Augmented Matrix is a matrix obtained by appending the columns of two given matrices.
Given the matrices A and B, where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 5 & 2 & 2 \end{bmatrix} \quad , B = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

the augmented matrix $(A|B)$ is written as

$$(A|B) = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 0 & 1 & 3 \\ 5 & 2 & 2 & 1 \end{bmatrix}$$

Inner Product The standard *inner product* (also called *dot product*) of two n-vectors is defined as the scalar

$$a^T b = a_1 b_1 + a_2 b_2 + ... + a_n b_n$$

Linear combinations. if $a_1....a_m$ are n-vectors, and $\beta_1...\beta_m$ are scalars, the $n - vector$

$$\beta_1 a_1 + ... + \beta_m a_m$$

is called a *linear combination* of vectors $a_1...a_n$. The scalars $\beta_1...\beta_m$ are called the *coefficients* of the lienar combination.

The inner product function. Suppose a is an $n - vector$. We can define scalar-valued function f of $n - vectors$, given by

$$f(x) = a^T x = a_1 x_1 + a_2 x_2 + ... + a_n x_n \tag{1}$$

for any $n - vectorx$. This function gives the inner product of its $n - vector$ argument x with some (fixed) $n - vector$.

Superposition and linearity. The inner product function f defined above satisfies property

$$\begin{aligned} f(\alpha x + \beta y) &= a^T(\alpha x + \beta y) \\ &= a^T(\alpha x) + a^T(\beta y) \\ &= \alpha(a^T x) + \beta(a^T y) \\ &= \alpha f(x) + \beta f(x) \end{aligned}$$

for all $n - vectors x, y$, and all scalars α, β. This property is called *superposition.* A function that satisfies the superposition property is called *linear.*

Superposition equality is thus

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \tag{2}$$

A function $f : \mathbf{R}^n \to \mathbf{R}$ is linear if satisfies

- *Homogenity.* For any n-vector x and any scala α, $f(\alpha x) = \alpha f(x)$
- *Additivity.* For any n-vector x and y, $f(x + y) = f(x) + f(y)$

Inner product representation of linear function A function is linear if it is defined as inner product of it's argument with some fixed vector.
$f(x) = a^T x$ for all x. $a^T x$ is inner product representation of f

Affine functions. A linear function plus a constant is called an *affine* function. A function $f : \mathbf{R}^n \to \mathbf{R}$ is affine if an only if it can be expressed as $f(x) = a^T x + b$