Ordinary Differential Equations: Skydiver

The free-fall velocity of a parachutist initially at rest can be estimated using the force balance:

$$\dot{v} = g - \frac{c_d}{m} v^2$$

where $g = 9.81 \frac{m}{s^2}$ = acceleration due to gravity and $m = 80\ kg$ is the parachutist's mass. During freefall, the drag coefficient $c_d = 0.25 \frac{kg}{m}$. When the parachutist opens the chute at time t_{open}, the drag coefficient becomes $c_d = 5 \frac{kg}{m}$.

a) What would be the steady-state velocity if the chute was not opened? What is the steady-state velocity after the chute opens?
b) Sketch (by hand) the phase portrait and anticipated solution.
c) Write a function to solve this ODE numerically from $t = 0{:}\,0.01{:}\,30\ s$. Create a plot of $v(t)$ vs t for $t_{open} = 0{:}\,5{:}\,20\ s$, putting all curves on a single plot. Use a relative and absolute tolerance of 10^{-9} (see the `ode45()` documentation).
d) Create a plot of $v(t)$ vs t for $m = 25{:}\,25{:}\,100\ kg$, putting all curves on a separate figure. Take $t_{open} = 15\ s$. Use a relative and absolute tolerance of 10^{-9}.

Skydiver Phase Portrait/Anticipated Solution

ODE and Initial Condition	Fixed Points (=Steady-State Values)	Stability
$\dot{v} = g - \frac{c_d}{m} v^2$ $v_0 = 0\frac{m}{s}, t = 0$		

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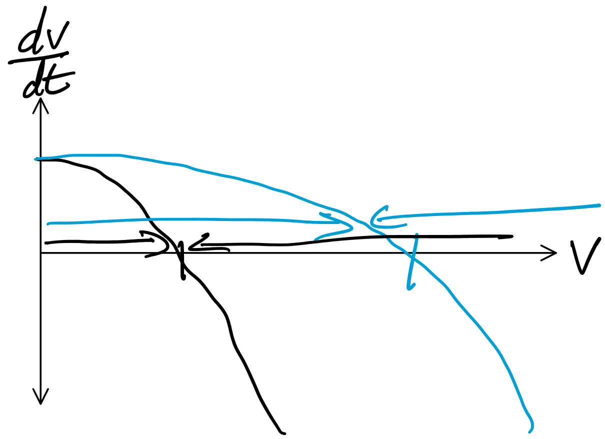
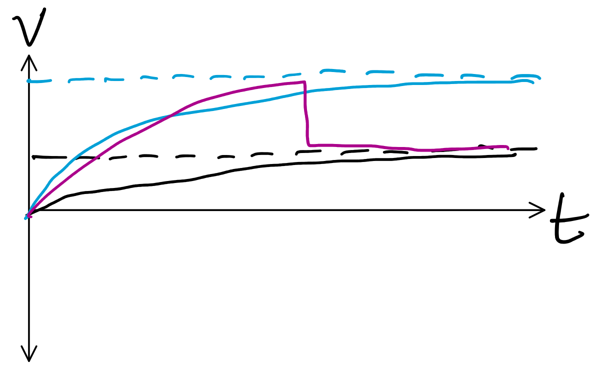
$\dot{v} = g - \frac{c_d}{m}v^2$ $v_0 = 0\frac{m}{s}, t = 0$	$V_{ss} = \sqrt{\frac{mg}{c_d}}$	Stable!





$$\cancel{\dot{v}} = g - \left(\frac{c_d}{m}\right)v^2 \rightarrow v_{ss} = \sqrt{\frac{gm}{c_d}}$$