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LETTER TO THE EDITOR

Trans–Planckian modes, back–reaction, and the Hawking process

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Abstract. Hawking's prediction of black–hole evaporation depends on the application of known physics to fantastically high energies — well beyond the Planck scale. Here, I show that before these extreme regimes are reached, another physical effect will intervene: the quantum backreaction on the collapsing matter and its effect on the geometry through which the quantum fields propagate. These effects are estimated by a simple thought experiment. When this is done, it appears that there are no matrix elements allowing the emission of Hawking quanta: black holes do not radiate.

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Hawking (1974, 1975) famously studied quantum fields in the presence of a gravitationally collapsing object, and predicted that black holes are not in fact black, but radiate with thermal spectra and eventually explode. This work raised many issues, and forced the development of far deeper understandings of quantum field theory in curved space–time than had previously been achieved. Whether black holes radiate or not, these deeper foundations seem secure. It is no exaggeration to say that almost every important paper in the past twenty–five years on quantum field theory in curved space–time has Hawking's papers as antecedents.

The strength of the argument for black–hole evaporation is at best dubious. The main difficulty, which was recognized almost from the outset, is that Hawking's analysis relies on the application of known physics to fantastically high energies, well beyond the Planck scale. In spite of this, the prediction of black–hole evaporation has often been regarded as a cornerstone of the theory, to such an extent that attempts to quantize gravity have been judged by whether they can reproduce Hawking's prediction in appropriate regimes (Rovelli 1996, Strominger and Vafa 1996, Balbinot and Fabbri 1999, Kummer and Vassilevich 1999; see however Belinski 1995 for another view). So it is worthwhile reconsidering Hawking's argument, not just for itself, but for its implications for more ambitious theories.

The kernel of the trans–Planckian problem is this. The predicted spectrum corresponds to a temperature $T_{\mathrm{H}} = \hbar c^3/8\pi Gm$ (where m is the mass), and so to wavelengths of characteristic size $\sim 8\pi Gm/c^2$. However, the field modes from which these arise have been exponentially red–shifted as the modes propagate away from the collapsed object, so that the original wave–lengths they corresponded to, in the distant past, are $\sim (Gm/c^2)\exp -u/(4m)$, where u is the retarded time. For a solar–mass black hole, the scale $4Gm/c^3$ is on the order of milliseconds, so in a fraction of a second the energies of the modes have passed any known physical scale — including the mass of the universe!

No resolution of the trans–Planckian problem is known. There have been a number of ingenious investigations which have supposed that the effect of trans–Planckian frequencies on the field theory might be modeled by a dispersive propagation of the quantum field (Jacobson 1991, 1993, Brout et al. 1995, Unruh 1995). However, all of these are guesses at what physics at very high energies will be, all significantly alter the propagation assumed by Hawking, and none seems able to reproduce Hawking's result without introducing trans–Planckian wave–numbers. It has also sometimes been suggested that, because of the very beautiful form of Hawking's results, there ought to be some sort of general invariance arguments leading to them, independent of trans–Planckian physics. However, at present no such argument is known.

In this letter, I will show that before the trans–Planckian regime is reached, another physical effect will have to be considered: the quantum back–reaction on the collapsing matter, and its effect on the space–time geometry through which the quantum field propagates. That this should need to be considered is at first surprising, since for almost all purposes gravitational fields can be adequately computed by considering their sources as classical objects. But the Hawking mechanism is not ordinary physics. It is a tiny quantum effect, and it turns out to be crucially influenced by quantum complementarity issues involving the geometry and the collapsing matter.

In what follows, I shall consider only the case of spherical symmetry explicitly, but it will be evident that the physical arguments are quite general and should apply more broadly. The conventions are those of Penrose and Rindler (1984–6), and of Schweber (1961). The metric has signature $+---$. For the most part, factors of G, $\hbar$ and c are given explicitly, but these are omitted where the expressions become too cumbersome.

In the space–time exterior to the collapsing body, we have the Schwarzschild metric in the familar form

$$\mathrm{d}s^2 = (1-2m/r)\mathrm{d}t^2 - (1-2m/r)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2\,, \tag{1}$$

and we also make use of the advanced and retarded time coordinates

$$v = t + r_* \qquad \text{and} \qquad u = t - r_*\,, \tag{2}$$

where $r_* = r - 2m + 2m\log((r-2m)/2m)$.

