

1.4 FEM error estimates

- what do we know about errors and convergence of the error when you keep refining the mesh?
- does it help to use finite elements of higher order?

a priori estimates

Cea's lemma:
$$\|u_h - u\| \leq \frac{\alpha}{\kappa} \inf_{\phi \in V_h} \|u - \phi\|$$

For finite element space of degree k and certain assumptions on the geometry (shape-regular, conforming, convex), and $u \in H^{k+1}(\Omega)$:

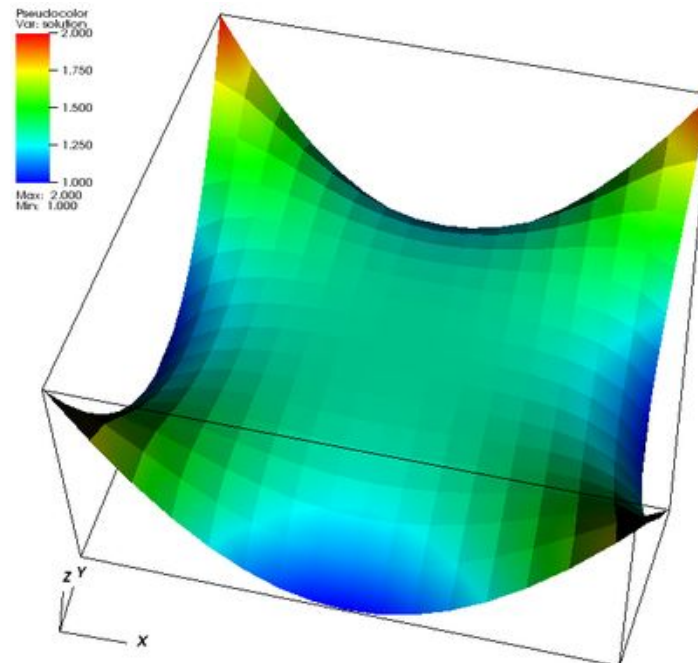
$$\|u - u_h\|_1 \leq Ch^k |u|_{k+1}$$

$$\|u - u_h\| \leq Ch^{k+1} |u|_{k+1}$$

So, for smooth problems and FE degree k , L2 norms converge with order $k+1$, and H1 norm with order k .

1.5 towards LAB 04

- key points:
 - Modified step-4 to check correctness
 - Using the method of manufactured solutions
 - Computing L2 and H1 errors and check convergence orders



Manufactured Solutions

- think of any function $u(x)$, the **exact solution**
- compute right-hand side to fulfill the PDE
- apply boundary conditions of exact solution
- solve the PDE numerically
- finally: measure the error between the computed and exact solution

Computing Errors

- Important for code verification!
- See step-7 for details
- We set up the problem with analytical solution and implement it as a Function<dim>
- Quantities of interest:
 - Break it down as one operation per cell and the “summation” (local and global error)
 - Need quadrature to compute integrals

Computing Errors

Code:

```
Vector<float> difference_per_cell (triangulation.n_active_cells());

VectorTools::integrate_difference (dof_handler,
                                   solution,           // solution vector
                                   Solution<dim>(),      // reference solution
                                   difference_per_cell,
                                   QGauss<dim>(3),      // quadrature
                                   VectorTools::L2_norm); // local norm

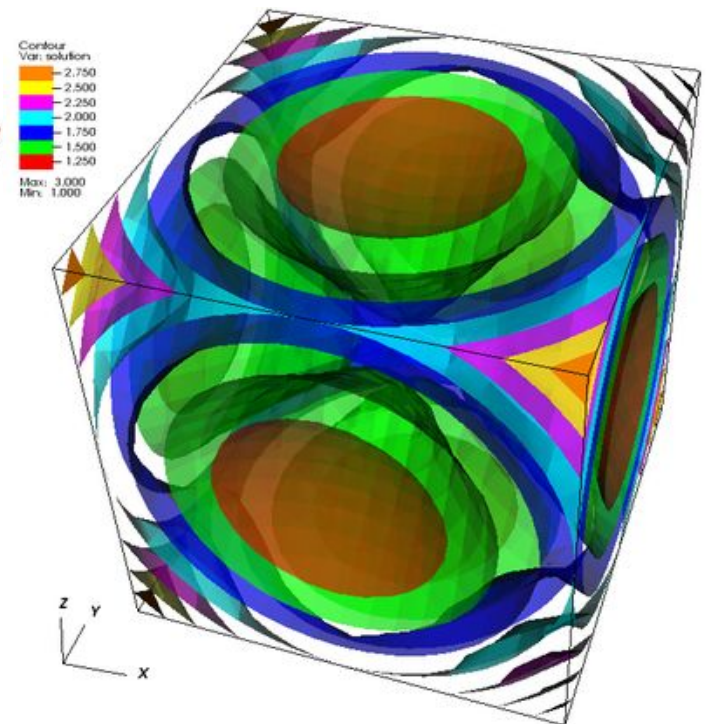
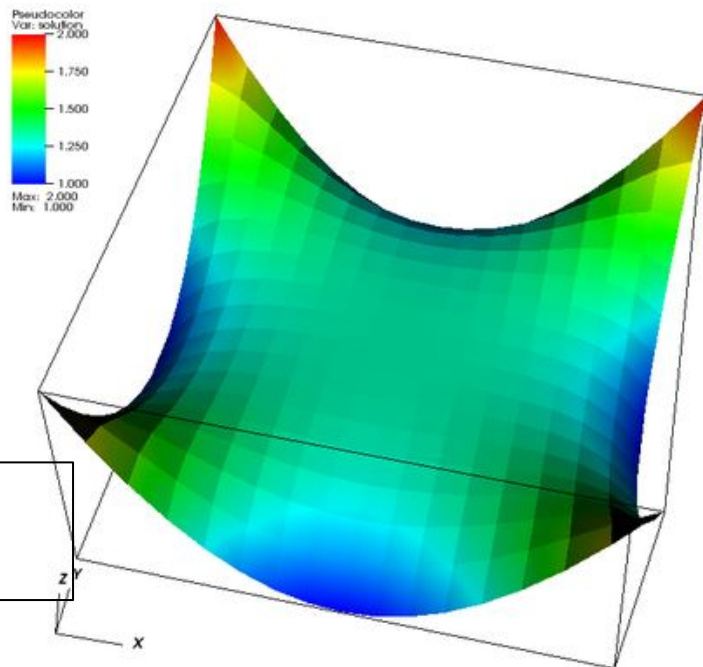
const double L2_error = VectorTools::compute_global_error(triangulation,
                                                           difference_per_cell,
                                                           VectorTools::L2_norm);
```

norms:

mean, L1_norm, L2_norm, Linfty_norm, H1_seminorm, H1_norm, ...

LAB 04

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Exercise time!