Hawking Modes And The Optimal Jumbler: Lessons On Unitarity

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Abstract

Based on an 'observer-centric methodology' which is conceptually and operationally most direct, we pinpoint the most basic origin of the spectral Planckianity of the asymptotic Hawking modes, which is generic to the conventional treatments of the evolving horizons. Analyzing spacetimes marked by the sole virtue of possessing causal horizons, we clarify in details how this spectral Planckianity is geometrically imposed on the exponentially-redshifted propagating Hawking modes through a 'holographically-illicit' 'jumbling mechanism' which is developed by an environment of the 'curvature modes' in the causal patch of the asymptotic observer. We verify that this mechanism is stable under broad variations of geometries, topologies and categories of the causal horizons, in all the Minkoswki, dS and AdS asymptotics. In the continuation, we elucidate the underlying microscopic physics of the actual phenomenon of the Hawking evaporation of causal horizons. The description is based on a novel holographic scheme of gravitational open quantum systems in which the degrees of freedom that build up the curvature of the observer's causal patch interact with the released Hawking modes, initially as environmental quanta, and later as quantum defects. Planckian dispersion of the modes can only be developed in the strict 'thermodynamic limit' of the quantum environment, the so-called "optimal jumbler". Finally, we characterize how this holographic microscopic formulation does realize the information-theoretic processing of unitarity.

I. MOTIVATION AND INTRODUCTION

'Quest' for the universal and nature-wise successful theory of *'quantum gravity'* is lively going on. The aim is the discovery of a complete theory of quantum gravity which not only is self-consistent and (gravitationally-meant) nonperturbative, but also being maximally generic in its phenomenological board of applicability, is *successfully predictive* on this physical nature that we commonly observe. Through the long course of the collaborative research we have conducted so far, a number of extremely deep lessons have been extracted on how conceptually distinguished the correct formulation of this aimed quantum gravity must be. The '*holographic way*' by which gravity and spacetime must intrinsically emerge is one example [25]. However, as the major lessons are yet incomplete, *a complete list of the 'correct principles' is still to be discovered*. But, no doubt, we aim to get there.

Up until now, in the void of direct or even helpfully indirect experimental data in quantum gravity, we have been mainly hinted by several methodologically significant '*theoretical laboratories*' on how to advance forward. Centered in the most exciting core of all these theoretical laboratories is the open question of discovering *the 'correct' and 'complete' microscopic formulation of the 'phenomenologically generic' black holes.* Its methodological significance is because their *horizons* are the best initiative zone where *the most characteristic* aspects of quantum physics do 'unavoidably' interpenetrate with those of classical gravity. But, apart from some 'alien' category of supersymmetric black holes, a lucid understanding of *the microscopic physics* of these objects is yet missing.

We must highlight that, our '*majorly incomplete*' understanding of the constitutional microscopic system of the generic black holes, and their so-produced quantum behavior, is not because the correct description of the near-"singularity"-region is unknown to us, but primarily *because we do not yet understand properly the 'horizon', namely the defining 'holographic screen' of these objects.* There are two good reasons for this 'priority discernment'. The minor reason is that, the horizon scale where quantum physics and classical gravity 'fuse' is decoupled from the quantum curvature corrections, for "large black holes". But, the major reason is that, black hole physics is, by means of holography, *a 'decodification' of the physics of the 'horizon many body system'.*

In understanding the correct microscopic system of causal horizons, the first arena where we need to clean up the illusions of the semiclassical description, and to replace them with the correct microscopic physics, is "*the information paradox*". This is indeed the aim of this study. Fortunately, there are very good introductory material on this subject [30], so the background can be escaped. Now, let us give a brief introduction to this paper.

To *'conclusively'* reinstate the *indestructibility of the initial-state information bits* in the Hawking evaporation of causal horizons, as a first step, we may better *'pinpoint'* the *'exact correct origin'* of their 'illusory Planckian-loss' in the conventional settings where black hole physics has been configurated. This 'root-problem identification' is the main subject of *the part 'A'* of this work. For this, we single out the original observation in [1], which was inspired by [2], and by 'a thorough re-analysis' unfold the important messages hidden in it. The most unique virtue of our study, compared to the related literature, is its *most straightforward methodology.* This method is '*speculation minimalistic*', and in being '*observer based*' is also operationally '*most direct*'.

The part 'A' includes chapters two to five of this study. First, in chapter two, we thoroughly work out our observer-centric analysis in the 'canonical example' of Schwarzschild spacetime (in arbitrary dimensions), and already extract the main *macroscopic lessons.* Incorporating '*all the relevant deformations*', within a broad spectrum of examples, we do *successfully* verify the robustness and the generality of our principal approach by applying it to 'the NUT hole and the Kerr black hole' in chapter three, then to 'the Schwarzschild de Sitter black hole, the Schwarzschild Anti-de Sitter black hole, and the cylindrical black hole' in chapter four, and finally to 'the Rindler horizon and the de Sitter horizon' in chapter five.

The part 'B', brings this work into a deeper level. '*First*', it carefully *unfolds* the *foundational 'microscopic lessons'* underlying the macroscopic outcomes of the part 'A'. '*Second*', it so develops the quantum structure of a *holographic microscopic formulation* that realizes *the unitarity of 'information processing' in the horizon evaporation.* '*Third*', to further *fixing 'the exact dynamics' of the evolving causal horizons*, it 'interconnects' this so-structured formalism with *'the observer-based fundamental theory' being proposed in* [33].

PART “A”

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‘BITS LOST’

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“THE HOW”

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II. ASYMPTOTIC HAWKING MODES FROM THE EVENT HORIZONS : THE CANONICAL EXAMPLE

In this beginning chapter of our research, we define our '*observer-centric method*' and utilize it to thoroughly analyze our '*canonical example*' as the first case-study among a much broader family of spacetimes that will be explored later in this study to confirm the generality and the robustness of *both our methodology and its conclusive statements.* This canonical case-study sets up a good physical basis for our novel understanding of the physical nature of the almost Planckian profile of the asymptotic Hawking radiation, whose in-depth information theoretic perspectives will be addressed in the part 'B' of this paper. Before working out in details our canonical example, let us first say a few words on the physical setting in which we develop all our forthcoming analyses.

We choose to configurate and develop our prototype analysis, and consecutively the information theoretic proposals concluded from it, on the basis of an *'observationally direct methodology'* which was originally and successfully utilized in [1]. We also decide to take a most generic stance in our study, and explore the landscape of *spacetimes which are characterized by the sole virtue of possessing a 'causal horizon', namely a (compact or noncompact) null hypersurface that joins two causally disconnected complementary regions of the geometry.* For every such spacetime, we carefully set up and work out a prototype *thought experiment* in which *'a sufficiently far' 'static observer' "Bob"* receives, and 'spectrally' analyzes, *'initially monochromatic' null rays of Hawking modes* which, starting from the immediate proximity of the horizon, have propagated toward his detector, all through '*his causal-patch geometry*'. The spectral analysis that Bob performs on the detected rays of Hawking modes is primarily on their *frequency distribution*, much like a radio astronomer, and subsequently on their *particle-energy distribution*, much like a particle-detector analysist. With this same thought experiment, we explore a broad class of spacetimes with variations in horizon geometry and topology, and also in the asymptotics, and *in this way probe into the information content of the asymptotic Hawking modes.* We finally show what this observer-centric analysis unfolds about the *the interplay of the 'Bobby geometry' and propagating Hawking modes.*

Having explained the general scheme of our analysis, we are now at *the right stage* to commence this basically distinct study of the information paradox by specific examples. Our *canonical case-study* is nothing but the simplest of all possible examples, that is, a classically-static spherically-symmetric spacetime possessing a quantum-evaporating compact event horizon, namely the *Schwartzchild black hole.* For the sake of maximal generality, we will conduct the analysis of this chapter in arbitrary number of spacetime dimensions, but will later stick to four spacetime dimensions in all our next examples. The metric of a one parameter family of Schwarzchild black holes in $D \geq 4$ spacetime dimensions reads as, [5],

$$ds^2 = (1 - \frac{R_H^{D-3}}{r^{D-3}})dt^2 - (1 - \frac{R_H^{D-3}}{r^{D-3}})^{-1}dr^2 - dS^2_{(D-2)} \tag{1}$$

in which dS_{D-2} is the metric of a $(D-2)$-dimensional sphere, and

$$R_H = (\frac{2Gm}{c^2(D-3)})^{\frac{1}{D-3}} = (\frac{2m}{D-3})^{\frac{1}{D-3}} \quad ; \quad c = G = 1 \tag{2}$$

is *the radius of the spacetime's event horizon.*

Now let us employ and work out our prototype '*thought experiment*' in this simple geometry. Suppose that at an arbitrary initial time '$t = t_{in}$' in the *Schwarzschild temporal coordinate*, a beam of *initially mono-frequency Hawking modes* begins to propagate radially with the speed of light from a point in the immediate proximity of the the (stretched) horizon, say at '$r_{in} = R_H + \epsilon$' with '$\epsilon/R_H \ll 1$', and is eventually received and spectrally analyzed by the *observer 'Bob'*, who is a *'late-time detecting' 'static observer'* located at a point '*sufficiently far*' from the quantum-evaporating stretched horizon. Let us further suppose that Bob's location is distant-enough from the *horizon's vacuum-entanglement "zone"* [22] by which we mean a horizon-exterior region, of a width of the order of the horizon radius, which gives regional support to those of the Hawking modes which are in a vacuum-based maximal entanglement with their partner modes in the black hole's interior. By these assumptions, let us take the Bob's receiving of the above-imagined null rays to be identified with a late-time spacetime event '$\mathcal{P}(t, r)$' with '$r/R_H \gg 1$'. The radial trajectory of this 'light' ray is given by the null geodesics,

$$ds^2 = (1 - \frac{R_H^{D-3}}{r^{D-3}})dt^2 - (1 - \frac{R_H^{D-3}}{r^{D-3}})^{-1}dr^2 = 0 \tag{3}$$

leading to the equation

$$\frac{dr}{dt} = 1 - \frac{R_H^{D-3}}{r^{D-3}}. \tag{4}$$

Solving this equation, say by using relation (2.141) in [6], we end up with the following two different solutions depending on the dimension of the spacetime being odd or even,

$$D = \text{odd}: \quad t = r + \frac{R_H}{n} Ln\left(\frac{r - R_H}{r + R_H}\right) + \frac{2R_H}{n} \sum_{k=1}^{n/2-1} P_k \cos\frac{2k}{n}\pi - \frac{2R_H}{n} \sum_{k=1}^{n/2-1} Q_k \sin\frac{2k}{n}\pi \tag{5}$$

$$P_k = \frac{1}{2} Ln\left(x^2 - 2x\cos\frac{2k}{n}\pi + 1\right), \quad Q_k = \arctan\frac{x - \cos\frac{2k}{n}\pi}{\sin\frac{2k}{n}\pi} \tag{6}$$

$$D = \text{even}: \quad t = r + \frac{R_H}{n} Ln\left(\frac{r - R_H}{R_H}\right) - \frac{2R_H}{n} \sum_{k=0}^{(n-3)/2} P_k \cos\frac{2k+1}{n}\pi - \frac{2R_H}{n} \sum_{k=0}^{(n-3)/2} Q_k sin\frac{2k+1}{n}\pi \tag{7}$$

$$P_k = \frac{1}{2} Ln\left(x^2 + 2x\cos\frac{2k+1}{n}\pi + 1\right), \quad Q_k = \arctan\frac{x + \cos\frac{2k+1}{n}\pi}{\sin\frac{2k+1}{n}\pi} \tag{8}$$

in which '$x \equiv r/R_H$' and '$n = D-3$'. Now upon employing the initial conditions and taking into account that by assumption '$r \gg R_H$' and '$\epsilon \ll R_H$', we find that to the leading order the trajectory is of the following form in both the cases (up to an irrelevant constant term),

$$r \simeq t - t_{in} + \frac{R_H}{D-3} \,\text{Ln}\left(\frac{\epsilon}{R_H}\right). \tag{9}$$

Now let us recollect and use this 'basic fact' that *even in a stationary spacetime the frequency of propagating modes as measured in terms of the proper time will be different at different points of space, and is being redshifted as it propagates outwardly reaching Bob at '$\mathcal{P}(t,r)$'*. The relation for the redshift between the light frequency 'Ω_{in}' at 'r_{in}' and its frequency received at time 't' by a '*static*' observer Bob located at '$r \gg R_H$' is given by [20],

$$\Omega \;=\; \Omega_{in}\,[g_{00}(r_{in})]^{1/2} \tag{10}$$

which, as a consequence of *the presence of the corresponding causal horizon*, leads to *a characteristic 'exponentially-redshifted' 'time-dependent' observed frequency* of the form,

$$\Omega = \Omega_{in}\left[1 - \left(\frac{R_H}{r_{in}}\right)^{D-3}\right]^{1/2} \simeq \Omega_{in}\left[(D-3)\frac{\epsilon}{R_H}\right]^{1/2} \simeq \Omega_{in}(D-3)^{\frac{1}{2}} \exp\left[\frac{-(D-3)(t - t_{in} - r)}{2R_H}\right] \tag{11}$$

