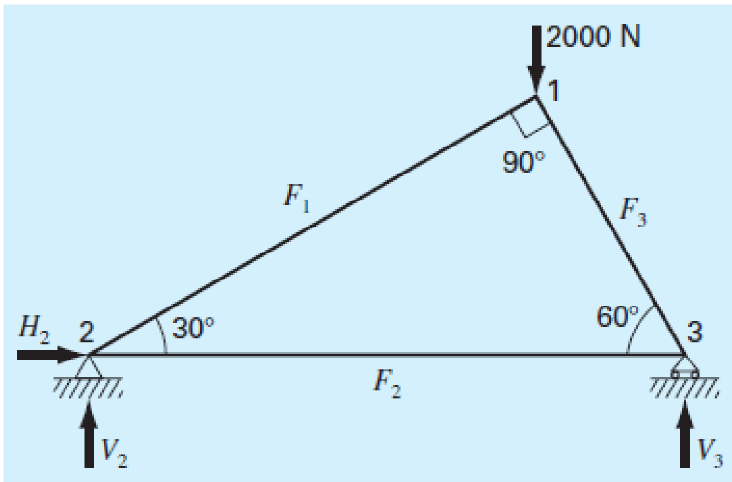
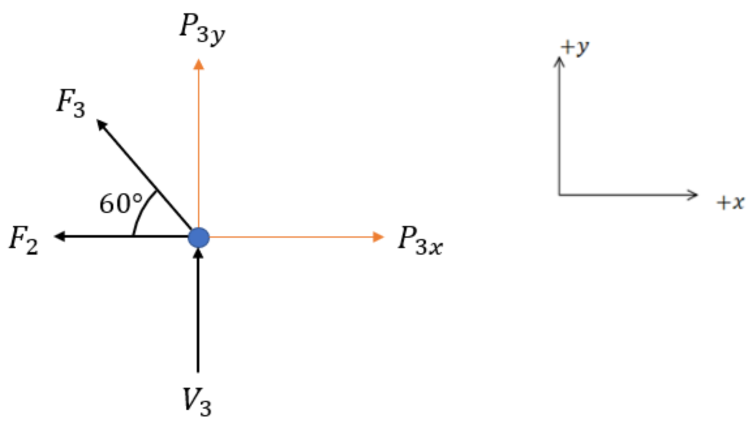
Linear Algebra: Truss

For the truss structure shown below, determine the unknown reactions (H_2, V_2, V_3) and internal forces (F_1, F_2, F_3) in the truss members by drawing FBDs at each vertex (node). The truss is in static equilibrium, so the sum of the horizontal and vertical forces at each node must be zero. Take $+y$ up and $+x$ right.



(Chapra 2011)

For example, the FBD and sum of the vertical and horizontal forces at Node 3 are:



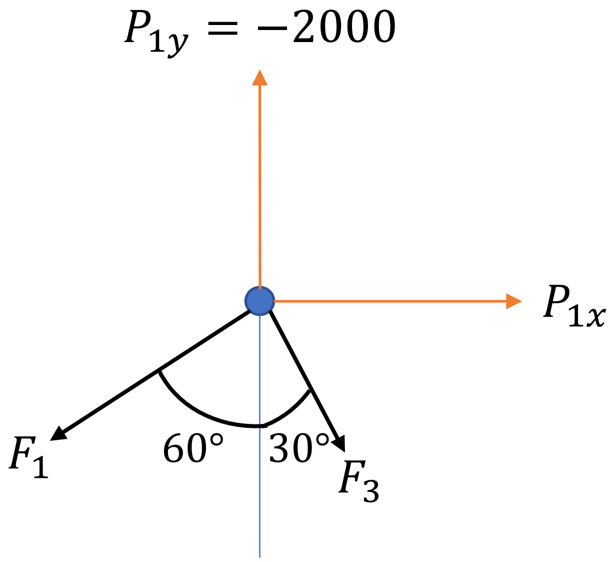
$$\Sigma F_x = 0 = -F_2 - F_3 \cos 60 + P_{3x}$$
$$\Sigma F_y = 0 = V_3 + F_3 \sin 60 \; + P_{3y}$$

$P_{3x} = 0$ and $P_{3y} = 0$ are externally applied forces on the node. Although their values are zero, including them on the FBD will be paramount later. Because the system is in static equilibrium, we can freely assume all of the forces' directions without penalty.

Objectives

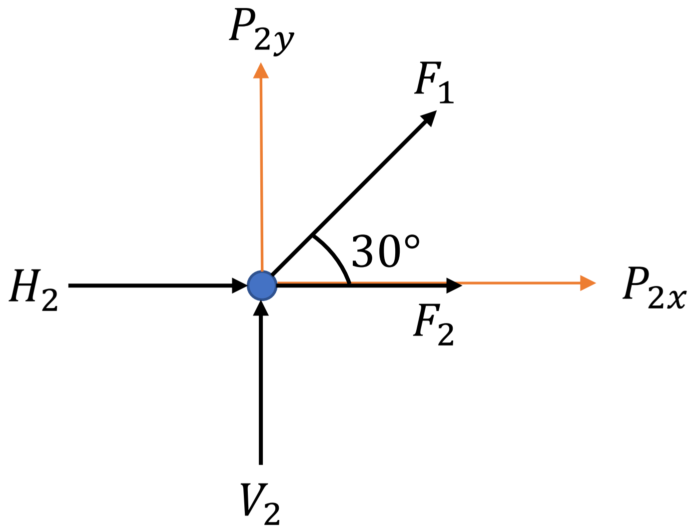
a) Draw the cause-effect diagram.
b) Derive the force balance at the other nodes, then place the equations in matrix form.
c) Solve for the unknown reactions and internal forces using the matrix inverse.
d) Determine and plot the internal and reaction forces when the externally applied vertical force at Node 1 varies from $P_{1y} = 0\ N$ to $+2000\ N$ (the force points upwards).
e) Determine the change in the internal and reaction forces if the vertical load at Node 1 is doubled to $P_{1y} = -4000\ N$ and a horizontal load of $P_{3x} = -1000\ N$ (the force points to the left) is applied at Node 3.

