Hawking radiation: a particle physics perspective.

Matt Visser*

Physics Department, Washington University, St. Louis, Missouri 63130-4899

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It has recently become fashionable to regard black holes as elementary particles. By taking this suggestion seriously it is possible to cobble together an elementary particle physics based estimate for the decay rate (black hole)$_i$ $\to$ (black hole)$_f$ + (massless quantum). This estimate of the spontaneous emission rate contains two free parameters which may be fixed by demanding that the high energy end of the spectrum of emitted quanta match a blackbody spectrum at the Hawking temperature. The calculation, though technically trivial, has important conceptual implications: (1) The existence of Hawking radiation from black holes is ultimately dependent only on the fact that massless quanta (and all other forms of matter) couple to gravity. (2) The thermal nature of the Hawking spectrum depends only on the fact that the number of internal states of a large mass black hole is enormous. (3) Remarkably, the resulting formula for the decay rate gives meaningful answers even when extrapolated to low mass black holes. The analysis strongly supports the scenario of complete evaporation as the endpoint of the Hawking radiation process (no naked singularity, no stable massive remnant).

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I. INTRODUCTION

It has recently become clear that the statistical description of the Hawking radiation process breaks down at low entropy [1]. Buoyed by this and other results a number of authors have investigated the utility of considering low mass black holes as elementary particles [2–4].

In the present paper, I propose taking the particle physics aspects of black hole physics seriously. Due to the absence of any fundamental theory of quantum gravity I will have to resort to physical reasoning to motivate an estimate for the decay rate

$$(\text{black hole})_i \to (\text{black hole})_f + (\text{massless quantum}) \tag{1.1}$$

For definiteness, one may take the massless quantum to be a photon, though neutrinos or linearized gravitons would do as well. A major goal of this paper will be to motivate Hawking radiation from a particle physics perspective, minimizing geometrodynamic aspects of the problem.

The estimate for $\Gamma_0(b_i \to b_f + \gamma)$ that I shall cobble together has several striking features:
(1) In the limit of a large black hole, where statistical techniques will be shown to be appropriate, the model predicts emission of a Maxwell–Boltzmann spectrum with two free parameters. These two parameters may be fixed by matching with Hawking's semiclassical calculation which is valid in this parameter regime [5,6].
(2) If the black hole interacts with a bath of photons, stimulated emission occurs. The spontaneous emission rate (appropriate for a black hole emitting radiation into a vacuum) is modified: $\Gamma = (1 + \langle n\rangle)\Gamma_0$. If the external photon bath is in equilibrium with itself at some temperature T_γ this enhancement factor gives $\Gamma(T_H, T_\gamma) = \{e^{\hbar\omega/T_\gamma}/(e^{\hbar\omega/T_\gamma} - 1)\}\Gamma_0(T_H)$. If furthermore the photon bath is also in equilibrium with the black hole itself, $(T_\gamma = T_H)$, then the emission and absorption spectra are equal to each other and are exactly Planckian.
(3) Once the two free parameters have been fixed in this manner, the resulting model continues to give sensible physical results even when extrapolated to small mass black holes —

strongly suggesting that the final stage of the Hawking radiation process is a cascade of gammas (and other massless quanta) leading to complete evaporation with no naked singularity and no stable massive remnant.

Units: $c = 1$; $k = 1$; $G = \ell_P/m_P = \ell_P^2/\hbar = \hbar/m_P^2$.

II. THE MODEL

A. Particle physics

Consider the two body decay $b_i \to b_f + \gamma$. The initial and final black holes are parameterized by mass M, spin J, and possess internal states whose number is denoted by N. The interaction vertex is just a photon coupling gravitationally to matter. Furthermore the decay is prohibited neither by phase space considerations nor by any conservation law. Therefore by general particle physics principles this decay *must* take place. The only question is at what rate. Without further ado, consider the estimate

$$\hbar\Gamma(b_i \to b_f + \gamma) = \xi \left(\frac{\hbar\omega}{m_P}\right)^2 \left(\frac{A_H}{\ell_P^2}\right) (\hbar\omega) \left(2 \; \frac{2J_f + 1}{2J_i + 1}\right) \frac{N_f}{N_i}. \tag{2.1}$$

The various terms in this estimate have the immediate physical interpretation:

(1) coupling constant: The dimensionless coupling constant describing matter coupling to gravity is $g = E/m_P$. The decay rate depends on the square of the coupling constant.

