Linear Interpolation

A Quick Review

Linear Interpolation

- Interpolation is used when we need to find a “missing value” in a data set

x	0	1	2	3
y	2	4	5	8

- What if we wanted to find y at $x = 2.5$?
- Can interpolate linearly, quadratically, etc.

Linear Interpolation Procedure

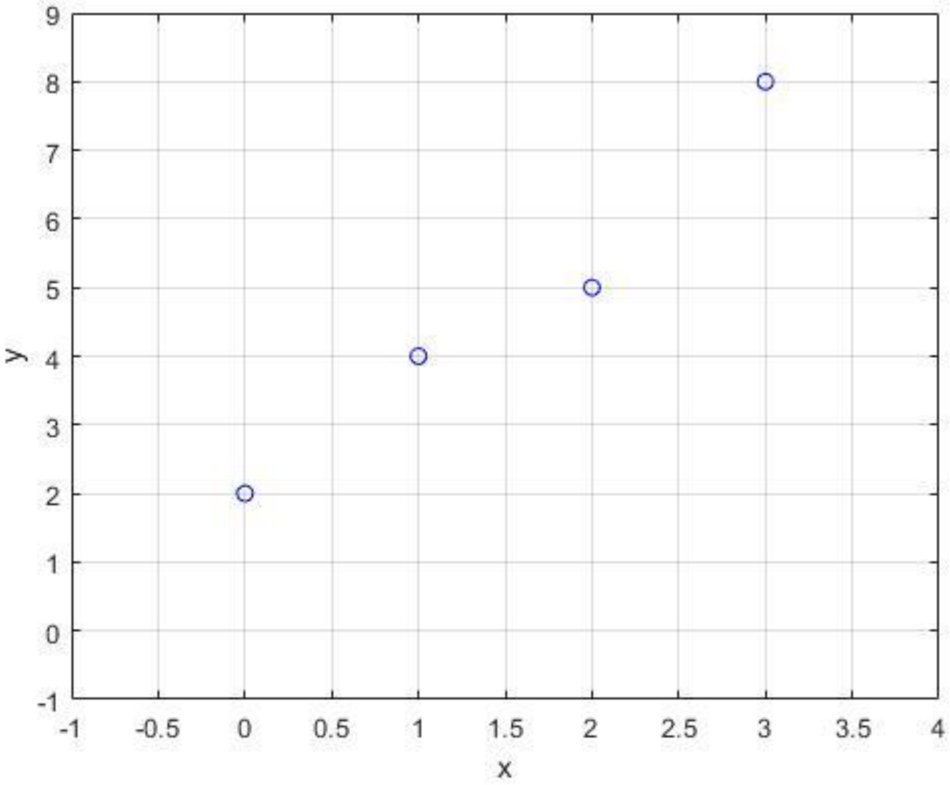
To find the interpolated value $\boldsymbol{f(b)}$ at a point $\boldsymbol{b}$:

