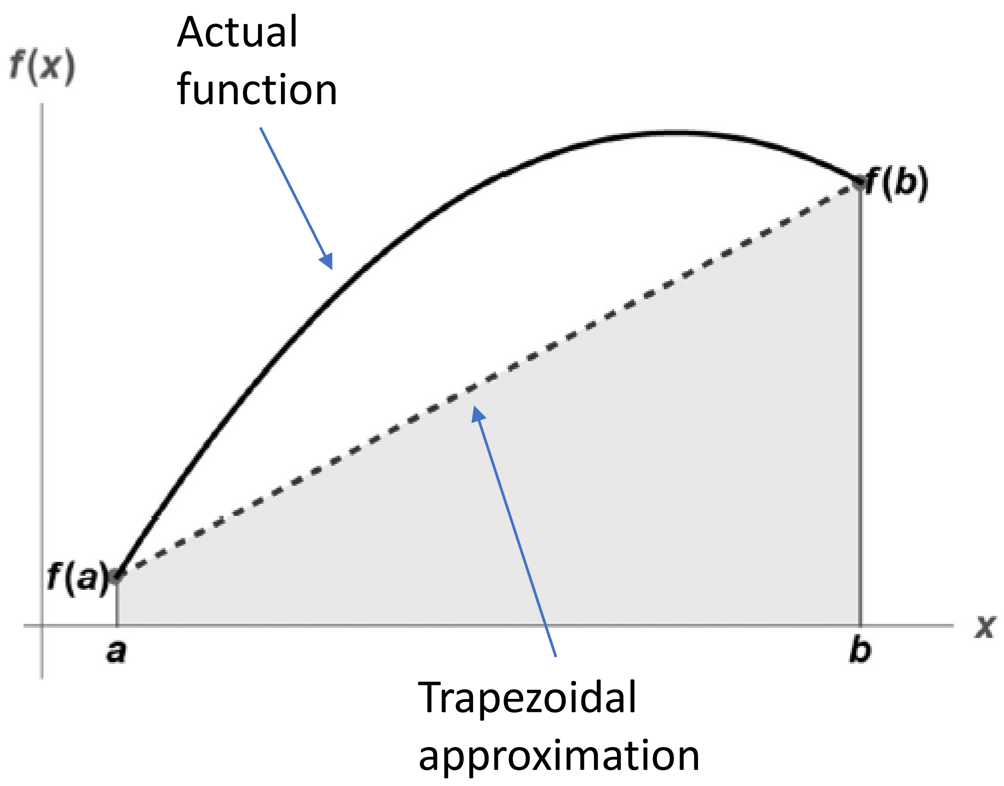
Trapezoidal Rule

A Quick Review

Trapezoidal Rule: Overview

- One of many numeric integration schemes
  - Riemann Sums
  - Simpson’s Rules
- Trapezoidal Rule: approximates the integral using trapezoids
- Many variations:
  - Single application
  - Composite Trapezoidal Rule
  - Trapezoid Rule for Unequally Spaced Data

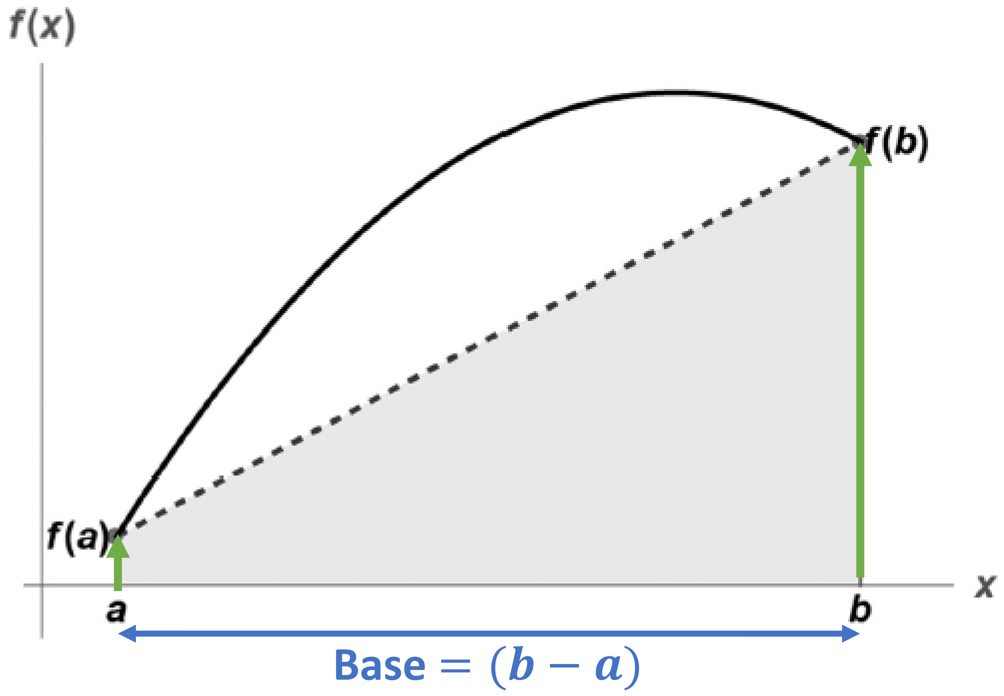


Trapezoidal Rule: Single Application

- Trapezoidal Rule:

