

Calculus: Step/Delta Integrals

Evaluate the following integrals. It helps to make a sketch. $H(t)$ is the unit step function and $\delta(t)$ is the delta function.

a) $\int_0^3 H(t-1) dt$

b) $\int_2^3 H(t-1) dt$

c) $\int_0^3 \delta(t-2) dt$

d) $\int_0^1 \delta(t-3) dt$

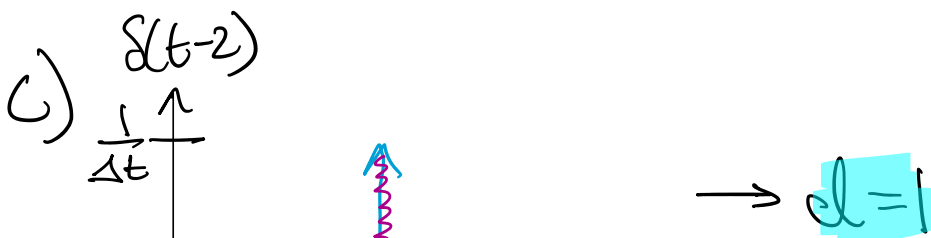
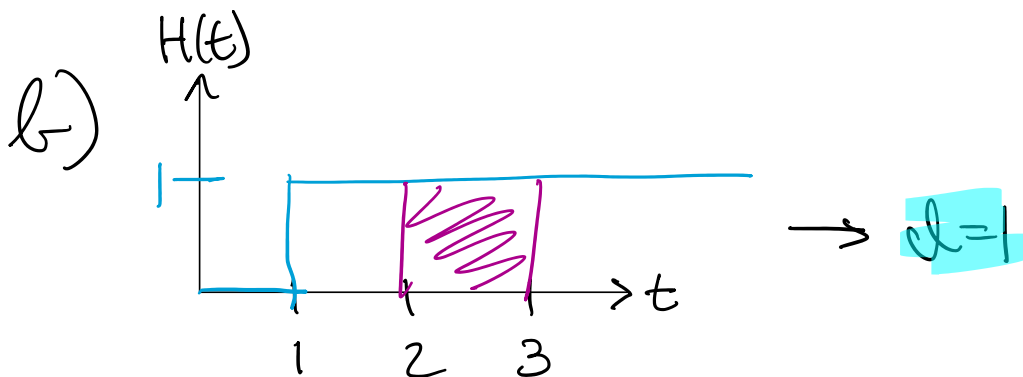
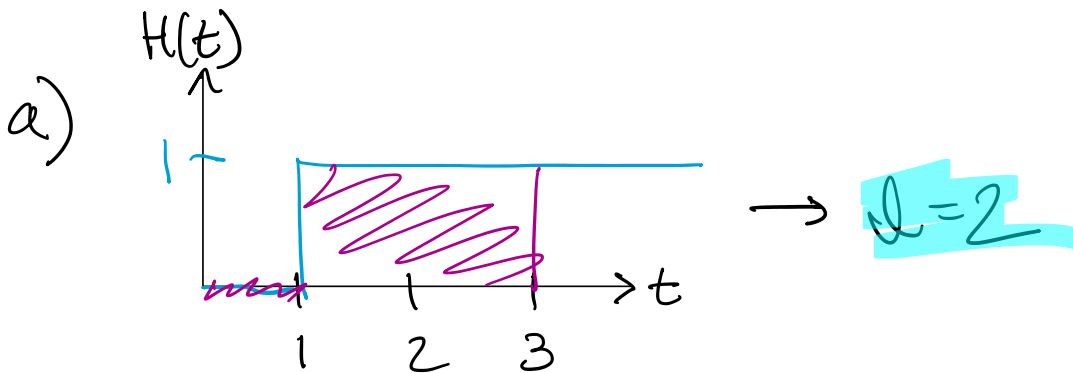
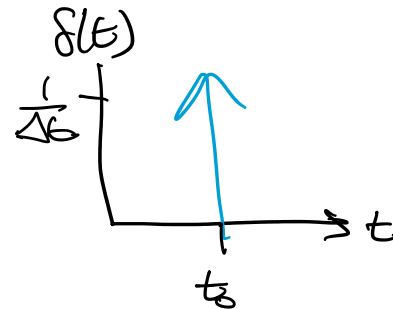
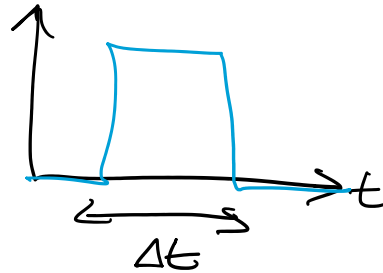
e) $\int_0^3 f(t)\delta(t-2) dt$, where $f(x)$ is an unknown function

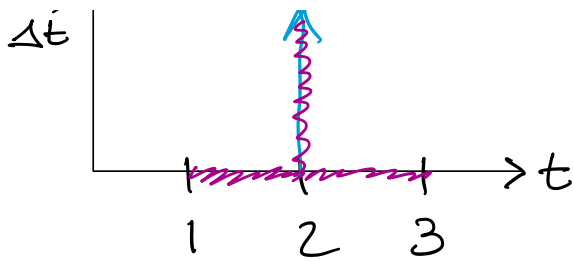
f) $\int_0^t \delta(t-t_0) dt$, where t_0 is an unknown constant

