Phase Portraits: Example 2

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ODE and Initial Condition	Fixed Points	Stability
$\dfrac{dy}{dt} = \sin(y)$	$y = n\pi$ $n = -2, -1, 0, 1, \dots$	$y = \begin{cases} a\pi, a = 1, 3, 5\dots & \text{stable} \\ b\pi, b = 0, 2, 4\dots & \text{unstable} \end{cases}$
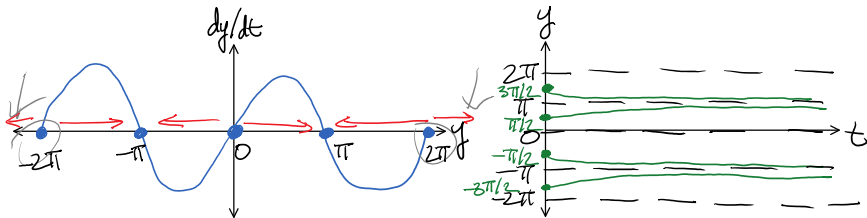
1) Draw the *phase portrait*. Sketch $\dfrac{dy}{dt}$ vs. y ($\dfrac{dy}{dt}$ on the y-axis, y on the x-axis)
2) Determine the fixed points.
   a. Find all the points where $\dfrac{dy}{dt} = 0$.
   b. Mark those points on the phase portrait.
3) Determine the stability of each fixed point.
   a. Draw a right arrow ($\rightarrow$) in all regions where $\dfrac{dy}{dt} > 0$
   b. Draw a left arrow ($\leftarrow$) in all regions where $\dfrac{dy}{dt} < 0$
   c. Fixed point is *stable* if the arrows converge to the point: $\rightarrow * \leftarrow$
   d. Fixed point is *unstable* if the arrows diverge from the point: $\leftarrow * \rightarrow$
4) Draw the anticipated solution based on completed phase portrait.
   a. $y(t)$ trends *towards* a *stable* fixed point
   b. $y(t)$ trends *away* from an *unstable* fixed point
   c. All fixed points are horizontal lines (by definition, a fixed point doesn't change with time, so that's just a horizontal line).