

# The Fate of a Five-Dimensional Rotating Black Hole via Hawking Radiation

Hidefumi NOMURA,<sup>1,\*</sup> Shijun YOSHIDA,<sup>1,\*\*</sup> Makoto TANABE,<sup>1,\*\*\*</sup> and  
Kei-ichi MAEDA<sup>1,2,3,†</sup>

<sup>1</sup>*Department of Physics, Waseda University, Tokyo 169-8555, Japan*

<sup>2</sup>*Advanced Research Institute for Science and Engineering, Waseda University,  
Tokyo 169-8555, Japan*

<sup>3</sup>*Waseda Institute for Astrophysics, Waseda University, Tokyo 169-8555, Japan*

(Received February 1, 2008)

We study the evolution of a five-dimensional rotating black hole emitting scalar field radiation via the Hawking process for arbitrary initial values of the two rotation parameters  $a$  and  $b$ . It is found that any such black hole whose initial rotation parameters are both nonzero evolves toward an asymptotic state  $a/M^{1/2} = b/M^{1/2} = \text{const}(\neq 0)$ , where this constant is independent of the initial values of  $a$  and  $b$ .

The conventional view of black hole evaporation is that, regardless of its initial state, Hawking radiation will cause a black hole to approach an uncharged, zero angular momentum state long before all its mass has been lost. For this reason, in some works, it is assumed that as a black hole evaporates close to the Planck scale, where quantum gravity is required to determine its evolution, the final asymptotic state is described by Schwarzschild solution.

However, Chambers, Hiscock and Taylor<sup>1)</sup> investigated, in some detail, the evolution of a Kerr black hole emitting scalar field radiation via the Hawking process, and showed that the ratio of the black hole's specific angular momentum to its mass,  $\tilde{a} = a/M$ , evolves toward a stable nonzero value ( $\tilde{a} \rightarrow 0.555$ ). This means that a rotating black hole will evolve toward a final state with non-zero angular momentum if there is a scalar field. In this Letter, we shall extend the analysis of Chambers, Hiscock and Taylor to a higher-dimensional case for the reasons described below. Considering the five-dimensional case specifically, we investigate the evolution of a five-dimensional rotating Myers-Perry (MP) black hole<sup>2)</sup> with two rotation parameters through scalar field radiation.

Recently, black holes in  $N (\geq 4)$  dimensions have attracted much attention. This is due to interest in the brane world scenario.<sup>3),4)</sup> From a phenomenological point of view, the most exciting possibility for the brane world scenario is that it might be possible to produce higher-dimensional mini-black holes in particle colliders, such as the CERN Large Hadron Collider (LHC), or to find them in cosmic ray events.<sup>5)</sup> A black hole produced in this manner would evaporate rapidly and emit many stan-

---

\*) E-mail: nomura@gravity.phys.waseda.ac.jp

\*\*) E-mail: shijun@waseda.jp

\*\*\*) E-mail: tanabe@gravity.phys.waseda.ac.jp

†) E-mail: maeda@gravity.phys.waseda.ac.jp

dard particles. Hence it would lose most of its mass and angular momenta through Hawking radiation<sup>6)</sup> or superradiance, which is intrinsic to a rotating or charged black hole.<sup>7),8)</sup> A few hot quanta emitted in the final Planck phase, which cannot be treated semiclassically, would not consist of the main part of the decay products.<sup>9)</sup> In most of the literature, the “spin-down” phase of black hole evolution, in which a black hole loses its angular momenta, is simply ignored, and a Schwarzschild black hole is assumed.

In generic particle collisions, however, the impact parameter will be non-zero. Therefore, most black holes produced in a collider would be rotating and could be described by a higher-dimensional MP solution,<sup>2)</sup> or other rotating objects, such as a black ring.<sup>10)</sup> For this reason, we focus on a “spin-down” phase through scalar field radiation. A five-dimensional rotating black hole possesses three Killing vectors:  $\partial t$ ,  $\partial\phi$ , and  $\partial\psi$ . Therefore a five-dimensional black hole has two rotation parameters. For a five-dimensional MP black hole with one rotation parameter, Ida, Oda and Park<sup>11)</sup> found the formulae for the black body factor in a low-frequency expansion and the power spectra of the Hawking radiation. However, if a brane is not infinitely thin but, rather, has a thickness in the order of a fundamental scale ( $\sim \text{TeV}$ ), we expect to exist a second component of angular momentum. We therefore study the case of two rotation parameters. Frolov and Stojković first derived expressions for the energy and angular momentum fluxes from a five-dimensional rotating black hole with two rotation parameters.<sup>12)</sup> In this work, we numerically evaluated the quantum radiation from a five-dimensional rotating black hole with two rotation parameters,  $a$  and  $b$ , which we assume to be positive, without loss of generality. We found that such a black hole evolves toward an asymptotic state characterized by stable values  $a = b \sim 0.1975 (8M/3\pi)^{1/2}$ , where  $M$  is the mass of the black hole. We also show that the asymptotic state can be described by  $a \sim 0.1183 (8M/3\pi)^{1/2}$  and  $b = 0$  if one of the initial rotation parameters is exactly zero.