1) Pick the 2 closest surrounding points, a and c

2) Fit a straight line between a and c

3) Evaluate $f(b)$ based on the fitted line and similar triangles

x	0	1	2	3
y	2	4	5	8

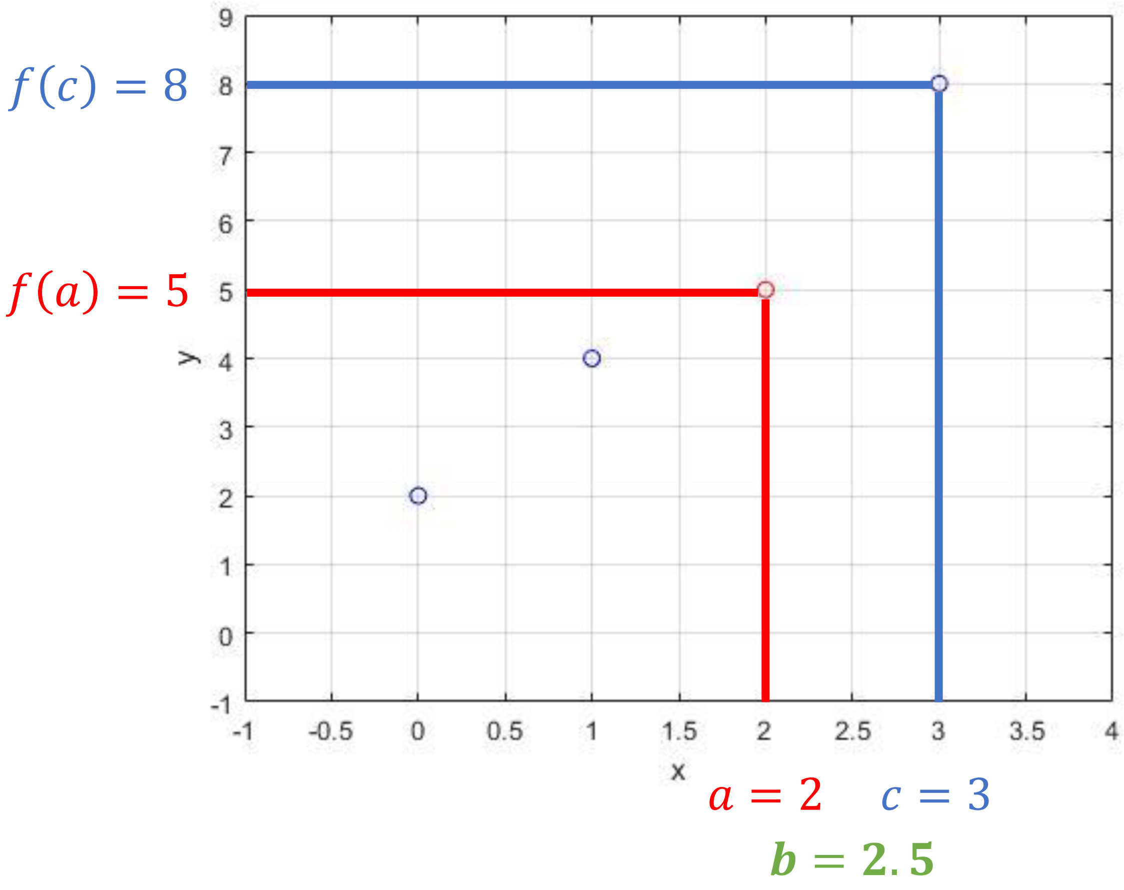


Linear Interpolation Procedure

To find the interpolated value $\boldsymbol{f(b)}$ at a point $\boldsymbol{b}$:

1) Pick the 2 closest surrounding points, a and c

- Corresponding y-values: $f(a)$, $f(c)$
- a and c should "sandwich" b

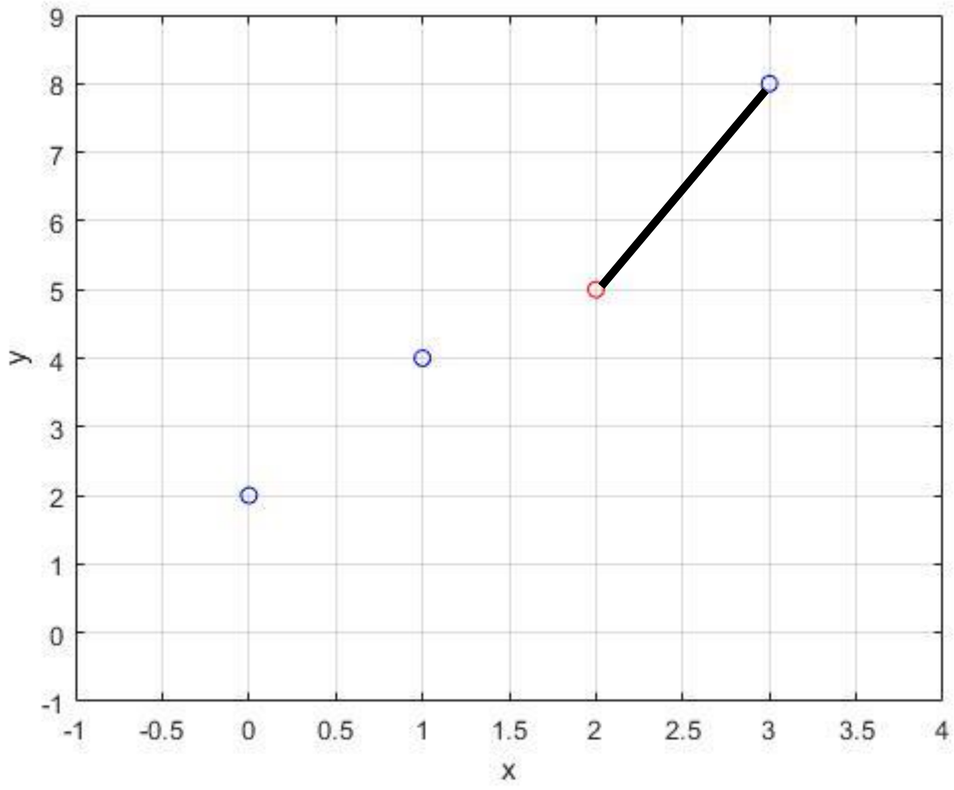


Linear Interpolation Procedure

To find the interpolated value $f(b)$ at a point b:

