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Bekenstein-Spectrum, Hawking-Temperature and Specific Heat of Schwarzschild Black Holes from Microscopic Chains

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Abstract

We study the thermodynamic consequences of a recently proposed description for a Schwarzschild black hole based on Euclidean $(D3, D3) + (\overline{D3}, \overline{D3})$ brane pairs described in terms of chain-like excitations. A discrete mass-spectrum of Bekenstein-type is inferred and upon identification of the black hole mass with the chain's energy the leading corrections to both Hawking-temperature and specific heat of the black hole are obtained. The results indicate that for small black holes the evaporation process will be considerably altered.

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1 Introduction

It has been argued [1] quite early that Quantum Gravity should give an equidistant discrete spectrum for the horizon area of a black hole. The logic being that the horizon area represents an adiabatic invariant which leads to a discrete spectrum upon quantisation with Bohr-Sommerfeld rules. Using the fact that for a neutral, non-rotating Schwarzschild black hole the horizon area A_H is related to its mass M_{BH} via its Schwarzschild radius $r_S = 2G_4 M_{BH}$, one obtains consequently the discrete Bekenstein mass-spectrum

$$M_{BH} \propto \sqrt{N}\ , \quad N \in \mathbf{N} \tag{1}$$

for the black hole. Such a spectrum has now been discussed and derived in many different ways [2], [3], [4]. In some approaches to a quantum treatment of black holes like e.g. the reduced phase space quantisation method [4], [5] this mass-spectrum gets augmented by an additional zero-point energy. This becomes important if one addresses the ultimate fate of the evaporating black hole but otherwise can safely be ignored for macroscopic black holes for which N will be extremely large.

The discreteness of the mass-spectrum implies a drastic departure from the thermal Hawking radiation spectrum [3]. Indeed energy can only be radiated off a macroscopic black hole at frequencies which are integer multiples of $\omega \simeq M_{Pl}^2/M_{BH}$ and are thus in principle observable at energies much lower than the Planck-scale. By detecting such quanta at various energies one might be able to distinguish experimentally between different approaches on how to quantise gravity. For instance Loop Quantum Gravity [6] predicts also a discrete area-spectrum which is however not equispaced [7].

Here, we want to focus on a recent approach which has its roots in String-Theory, and in which the microscopic black hole states get identified with long chains living on the worldvolume of two dual Euclidean brane pairs [8], [9]. We will show that this approach leads directly to a discrete Bekenstein mass-spectrum for the D=4 Schwarzschild black hole as well. Moreover, upon identification of the chain's energy with the black hole's mass we will be able to derive within this approach the leading corrections to the black hole's Hawking temperature and its specific heat while the leading terms will coincide with the standard results for Hawking-temperature and specific heat known from black hole thermodynamics. For this coincidence it is important that in the chain approach the chain's entropy is determined unambiguously in terms of the Bekenstein-Hawking entropy, i.e. there is no ambiguity resulting from an undetermined proportionality constant in this relation.

Following the proposal of [8] for the counting of microstates of D=4 spacetimes possessing event horizons with spherical boundary S^2_H (more precisely the boundary of the black hole, S^2_H, is defined as the intersection of the future event horizon $\mathcal{H}^+$ with a partial Cauchy surface ending at spatial infinity $\mathcal{I}^0$) one has to introduce a doublet of Euclidean dual brane pairs $(E_1, M_1) + (E_2, M_2)$. In each pair E_i and M_i are orthogonal to each other and wrap together S^2_H plus the whole internal compact space (6-dimensional for type II String-Theory, resp. 7-dimensional for M-Theory). In the low-energy limit where supergravity is valid $(E_1, M_1)+(E_2, M_2)$ acting as sources lead to a unique D=10/11 background solution of D=10/11 supergravity. The D=4 spacetime mentioned above is then identified with the D=4 external part of the D=10/11 background solution. Thus starting from type II String-Theory it was argued in [10] that for uncharged Schwarzschild black holes one needs a doublet $(E_1, M_1)+(\overline{E}_1, \overline{M}_1)$ consisting of a dual brane pair and its anti-brane equivalent. This configuration has no charges. Moreover, to get a non-dilatonic black hole one should take $E_1 = M_1 = D3$ which is the only non-dilatonic Dp-brane. Further evidence for the identification of the doublet $(D3, D3) + (\overline{D3}, \overline{D3})$ with a D=4 Schwarzschild black hole was given in [10].

