Systems of 1^{st} Order ODEs

A Quick Review

Systems of 1st Order ODEs

- A 2nd order (or higher) ODE must be converted to a system of 1st order ODEs to be solved via `ode45()`

ode45

Solve nonstiff differential equations — medium order method

Syntax

```
[t,y] = ode45(odefun,tspan,y0)
[t,y] = ode45(odefun,tspan,y0,options)
[t,y,te,ye,ie] = ode45(odefun,tspan,y0,options)
sol = ode45( __ )
```

Description

`[t,y] = ode45(odefun,tspan,y0)`, where `tspan = [t0 tf]`, integrates the system of differential equations $\boxed{y' = f(t, y)}$ from `t0` to `tf` with initial conditions `y0`. Each row in the solution array `y` corresponds to a value returned in column vector `t`.

- `ode45()` only solves 1st order ODEs!

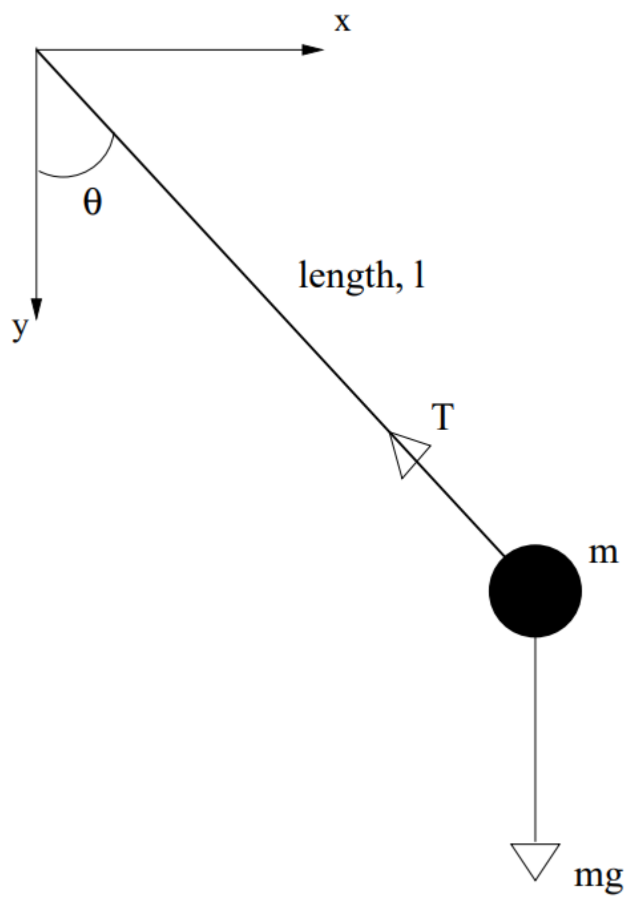
Systems of 1st Order ODEs

- Pendulum equation of motion:

$$ml\ddot{\theta} + mg\sin\theta = 0$$

$$\rightarrow \boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$$

How can we rewrite this using only 1st order ODEs?



Systems of 1st Order ODEs

- Pendulum EOM: $\ddot{\theta} + \frac{g}{l}\sin\theta = 0$
- Introduce a new vector z:

$$z = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\rightarrow \dot{z} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

- Notice $\dot{z}(1) = \dot{\theta} = z(2)$
- Notice $\dot{z}(2) = \ddot{\theta} = -\frac{g}{l}\sin\theta \rightarrow \dot{z}(2) = \ddot{\theta} = -\frac{g}{l}\sin z(1)$

Systems of 1st Order ODEs

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- Introduce a new vector z:

$$z = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\rightarrow \dot{z} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \rightarrow \boxed{\dot{z} = \begin{bmatrix} z(2) \\ -\frac{g}{l}\sin z(1) \end{bmatrix}}$$

- $\dot{z}$ is the ODE we'll use in `ode45()`