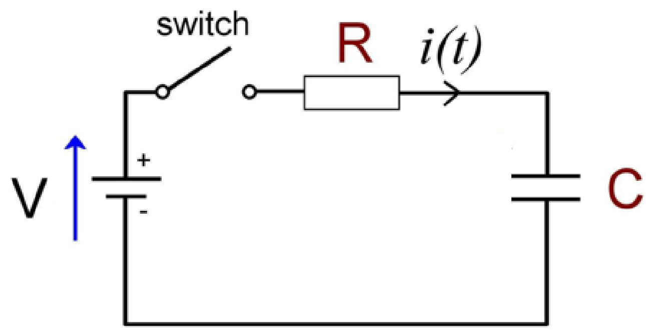
Linear Algebra: RC Circuit

The figure below comprises an RC circuit because it contains a resistor and a capacitor.



When the switch is closed, the voltage across the resistor over time can be modeled by:

$$V_R = Ve^{\left(-\frac{t}{RC}\right)}$$

where V_R and V are the resistor and battery voltages (V), respectively, $R = 2.5\ M\Omega$ is the resistance, and C is the capacitance (F). You build an RC circuit, but the labeling on the capacitor and battery have degraded and are no longer legible. Nevertheless, you close the switch and periodically measure the resistor's voltage. The experimental data and the value of R are contained in the `ME2004_RCData.mat` file.

a) Linearize the above equation.
b) Determine V and C.
c) Based on your analysis, predict V_R at $t = 2.5\ s$ and $t = 27\ s$.

Linearizing $V_R = Ve^{\left(-\frac{t}{RC}\right)}$:

Step	Result
Identify $Y = a_1X + a_0$ terms	$\underbrace{\quad}_{Y} = \underbrace{\quad}_{a_1}\underbrace{\quad}_{X} + \underbrace{\quad}_{a_0}$

Linearizing $V_R = Ve^{\left(-\frac{t}{RC}\right)}$:

Step	Result
Take natural log of both sides	$\ln(V_R) = \ln(V) - \frac{t}{RC}$
Reorder terms	$\ln(V_R) = -\frac{t}{RC} + \ln(V)$
Identify $Y = a_1X + a_0$ terms	$\underbrace{\ln(V_R)}_{Y} = \underbrace{\left(\frac{-1}{RC}\right)}_{a_1}\underbrace{t}_{X} + \underbrace{\ln(V)}_{a_0}$