

Secular evolution of galaxies and galaxy clusters in decaying dark matter cosmology

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If the dark matter sector in the Universe is composed by metastable particles, galaxies and galaxy clusters are expected to undergo significant secular evolution from high to low redshift. We show that the decay of dark matter, with a lifetime compatible with cosmological constraints, can be at the origin of the observed evolution of the Tully-Fisher relation of disk galaxies and alleviate the problem of the size evolution of elliptical galaxies, while being consistent with the current observational constraints on the gas fraction of clusters of galaxies.

Introduction. In the standard Λ cold dark matter (Λ CDM) model the dynamics of the Universe in the present epoch is dominated by dark matter (DM) and dark energy. The DM, which plays a crucial role in the growth of structure, does not have appreciable interactions with radiation, and cannot be in the form of ordinary baryonic matter as deduced from considerations of big-bang nucleosynthesis together with observations of the anisotropies in the cosmic microwave background (CMB).

The Λ CDM model further assumes that the DM particles are stable, and that any interactions have a negligible effect on the cosmological evolution. This model is extremely successful at reproducing the matter distribution on Mpc scales and above. However, on the smallest scales, where galaxies are formed, the simulations of structure growth seem to disagree with the observations of real galaxies [2]. Numerical simulations of dark halo formation in the Λ CDM cosmology result in highly cusped density distributions and abundance of substructure [3]. In contrast, the number of satellite dwarf spheroidal galaxies identified in the local group is lower than the number of subhalos found in simulations [4], and there is evidence that the halo of the Milky Way is not cusped at all [5]. Also, the recent findings that all dwarf spheroidal galaxies in the Milky Way, ranging in luminosity by more than 4 orders of magnitude, have DM halos with a common mass ($\sim 10^7 M_\odot$), do not follow from the standard CDM scenario [6]. At larger scales, recent high-redshift surveys indicate a structural and kinematical evolution of galaxies, which has not obvious explanation in the standard Λ CDM structure formation scenario. In particular, there is growing evidence that the Tully-Fisher (TF) [7] relation for disc galaxies (relating their stellar mass, M_* , and rotation speed) evolves with redshift, z , in the sense that higher- z galaxies have higher rotation speeds than local galaxies of similar M_* [8, 9]; in addition, elliptical galaxies of given M_* are found to be more compact at higher z [10].

Astrophysical processes, undoubtedly important, could alleviate the tension between (sub-)galactic struc-

ture observations and the predictions from simulations [11]. In particular, baryonic processes might affect significantly disc galaxies, which at high redshift are characterized by high star formation rate [8]. Elliptical galaxies are found to be quiescent at $z \sim 2$, but several mechanisms have been proposed to explain the size evolution of elliptical galaxies in the context of standard Λ CDM [12]. However, given the dominant role played by DM in the growth of structure, a compelling possibility is that the DM particles are not as cold, stable, or inert as it is assumed in the Λ CDM scenario. Warmer or strongly self-interacting DM particles produce more constant density cores with a higher minimum mass for DM haloes [13], although the absence of low mass haloes at large redshift would affect the reionization history [14]. However, since the formation of structure at larger, galactic, scales coincides with the CDM scenario the issues with the fast evolution of early-type galaxies would remain.

Forfeiting the stability assumption, if the particles composing the DM sector decay with a lifetime of a few tens of Gyr (consistent with observations as discussed below), the formation of structure would proceed until recently, $z \gtrsim 1$, as in the Λ CDM model, successfully reproducing the matter distribution on Mpc scales and above [15]. Interestingly, the depletion of a fraction of the DM due to subsequent decays affects the evolution of structure at galactic and subgalactic scales [15–18]. Cen [16] discussed the possibility that decaying dark matter (DDM) could solve the problem of the central cusp in DM halos and the overabundance of dwarf galaxies, but he also indicated some astrophysical implications of this scenario on the scale of galaxies and clusters of galaxies that could be tested observationally. In this paper, we explore whether the recent observations of the evolution of disc and elliptical galaxies from $z \sim 2$, and the available data on the gas fraction of clusters from $z \sim 1$ can be fit within a DDM scenario consistent with recent cosmological constraints.

Viable DM candidates with lifetimes on cosmological scales frequently appear in particle physics models be-

yond the standard model. In fact, much like CP is not a symmetry of nature, there is no fundamental reason that requires the discrete symmetries usually associated with the stability of DM, such as R-parity in supersymmetric extensions of the standard model, to hold. For instance, in supersymmetric models where a gravitino is the lightest supersymmetric particle, a subset of R-parity violating couplings allows a fast enough decay of the next-to-lightest-supersymmetric particle to avoid conflicts with big-bang nucleosynthesis predictions, without inducing the decay of the proton at an unacceptable rate [19]. Like in the R-symmetric case, the required relic density can result from thermal processes, and, in some scenarios, its decay products could be observed in cosmic-ray detectors [20]. Also, DDM has been recently linked to the origin of neutrino masses [21].

