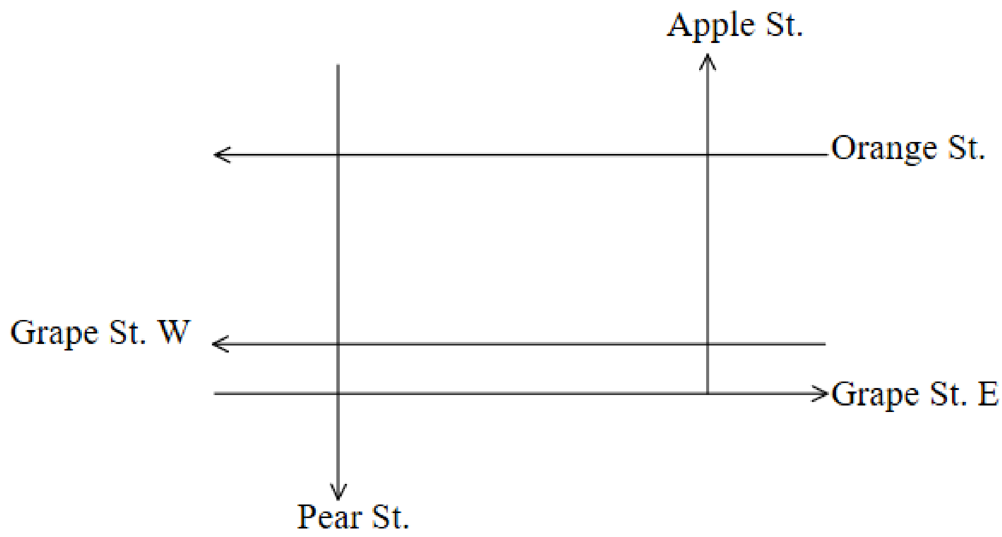
<u>Linear Algebra: Traffic Network</u>

Denizens of Fruit Village often commute through this road network:



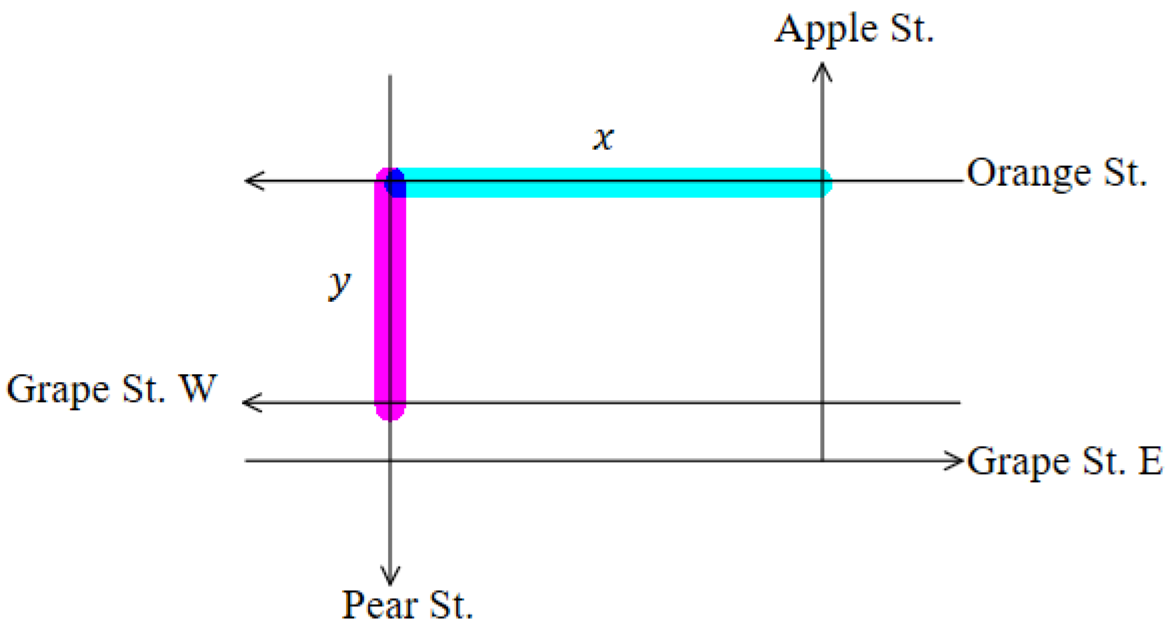
The traffic volume in a 1-hour window during rush hour was collected and tabulated:

