: The viscosity field when interpolated onto the mesh (internally, the “exact” viscosity field – large inside a circle, small outside – is used), and overlaid to it some velocity vectors. Right: The pressure with its oscillations along the interface. The oscillations become more localized and smaller as the mesh is refined.

```

set Dimension = 2

set Start time = 0
set End time = 0

set Output directory = output
set Pressure normalization = volume

##### Parameters describing the model

subsection Geometry model
  set Model name = box

  subsection Box
    set X extent = 2
    set Y extent = 2
  end
end

subsection Model settings
  set Prescribed velocity boundary indicators = 0,1,2,3
  set Tangential velocity boundary indicators =
  set Zero velocity boundary indicators =
end

subsection Material model
  set Model name = Inclusion

  subsection Inclusion
    set Viscosity jump = 1e3
  end
end

```



```

subsection Gravity model
  set Model name = vertical
end

##### Parameters describing the temperature field

subsection Boundary temperature model
  set Model name = box
end

subsection Initial conditions
  set Model name = perturbed box
end

##### Parameters describing the discretization

subsection Discretization
  set Stokes velocity polynomial degree      = 2
  set Use locally conservative discretization = false
end

subsection Mesh refinement
  set Initial adaptive refinement            = 0
  set Initial global refinement              = 6
end

##### Parameters describing the what to do with the solution

subsection Postprocess
  set List of postprocessors = DuretzEtAl error , visualization
end

```

## 7 Extending Aspect

ASPECT is designed to be an extensible code. In particular, the program uses a plugin architecture in which it is trivial to replace or extend certain components of the program:

- the material description,
- the geometry,
- the gravity description,
- the initial conditions,
- the boundary conditions,
- the functions that postprocess the solution.

We will discuss the way this is achieved in Section 7.1. Changing the core functionality, i.e., the basic equations (1)–(3), and how they are solved is arguably more involved. We will discuss this in Section 7.2.

In either of these two cases, you will need to extend the source code of the program. Since ASPECT is written in C++ using the DEAL.II library, you will have to be proficient in C++. You will also likely have to familiarize yourself with this library for which there is an extensive amount of documentation:

- The manual at <http://www.dealii.org/developer/doxygen/deal.II/index.html> that describes in detail what every class, function and variable in DEAL.II does.
- A collection of modules at <http://www.dealii.org/developer/doxygen/deal.II/modules.html> that give an overview of whole groups of classes and functions and how they work together to achieve their goal.
- The DEAL.II tutorial at <http://www.dealii.org/developer/doxygen/tutorial/index.html> that provides a step-by-step introduction to the library using a sequence of several dozen programs that introduce gradually more complex topics. In particular, you will learn DEAL.II's way of *dimension independent programming* that allows you to write the program once, test it in 2d, and run the exact same code in 3d without having to debug it a second time.
- The step-31 and step-32 tutorial programs at [http://www.dealii.org/developer/doxygen/deal.II/step\\_31.html](http://www.dealii.org/developer/doxygen/deal.II/step_31.html) and [http://www.dealii.org/developer/doxygen/deal.II/step\\_32.html](http://www.dealii.org/developer/doxygen/deal.II/step_32.html) from which ASPECT directly descends.
- The DEAL.II Frequently Asked Questions at [http://dealii.sourceforge.net/index.php/Deal.II\\_Questions\\_and\\_Answers](http://dealii.sourceforge.net/index.php/Deal.II_Questions_and_Answers) that also have extensive sections on developing code with DEAL.II as well as on debugging. It also answers a number of questions we frequently get about the use of C++ in DEAL.II.
- Several other parts of the DEAL.II website at <http://www.dealii.org/> also have information that may be relevant if you dive deeper into developing code. If you have questions, the mailing lists at <http://www.dealii.org/mail.html> are also of general help.
- A general overview of DEAL.II is also provided in the paper [BHK07].

As a general note, by default ASPECT utilizes a DEAL.II feature called *debug mode*, see also the introduction to this topic in Section 4.3. If you develop code, you will definitely want this feature to be on, as it will capture the vast majority of bugs you will invariably introduce in your code.

When you write new functionality and run the code for the first time, you will almost invariably first have to deal with a number of these assertions that point out problems in your code. While this may be annoying at first, remember that these are actual bugs in your code that have to be fixed anyway and that are much easier to find if the program aborts than if you have to go by their more indirect results such as wrong answers. The Frequently Asked Questions at [http://dealii.sourceforge.net/index.php/Deal.II\\_Questions\\_and\\_Answers](http://dealii.sourceforge.net/index.php/Deal.II_Questions_and_Answers) contain a section on how to debug DEAL.II programs.

The downside of debug mode, as mentioned before, is that it makes the program much slower. Consequently, once you are confident that your program actually does what it is intended to do – **but no earlier!** –, you may want to switch to optimized mode that links ASPECT with a version of the DEAL.II libraries that uses compiler optimizations and that does not contain the `assert` statements discussed above. This switch can be facilitated by editing the top of the ASPECT `Makefile` and recompiling the program.

In addition to these general comments, ASPECT is itself extensively documented. You can find documentation on all classes, functions and namespaces starting from the [doc/doxygen/index.html](http://www.dealii.org/developer/doxygen/index.html) page.

