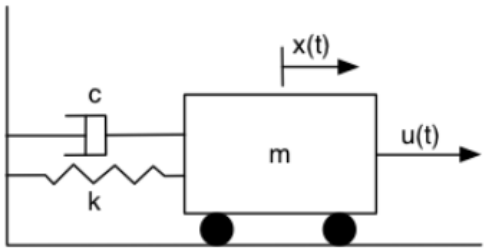
Ordinary Differential Equations: Mass-Spring-Damper Design

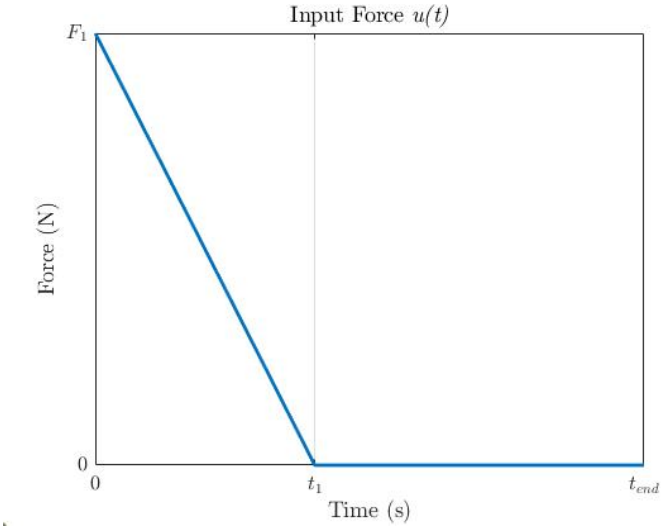
Consider the spring-mass-damper system described by the following differential equation and initial conditions:



$$m\ddot{x} + c\dot{x} + kx = u(t)$$

$$x = x_0, \dot{x} = \dot{x}_0 \text{ when } t = 0$$

The input force $u(t)$ starts at some magnitude F_1 and decreases linearly until it hits the x-axis at time t_1, after which $u(t) = 0$ until t_{end}, as illustrated by the figure below:



Find the values of c which satisfy the following requirements:

1. The cart's maximum forward (positive) velocity after t_1 must be $0.4 \leq v \leq 0.5\,\frac{m}{s}$.
2. $c < 2\sqrt{mk}$

Use `ode45()` and a time vector with a step size of $0.01\ s$. Change the relative and absolute tolerances to `1e-6` each. Use the following parameters:

```
m = 60;              % Cart's mass [kg] (scalar)
k = 55;              % Spring stiffness [N/m] (scalar)
F1 = 45;             % Initial magnitude of the input force [N] (scalar)
t1 = 10;             % Time at which the input force reaches 0 [s] (scalar)
t_end = 25;          % Final simulation time [s] (scalar)
IC = [0 0];          % Initial position and velocity (vector)
```