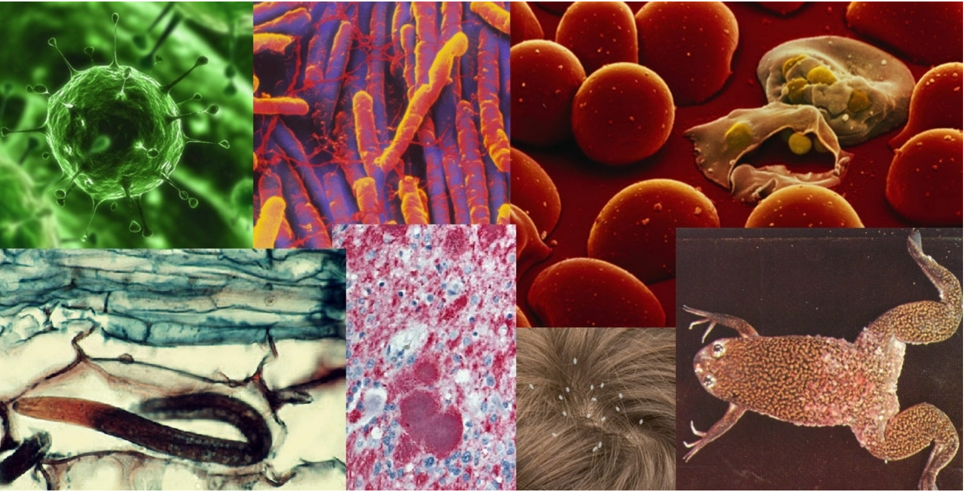
Epidemics in Networks
Introduction

Joel C. Miller & Tom Hladish

20–22 July 2016

Infectious Diseases



Common thread:

enter a host → multiply in host → spread to another host

→

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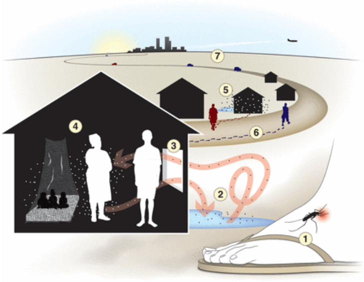

Recently eliminated diseases

- Smallpox

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- Smallpox
  Eliminated by vaccination

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- Rinderpest (livestock)
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- SARS
  Eliminated by contact reduction
- Influenza A H1N1, pre-swine flu version
  Outcompeted by new strain

Nearly eliminated diseases

- Polio

Nearly eliminated diseases

- Polio
- Guinea Worm

Recent emerging diseases

- HIV

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- MERS

Recent emerging diseases

- HIV
- SARS
- 2009 Influenza A H1N1
- Ebola
- MERS
- Zika

Why model disease spread?

We want to build mathematical models for infectious disease spread that:

- Predict future disease dynamics so that policy makers can prepare resources.
- Identify critical/efficient targets for intervention.
- Identify gaps in our knowledge.

Art is a lie that makes us realize truth

Pablo Picasso

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It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.
A. Einstein

Art is a lie that makes us realize truth

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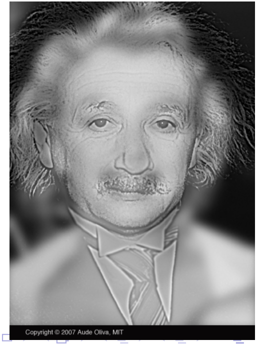
Everything should be made as simple as possible, but not simpler.
"A. Einstein"



How complex should a model be?

Modeling $\neq$ mountain climbing

$$\boxed{\begin{aligned}\dot{S} &= -\beta k I S \\ \dot{I} &= \beta k I S - \gamma I \\ \dot{R} &= \gamma I\end{aligned}} \neq$$

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- Sometimes intuition is good enough — it's usually a simple mathematical model.

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- Only include things that could affect decisions/improve policy.
- Sometimes intuition is good enough — it's usually a simple mathematical model.
- But when there are feedbacks or opposing effects, I don't trust mine.

Disease spread

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- Relation between mode of transmission and population structure.

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- Relation between mode of transmission and population structure.
- How the immune system responds to exposure.

