Ordinary Differential Equations: Epidemic

A basic model of an epidemic adheres to the input-output relation:

$$\boxed{\begin{matrix}\textbf{Susceptible}\\ \boldsymbol{S}\end{matrix}} \overset{\boldsymbol{aSI}}{\Longrightarrow} \boxed{\begin{matrix}\textbf{Infected}\\ \boldsymbol{I}\end{matrix}} \overset{\boldsymbol{rI}}{\Longrightarrow} \boxed{\begin{matrix}\textbf{Recovered}\\ \boldsymbol{R}\end{matrix}}$$

where

S = the number of currently healthy but susceptible individuals (people)
I = the number of infected individuals (people)
R = the number of recovered individuals (people)
a = infection rate
r = recovery rate

Thermal systems have an energy balance (conservation of energy); mechanical systems have a force balance (conservation of momentum). Likewise, the rate of change of a state variable in this model (S, I, R) can be written by a population balance. If one considers each box as a "boundary", population balances can be written. For example, the balance around the S box is:

$$\frac{dS}{dt} = (in) - (out) \rightarrow \frac{dS}{dt} = 0 - aSI = -aSI$$

A city initially has 10,000 people, all of whom are susceptible. Then, a single infectious individual enters the city at $t = 0$. Use the following estimates for the parameters:

$a = 0.002/7$ (1/(person·day))
$r = 0.15$/day

a) Perform the population balances for the I and R state variables to obtain a complete description of the epidemic model.
b) Verify the units of a and r.
c) Compute the progression of the epidemic via `ode45()`.
d) Physically interpret your results.