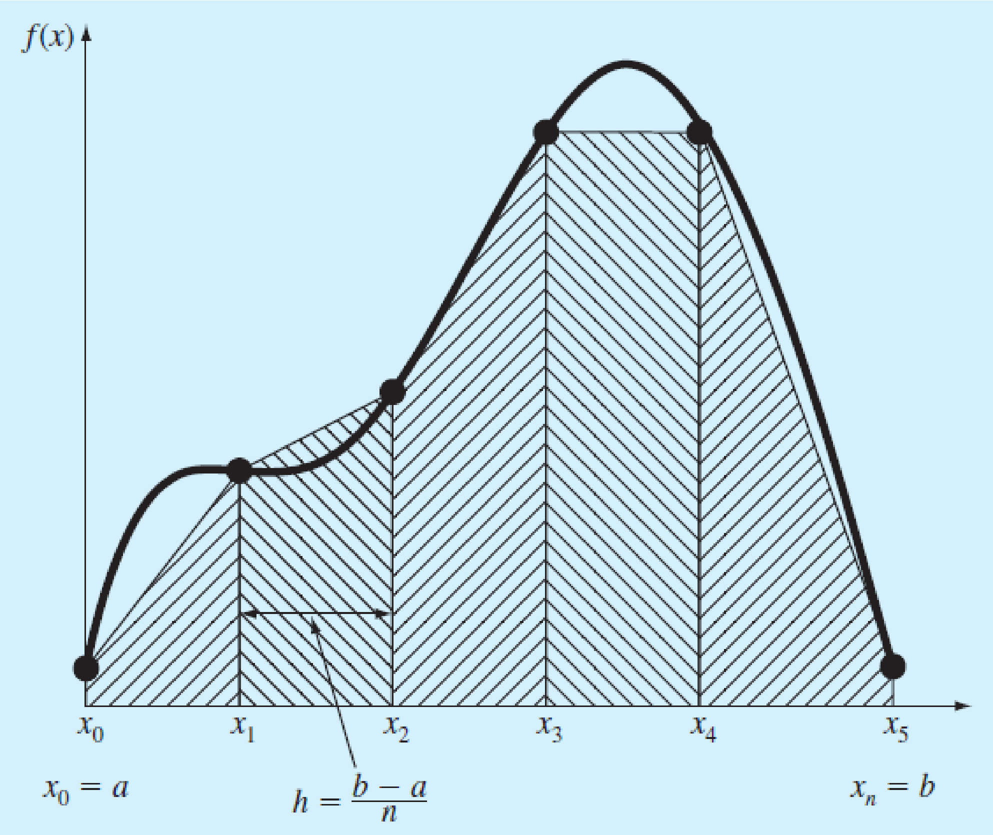
$$I = \int_a^b f(x)\ \ dx$$

$$\approx (b - a)\left(\frac{f(a) + f(b)}{2}\right)$$



Composite Trapezoidal Rule

- Divide the interval $[a, b]$ into multiple *equally spaced* segments
- n equally spaced points → $(n-1)$ trapezoids
- h = base of each segment $= \frac{b-a}{n}$
- Apply the Trapezoid Rule in each segment