## 7.1 Materials, geometries, gravitation and other aspects of the model: plugins

The most common modification you will probably want to do to ASPECT are to switch to a different material model (i.e., have different values of functional dependencies for the coefficients  $\eta, \rho, C_p, \dots$  discussed in Section 2.2); change the geometry; change the direction and magnitude of the gravity vector  $\mathbf{g}$ ; or change the initial and boundary conditions.

To make this as simple as possible, all of these parts of the program have been separated into modules that can be replaced quickly and where it is simple to add a new implementation and make it available to the rest of the program and the input parameter file. The way this is achieved is through the following two steps:

- The rest of the program only communicates with material models, geometry descriptions, etc., through a simple and very basic interface. These interfaces are declared in the files `include/aspect/material_model/interface.h`, `include/aspect/geometry_model/interface.h`, etc., header files. These classes are always called `Interface`, are located in namespaces that identify their purpose, and their documentation can be found from the general class overview in [doc/doxygen/classes.html](http://doc/doxygen/classes.html).

To show an example of a rather minimal case, here is the declaration of the `aspect::GravityModel::Interface` class (documentation comments have been removed):

```
class Interface
{
public:
    virtual ~Interface();

    virtual
    Tensor<1,dim>
    gravity_vector (const Point<dim> &position) const = 0;

    static void declare_parameters (ParameterHandler &prm);

    virtual void parse_parameters (ParameterHandler &prm);
};
```

If you want to implement a new model for gravity, you just need to write a class that derives from this base class and implements the `gravity_vector` function. If your model wants to read parameters from the input file, you also need to have functions called `declare_parameters` and `parse_parameters` in your class with the same signatures as the ones above. On the other hand, if the new model does not need any run-time parameters, you do not need to overload these functions.<sup>11</sup>

Each of the categories above that allow plugins have several implementations of their respective interfaces that you can use to get an idea how to implement a new model.

- At the end of the file where you implement your new model, you need to have a call to the macro `ASPECT_REGISTER_GRAVITY_MODEL`. For example, let us say that you had implemented a gravity model that takes actual gravimetric readings from the GRACE satellites into account, and had put everything that is necessary into a class `aspect::GravityModel::GRACE`. Then you need a statement like this at the bottom of the file:

```
ASPECT_REGISTER_GRAVITY_MODEL
(GRACE,
 "grace",
 "A_gravity_model_derived_from_GRACE"
 "data. _Run-time_parameters_are_read_from_the_parameter_"
 "file_in_subsection_'Radial_constant'." );
```

Here, the first argument to the macro is the name of the class. The second is the name by which this model can be selected in the parameter file. And the third one is a documentation string that describes the purpose of the class (see, for example, Section 5.10 for an example of how existing models describe themselves).

This little piece of code ensures several things: (i) That the parameters this class declares are known when reading the parameter file. (ii) That you can select this model (by the name “grace”) via the run-time parameter `Gravity model/Model name`. (iii) That ASPECT can create an object of this kind when selected in the parameter file.

<sup>11</sup>At first glance one may think that only the `parse_parameters` function can be overloaded since `declare_parameters` is not virtual. However, while the latter is called by the class that manages plugins through pointers to the interface class, the former function is called essentially at the time of registering a plugin, from code that knows the actual type and name of the class you are implementing. Thus, it can call the function – if it exists in your class, or the default implementation in the base class if it doesn’t – even without it being declared as virtual.

Note that you need not announce the existence of this class in any other part of the code: Everything should just work automatically. Note also that the existing implementations of models of the gravity and other interfaces declare the class in a header file and define the member functions in a .cc file. This is done so that these classes show up in our doxygen-generated documentation, but it is not necessary: you can put your entire class declaration and implementation into a single file as long as you call the macro discussed above on it. This single file is all you need to touch to add a new model.

The procedure for the other areas where plugins are supported works essentially the same, with the obvious change in namespace for the interface class and macro name.

In the following, we will discuss the requirements for individual plugins.

### 7.1.1 Material models

The material model is responsible for describing the various coefficients in the equations that ASPECT solvers. To implement a new material model, you need to overload the `aspect::MaterialModel::Interface` class and use the `ASPECT_REGISTER_MATERIAL_MODEL` macro to register your new class. The implementation of the new class should be in namespace `aspect::MaterialModel`.