(2) surface enhancement: The emitted photon can couple to the black hole at any point on its horizon. Since the coupling is gravitational one can plausibly argue that different pieces of the horizon, of order a Planck length squared in area, will couple independently and incoherently to the emitted radiation. This implies a decay rate proportional to the area of the event horizon.

(3) phase space: The volume of two particle phase space is proportional to $\hbar\omega$.

(4) spin factors: These arise from a sum over squares of Clebsch–Gordon coefficients.

(5) statistical factors: One should average over initial states and sum over final states. The

number of internal states of a black hole of mass M has been denoted $N(M)$.

An overall dimensionless numerical factor ξ is left undetermined by this argument.

To reiterate the basic ingredient of this estimate: If a decay is (a) kinematically allowed, (b) does not violate conservation laws, and (c) the initial and final states are coupled, then that decay *will* and in fact *must* take place.

B. Kinematics

Consider the two body decay $b_i \to b_f + \gamma$. Work in the rest frame of the initial state. Elementary kinematics yields the energies of the final state particles

$$\hbar\omega = \frac{M_i^2 - M_f^2}{2M_i}; \qquad E_f = \frac{M_i^2 + M_f^2}{2M_i} = \sqrt{M_f^2 + (\hbar\omega)^2}. \tag{2.2}$$

C. Maxwell–Boltzmann spectrum

With malice aforethought, one defines $N = e^S$. Suppose now that for $M_{(i/f)} >> m_P$, one can show $N_{(i/f)} >> 1$. Then $S_{(i/f)} >> 1$, and one may adopt a statistical approach, treating $M_{(i/f)}$, $N_{(i/f)}$, $S_{(i/f)}$ and $\hbar\omega$ as effectively continuous variables.

Combining this statistical approximation with two body kinematics gives for the initial-final state factor

$$\begin{aligned} \frac{N_f}{N_i} \equiv \exp\left(S_f - S_i\right) &\approx \exp\left(\frac{\partial S}{\partial M}[M_f - M_i]\right) \\ &\approx \exp\left(-\frac{\partial S}{\partial M}\hbar\omega\right) \equiv \exp(-\hbar\omega/T). \end{aligned} \tag{2.3}$$

Where, with further malice aforethought, one defines $T^{-1} \equiv (\partial S/\partial M)|_{M_i}$. At this stage this is merely a definition but it is instructive to see that the gross features of a thermal spectrum are already emerging without recourse to curved space quantum field theory, Euclidean continuations of the metric, or similar arcane.

Up to this point, the argument of this paper has provided the estimate

$$\Gamma_0(b_i \to b_f + \gamma) = 2\xi \left(\frac{\hbar\omega}{m_P}\right)^2 \left(\frac{A_H}{\ell_P^2}\right) \omega \, \left(\frac{2J_f+1}{2J_i+1}\right) \exp(-\hbar\omega/T). \tag{2.4}$$

Perhaps more interestingly, the decay rate into the frequency *interval* $[\omega, \omega + d\omega]$ can be written as

$$d\Gamma_0(b_i \to b_f + \gamma) = 2\xi \left(\frac{\hbar\omega}{m_P}\right)^2 \left(\frac{A_H}{\ell_P^2}\right) \left(\frac{2J_f+1}{2J_i+1}\right) \exp(-\hbar\omega/T) \, d\omega. \tag{2.5}$$

Therefore, the power radiated into this frequency range is

$$dP(\omega) = \hbar\omega \; d\Gamma_0 = 2\xi A_H \, (\hbar\omega)^3 \left(\frac{2J_f+1}{2J_i+1}\right) \exp(-\hbar\omega/T) \, d\omega, \tag{2.6}$$

which is the Maxwell–Boltzmann spectrum promised in the introduction. The two parameters (ξ, T) are now fixed by appealing to the known behaviour of the Hawking radiation in the high frequency limit. Taking $J_i = J_f = 0$, comparison with Hawking's original calculation [5,6] or any of a number of subsequent calculations [7,8] shows that $\xi = 1/(3\pi)$ and $T = T_H$.

D. Stimulated emission

Suppose now that instead of radiating into a vacuum, the black hole is radiating into a bath of photons. If a certain mode is already inhabited by n photons it is well known that the rate $n \to n+1$ is enhanced by stimulated emission: $\Gamma_{n\to n+1} = (1+n)\Gamma_{0\to 1}$. After averaging over n the total emission rate is

$$\Gamma = \sum_n p_n \Gamma_{n\to n+1} = (1 + \langle n \rangle)\Gamma_{0\to 1}. \tag{2.7}$$

Now if the photon bath is in equilibrium with itself at some temperature T_γ Bose statistics implies that $\langle n \rangle = [\exp(\hbar\omega/T_\gamma) - 1]^{-1}$. The total emission rate for the black hole is then

$$\Gamma(T_H, T_\gamma) = \frac{e^{\hbar\omega/T_\gamma}}{e^{\hbar\omega/T_\gamma} - 1} \Gamma_0(T_H) \tag{2.8}$$

If furthermore the photon bath is also in equilibrium with the black hole itself, ($T_\gamma = T_H$), then the emission and absorption spectra are equal to each other and are both exactly Planckian.