2) Fit a (straight) line between a and c

- (Simple) Curve fitting!
- This is as straightforward as it sounds
- Gets more complex for higher-order interpolations



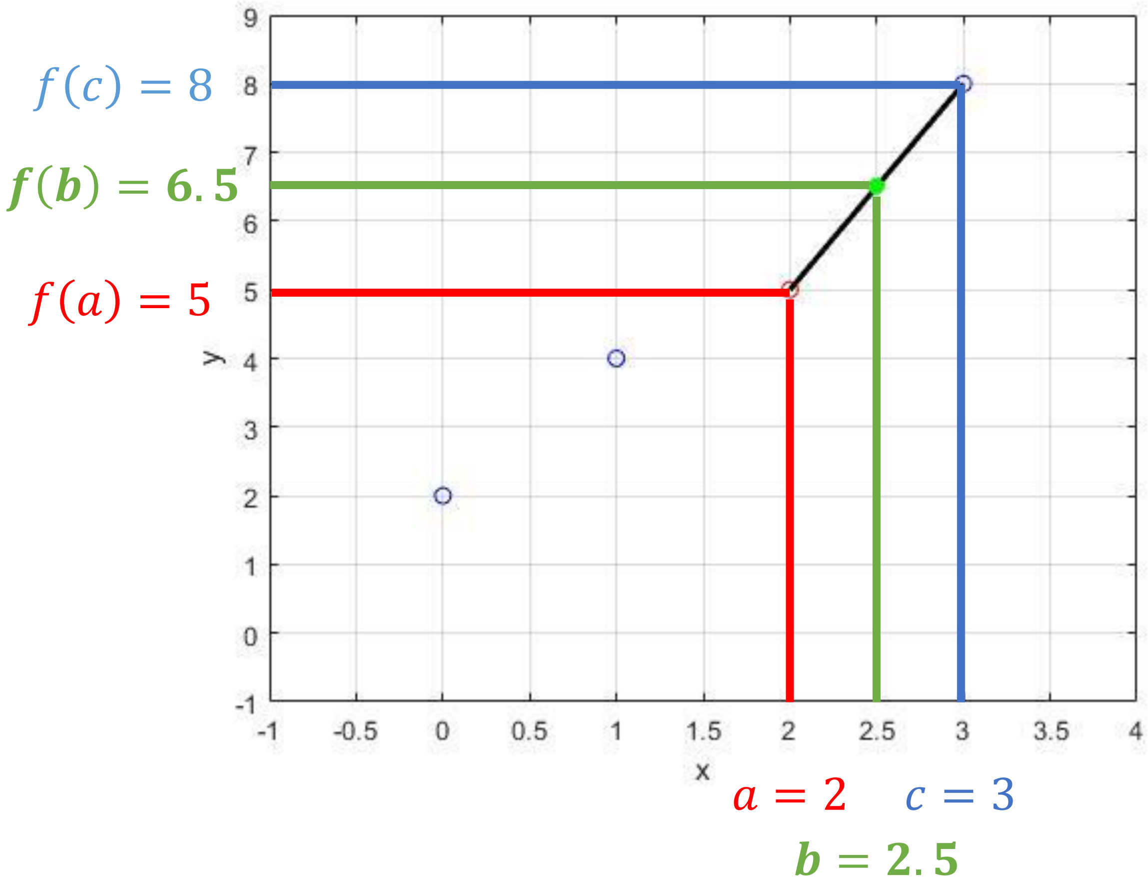
Linear Interpolation Procedure

To find the interpolated value $f(b)$ at a point b:

