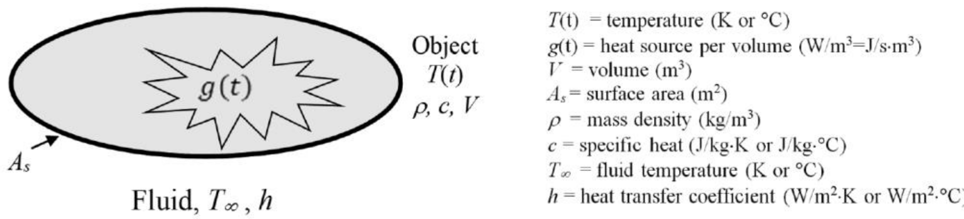
Ordinary Differential Equations: Lumped Thermal Mass

Consider the cooling of a hot object initially at temperature T_0 in cold air at T_∞ with heat transfer coefficient h.



Assume that the lumped thermal capacity model, as described in Vick Ch. 7.2.4, is valid. Applying Conservation of Energy yields the differential equation:

$$\rho c V \frac{dT}{dt} = -hA_s(T - T_\infty) + gV$$

a) Draw the cause-effect diagram for this physical problem.
b) Determine the steady-state solution (aka, find the fixed point). Is it stable or unstable? Sketch the phase portrait and anticipated solution.
c) Write a function to numerically solve for temperature as a function of time. The system parameters and forcing functions should be inputs, and the time and temperature vectors should be outputs.
d) Explore the effect of the heat transfer coefficient, h. Plot temperature versus time for $h = 0\,\frac{W}{m^2\,°C}$ (no heat transfer), $10\,\frac{W}{m^2\,°C}$ (still day), $25\,\frac{W}{m^2\,°C}$ (typical day), and $75\,\frac{W}{m^2\,°C}$ (hurricane). Put all curves on a single graph. Carry out the integration long enough that the temperature begins to level out at steady state and use a step size of $1\ s$. The other parameters are $V = 10^{-6}\ m^3, A_s = 10^{-4}\ m^2, \rho = 1000\,\frac{kg}{m^3}, c = 500\,\frac{J}{kg}, T_\infty = 25\ °C, T_0 = 400\ °C$, and $g(t) = 0\,\frac{W}{m^3}$.
e) Explore the effect of a *constant* heat source strength, $g(t)$. Plot the solution for $g(t) = 0, 10^5, 2 * 10^5$, and $3 * 10^5\,\frac{W}{m^3}$. Put all curves on a single graph. Use the parameter values from part (d) with $h = 25\,\frac{W}{m^2\,°C}$.

f) Explore the effect of a *pulsed* heat source, defined by:

$$g(t) = g_c \frac{1}{\Delta t}\left(H(t) - H(t - \Delta t)\right)$$

$g_c =$ heat source strength $\left(\frac{J}{m^3}\right)$, $\Delta t =$ pulse time (s), and $H(t) =$ unit step/Heaviside function. Plot the temperature of the thermal mass for:

$g_c = 4 * 10^7 \frac{J}{m^3}$
$\Delta t = 400, 200, 100\ s$
$T_0 = T_\infty = 25\ °C$
$h = 25 \frac{W}{m^2\ °C}$

The remaining parameters are the same as in part (d). For this part, employ a relative and absolute `ode45()` tolerance of `10^-9`. Create a subplot containing the heat source $g(t)$ on the upper subplot and $T(t)$ on the lower subplot. Physically interpret your results.

Lumped Thermal Mass Phase Portrait/Anticipated Solution

ODE and Initial Condition	Fixed Points (=Steady-State Values)	Stability
$\rho c V \dfrac{dT}{dt} = -hA_s(T - T_\infty) + gV$ $T_0 = 400\ °\mathrm{C}, t = 0$		

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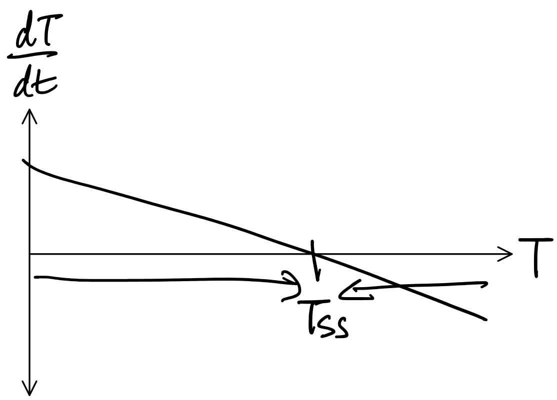

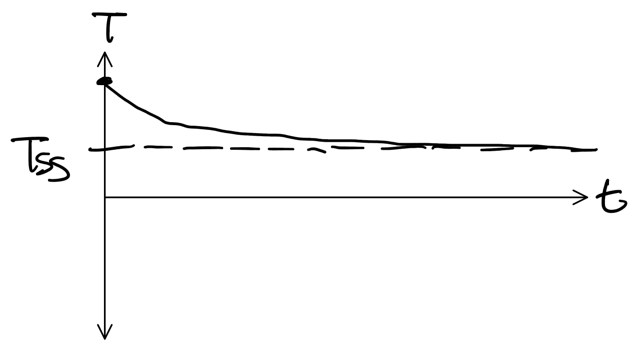

$$\cancel{\rho c V \frac{dT}{dt}} = -hA_s(T - T_\infty) + gV$$

$$\longrightarrow \quad T = T_\infty + \frac{gV}{hA_s}$$