The general scheme of Hawking's computation is this. A massless quantum field $\hat{\phi}$ is to be investigated on the space–time corresponding to a gravitationally collapsing object. The state of the field in the far past is quiescent, say vacuum for simplicity. In principle, then, one should work out the field operators in the far future in terms of those in the far past, and from this one can read off the particle–content, stress–energy, etc., of the state in the far future.

The leading contribution to the field operators in the future far from the object and after collapse has substantially occurred will be the geometric–optics approximation. We may write this as

$$\hat{\phi}^0_{\mathrm{f}}(u) = \hat{\phi}^0_{\mathrm{p}}(v(u))\,, \tag{3}$$

where the subscripts f, p stand for future and past, the superscript indicates that the fields have been conformally rescaled to attain finite limits at $\mathcal{I}^{\pm}$, and $u \mapsto v(u)$ is the mapping of surfaces of constant phase, from $u = \text{const}$ in the future to $v = v(u)$ in the past.

Equation (3) is remarkable in that it has the same form as a "moving mirror" model: a massless field propagating in two–dimensional Minkowski space and reflecting from a perfect mirror whose trajectory is given by $v = v(u)$ in null coordinates (Fulling and Davies 1976, Davies and Fulling 1977). (The moving mirror models were developed after Hawking's work, however, and in part to help understand it.) Using standard formulas from these models, we may read off the particle–content and stress–energy. We find

$$\langle \widehat{T}^{\mathrm{ren}}_{uu}\rangle = (12\pi r^2)^{-1}\hbar\left(-(1/2)(v''/v')' + (1/4)(v''/v')^2\right)\,. \tag{4}$$

Note that the key quantity that enters is the fractional acceleration v''/v'. This is the center of the Hawking mechanism. The aspect of the space–time geometry which controls the Hawking mechanism is v''/v', and measurements of Hawking quanta are essentially probes of this geometric quantity.

According to classical collapse theory, one has, at late retarded times

$$v(u) \sim -(Gm/c^3)\exp -u/(4m)\,, \tag{5}$$

where the prefactor Gm/c^3 has been inserted for dimensional reasons only; one has no control over the constant in front of the exponential at this level of analysis. From this relation and equation (4), one immediately has $\langle \widehat{T}^{\rm ren}_{uu}\rangle = (48\pi r^2)^{-1}\hbar(c^3/4Gm)^2$. This is one of Hawking's predictions, and others can be similarly recovered.

The relation (5) is the one which gives rise to the trans–Planckian problems. But exponential relations like (5) are never accepted unreservedly in physics. While they may apply within a particular model, no model holds at arbitrarily small scales, and eventually one must ask what aspect of the model breaks down. Here we shall see that there are quantum limitations.

We shall show that it is necessary to consider the quantum back–reaction on the collapsing matter and the space–time geometry. Since this is precisely a question of how geometry is affected by quantization, and we have at present no reliable theory of quantum gravity, any investigation along these lines involves some speculation. However, here we are not concerned with gravitational fields which are locally very strong in any invariant sense, nor (it will turn out) with Planck–scale physics, so it seems reasonable that we should be able to apply conventional physical principles to understand them.

Accordingly, we shall consider a simple thought–experiment to measure v''/v'. The idea is direct. We imagine sending massless particles of a given frequency into the collapsing body, slightly before the point where they would inevitably be captured. The particles emerge, and we measure their red–shifts, or more precisely certain ratios of these, to compute v''/v'. See figure 1.

In this experiment, and in any real experiment, one does not measure v''/v' at an instant; one measures an average of it over some finite time. For our present purposes, the time scale of interest is $\sim Gm/c^3$, the characteristic time for the emission of a Hawking quantum. If v''/v' is to be measured over such a time, then the quanta used must, on arrival, have frequencies $\gtrsim c^3/(Gm)$. However, this means that the initial quanta must have had frequencies of order $\gtrsim (c^3/Gm)(v')^{-1} \sim (c^3/Gm)\exp +u/(4m)$. We thus very quickly pass a point where

$$(\hbar c^3/Gm)(v')^{-1} \gtrsim mc^2\,, \tag{6}$$

that is, where the energies of the incoming quanta must exceed that of the collapsing object. By this point, measurements of v''/v' of sufficient precision to correspond to the geometry determining the Hawking phenomenon would require disturbances of the energy of the same order as the energy of the collapsing object itself, and the whole notion of probing a background geometry has clearly broken down.

