Matrix Terminology Review

ME 2004

Outline

1.1: Scalars, Vectors, and Matrices

Scalar

- Scalar: single number.
  - Negatives, fractions

- Some common physical constants:
  - $g = 9.81 \frac{m}{s^2}$ (acceleration due to gravity)
  - $c = 3 * 10^8 \frac{m}{s}$ (speed of light in a vacuum)
  - $\pi = 3.14159 \ldots$
  - $e = 2.718 \ldots$

- Dimensions: 1x1

Vector

- Vector: collection of scalars. Has either 1 row or 1 column.

- Row vector: scalars arranged in one row
  - $masses = [2\ 5\ 8\ 10] = [2, 5, 8, 10]$
- Column vector: scalars arranged in one column
  - $masses = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 10 \end{bmatrix} = [2;\ 5;\ 8; 10]$

- Dimension: 1xn (row vector), nx1 (column vector)

Matrix

- Matrix: collection of vectors. Has multiple rows and columns.
  - Collection of vertically concatenated row vectors
  - Collection of horizontally concatenated column vectors

- $A = \begin{bmatrix} 0 & -3 \\ \pi & e \end{bmatrix} = \begin{bmatrix} [0 & -3] \\ [\pi & e] \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ \pi \end{bmatrix} \begin{bmatrix} -3 \\ e \end{bmatrix} \end{bmatrix}$

- Dimension: mxn
  - Matrix is square if $m = n$

1.2: Matrix Operations

Transpose

- Transpose: flip the rows and columns of A.

$$\left(A^T\right)_{ij} = A_{ji}$$

(may see A' instead of A^T)

- Example: $\begin{bmatrix} -1 & 3 & -5 \\ 7 & 0 & 4 \end{bmatrix}^T = \begin{bmatrix} -1 & 7 \\ 3 & 0 \\ -5 & 4 \end{bmatrix}$
- Converts row vector $\leftrightarrow$ column vector

Addition (and Subtraction)

- If A and B have the same size, form $C = A + B$ by adding corresponding elements
- Example: $C = \begin{bmatrix} -1 & 7 \\ 3 & 0 \\ -5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -9 \\ 15 & 28 \\ 6 & 16 \end{bmatrix} \rightarrow C = \begin{bmatrix} -3 & -2 \\ 18 & 28 \\ 1 & 20 \end{bmatrix}$
- Properties:
  - Commutative: $A + B = B + A$
  - Associative: $(A + B) + C = A + (B + C) = A + B + C$
  - Transpose: $(A + B)^T = A^T + B^T$

Scalar Multiplication/Division

- If α is a scalar, form $C = \alpha A$ by multiplying every element of A by α

- Example: $C = (3)\begin{bmatrix} 2 & 0 & -4 \end{bmatrix} \rightarrow C = \begin{bmatrix} 6 & 0 & -12 \end{bmatrix}$

- Properties:
  - $\alpha(A + B) = \alpha A + \alpha B$
  - $(\alpha + \beta)A = \alpha A + \beta A$

Vector/Matrix Multiplication

- To form $C = AB$, A must be mxn and B must be nxp. C will be mxp
  - Inner dimensions must match
- To form C_{ij}, multiply corresponding elements from the ith row of A and the jth column of B, then add them

$$C_{ij} = \sum_{z=1}^{n} A_{iz}B_{zj} \qquad (i = 1 \ldots m, j = 1 \ldots p)$$

- In general: $AB \neq BA$

Vector/Matrix Multiplication

- Example: $A = \begin{bmatrix} -1 & 7 \\ 3 & 0 \\ -5 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -9 \\ 3 & 10 \\ 6 & 0 \end{bmatrix}^T \rightarrow C = 3\text{x}3$
- To find C_{23}: $C_{23} = A_{21}B_{13} + A_{22}B_{23} = (3)(-2) + (0)(-9) = -6$

$$A = \begin{bmatrix} -1 & 7 \\ \boxed{3 \quad 0} \\ -5 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 & \boxed{\begin{matrix} -2 \\ -9 \end{matrix}} \\ 0 & 10 & \end{bmatrix}$$

Matrix Inverse

- Identity matrix: contains 1 along the main diagonal, 0 elsewhere

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The matrix Z which satisfies $AZ = I$ is the inverse of matrix A
- Z exists (A is invertible) if A:
  - Is square
  - Has pivots in each row and column
  - Has a nonzero determinant
  - …etc.

Matrix Division

- Consider matrices A, B, and C such that $AB = C$
- To find A, we CANNOT do $A = \frac{C}{B}$
- But we can do $A = CB^{-1}$
  - Obviously B^{-1} needs to exist
  - Different than $A = B^{-1}C$

Summary

- Scalar: 1x1
- Vector: collection of scalars
  - Row vector: horizontally arranged scalars. $1\mathrm{x}n$
  - Column vector: horizontally arranged scalars. $n\mathrm{x}1$
- Matrix: collection of vectors. $m\mathrm{x}n$

Summary

- Transpose: swaps rows and columns of a vector/matrix
- Addition/subtraction: operands must be same size. Resulting vector/matrix is the same size as the operands
- Scalar multiplication: resulting vector/matrix is the same size as the vector/matrix that is being multiplied
- Matrix multiplication: inner dimensions must match
  - If A is mxn and B is nxp, C is mxp
- Matrix inverse: only exists for square matrices (and other conditions)
- Matrix division: does not exist (multiply by the inverse instead)