Linear combinations. if $a1....am$ are n-vectors, and $\beta_1...\beta_m$ are scalars, the $n-vector$

$$\beta_1 a_1 + ... + \beta_m a_m$$

is called a *linear combination* of vectors $a_1...a_n$. The scalars $\beta_1...\beta_m$ are called the *coefficients* of the lienar combination.

The inner product function. Suppose a is an $n-vector$. We can defien scalar-valued function f of $n-vectors$, given by

$$f(x) = a^T x = a_1 x_1 + a_2 x_2 + ... + a_n x_n \tag{1}$$

for any $n-vectorx$. This function gives the inner product of its $n-vector$ argument x with some (fixed) $n-vector$.

Superposition and linearity. The inner product function f defined above satisfies property

$$\begin{aligned} f(\alpha x + \beta y) &= a^T(\alpha x + \beta y) \\ &= a^T(\alpha x) + a^T(\beta y) \\ &= \alpha(a^T x) + \beta(a^T y) \\ &= \alpha f(x) + \beta f(x) \end{aligned}$$

for all $n-vectorsx, y$, and all scalars α, β. This property is called *superposition.* A function that satisfies the superposition property is called *linear.*

Superposition equality is thus

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \tag{2}$$