Calculus: Step/Delta Integrals

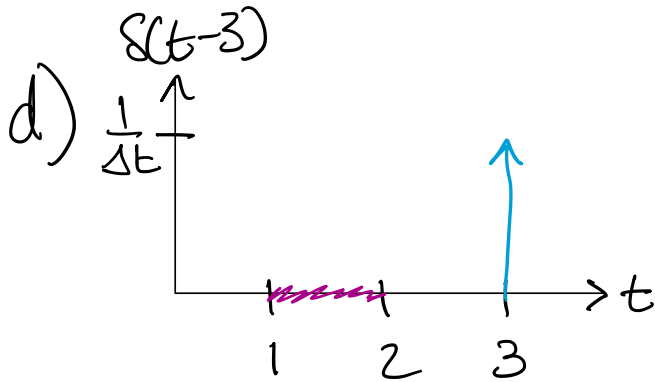
Evaluate the following integrals. It helps to make a sketch. $H(t)$ is the unit step function and $\delta(t)$ is the delta function.

- a) $\int_0^3 H(t-1) dt$
- b) $\int_2^3 H(t-1) dt$
- c) $\int_0^3 \delta(t-2) dt$
- d) $\int_0^3 f(t)\delta(t-2) dt$, where $f(x)$ is an unknown function
- e) $\int_0^3 f(t)\delta(t-2) dt$, where $f(x)$ is an unknown function
- f) $\int_0^t \delta(t-t_0) dt$, where t_0 is an unknown constant

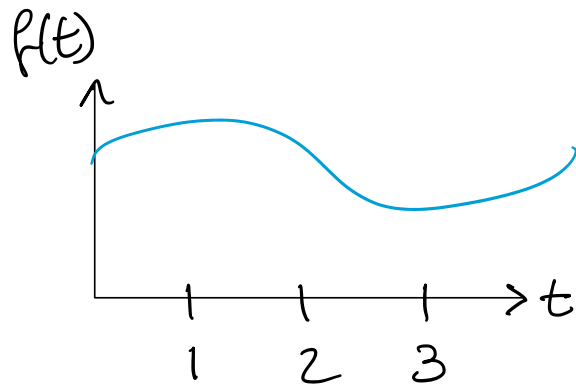
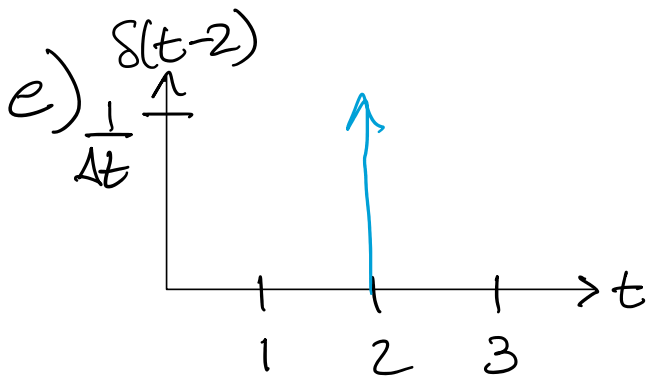




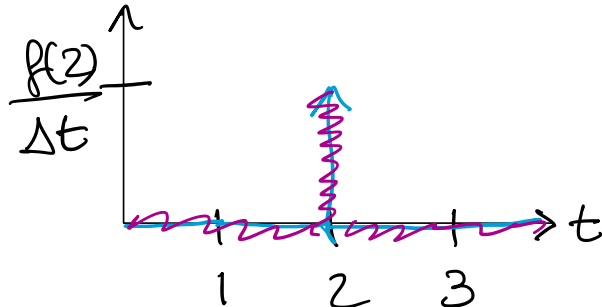
$$\rightarrow d=1$$



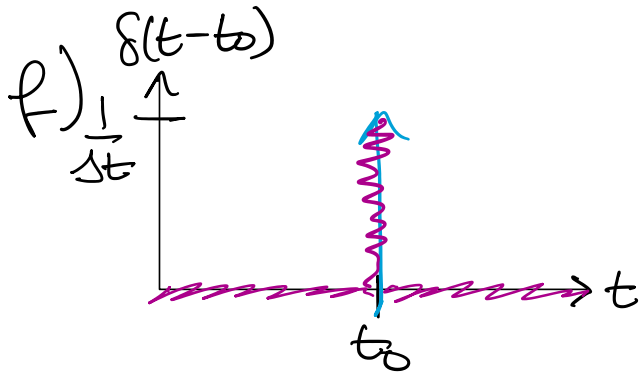
$$\rightarrow d=0$$



$$g(t) = \delta(t-2) \cdot f(t)$$



$$\rightarrow d=f(2)$$



Region 1: $t < t_0$

$$\rightarrow d = 0$$

Region 2: $t = t_0$

$$\rightarrow d = 0 + 1 = 1$$

Region 3: $t > t_0$

$$\rightarrow d = 0 + 1 + 0 = 1$$

$$\rightarrow d(t) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

$$u(t) = H(t - t_0)$$