Solving Linear Algebra Problems in MATLAB

ME 2004

Outline

- 1.1: Five-Step Process

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Five-Step Process

Solving a linear algebra problem:

1) Analyze the system holistically. What type of system are you analyzing (electrical, mechanical, structural, chemical, etc.)?
2) Apply the relevant engineering laws/first principles to obtain a series of equations.
3) Put the equations in $Ax = b$ form.
4) Solve by hand or in MATLAB.
5) Apply a series of test cases to check your results.

Five-Step Process

1) Analyze the system holistically. What type of system are you analyzing (electrical, mechanical, structural, chemical, etc.)?
   - Helps frame the problem
   - Recall any important equations, subtleties, etc.

Five-Step Process

2) Apply the relevant engineering laws/first principles to obtain a series of equations.

System Type	Potentially Relevant Equation
Structural	$\Sigma F = ma$
Static Equilibrium	$\Sigma F = 0$
Spring Systems	$F = kx$
Electrical	$V = IR\ (Ohm's\ Law), \Sigma V = 0\ (KVL), \Sigma i_{in} = \Sigma i_{out}\ (KCL)$
Thermal circuits	$T = QR$

Five-Step Process

3) Put the equations in $Ax = b$ form.
   - Place variables in the same order in each row
   - Write in missing variables with a 0 coefficient
   - Write in the "1" for variables with a coefficient of 1
   - Triple-check signs
   - Be neat and work slowly!

$$\begin{aligned} -3x_3 + 5x_1 + 2x_2 &= 1 \\ x_2 + x_3 &= 2 \\ 2x_1 - 6x_3 - 2 &= 2 \end{aligned} \Rightarrow \begin{aligned} 5x_1 + 2x_2 - 3x_3 &= 1 \\ 0x_1 + 1x_2 + 1x_3 &= 2 \\ 2x_1 + 0x_2 - 6x_3 &= 4 \end{aligned} \Rightarrow \underbrace{\begin{bmatrix} 5 & 2 & -3 \\ 0 & 1 & 1 \\ 2 & 0 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}}_{b}$$

Five-Step Process

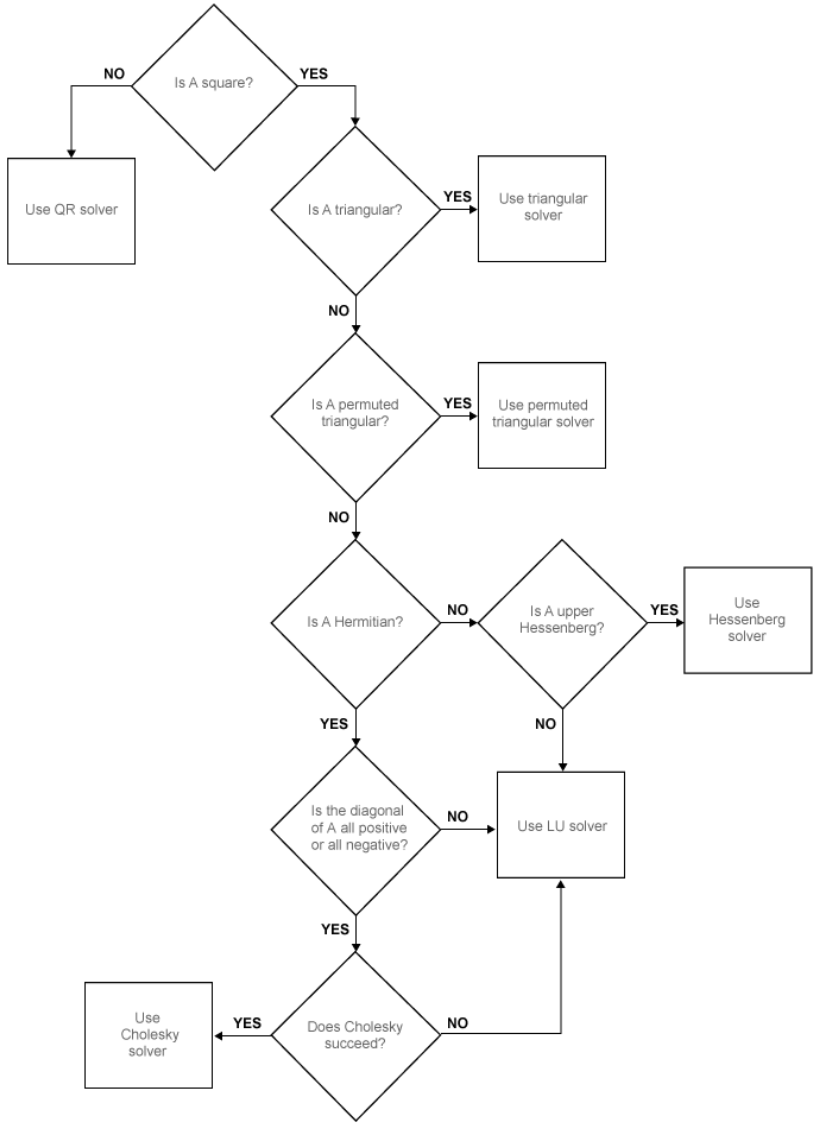
4) Solve by hand or in MATLAB.
   - Gauss-Jordan Elimination (by hand or MATLAB `rref()`)
   - MATLAB backslash operator, \ (AKA `mldivide()`)
   - Other methods

Command Window

```
>> A = [5 2 -3; 0 1 1; 2 0 -6];
>> b = [1 2 4]';
>> x = A\b

x =

   -1.9000
    3.3000
   -1.3000

>> rref([A b])

ans =

    1.0000         0         0   -1.9000
         0    1.0000         0    3.3000
         0         0    1.0000   -1.3000
```

Five-Step Process

- Backslash operator is not a single operation, but a multistep computation depending on A
- In MATLAB, `A\b` roughly equals `inv(A)*b`
- Can also compute least-squares regression for underdetermined systems (later)



Five-Step Process

5) Apply a series of test cases to check your results.
   - Check the “zero case:” if there’s no input, there shouldn’t be a change in the output!
   - Perform parameter studies (including plots)
   - Perform “back-of-the-envelope” sanity checks

Summary

1) Analyze the system holistically. What type of system are you analyzing (electrical, mechanical, structural, chemical, etc.)?
2) Apply the relevant engineering laws/first principles to obtain a series of equations.

3) Put the equations in $Ax = b$ form.
   - Slow and steady wins the race!
4) Solve ~~by hand~~ or in MATLAB.
   - Use the backslash operator in MATLAB
5) Apply a series of test cases to check your results.