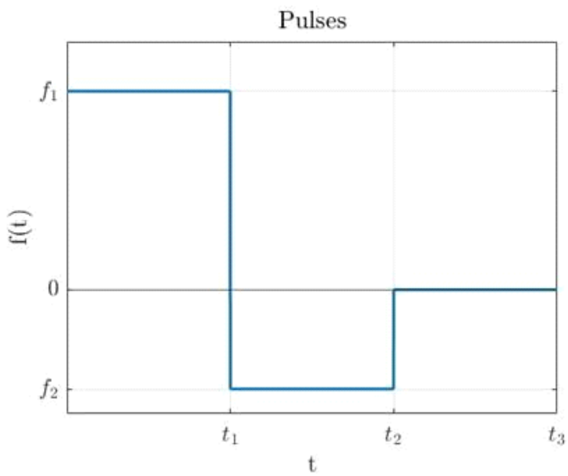
Calculus: Pulses

Consider the function $f(t)$ shown.



H(t)

a) Determine an expression for $f(t)$ over the entire range $0 \le t \le t_3$ in terms of the variables shown in the figure.

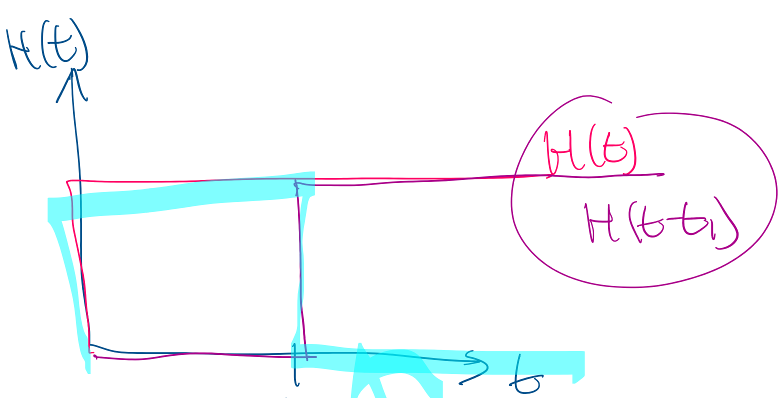
b) Determine an expression for the integral $I(t) = \int_{t_o=0}^{t} f(t_o)\, dt_o$ over the entire range $0 \le t \le t_3$ in terms of the variables shown in the figure.

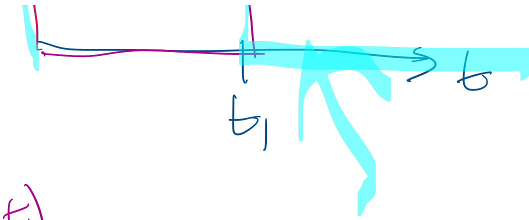
c) Create a 2x1 subplot. Plot $f(t)$ on the upper subplot and $I(t)$ on the lower subplot. Take $f_1 = 10, f_2 = -5, t_1 = 1, t_2 = 2,$ and $t_3 = 3$.

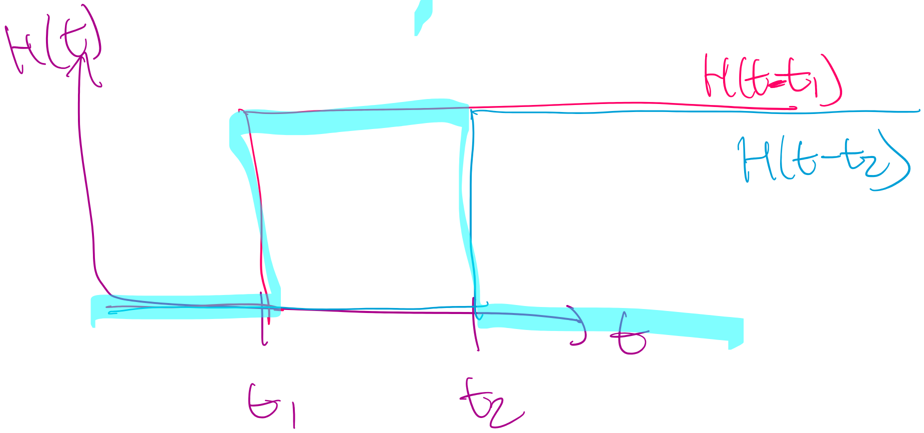
Since the integrand $f(t)$ is a three-part function, the integral $I(t)$ is best evaluated as a three-part function.