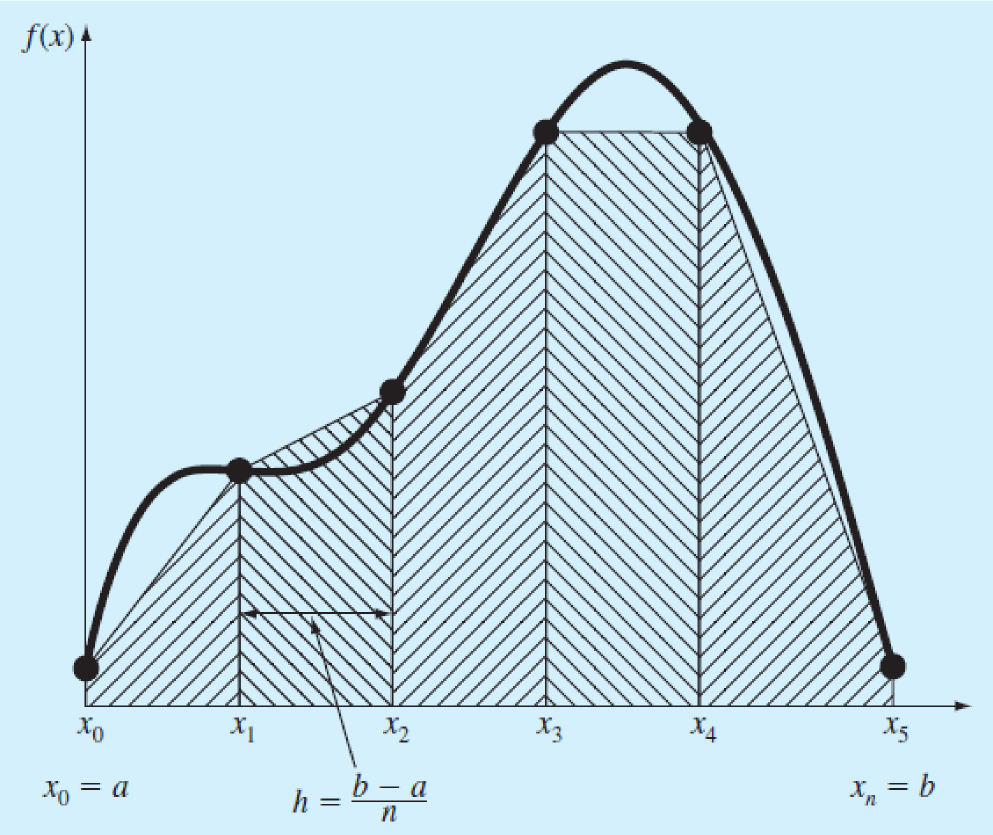


Composite Trapezoidal Rule

$$I = h\frac{\left(f(x_0) + f(x_1)\right)}{2} + \cdots + h\frac{\left(f(x_{n-1}) + f(x_n)\right)}{2}$$

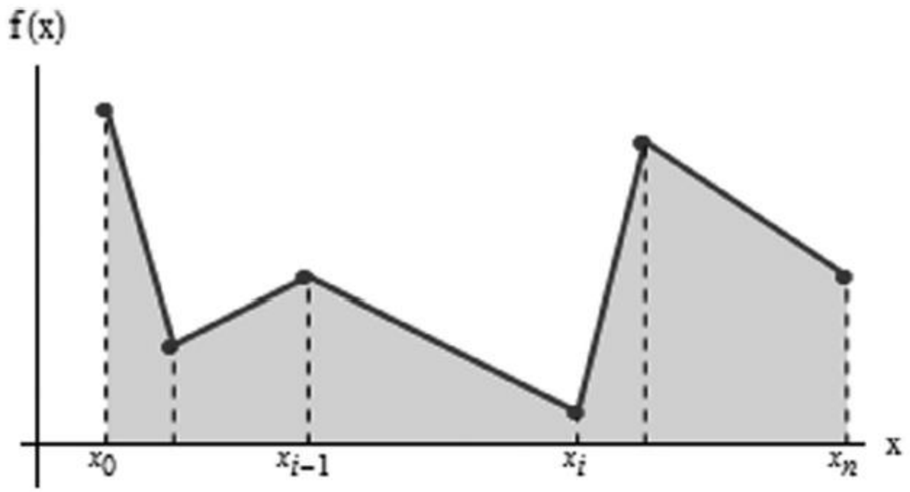
$$\rightarrow \boxed{I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]}$$

- Interior points weighted twice as heavily as the endpoints



Trapezoidal Rule for Unequal Segments

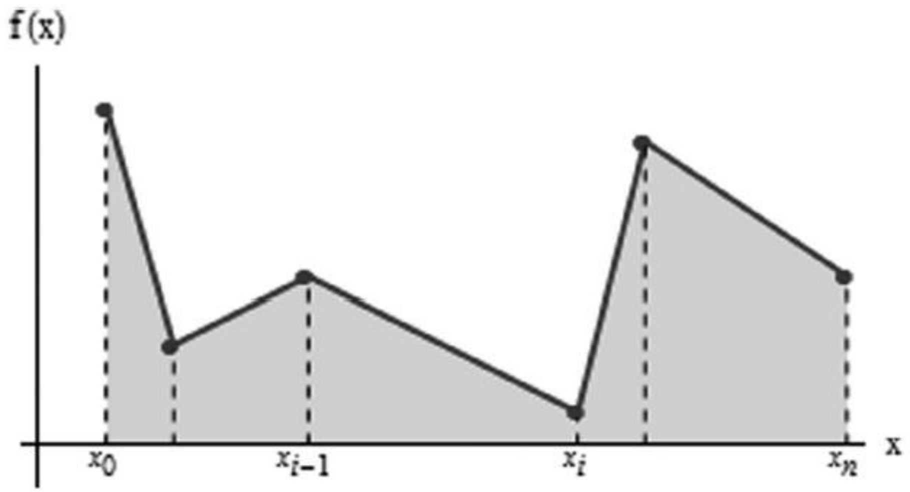
- $[a, b]$ contains multiple *UNequally* spaced segments
- n equally spaced points →$(n - 1)$ trapezoids
- h is no longer constant
- Apply the Trapezoid Rule in each segment