Mode of transmission

Potential spread mechanisms:

- Water & food contamination

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Cholera

Mode of transmission

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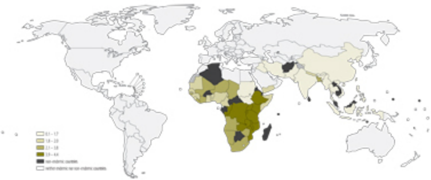

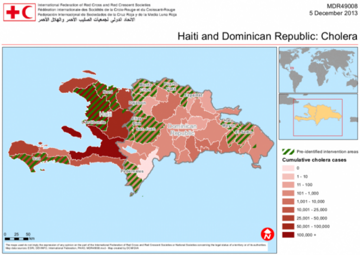

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Guinea Worm

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- Vectors

Malaria, Chikungunya, Dengue, West Nile, Zika

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Chagas

Mode of transmission

Potential spread mechanisms:

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Lyme

Mode of transmission

Potential spread mechanisms:

- Water & food contamination
- Vectors

also Anaplasmosis, Babesiosis, Borrelia, Rocky Mountain Spotted Fever, Crimean-Congo Hemorrhagic Fever, . . .

Mode of transmission

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- Vectors
- Direct contact

Influenza, SARS, MERS, Ebola, ...

Mode of transmission

Potential spread mechanisms:

- Water & food contamination
- Vectors
- Direct contact

HIV, Gonorrhea, Chlamydia, Zika, ...

Immune response

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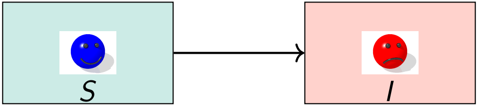

HIV, Tuberculosis, Hepatitis,

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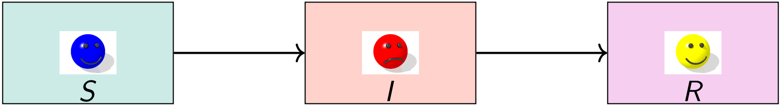
- Gains permanent immunity: SIR



Measles, Mumps, Rubella, Pertussis, . . .

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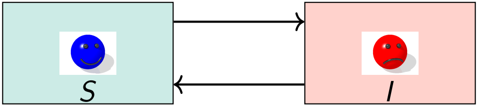

Many parasites (e.g., lice), Many bacteria, Many STDs, ...

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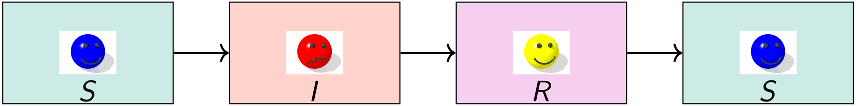

Dengue (sort of), Pertussis, Influenza,

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Network definition

- A network is a collection of nodes which are joined into pairs by edges.
- Two nodes that are joined together are called neighbors. The number of neighbors a given node has is its degree, k.
- There is no real difference between the definitions of "network" and "graph".
- I will tend to use the terminology "partner" for neighbor and "partnership" for edge.

Network Properties

There are a number of things we can measure:

- Degree distribution: $P(k)$, the proportion of nodes with degree k.

High degree nodes tend to be infected early and in turn infect more nodes. So the early growth is more affected by the presence of high-degree nodes than by the average degree.

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- Clustering: frequency of short cycles [not common in sexual networks].

Clustering tends to slow the spread of a disease, but often does not significantly affect whether a disease occurs or how large it gets.

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Changing partnerships reduces the effect of local “susceptible depletion”

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- Assortativity: Individuals may actively select similar partners. In particular, partners with similar degree.

Assortative mixing by degree tends to make it easier for a disease to get established because the core of high-degree nodes provides a good place to spread. However, it often reduces the total size of the epidemic because the low degree nodes tend to connect only to low degree nodes.

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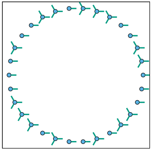
- Edge weights: some edges may have higher transmission probabilities than others.

Edge weights and many other effects are generally less significant (but what if weights inversely correlated with degree?)

Configuration Model

The simplest model capturing a heterogeneous degree distribution:



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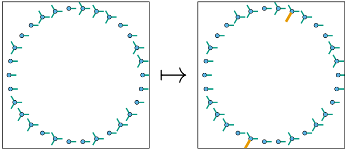

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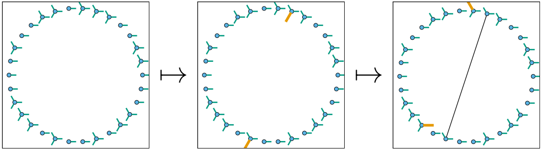