Node 1:



$$\Sigma F_x = 0 =$$
$$\Sigma F_y = 0 =$$

Node 2:



$$\Sigma F_x = 0 =$$
$$\Sigma F_y = 0 =$$

Equations:

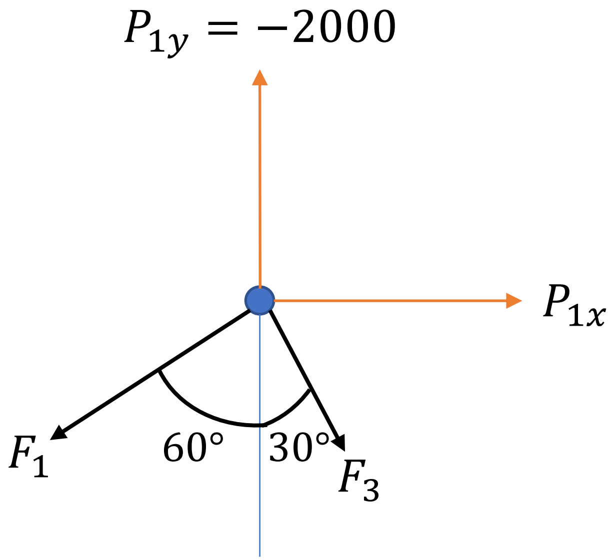
$$Node\ 1 \begin{cases} F_1 \sin 60 - F_3 \sin 30 = P_{1x} \\ F_1 \cos 60 + F_3 \cos 30 = P_{1y} \end{cases}$$

$$Node\ 2 \begin{cases} -F_1 \cos 30 - F_2 - H_2 = P_{2x} \\ -F_1 \sin 30 - V_2 = P_{2y} \end{cases}$$

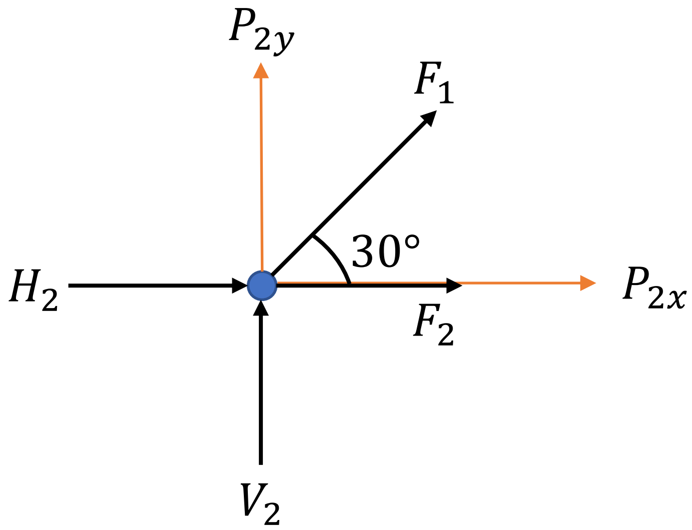
$$Node\ 3 \begin{cases} F_2 + F_3 \cos 60 = P_{3x} \\ -F_3 \sin 60 \ - V_3 = P_{3y} \end{cases}$$

$$\underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{b}$$

Node 1:



Node 2:



$$\Sigma F_x = 0 = -F_1 \sin 60 + F_3 \sin 30 + P_{1x}$$
$$\Sigma F_y = 0 = -F_1 \cos 60 - F_3 \cos 30 + P_{1y}$$

$$\Sigma F_x = 0 = F_1 \cos 30 + F_2 + H_2 + P_{2x}$$
$$\Sigma F_y = 0 = F_1 \sin 30 + V_2 + P_{2y}$$

Equations:

$$Node\ 1 \begin{cases} F_1 \sin 60 - F_3 \sin 30 = P_{1x} \\ F_1 \cos 60 + F_3 \cos 30 = P_{1y} \end{cases}$$

$$Node\ 2 \begin{cases} -F_1 \cos 30 - F_2 - H_2 = P_{2x} \\ -F_1 \sin 30 - V_2 = P_{2y} \end{cases}$$

$$Node\ 3 \begin{cases} F_2 + F_3 \cos 60 = P_{3x} \\ -F_3 \sin 60 \ - V_3 = P_{3y} \end{cases}$$

$$\underbrace{\begin{bmatrix} \sin 60 & 0 & -\sin 30 & 0 & 0 & 0 \\ \cos 60 & 0 & \cos 30 & 0 & 0 & 0 \\ -\cos 30 & -1 & 0 & -1 & 0 & 0 \\ -\sin 30 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & \cos 60 & 0 & 0 & 0 \\ 0 & 0 & -\sin 60 & 0 & 0 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{2x} \\ P_{2y} \\ P_{3x} \\ P_{3y} \end{bmatrix}}_{b}$$