Now the equation (11) manifests that, *despite being in a static spacetime*, the frequency of the wave modes measured by Bob at some fixed '$r \gg R_H$' varies nontrivially with time 't'. *Bob will not see a monochromatic radiation, even though the beams were initially signaled monochromatically from the immediate proximity of the horizon.* This 'time-dependence' of the mode frequencies as observed by Bob is 'a crucial point' that can be clearly understood by the following reasoning. If we imagine a wave packet,

$$\Phi(t,r) \propto \exp(i\theta(t,r)) \tag{12}$$

centered on this null ray, the instantaneous frequency would be related to its phase by

$$\Omega = \frac{\partial\theta}{\partial t} \tag{13}$$

Now integrating (11) with respect to the time variable 't', the relevant wave mode is found to be,

$$\Phi(t,r) \propto \exp\left[i\int dt\ \Omega\right] \propto \exp\left[-\frac{2R_H}{(D-3)^{1/2}}\ i\Omega_{in}\ \exp(-\frac{(D-3)(t-t_{in}-r)}{2R_H})\right] \tag{14}$$

Therefore, *our asymptotic static Bob who is using his Schwarzchild time coordinate* 't' will Fourier decompose these modes with respect to the frequency 'ω', defined by his same time coordinate, in the following form,

$$\Phi(t,r) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ e^{-i\omega t} f(\omega) \tag{15}$$

in which,

$$f(\omega) = \int_{-\infty}^{\infty} dt\ e^{i\omega t}\ \Phi(t,r) \propto \int_0^{\infty} dx\ x^{\frac{-2i\omega R_H}{D-3}-1}\ \exp(-\frac{2R_H}{D-3} ix\Omega_{in}) \tag{16}$$

and '$x = \exp\left[(D-3)(-t+t_{in}+r)/2R_H\right]$'. Rotating the contour in the above integral to the imaginary axis, i.e. '$x \to y = ix$', we have,

$$f(\omega) \propto e^{-\pi\omega R_H/(D-3)} \int_0^{i\infty} dY\ Y^{z-1} e^{-Y} \tag{17}$$

in which,

$$z = -2i\omega R_H/(D-3) \quad \text{and} \quad Y = -2R_H\Omega_{in} y/(D-3)$$

Now using the the representations of the Gamma function and that $|\Gamma(ix)|^2 = \pi/(x \sinh \pi x)$ the associated frequency spectrum is found to be [1],

$$|f(\omega)|^{-2} \propto \exp(\frac{4\pi R_H\ \omega}{D-3}) - 1 \tag{18}$$

Manifesting the explicit dependence on the Newton constant 'G' and the light velocity 'c', the result for the *frequency spectrum of the distantly observed Hawking Modes* becomes,

$$|f(\omega)|^2 \propto \left[\exp\,(\frac{\omega}{\omega_0}) - 1\right]^{-1} \tag{19}$$

in which the *characteristic frequency* ω_0 reads as,

$$\omega_0 = \frac{c^3}{4\pi G} \cdot \frac{D-3}{R_H} \tag{20}$$

which depends non-trivially only on the horizon scale and the dimensionality of the spacetime, but is otherwise universal, *as it must be.*

Now, let us make a preliminary evaluation of the *'meanings' of the results (19,20)* we have just obtained, and also take a first look into the *'physics lessons'* that are extractable from it, while we postpone a thorough elucidation of these points to the specific chapters on the black hole information paradox which will be presented in *the part 'B'* of this paper.

Equations (19,20) conclude what a 'late-time detecting' 'asymptotic static observer' Bob *'experimentally'* obtains for the frequency spectrum of the Hawking modes that were monochromatically released in the stretched horizon band and then propagated as radial null rays all through *horizon's exterior bulk* to be received by his static detector. Now, one principal fact that we instantly recognize is the following:

The observed 'spectral distribution' that we found up here for the distantly propagated Hawking modes, equations (19,20), is nothing but the very 'Planckian frequency spectrum', although 'the procedure' that we followed to obtain it was 'a solely classical one'.

That is, *we must highlight*, not only the final result for the asymptotically observed frequency spectrum is a manifestly classical formula with no '$\hbar$-dependence' in it, but also nowhere in the entire thought-experiment analysis we conducted to obtain this Planckian spectrum of the Hawking modes quantum physics played any role whatsoever. No doubt, on the other hand, the primal emergence of the Hawking modes out of the near-horizon vacuum is an *intrinsically quantum phenomenon*, as we know very well.

As '*one more highlight*', we must appreciate the very crucial point that:

Evaluating information theoretically, already in the classically obtained equations (19,20), 'the very essence of the information loss' is clearly present. That is, the asymptotically observed 'Planckian multi-chromaticity' of the 'initially monochromatic' Hawking beams 'is' itself the very same phenomenon of the semiclassical information loss.

So, the horizon evaporation is a quantum effect, but the procedure which "*jumbles*" the initial-state information content of the Hawking modes during their propagation in the horizon-exterior bulk has nothing to do with quantum effects, in fact (as we will later elaborate on), it turns out that:

The asymptotically-observed 'Planckian-loss of information' can only occur in a strict "environmental thermodynamic limit" which will be specified later in this work. Once the Hawking modes are quantum sourced in the proximity of the stretched horizon, the physics that so-'optimally' 'jumbles' the initial beams of 'information-theoretically' 'contentful' modes into the spectrally-Planckian 'contentless' asymptotic modes is an 'observer-bulk' 'geometric mechanism' actualizable in the 'thermodynamic limit' of a 'microscopic geometric environment'.

Further, we ask if there also comes *'an actual or effective thermalization'* into this optimal jumbling of the radiated-out information bits. Intuitively, this question is natural to be raised, knowing the fact that the horizon itself is a well-defined thermal system with a finite temperature, and then also knowing that a Planckian profile information loss is so typical of a black body radiation. If that is so, then a simple combination of the dimensional analysis together with the universality of this phenomenon implies that the *associated temperature of the asymptotic radiation* has to be proportional to a universal constant of the dimension '$\hbar$', namely, *proportional to the '$\hbar$' itself*. That this '*associated temperature*' is an unavoidably quantum effect is a second crucial fact which we will later take notice of, in order to unfold *the actual 'information-theoretic physics' underlying the Hawking radiation in the part 'B' of this paper.*

As the simplest way that the already anticipated quantum thermal character of the asymptotic Hawking radiation becomes manifest in our '*thought-experiment approach*', let us go back once more to the equations (19,20) for the frequency spectrum of the distantly propagated modes as detected by Bob. This time however, let us suppose that our asymptotic observer Bob, who was up until now a sole radio astronomer, has just renewed his device from a radio-frequency detector to a quantum-particle detector, to see if he would also detect *an asymptotic Planckian 'power-spectrum' of quantum beeps.* By experimentally doing so in his lab, and concluding the measurement of the energy distribution of the detected beeps, his so-obtained experimental plot on the power spectrum of the distantly-propagated Hawking modes will fit a distribution formula which is found by simply re-expressing the classical distribution (19,20) in terms of the quantum energy eigenvalues '$E = \hbar\omega$', that is [7],

$$|f(\omega)|^2 \propto \left[\exp(\frac{\hbar\omega}{\hbar\omega_0}) - 1\right]^{-1} \propto \left[\exp(\frac{E}{k_B T}) - 1\right]^{-1}. \tag{21}$$

Therefore, the experimentally-plotted power spectrum of the asymptotic Hawking modes would be the Planckian blackbody distribution at the well-defined temperature

$$T = \frac{\hbar}{4\pi} \cdot \frac{D-3}{R_H} \tag{22}$$

which is indeed the same result obtained in the literature by independent methods [8], and for '$D = 4$' does reduce to the Hawking temperature for a 4-d Schwarzschild black hole [1]. Let us highlight in passing a subtle point worthy of a special notice. Note that all we have seen so far is that, the classically-sourced maximal dispersion of the initial-state information bits, and their effective quantum thermalization, do consistently come together in the system of 'propagating Hawking modes' interacting with the 'horizon-exterior geometry', *but 'not that it is this thermalization that causes that dispersion'.* These two statements are *both logically and physically* very different from one another and we only approve the former one. To become crystal clear, note that *by turning off $\hbar$, the thermalization does banish, but yet the classical geometric procedure of 'the information-theoretic jumbling', given by (19,20) totally survives.* That is, the classical geometric dispersion of the information bits and the effective internal quantum thermalization of those modes are, 'causally speaking', independent phenomena.

Before moving to the next chapter, let us further bring to the reader's attention an indirect historic remark, a point which is in a beautiful conceptual accord with our discussions up until now, and is moreover insightful by itself. It was priorly observed in [2] that by Fourier decomposing the complex Minkowski plane wave of a massless scalar field, '$\Phi(t, \mathbf{x}) \propto e^{-i(\mathbf{k}.\mathbf{x}-\omega t)}$', with respect to the proper time of a uniformly accelerated observer, one is interestingly led to a similar Planckian spectrum as seen by the observer. Therefore on the basis of '*the equivalence principle*', which allows replacing an accelerated observer with constant proper acceleration 'g' in flat spacetime with a static observer in an actual gravitational field of the same characteristic strength, one must have intuitively expected to obtain the very same classical result (19,20). Indeed, in ordinary (c.g.s) units, it was shown that [2],

$$P(\Omega) \propto \frac{1}{g}\,[\exp(\frac{\Omega}{g/2\pi}) - 1]^{-1} \tag{23}$$

where 'g' is the proper acceleration of the observer and 'Ω' is the frequency of the decomposed wave component measured by the same observer. The Planckian spectrum (23) is also a classical result, with no '$\hbar$' included in it. Moreover, the very procedure that had led to it was also entirely classical. So it is a merely classical phenomenon by all its nature. But yet indeed, it does precisely resemble its very twin result in the Unruh effect which is nevertheless a quantum field theoretic effect by its nature. Now once more, by simply rewriting the frequency-spectrum formula (23) in the form of an energy-spectrum, being expressed in terms of the massless quantum bits of energy which constitute the associated beams, we get,

$$P(E) \propto \frac{1}{g}\,[\exp(\frac{E}{\hbar g/2\pi}) - 1]^{-1} \tag{24}$$

which is again a Planckian blackbody distribution at the well-defined temperature,

$$T = \hbar\,\frac{g}{2\pi} \tag{25}$$

This result indeed resembles very well, as it should, the closely-related case of the Rindler's observer which we will later directly address in this paper. Indeed, the specific case of the Rindeler spacetime is specially interesting as it simulates the near horizon region of the typical black holes, if as demanded by the equivalence principle, nothing special is recorded by an infalling observer while crossing the horizon. But independent of its significance, already with this early result we can do a little bit better.

By invoking again '*the equivalence principle*' in equating the temperature (24) with the Hawking temperature of a Schwarzschild black hole (22), also re-manifesting the dependences on the constants 'c' and 'G', we obtain

$$g = \frac{c^4}{4GM} \tag{26}$$

which is nothing but the surface gravity of a Schwarzschild black hole, fulfilling our preceding intuitive expectation.

As a fact that we will verify 'example-wise', and will also conceptually discuss later in this paper, let us finally mention the fact that:

The presence of a causally-disconnecting null-boundary, to be called "a causal horizon", is central to the Planckian jumbling of the information bits.

III. ASYMPTOTIC HAWKING MODES IN STATIONARY SPACETIMES : THE NUT HOLE AND THE KERR BLACK HOLE

Let us begin with a brief wording on our motive in probing all the further examples which in the rest of the part 'A' of this work we will be considering. The *'directly observer-based' 'thought-experiment method'* that we successfully employed to obtain and analyze the semiclassical asymptotic aspects of the Hawking radiation in the canonical example of the Schwarzschild black hole, is both 'concept-wise' and 'formulation-wise' *intrinsically different* from the standard approaches to this subject. Therefore, we do need to make sure if, beyond merely being a computational coincidence or a simplest-case matching, we can trust this approach as a firmly-based and generally-applicable *'root methodology'* to further investigate the physical nature of the Hawking radiation, and to formulate the correct theory of the quantum-evolving horizons. Thus, we must examine if this approach successfully, and also impartially, goes through for the *'generic spacetimes' possessing 'causal horizons'*. Of course, we would not need to work out every possible example individually, but instead we can manifest the robustness and generality of this approach by verifying that it is *successfully 'stable' under all the 'major physical deformations' applied both on the spacetime geometry and on the virtues of causal horizons.*

To accomplish this verification, we will be carefully selecting and scanning over a good number of broadly-different examples which are marked by *extreme physical distinctions* in the local, global, and in the causal structure of the spacetime. In particular, we consider deformations on the characteristics such as the symmetries, asymptotics and complexities of spacetime, as well as the horizon topologies and the very nature of the causal horizons. By this sufficiently-broad investigation, we will indeed verify the '*success*' of this methodology *over the landscape of all the most major and relevant physical deformations*. It must be highlighted that this verification is for us a necessity, not just for the value of the method by itself, but primarily because *our aim is to discover the correct and complete resolution of the black hole information paradox, and to develop the correct microscopic theory of 'the evolving horizons', for which in the part 'B' of this study we will take instrumental lessons obtained here by this methodology.*

And now a final word on the related generalities needs to be said. As a by-product which is conceptually interesting even all-independent of the Hawking radiation aspects, one confirms by this broad analysis the following statement. *In order for an initially monochromatic beam of null rays to become a system of spectrally-Planckian multi-frequency modes as observed by an asymptotic observer, the pretense of a null horizon in the associated remote-past of the observer is necessary.*

In this chapter, we examine our methodology in obtaining the characteristics of the asymptotic Hawking radiation in the case of *'stationary spacetimes' with event horizons*. In doing so, we follow the *'selection strategy'* that was just explained up here. That is, we select a pair of examples which *'deformation-wise' are physically 'complementary' to each other*. The two complementary spacetimes that we analyze in the following are *the NUT hole* versus the *Kerr black hole*. Because in probing each example, we will realize and analyze the same *direct thought experiment* based on the spectral analysis of an asymptotic observer Bob, we will not overweight the presentations with loads of repetitive details, unless being conceptually necessary. In other words, we will be brief but not too brief. The two examples being complementary, our detailing in the NUT hole example will still be a little more involved, because in the case of the Kerr black hole the details were already worked out in the primary reference of this study [1].