$$f(t) = \begin{cases} f_1 & 0 \le t \le t_1 \\ f_2 & t_1 \le t < t_2 \\ 0 & t_2 \le t < t_3 \end{cases}$$

$$f(t) = f_1\left(H(t) - H(t-t_1)\right) + f_2\left(H(t-t_1) - H(t-t_2)\right)$$







b) Region 1: $0 \le t \le t_1$

$$u(t) = \int_0^t f_1 \, dt_0 = f_1 \int_0^t dt_0 = f_1 \left[t_0\right]_0^t = f_1 t$$

Region 2: $t_1 \le t < t_2$

$$u(t) = \underline{\int_0^{t_1} f_1 \, dt_0} + \int_{t_1}^t f_2 \, dt_0 = f_1 t_1 + f_2 \int_{t_1}^t dt_0$$

$$= f_1 t_1 + f_2 \left[t_0\right]_{t_1}^t = f_1 t_1 + f_2(t - t_1)$$

Region 3: $t_2 \le t < t_3$

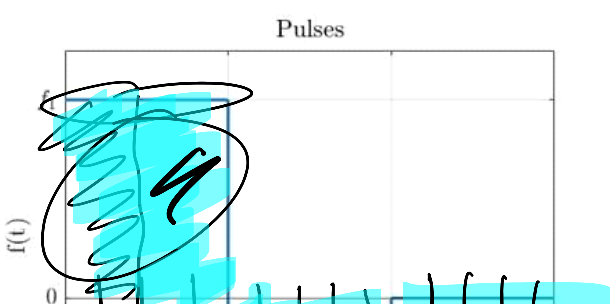
Region 3: $t_2 \le t < t_3$

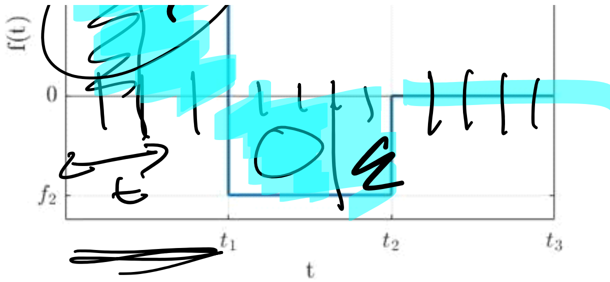
$$u(t) = \int_0^{t_1} f_1 \, dt_0 + \int_{t_1}^{t_2} f_2 \, dt_0 + \cancel{\int_{t_2}^{t} 0 \, dt_0}$$

$$\rightarrow u(t) = f_1 t_1 + f_2 (t_2 - t_1)$$

$$\rightarrow u(t) = \begin{cases} f_1 t & 0 \le t < t_1 \\ f_1 t_1 + f_2 (t - t_1) & t_1 \le t < t_2 \\ f_1 t_1 + f_2 (t_2 - t_1) & t_2 \le t < t_3 \end{cases}$$

$$u(t) = f_1 t \left[H(t) - H(t - t_1) \right] + \left[f_1 t_1 + f_2 (t - t_1) \right] \left[H(t - t_1) - H(t - t_2) \right]$$
$$+ \left[f_1 t_1 + f_2 (t_2 - t_1) \right] \left[H(t - t_2) - H(t - t_3) \right]$$





b) Region 1: $0 \le t < t_1$

$$v(t) = \int_0^t f_1 \, dt_0 = f_1 t$$

Region 2: $t_1 \le t < t_2$

$$v(t) = f_1 t_1 + f_2 (t - t_1)$$

Region 3: $t_2 \le t < t_3$

$$v(t) = f_1 t_1 + f_2 (t_2 - t_1)$$

$$\rightarrow v(t) = \begin{cases} f_1 t & 0 \le t < t_1 \\ f_1 t_1 + f_2 (t - t_1) & t_1 \le t \le t_2 \\ f_1 t_1 + f_2 (t_2 - t_1) & t_2 \le t < t_3 \end{cases}$$