Ordinary Differential Equations

A Quick Introduction

Ordinary Differential Equations (ODEs)

- Crown jewel of Mechanical Engineering (and most other fields)
- Formal course in Differential Equations NOT REQUIRED for this unit!
- This class emphasizes:
  - Visualizing ODEs
  - Hand-sketching solutions to simple ODEs
  - Using built-in MATLAB solvers for complex ODEs

- Will only learn a few hand-solving techniques – save the rest for the math department!

Motivation

- Many ME classes focus on *static analysis*: computing necessary forces, etc. when given static (time invariant) input loads
- Static analysis might be valid for your engineering problem, but many real problems involve time-varying inputs
- It's more useful to know the *entire time history*

The figure shown is a geared countershaft with an overhanging pinion at *C*. Select an angular-contact ball bearing from Table 11–2 for mounting at *O* and an 02-series cylindrical roller bearing from Table 11–3 for mounting at *B*. The force on gear *A* is *FA* 5 600 lbf, and the shaft is to run at a speed of 420 rev/min. Solution of the statics problem gives force of bearings against the shaft at *O* as **R***O* = 2387**j** + 467**k** lbf, and at *B* as **R***B* = 316**j** − 1615**k** lbf. Specify the bearings required, using an application factor of 1.2, a desired life of 40 kh, and a combined reliability goal of 0.95, assuming distribution data from manufacturer 2 in Table 11–6.

Problem 11–32 Dimensions in inches.

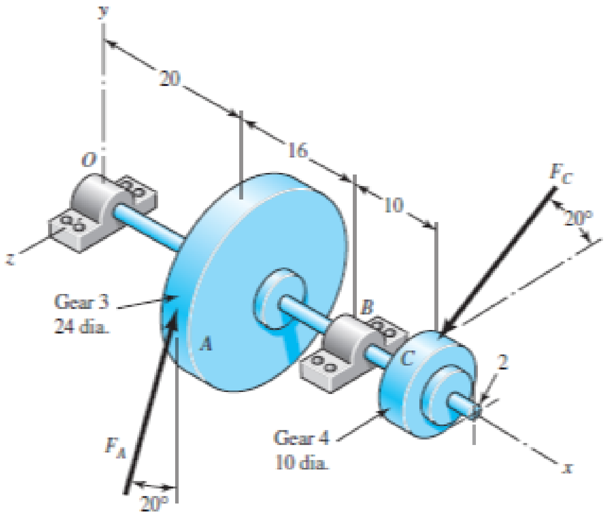


Table 11–2 Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

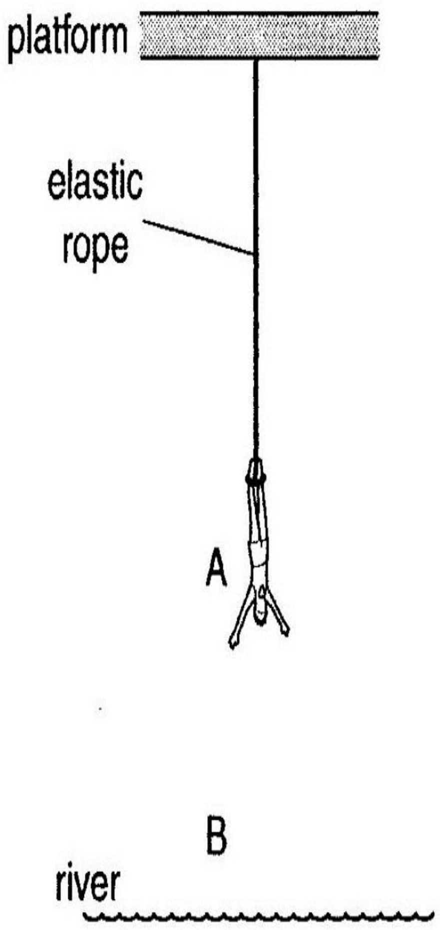
Motivation

- We study ODEs because many of the fundamental laws in engineering (conservation, continuity, etc.) are expressed as rates of change
- Engineering problems are inherently modeled as ODEs and we must solve them to find our desired quantity (such as position vs. time, $x(t)$)

Law	**Equation**	**Physical Area**
Fourier's law	$q = -k\dfrac{dT}{dx}$	Heat conduction
Fick's law	$J = -D\dfrac{dc}{dx}$	Mass diffusion
Darcy's law	$q = -k\dfrac{dh}{dx}$	Flow through porous media
Ohm's law	$J = -\sigma\dfrac{dV}{dx}$	Current flow
Newton's viscosity law	$\tau = \mu\dfrac{du}{dx}$	Fluids
Hooke's law	$\sigma = E\dfrac{\Delta L}{L}$	Elasticity

These are some common rate equations you might stumble upon in ME. All of them have derivatives!

Modeling Process



Physical Law $\Sigma F = ma$

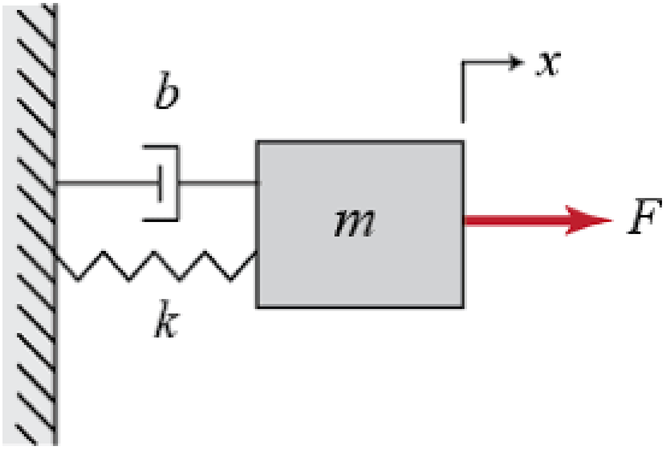
ODE
$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

Solution
$$v(t) = \sqrt{\frac{mg}{c_d}} \tanh\left(\sqrt{\frac{g c_d}{m}}\, t\right)$$
(Analytical)

$$v_{i+1} = v_i + \left(g - \frac{c_d}{m} v^2\right) \Delta t$$
(Numerical)

Definitions

- *Order:* highest derivative in the ODE.



Mass-spring-damper system.
You'll see this frequently in this class + your upper-level classes.

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t)$$

2nd order system because the highest derivative is a 2nd derivative

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

Equivalent system representation using *dot notation*:

$$\frac{d^2x}{dt^2} = \ddot{x}, \frac{dx}{dt} = \dot{x}$$