Model. Let us consider DM particles that decay into relativistic particles with a characteristic lifetime τ . We will assume that the decay products are electromagnetically noninteracting so that only the cosmological evolution is modified with respect to the standard Λ CDM model, through the coupling between the matter and the radiation sectors. In particular, observations of the anisotropies in the CMB put a stringent lower bound on τ , since DDM affects the evolution of the cosmological perturbations at late times enhancing the integrated Sachs-Wolfe effect [22]. Using first year WMAP observations the authors in [23] conclude that a particle making the whole of the DM should have a lifetime $\tau \gtrsim 52$ Gyr (95.4% C.L.). A recent study combining the latest WMAP data, together with type-Ia supernovae, large scale structure and weak lensing observations, strengthens the limit to $\tau \gtrsim 100$ Gyr [24]. However, some of these datasets have been analyzed with the assumption that the Λ CDM model is correct, which could bias comparisons between models with a markedly different cosmological evolution [25]. Moreover, the fits to the CMB datasets assume that the primordial fluctuations are purely adiabatic, while a sizable admixture of isocurvature perturbations is allowed by observations [26]. In that case, the power at low multipoles could be reduced further weakening the constraints from the integrated Sachs-Wolfe effect. In view of these facts, we will consider two benchmark models: a DDM model with $\tau \sim 52$ Gyr, and a more conservative scenario with $\tau \sim 100$ Gyr.

The decay of DM couples its time evolution with that of radiation (r):

$$\begin{aligned} \dot{\rho}_r + 4H\rho_r &= \frac{1}{\tau}\rho_{\text{ddm}} \\ \dot{\rho}_{\text{ddm}} + 3H\rho_{\text{ddm}} &= -\frac{1}{\tau}\rho_{\text{ddm}}, \end{aligned} \quad (1)$$

while $\rho_b \propto a^{-3}$, and $\rho_\Lambda = \rho_\Lambda^0$ as in the Λ CDM model, and $H^2 \equiv \dot{a}/a = 8\pi G/3(\rho_b + \rho_\Lambda + \rho_r + \rho_{\text{ddm}})$ is the expansion rate. The superscript 0 refers to a quantity at present, and $a^0 = 1$. Integrating Eq. (1)

we find the look-back time corresponding to a redshift z , $t_0 - t_1 = 1/H_0 \int_{-\log(1+z)}^0 d\eta / \sqrt{\rho/\rho_{\text{crit}}^0}$, where $\eta \equiv 1/a = -\log(1+z)$. To make contact with observations, we also need the luminosity distance, $d_L(z) = (1+z)/H_0 \int_{-\log(1+z)}^0 e^{-\eta} d\eta / \sqrt{\rho/\rho_{\text{crit}}^0}$, which is related to the angular distance by $d_A = d_L/(1+z)^2$.

In this framework, we want to study the secular evolution of galaxies and galaxy clusters due to the decay of DM. In the absence of other evolutionary effects, from high to low redshift, galaxies should expand and their rotation speed should diminish, and the gas mass fraction in clusters should increase, as a consequence of the variation of mass in DM. Interestingly, the recent observations of the evolution of the TF relation for disc galaxies, and the findings that elliptical galaxies of a given M_* are found to be more compact at higher z , appear qualitatively in agreement with this expected trend. On the other hand, there is no evidence for an evolution with z of the gas fraction f_{gas} of clusters, though—as we show below—current data cannot exclude a variation of f_{gas} of the order of $\sim 20\%$ at $z \lesssim 1$.

Given that we consider particle lifetimes τ longer than the age of the Universe, we can safely assume that structure formation at high z is unaffected by the decay. Once a DM halo has decoupled from the Hubble flow and virialized at some time t_i , its structure and dynamics will be affected by the DDM density evolving as

$$\rho_{\text{ddm}}^h(t) = \rho_{\text{ddm}}^h(t_i) \exp[-(t - t_i)/\tau]. \quad (2)$$

Quantitatively, Cen [16] considered the case in which one-half of the DM particles decay into relativistic particles from the time of halo formation to the present: the depletion of such a high fraction of DM corresponds to too short a lifetime, $\tau \lesssim 19$ Gyr. Here, we reconsider the astrophysical consequences of DDM by assuming lifetimes consistent with recently determined cosmological constraints, comparing the predictions of the DDM model with state-of-the-art observations of galaxies and galaxy clusters.