3) Evaluate $f(b)$ based on the fitted line and similar triangles

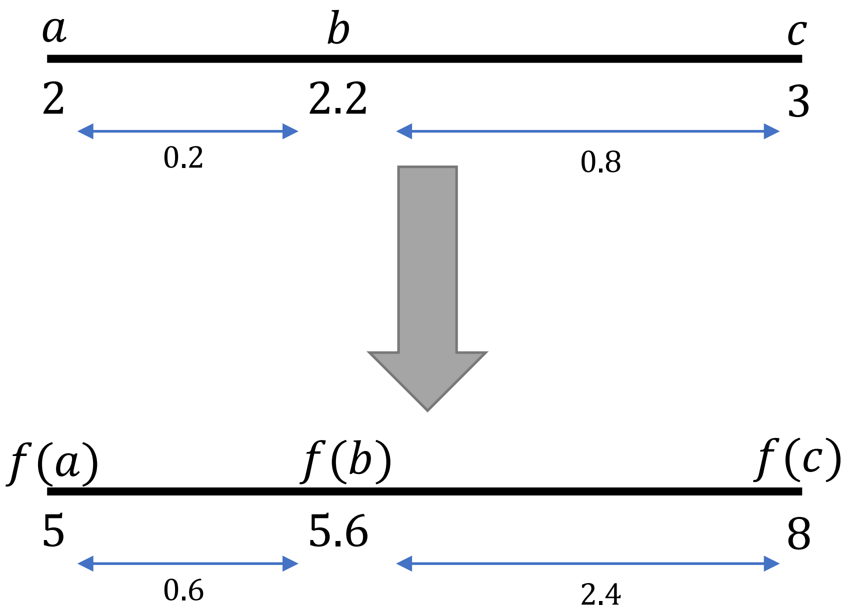
$$\boxed{f(b) = f(a) + \frac{f(c) - f(a)}{c - a}(b - a)}$$

Code this in MATLAB!