Specifically, your new class needs to implement the basic interface:

```
template <int dim>
class aspect::MaterialModel::Interface
{
public:
    // Physical parameters used in the basic equations
    virtual double viscosity (const double temperature,
                             const double pressure,
                             const SymmetricTensor<2,dim> &strain_rate,
                             const Point<dim> &position) const = 0;

    virtual double density (const double temperature,
                           const double pressure,
                           const Point<dim> &position) const = 0;

    virtual double compressibility (const double temperature,
                                    const double pressure,
                                    const Point<dim> &position) const = 0;

    virtual double specific_heat (const double temperature,
                                  const double pressure,
                                  const Point<dim> &position) const = 0;

    virtual double thermal_expansion_coefficient (const double temperature,
                                                  const double pressure,
                                                  const Point<dim> &position) const;

    virtual double thermal_conductivity (const double temperature,
                                         const double pressure,
                                         const Point<dim> &position) const
    = 0;

    // Qualitative properties one can ask a material model
    virtual bool
    viscosity_depends_on (const NonlinearDependence::Dependence dependence) const = 0;

    virtual bool
    density_depends_on (const NonlinearDependence::Dependence dependence) const = 0;

    virtual bool
    compressibility_depends_on (const NonlinearDependence::Dependence dependence) const = 0;

    virtual bool
    specific_heat_depends_on (const NonlinearDependence::Dependence dependence) const = 0;
```

```

virtual bool
thermal_conductivity_depends_on (const NonlinearDependence::Dependence dependence) const = 0;

virtual bool is_compressible () const = 0;

// Partial derivatives of physical parameters
virtual double
viscosity_derivative (const double          temperature ,
                     const double          pressure ,
                     const Point<dim>      &position ,
                     const NonlinearDependence::Dependence dependence) const;

virtual double
density_derivative (const double          temperature ,
                   const double          pressure ,
                   const Point<dim>      &position ,
                   const NonlinearDependence::Dependence dependence) const;

virtual double
compressibility_derivative (const double          temperature ,
                           const double          pressure ,
                           const Point<dim>      &position ,
                           const NonlinearDependence::Dependence dependence) const;

virtual double
specific_heat_derivative (const double          temperature ,
                          const double          pressure ,
                          const Point<dim>      &position ,
                          const NonlinearDependence::Dependence dependence) const;

virtual double
thermal_conductivity_derivative (const double          temperature ,
                                const double          pressure ,
                                const Point<dim>      &position ,
                                const NonlinearDependence::Dependence dependence) const;

// Reference quantities
virtual double reference_viscosity () const = 0;

virtual double reference_density () const = 0;

virtual double reference_thermal_expansion_coefficient () const = 0;

// Auxiliary material properties used for postprocessing
virtual
double
seismic_Vp (const double          temperature ,
            const double          pressure) const;

virtual
double
seismic_Vs (const double          temperature ,
            const double          pressure) const;

virtual
unsigned int
thermodynamic_phase (const double          temperature ,
                    const double          pressure) const;

// Functions used in dealing with run-time parameters
static
void
declare_parameters (ParameterHandler &prm);

virtual

```

```

    void
    parse_parameters (ParameterHandler &prm);
};

```

Here, the first set of functions refer to the coefficients  $\eta, C_p, k, \rho$  in equations (1)–(3), each as a function of temperature, pressure, position and, in the case of the viscosity, the strain rate. Implementations of these methods may of course choose to ignore dependencies on any of these arguments. The second set of functions describes the nonlinear dependence of the various coefficients on pressure, temperature, or strain rate, and the next block then provides the numerical values of these dependencies. This information will be used in future versions of ASPECT to implement a fully nonlinear solution scheme based on, for example, a Newton iteration. The remaining functions are used in postprocessing as well as handling run-time parameters. The exact meaning of these member functions is documented in the [aspect::MaterialModel::Interface class documentation](#). Note that some of the functions listed above have a default implementation, as discussed on the documentation page just mentioned.

The function `is_compressible` returns whether we should consider the material as compressible or not, see Section 2.6.1 on the Boussinesq model. As discussed there, incompressibility as described by this function does not necessarily imply that the density is constant; rather, it may still depend on temperature or pressure. In the current context, compressibility simply means whether we should solve the continuity equation as  $\nabla \cdot (\rho \mathbf{u}) = 0$  (compressible Stokes) or as  $\nabla \cdot \mathbf{u} = 0$  (incompressible Stokes).

The purpose of the last two functions has been discussed in the general overview of plugins above.

### 7.1.2 Geometry models

The geometry model is responsible for describing the domain in which we want to solve the equations. A domain is described in DEAL.II by a coarse mesh and, if necessary, an object that characterizes the boundary. Together, these two suffice to reconstruct any domain by adaptively refining the coarse mesh and placing new nodes generated by refining cells onto the surface described by the boundary object. The geometry model is also responsible to describe to the rest of the code which parts of the boundary represent Dirichlet-type (fixed temperature) or Neumann-type (no heat flux) boundaries for the temperature, and where the velocity is considered zero or tangential to the boundary. This information is encoded in functions that return which boundary indicators represent these types of boundaries; in DEAL.II, a boundary indicator is a number attached to each piece of the boundary that can be used to represent the type of boundary a piece belongs to.

To implement a new geometry model, you need to overload the `aspect::GeometryModel::Interface` class and use the `ASPECT_REGISTER_GEOMETRY_MODEL` macro to register your new class. The implementation of the new class should be in namespace `aspect::GeometryModel`.

Specifically, your new class needs to implement the following basic interface:

```

template <int dim>
class aspect::GeometryModel::Interface
{
public:
    virtual
    void
    create_coarse_mesh (parallel::distributed::Triangulation<dim> &coarse_grid) const = 0;

    virtual
    double
    length_scale () const = 0;

    virtual
    double depth(const Point<dim> &position) const = 0;

    virtual
    Point<dim> representative_point(const double depth) const = 0;

    virtual

```

```

    double maximal_depth() const = 0;

    virtual
    std::set<types::boundary_id_t>
    get_used_boundary_indicators () const = 0;

    static
    void
    declare_parameters (ParameterHandler &prm);

    virtual
    void
    parse_parameters (ParameterHandler &prm);
};

```

The kind of information these functions need to provide is extensively discussed in the documentation of this interface class at [aspect::GeometryModel::Interface](#). The purpose of the last two functions has been discussed in the general overview of plugins above.