Results. On the scale of galaxies the decay of DM has the effect of an adiabatic expansion of the DM halo, because τ is much larger than the galactic dynamical time [16]. To explore quantitatively the consequences of this process on the structure and kinematics of galaxies we have performed N-body simulations of the evolution of the collisionless systems representing the galactic DM distribution, taking into account the finite lifetime of the DM particles. We adapted the parallel N-body code FVFPS [27], so that the mass of each DM particle in the N-body simulation varies in time according to the exponential decay law: given the collisionless nature of the simulated system, this is an effective way to model the decrease of the number of DM particles due to the decay. We assume that at some initial redshift z_i the DM halo is in equilibrium, with isotropic velocity distribution

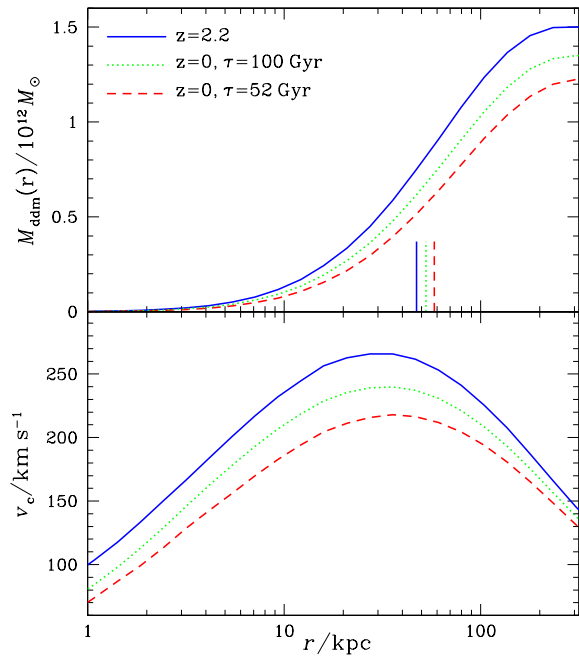


FIG. 1: Initial ($z = 2.2$) and final ($z = 0$) DM mass enclosed within radius r (upper panel) and circular speed (bottom panel) of the two DDM simulations with $\tau = 52$ and 100 Gyr. The vertical lines indicate the initial and final half-mass radius r_{50} . The initial distribution is the same in the two cases.

and a spherically symmetric Navarro, Frenk, and White [28] density distribution

$$\rho_{\text{ddm}}^h(r) = M_0 \frac{\exp\left[-(r/r_{\text{vir}})^2\right]}{r(r+r_s)^2}, \quad (3)$$

where r_s is the scale radius, M_0 is a reference mass, and we adopt an exponential cutoff to truncate the distribution smoothly at the virial radius r_{vir} : the total DM mass is $M_{\text{ddm}} = 4\pi \int_0^\infty \rho_{\text{ddm}}^h(r) r^2 dr$. We have performed several simulations with different choices of the parameters, but here we present results for a representative pair of simulations with 5×10^5 particles, $r_{\text{vir}}/r_s = 10$, $r_s = 15$ kpc and $M_{\text{ddm}} = 1.5 \times 10^{12} M_\odot$ starting at redshift $z_i = 2.2$ (which we choose to compare with observations). The two simulations differ only in the value of τ , which is 52 and 100 Gyr in each case. We have verified that the N-body system does not evolve significantly in the absence of DM decay. In Fig. 1 we plot the initial ($z = 2.2$) and final ($z = 0$) DDM mass profiles (upper panel) and circular speed (v_c) profiles of the simulated halo. In both simulations the final density distribution is very well represented by a spherical Navarro, Frenk, and White profile with $r_{\text{vir}}/r_s \simeq 10$. For $\tau = 52$ Gyr the final system has $\sim 18\%$ lower DM mass, 23% larger half-mass radius r_{50} and $\sim 22\%$ smaller maximum v_c than the initial system, while proportionally smaller variations are

