Root Finding via Newton-Raphson

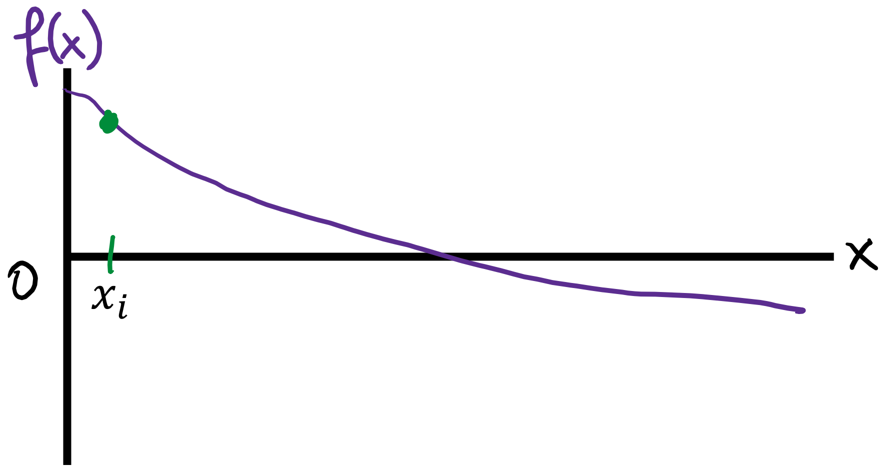
A Quick Review

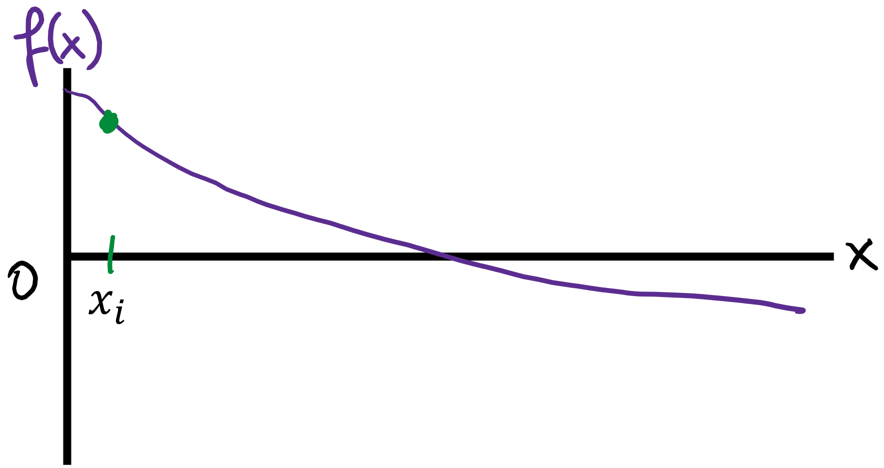
Newton-Raphson Method: Overview

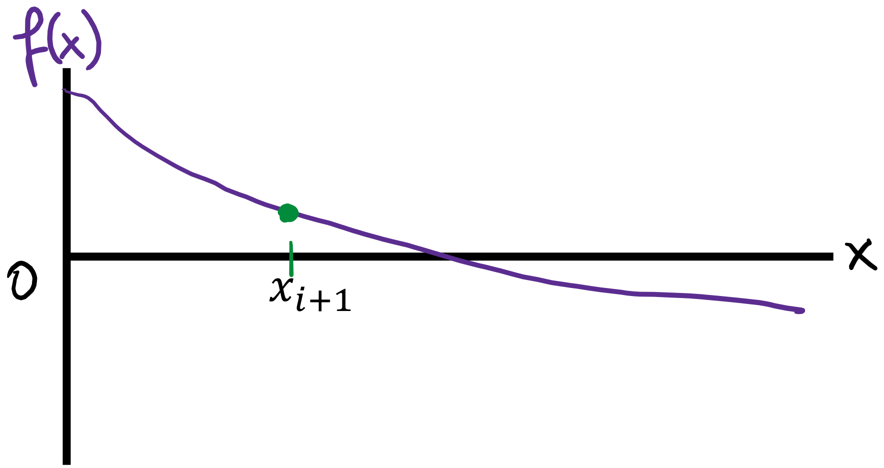
- Newton-Raphson/Newton’s Method: another root finding technique
- Premise: combine an initial starting point with the derivative of the function to iteratively trace the root
  - WARNING: May diverge!!! (unable to locate root)
- Called an *open method* because the user must supply the algorithm with a singular starting point
  - Contrast: Bisection is a *bracketing method* because it needs two initial guesses which bracket the root

