Root Finding: Critical Channel Depth

Water is flowing in a trapezoidal channel at a volumetric flow rate of $Q\left(\frac{m^3}{s}\right)$. The width of the channel at the surface is $B = 3 + y$ meters, where y is the critical depth (depth where the specific energy of the flow is minimized) and the cross-sectional area of the channel is $A_c = \frac{y^2}{2} + 3y$ m^2. The critical depth y for such a channel must satisfy the equation:

$$0 = 1 - \frac{Q^2 B}{g A_c^3} = 1 - \frac{Q^2(3+y)}{g\left(\frac{y^2}{2} + 3y\right)^3}$$

where $g = 9.81\frac{m}{s^2}$ = acceleration due to gravity.

a) Given $Q = 20\frac{m^3}{s}$, estimate the critical depth graphically.

b) Given $Q = 20\frac{m^3}{s}$, estimate the critical depth via the Bisection Method with initial guesses $x_l = 0.5$ and $x_u = 2.5$. Iterate until the percent relative error falls under 1% or the number of iterations exceeds 15.

c) Estimate the critical depths via the Bisection Method when Q varies from $20\frac{m^3}{s}$ to $40\frac{m^3}{s}$ in increments of $2.5\frac{m^3}{s}$. Supply your own initial guesses. Iterate until the percent relative error falls under 1% or the number of iterations exceeds 15. Create a plot of Q versus y.