Trapezoidal Rule for Unequal Segments

$$I = h_1 \frac{\left(f(x_0) + f(x_1)\right)}{2} + \cdots + h_n \frac{\left(f(x_{n-1}) + f(x_n)\right)}{2}$$

$$\rightarrow \boxed{I = \sum_{i=1}^{n} h_i \frac{\left(f(x_{i-1}) + f(x_i)\right)}{2}}$$

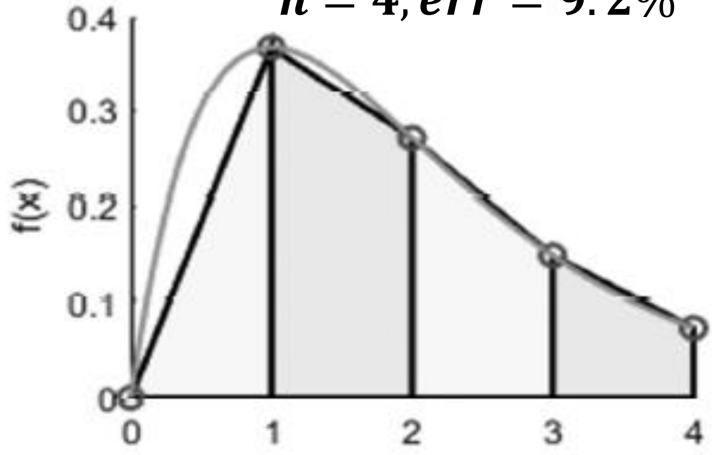


- $h_i = x_i - x_{i-1}$ = base of each segment
  - Reduces to *Composite Trapezoidal Rule* for constant h_i

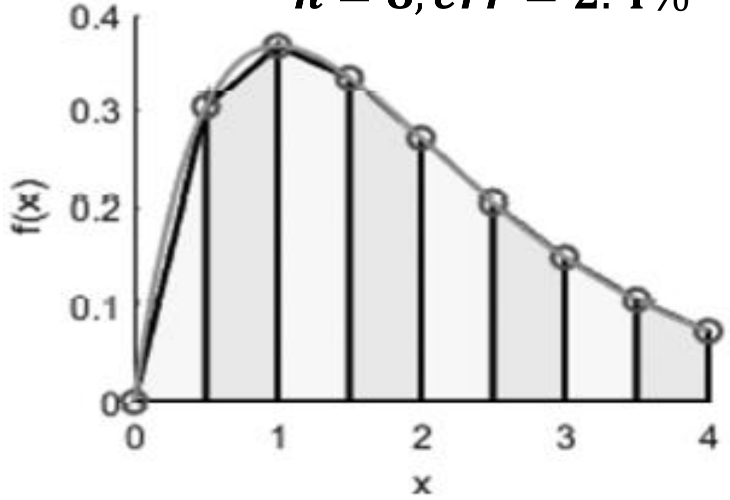
Trapezoidal Rule Error

- Trapezoidal Rule (any scheme) uses a *line* to approximate I $\rightarrow$ <u>perfectly approximates</u> $f(x) \ni f''(x) = 0$
- Error is on the order of h^2
  - Halving $h \rightarrow$ error $\sim$quartered
  - Doubling $n \rightarrow$ error $\sim$quartered
- Over/underestimates can occur!

$\boldsymbol{h = 4, err = 9.2\%}$



$\boldsymbol{h = 8, err = 2.4\%}$



`trapz` Function

- For x vectors with unit spacing ($h = 1$):

```
I = trapz(y)
```

- For x vectors with non-uniform and/or non-unit spacing:

```
I = trapz(x,y)
```

- As always, read the documentation!!!

Composite Trapezoidal Rule Example

x	0	1	2	3
y	10	18	31	50

4 data points $\rightarrow$ 3 trapezoids $(n = 3)$

$$h = \frac{b - a}{n} = \frac{3 - 0}{3} = 1$$

Composite Trapezoidal Rule Example

x	0	1	2	3
y	10	18	31	50

$$I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

$$= \frac{1}{2}\left[f(0) + 2\big(f(1) + f(2)\big) + f(3)\right]$$

$$= \frac{1}{2}(10 + 2(18 + 31) + 50) \rightarrow \boxed{I = 79}$$

Composite Trapezoidal Rule Example

x	0	1	2	3
y	10	18	31	50

Command Window

```
>> x = 0:3; y = [10 18 31 50];
>> I = trapz(y)

I =

    79

>> I = trapz(x,y)

I =

    79

fx >>
```