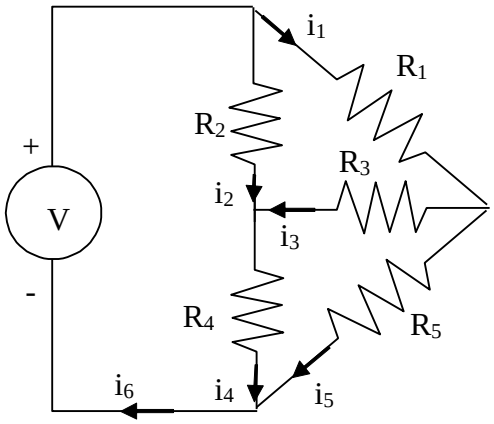
Linear Algebra: Wheatstone Bridge

The circuit shown in the figure is known as a Wheatstone bridge. It consists of five resistances and one applied voltage.



Kirchhoff's Voltage Law (KVL) applied to each loop in the circuit shown gives:

$$V - R_2 i_2 - R_4 i_4 = 0$$
$$-R_2 i_2 + R_1 i_1 + R_3 i_3 = 0$$
$$-R_4 i_4 - R_3 i_3 + R_5 i_5 = 0$$

Conservation of charge (aka Kirchhoff's Current Law, KCL) applied at each node in the circuit gives:

$$i_6 = i_1 + i_2$$
$$i_2 + i_3 = i_4$$
$$i_1 = i_3 + i_5$$
$$i_4 + i_5 = i_6$$

Note that only 3 of these 4 equations are linearly independent.

Objectives

a) Draw the cause/effect diagram.

b) Write a function M-file that accepts as inputs the applied voltage, `V`, and a vector containing the values of the resistances, **R**=`[R1,R2,R3,R4,R5]`. The function should solve for the six currents and return these values in a vector as output.

c) Use your function to find the currents for the following cases:

	V (volts)	R_1 (kΩ)	R_2 (kΩ)	R_3 (kΩ)	R_4 (kΩ)	R_5 (kΩ)
Case 1	0	1	1	1	1	1
Case 2	100	1	1	1	1	1
Case 3	100	1	5	2	10	5

d) Use your function to study the effect of the applied voltage on the currents. On a single graph, plot the 6 currents as a function of voltage for V = 0 to 500 volts. Use resistance values from Case 3. If the maximum allowable current is I_{max} = 0.08 amps, what is the maximum allowable voltage, V_{max}?

e) Use your function to study the effect of R_1 on the currents. On a single graph, plot the 6 currents as a function of R_1 for R_1 = 0 to 1 kΩ. Use the R_2-R_5 values from Case 3. If the minimum allowable I_3 is $I_{3,min}$ = 0.005 A, what is $R_{1,max}$?