We start with the quantum radiation of a massless scalar field  $\Phi$ , which is minimally coupled, for a five-dimensional MP black hole with two rotation parameters,<sup>12)</sup> (see also Ref. 13) for details). To quantize the scalar field, we expand it as  $\Phi = R(r)\Theta(\theta)e^{im\phi}e^{in\psi}e^{-i\omega t}$ . For the vacuum state, we adopt the (past) Unruh vacuum state  $|U^-\rangle$ , which mimics the state of collapse of a star to a black hole.<sup>7)</sup> Calculating the vacuum expectation value of the energy-momentum tensor of the scalar field, we can evaluate the emission rates of the total energy and angular momenta, which give the changes of the black hole mass  $M$  and angular momenta  $J_\phi$  and  $J_\psi$  as

$$\dot{M} = -\pi \sum_{lmn} \int_0^\infty d\omega \frac{\omega^2}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \quad (1)$$

$$\dot{J}_\phi = -\pi \sum_{lmn} \int_0^\infty d\omega \frac{m\omega}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \quad (2)$$

$$\dot{J}_\psi = -\pi \sum_{lmn} \int_0^\infty d\omega \frac{n\omega}{\omega_+} \frac{\Gamma_{lmn}}{e^{2\pi\omega_+/\kappa} - 1}, \quad (3)$$

where  $\omega_+ = \omega - m\Omega_\phi - n\Omega_\psi$ ,  $\kappa = (r_+^2 - r_-^2)/2Mr_+$ ,  $l$  is the eigenvalue of the angular function  $\Theta(\theta)$ , and  $\Gamma_{lmn}$  is the greybody factor, which is identical to the absorption probability of the incoming wave of the corresponding mode. The values  $r_+$  and  $r_-$  represent the event horizon and the inner horizon of the black hole, respectively. The quantities  $\Omega_\phi = a/(r_+^2 + a^2)$  and  $\Omega_\psi = b/(r_+^2 + b^2)$  are the two angular velocities at the horizon  $r_+$ . The superradiance modes are given by the condition  $0 < \omega < m\Omega_\phi + n\Omega_\psi$ . From this condition, we find the interesting feature that a counter-rotating particle can be created by superradiance (i.e. if  $\Omega_\phi \gg \Omega_\psi$  and  $m \geq 1$ ) because the superradiance condition is satisfied for a counter-rotating particle ( $n < 0$ ) (see Ref. 13) for details).

Using the above formula for the quantum creation of a scalar field, we investigate the evolution of a five-dimensional MP black hole with two rotation parameters. From the condition for the existence of horizon(s), we obtain the condition  $a + b \leq r_s$  constraining the angular momenta, where  $r_s$  is a typical scale length which is related to the gravitational mass  $M$  of the black hole as  $r_s^2 = 8M/3\pi$ .

As shown by Page,<sup>14)</sup> it is convenient to introduce scale invariant rates of change for the mass and angular momenta of an evaporating black hole as

$$f \equiv -r_s^2 \dot{M}, \quad g_a \equiv -\frac{r_s}{a_*} \dot{J}_\phi, \quad \text{and} \quad g_b \equiv -\frac{r_s}{b_*} \dot{J}_\psi, \quad (4)$$

where  $a_* = a/r_s$  and  $b_* = b/r_s$ . In terms of the scale invariant functions  $f$ ,  $g_a$ , and  $g_b$ , the time evolution equations for  $a_*$  and  $b_*$  are given by

$$\frac{\dot{a}_*}{a_*} = -\frac{8}{3\pi} \frac{f h_a}{r_s^4} \quad \text{and} \quad \frac{\dot{b}_*}{b_*} = -\frac{8}{3\pi} \frac{f h_b}{r_s^4}, \quad (5)$$

where the dimensionless functions  $h_a$  and  $h_b$  are defined as

$$h_a \equiv \frac{d \ln a_*}{d \ln M} = \frac{3}{2} \left( \frac{g_a}{f} - 1 \right) \quad \text{and} \quad h_b \equiv \frac{d \ln b_*}{d \ln M} = \frac{3}{2} \left( \frac{g_b}{f} - 1 \right). \quad (6)$$

