Augmented Matrix is a matrix obtained by appending the columns of two given matrices.
Given the matrices A and B, where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 5 & 2 & 2 \end{bmatrix} \quad , B = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

the augmented matrix $(A|B)$ is written as

$$(A|B) = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 0 & 1 & 3 \\ 5 & 2 & 2 & 1 \end{bmatrix}$$

Augmented Matrix can be used to represent sytem equations. For example :

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 - 5x_3 &= 0 \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

The basic method for solving a linear system is to perform appropiate algebraic operations on the system that do not alter the solution set and that produce a succession of increasingly simple system, until a point is reached where it can be ascertained whether the system is consistent, and if so, what its solutions are. Typically, the algebraic operations are as follows:

1. Multiply an equation through a a nonzero constant
2. Interchange two equations
3. Add a constant time one equation to another

Since the rows (horizontal lines) of an augmented matrix correspond to the equations in the associated system, these three operations correspond to the following operation on the rows of the augmented matrix

1. Multiply a row through a a nonzero constant
2. Interchange two rows
3. Add a constant time one row to another

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

Add -2 times the first equation to all equation

Add -2 times the first row to all row

$$\begin{aligned} -x - y - 2z &= -9 \\ 2y - 7z &= -17 \\ x + 4y - 9z &= -18 \end{aligned} \qquad \begin{bmatrix} -1 & -1 & -2 & -9 \\ 0 & 2 & -7 & -17 \\ 1 & 4 & -9 & -18 \end{bmatrix}$$

Inner Product The standard *inner product* (also called *dot product*) of two n-vectors is defined as the scalar

$$a^T b = a_1 b_1 + a_2 b_2 + ... + a_n b_n$$

Linear combinations. if $a_1....a_m$ are n-vectors, and $\beta_1...\beta_m$ are scalars, the $n - vector$

$$\beta_1 a_1 + ... + \beta_m a_m$$

is called a *linear combination* of vectors $a_1...a_n$. The scalars $\beta_1...\beta_m$ are called the *coefficients* of the lienar combination.

The inner product function. Suppose a is an $n - vector$. We can define scalar-valued function f of $n - vectors$, given by

$$f(x) = a^T x = a_1 x_1 + a_2 x_2 + ... + a_n x_n \tag{1}$$

for any $n - vectorx$. This function gives the inner product of its $n - vector$ argument x with some (fixed) $n - vector$.

Superposition and linearity. The inner product function f defined above satisfies property

$$\begin{aligned} f(\alpha x + \beta y) &= a^T(\alpha x + \beta y) \\ &= a^T(\alpha x) + a^T(\beta y) \\ &= \alpha(a^T x) + \beta(a^T y) \\ &= \alpha f(x) + \beta f(x) \end{aligned}$$

for all $n - vectors x, y$, and all scalars α, β. This property is called *superposition.* A function that satisfies the superposition property is called *linear.*

Superposition equality is thus

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \tag{2}$$

A function $f : \mathbf{R}^n \to \mathbf{R}$ is linear if satisfies

- *Homogenity.* For any n-vector x and any scala α, $f(\alpha x) = \alpha f(x)$
- *Additivity.* For any n-vector x and y, $f(x + y) = f(x) + f(y)$

Inner product representation of linear function A function is linear if it is defined as inner product of it's argument with some fixed vector.
$f(x) = a^T x$ for all x. $a^T x$ is inner product representation of f

Affine functions. A linear function plus a constant is called an *affine* function. A function $f : \mathbf{R}^n \to \mathbf{R}$ is affine if an only if it can be expressed as $f(x) = a^T x + b$