A. The NUT Hole

We begin with the example of the NUT hole spacetime, because based on its physical aspects, it is in fact the most straightforward generalization of the Schwarzschild spacetime. Reading from the symmetries of the metric of the spacetime, the NUT geometry is a stationary axially-symmetric solution of the Einstein equations. But, physically behaving, the NUT spacetime is in fact a spherical spacetime. To appreciate the discernment here, let us remember that the *physical symmetries* of a spacetime must be *'primarily'* identified with the transformations under which the *physical 'gauge-invariant' 'observables'* of the spacetime are invariant. But as the spacetime metric is not directly a physical observable, its symmetries do not need to necessarily coincide with *the physical symmetries* of the spacetime. NUT spacetime does realize this distinction, because unlike its metric tenser, 'all' of its 'curvature invariants' are indeed spherically symmetric.

Then, there is also a *'secondary methodology'* by which the physical symmetries of a spacetime can be identified. In this *observer-based* methodology, we identify the symmetries of the spacetime by the invariances of *the physical observables which are intrinsically defined with respect to a global family of 'observers' who are represented by a congruence of 'timelike curves' treading the spacetime.* In this approach, known as 'the threading decomposition of the spacetime', Einstein equations are being inspiringly re-expressed, in an exact form, as a nonlinear version of the Maxwell equations formulated in terms of the "gravitoelectric" and the "gravitomagnetic" fields which are defined for the family of observers. In this view, the mass parameter *'m'* becomes the gravitoelectric charge, but then the spacetime can be as well endowed with a 'gravitomagnetic monopole' of charge *'ℓ'*.

In this sense, *the NUT spacetime, which is being fixed by the two parameters* (m, ℓ), becomes the most straightforward and the simplest generalization of the Schwarzschild geometry, because now *in the 'center' of the spacetime, both a gravitoelectric monopole and a gravitomagnetic monopole are present, manifestly respecting the physically-defined spherical symmetry.* As such, one indeed shows that, all the physical observables, in particular the gravitoelectric and gravitomagnetic fields, of the NUT hole spacetime are spherically symmetric [15].

The metric of the NUT spacetime with the charges '(m,l)' can be best written in the following form [14, 15],

$$ds^2 = f(r)(dt - 2l\cos\theta d\phi)^2 - \frac{dr^2}{f(r)} - (r^2 + l^2)d\Omega^2 \quad ; \quad f(r) = \frac{r^2 - 2mr - l^2}{r^2 + l^2} \tag{27}$$

The spacetime being physically spherical, there indeed are radial null geodesics which are given by [14],

$$\frac{dr}{dt} = \frac{r^2 - 2mr - l^2}{r^2 + l^2} \equiv \frac{(r - r_+)(r - r_-)}{r^2 + l^2} \tag{28}$$

in which '$r_\pm = m \pm (m^2 + l^2)^{1/2}$' are the two well-known event horizons of the NUT solution. The above equation can be integrated as,

$$t = r + r_+ \ln\left|\frac{r}{r_+} - 1\right| + r_- \ln\left|\frac{r}{r_-} - 1\right|. \tag{29}$$

which are the physically-anticipated radial null geodesic of the spacetime.

It should be highlighted that in the case of the NUT metric *there are no intrinsic singularities hidden behind the horizon*, justifying the given name NUT hole. Also, it is to be noted that, the two horizons of the geometry, separate the stationary NUT regions '$(r < r_-$ and $r > r_+)$' from the time-dependent Taub regions '$(r_- < r < r_+)$' of the so-called Tuub-NUT spacetime which are causally disconnected [14].

Now let us apply and examine our directly observer-based thought-experiment methodology to this causally-nontrivial spacetime. As set in the canonical example, we consider a beam of initially-monochromatic light rays of Hawking modes that propagate radially from an initial point in the immediate proximity of *the outer event horizon* '$(r_+ + \epsilon)$' at time '$t = t_{in}$', to be received by the asymptotic observer Bob at the late-time event '$\mathcal{P}(t,r)$', with '$r \gg r_+$' and '$\epsilon \ll r_+$'. The equation of such trajectory has the following form,

$$r \simeq t - t_{in} + r_+ \ln\left(\frac{\epsilon}{r_+}\right) \tag{30}$$

The redshifted frequency Ω will be related to the initial frequency Ω_{in} in at r_{in} by

$$\Omega = \Omega_{in}\left[g_{00}(r = r_+ + \epsilon)\right]^{1/2} \simeq \Omega_{in}(\frac{\epsilon}{r_+})^{1/2} \simeq \Omega_{in}\exp\left(\frac{-t + t_{in} + r}{2r_+}\right) \tag{31}$$

By applying the very same detailed procedure used in the Schwarzschild example now to NUT spacetime, we finally obtain the following power spectrum for a wave packet which has scattered off the outer event horizon of the NUT hole reaching the observer Bob *at late times and at '$r \gg r_+$'*,

$$|f(\omega)|^2 \propto \left[\exp\left(\frac{1}{4\pi(m+(m^2+l^2)^{1/2})}\right)\omega - 1\right]^{-1} \tag{32}$$

The result (32) does confirm the validity of our method in its application to the NUT hole spacetime. Indeed, it manifestly is the *classically-obtained Planckian frequency spectrum* of the distantly-measured Hawking modes, which if being re-expressed in terms of the energy quanta received in Bob's particle detector (as we discussed in the previous chapter), leads to a standard Planckian radiation at the well-defined temperature,

$$T = \frac{\hbar}{4\pi\left[m+(m^2+l^2)^{1/2}\right]} \tag{33}$$

Let us notice that indeed (33) matches with the standard result in the literature [10, 11], and also that it reduces to that of the Schwarzschild case when '$l = 0$'.

B. The Kerr Black Hole

The second case is the Kerr black hole. The method has already been worked out in [1]. Kerr black hole is a two-parameter stationary spacetime, which being axially symmetric both mathematically and physically, does not have radial null geodesics and so one should use principal null congruences to apply the Fourier decomposition method. It also contains two horizons which not only divide the spacetime into causally disconnected regions but also hide an intrinsic singularity from the outer region. Another point is the fact that, the infinite redshift surface and the event horizon do not coincide. This will require a careful employment of the above procedure taking into account the existence of the ergosphere region from inside which the outgoing null rays propagate to the observers at late time and large distances. Working out all these details, we are led to the Planckian spectrum [1],

$$|f(\omega)|^2 \propto \left[\exp\left(\frac{4\pi(m^2+m(m^2-a^2)^{1/2})}{(m^2-a^2)^{1/2}}\right)\omega - 1\right]^{-1} \tag{34}$$

at the temperature,

$$T = \hbar\,\frac{(m^2-a^2)^{1/2}}{4\pi(m^2+m(m^2-a^2)^{1/2})}$$

which indeed reduces to the result for the Schwarzschild black hole for '$a = 0$'.

IV. ASYMPTOTIC HAWKING MODES IN MUCH BROADER SETTINGS: VARIATIONS OF GEOMETRIES, TOPOLOGIES AND ASYMPTOTICS

We need to highlight in the very beginning of this chapter that everywhere in our observer-based methodology, by the asymptotic observer Bob what we collectively mean is, either an ideally-asymptotic observer, or simply an observer whose static detector is located at a *sufficiently-far 'finite distance'* from the causal horizon, and receives the bulk-propagated Hawing modes at a *sufficiently-late 'finite time'*. Appreciating this point of '*sufficient finiteness*' will be important, and in fact essential for us, because of the following two major reasons.

'*The first reason*' is that there are physically important settings in each of which there is a strict '*radial cutoff*' on the observer's location, set by the presence of a spacetime boundary which is itself located at some finite distance from the radiating horizon. One obvious example, which both theoretically and phenomenologically is a significant one, is the case of *an evolving black hole in de Sitter spacetimes* in which the finite-size cosmological horizon serves as the spacetime boundary. In fact, the specific example of the *'Schwarzschild-de Sitter black hole'* which we study here is the 'canonical case' of these examples. As being intuitively anticipated, in all these settings, the black holes whose size are 'comparable' with that of the spacetime bubble do not experience any effective evaporation, because of the thermalization between the two horizons, but the black hole which are sufficiently smaller than the spacetime's boundary horizon do source a net-positive Hawking radiation, and so does evaporate monotonically.

'*The second reason' is however an 'asynptotics-independent' 'universal one' which is intrinsically 'observer holographic' and has a significant 'information theoretic' impact.* Better elaborations on this crucial point will be postponed to the part 'B', and much more extensively, to the microscopic theory conjectured in [33], but yet we most briefly mention it. It turns out that in order to *correctly and conclusively* resolve the information paradox, based on a *'maximally-holographic' complete microscopic theory*, the formulation must be developed on the basis of *'Bob-type' observers whose 'causal accommodation' of the horizon history is of 'finite-size'*. Only then, the infinite limit can also be *successful*.

Having probed the Schwarzschild, the NUT and the Kerr horizons, let us now move on to *the second class* of our '*deformational analysis*'. Here we examine our directly observer-based methodology under some *'more radical relevant deformations'* in the physical settings in which the asymptotic behavior of the horizon Hawking radiation can be configured. In this chapter, we examine *three such physical deformations which form a threefold complementarity.*

In the first two of our threefold examples, we scan over *the different 'global phases' of the 'observer's causal-patch geometry*', realized by *the two possible deformations of the 'spacetime asymptotics'*, but then we keep the characteristics of the event horizon intact. These two asymptotically-complementary examples are Schwarzschild black holes with the dS asymptotics, and with the AdS asymptotics, respectively. We must highlight that *the very question of the impact of the spacetime asymptotics on both the 'semiclassical' and the 'actual' 'spectrality' of the Hawking modes is nontrivial in our approach.* This is because our method is 'directly' based on the asymptotic observers, so is *'a global one'* by its very conception. Metaphorically, the comparison between the path integral versus the Hamiltonian definitions of quantum mechanics may be mentioned as an analogy.

In the third example, which is in the 'deformational sense' complementary to both of the afore-mentioned examples, we consider the opposite setting in which we freeze the asymptotics of the spacetime (as automatically decided by the solution), but then *the topology of the event horizon is being maximally deformed.* By a 'maximal deformation' of the horizon topology, we mean that we alter its physically most important topological characteristic, namely the *'compactness virtue' of the causal screen.* Here we address, by a well-chosen case-study, the spacetimes with *'noncompact causal surfaces, namely, causal domain walls of infinite area,* which are event horizons of finite temperature, and so radiate-out Hawking modes. Then by a similar method, we obtain and spectrally analyze the asymptotic radiation as semiclassically observed by the 'Bobby observers'. The specific spacetime that we choose, is the simplest and best representative of this class of examples, namely *the asymptotically-AdS cylindrical black hole* whose solution and thermal behavior were initially obtained in [13].

To make the presentation of the following examples compact, let us start with the general form of a class of spacetime metrics that collectively encodes the solutions of the three examples which we probe in this chapter. This general metric, in terms of the mass parameter 'm' and the cosmological constant 'Λ', is given by [16, 17],

$$ds^2 = \left(k - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right)dt^2 - \left(k - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 - r^2 d\Omega_k^2 \tag{35}$$

The curvature parameter 'k' takes the values $\{-1, 0, +1\}$, so that '$d\Omega^2_{(k)}$' is the metric on a Riemann surface '$\Sigma_{(k)}$' of constant Gaussian curvature k. The surface '$\Sigma_{(k)}$', which can be of different topologies, will be the event horizon for the corresponding black hole solutions. When expressed in conventional angular local coordinates '(θ, ϕ)', the area element '$d\Omega^2_{(k)}$' takes the following forms,

$$d\Omega^2_{(k)} = \begin{cases} d\theta^2 + \sin^2(\theta) d\phi^2, & k = 1 \\ d\theta^2 + d\phi^2, & k = 0 \\ d\theta^2 + \sinh^2(\theta) d\phi^2, & k = -1 \end{cases}$$

The AdS cylindrical black hole corresponds to the above metric with '$k = 0$' and with a negative 'Λ', while both the Schwarzschild AdS and the Schwarzschild dS spacetimes are given by this same metric with negative and positive values of 'Λ' respectively, upon taking the choice '$k = 1$'.

A. The Schwarzschild horizon in the 'de Sitter Phase'

The *'de Sitter phase of quantum gravity'*, that is, *the gravitational life with '$\Lambda > 0$'*, is *drastically distinguished* from the other two phases with '$\Lambda \leq 0$', because of two reasons. '*First*', the physical supersymmetric enhancements are not possible in the de Sitter phase. '*Second*', the de Sitter phase of quantum gravity must be 'holographically' formulated by quantum theories which are strictly 'finite-dimensional' [24, 27]. The theory of black holes in this phase is not only a physical sector of it, but also serves as a precious laboratory in developing the full theory of the '*de Sitter quantum gravity*'. Here we employ our principal method to the de Sitter phase. The motive is as before to formulate the unitarity of the horizon evolution by the lessons we make use of, in the part 'B' of this work.

