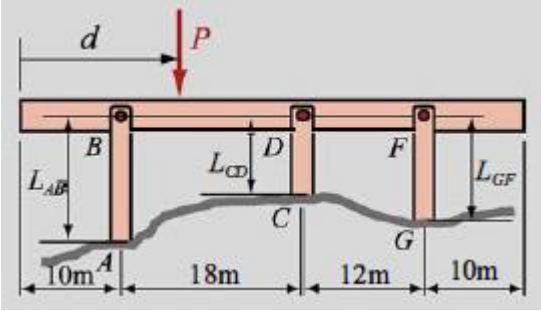
Linear Algebra: Column Elongation

A bridge is modeled by a rigid horizontal bar supported by three elastic vertical columns as shown:



(Gilat 2013)

A force $P = 40\ kN$ applied to the rigid bar at a distance d from the leftmost end of the bar represents a car on the bridge. The forces in columns F_{AB}, F_{CD}, and F_{GF} can be determined from the solution of the following system of three equations:

$$F_{AB} + F_{CD} + F_{GF} = -P$$

$$10F_{AB} + 28F_{CD} + 40F_{GF} = -dP$$

$$12L_{AB}F_{AB} - 30L_{CD}F_{CD} + 18L_{GF}F_{GF} = 0$$

Once the force in a column is known, its elongation δ can be determined by:

$$\delta = \frac{FL}{EA}$$

Where $E =$ modulus of elasticity (how stiff a material is) and $A =$ cross-sectional area. E and A are uniform for all 3 columns, but F and L are not.

If $\delta > 0$, the column is elongated and is said to be in *tension.* If $\delta < 0$, the column is in *compression*.

P, E, A, L_{AB}, L_{CD}, and L_{GF} are stored in the `ME2004_BarParameters.mat` data file. This can be loaded into MATLAB by issuing:

```
load('ME2004_BarParameters.mat')
```

in your script.

a) Arrange the first three equations in matrix form.
b) Write a user-defined MATLAB function to solve the system of equations. The function should accept P, d, and the three column lengths. It should output the three forces and elongations.
c) Sweep d across the length of the beam and compute the associated forces/elongations (i.e., repeatedly call your function to compute the forces/elongations as d varies)
d) Plot the forces on one subplot and the elongations on another subplot.
e) It is undesirable for the columns to be significantly elongated. A safety standard dictates that the deflection of any column should remain under $\delta_{max} = 0.003\ m$ always. Is this satisfied? If so, compute the *factor of safety*:

$$FS = \left|\frac{\delta_{max}}{\delta_{close}}\right|$$

where δ_{close} is the δ closest to δ_{max} across all three columns. If not, over what d is the standard upheld?
f) Comment on your results.