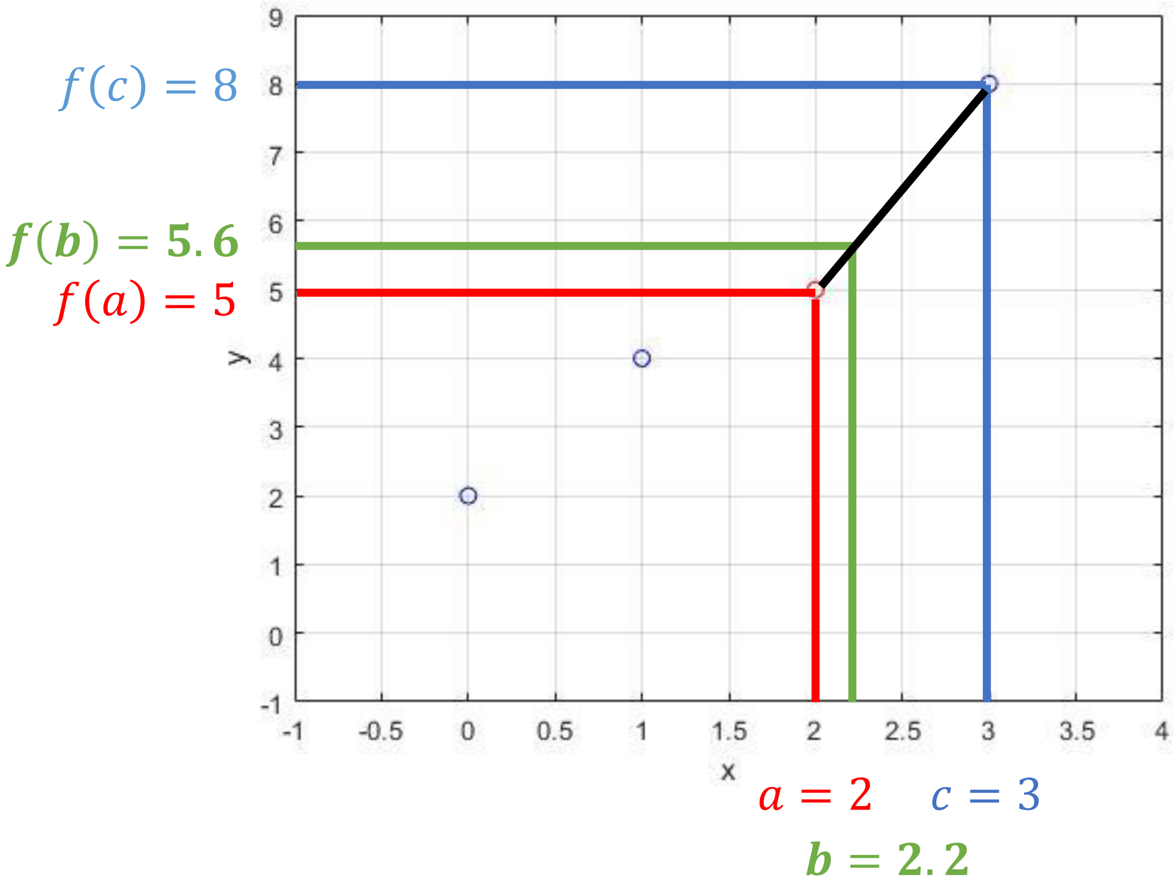


Cool Property of Linear Interpolation

- Because of linearity, $f(b)$ is the same distance from $f(a)$ and $f(c)$ as b is from a and c

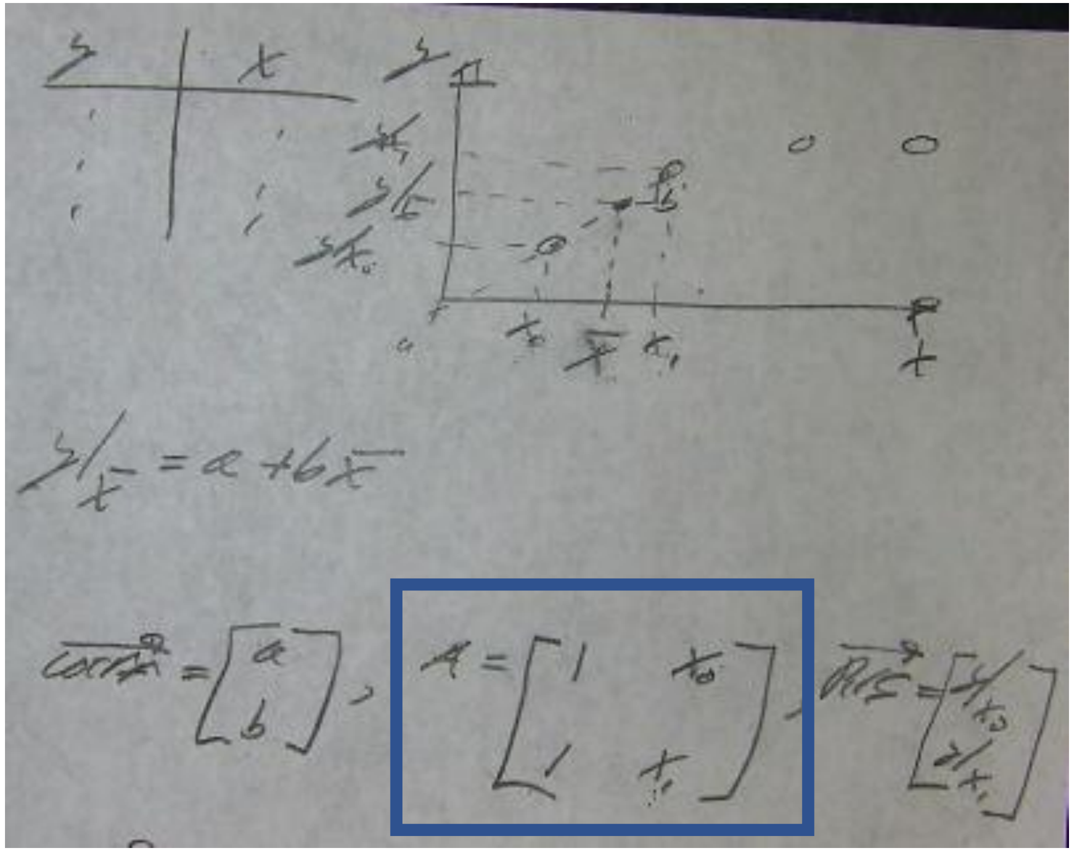


"Distance ratio" is preserved!



Aside

- You may have learned to interpolate via a linear system of equations ($Ax = b$)
- A matrix is called the *Vandermonde Matrix*
- Be wary in practical applications because A is VERY sensitive
    - Matrix *condition number*



Food For Thought 

- Linear interpolation formula includes a finite-difference approximation of $\frac{dy}{dx}$
  - Where have you seen this before?
- Finite-difference approximations are derived from the Taylor Series
  - What happens to the accuracy if you add more terms of the Taylor Series to the interpolation formula?
- The smaller the interval between data points, the better the approximation. Why?
  - Why don’t we make the interval something like $\Delta x = 0.000000000000000001$ for every interpolation?