Brief Algorithm Overview

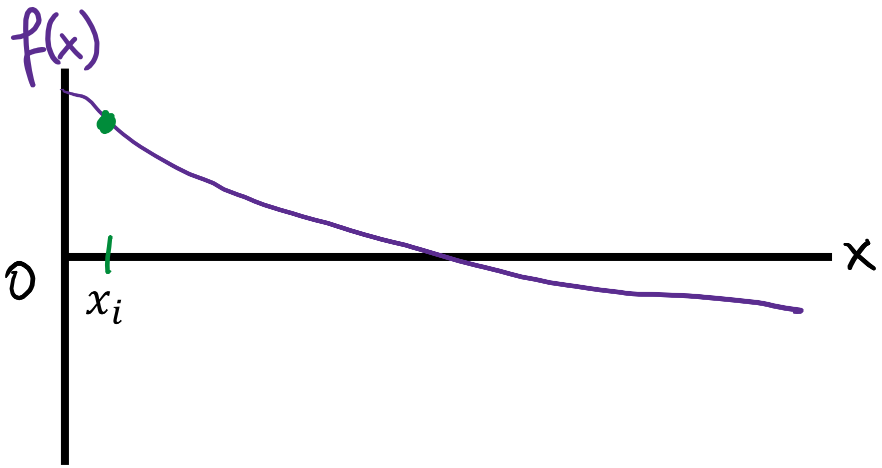
- Input initial guess x_i and compute $f'(x)$
- From the starting point x_i, extend the tangent $f'(x_i)$ until it hits the x-axis
- This new location is the new estimate of the root, x_{i+1}
- Compute $|f(x_{i+1})|$ and iterate if the stopping criterion is not met


f(x)
0
x
x_i


f(x)
0
x
x_i


f(x)
0
x
x_{i+1}

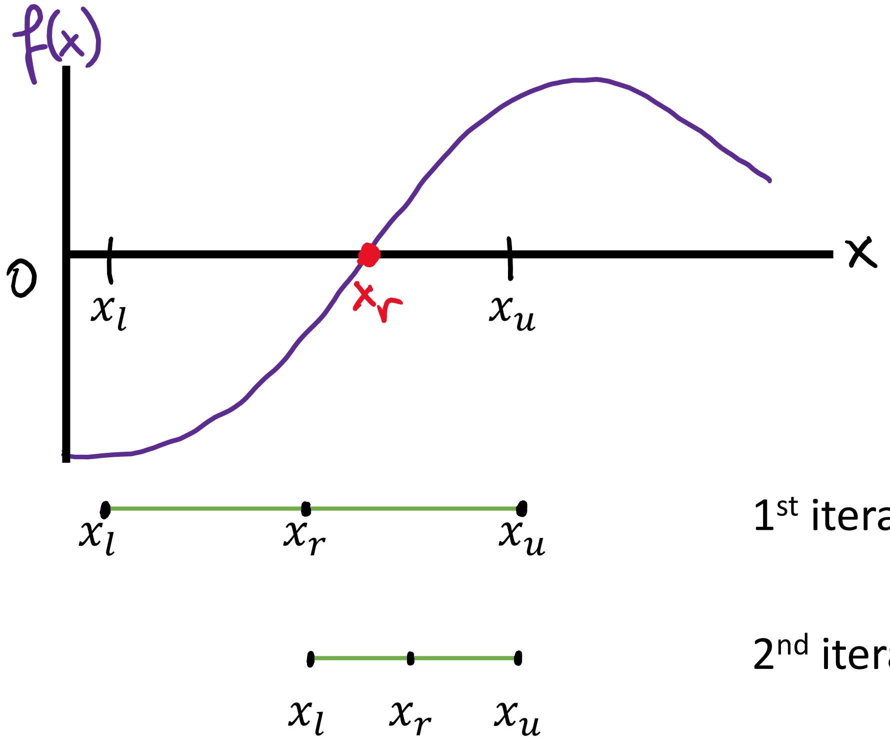
Mathematics Behind Newton-Raphson



- Definition of slope:

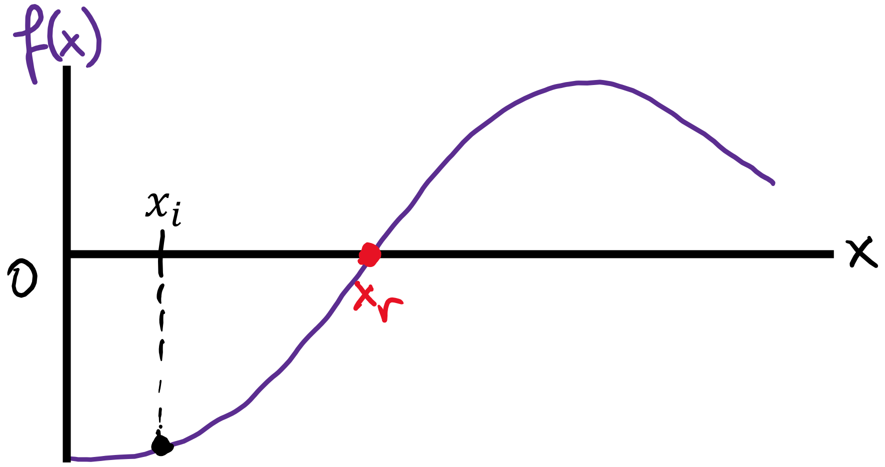
- Rearranged:

Pitfalls of Newton-Raphson



1st iteration

2nd iteration



Bisection always converges (given a valid initial bracket)!

NR may diverge!

Pitfalls of Newton-Raphson

- Newton-Raphson equation:

- What happens if $f'(x_i) = 0$? Division-by-zero error!

Bisection vs. Newton-Raphson

Consider Picking <u>Bisection</u> If…

Consider Picking <u>NR</u> If…

Food For Thought 

- What *graphically* happens when you get a division-by-zero error?
- Can you generate a function $f(x)$ which causes bisection to converge <u>faster</u> than NR?
- Can you derive the function $f(x)$ which causes NR to cycle infinitely around a point?