We so begin the analysis of this chapter with the example of the Schwarzschild-dS horizon. Given the metric which was identified before, the radial null geodesics are integrated from,

$$\frac{dr}{dt} = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} \tag{36}$$

If we restrict our attention to the case '$\Lambda > 0$' and '$9M^2G^2\Lambda < 1$' [4], the above equation can be integrated as,

$$t = r + \alpha Ln\left|\frac{r - r_H}{r_H}\right| + \beta Ln\left|\frac{r - r_C}{r_C}\right| + \gamma Ln\left|\frac{r - r_U}{r_U}\right| \tag{37}$$

in which,

$$\alpha = \frac{3r_H}{\Lambda(r_C - r_H)(r_H - r_U)}, \quad \beta = \frac{-3r_C}{\Lambda(r_C - r_H)(r_C - r_U)}, \quad \gamma = \frac{-3r_U}{\Lambda(r_C - r_U)(r_H - r_U)}$$

and

$$r_H = \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{1}{3}\cos^{-1}(3M\sqrt{\Lambda}) + \frac{\pi}{3}\right], r_C = \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{1}{3}\cos^{-1}(3M\sqrt{\Lambda}) - \frac{\pi}{3}\right], r_U = -(r_H + r_C)$$

are the roots of the equation '$dr/dt = 0$'. In fact, both 'r_H' and 'r_C' have positive values, thereby defining the two horizons of the spacetime which are albeit of very different physical natures. The larger root 'r_C' is the '*de Sitter cosmological horizon*' but the smaller root 'r_H' is the black hole *event horizon*. The negative root 'r_U' is unphysical. Under the condition '$3M\sqrt{\Lambda} < 1$', the Schwarzschild black hole is sitting *inside* the cosmological horizon. Naturally, we must be able to obtain the Hawking radiation related for either of the horizons. Here, we only consider the case of the emission from the event horizon, and later in the next chapter will come back to that of the cosmological horizon. By utilizing our directly observer-based thought experiment as before, we consider an outgoing radial light ray which propagates from a point very close to inner horizon '$(r_{in} = r_H + \epsilon)$' at '$t = t_{in}$' to the 'detecting event' '$\mathcal{P}(t, r)$' where '$r_H \ll r \ll r_C$' and '$\epsilon \ll r_H$', for an observer Bob of the finite-distance '$(r - r_H)$' from the event horizon. With the required initial condition, we will find the corresponding trajectory in the following form,

$$r \simeq t - t_{in} + \alpha \text{ Ln } \frac{\epsilon}{r_H} \tag{38}$$

So, the redshifted frequency 'Ω' will be related to the initial frequency at '$r = r_H + \epsilon$' by,

$$\Omega \cong \Omega_{in} \;\exp(-\frac{t - t_{in} - r}{2\alpha}) \tag{39}$$

Similar to the computations before, we find the asymptotic frequency spectrum to be,

$$|f(\omega)|^2 \propto \left[\exp \frac{12\pi r_H \omega}{\Lambda(r_C - r_H)(r_H - r_U)} - 1\right]^{-1} \tag{40}$$

which in terms of the energy modes becomes a Planckian power spectrum at the temperature,

$$T = \frac{\hbar}{2\pi} \frac{\Lambda(r_C - r_H)(r_H - r_U)}{6 r_H} \tag{41}$$

This again coincides with the known result obtained from the conventional approach [4]. Now, in this same case of the Schwarzschild dS spacetime, we may want to see if we can successfully do our directly observer-based spectral analysis of the asymptotic Hawking radiation for the modes which are initially emanated from the cosmological horizon. We will postpone answering this question to the next chapter.

B. The Schwarzschild-AdS Horizons

In this section, we switch to the '$\Lambda < 0$ phase', and work out the example of the Schwarzschild horizon with the AdS asymptotics. Given the metric which we identified before, the location of the event horizon is determined by the zeros of the cubic equation,

$$f(r) = r^3 + b^2 r - 2mb^2 = 0 \quad ; \quad b^2 \equiv -\frac{3}{\Lambda} \tag{42}$$

As the function 'f' is strictly monotonic, this equation has only one root, given by [9, 19],

$$r_H = \frac{2b}{\sqrt{3}} \;\sinh\left[\frac{1}{3}\sinh^{-1}(3\sqrt{3}\,\frac{m}{b})\right] \tag{43}$$

Indeed, in this example a black hole horizon is always present. By expanding 'r_H' in term of 'm', with '$b \gg 3/m$', we have '$r_+ = 2m(1 - 4m^2/b^2 + \cdots)$', which shows that the '$\Lambda < 0$' has a shrinking effect on the horizon scale. For the outgoing light rays, the radial geodesics adopt the simple form,

$$\frac{dr}{dt} = 1 - \frac{2M}{r} + \frac{r^2}{b^2} \equiv \frac{1}{b^2 r}(r - r_H)(r^2 + r r_H + \rho^2) \tag{44}$$

The solution is given by,

$$t = \frac{b^2 r_H}{2r_H^2 + \rho^2}\left[\mathrm{Ln}(\frac{r - r_H}{r_H}) - \frac{1}{2}\mathrm{Ln}(r^2 + r r_H + \rho^2) + \gamma \tan^{-1}(\frac{2r + r_H}{\beta})\right] \tag{45}$$

where

$$\alpha = \rho^2/r_H \quad ; \quad \beta^2 = 4\rho^2 - r_H^2 \quad ; \quad \gamma = (2\alpha + r_H)/\beta \quad ; \quad \rho^2 = 2mb^2/r_H$$

Considering a light ray propagating from '$r_{in} = r_H + \epsilon$' at '$t = t_{in}$' to the detecting event '$\mathcal{P}(t, r)$' where '$r_H \ll r \ll b$' and '$\epsilon \ll r_H$', the last equation becomes,

$$t - t_{in} \simeq -\frac{b^2\, r_H}{2r_H^2 + \rho^2}\, \mathrm{Ln}\frac{\epsilon}{r_H} \tag{46}$$

The redshifted frequency Ω will be related to the frequency at r_{in} by,

$$\Omega \simeq \Omega_{in}\left[\frac{(r_{in} - r_H)(r^2 + r r_H + \rho^2)}{b^2 r_H}\right]^{1/2} \simeq \Omega_{in} \exp\left[-\frac{2r_H^2 + \rho^2}{2b^2 r_H}(t - t_{in})\right] \tag{47}$$

As in the examples before, we find that the power spectrum for a wave packet scattered off the event horizon and travelled to infinity has the Plankian form at the temperature,

$$T = \frac{1}{4\pi}\frac{2r_H^2 + \rho^2}{b^2\, r_H} \tag{48}$$

This again matches with the result from the conventional approach [18], and does reduce to that of the asymptotically-flat Schwarzschild horizon for '$b \to \infty$'.

C. The Cylindrical Horizons

The third example that we probe in this section is the black hole with cylindrical event horizon and with the AdS asymptotics [13]. By a coordinate transformation, the metric can be brought to the form,

$$ds^2 = (b^2 r^2 - \frac{a}{br})dt^2 - (b^2 r^2 - \frac{a}{br})^{-1} dr^2 - b^2 r^2 dz^2 - r^2 d\phi^2$$

where '$a \equiv 4M$' and '$b^2 \equiv -\Lambda/3$'. This is the metric of a static black hole whose event horizon is at '$br = a^{1/3}$', and has a curvature singularity at '$r = 0$'. Similar to the examples before, we consider radially ingoing light rays which propagate from the proximity of the event horizon '$r_{in} = a^{1/3}/b - \epsilon$', at '$t = t_{in}$', to the detecting event '$\mathcal{P}(t, r)$' where '$rb \ll a^{1/3}$' and '$\epsilon \ll r_+$'. The equation governing the radial ingoing light rays in this space, can be written as follows,

$$\frac{dr}{dt} = -(b^2 r^2 - \frac{a}{br}) \tag{49}$$

By solving this equation (Relation 2.145 in [6]), we obtain,

$$t = \frac{1}{a^{1/3}b}\left[\frac{1}{6}\,\mathrm{Ln}\left(\frac{b^2r^2 + bra^{1/3} + a^{2/3}}{(br - a^{1/3})^2}\right) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2br + a^{1/3}}{-a^{1/3}\sqrt{3}}\right)\right] \tag{50}$$

Introducing '$a' = a^{1/3}$', with '$r \ll a'$' and '$\epsilon \ll r_+$', the above relation becomes,

$$t - t_{in} \simeq -\frac{1}{3ba'}\log\frac{b\epsilon}{a'\sqrt{3}} \tag{51}$$

The frequency 'Ω' will be related to the initial frequency 'Ω_{in}' at 'r_{in}' by

$$\Omega(r \ll r_+) = \Omega_{in}\left[\frac{br(b^3r_{in}^3 - a^{1/3})}{br_{in}(b^3r^3 - a^{1/3})}\right]^{1/2} \simeq \Omega_{in}\left(\frac{br}{a^{1/3}}\right)^{1/2}\cdot\left(\frac{\sqrt{3}b\epsilon}{ba^{1/3}}\right)^{1/2} \simeq \Omega_{in}\exp\left[-\frac{3}{2}ba^{1/3}(t - t_{in})\right] \tag{52}$$

Now if we repeat the analyses of the compact-horizon cases for the cylindrical horizon, the asymptotic spectrality of the null rays will be thermally Planckian as follows,

$$|f(\omega)|^2 \propto \left[\exp\left(\frac{4\pi}{3ba^{1/3}}\right)\omega - 1\right]^{-1} \quad ; \quad T = \frac{3\hbar ba^{1/3}}{4\pi} \tag{53}$$

as was also obtained in [12, 13].

V. HAWKING RADIATION FROM THE OBSERVER-BASED HORIZONS

An *intrinsically-distinguished category of causal horizons* in classical and quantum gravity are the *'observer-based horizons'*. These are well-defined horizons which causally 'luck in' their 'defining observers'. The Rindler horizon of an accelerating observer in the Minkowski spacetimes is the canonical example of these causal bubbles. A global spacetime can be patch-wise partitioned and also re-assembled by a number of such observer-based causal patches. Once more, the canonical example is the '*causal deconstruction* of the Minkowski spacetime into four Rindler wedges. We must highlight that, his is not a physically content-empty deconstruction of the global spacetime, because between these complementary causal patches there can be fruitful and strong 'physical interplays'. For example, the 'vacuum-ness' virtue of the Minkowski empty spacetime is faithfully related with the 'maximal entanglements' that interconnect the partnering pairs of the Rindler wedges. These 'regional' maximal entanglements, being re-verbalized in an underlying abstract quantum formulation of the geometry, can be proposed as the fundamental fabricators of the physical spacetime [29].

Let us highlight that by *'globally reassembling'* an entire generic *spacetime* with a collection of sufficiently-many *observer-based* '*causal units'* can be a very natural and very productive methodology in developing *the complete theory of quantum gravity for the spacetimes which are phenomenologically relevant to us.* A very intriguing observer-based framework of this kind, based on the identification of '*causal diamonds*' as the constituting 'causal units', has been already proposed and developed [27]. Moreover, an inter-related but physically and mathematically new initiative proposal is being put forward in the forthcoming paper [33]. Any candidate formulation in this paradigm of quantum gravity should begin with proposing the '*unit holographic microscopic theory' in correspondence with each one single 'causal unit'.* So, as the simplest 'model theory', one could begin with initiating a microscopic theory of one Rindler patch [28]. But a much more gratifying '*unit theory*' would be the microscopic theory of the '*dS causal patch*', namely the '*dS static patch*', as for example proposals in [24, 27]. After obtaining *'the good theory' of de Sitter static patch*, we could then advance to the *microscopic theory of the global de sitter spacetime* by the 're-assembling methodology'. But, the *correct microscopic theory of the dS causal patch* is what we need phenomenologically.

Within all these understandings and scopes, through the rest of the current chapter we will consider both of these 'canonical observer-based horizons'. *The aim and the plan would be the same as before.* We employ our directly observer-based methodology to study the semiclassical behavior of the *'distantly propagated' Hawking modes.* Besides the further verification of our root-methodology, this study will clarify to two facts. *First*, as a by-product, it independently reconfirm the important statement that, the observer-based causal horizons does behave as the event horizons do, in their semiclassically coarse-grained, and most-expectedly also in their very fine-grained aspects of the Hawking radiation. But *second*, the important outcomes for us will be as before *the universal 'distinguished lessons' on the mechanism of unitarity in the quantum evolution of the causal horizon* that we take from this macroscopic analysis here, and will so utilize in the part 'B' of this study, and as well, in the forthcoming initiative work on the *fundamental microscopic theory of the evolving horizons* [33].