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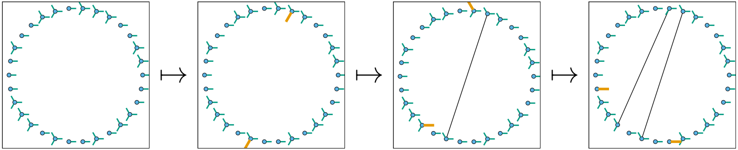

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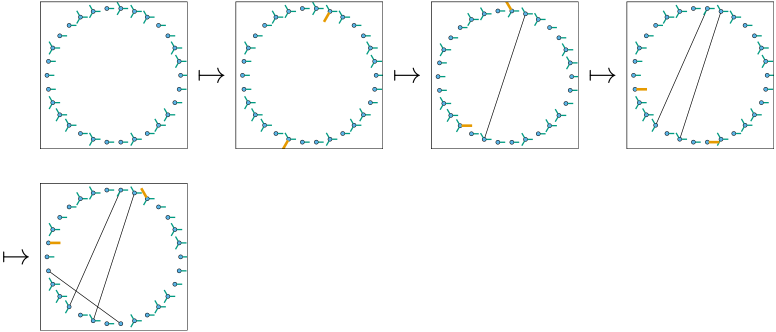

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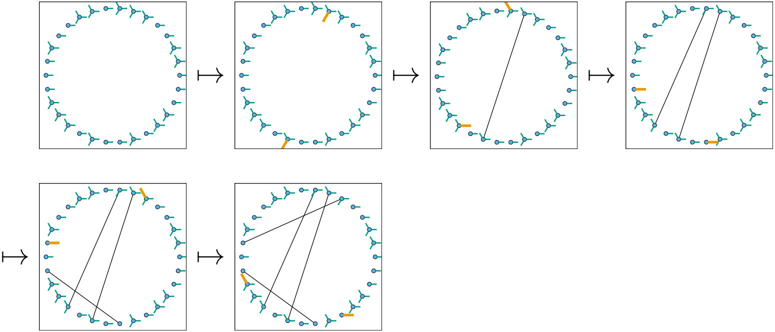

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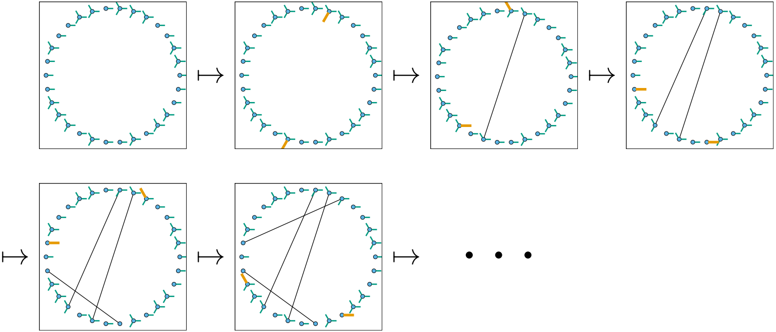

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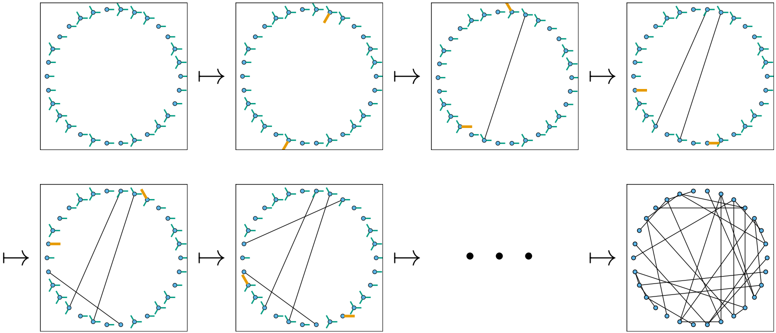

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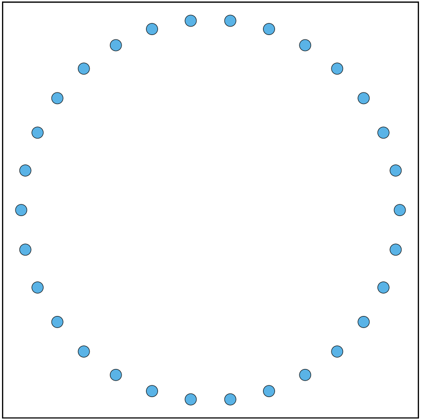