### 7.1.3 Gravity models

The gravity model is responsible for describing the magnitude and direction of the gravity vector at each point inside the domain. To implement a new gravity model, you need to overload the `aspect::GravityModel::Interface` class and use the `ASPECT_REGISTER_GRAVITY_MODEL` macro to register your new class. The implementation of the new class should be in namespace `aspect::GravityModel`.

Specifically, your new class needs to implement the following basic interface:

```

template <int dim>
class aspect::GravityModel::Interface
{
public:
    virtual
    Tensor<1,dim>
    gravity_vector (const Point<dim> &position) const = 0;

    static
    void
    declare_parameters (ParameterHandler &prm);

    virtual
    void
    parse_parameters (ParameterHandler &prm);
};

```

The kind of information these functions need to provide is discussed in the documentation of this interface class at [aspect::GravityModel::Interface](#) but is likely already obvious. The purpose of the last two functions has been discussed in the general overview of plugins above.

### 7.1.4 Initial conditions

The initial conditions model is responsible for describing the initial temperature distribution throughout the domain. It essentially has to provide a function that for each point can return the initial temperature. Note that the model (1)–(3) does not require initial values for the pressure or velocity. However, if coefficients are nonlinear, one can significantly reduce the number of initial nonlinear iterations if a good guess for them is available; consequently, ASPECT initializes the pressure with the adiabatically computed hydrostatic pressure, and a zero velocity. Neither of these two has to be provided by the objects considered in this section.

To implement a new initial conditions model, you need to overload the `aspect::InitialConditions::Interface` class and use the `ASPECT_REGISTER_INITIAL_CONDITIONS` macro to register your new class. The implementation of the new class should be in namespace `aspect::InitialConditions`.

Specifically, your new class needs to implement the following basic interface:

```
template <int dim>
class aspect::InitialConditions::Interface
{
public:
    virtual
    double
    initial_temperature (const Point<dim> &position) const = 0;

    static
    void
    declare_parameters (ParameterHandler &prm);

    virtual
    void
    parse_parameters (ParameterHandler &prm);
};
```

The meaning of the first class should be clear. The purpose of the last two functions has been discussed in the general overview of plugins above.

### 7.1.5 Temperature boundary conditions

The boundary conditions are responsible for describing the temperature values at those parts of the boundary at which the temperature is fixed (see Section 7.1.2 for how it is determined which parts of the boundary this applies to).

To implement a new boundary conditions model, you need to overload the `aspect::BoundaryTemperature::Interface` class and use the `ASPECT_REGISTER_BOUNDARY_TEMPERATURE_MODEL` macro to register your new class. The implementation of the new class should be in namespace `aspect::BoundaryTemperature`.

Specifically, your new class needs to implement the following basic interface:

```
template <int dim>
class aspect::BoundaryTemperature::Interface
{
public:
    virtual
    double
    temperature (const GeometryModel::Interface<dim> &geometry_model,
                const unsigned int boundary_indicator,
                const Point<dim> &location) const = 0;

    virtual
    double minimal_temperature () const = 0;

    virtual
    double maximal_temperature () const = 0;

    static
    void
    declare_parameters (ParameterHandler &prm);

    virtual
    void
    parse_parameters (ParameterHandler &prm);
};
```

The first of these functions needs to provide the fixed temperature at the given point. The geometry model and the boundary indicator of the particular piece of boundary on which the point is located is also given as a hint in determining where this point may be located; this may, for example, be used to determine if a point is on the inner or outer boundary of a spherical shell. The remaining functions are obvious, and are



also discussed in the documentation of this interface class at [aspect::BoundaryTemperature::Interface](#). The purpose of the last two functions has been discussed in the general overview of plugins above.

### 7.1.6 Postprocessors: Evaluating the solution after each time step

Postprocessors are arguably the most complex and powerful of the plugins available in ASPECT. They are executed once at the end of each time step and, unlike all the other plugins discussed above, there can be an arbitrary number of active postprocessors in the same program (for the plugins discussed in previous sections it was clear that there is always exactly one material model, geometry model, etc.).

**Motivation.** The original motivation for postprocessors is that the goal of a simulation is of course not the simulation itself, but that we want to do something with the solution. Examples for already existing postprocessors are:

- Generating output in file formats that are understood by visualization programs. This is facilitated by the [aspect::Postprocess::Visualization](#) class.
- Computing statistics about the velocity field (e.g., computing minimal, maximal, and average velocities), temperature field (minimal, maximal, and average temperatures), or about the heat fluxes across boundaries of the domain. This is provided by the [aspect::Postprocess::VelocityStatistics](#), [aspect::Postprocess::TemperatureStatistics](#), [aspect::Postprocess::HeatFluxStatistics](#) classes, respectively.

Since writing this text, there may have been other additions as well.