A. Hawking Modes From The de Sitter Cosmological Horizon

Let us begin the study of this chapter with the de Sitter cosmological horizon, while for a discussion of the comparative physics of the de Sitter spacetime in different patches we refer to [32]. The static patch of the de Sitter spacetime is defined by the metric,

$$ds^2 = \left(1 - \frac{\Lambda}{3}r^2\right) dt^2 - \left(1 - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 - r^2 d\Omega^2$$

The *cosmological horizon* is the causally encompassing '$\mathbb{S}_2$'-surface with the *finite radius* '$R = \sqrt{3/\Lambda}$'. Therefore, the causal patch has also the finite radial extension '$r \in (0, R)$'. The static observer Bob is located *inside* the cosmological Horizon at a point sufficiently far from it. To configurate our principal thought experiment, now consider an ingoing radial light ray which is released at '$r_{in} = R - \epsilon$', with '$\epsilon \ll R$', in the proximity of the cosmological horizon at the initial time '$t = t_{in}$', and so propagates toward the distantly located Bob who much later receives those modes at the spacetime event '$\mathcal{P}(r, t)$' with '$r \ll R = \sqrt{3/\Lambda} \equiv H^{-1}$'. The trajectory of the radial light ray can be written in the form,

$$\frac{dr}{dt} = -\left(1 - \frac{\Lambda}{3}r^2\right) \tag{54}$$

The integrated trajectory, given the required initial condition, is,

$$t - t_{in} = \frac{-1}{2H}\left(\log\frac{1 + Hr}{1 - Hr} - \log\frac{1 + Hr_{in}}{1 - H_{in}r}\right) \simeq \frac{-1}{2H}\log\frac{H\epsilon}{2} \tag{55}$$

So, the relation between the Bob's observed frequency and the initial frequency is given by,

$$\Omega(r \ll H^{-1}) = \Omega_{in}\left(\frac{1 - H^2 r_{in}^2}{1 - H^2 r^2}\right)^{1/2} \simeq \Omega_{in}\ (2H\epsilon)^{1/2} \simeq 2\ \Omega_{in}\ e^{-H(t - t_{in})} \tag{56}$$

Now by repeating the same analysis we performed in the case of black hole horizons, we find in exactly the same way that the spectrum of the modes received by Bob follows the distribution,

$$|f(\omega)|^2 \propto \left[\exp(\frac{2\pi\omega}{H}) - 1\right]^{-1} \tag{57}$$

which in terms of the energy quanta is a Planckian power spectrum at Hawking temperature,

$$T = \frac{\hbar}{2\pi} \cdot \frac{1}{R} \tag{58}$$

which is the result initially obtained in [4].

As a followup, let us be back again to the example of the Schwarzschild de Sitter spacetime that we considered in the previous chapter, and redo the analysis we did there, for the light rays which propagates toward the interior observer from the near cosmological event horizon. Considering an ingoing light ray which starting from a point very close to the outer event horizon '$r_{in} = r_C - \epsilon$', the trajectory would have the following form,

$$r \simeq -(t - t_{in}) + \beta \, \mathrm{Ln}\frac{\epsilon}{r_H} \tag{59}$$

And so, the frequency Ω will be related to the initial frequency 'Ω_{in}' at 'r_{in}' by

$$\Omega(r_H \ll r \ll r_C) \simeq \Omega_{in} \left(\frac{2\epsilon}{r_C}\right)^{1/2} \simeq \Omega_{in} exp \left(\frac{t - t_{in} - r}{2\beta}\right) \tag{60}$$

Therefore, we will obtain a Planckian power spectrum defined at the temperature,

$$T = \frac{\hbar}{2\pi} \frac{\Lambda(r_C - r_H)(r_C - r_U)}{6r_H} \tag{61}$$

B. Rindler Observer And Hawking Modes

Now we come to our second canonical example, namely the Hawking radiation of the Rindler horizon. Besides the highlighted points that we briefly explained in the beginning of this chapter on the significance of understanding the microscopic theory of the observer based horizons, in particular the Rindler horizon as possibly the simplest 'model theory', there is still one more dimension into the distinguished relevance of the Rindler spacetime, to be mentioned. This important aspect is the fact that, *for a large variety of phenomenologically interesting black holes, the near horizon region is given by the Rindler spacetime.* Therefor, methodologically speaking, the correct microscopic understanding of the *"Rindler quantum information theory* might be a '*rescaled basis*' for the the full theory. Naturally assuming that the microscopic theories of 'all the causal horizons' must be the same, then this near-horizon "Rindler-ness", may suggest *an intrinsic 'self-similarity'* in the underlying microscopic theory of black holes.

Here we apply our root-methodology to study the semiclassical spectral behavior of the Hawking modes released by a Rindler horizon. Because we have earlier discussed this case based on the equivalence principle, our presentation in here would be brief.

Consider a congruence of worldlines for a family of observers moving with constant acceleration 'a' in Minkowski spacetime. In terms of the conventional coordinates '(τ, ξ)' parametrizing this family of worldlines [14], the metric takes the form,

$$ds^2 = dt^2 - dx^2 = (1 + a\xi)^2 d\tau^2 - d\xi^2$$

The right-hand side form of the above metric defines the Rindler spacetime. In fact, the proper coordinate system is incomplete and covers only a quarter of the Minkowski spacetime '$x > |t|$', therefore one of its *Rindler wedges*. Indeed, this is the subdomain of Minkowski spacetime which is causally accessible to a uniformly accelerated observer, namely, it is *the causal patch of the accelerating observer, bounded by the corresponding observer-based causal horizon.* The Rindler observer perceives this causal horizon at proper distance 'a^{-1}'. Therefore, because of the presence of a causal horizon for the Rindler observer, we may anticipate that there must be a typical Hawking emission associated to it. Let us apply our directly observe-based method to see if this intuition is correct, and if so, then obtain the spectral behavior of the distantly propagated Hawking modes. To address this question, we consider as before a radial light ray which propagates from '$\xi_{in} = -a^{-1} + \epsilon$' at '$\tau = \tau_{in}$' to an observer located in the spacetime position '(ξ, τ)', where '$\xi \gg a^{-1}$' and '$\epsilon \ll a^{-1}$'. The trajectory of such radial light rays is given by the equation,

$$\frac{d\xi}{d\tau} = 1 + a\xi \tag{62}$$

so that, by the required initial condition the integrated trajectory will be,

$$\xi - \xi_{in} = \frac{1}{a} \log\left(\frac{1 + a\xi_{in}}{1 + a\xi}\right) \simeq \frac{1}{a} \log\left(\frac{\epsilon}{\xi}\right) \tag{63}$$

So, the observed frequency 'Ω at $\xi \gg -a^{-1} + \epsilon$' will be related to initial frequency 'Ω_{in}' at '$\xi_{in} = -a^{-1} + \epsilon$' by,

$$\Omega(\xi \gg -a^{-1}) = \Omega_{in}\left(\frac{1 + a\xi_{in}}{1 + a\xi}\right) \simeq \Omega_{in}(\frac{\epsilon}{\xi}) = \Omega_{in} e^{-a(\tau - \tau_{in})} \tag{64}$$

Now, very similarly, this Planckian spectrum for the observed Hawking modes is obtained,

$$|f(\omega)|^2 \propto \left[\exp\left(\frac{2\pi\omega}{a}\right) - 1\right]^{-1} \tag{65}$$

which corresponds to the Hawking temperature,

$$T = \frac{\hbar a}{2\pi} \tag{66}$$

Which is the standard Unruh temperature [21].

PART “B”

:

‘BITS BACK’

....

THE “1”ST MOVEMENT

....

VI. UNITARITY, HOLOGRAPHY AND HORIZONS

A. The Unitarity of Information Processing In Bob's View : The Paradox

Suppose that an asymptotic observer '*Bob*' is monitoring in all-details the entire history of a well-defined physical event in which a '*large black hole*' is formed out of a sufficiently high-energy initial scattering, or from the gravitational collapse of some sufficiently massive 'quantum matter'. The so-formed black hole then 'evaporates', very softly but continually, by releasing the quantum-sourced Hawking modes in the proximity of its (stretched) horizon, until finally, it becomes all-annihilated into the totality of an asymptotic Hawking radiation collectable by Bob. By the 'largeness' of the black hole we mean that the radius of the initially-formed horizon, to be called "*the primal horizon*", is so enormous in Planck units that no quantum gravitational corrections would matter at the horizon scale (up until the horizon becomes of the Planckian sizes, sufficiently-close to its total evaporation). To have the right setting for the information paradox, we also assume that the initial state of the quantum matter is set to be, and does stably remain to be, in *a 'pure state;*, before the primal horizon is being formed,. That is, we take that the von Neumann entropy of the total quantum system in the '*pre-horizon-era*' vanishes, as an *initial-state condition*,

$$S^{\text{Initial State}}_{\text{(von Neumann)}} = 0 \tag{67}$$

Now let us see, from *Bob's information-theoretic view*, the '*post-horizon-formation era*'. Bob's main assumption is that the evolution of the black hole, from the initial scattering or collapse to its total annihilation, is microscopically processed in a way that is *completely 'consistent with' all the structure and the laws of quantum physics.* Indeed, given all his experiences in string theory and quantum gravity, he does not have any serious evidence or even any significant motive to postulate any statement in contradiction to this assumption. To the other extreme, he does have a large manifold of evidences for this one assumption. For example, all black holes which are addressable by the AdS/CFT correspondence are confirmed to be so. Bob will so be assuming that this quantum gravitational phenomenon evolves in consistency with the 'unitary dynamics of a quantum system'. *A unitary dynamics preserves the information*, so, all the *'information bits'* that identify the exact *initial-state* of the pre-horizon quantum system *must be indestructible.*

The delicate central question to be addressed as of now is *if the 'must-be' unitarity of the information processing is indeed validated by Bob.* To the 'asymptotic observations' of Bob, *the so-formed 'horizon'* behaves by all means as a thermal physical 'membrane' to which a thermodynamical entropy, the Bekenstein-Hawking entropy, equal to one quarter of its area in Planck units, is associated,

$$S_{B.H} = \frac{A_{\text{ Horizon}}}{4l_p^2} \tag{68}$$

Because now, a 'quantum-finite' positive entropy has suddenly developed in the gravitational system, Bob must be immediately concerned if the principle of information indestructibility is being held, due to the defining relation between entropy and information. Fortunately, by simply recollecting and utilizing *the very defining virtue of a black hole*, he gets the right clue out of this worry. A black hole is a 'maximally entropic' gravitating system, entirely enclosed by a spacetime hypersurface, *the horizon*, that acts as a *causal boundary* to all the events in the complementary part of the spacetime geometry. As such, the internal system of the black hole, that which is localized in *the horizon's 'interior'*, is causally disconnected from the asymptotic Bob, and so *does not belong to his own 'causal patch'*. Now, by merely utilizing this conceptual point, Bob understands that the demand for the indestructibility of the information bits becomes equivalent to the following statement:

The thermodynamical Bekenstein-Hawking entropy (68) associated to the 'primal horizon', now being interpreted as the holographic 'statistical entropy' of the horizon-interior system, must necessarily 'count' the total number of black hole's microstates to be exact-equal to the total number of the initial-state information bits.

Let the *'integer'* 'N', which is taken to be a sufficiently large number, denote for us '*the total number of the initial-state information bits*',

$$N \equiv I^{\text{ Initial State}} \tag{69}$$

Then, in Bob's view, *the principle of information indestructibility becomes these statements*,

$$S_{\text{ Primal Horizon}} = I^{\text{ Initial State}} \tag{70}$$

$$A_{\text{ Primal Horizon}} = 4l_p^2 \cdot N \tag{71}$$

As implied by the principal equalities (70,71), Bob also concludes that, *the initial-state information build up, 'bit by bit', the total Hilbert space of the primal black hole, by a one to one mapping that takes those bits to the black hole microstates*, that is,

$$\dim[\mathcal{H}_{\text{ Primal Black Hole}}] = \dim[\mathcal{H}_{\text{ Primal Horizon}}] = \exp[\ \mathrm{I}^{\text{ Initial State}}\ ] = \mathrm{e}^{\mathrm{N}} \tag{72}$$

So far, so good. But as Bob also experimentally confirms by his detector, the primal horizon is *quantum unstable*. It decays by quantum vacuum fluctuations in the proximity of the (stretched) horizon. It so shrinks, very very softly but monotonically, by a continual release of the Hawking modes. As the horizon decays such, the dimension of the black hole's Hilbert space, '$\mathcal{H}^{\text{ Interior}}$', also shrinks in accordance with the Bekenstein-Hawking entropy formula. But then after this evaporation process is completed, once mrs the entirety of the spacetime geometry becomes causally connected. Therefore, Bob will be able to check whether the initial-state information are all preserved, upon collecting the totality of the '*final-state*' asymptotic Hawking modes. Now, to manifest the core of the problem, let us see *how the dimension of '$\mathcal{H}^{\text{Interior}}_{(t)}$' at an arbitrary time '$t$' after the formation of the primal horizon would shrink upon the release of a 'spectrally-typical' Hawking mode.* Also, to be more specific, let us consider the case of a Schwarzschild black hole in four spacetime dimensions in the rest of this study, knowing that an obviously similar analysis goes through in other spacetime dimensions, or for more complicated geometries. At a time 't' of this quantum evaporation process, the horizon-interior system and the exterior spacetime geometry can be sufficiently-well described by those of a Schwarzschild geometry which is endowed with a very-slowly varying time-dependent mass '$M(t)$', together with its corresponding quantities for the horizon radius '$R(t)$', the temperature '$T(t)$', and the black hole entropy '$S_{\text{Interior}}(t)$',

$$R(t) = 2l_p^2\, M(t) \quad ; \quad T(t) \equiv \beta^{-1}(t) = \frac{1}{8\pi l_p^2\, M(t)} \quad ; \quad S_{\text{Interior}}(t) = 4\pi l_p^2\, M^2(t) \tag{73}$$