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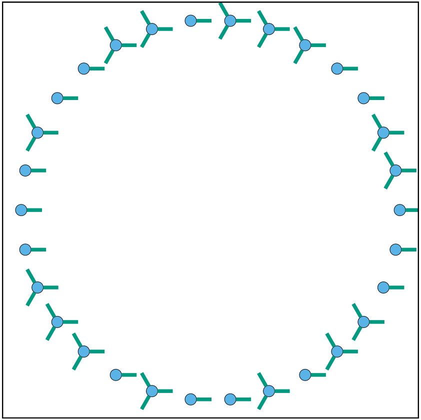

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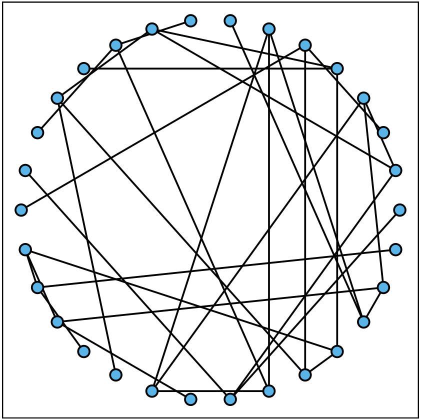

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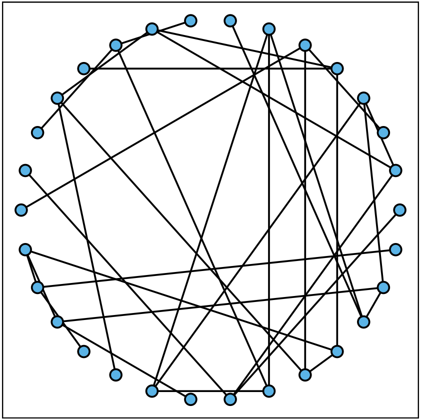

The probability a partner has degree k is $P_n(k) = kP(k)/\langle K \rangle$.

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- In fact $P_n(k) = kP(k)/\langle K \rangle$ where $\langle K \rangle$ is the average degree.
- Note that the degrees of a random individual and a random neighbor of a random individual have different distributions, but a random neighbor and a random neighbor's random neighbor are both from $P_n(k)$.

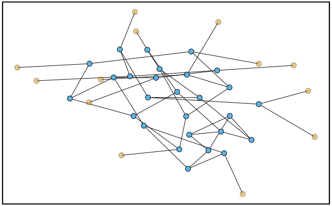
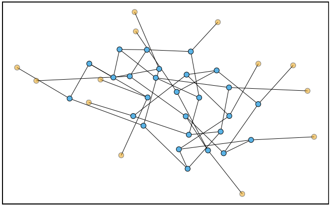
Size Bias

I cannot stress enough that if $P(k)$ is the probability a random individual has k partners, then

$$P_n(k) = kP(k)/\langle K \rangle$$

is the probability a random partner has k partners.

"Annealed" version

 $\mapsto$ 

- The <u>annealed network</u> version assumes that at every moment the network looks like a Configuration model network.
- However, at every moment, an individual changes all of its partners.
- In practice this is appropriate if partnerships are so short or disease transmission so rare that an individual is unlikely to ever transmit to the same individual twice or transmit back to its infector.
- People who use the term "annealed network" call the static version a "quenched network".

Social networks

- facebook
- linkedin
- twitter
- ...

These may be more appropriate for spread of ideas or opinions.

Contact networks

- The network of physical interactions.
- Often highly clustered.
- Appropriate for respiratory diseases.
- Often measured by giving people devices that measure physical proximity.

Sexual networks

- Appropriate for sexually transmitted diseases.
- Often low clustering.
- Often highly heterogeneous.
- Transient partnerships may play a large role.

Location–Location networks

- Cities connected by travel of people between them [spread of H1N1].
- Farms connected by movement of animals [foot and mouth].
- Habitats connected by bird migrations [West Nile].

Empirical networks

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- Romantic networks [10]

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- Seasonal population movements [16]: study of seasonal population movements for malaria control (phone data, census, satellite imagery).

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- South Africa: Simulation by George Seage's group at HSPH for HIV transmission ($\approx$ 6 million?)

Lots of things to think about

For SIR, we are typically interested in

- $\mathcal{P}$, the probability of an epidemic.
- $\mathcal{A}$, the "attack rate": the fraction infected (better named the attack ratio)
- $\mathcal{R}_0$, the average number of infections caused by those infected early in the epidemic.
- $I(t)$, the time course of the epidemic.

For SIS, we are typically interested in

- $\mathcal{P}$
- $I(\infty)$, the equilibrium level of infection
- $\mathcal{R}_0$
- $I(t)$

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