However, postprocessors can be more powerful than this. For example, while the ones listed above are by and large stateless, i.e., they do not carry information from one invocation at one timestep to the next invocation,<sup>12</sup> there is nothing that prohibits postprocessors from doing so. For example, the following ideas would fit nicely into the postprocessor framework:

- *Passive tracers:* If one would like to follow the trajectory of material as it is advected along with the flow field, one technique is to use tracer particles. To implement this, one would start with an initial population of particles distributed in a certain way, for example close to the core-mantle boundary. At the end of each time step, one would then need to move them forward with the flow field by one time increment. As long as these particles do not affect the flow field (i.e., they do not carry any information that feeds into material properties; in other words, they are *passive*), their location could well be stored in a postprocessor object and then be output in periodic intervals for visualization. In fact, such a passive tracer postprocessor is already available.
- *Surface or crustal processes:* Another possibility would be to keep track of surface or crustal processes induced by mantle flow. An example would be to keep track of the thermal history of a piece of crust by updating it every time step with the heat flux from the mantle below. One could also imagine integrating changes in the surface topography by considering the surface divergence of the surface velocity computed in the previous time step: if the surface divergence is positive, the topography is lowered, eventually forming a trench; if the divergence is negative, a mountain belt eventually forms.

In all of these cases, the essential limitation is that postprocessors are *passive*, i.e., that they do not affect the simulation but only observe it.

**The statistics file.** Postprocessors fall into two categories: ones that produce lots of output every time they run (e.g., the visualization postprocessor), and ones that only produce one, two, or in any case a small and fixed number of often numerical results (e.g., the postprocessors computing velocity, temperature, or heat flux statistics). While the former are on their own in implementing how they want to store their data

---

<sup>12</sup>This is not entirely true. The visualization plugin keeps track of how many output files it has already generated, so that they can be numbered consecutively.

to disk, there is a mechanism in place that allows the latter class of postprocessors to store their data into a central file that is updated at the end of each time step, after all postprocessors are run.

To this end, the function that executes each of the postprocessors is given a reference to a `dealii::TableHandler` object that allows to store data in named columns, with one row for each time step. This table is then stored in the `statistics` file in the directory designated for output in the input parameter file. It allows for easy visualization of trends over all time steps. To see how to put data into this statistics object, take a look at the existing postprocessor objects.

Note that the data deposited into the statistics object need not be numeric in type, though it often is. An example of text-based entries in this table is the visualization class that stores the name of the graphical output file written in a particular time step.

**Implementing a postprocessor.** Ultimately, implementing a new postprocessor is no different than any of the other plugins. Specifically, you'll have to write a class that overloads the `aspect::Postprocess::Interface` base class and use the `ASPECT_REGISTER_POSTPROCESSOR` macro to register your new class. The implementation of the new class should be in namespace `aspect::Postprocess`.

In reality, however, implementing new postprocessors is often more difficult. Primarily, this difficulty results from two facts:

- Postprocessors are not self-contained (only providing information) but in fact need to access the solution of the model at each time step. That is, of course, the purpose of postprocessors, but it requires that the writer of a plugin has a certain amount of knowledge of how the solution is computed by the main `Simulator` class, and how it is represented in data structures. To alleviate this somewhat, and to insulate the two worlds from each other, postprocessors do not directly access the data structures of the simulator class. Rather, in addition to deriving from the `aspect::Postprocess::Interface` base class, postprocessors also derive from the `aspect::Postprocess::SimulatorAccess` class that has a number of member functions postprocessors can call to obtain read-only access to some of the information stored in the main class of ASPECT. See [the documentation of this class](#) to see what kind of information is available to postprocessors.
- Writing a new postprocessor typically requires a fair amount of knowledge how to leverage the DEAL.II library to extract information from the solution. The existing postprocessors are certainly good examples to start from in trying to understand how to do this.

Given these comments, the interface a postprocessor class has to implement is rather basic:

```
template <int dim>
class aspect::Postprocess::Interface
{
public:
    virtual
    std::pair<std::string, std::string>
    execute (TableHandler &statistics) = 0;

    virtual
    void
    save (std::map<std::string, std::string> &status_strings) const;

    virtual
    void
    load (const std::map<std::string, std::string> &status_strings);

    static
    void
    declare_parameters (ParameterHandler &prm);

    virtual
    void
    parse_parameters (ParameterHandler &prm);
```

```
};
```

The purpose of these functions is described in detail in the documentation of the [aspect::Postprocess::Interface](#) class. While the first one is responsible for evaluating the solution at the end of a time step, the `save/load` functions are used in checkpointing the program and restarting it at a previously saved point during the simulation. The first of these functions therefore needs to store the status of the object as a string under a unique key in the database described by the argument, while the latter function restores the same state as before by looking up the status string under the same key. The default implementation of these functions is to do nothing; postprocessors that do have non-static member variables that contain a state need to overload these functions.

Give a simple example, e.g. evaluating a point value.