2 Chains From Branes

Let us now explain as a specific example of the more general proposal made in [8] the connection between the aforementioned Euclidean branes, chain states and the entropy of the D=4 Schwarzschild black hole. We will start with type IIB String-Theory on a ten-dimensional Lorentzian manifold $\mathcal{M}^{(1,3)} \times T^6$, i.e. a torus compactification from ten to four dimensions. It will be convenient to think of the T^6 as the product $T^2 \times T^4$. Moreover we will choose for $\mathcal{M}^{(1,3)}$ the standard Schwarzschild metric solution and denote the boundary of the D=4 black hole by S^2_H. We will next wrap a Euclidean $D3$ around $S^2_H \times T^2$ and another one around the remaining internal T^4. For technical reasons (in order to avoid a mismatch overall factor of two in the derivation of the entropy) and for physical reasons (neutral, i.e. uncharged black holes can only be obtained from brane-antibrane pairs whose Ramond-Ramond (RR) charge cancels) we have to wrap in exactly the same way another $\overline{D3}$ around $S^2_H \times T^2$ and a second $\overline{D3}$ around T^4. Evidence that indeed the backreaction of this Euclidean brane pair doublet can generate a D=10 background including in its external part the D=4 Schwarzschild metric was given in [10]. The background (as well as the brane configuration) breaks all supersymmetry and can alternatively be characterized as a black D6-brane in its ultra non-extreme limit. In this limit the black D6-brane looses its magnetic RR 2-form charge while the dilaton becomes constant thus giving a non-dilatonic vacuum solution.

Following [8], it is then easy to see that for this set-up of Euclidean branes the Bekenstein-Hawking (BH) entropy of the D=4 Schwarzschild black hole can be expressed purely in terms of the Nambu-Goto actions $S_{D3}, S_{\overline{D3}}$ of the two brane pairs as

$$\mathcal{S}_{BH} \equiv \frac{A_H}{4G_4} = (S_{D3})_{S^2\times T^2}(S_{D3})_{T^4} + (S_{\overline{D3}})_{S^2\times T^2}(S_{\overline{D3}})_{T^4} \ . \tag{2}$$

The crucial point now is to think of the tension τ_{D3} of a Euclidean $D3$-brane as the inverse of a fundamental smallest volume unit v_{D3}

$$\tau_{D3} = \frac{1}{v_{D3}} \tag{3}$$

which is an interpretation more adapt to a Euclidean brane as it treats all worldvolume dimensions equally (as opposed to an interpretation as a mass per unit volume which allocates a special role to the time-direction). This interpretation of a brane's tension follows also from the 'brane worldvolume uncertainty relation' [13] as explained in [14], [15]. Consequently a Euclidean $D3$-brane or likewise the $\overline{D3}$-antibrane can be thought of as a lattice made out of cells with volume v_{D3}. The number N_{D3} of such cells is then precisely measured by the brane's Nambu-Goto action

$$N_{D3} = \tau_{D3}\int d^4x\sqrt{\det g} = \frac{\text{Volume of Euclidean}\, D3}{v_{D3}} \tag{4}$$

and similarly for $N_{\overline{D3}}$. Therefore the D=4 Schwarzschild black hole's BH-entropy becomes simply an integer $N \in \mathbf{N}$

$$\mathcal{S}_{BH} = (N_{D3})_{S^2\times T^2}(N_{D3})_{T^4} + (N_{\overline{D3}})_{S^2\times T^2}(N_{\overline{D3}})_{T^4} = N \tag{5}$$

which stands for the total number of cells contained in the combined worldvolume of the $(D3, D3) + (\overline{D3}, \overline{D3})$ doublet.

In order to derive the black hole's BH-entropy by counting an appropriate set of microstates, it was then proposed in [8] to consider long chains[2] composed out of $(N-1)$ links on the N cell worldvolume lattice formed by the $(D3, D3) + (\overline{D3}, \overline{D3})$ doublet. A quantum-mechanical (a Gibbs-correction factor was included to account for the quantum

[2]Short chains on the other hand, composed out of two links, were used in [16] to construct standard model fields in warped backgrounds [17].

mechanical indistinguishability of the bosonic cells) counting then delivered an entropy for the chains

$$\mathcal{S}_c = N - \frac{1}{2}\ln N - \ln\sqrt{2\pi} + \mathcal{O}\Big(\frac{1}{N}\Big) \ . \tag{6}$$

By virtue of the identification (5) the chains living on the black hole's horizon thus exhibit an entropy

$$\mathcal{S}_c = \mathcal{S}_{BH} - \frac{1}{2}\ln \mathcal{S}_{BH} - \ln\sqrt{2\pi} + \mathcal{O}\Big(\frac{1}{\mathcal{S}_{BH}}\Big) \tag{7}$$

and are therefore good candidates to explain both the black hole's BH-entropy and the known logarithmic corrections [11], [12] thereof. Note that the factor multiplying the logarithm is 1/2 in accordance with the results of [12].

3 Bekenstein Mass Spectrum and Temperature

Let us now see what black hole mass spectrum follows from this proposal. By expressing $\mathcal{S}_{BH}$ for a Schwarzschild black hole in terms of its mass M_{BH}

$$\mathcal{S}_{BH} = 4\pi G_4 M_{BH}^2 \tag{8}$$

one infers from the discreteness of the entropy (5) that the black hole's mass-spectrum becomes discrete (quantized) as well

$$M_{BH}(N) = \frac{\sqrt{N}}{\sqrt{4\pi G_4}} \tag{9}$$

and turns out to be precisely of Bekenstein-type. This coincidence is interesting because to arrive at this result we have only used the geometrical interpretation of the brane's tension as the inverse of a smallest volume unit while its standard derivation uses the argument that the black hole's horizon area behaves as an adiabatic invariant and can therefore be quantized according to the Bohr-Sommerfeld rule [1]. Notice also that our derivation did not require the notion of the chains yet.