	Grape St. E	**Grape St. W**	**Pear St.**	**Apple St.**	**Orange St.**
# Vehicles Entering	80	50	65	*N/A*	40
# Vehicles Exiting	30	*?*	70	55	75

Recall *Kirchhoff's Current Law:* the current flowing into a junction equals the current flowing out of that junction. Civil engineers often extend KCL to road networks: the number of vehicles entering an intersection equals the number of vehicles leaving the intersection.

a) Why is the "into Apple St." entry *N/A*?
b) The total number of vehicles entering the whole network must equal the total number of vehicles leaving. Compute the number of vehicles exiting Grape St. W.

Fruit Village needs you to determine the traffic in blocks between intersections, such as x and y:



c) Write the equations describing the flows in the blocks lying between intersections. As a starter, analyze the Orange-Pear intersection:

$$(\# \, Vehicles \, Entering) = (\# \, Vehicles \, Exiting)$$

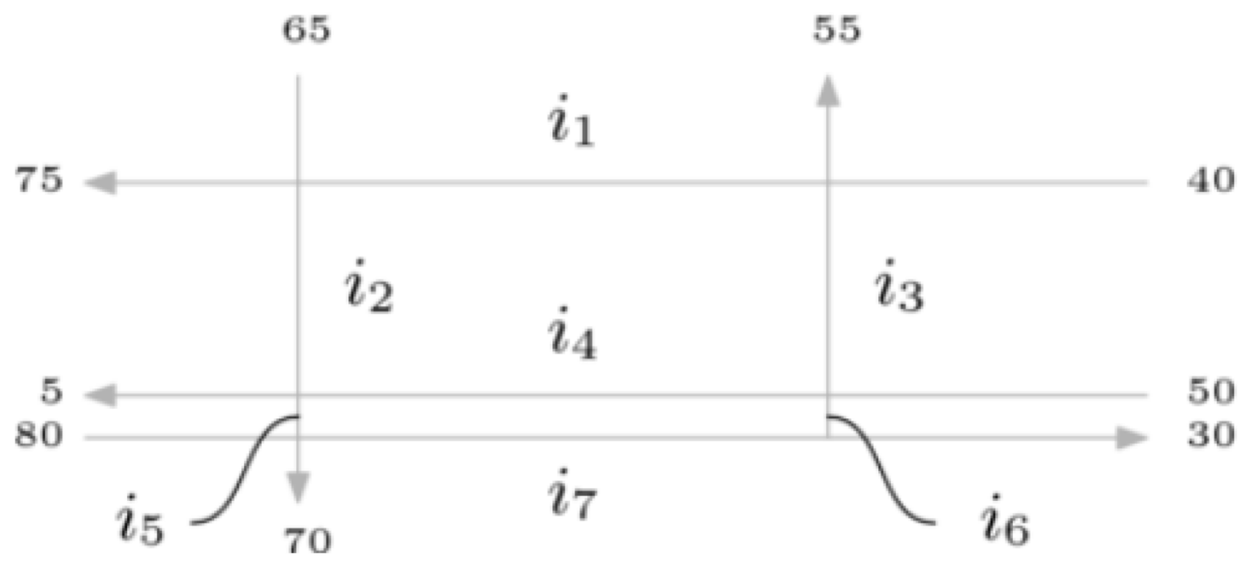
$$(x + 65) = (y + 75)$$

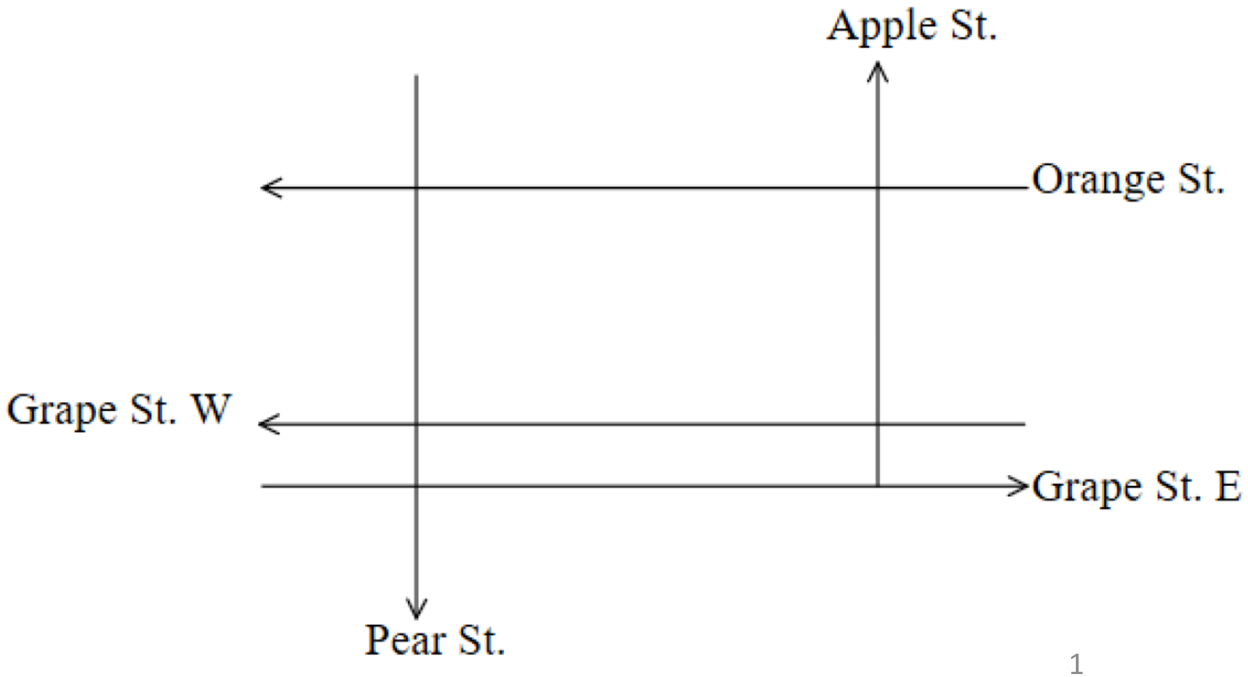
$$\rightarrow x = y + 10$$

Here, x vehicles enter the intersection (because the cars on Orange St. travel westbound) and y vehicles leave the intersection (because the cars on Pear St. travel southbound).

d) Arrange the equations in matrix form and solve.
e) What is the physical interpretation of your solution?
f) Suppose the *BerryGoodRoads, Inc.* company begins construction on Grape St. E between Apple St. and Pear St. What is the minimum number of vehicles allowed on that block which maintains the network's overall traffic flow?

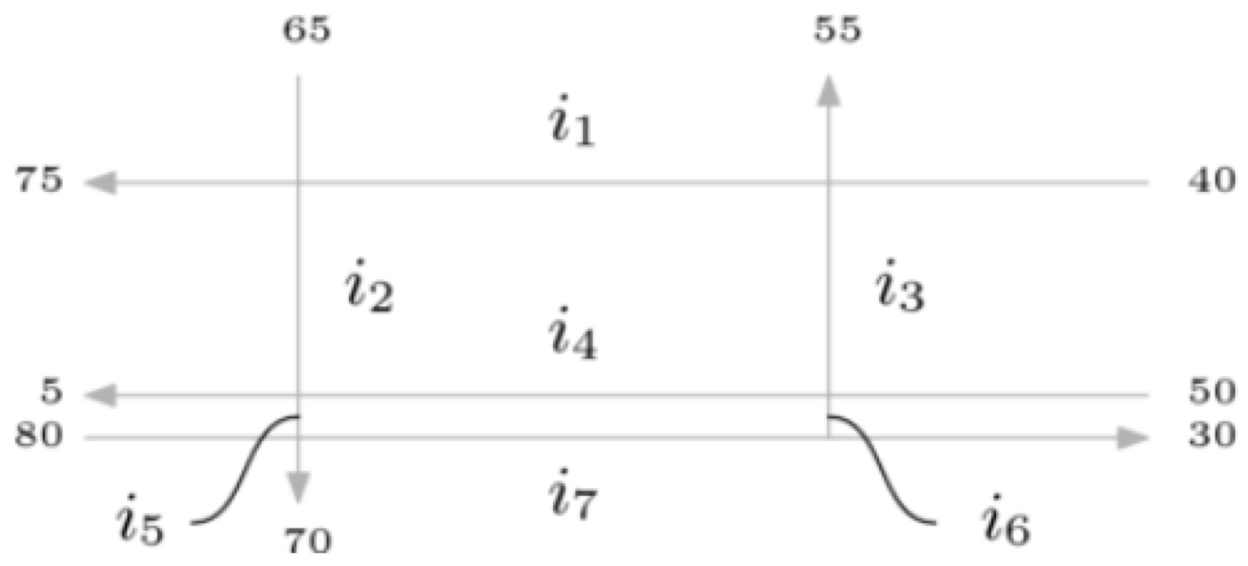
Intersection	Equation ($\# \, in = \# \, out$)
Orange-Pear	$i_1 + 65 = i_2 + 75$
Grape W-Pear	
Grape E-Pear	
Grape E-Apple	
Grape W-Apple	
Orange-Apple	