## 7.2 Extending the basic solver

The core functionality of the code, i.e., that part of the code that implements the time stepping, assembles matrices, solves linear and nonlinear systems, etc., is in the `aspect::Simulator` class (see the [doxygen documentation of this class](#)). Since the implementation of this class has more than 3,000 lines of code, it is split into several files that are all located in the `source/simulator` directory. Specifically, functionality is split into the following files:

- `source/simulator/core.cc`: This file contains the functions that drive the overall algorithm (in particular `Simulator::run`) through the main time stepping loop and the functions immediately called by `Simulator::run`.
- `source/simulator/assembly.cc`: This is where all the functions are located that are related to assembling linear systems.
- `source/simulator/solver.cc`: This file provides everything that has to do with solving and preconditioning the linear systems.
- `source/simulator/initial_conditions.cc`: The functions in this file deal with setting initial conditions for all variables.
- `source/simulator/checkpoint_restart.cc`: The location of functionality related to saving the current state of the program to a set of files and restoring it from these files again.
- `source/simulator/helper_functions.cc`: This file contains a set of functions that do the odd thing in support of the rest of the simulator class.
- `source/simulator/parameters.cc`: This is where we define and read run-time parameters that pertain to the top-level functionality of the program.

Obviously, if you want to extend this core functionality, it is useful to first understand the numerical methods this class implements. To this end, take a look at the paper that describes these methods, see [\[KHB12\]](#). Further, there are two predecessor programs whose extensive documentation is at a much higher level than the one typically found inside ASPECT itself, since they are meant to teach the basic components of convection simulators as part of the DEAL.II tutorial:

- The step-31 program at [http://www.dealii.org/developer/doxygen/deal.II/step\\_31.html](http://www.dealii.org/developer/doxygen/deal.II/step_31.html): This program is the first version of a convection solver. It does not run in parallel, but it introduces many of the concepts relating to the time discretization, the linear solvers, etc.
- The step-32 program at [http://www.dealii.org/developer/doxygen/deal.II/step\\_32.html](http://www.dealii.org/developer/doxygen/deal.II/step_32.html): This is a parallel version of the step-31 program that already solves on a spherical shell geometry. The focus of the documentation in this program is on the techniques necessary to make the program run in parallel, as well as some of the consequences of making things run with realistic geometries, material models, etc.

Neither of these two programs is nearly as modular as ASPECT, but that was also not the goal in creating them. They will, however, serve as good introductions to the general approach for solving thermal convection problems.

**Note:** Neither this manual, nor the documentation in ASPECT makes much of an attempt at teaching how to use the DEAL.II library upon which ASPECT is built. Nevertheless, you will likely have to know at least the basics of DEAL.II to successfully work on the ASPECT code. We refer to the resources listed at the beginning of this section as well as references [BHK07, BHK12].

## 8 Future plans for Aspect

We have a number of near-term plans for ASPECT that we hope to implement soon:

- *Iterating out the nonlinearity:* In the current version of ASPECT, we use the velocity, pressure and temperature of the previous time step to evaluate the coefficients that appear in the flow equations (1)–(2); and the velocity and pressure of the current time step as well as the previous time step’s temperature to evaluate the coefficients in the temperature equation (3). This is an appropriate strategy if the model is not too nonlinear; however, it introduces inaccuracies and limits the size of the time step if coefficients strongly depend on the solution variables.

To avoid this, one can iterate out the equations using either a fixed point or Newton scheme. Both approaches ensure that at the end of a time step, the values of coefficients and solution variables are consistent. On the other hand, one may have to solve the linear systems that describe a time step more than once, increasing the computational effort.

We have started implementing such methods using a testbase code, based on earlier experiments by Jennifer Worthen [Wor12]. We hope to implement this feature in ASPECT early in 2012.

- *Faster 3d computations:* Whichever way you look at it, 3d computations are expensive. In parallel computations, the Stokes solve currently takes upward of 90% of the overall wallclock time, suggesting an obvious target for improvements based on better algorithms as well as from profiling the code to find hot spots. In particular, playing with better solver and/or preconditioner options would seem to be a useful goal.
- *Particle-based methods:* It is often useful to employ particle tracers to visualize where material is being transported. While conceptually simple, their implementation is made difficult in parallel computations if particles cross the boundary between parts of the regions owned by individual processors, as well as during re-partitioning the mesh between processors following mesh refinement. Eric Heien is working on an implementation of such passive tracers.
- *More realistic material models:* The number of material models available in ASPECT is currently relatively small. Obviously, how realistic a simulation is depends on how realistic a material model is. We hope to obtain descriptions of more realistic material descriptions over time, either given analytically or based on table-lookup of material properties.
- *Incorporating latent heat effects:* Real materials undergo phase transitions at certain pressures and temperatures, and these phase transitions release or take up energy (i.e., heat). The terms that need to be added to the temperature equation (3) are not very difficult but one needs a description of the latent heat based on the Clapeyron slope as a function of temperature and pressure [CY85, STO01], which we currently don’t have. If someone contributes such a description we’ll be happy to add the relevant terms into the model.
- *Prescribing boundary velocities:* One might want to prescribe the velocity at the outer boundary, for example using GPS-inferred plate velocities.

- *Melting*: An important part of mantle behavior is melting. Melting not only affects the properties of the material such as density or viscosity, but it also leads to chemical segregation and, in fact, to the flow of two different fluids (the melt and the rock matrix) relative to each other. Modeling this additional process would yield significant insight.
- *Converting output into seismic velocities*: The predictions of mantle convection codes are often difficult to verify experimentally. On the other hand, simulations can be used to predict a seismic signature of the earth mantle – for example the location of transition zones that can be observed using seismic imaging. To facilitate such comparisons, it is of interest to output not only the primary solution variables but also convert them into the primary quantity visible in seismic imaging: compressive and shear wave velocities. Implementing this should be relatively straightforward if given a formula or table that expresses velocities in terms of the variables computed by ASPECT.

