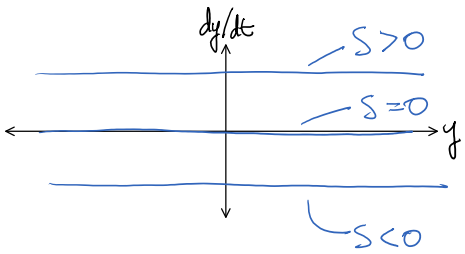
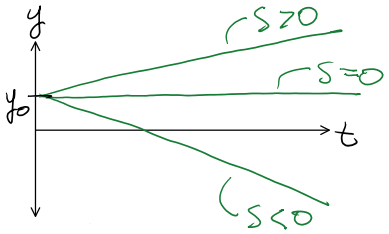
Phase Portraits: Example 3

Friday, July 3, 2020 1:36 PM

ODE and Initial Condition	Fixed Points	Stability
$\frac{dy}{dt} = S$ $y = y_0, t = 0$	$S=0$: any y	$S=0$: stable





1) Draw the *phase portrait.* Sketch $\frac{dy}{dt}$ $vs.$ y ($\frac{dy}{dt}$ on the y-axis, y on the x-axis)
2) Determine the fixed points.
   a. Find all the points where $\frac{dy}{dt} = 0$.
   b. Mark those points on the phase portrait.
3) Determine the stability of each fixed point.
   a. Draw a right arrow (→) in all regions where $\frac{dy}{dt} > 0$
   b. Draw a left arrow (←) in all regions where $\frac{dy}{dt} < 0$
   c. Fixed point is *stable* if the arrows converge to the point: →*←
   d. Fixed point is *unstable* if the arrows diverge from the point: ←*→
4) Draw the anticipated solution based on completed phase portrait.
   a. $y(t)$ trends *towards* a *stable* fixed point
   b. $y(t)$ trends *away* from an *unstable* fixed point
   c. All fixed points are horizontal lines (by definition, a fixed point doesn't change with time, so that's just a horizontal line).