Now, let us suppose that at 'about' this time 't' one spectrally-typical Hawking mode is released. The emission of this quanta reduces the entropy of the interior black hole system by the following amount,

$$\delta S_{\text{ Interior}}(t) = -\beta(t)\ \cdot\ \varepsilon_{\text{ H.M}}(t) \tag{74}$$

in which '$\varepsilon_{\text{H.M}}(t)$' denotes the energy of the released Hawking mode at the emission time 't'. Being spectrally thermal, the emitted Hawking mode is of the energy,

$$\varepsilon_{\text{H.M}}(t) = T(t) \tag{75}$$

and so from (74) we learn that,

$$\delta S_{\text{Interior}}(t) = -1 \tag{76}$$

Namely, each typical Hawking mode, once being released, carries away one unit of entropy from the black hole. Combining this with the unitarity conditions (70,71), we obtain,

$$S_{\text{ Interior}}(t) = N - N'(t) \tag{77}$$

$$\dim[\mathcal{H}_{\text{interior}}(t)] = e^{N-N'(t)} \tag{78}$$

with '$N'(t)$' denoting the total number of the Hawking modes released-out up until a time 't' after the formation of the primal horizon. Of course we know that, the released Hawking modes need not to be all exactly-typical but only spectrally Planckian, mostly carrying $O(1)$ entropies. However, doing a statistically more careful analysis with all details included, we would get the same result for '$S_{\text{ Interior}}(t)$' as stated above. From (77,78) Bob concludes that, *the primal black hole will be 'all annihilated' after radiating out a total number of 'N' thermal Hawking quanta, namely as many as the total number of the initial-state information bits.* Now, let us suppose that Bob does patiently his monitoring for an enormously-long time scale which turns out to be of the order of '$N^{\frac{3}{2}}$', until the primal black hole gets entirely quantum-annihilated. If so, then the*'final state'* of the system would be an *asymptotic radiation* characterized by a *'thermal density matrix $\hat{\rho}$'* defined at the temperature,

$$T = (4\sqrt{\pi} l_p)^{-1} N^{-\frac{1}{2}} \tag{79}$$

whose von Neumann entropy, '$S \equiv -\text{Tr}[\hat{\rho}\ \log(\hat{\rho})]$', equals to,

$$S^{\text{(Thermal State)}}_{\text{Asymptotic Radiation}} = N \tag{80}$$

and whose expectation value of the total number-operator as well equals to the same 'N'. Now, such a quantum many body system, in which the von Neumann entropy satires its upper bound in being equal to the total number of the microscopic degrees of freedom of the system, must be *in the 'thermal version' of the 'maximally uncertain quantum state' which is information-theoretically contentless.* That is, the final state we be such,

$$I^{\text{ Final State}}_{\text{ (Thermal State)}} = 0 \tag{81}$$

That would then mean that, all the initial-state bits of information would have been lost, in a maximum violation of unitarity. *This enormous 'quantum information mismatch' does clearly call Bob for a resolution.*

B. Geometric Planckianity And The 'Optimal Jumbler' : The Microscopic Theory

To be able to *correctly resolve* the afore-explained 'quantum information mismatch', which would violate unitarity as enormously as possible, it would be most natural to begin with 'correctly pinpointing' the exact origin of the spectral Planckianity of the asymptotically-observed Hawking radiation, as the very first step. *There, in the very origin of the problem, is the key to the solution.*

In the part '*A*' of this study, we addressed this question by a root methodology which was in physical speculations and the underlying assumptions was most minimalistic, and as an '*observer-centric approach*' was both conceptually and operationally most direct. We also verified the generality and stability of that method by scanning over a broad class of causally-nontrivial spacetimes that collectively constituted a landscape of all the major deformations. What we methodically demonstrated and empirically validated as being a general physical 'theorem', is the following statement:

An 'initial-state system' of the near-horizon 'mono-frequency' Hawking modes becomes a 'finite-state system' of the spectrally-Planckian 'multi-frequency' Hawking modes, merely as the direct geometric result of the 'geodesic propagation' of the frequency modes through the so-curved continuum of the asymptotic observer's causal patch.

Now, to advance further this result of the part 'A', we must first appreciate that, *this 'maximal dispersion of the frequency modes' is equivalent to the quantum-information mismatch itself.* That is, the information loss is the very semiclassical observation that, even if the 'initial system of the near-horizon frequency modes' is prepared in a pure quantum state, its 'geometrically-driven' 'final-state system of the asymptotic frequency modes' is found to be in a 'Planckian-profile' 'mixed quantum state'. As we found there, *this maximal dispersion of the frequency modes is a 'characteristically classical' 'geometric' effect which 'disturbs' a microscopic 'quantum system'.* The underlying quantum-ness of 'the affected system' is because, not only the modes are quantum-released, but also they inter-process 'an almost-thermalization' whose temperature is '$\hbar$'-proportional.

Having known those basic points, we are finally at the right stage to pinpoint the correct origin of the information paradox. The final picture will become transparent in several steps. Let us primarily ask the following question.

'How', namely by which 'underlying microscopic mechanism', is that 'conversion' of the initial system of the near-horizon mono-frequency modes into a final system of the 'spectrally-Planckian multi-frequency asymptotic modes' can be caused?

In answering this question, this much of the microscopic mechanism is clear. That which operates on the quantum system of frequency modes, and maximally disperses them, must be the '*classical limit*' of a '*geometric environment*'. In a better wording, the frequency modes constitute '*an open quantum system*' interacting with the *'quantum environment' of some 'characteristically-geometric' 'degrees of freedom'*, so that as the collective outcome of this '*environment-system entanglement*', *in a certain 'classical environmental limit'*, the geometric effect by which the modes are optimally dispersed can be simulated. Being so, we will be calling this 'optimally-dispersing environment', the *"Optimal Jumbler"*.

As the next move forward, we must formulate the global structure of an associated *Hilbert space scheme* which '*quantum microscopically*' formulates this physical picture we have been led to. '*First*', at any arbitrary time during the horizon evaporation, there is one Hilbert space accounting for the totality of *the 'already-released' Hawking modes* which, as an evolving 'open quantum system', must be itself tensor-factorially accommodated into a '*total Hilbert space*' defining an entire closed quantum system. Let us highlight that we define this 'total Hilbert space' for *'the smallest' set of the microscopic degrees of freedom which can be 'consistently' treated as a 'closed system' in Hawking evaporation*. So,

$$\mathcal{H}^{(t)}_{\text{Hawking Modes}} \subset_{\otimes_{(t)}} \mathcal{H}^{(\text{Minimum})}_{\text{Total}} \tag{82}$$

As '$\mathcal{H}^{(t)}_{\text{Hawking Modes}}$' defines *'all'* the Hawking modes available in the '*Bob's causal patch*' up until a time 't' after the 'the formation of the primal-horizon', its dimension is increasing as far as the black hole evaporation goes on. Therefore, in (82), the left-hand side is explicitly *time-dependent*, but the right-hand side is '*static*'.

Next, being back to (82), we frame the subsystem *'complementary'* to the Hawking Modes,

$$\mathcal{H}_{(t)}^{\text{Complementary}} \subset_{\otimes_{(t)}} \mathcal{H}_{\text{Total}}^{(\text{Minimum})} \tag{83}$$

and in accordance with the physical description given before, '*demand that*' *it must develop the "Optimal Jumbler" in a 'specific environmental limit'*.

As '*a marked note*', let us already clarify *a point of foundational importance.* By thinking more fundamentally and more ambitiously, one may begin the formulation of the theory with *doing better* than cutting the *'defining frame' of the total microscopic system* down to the 'so-minimalistically defined' '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$'. Let us remember that '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$' is defined to be the *'smallest possible'* Hilbert space that can be *consistently* identified as a 'closed quantum system' which accommodates '*all*' the Hawking modes ever released-out into the asymptotic-observer Bob's causal patch, during the entire evolution. Although a 'minimum frame-cut' for the total system clearly brings some advantages, in order to be having a more fundamental, more unified and in fact '*a better detailedly-fixed theory*', it would be much better for us to instead begin the formulation with a *'more natural' identification of the* '$\mathcal{H}_{\text{Total}}$' which turns out to be a much much bigger state-space than '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$'. To see better this central point, we must remember that *our principal methodology in defining quantum gravity is being fundamentally 'observer-centric'*, which in this context means that we must entirely take the physical stance of the asymptotic observer Bob, and so then try to develop a complete microscopic theory of evolving horizons *'as he would do' in his 'sufficiently-large causal diamond'.* That is, we must redefine *the total microscopic system* of this scenario to be *any, or a smallest choice, of the 'Bob's causal diamonds' which accommodates the entire history of the evolution of the black hole*, from the initial high-energy scattering or collapse to the completion of its Hawking emission [23, 33]. Because a causal diamond must be physically treated as a consistent closed universe [27], the Hilbert space of Bob's causal diamond defines the *"Theory of Everything"*, the *"TOE"*, for his causal universe in which '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$' is placed *as a (much smaller) 'sub-theory'*. Here, we will limit our analysis to the '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$-theory', and refer the interested readers to consult the forthcoming work on *Bob's unified theory* [33]. Let us simply restate that in that theory, the relation (82) is being completed with the following one,

$$\mathcal{H}_{\text{Total}}^{(\text{Minimum})} \subset_{\otimes} \mathcal{H}_{(\text{TOE})}^{\text{Bob}} \tag{84}$$

Now, let us be back to the *defining relation* (83), and ask the following important question: *What is the physical identity of the microscopic degrees of freedom defined by* $\mathcal{H}_{(t)}^{\text{Complementary}}$ *?* To answer to this question correctly, we need to understand more clearly the *'fine-grained' microscopic nature* of the '*classical geometric mechanism*' which, in the *coarse-grained collective way* that was found in the part 'A', disperses the initial system of mono-frequency Hawking modes into a final system of spectrally-Planckian modes. The "AHA" experience is simply realized by recollecting that this *'environmental' dispersing mechanism* is developed as the *direct effect* of the *'background curvature' of the asymptotic-observer Bob's 'causal patch'* on the beams of Hawking modes which are propagating *away from the null horizon.* As such, the underlying fine-grained structure is '*inversely*' obtained by *'deconstructing' the 'background curvature' of Bob's causal diamond* with a null-horizon causal boundary down to a Hilbert space of *quantum 'curvature modes' which build up that geometric environment.* Being translated and promoted into 'an exact statement' about *the 'Hilbert space systems' defined by the relations (82,83)*, we are led to the following principal identification:

'$\mathcal{H}_{(t)}^{\text{Complementary}}$*', defined by the relations (82,83) at any given time 't' of the Hawking evaporation process, must be physically identified as the Hilbert space of the entirety of 'microscopic degrees of freedom' that 'build up' the'curvature' of the asymptotic-observer Bob's causal diamond with a null-horizon causal boundary, at the corresponding time 't' . That is,*

$$\mathcal{H}_{(t)}^{\text{Complementary}} \equiv \mathcal{H}_{\text{Bobby Curvature Modes }(t)}^{[\text{Causal Diamond}]} \equiv \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}} \tag{85}$$

In accordance to this principal physical identification, we will be calling the degrees of freedom of '$\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}$*' the "Bobby curvature modes".*

From the identification (85), together with the relations (83) and (82), we learn that our *'minimum-total Hilbert space'* admits the following tensor-factorial *decomposition*, as an exact equality which must hold at any time during the entire Hawking evaporation,

$$\mathcal{H}_{\text{Total}}^{(\text{Minimum})} = \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}} \otimes \mathcal{H}_{\text{Hawking Modes}}^{(t)} \tag{86}$$

In the rest of this paper, we will be referring to *the quantum many body system* which is defined by the 'triplet system' of '$(\mathcal{H}_{\text{Hawking Modes}}^{(t)}, \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}, \mathcal{H}_{\text{Total}}^{(\text{Minimum})})$' the "*Hilbert space triplet*", or simply "*the quantum triplet*".

Having known the physical identification (85), and the Hilbert space structure (86), now we must revisit the *'Optimal-Jumbler Correspondence' demand* which we introduced earlier, and so dissect its '*defining physical conditions*'. Let us first *restate that demand* as follows:

$$\text{Optimal Jumbler} \ : \ \exists \text{ A Specific Semiclassical Limit of The Quantum Triplet s.t :}$$
$$\text{Optimal Jumbler} = \left(\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}\right)_{\text{In That Specific Semiclassical Limit}} \tag{87}$$

So then we ask: What physical conditions do identify the 'Optimal Jumbler Limit' as being phrased in the 'correspondence demand' (87)? The question, in better wording, is the following: *What are the necessary and sufficient conditions to be imposed on the quantum many body system defined by our 'Hilbert space triplet' so that the 'Optimal Jumbler limit' of the demand (87) is actualized?*

The correct answer is being manifested in just two steps. '*First*', it is so manifest from the *instrumental relations between the initial frequencies versus the final frequencies* in all the examples we studied in the part 'A' that, *the 'optimal Jumbling' of the Bobby bulk Hawking modes is environmentally developed by the direct effect of the background curvature of the Bob's causal patch, 'only because', 'an exponentially-enhanced redshifting' is being imposed on the propagating modes by the very presence of the corresponding causal horizon.* That is, the 'Planckian-type dispersion' of the propagating modes requires 'as a necessary condition' that the Bob's causal geometry has been curved in the presence of a null causal screen. In fact, one reason for our explicitness in choosing the name "*Bobby Causal Curvature*" in (85) and (86) was to make this point crystal clear. Therefore, by implementing this basic understanding into the exact microscopic physics of the 'Hilbert space triplet', we now state '*the first necessary condition*' for the actualization of the specific semiclassical limit that corresponds to the '*Optimal Jumbler*':

'*C.1*'. *The semiclassical 'Optimal Jumbler Limit' can be actualized by '*$(\mathcal{H}^{(t)}_{\text{Hawking Modes}}, \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}, \mathcal{H}^{(\text{Minimum})}_{\text{Total}})$*', if as 'the first necessary condition', this triplet corresponds to a 'quantum many body system' that consistently defines, and all-correctly accounts for, the 'curved geometry' of the sufficiently large 'causal diamond of an asymptotic observer' which has as its 'causal boundary' the null horizon.*

But, this necessary condition is not yet a sufficient one. Namely, based on the 'universally-verified' lessons we took from the analyses of the part 'A' of this study, we still need one more condition to impose on the triplet system for the Optimal Jumbler to be actualized by it. This '*second necessary condition*' will be the direct quantum microscopic implementation of the following statement into the 'Hilbert space triplet':

The geometric Planckianity of the Optimal Jumbler, abstractly and microscopically understood as the 'Planckian-optimal spectral-dispersion' of the 'frequency modes' of an 'open quantum system', can be actualized only in the strict "thermodynamic limit" of its 'quantum environment' defined and constituted by the 'Bobby Curvature Modes'.