We now discuss the evolution of  $a_*$  and  $b_*$ , as determined through the numerical evaluation of  $f$ ,  $g_a$  and  $g_b$ . Henceforth, we use units such that  $r_s = 1$ . In the dynamical system (5), a fixed point plays an important role. It is defined by  $h_a = 0$  and  $h_b = 0$ . Note that  $f$  is positive definite. If  $h_a$  ( $h_b$ ) is positive, then  $a_*$  ( $b_*$ ) decreases, while if  $h_a$  ( $h_b$ ) negative, then  $a_*$  ( $b_*$ ) increases. Because  $h_a$  ( $h_b$ ) depends not only on  $a_*$  ( $b_*$ ) but also on  $b_*$  ( $a_*$ ),  $h_a = 0$  ( $h_b = 0$ ) gives a curve in the  $a_*$ - $b_*$  plane. Since there is symmetry between  $a_*$  and  $b_*$ , the fixed point should be symmetrical, too.

We first discuss the behavior of the mass and angular momentum loss rates in the case  $a = b$  (and hence  $a_* = b_*$ ). Fig. 1 displays the mass loss rate  $f(a_*)$  in terms of  $a_*$  ( $= b_*$ ). The mass loss rate through the scalar radiation is more effective at smaller values of  $a_*$ . We depict the angular momentum loss rate  $g_a(a_*)$  ( $= g_b(a_*)$ ) in Fig. 2. The function  $g_a(a_*)$  has a maximum at  $a_* = a_*^{(\max)} \approx 0.3844$ . We plot the function  $h_a(a_*)$  ( $= h_b(a_*)$ ) in Fig. 3. We find  $h_a(a_*) = 0$  at  $a_* = a_*^{(\text{cr})} \approx 0.1975$ , which is a fixed point in the present dynamical system. An important property of the function  $h_a(a_*)$  is that  $h_a(a_*) < 0$  [ $h_a(a_*) > 0$ ] for  $a_* < a_*^{(\text{cr})}$  [ $a_* > a_*^{(\text{cr})}$ ].

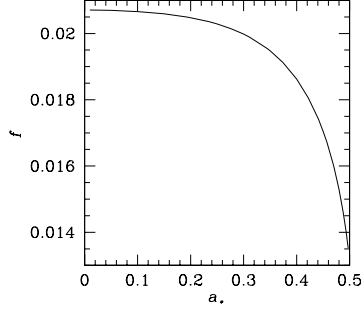


Fig. 1. The scale invariant quantity  $f$ , which represents the mass loss rate, as a function of  $a_*$  for the case  $a_* = b_*$ . The function  $f$  is positive definite by definition.

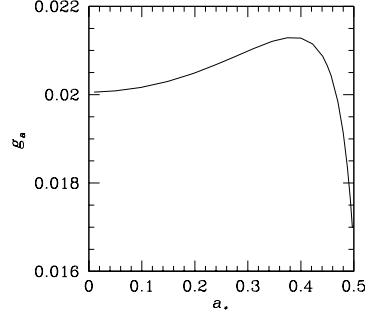


Fig. 2. The scale invariant quantity  $g_a$ , which represents the loss rate of the angular momentum  $J_\phi$ , as a function of  $a_*$  for the case  $a_* = b_*$ , for which  $g_a = g_b$ . The function  $g_a$  is positive definite by definition.

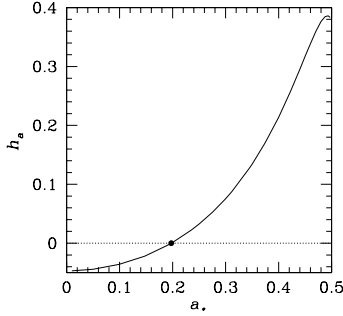


Fig. 3. The scale invariant quantity  $h_a$ , which represents the change rate of  $a_*$ , as a function of  $a_*$  for the case  $a_* = b_*$ , for which  $h_a = h_b$ . The function  $h_a(a_*)$  has a zero at  $a_* = a_*^{(\text{cr})} \simeq 0.1975$  (a black spot).

As a result, the fixed point  $(a_*, b_*) = (a_*^{(\text{cr})}, a_*^{(\text{cr})})$  is stable along the line  $a_* = b_*$ . Hence, a black hole formed with equal rotation parameters,  $a_* = b_* \neq 0$ , will eventually reach an asymptotic state characterized by  $a_* (= b_*) = a_*^{(\text{cr})}$ , through scalar field radiation.