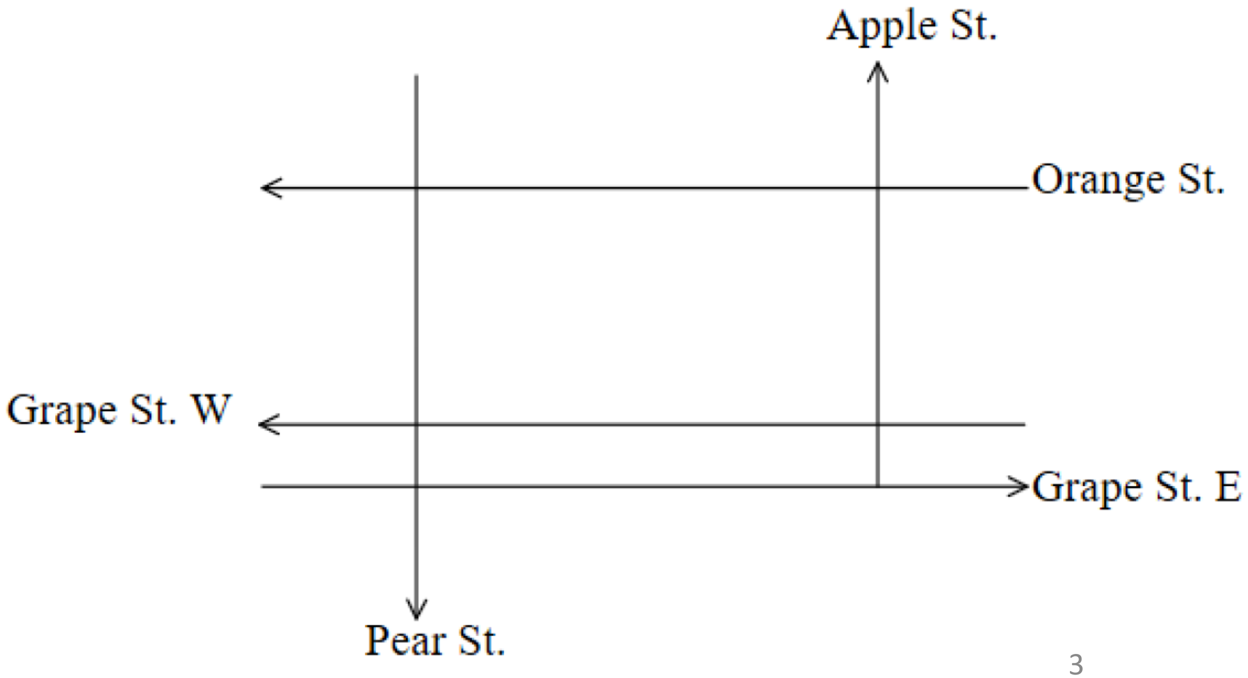




$$\underbrace{\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{b}$$

Intersection	Equation $(\# \, in = \# \, out)$
Orange-Pear	$i_1 + 65 = i_2 + 75$
Grape W-Pear	$i_2 + i_4 = i_5 + 5$
Grape E-Pear	$i_5 + 80 = i_7 + 70$
Grape E-Apple	$i_7 = i_6 + 30$
Grape W-Apple	$i_6 + 50 = i_3 + i_4$
Orange-Apple	$i_3 + 40 = i_1 + 55$





$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10 \\ 5 \\ -10 \\ 30 \\ 50 \\ -15 \end{bmatrix}}_{b}$$