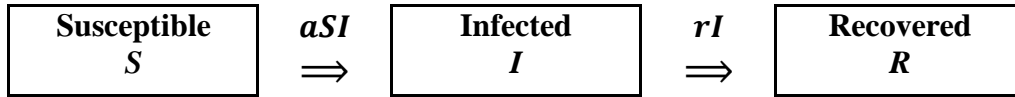


Ordinary Differential Equations: Epidemic

A basic model of an epidemic adheres to the input-output relation:



where

S = the number of currently healthy but susceptible individuals (people)

I = the number of infected individuals (people)

R = the number of recovered individuals (people)

a = infection rate

r = recovery rate

Thermal systems have an energy balance (conservation of energy); mechanical systems have a force balance (conservation of momentum). Likewise, the rate of change of a state variable in this model (S, I, R) can be written by a population balance. If one considers each box as a “boundary”, population balances can be written. For example, the balance around the S box is:

$$\frac{dS}{dt} = (in) - (out) \rightarrow \frac{dS}{dt} = 0 - aSI = -aSI$$

A city initially has 10,000 people, all of whom are susceptible. Then, a single infectious individual enters the city at $t = 0$. Use the following estimates for the parameters:

$$a = 0.002/7 \text{ (1/(person·day))}$$

$$r = 0.15/\text{day}$$

- Perform the population balances for the I and R state variables to obtain a complete description of the epidemic model.
- Verify the units of a and r .
- Compute the progression of the epidemic via `ode45()`.
- Physically interpret your results.