In order to investigate the more generic case ( $a \neq b$ ), we have to analyze Eq. (5). For this purpose, we depict the contour plots of  $f$  and  $g_a$  in Figs. 4 and 5, respectively. ( $g_b$  is obtained by exchanging the axes for  $a_*$  and  $b_*$  in Fig. 5.)

In the  $a_*$ - $b_*$  plane, the region in which  $a_* + b_* > 1$  is forbidden, because there is no horizon (the black region in Figs. 4 and 5). In Fig. 4, there are two bright regions (one for large  $a_*$  and small  $b_*$ , and one for small  $a_*$  and large  $b_*$ ), where  $f$  becomes large. This means that the creation rate is high in these regions. In Fig. 5, there is only one bright region (for large  $a_*$  and small  $b_*$ ). Therefore, the angular momentum  $J_\phi$  is emitted effectively only in this region. This is the superradiance effect. For the angular momentum  $J_\psi$ , if  $b_*$  is large, we find effective emission. This means that the superradiance modes give a dominant contribution to the particle creation.

There is one interesting observation here: If the two rotation parameters are equal (i.e.  $a_* = b_*$ ), the emission rates are suppressed even if the black hole is in

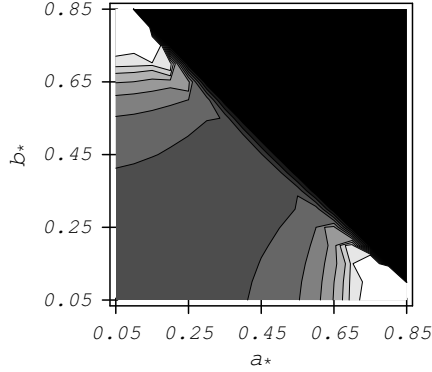


Fig. 4. The contours of  $f$  in the  $a_*$ - $b_*$  plane. The darkest and brightest regions correspond to zero and  $f_{\max}$  (the maximum of  $f$ ), which is given by  $f_{\max} \simeq f(0.85, 0.05) = 4.349$ , respectively. The difference between two contours is  $f_{\max}/10$ . The black region is forbidden, because there is no horizon in this region.

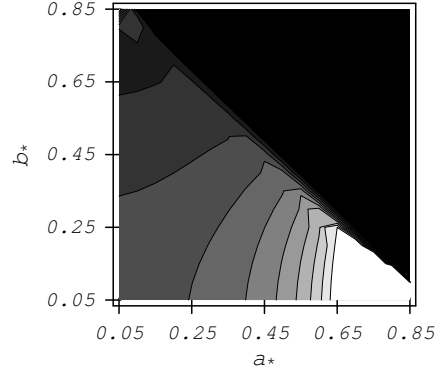


Fig. 5. The contours of  $g_a$  in the  $a_*$ - $b_*$  plane. The black and white regions correspond to zero and  $g_{a,\max}$  (the maximum of  $g_a$ ), which is given by  $g_{a,\max} \simeq f(0.85, 0.05) = 5.92467$ , respectively. The difference between two contours is  $g_{a,\max}/10$ . The black region is forbidden, because there is no horizon in this region.

a maximally rotating state ( $a_* = b_* = 0.5$ ). In the case  $a = b$ , something strange seems to happen, and the system behaves like a “spherically symmetric” black hole. In fact, the angular equation for  $\Theta(\theta)$  in this case is exactly the same as that for the Schwarzschild black hole.<sup>12)</sup> This may suppress the superradiance effect. This is consistent with the result given in Ref. 15), the efficiency of energy extraction for a MP black hole is very small in the case that the rotation parameters are equal.

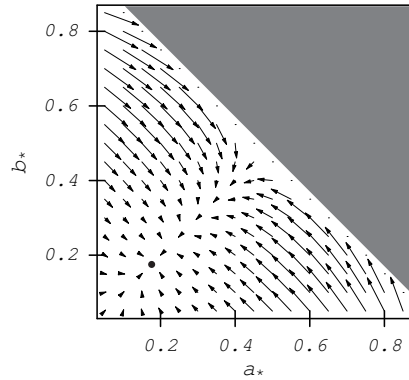


Fig. 6. The vector field describes the direction in which  $a_*$  and  $b_*$  evolve, i.e.  $(\dot{a}_*, \dot{b}_*)$ . For any initial values of  $a_*$  and  $b_*$ , the system evolves toward  $a_* = b_* = 0.1975$  (the black spot), which is a stable fixed point. The shaded region is forbidden.