found in the case $\tau = 100$ Gyr. Our $\tau = 52$ Gyr model at $z = 0$ is a Milky Way like DM halo, with total mass $\sim 1.2 \times 10^{12} M_\odot$ and $v_{c,\text{max}} \sim 216 \text{ km s}^{-1}$. Assuming that most of the galactic stellar mass is in place at $z = 2.2$ our results suggest that, in the absence of other evolutionary effects, a $z \sim 2.2$ disk galaxy with stellar mass similar to that of the Milky Way should have $\sim 22\%$ larger $v_{c,\text{max}}$ than the Milky Way (the variation in $v_{c,\text{max}}$ might be smaller if the baryonic mass contributes significantly within the radius at which v_c peaks). As DDM implies secular evolution for all galaxies, it predicts that the $M_*-v_{c,\text{max}}$ relation of galaxies must evolve with z . Assuming that the maximum circular velocity of the halo can be taken as a proxy of the maximum line-of-sight rotation speed of observed disk galaxies, the DDM scenario predicts a higher normalization of the TF relation of $\sim 22\%$ in rotation speed at $z \simeq 2.2$ than at $z = 0$ if $\tau = 52$ Gyr. Remarkably, the observationally estimated offset between the $z \sim 2.2$ and the $z \sim 0$ TF relation is $23\% \pm 6\%$ [8].

As outlined above, another secular evolution effect predicted by the DDM model is that galaxies should become more extended from high to low z . It is then tempting to try to explain in this framework the recent finding that early-type galaxies of similar M_* appear to have smaller half-light radius R_e at high z than at $z = 0$, with an increase in size of at least a factor of $\sim 2 - 3$ from $z \sim 2$ to the present [10]. To make predictions on the evolution of R_e in a DDM model, we ran a few simulations similar to those described above, but in which we allow for the presence of a stellar component embedded in the DDM halo. We found that R_e increases roughly proportionally to r_{50} , so we can safely use the results of our DDM-only simulations. Even our DDM with $\tau = 52$ Gyr predicts an increase in size of $\sim 24\%$ and thus cannot explain the observed dramatic increase in size. As in standard Λ CDM cosmology, also in DDM cosmology other effects must be invoked in order to explain these observations. Nevertheless, it is interesting that the problem of the size evolution of early-type galaxies is alleviated in DDM, especially given the possibility that the sizes of high- z galaxy could be underestimated in current observations [29].

The decay of DM has a straightforward implication for the evolution of the gas fraction f_{gas} in clusters of galaxies: in a cluster of fixed gas mass we expect the gas fraction to evolve in time as $f_{\text{gas}} \propto [1 + k \exp(-t/\tau)]^{-1}$, with k constant. In Fig. 2, we compare this prediction for $\tau = 52$ Gyr with the gas mass fraction measurements from [30], which represent the largest sample available with the smallest relative statistical error (median value of about 13%) and limited intrinsic scatter (the weighted mean scatter around the best-fit Λ CDM model is $\sim 7\%$). Considering that all the dependence on the cosmology can be written as $f_{\text{gas}} \propto d_L(z)^{1.5}$ [30, 31], we plot the values, originally estimated in a Λ CDM universe, corrected by the factor $d_{L,\text{ddm}}^{1.5}/d_{L,\text{ACDM}}^{1.5}$. We show that the ob-

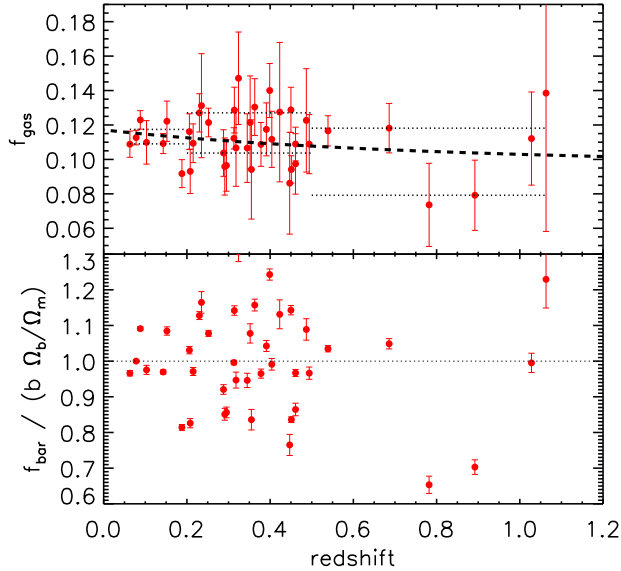


FIG. 2: *Upper panel:* Distributions of the gas mass fraction within an overdensity of 2500 with respect to the critical density of the Universe at the cluster’s redshift in a sample of massive, X-ray luminous and relaxed objects from Table 3 in [30]. The dashed line indicates the expected evolution in a DDM scenario normalized to the value that minimizes the total χ^2 in the distribution ($\chi^2 = 41.9$ with 42 data points). The dotted lines show the 1st and 3rd quartile of the distribution of the clusters at $z < 0.2$, $0.2 < z < 0.5$, and $z > 0.5$. *Bottom panel:* Distribution of the cluster baryon fraction, $f_{\text{bar}} = f_{\text{gas}} + f_{\text{star}}$, with respect to the expected cosmic baryonic value in a DDM scenario. To estimate f_{bar} , we consider a depletion parameter $b = 0.83$ and a contribution in stars of $f_{\text{star}} = 0.16h_{70}^{0.5} f_{\text{gas}}$ (as in [30]).