To end this section, let us repeat something already stated in the introduction:

**Note:** ASPECT is a community project. As such, we encourage contributions from the community to improve this code over time. Obvious candidates for such contributions are implementations of new plugins as discussed in Section 7.1 since they are typically self-contained and do not require much knowledge of the details of the remaining code. Obviously, however, we also encourage contributions to the core functionality in any form!

## 9 Finding answers to more questions

If you have questions that go beyond this manual, there are a number of resources:

- For questions on the source code of Aspect, portability, installation, etc., use the ASPECTdevelopment mailing list at [aspect-devel@geodynamics.org](mailto:aspect-devel@geodynamics.org). Information about this mailing list is provided at <http://geodynamics.org/cgi-bin/mailman/listinfo/aspect-devel>. This mailing list is where the Aspect developers all hang out.
- Aspect is primarily based on the deal.II library (the dependency on Trilinos and p4est is primarily through deal.II, and not directly visible in the Aspect source code). If you have particular questions about deal.II, contact the mailing lists described at <http://www.dealii.org/mail.html>.
- In case of more general questions about mantle convection, you can contact the CIG mantle convection mailing lists at [cig-mc@geodynamics.org](mailto:cig-mc@geodynamics.org). Information about this mailing list is provided at <http://geodynamics.org/cgi-bin/mailman/listinfo/cig-MC>.
- If you have specific questions about Aspect that are not suitable for public and archived mailing lists, you can contact the primary developers:
  - Wolfgang Bangerth: [bangerth@math.tamu.edu](mailto:bangerth@math.tamu.edu).
  - Timo Heister: [heister@math.tamu.edu](mailto:heister@math.tamu.edu).

## References

- [BHK07] W. Bangerth, R. Hartmann, and G. Kanschat. deal.II – a general purpose object oriented finite element library. *ACM Trans. Math. Softw.*, 33(4):24, 2007.
- [BHK12] W. Bangerth, T. Heister, and G. Kanschat. *deal.II Differential Equations Analysis Library, Technical Reference*, 2012. <http://www.dealii.org/>.
- [BRV<sup>+</sup>04] J. Badro, J.-P. Rueff, G. Vankó, G. Monaco, G. Fiquet, and F. Guyot. Electronic transitions in perovskite: Possible nonconvecting layers in the lower mantle. *Science*, 305:383–386, 2004.
- [BWG11] C. Burstedde, L. C. Wilcox, and O. Ghattas. **p4est**: Scalable algorithms for parallel adaptive mesh refinement on forests of octrees. *SIAM J. Sci. Comput.*, 33(3):1103–1133, 2011.
- [CY85] U. R. Christensen and D. A. Yuen. Layered convection induced by phase transitions. *J. Geoph. Res.*, 90:10291–10300, 1985.
- [DMGT11] T. Duretz, D. A. May, T. V. Garya, and P. J. Tackley. Discretization errors and free surface stabilization in the finite difference and marker-in-cell method for applied geodynamics: A numerical study. *Geoch. Geoph. Geosystems*, 12:Q07004/1–26, 2011.
- [H<sup>+</sup>11] M. A. Heroux et al. Trilinos web page, 2011. <http://trilinos.sandia.gov>.
- [HBH<sup>+</sup>05] M. A. Heroux, R. A. Bartlett, V. E. Howle, R. J. Hoekstra, J. J. Hu, T. G. Kolda, R. B. Lehoucq, K. R. Long, R. P. Pawlowski, E. T. Phipps, A. G. Salinger, H. K. Thornquist, R. S. Tuminaro, J. M. Willenbring, A. Williams, and K. S. Stanley. An overview of the Trilinos project. *ACM Trans. Math. Softw.*, 31:397–423, 2005.
- [KHB12] M. Kronbichler, T. Heister, and W. Bangerth. High accuracy mantle convection simulation through modern numerical methods. *submitted*, 2012.
- [MQL<sup>+</sup>07] L. Moresi, S. Quenette, V. Lemiale, C. Meriaux, B. Appelbe, and H. B. Mühlhaus. Computational approaches to studying non-linear dynamics of the crust and mantle. *Phys. Earth Planet. Interiors*, 163:69–82, 2007.
- [SP03] D.W. Schmid and Y.Y. Podladchikov. Analytical solutions for deformable elliptical inclusions in general shear. *Geophysical Journal International*, 155(1):269–288, 2003.
- [STO01] G. Schubert, D. L. Turcotte, and P. Olson. *Mantle Convection in the Earth and Planets, Part 1*. Cambridge, 2001.
- [Wor12] J. Worthen. *Inverse Problems in Mantle Convection: Models, Algorithms, and Applications*. PhD thesis, University of Texas at Austin, in preparation, 2012.
- [Zho96] S. Zhong. Analytic solution for Stokes’ flow with lateral variations in viscosity. *Geophys. J. Int.*, 124:18–28, 1996.

## Index of run-time parameter entries

The following is a listing of all run-time parameters that can be set in the input parameter file. They are all described in Section 5 and the listed page numbers are where their detailed documentation can be found. A listing of all parameters sorted by the section name in which they are declared is given in the index on page 60 below.