E. The Bekenstein entropy estimate

From an analogy between the classical law of area growth for event horizons and and the second law of thermodynamics, Bekenstein [9] argued that the physical entropy of a (large) black hole is

$$S = \frac{1}{4\eta} \frac{A_H}{\ell_P^2}. \tag{2.9}$$

Here η is an undetermined numerical constant. Equivalently, the Bekenstein estimate for the number of internal states of a (large) black hole is

$$N = \exp\left(\frac{1}{4\eta} \frac{A_H}{\ell_P^2}\right). \tag{2.10}$$

By appealing to either curved space quantum field theoretic calculations [5,6] or to manipulations involving Euclidean analytic continuations of the manifold [7], Bekenstein's estimate may be verified, and the constant η found to be $\eta = 1$. For any reasonably large black hole, this immediately implies $N >> 1$, thereby justifying the statistical approximation adopted above.

F. Comments

This calculation, though technically trivial, teaches one several very important things. (1) The existence of Hawking radiation from black holes is ultimately dependent only on the fact that photons (and all other forms of matter) couple to gravity. (2) The thermal nature of the Hawking spectrum depends only on the fact that the number of internal states of a large mass black hole is enormous. (3) The formal definitions $S = \ln N$ and $T^{-1} \equiv (\partial S/\partial M)$ really do correspond to the physical entropy and temperature of the black hole. (4) These conclusions are insensitive to the precise nature of the black hole: Schwarzschild, Reissner–Nordstrom, Kerr–Newman, or dilatonic. (5) Curved space quantum field theory calculations are required only when one wishes to explicitly calculate the entropy (or, equivalently, the number of internal states) as a function geometrodynamic parameters.

III. THE ISSUE OF THE FINAL STATE

A. Generalities

One of the most intriguing aspects of this microscopic modelling of the Hawking radiation process is that the resulting formalism continues to give meaningful and quite physical answers when extrapolated to small black holes (where the statistical and semiclassical arguments usually employed fail utterly).

All the evidence assembled up to this point indicates that one should take the notion of the number of internal states of a black hole seriously — as though N were a real physical parameter. Proceed by *really* taking this notion of the number of internal states seriously. One notes that the number of internal states is by definition an integer. One infers a discrete spectrum for the allowed entropy.

$$S(N) = \ln N. \tag{3.1}$$

For a general black hole, the entropy will be related to the mass, and to other parameters such as charge, spin, and the presence or absence of a dilaton field. The quantization of entropy implies a quantization of mass $M(N)$, with the precise form of $M(N)$ depending on the type of black hole in question. The spontaneous decay rate is then

$$\Gamma_0(b_i \to b_f + \gamma) = \frac{2}{3\pi} \left(\frac{M(N_i)^2 - M(N_f)^2}{2\hbar M(N_i)} \right)^3 A_H(N_i) \, \frac{2J_f + 1}{2J_i + 1} \, \frac{N_f}{N_i}. \tag{3.2}$$

For a small black hole this implies the onset of a cascade process with characteristic time scale of order the Planck time. When does one expect this formula to break down? If the emitted photons have energy greater than or of order the Planck mass, then their Compton wavelength is less than their “Schwarzschild radius”, and one would expect the emitted photons to “disappear down their own event horizons” rendering the present estimate unreliable. Conversely, if one can show that the energy gaps are small compared to the Planck scale, and if the average energy of emitted photons is small compared to the Planck scale, then there is no obstruction to asserting the validity of this formula down to the lowest mass scales.

