Linear Algebra: Nonlinear Regression

Consider the data tabulated below (also contained in the `ME2004_NonlinearRegressionData.mat` file):

x	0.5	1	2	3	4
y	10.4	5.8	3.3	2.4	2

The data can be modeled by the following equation:

$$y = \left(\frac{a + \sqrt{x}}{b\sqrt{x}} \right)^2$$

a) Linearize the above equation.
b) Determine the parameters a and b.
c) Based on your analysis, predict y at $x = 1.6$ and $x = 3.5$.

Linearizing $y = \left(\frac{a+\sqrt{x}}{b\sqrt{x}}\right)^2$:

Step	Result
Identify $Y = a_1 X + a_0$ terms	$\underbrace{\quad}_{Y} = \underbrace{\quad}_{a_1}\underbrace{\quad}_{X} + \underbrace{\quad}_{a_0}$

Linearizing $y = \left(\frac{a+\sqrt{x}}{b\sqrt{x}}\right)^2$:

Step	Result
Square root both sides	$\sqrt{y} = \frac{a+\sqrt{x}}{b\sqrt{x}}$
Separate the RHS into two terms	$\sqrt{y} = \frac{a}{b\sqrt{x}} + \frac{\sqrt{x}}{b\sqrt{x}} = \frac{a}{b\sqrt{x}} + \frac{1}{b}$
Identify $Y = a_1 X + a_0$ terms	$\underbrace{\sqrt{y}}_{Y} = \underbrace{\left(\frac{a}{b}\right)}_{a_1}\underbrace{\left(\frac{1}{\sqrt{x}}\right)}_{X} + \underbrace{\frac{1}{b}}_{a_0}$