Additional refinement times, 34  
Adiabatic surface temperature, 9, 24  
alpha, 27  
Amplitude, 30  
Angle, 30  
  
beta, 27  
  
CFL number, 25  
Coarsening fraction, 34  
Composition, 32  
Compressible, 32  
ComputePhases, 32  
cR, 27  
  
Dimension, 18  
  
End time, 25  
exponential.P, 34  
exponential.T, 34  
  
Filename for initial geotherm table, 30  
Fixed temperature boundary indicators, 35, 52  
  
Gravity, 32  
  
Include adiabatic heating, 35  
Include shear heating, 35  
Initial adaptive refinement, 34  
Initial global refinement, 34, 41  
Inner radius, 28  
Inner temperature, 26  
  
List of postprocessors, 15, 36, 53  
  
Magnitude, 29  
Model name, 26, 28, 29, 31, 41, 48, 50, 51  
  
Non-dimensional depth, 30  
Number of grouped files, 21, 37  
  
Opening angle, 28  
Outer radius, 29  
  
Outer temperature, 26  
Output directory, 15, 22, 25  
Output format, 19, 37  
  
Path to model data, 32  
Prescribed velocity boundary indicators, 36  
Pressure normalization, 8, 40  
  
Radiogenic heating rate, 36  
Reference density, 31, 32  
Reference specific heat, 31, 33  
Reference temperature, 31, 33  
ReferenceViscosity, 33  
Refinement fraction, 35  
Resume computation, 22, 25  
  
Sigma, 30  
Sign, 30  
Start time, 25  
Stokes velocity polynomial degree, 27, 41  
Strategy, 35  
Surface pressure, 8, 25  
  
Tangential velocity boundary indicators, 36  
Temperature polynomial degree, 27  
Thermal conductivity, 31, 33  
Thermal expansion coefficient, 31, 33  
Time between graphical output, 37  
Time steps between mesh refinement, 35  
  
Use locally conservative discretization, 27  
Use years in output instead of seconds, 7, 26  
  
Viscosity, 32  
Viscosity increase lower mantle, 33  
ViscosityModel, 33  
  
X extent, 28  
  
Y extent, 28  
  
Z extent, 28  
Zero velocity boundary indicators, 36

## Index of run-time parameters with section names

The following is a listing of all run-time parameters, sorted by the section in which they appear. To find entries sorted by their name, rather than their section, see the index on page 59 above.

- Adiabatic surface temperature, 9, 24
- Boundary temperature model
  - Model name, 26
  - Spherical constant
    - Inner temperature, 26
    - Outer temperature, 26
- CFL number, 25
- Dimension, 18
- Discretization
  - Stabilization parameters
    - alpha, 27
    - beta, 27
    - cR, 27
  - Stokes velocity polynomial degree, 27, 41
  - Temperature polynomial degree, 27
  - Use locally conservative discretization, 27
- End time, 25
- Geometry model
  - Box
    - X extent, 28
    - Y extent, 28
    - Z extent, 28
  - Model name, 28, 41, 50
  - Spherical shell
    - Inner radius, 28
    - Opening angle, 28
    - Outer radius, 29
- Gravity model
  - Model name, 29, 41, 51
  - Radial constant
    - Magnitude, 29
- Initial conditions
  - Model name, 29, 51
  - Spherical gaussian perturbation
    - Amplitude, 30
    - Angle, 30
    - Filename for initial geotherm table, 30
    - Non-dimensional depth, 30
    - Sigma, 30
    - Sign, 30
- Material model
  - Model name, 31, 48
  - Simple model
    - Reference density, 31
    - Reference specific heat, 31
    - Reference temperature, 31
    - Thermal conductivity, 31
    - Thermal expansion coefficient, 31
    - Viscosity, 32
  - Table model
    - Composition, 32
    - Compressible, 32
    - ComputePhases, 32
    - Gravity, 32
    - Path to model data, 32
    - Reference density, 32
    - Reference specific heat, 33
    - Reference temperature, 33
    - Thermal conductivity, 33
    - Thermal expansion coefficient, 33
- Mesh refinement
  - Additional refinement times, 34
  - Coarsening fraction, 34
  - Initial adaptive refinement, 34
  - Initial global refinement, 34, 41
  - Refinement fraction, 35
  - Strategy, 35
  - Time steps between mesh refinement, 35
- Model settings
  - Fixed temperature boundary indicators, 35, 52
  - Include adiabatic heating, 35
  - Include shear heating, 35
  - Prescribed velocity boundary indicators, 36
  - Radiogenic heating rate, 36
  - Tangential velocity boundary indicators, 36
  - Zero velocity boundary indicators, 36
- Output directory, 15, 22, 25
- Postprocess
  - Depth average
    - Time between graphical output, 37
  - List of postprocessors, 15, 36, 53
  - Visualization
    - Number of grouped files, 21, 37



Output format, [19](#), [37](#)  
Time between graphical output, [37](#)  
Pressure normalization, [8](#), [40](#)  
Resume computation, [22](#), [25](#)  
Start time, [25](#)  
Surface pressure, [8](#), [25](#)  
Use years in output instead of seconds, [7](#), [26](#)