In order to see the evolution of a black hole in the  $a_*$ - $b_*$  plane, we plot the vector field  $(\dot{a}_*, \dot{b}_*)$  with arrows in Fig. 6. From this figure, we see how the values of  $a_*$  and  $b_*$  evolve toward the fixed point. We can also prove that the fixed point is a stable attractor (see Ref. 13) for details).

In Fig. 6 the arrows far from the symmetry line of  $a_* = b_*$  are very large. Then, if the initial value of  $a_*$  ( $b_*$ ) is large, while that of  $b_*$  ( $a_*$ ) is small,  $a_*$  and  $b_*$  first approach the same value. Near the fixed point  $(a_*^{(\text{cr})}, a_*^{(\text{cr})})$ , the arrows are very small, which means that the evolution toward the fixed point is slow. We thus find that after reaching a state with  $a_* = b_*$ ,  $a_*$  and  $b_*$  eventually evolve together toward the fixed point

$(a_*^{(\text{cr})}, a_*^{(\text{cr})}) \approx (0.1975, 0.1975)$ . This means that any rotating black hole with two non-zero rotation parameters will evolve toward a final state with the same specific angular momenta,  $a_* = b_* = a_*^{(\text{cr})} \approx 0.1975$ . For a black hole with only one non-trivial rotation parameter, i.e.  $a \neq 0$  and  $b = 0$  exactly, we obtain the stable fixed point from the equation  $h_a(a_*, 0) = 0$ , which yields  $a_* \approx 0.1183$ .

Finally, consider the evaporation time of the black hole. In the above analysis, we showed that our dynamical system (5) has one stable attractor, which can be reached through quantum particle production. However, the black hole may evaporate away before this fixed point is reached. Whether this happens depends on the evaporation time and the evolution time in the  $a_*$ - $b_*$  plane. We can evaluate the evaporation time scale  $\tau_M$  using the emission rate of the black hole mass as  $\tau_M = -M/\dot{M}$ , and we can evaluate the evolution time scale  $\tau_{a_*}$  using the evolution equation (5) as  $\tau_{a_*} = a_*/|\dot{a}_*|$ .

We thus find that  $\tau_M/\tau_{a_*} = 8|h_a|/(3\pi) \sim O(1)$ . However, this does not mean that the black hole will evaporate away before reaching the fixed point. If the integrated evaporation time, which depends on the initial mass of the black hole, is much longer than the evolution time, we have enough time to realize the final state described by the fixed point. Therefore, we conclude that if a black hole has a mass that is larger than the fundamental Planck mass scale, its two specific angular momenta will eventually become equal when it evaporates away.

### Acknowledgements

We would like to thank J. Koga and T. Torii for useful discussions. This work was partially supported by the Grant-in-Aid for Scientific Research Fund of the MEXT (No. 17540268), by the Waseda University Grant for Special Research Projects, and by a Grant for The 21st Century COE Program (Holistic Research and Education Center for Physics Self-organization Systems) at Waseda University.

- 
- 1) C. M. Chambers, W. A. Hiscock and B. Taylor, Phys. Rev. Lett. **78** (1997), 3249.
  - 2) R. C. Myers and M. J. Perry, Ann. of Phys. **172** (1986), 304.
  - 3) N. Arkani-Hamad, S. Dimopoulos and G. Dvali, Phys. Lett. **B429** (1998), 263.
  - 4) L. Randall and R. S. Undrum, Phys. Rev. Lett. **83** (1999), 3370; *ibid.* **83** (1999), 4690.
  - 5) L. A. Anchordoqui, H. Goldberg and A. D. Shapere, Phys. Rev. D **66** (2002), 024033.
  - 6) S. W. Hawking, Commun. Math. Phys. **43** (1975), 199.
  - 7) W. G. Unruh, Phys. Rev. D **10** (1974), 3194.
  - 8) G. W. Gibbons, Commun. Math. Phys. **44** (1975), 199.
  - 9) S. B. Giddings and S. Thomas, Phys. Rev. D **65** (2002), 056010.
  - 10) R. Emparan and H. S. Reall, Phys. Rev. Lett. **88** (2002), 101101.
  - 11) D. Ida, K. Oda, and S. C. Park, Phys. Rev. D **67** (2003), 064025.
  - 12) V. Frolov and D. Stojković, Phys. Rev. D **67** (2003), 084004.
  - 13) H. Nomura, S. Yoshida, M. Tanabe and K. Maeda, in preparation.
  - 14) D. N. Page, Phys. Rev. D **46** (1992), 1312.
  - 15) M. Nozawa and K. Maeda, Phys. Rev. D **71** (2005), 084028.