Here, the 'thermodynamic limit', or the 'continuum limit', is meant exactly as in the (classical or quantum) statistical physics of many body systems, namely the exact limit in which the *total number of the microscopic degrees of freedom* of the system goes to *infinity.* In accordance, '*the second necessary condition*' for the microscopic actualization of the *Optimal Jumbler Limit* is stated as follows:

'*C.2*'. *The Optimal Jumbler can be actualized by the triplet quantum system (86) only and only in the strict 'thermodynamic limit' of the environment of the 'Bobby Curvature Modes',*

$$\dim[\, \mathcal{H}^{\text{Bobby Causal Curvature}}_{(\text{ Optimal Jumbler Limit})}\,] \;=\; \infty \tag{88}$$

Therefore, for the 'Bobby-bulk geometry' of a spacetime possessing a quantum-evaporating horizon to *'jumble' the initial-state bits of information* to the Planckian effect (with or without the graybody factors or all the other perturbative corrections included in it), *both of the conditions* 'C1' *and* 'C2' *must be simultaneously actualized by the quantum triplet.* However, it is simply a matter of paying enough attention to the '*physical content of the condition* C1' to become clear about the following fact:

'Fact'.: Once the Hilbert space triplet '$(\mathcal{H}^{(t)}_{\text{Hawking Modes}}, \mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}, \mathcal{H}^{(\text{Minimum})}_{\text{Total}})$' *validates the condition* 'C1'*, then the condition* 'C2' *can not be actualized by it.*

How do we most clearly see that a microscopic validation of '*C1*' invalidates '*C2*'? By simply applying the universal statements of holography [25], and as well the Jacobson's re-derivation of the Einstein's equations [26], to the 'Bobby-causal type' configurations as phrased in the exact statement of '*C1*', it is being concluded that, for the quantum many body system defined by the Hilbert space triplet to realize the condition '*C1*', it must satisfy the following *holographic demand on the dimension of the total quantum system*,

$$\dim[\mathcal{H}_{\text{primal Horizon}}] = \exp(\frac{A_{\text{Primal Horizon}}}{4l_p^2}) = e^N = \dim[\ \mathcal{H}^{(\text{Minimum})}_{\text{Total}}\ ] \tag{89}$$

Now, although the right hand side of the *'holographic equality'* (89) can be arbitrarily large, must be always a '*finite*' value. Indeed, the area of the primal horizon in Planck units can be taken as giant as we want, but it should always be *finitely large*, for one reason at least, simply because the strict limit of an *'infinite' horizon radius* is *physically inconsistent* with the co-presence of an *asymptoticly far observer 'Bob'* to collect and to analyze the asymptotic Hawking radiation. Because of this '*strict finiteness*', and by utilizing the equalities (83) and (85) which identify '$\mathcal{H}^{\text{Bobby Causal Curvature}}$' as a subspace of '$\mathcal{H}^{(\text{Minimum})}_{\text{Total}}$', we conclude that,

$$\dim[\ \mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}\ ]\ <\ \infty\quad ;\quad \forall t \tag{90}$$

This does contradict (88) which is the statement of the necessary condition '*C2*'. This proves that a simultaneous validation of the two necessary conditions of an '*optimal jumbler environment*' for the horizon released Hawking modes, the conditions '*C1*' and '*C2*', is physically impossible. Therefore we have learned the following lesson:

A holographically correct '$\mathcal{H}^{\text{Bobby Causal Curvature}}$' can never be an 'optimal jumbler'. Therefore, the unitarity of the 'information processing' in the Hawking evaporation of the causal horizons is guaranteed if 'holography' is implemented microscopically - 'as it must'.

It may be worth to highlight on one independent point. We have been explicit in the condition 'C2' on the statement that, the optimal jumbler can only be actualized if the Bobby curvature environment is infinitely large in its microscopic degrees of freedom. This point may deserve a distinct attention. By naturally taking this statement beyond its specific quantum gravity context into 'quantum information theory', it becomes an '*information theoretic conjecture*' which we do propose.

This information theoretic conjecture is the abstract general statement that, the *'Planckian-effect dispersion' of any set of frequency modes in any subsystem of a given total closed system can not physically happen as far as the total system is finite dimensional.* By the term 'Planckian-effect dispersion' here, we collectively mean any *'environmentally sourced'* dispersion of frequency modes which is of the *Planckian type*, incorporating the possible graybody factors and any possible type of perturbative quantum corrections in it. We should also note that this information theoretic statement is consistent with the 'Page phenomenon' which is generic to the 'finitely-large' quantum many body systems [31].

Now, let us be back to the central subject of this section, and restate our 'main conclusion'. In short, we have learned that, *the 'optimal jumbler' can only be actualized in a limit of Bob's causal-curvature environment that is 'holographically illicit'. Therefore, there can never be any information loss in holographic microscopic systems.*

C. The Holographic Way To Quantum Gravitational Unitarity

Finally, we are *at the right stage* to clarify in more details how *holography* must be implemented into the *observer-centric microscopic theory* of the triplet quantum many body system $(\mathcal{H}^{(t)}_{\text{Hawking Modes}}, \mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}, \mathcal{H}^{(\text{Minimum})}_{\text{Total}})$ which formulates the '*minimum' total system*' of a quantum-evolving causal horizon during its entire history.

Naturally anticipated, it becomes manifest that *the correct microscopic theory of the Hawking-evaporating horizons as described in the asymptotic observer Bob's causal patch* is being formulated based on an intrinsically-holographic '*'quantum duality' between the 'horizon-interior system' and the causally-independent system of 'Bobby curvature modes'.* That, in one way or another, the quantum systems in the '*interior*' and the '*exterior*' of a causal horizon must be physically *'mirroring'* one another is a major idea in quantum gravity that dates back to the seminal papers which proposed the principle of *"observer complementarily"* [21]. Indeed, the *'quantum duality'* known as observer complementarity finds its deepest roots in the black hole information physics which is the core subject of this study as well.

Back then, it was shown that for *consistently* joining the natural postulate of *the 'quantum unitarity of the Hawking evaporation process'* with the 'equivalence-principle demand' of the '*empty-space vacuum-ness' of the horizon to the infalling observer's observation*, we must also postulate a *quantum duality* between a defining pair of Hilbert spaces in the interior and in the exterior of the causal horizon. However, many physical and mathematical details of the corresponding '*duality map*' have been a matter of both minor and major revisits up until present [21], which must be yet discovered. Here we state and utilize our distinct *'version'* of this *"inside-outside duality"*. The duality that we frame here will also be at the level of an exact equivalence between a pair of Hilbert spaces in the two causal sides of the horizon, but the detailed duality map between the degrees of freedom on both sides will be an interesting theme for future works. The exact foundation of this 'inside-outside duality' proposal will be unfolded from first principles by the forthcoming conjectural paper [33]. Here, we will briefly elaborate on the reasoning behind this proposal, and then exactly state and utilize it.

Let us remember that '$\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}$' is by definition the quantum system of all the *'Bobby curvature modes'* at the given time, namely it is the system of all those degrees of freedom that microscopically 'build up' the 'time-t' '*curvature*' of the causal patch of an asymptotic observer Bob who is monitoring the entire evolution of the horizon. Let us also remember that, the evolving horizon monotonically shrinks by a slow release of Hawking modes into the Bob's causal patch, so that the black hole's mass continually decreases up until it vanishes entirely. In accordance, the curvature of the Bob's causal geometry, which is directly sourced by the decreasing mass of the black hole, is also being driven to vanish softly but monotonically. For example, in our canonical model, the bulk spacetime curvature is simply modeled by that of the Schwarzschild metric with a monotonically-decreasing time-dependent mass profile is it. Now, it is holographically clear that the *total number of the 'Bobby curvature modes'* must be counted by the area in Planck units of the causal horizon, monodically decreasing from its initial value 'N' to zero. Therefore, let us introduce a one-parameter family of *integers* that serve as a holographic measure of the time-dependent area of the evaporating horizon in Planck units,

$$N(t) \equiv \frac{\text{Horizon Area(t)}}{4l_p^2} \quad ; \quad N(t) \in \mathbb{N} \quad ; \quad \forall t \tag{91}$$

Now, the dimension of the Hilbert space of the curvature modes is given by,

$$\dim[\ \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}\ ] = e^{N(t)} \qquad \forall t \tag{92}$$

which evolves in accordance with the following boundary conditions on '$N(t)$' ,

$$N(0) = N = I_{\text{ Initial State}} \quad ; \quad N(t > t^{\star}) = 0 \tag{93}$$

with '$t^{\star}$' denoting the time when the evaporation is completed.

Indeed, on the account of the to-be proposed '*inside-outside duality*', there is '*another Hilbert space*', which although *is defined all-independently* of the triplet '$(\mathcal{H}^{(t)}_{\text{Hawking Modes}}, \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}, \mathcal{H}^{(\text{Minimum})}_{\text{Total}})$', and does not belong to it directly, has the exact same dimension at any time during the entire evolution of the horizon. Obviously, by the first statement of holography [25], this 'definition-wise independent' Hilbert space is that which defines *the microstates of the black hole in the 'interior' of its causal horizon.* That is, one has this all-time valid holographic equality of dimensions,

$$\dim[\ \mathcal{H}^{(t)}_{\text{Horizon Interior}}\ ] = e^{N(t)} = \dim[\ \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}\ ] \quad ; \quad \forall t \tag{94}$$

This holographic equality of dimensions for two independently-defined Hilbert spaces which belong to *a pair of 'causal-complementary' regions of the same global spacetime*, signals that they may be also a much deeper physical connection between them. This anticipation turns out to be indeed credential, and all that it takes to unfold its physical statement is to pinpoint yet another instrumental role played by *holography* for the physics of horizons. *The central holographic point* is that we have a setting in which *one same 'holographic screen' [25]* is commonly shared by *the two "geometrically-regional" quantum theories" that are defined by* '$\mathcal{H}^{(t)}_{\text{Horizon Interior}}$' *and* '$\mathcal{H}^{(t)}_{\text{Bobby Causal Curvature}}$'. On one hand, the quantum system of '*black hole microstates*' living on '$\mathcal{H}^{(t)}_{\text{Horizon Interior}}$' possesses as its *'system-defining' holographic screen* the causal boundary of the '*horizon interior*', which is identical to its 'region defining' topological boundary, namely '*the horizon surface*'. On the other hand, *Bob's 'TOE'* which includes the complete quantum gravitational physics of his causal patch is also an intrinsically-holographic theory [27]. Namely, Bob's 'TOE' which lives on '$\mathcal{H}^{\text{Bob}}_{(\text{TOE})}$' introduced before, is defined based on the 'holographic screen' of any sufficiently-large causal diamond that accommodates the entire evolution of the horizon.

As Bob's holographic screen, whose area in Planck units must be at least of the order of '$N^{\frac{3}{2}}$', will be enormously larger that the '*primal horizon*' whose holographic size is 'N', we may initially think that it should have nothing to do with the primal horizon which is the holographic screen for the the theory of black hole microstates, albeit apart from an uninteresting 'big-theory sub-theory' relationship. *But in fact the correct physics is going to be fruitfully different* [33]. As it will be shown there '*from first principles*', one has a physically beautiful, meaningful, and well-defined sense in which *the entire Bob's 'TOE' is "perfectly reducible" in its black hole sector to the the very 'total system' of our quantum triplet, namely the 'minim-total theory' living on '$\mathcal{H}^{(\text{Minimum})}_{\text{Total}}$'* whose dimension has been set by the primal horizon as (89). Now, the '$\mathcal{H}^{(\text{Minimum})}_{\text{Total}}$ quantum theory' has in it a *subsystem theory* defined on '$\mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}$' which accounts for the '*curvature modes*' living in Bob's causal diamond. This 'subsystem theory', whose dimension is being holographically counted by the area in Planck units of its 'region-defining-topological causal boundary' as in (92), must be by itself a well-defined holographic theory. Obviously and unavoidably, *the 'system-defining' holographic screen for the '$\mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}$ theory' is the 'time-t' evolving horizon.* The clear conclusion is the following statement,

The two holographic quantum theories living on '$\mathcal{H}^{(t)}_{\text{Horizon Interior}}$' and '$\mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}$', being a pair of causally-independent quantum many body systems which nevertheless 'share' one same 'system-defining' holographic screen, are 'quantum dual' to one another, and so their exact Hilbert space physics must be 'duality-wise' mappable to one another. That is, the following statement of 'inside-outside duality' does exactly hold,

$$\mathcal{H}^{(t)}_{\text{Dual Horizon Interior}} \equiv \mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)} \quad ; \quad \forall t \tag{95}$$

Based on the all-time-exact 'outside-inside duality' (95), the 'internal degrees of freedom' that microscopically construct the (time-dependent) black hole system in the horizon interior, and the 'quantum curvature modes' in the causal patch of the asymptotic observer Bob, are 'mapped to', and are exactly '*reconstructable from*' 'one other', similar to [21]. One point to be highlighted here is that, this pair of systems in the inside and in the outside are always 'holographic-subsets' of the total microscopic degrees of freedom that define the 'primal horizon'. To have a more complete view of the foundations and physical discussions of the proposed 'inside-outside duality' we refer to the forthcoming study [33].

