# Root Finding: Population Study

Many fields of engineering require accurate population estimates. For example, transportation engineers might find it necessary to determine separately the population growth trends of a city and adjacent suburb. The population of the city is over time is expressed by:

$$P_c(t) = P_{c,max}e^{-k_c t} + P_{c,min}$$

Meanwhile, the suburban population adheres to:

$$P_s(t) = \frac{P_{s,max}}{1 + \left(\frac{P_{s,max}}{P_0 - 1}\right)e^{-k_s t}}$$

where $P_{c,max}, k_c, P_{c,min}, P_{s,max}, P_0$, and $k_s$ are empirically derived parameters.

a) Just based on the forms of the equations, determine which population grows and which population dwindles over time.
b) Determine the time (in years) and corresponding values of $P_c(t)$ and $P_s(t)$ when the suburbs are $20\%$ larger than the city.
c) Determine the time (in years) and corresponding values of $P_c(t)$ and $P_s(t)$ when the suburbs are $X\%$ larger than the city, where $X$ ranges from $0: 25: 100$.
d) Estimate the maximum possible value of $X$.

For all parts, use the following parameters:

| Parameter | Value |
|---|---|
| $P_{c,max}$ | $80{,}000\ people$ |
| $k_c$ | $0.05\frac{1}{year}$ |
| $P_{c,min}$ | $110{,}000\ people$ |
| $P_{s,max}$ | $320{,}000\ people$ |
| $P_0$ | $10{,}000\ people$ |
| $k_s$ | $0.09\frac{1}{year}$ |