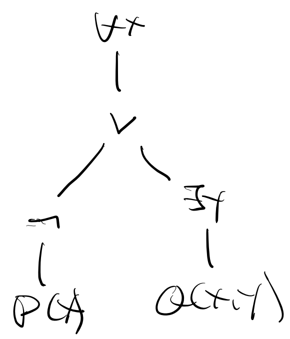
Example of using the Tree Method for First Order Logic.
Consider the following argument with premise p1 and conclusion c
I've drawn the trees for them below (as well as the tree for not(c))

(p1) $\forall x(P(x) \to \exists y\, Q(x,y))$

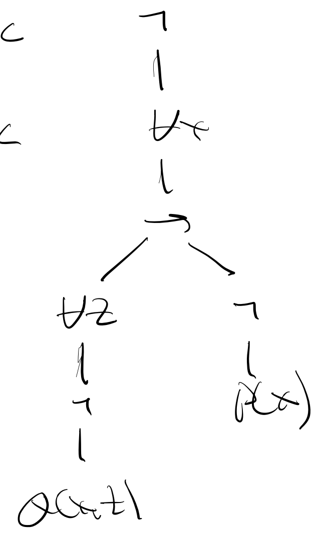
(c) $\forall x(\forall z\, \neg Q(x,z)) \to \neg P(x)$

p1



¬c

c

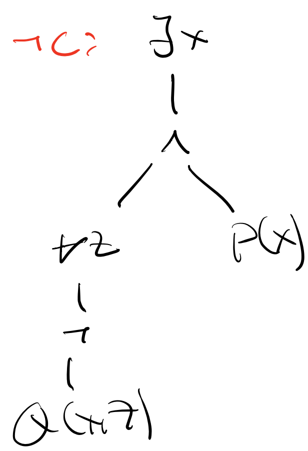


The first step in proving this is a valid argument is to simplify p1 and the negation of the conclusion not(c)
Let's start by simplifying not(c)

$$\neg c: \neg(\forall x((\forall z\, \neg Q(x,z)) \to \neg P(x)))$$
$$\neg(\forall x(\neg(\forall z\, \neg Q(x,z)) \vee \neg P(x)))$$
$$\exists x\, \neg(\neg(\forall z\, \neg Q(x,z)) \vee \neg P(x))$$
$$\exists x\, (\forall z\, \neg Q(x,z)) \wedge P(x)$$
$$\neg c: (\forall z\, \neg Q(a,z)) \wedge P(a)$$

↑ simplified ¬c

¬c:

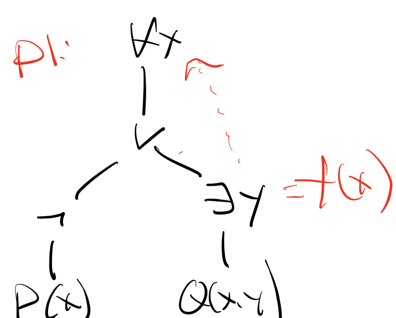


tree for simplified ¬c

Next we simplify the premise p1 and draw its tree

$$p1: \forall x(P(x) \to \exists y\, Q(x,y))$$
$$\forall x(\neg P(x) \vee \exists y\, Q(x,y))$$
$$p1: \forall x(\neg P(x) \vee Q(x, f(x)))$$

p1:



We have now simplified p1 and not(c) and in the process introduced a skolem constant a and skolem function f(x)
When we use the tree method we can replace any universally quantified variable with any term constructed from
The skolem constants and functions. In this case, the universe of "ground terms" is.
a f(a) f(f(a)) f(f(f(a))) …

$\mathcal{U}$ = ground terms
$= \{a, f(a), f(f(a)), f^3(a), f^4(a) \ldots\}$

Now we can use the tree method. The rules are the same as for propositional logic

1. When you have a formula U ^ V. Check it off and stack U and V beneath it on their own
2. When you have a formula U v V. Check it off and create a branch for U and a branch for V
3. When you have a term U and it's negation -U on the same branch, cross it off

There is one more rule for first order logic

4. When you have a formula forall x Q(x). You can copy the formula Q(x) and replace x with any ground term

In this case the ground terms are a,f(a),f(f(a)),…

You pick the substitutions to help you generate contradictions … U and -U on a branch…

p1: $\forall x\,(-P(x) \vee Q(x, f(x)))$

$\neg c$: $(\forall z\ -Q(a,z)) \wedge P(a)$ ✓

conjunction rule

$\forall z\ -Q(a,z)$

$P(a)$

$-P(a) \vee Q(a, f(a))$ p1 ie $x=a$

disjunction rule

$-P(a)$ ✗ | $Q(a,f(a))$

$-Q(a,f(a))$ ✗ $z=f(a)$ ie