Ordinary Differential Equations: Population Model

The earth's population can be estimated using the following simplified model:

$$\frac{dP}{dt} = aP - bP^2$$

where $a > 0$ is the birth rate parameter and $b > 0$ is the death rate parameter.

a) Determine the steady-state population.
b) Sketch the phase portrait and anticipated solution.
c) When the time variable t is measured in years, experimental evidence suggests that the parameters a and b in the population model are approximately $a = 0.028 \frac{1}{year}$ and $b = 2.9 \times 10^{-12} \frac{1}{people*year}$. Starting from a population of approximately P_0 = 100 million people in the year 1800, compute and plot the earth's population as a function of time. Use ode45() with a time vector step size of 1 year. Run your simulation until the population appears to level off. Using these parameters, how many people will we eventually have on this earth? At what year will we have reached 99% of our maximum population?
d) Estimating population is an inexact science. Examine the effect of the birth rate parameter by plotting the earth's population for $a = 0.025, 0.03,$ and 0.035 with $b = 2.9 \times 10^{-12} \frac{1}{people*year}$. Put all curves on a single graph.
e) Examine the effect of the death rate parameter by plotting the earth's population for $b = 2.5 \times 10^{-12}, 3 \times 10^{-12}$ and 3.5×10^{-12} with $a = 0.029 \frac{1}{year}$. Put all curves on a single graph.