served distribution is consistent with the predicted variation as function of redshift of about 15% between $z = 1.2$ and $z = 0$. Moreover, as shown in the bottom panel of Fig. 2, we obtain that the estimated cluster baryon fraction is well in agreement with the expected cosmic value $f_{\text{bar}} / (b\Omega_{\text{b,ddm}}/\Omega_{\text{m,ddm}}) = 0.988 \pm 0.018$ (error-weighted mean and relative error; median: 0.995). Following [30], here we assume a depletion parameter $b = f_{\text{bar}}/(\Omega_{\text{b}}/\Omega_{\text{m}})$ within R_{2500} (i.e. the cluster radius encompassing a mean total density 2500 times the critical density of the Universe at the cluster’s redshift) of 0.83. This parameter represents the relative amount of cosmic baryons that are thermalized within the cluster potential. It can presently only be inferred from numerical simulations, where both the input cosmic baryons are known and the amount of them accreting into the cluster dark matter halo can be traced. When measured over representative regions of a galaxy cluster, this parameter is estimated to be lower than unity indicating that a minority of cosmic baryons are not shock heated during the process of accretion on the dark matter halo. However, as discussed in [31, 32],

the estimate of the depletion parameter b depends on many aspects of the hydrodynamical simulations investigated. For example, (i) grid-based, shock-capturing numerical codes tend to estimate a value b larger by up to 10 percent than what is measured in smoothed particle hydrodynamics; (ii) radiative processes, such as cooling and star formation with feedback provided from galactic winds, redistribute the baryons within the virial radius, in particular, in the cluster core, through the conversion of gas into stars and the replacement of the condensing gas with gas from the outer regions, with a net effect that reduces the gas mass fraction and increases slightly the total baryon fraction with respect to the runs where such processes were not considered; (iii) the evolution with cosmic time of b is expected to be very mild and less than 5 percent at $z = 1$ with respect to the local value. To mitigate, at least partially, these uncertainties, observers prefer to select only hot, massive, and relaxed galaxy clusters that are expected to be dominated energetically by gravitational collapse and should be well represented from simulations with no radiative processes included, as done in the present study by following the prescription in [30] (see also discussion on the estimate of b in [31]).

Conclusion. We have shown that the current observational constraints on the secular evolution of galaxies and galaxy clusters are consistent with a DDM cosmology with a lifetime larger than $\tau \sim 52$ Gyr, which is the shortest DM decaying timescale that can be reconciled with CMB observations. This finding is remarkable, because the cosmological constraints on τ are completely independent of the set of observations of galaxies and galaxy clusters at different redshifts used for comparison, so DM decaying with the shortest timescale allowed by cosmology could, in principle, give a redshift evolution of galaxies and galaxy clusters so strong to be excluded by current observations. Our results imply that a DDM scenario (with DM lifetime $\tau \gtrsim 52$ Gyr) is not disfavored with respect to the standard stable DM scenario, on the basis of astrophysical as well cosmological arguments.

We find it suggestive that DDM could explain naturally the observed redshift-evolution of the TF relation of disc galaxies, though this cannot be considered, *per se*, evidence that DM decays. In fact, in our exploration we isolated the effect of DM decay by neglecting other processes, such as baryonic physics and galaxy interactions, that —though poorly constrained— are expected to influence the dynamical evolution of disc galaxies.

The gas mass fraction in galaxy clusters can provide a robust test of DDM models once both the statistical errors and the intrinsic scatter can be constrained well below the expected change of the order of 15% in the redshift range 0 – 1.2. In addition, precise observations of ultrafaint dwarfs with a wide-field spectrograph such as Gaia or SIM Lite [33] will shed light on the DM distribution at subgalactic scales, where the DDM could also

play a distinctive role.

We are grateful to S. Räsänen and U. Sarkar for useful conversations. FF and CN acknowledge the hospitality of the Galileo Galilei Institute where part of this work was completed. SE acknowledges the financial contribution from Contract Nos. ASI-INAF I/023/05/0 and I/088/06/0. FF was partially supported by the NSF under Contract No. PHY-0855580 and by the DOE under Contract No. DE-FG02-91ER40628.

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