It is precisely the exponential increase in the red–shift which signals the approach to the horizon. So this quantum limitation seems to probe the limits of distant experiments to reveal the geometry of a black hole.

This argument suggests that there is a quantum complementarity between: (a) measurements of v''/v' to the precision which detection of Hawking quanta would imply; and (b) the total energy of the collapsed object. Analyzing a thought experiment can never *prove* a complementarity, of course, but in this case the experiment is so natural, and seems so much to go to the heart of black–hole formation, that we consider the suggestion a very strong one. It is also possible to give a mathematical corroboration of this, using canonical quantization. This will be done elsewhere.

This argument implies that black holes do not radiate. This is because detection of Hawking quanta would be equivalent to a measurement of v''/v', which could not be accomplished without giving rise to a large spread in the energy of the hole, a macroscopically large spread. Hawking radiation is then forbidden by conservation of energy: there are no matrix elements available for transition from the state of the collapsing hole to a state with emitted quanta, as that final state must involve a too–large spread in the energy of the hole.

This state of affairs is parallel to the familiar one for transitions in elementary quantum systems. If a Hamiltonian H_0 is perturbed by a term ΔH which is considered to give rise to transitions, then the final H_0–width is typically of the order of $\Delta E_{\rm rms} = \sqrt{(\langle(\Delta H)^2\rangle - \langle\Delta H\rangle^2)}$ (since it is an H–eigenstate). In the present case, because the collapsed object is very nearly at the black–hole state, there is no way of mining enough energy from it to give it a width sufficient to accomodate the production of a Hawking quantum. All of the internal energy of the object is red–shifted almost infinitely relative to infinity, and, since the object is very nearly a black hole, the potential energy available to infinity is nearly exhausted.

These considerations are essentially new, and are not addressed by earlier analyses on conservation of energy in the Hawking process. Those have been of two sorts. First, as Hawking pointed out, quantum field theory predicts a negative flux of energy across the event horizon which counterbalances the energy carried off by Hawking quanta. While true, this simply does not address what accomodation the quantum state of the collapsing matter must make to the emission of the radiation. In the analogy of the previous paragraph, it is the statement that $\langle\Delta H\rangle = 0$.

The second sort of energy conservation which has been considered has been a semi–classical approximation, $R_{ab} - (1/2)Rg_{ab} = -8\pi G\left(T_{ab}^{\rm classical} + \langle\widehat{T}_{ab}^{\rm ren}\rangle\right)$. However, such approximations have as their hypothesis that the quantum character of corrections can be ignored.

To summarize: We have used a simple physical model to investigate the nature of the effects of the quantum back–reaction on the Hawking process. This model strongly suggests that there is a quantum complementarity forbidding simultaneous measurement of (a) that aspect of the geometry of space–time controlling the production of Hawking quanta and revealed by their detection, and (b) the energy of the collapsing object. This complementarity becomes significant at a scale

$$v' \sim (m_{\rm Pl}/m)^2 \,, \tag{7}$$

where $v'(u)$ is the red–shift factor for massless particles arriving at retarded time u (from equation (6)). This point is rapidly reached in the collapse process, and beyond it the emission of Hawking quanta is forbidden by energy conservation.

In this Letter, I have doubted that Hawking's prediction of thermal radiation from black holes is correct. Yet I hope that I have made my debt to, and admiration for, his work clear.

I thank Ted Jacobson for bringing the problem of trans–Planckian modes to my attention, and for patient answers to questions.

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Figure captions

Figure 1. Conformal diagram showing the experiment to measure the fractional acceleration v''/v'. The collapsing body is to the left (bounded by the dotted line). Massless particles of given frequency are directed into the body, and ratios of their red–shifts are measured in the distant future. As the event horizon (represented by the dashed line) is approached, the red–shifts increase exponentially, necessitating exponentially increasing initial frequencies.