One may normalise N so that $N = 1$ corresponds to the lowest mass black hole of the given type. Then the final decay from the $N = 1$ state should be handled by means different from the above. For instance, for the lowest mass neutral black hole, consider the decay into two gammas $b_1 \to \gamma + \gamma$. One may easily estimate

$$\Gamma(b_1 \to \gamma + \gamma) = \zeta \left(\frac{M_1}{2m_P}\right)^4 \left(\frac{A}{\ell_P^2}\right)^2 \frac{M_1}{2\hbar}. \tag{3.3}$$

The only novelty in this estimate is that the amplitude is second order in the coupling constant $g = (M_1/2)/m_P$, so that the decay rate is fourth order. Additionally each photon can couple independently somewhere on the horizon so there are two area enhancement factors. Naturally the dimensionless constant ζ is not calculable by present techniques. From a variation of the discussion in the previous paragraph, this particular estimate is of course trustworthy if and only if $(M_1/2) << m_P$. Again, I wish to emphasise: (a) photons couple to gravity, (b) kinematics allows the decay, and (c) no conservation laws are violated, therefore the decay *must* take place. These ideas generalize: For instance, for the lowest mass singly charged black hole b_1^+ the same sort of analysis can be applied *mutatis mutandis* to the decay $b_1^+ \to e^+ \bar{\nu}_e$.

This analysis rather strongly suggests that the endpoint of the Hawking evaporation process is a cascade of gammas (and other light particles), emitted as the black hole descends to the $N = 1$ state, followed by complete evaporation into two light particles (no naked singularities, no stable massive remnants).

B. Schwarzschild black holes

Consider now an ordinary unadorned Schwarzschild black hole. The mass spectrum alluded to previously is easily seen to be

$$M^2(N) = m_P^2 \, \frac{\ln(1+N)}{4\pi}. \tag{3.4}$$

For large N, the mass gap between neighbouring states is of order $\delta(M^2) = m_P^2/N$, so that (as expected) the mass of large mass black holes is effectively a continuous parameter. The spontaneous decay rate is

$$\Gamma_0(b_i \to b_f + \gamma) = \frac{4}{3}(4\pi)^{-5/2}\omega_P \left[\ln\left(\frac{1+N_i}{1+N_f}\right)\right]^3 [\ln(1+N_i)]^{-1/2}\,\frac{N_f}{N_i}. \tag{3.5}$$

It may easily be verified that all of the emitted gammas have energy much less than the Planck mass. The highest energy gamma from this cascade arises in the $(N=2) \to (N=1)$ decay for which $\hbar\omega = m_P\left[\sqrt{(\ln 3)/4\pi} - \sqrt{(\ln 2)/4\pi}\right] = (0.0608...)m_P$.

The putative final decay into two gammas $b_1 \to \gamma + \gamma$, produces photons of energy $E = (1/2)M_1 = (1/2)\sqrt{(\ln 2)/4\pi}\; m_P = (0.1174...)m_P$, which is indeed safely smaller than the Planck scale.

The lesson to be learned is this: from a particle physics perspective the evaporation of a Schwarzschild black hole can be plausibly tracked all the way to the final state without ever having to directly confront Planck scale physics.

C. Kerr-Newman black holes

Once the mass of a Schwarzschild black hole drops below $M \approx m_P^2/m_e$, the Hawking temperature rises above the mass of an electron $T_H > m_e$. Once this happens the Hawking radiation will include electrons and positrons as well as photons (and gravitons and neutrinos). Since electrons and positrons carry off both spin and charge, it is clear that any truly realistic description of black hole evaporation cannot be modelled solely using Schwarzschild geometry, but must at some level include the effects of charge and spin.

Consider a charged, spinning black hole described by the Kerr–Newman geometry. Adopt units wherein electric charge is measured in mass units, *ie* $q^2/(4\pi\epsilon_0) \equiv GQ^2$, so that the charge on an electron is $Q_e \equiv \sqrt{\alpha}m_P$. I shall for the time being content myself with writing down the spectrum. Starting from the standard result for the entropy [1]

$$S = \frac{2\pi M^2}{m_P^2}\left\{1 - \frac{Q^2}{2M^2} + \sqrt{1 - \frac{Q^2}{M^2} - \frac{L^2}{G^2M^4}}\right\}, \tag{3.6}$$

straightforward manipulations yield

$$M^2 = m_P^2 \frac{\pi}{S} \left\{ \left(\frac{S}{2\pi} + \frac{Q^2}{2m_P^2} \right)^2 + \frac{L^2}{\hbar^2} \right\}. \tag{3.7}$$

The mass spectrum is then simply

$$M^2(N,Z,J) = m_P^2 \frac{\pi}{\ln(1+N)} \left\{ \left(\frac{\ln(1+N)}{2\pi} + \frac{Z^2\alpha}{2} \right)^2 + J(J+1) \right\}. \tag{3.8}$$

Note that $J = Z = 0$ reproduces the Schwarzschild case, that $N = Z = J = 0$ is the vacuum state, and that $N = 0$ with $Z \neq 0 \neq J$ is forbidden. Verifying that the mass gaps are small is an exercise in tedious integer algebra.