Now, *what about the open quantum system of the Hawking modes living in the 'Bobby bulk'?* Because the two quantum systems living on '$\mathcal{H}^{(t)}_{\text{Hawking Modes}}$' and '$\mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}$' are complementary pairs of our quantum triplet whose total-system dimension is '*statically*' fixed by (89), and given that the time-dependent dimension of '$\mathcal{H}^{\text{Bobby Causal Curvature}}_{(t)}$' is set by the equality (92), we learn that the total dimension of the 'Hawking-mode subsystem' evolves as follows,

$$\dim[\, \mathcal{H}^{(t)}_{\text{Hawking Modes}}\,] = \exp[N - N(t)] \equiv e^{N'(t)}$$
$$\text{With The Boundary Conditions :}$$
$$N'(0) = 0 \quad ; \quad N'(t > t^{\star}) = N = I_{\text{ Initial State}} \tag{96}$$

According to the time-dependent dimension (96), as the horizon evaporates, the subsystem of the released Hawking modes expands in population from '0' to a total number of '$I^{\text{ Initial State}}$' 'all-asymptotic modes' once the horizon becomes entirely annihilated. So, as being mirrored in (92,93,96), in the process of horizon evaporation, *the initial* '$I_{\text{ Initial State}}$' *Bobby curvature modes* (which themselves are manifested as 'out-projections' of the horizon modes by the holographic map) *become 'one-by-one' converted into* the 'lastly-all-asymptotic' '$I_{\text{ Initial State}}$' *Hawking modes* as the '*final state*' of the system. The extremely important point is that, as was 'holographically proved' in the previous section, *during this holographic 'conversion of the modes' the initial-state bits of information are all indestructibly preserved, and so are being 'unitarily processed' as the horizon evaporates.* Because of this '*holographic unitarity*', the '*final state*' of this evolving system will be in a *pure quantum state*, that is, in terms of its von Neumann entropy it is characterized by the '*purity condition*',

$$S^{\text{Final State}}_{(\text{vonNeumann})} = 0$$
$$I^{\text{ Initial State}} = I^{\text{ Initial State}} \tag{97}$$

Next, let us describe with some more physical detailing, how this '*purity of the final state*' of the 'Hilbert space triplet' is processed in the form of a quantum many body system of the '$N = I^{\text{ Initial State}}$' microscopic modes (which by being holographically projected-out) live in the 'Bobby causal bulk'. We ask this same question both '*regional-wise*', namely in terms of the spatial distribution of the modes, and '*era-wise*', namely in terms of the temporal phases of the quantum evolution of the horizon.

As a highlighting foreword to the following discussion, to be holographically 'in good shape', we better think of the '*primal horizon*' as a causal boundary which serves the '$\mathcal{H}_{\text{Total}}^{(\text{Minimum})}$*-theory*' as its *'system-defining' holographic screen*, but here more like an *"unstable brane"*. Understood this way, all the '$N = I^{\text{ Initial State}}$' microscopic modes that constitute this 'minimum-total theory' are *fundamentally and identically* the degrees of freedom of the 'primal-horizon system', which upon being projected-out into the Bobby causal patch by the 'holographic map', manifest as the Bobby curvature modes or the Hawking modes, albeit *in an 'unitarily time-dependent', 'interchanging' manner.*

Let us begin with describing how the projected distribution of the microscopic modes goes '*Regional-wise*'. The Hilbert space triplet '$(\mathcal{H}_{\text{Hawking Modes}}^{(t)}, \mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}, \mathcal{H}_{\text{Total}}^{(\text{Minimum})})$' identifies a total number of 'N' microscopic degrees of freedom in the Bob's causal patch which are '*subsystem-wise*' *distributed* among its interacting, simultaneously evolving, complementary quantum subsystems '$\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}$' and '$\mathcal{H}_{\text{Hawking Modes}}^{(t)}$' in a 'unitarily-driven' '$(N(t), N'(t) = N - N(t))$'-profile. *The point* is that the (holographically projected) '*regional localization*' of these 'N' degrees of freedom is done in a way that characteristically distinguishes between the modes distributed in the two complementary subsystems. Let us see this characteristic difference. The '$\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}}$-subsystem' is constituted from the modes that by definition build up the curvature of the Bob's causal patch. So, simply by their physical identification, it is crystal clear that those modes must have a *'radially-inhomogeneous' distribution in the 'Bobby bulk'.* That *the localization of the '$N(t)$' Bobby curvature modes* must be regionally marked with a *'radially-decaying' profile of the 'mode-density'* is manifested by the 'output-fact' that the *curvature-invariants* of the spacetime (which must be 'directly built-up by them) are *radially enhanced by nearing the horizon.* The most familiar example of this behavior is the Kretschmann invariant of Schwarzschild spacetime with a '$1/r^6$-falling' profile. Moreover, the distribution of the Bobby curvature modes must be *'dominantly' concentrated in a 'localization band*', being an 'order-one subregion' of a so-named *"horizon nabe"* whose independent definition will be soon given in what follows. Not only the rapidly-decreasing power-law profile of the curvature invariants shows this 'horizon-nearby dominance' of the Bobby curvature modes, but also we will see a re-confirmation of this fact in the discussion below on the semiclassical finite-distance spectrum of the Hawking modes.

In contrast, the '$\mathcal{H}^{(t)}_{\text{Hawking Modes}}$-subsystem' is composed of the '$N'(t) = N - N(t)$' released Hawking modes which are dynamically classified into *two 'sub-groups of modes'* with '*complementary regional localization', and also distinct physical behavior.* To clearly see this inter-grouping of Hawking modes in our observer-centric approach, we must go back to the principal analysis of the part 'A', but this time redo it for *an observer Bob who is located in an 'arbitrarily finite' distance from the horizon.* By the 'geometric mechanism' with which these modes become semiclassically '*jumbled*' while propagating in the 'Bobby bulk', the following fact is clear. *Given a spacetime with a 'null causal horizon', there must be a 'finite' 'crossover scale' "R_C", so that the emitted Hawking modes which propagate-away by distances "crossover-wise beyond" this characteristic scale are observed by the local observer Bob to be (semiclassically) 'Planckian dispersed'* (up to unimportant finite-size corrections). In the "mono-scale" Schwarzschild spacetime, 'R_C' can only depend on the horizon radius, and therefore it must be simply proportional to it, namely, '$R_C \propto R_H \propto N^{\frac{1}{2}}$'. But even in "multi-sclae spacetimes", it is still obvious that, *the most dominant dependence of 'R_C'* must be carried by the horizon scale in the same linear manner. Moreover, being mainly a result of the 'extremely dominant' role that *the 'exponential-redshifting virtue' of the null horizon* plays in this semiclassical dispersion, it can be shown that (modulo all the sub-leading dependences), '$R_C \sim a \cdot R_H \sim a \cdot l_p\, N^{\frac{1}{2}}$', with '$a$' being a constant, typically of 'order-one'. To fix 'a', we will need the microscopic theory, but its value is not important to us. Let us name the '*over-horizon region*' '$R \lesssim R_C$' "*the (horizon) nabe*", and so the region beyond it, "*the asymptotic territory*". Now, the fact that the semiclassical dispersion of Hawking modes is being developed already in the 'nabe', does reconfirm that the 'Bobby curvature modes' are also concentrated in the nabe. So, we conclude that *'the dominant' 'regional distribution' of the modes of the quantum triplet* is of the form,

$$\mathcal{H}_{(t)}^{\text{Bobby Causal Curvature}} \approxeq \mathcal{H}_{(t)}^{\text{Nabe Curvature Modes}}$$

$$\mathcal{H}^{(t)}_{\text{Hawking Modes}} \approxeq \mathcal{H}^{(t)}_{\text{Nabe Hawking Modes}} \otimes \mathcal{H}^{(t)}_{\text{Asymptotic Hawking Modes}} \tag{98}$$

'*Era-wise*', the quantum triplet system begins its unitary evolution in the '*pre-Page-time*' during which an enormous '*environment*' of the nabe curvature modes interacts with the nabe Hawking modes. Then at a time scale of about the '*the Page time*' [31], the system turns to its '*post-Page-time*' era during which the nabe curvature modes become a small system of "interacting *defects*" among the released Hawking modes in the nabe.

The system evolves *unitarily*, until by a time scale of the order '$N^{\frac{3}{2}}$', it ends up with '$N = I^{\text{Initial State}}$' asymptotic Hawking modes at Bob's disposal, defining a *'pure' 'final state'*. Recollecting and joining all the stated points so far, we obtain *the following structure for "the triplet Bobby quantum many body system" that holographically formulates the unitary evolution of the quantum-evaporating causal screens*,

$$\begin{aligned} \mathcal{H}^{(\dim=e^{N})}_{\text{Minimum Total}} &= \mathcal{H}^{(\dim=e^{N(t)})}_{\text{Dual Horizon Interior}} \otimes_{(t)} \mathcal{H}^{(\dim=e^{N-N(t)})}_{\text{Hawking Modes}} \\ \approxeq\ \mathcal{H}^{(\dim=e^{N(t)})}_{\text{Nabe Curvature Modes}} &\otimes_{(t)} \mathcal{H}^{(\dim=e^{N-N(t)-N_R(t)})}_{\text{Nabe Hawking Modes}} \otimes_{(t)} \mathcal{H}^{(\dim=e^{N_R(t)})}_{\text{Asymptotic Hawking Modes}} \end{aligned} \tag{99}$$

This 'separation of scales' which is seen in the holographic projection of the microscopic modes into the 'Bobby bulk' must have a very meaningful reflection back on their defining root system, namely the quantum many body system of the primal horizon, which calls for further research. Clearly, the detailed interactions and the exact unitary dynamics of (99) is still to be discovered. *This mission will begin to be developed from first principles by* [33].

VII. A BRIEF CONCLUSION

By an observer-centric methodology, we clarified that Hawking evaporation is protected against any loss of the information bits, unless we 'holographic-illicitly' configurate the system in the strict thermodynamic limit of the Bobby curvature modes. So, the correct *'way-out'* of the information paradox turns out to be just the same as the correct *'way-in'* to the very foundation of quantum gravity, that is, '*the way of holography'*. Based upon this, we elucidated *the physics of a microscopic holography* that successfully realizes the *unitarity of information processing* in quantum evolution of causal horizons.

However, the corresponding *'triplet quantum many body system'*, summarized in (99), clearly needs to be advanced to become a complete microscopic description. In particular, still there are *three main aspects* of this microscopic system to be known. First, *the detailed 'microscopic interactions' between all the different sets of modes* needs to be fixed. Second, *the 'exact unitary operator' that drives the dynamics of the entire system, together with the individual 'dynamics of each one of the evolving subsystems'* must be determined. Third, we still need to unfold *how the 'purity of the final state' of the system is being 'imprinted' on the 'fine-grained multimode quantum correlations' of the Hawking modes.*

As a hint forward, by the 'holographically-driven' way in which Bobby curvature modes are being dynamically 'converted into' the Hawking modes, and also by their holographic origin as the 'identical' microscopic degrees of freedom of the 'many body system of the primal horizon' as the 'Holographic screen', it is very natural to anticipate that, in the triplet quantum system there must be an underlying scheme of *'impartial physical unification' between all those sets of modes.* Namely, *the 'fundamental' microscopic description of the system must be such that all the modes are being physically considered as identical entities.* Moreover, both from the principal methodology of this study, and from several independent quantum gravitational reasonings, it becomes transparent that *this fundamental holographic microscopic formulation must be 'observer-centric' in its very 'conception and formulation'.* Indeed, it becomes transparent to us that by accommodating our triplet quantum system into the following '*quintet quantum many body system*',

$$\Big(\mathcal{H}_{\text{Hawking Modes}}, \mathcal{H}^{\text{Bobby Causal Curvature}}, \mathcal{H}^{\text{(Minimum)}}_{\text{Total}}, \mathcal{H}^{\text{Bobby Environment}}, \mathcal{H}^{\text{Bob}}_{\text{(TOE)}} \Big)$$

and further by *'axiomatically' following a specific set of foundational physical principles*, one can indeed accomplish *the holographic fundamental unified theory* as envisioned in here [33].

Acknowledgments

Javad Koohbor and Mohammad Nouri-Zonoz would like to thank University of Tehran for supporting this research project under the grants provided by the research council. Mohammad Nouri-Zonoz also thanks the Albert Einstein Center for Fundamental Physics, University of Bern, for kind hospitality and supporting his visit during which part of this study was carried out. Alireza Tavanfar thankfully acknowledge the support of his research by the Brazilian Ministry of Science, Technology and Innovation (MCTI-Brazil). Alireza Tavanfar would also like to thank CERN Physics Department, where part of the ideas in the part B of this research were initially developed, for both hospitality and support.

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