After spitting out a cascade of gammas, charged leptons, neutrinos, quarks, *etc*, the black hole will minimize its mass by eventually settling to a $N = 1$, $J = 0$ state (possibly with $Q \neq 0$). The most obvious manner in which the black hole might then lose its electric charge, emission of an electron, is energetically forbidden since it requires the black hole to increase its spin. Thus subsequent evolution depends on whether or not the particle physics spectrum contains any charged scalars. If elementary charged scalar particles exist then there is no barrier to the black hole losing its electric charge by spitting out these charged scalars, thereby descending to the $N = 1$, $J = Z = 0$ state considered previously. On the other hand, if charged scalars do not exist, the lowest order process leading to charge loss is the semi–gravitational semi–weak three body decay $b_i^Z \to b_f^{Z-1} + e^+ + \bar{\nu}_e$ which proceeds via a virtual W^+. This process will eventually reduce the black hole to the $N = 1$, $Z = 1$, $J = 0$ state, where the two body decay $b_1^+ \to e^+ \bar{\nu}_e$ can take over. Again, one sees the same basic physics in operation: Gravity couples to everything, thus, when kinematics and superselection rules permit the decay, that decay *will* take place.

The only hope for a stable massive remnant is if the decay is forbidden by a conservation law. For instance, if the black hole is a magnetic monopole, and if no "ordinary" particles carry magnetic charge, then the lowest mass black hole corresponding to a given magnetic charge must be stable.

D. Dilatonic black holes

Consider a spinning charged axionic dilatonic black hole. Such a black hole is a solution of the combined gravitational, electromagnetic, axion and dilaton equations of motion. The spin zero case has been discussed by Gibbons and Maeda [10], and more recently by Garfinkle, Horowitz, and Strominger [11]. Further variations on the spin zero theme may be found in references [12–16]. Generalizations to nonzero spin were first explored by Horne and Horowitz [17], and the complete solution worked out by Sen [18]. The entropy of such a black hole is

$$S = \frac{2\pi M^2}{m_P^2} \left\{ 1 - (Q^2/2M^2) + \sqrt{[1-(Q^2/2M^2)]^2 - (L^2/G^2 M^4)} \right\}. \tag{3.9}$$

As a function of entropy and charge,

$$M^2 = \frac{Q^2}{2} + m_P^2 \left\{ \frac{S}{4\pi} + \frac{\pi L^2}{S \hbar^2} \right\}. \tag{3.10}$$

One easily infers the mass spectrum

$$M(N,Z,J)^2 = m_P^2 \left\{ \frac{Z^2 \alpha}{2} + \frac{\ln(1+N)}{4\pi} + \frac{\pi J(J+1)}{\ln(1+N)} \right\}. \tag{3.11}$$

This class of black holes is in fact just as easy to analyse as the Kerr–Newman case. Note that $J = Z = 0$ reproduces the Schwarzschild case, that $N = Z = J = 0$ is the vacuum state, and that $N = 0$ with $Z \neq 0 \neq J$ is forbidden. As for the Kerr–Newman geometry, verifying that the mass gaps are desirably small is an exercise in tedious integer algebra. The discussion of the decay cascade may be carried over *mutatis mutandis*.

IV. DISCUSSION

In the absence of a reliable theory of fully quantized gravity the problem of dealing with the final stages of Hawking evaporation via an *ab initio* calculation appears to be a completely hopeless task. The arguments presented in this note seek to sidestep the whole issue by developing a phenomenological particle physics based model for the Hawking evaporation process. The model developed herein adequately accounts for the Hawking radiation process from heavy black holes — albeit with two free parameters that have to be fixed by comparison with Hawking's semiclassical calculation.

Having obtained the model, the model yields finite meaningful results even when extrapolated to small black holes. The model rather strongly supports the notion that the final state of the Hawking radiation process is complete evaporation. It appears that, from a particle physics perspective, the evaporation of a black hole can be plausibly tracked all the way to the final state without ever having to directly confront Planck scale physics. Because the model treats Hawking radiation as a cascade of quite ordinary particle physics decays, the model automatically preserves quantum coherence. Any initially pure quantum state will evolve to some pure (but complicated) final state.

Insofar as the ideas presented in this paper make any sense it now becomes absolutely critical to face head on the question: "What exactly *are* the internal states of a black hole?".

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* Electronic mail: visser@kiwi.wustl.edu

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