Before proceeding let us note that there is a very interesting observation related to the Bekenstein spectrum (9). As pointed out first in [3] this type of mass-spectrum offers an experimental verification well below the Planck-scale. For microscopically small Planck-sized black holes with N not much bigger than one, the energy radiated off the hole when jumping down from one energy level to the next is of order the Planck-scale

$$\Delta E_N \equiv M_{BH}(N) - M_{BH}(N-1) \simeq \frac{1}{\sqrt{4\pi G_4}} \simeq M_{Pl} \ . \tag{10}$$

However, when one considers macroscopically large black holes for which $N \gg 1$ then a level jump is accompanied by an energy-loss ($M_\odot$ denotes the mass of the sun)

$$\Delta E_N \simeq \frac{1}{4\sqrt{\pi G_4 N}} = \frac{1}{8\pi}\frac{M_{Pl}^2}{M_{BH}} = 1.3 \times 10^{-10}\frac{M_\odot}{M_{BH}}\,\text{eV} \tag{11}$$

which can be considerably smaller than Planck-scale and therefore possibly detectable. For instance primordial black holes with a lifetime of order the present age of the universe have a mass $M_{BH} = 2.5 \times 10^{-19} M_\odot$ and would emit quanta at an energy of $\Delta E_N = 0.5$ GeV.

Let us now come to the chains and examine their temperature in a microcanonical ensemble approach. To this end we have to determine the chain's energy which would ideally follow from a microscopic Hamiltonian governing its dynamics. As this is still largely unknown we will proceed differently. Since we know that at a microscopic level in the approach proposed in [8] the black hole resolves into a chain, what an observer at spacelike infinity measures as the black hole's mass M_{BH} is nothing else but the chain's energy E_c. We are therefore led to identify E_c with M_{BH} at leading order in $1/N$. Moreover we expect that there could be subleading corrections in this identification at relative level $1/N$. For instance we know that a fundamental string at very high excitation level $n \gg 1$ can be thought of as a random walk [18] and becomes therefore very similar to a chain. Therefore as for the string whose energy $E \propto \sqrt{n+c} = \sqrt{n}(1 + c/2n + \mathcal{O}(1/n^2))$ (c being a constant of $\mathcal{O}(1)$ depending on the type of string one is considering) receives subleading corrections at order $1/n$ we would expect that also the chain energy might receive similar corrections[3]. We will therefore write (a being a constant)

$$E_c(N) = M_{BH}(N)\Big(1 + \frac{a}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big) = \frac{\sqrt{N}}{\sqrt{4\pi G_4}}\Big(1 + \frac{a}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big)\,. \tag{12}$$

Knowing the energy and entropy for the chain then allows us to determine the chain's temperature T_c in a microcanonical ensemble approach

$$\frac{1}{T_c} = \frac{\partial \mathcal{S}_c}{\partial E_c} = \frac{dN}{dE_c}\frac{d\mathcal{S}_c}{dN} = 4\sqrt{\pi G_4}\sqrt{N}\left(1 + (a - \frac{1}{2})\frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\right) \tag{13}$$

where we regard N as a quasi-continuous parameter. By using the Bekenstein mass-spectrum for the black hole, the chain temperature can be expressed through the hole's

[3]Corrections of relative order $1/n$, $n \in \mathbf{N}$ are also known to arise in other approaches which treat the black hole's area like a harmonic oscillator and consequently obtain a 'zero-point correction' $A_H \propto (n + 1/2)$ for the horizon area which translates into a $1/4n$ correction for M_{BH}.

mass as

$$\frac{1}{T_c} = \frac{1}{T_H} + \frac{(2a-1)}{M_{BH}} + \mathcal{O}\Big(\frac{M_{Pl}^2}{M_{BH}^3}\Big) \tag{14}$$

where

$$\frac{1}{T_H} = 8\pi G_4 M_{BH} = 8\pi \frac{M_{BH}}{M_{Pl}^2} \tag{15}$$

is the Hawking-temperature of the Schwarzschild black hole. Thus the chain's temperature will equal the Hawking temperature for large N (which was actually clear from the fact that at leading order the chain's energy and entropy coincide with the standard black hole entities) but will in general deviate from it the more the smaller N becomes and therefore the smaller the black hole's mass M_{BH} becomes. This clearly indicates that the black hole's evaporation process will be considerably altered as compared to the standard view once the black hole becomes sufficiently small. Indeed the first and the second term on the rhs of (14) show opposing dependences on M_{BH} such that if $a < 1/2$ the chain's temperature will diverge already at some finite M_{BH} value as opposed to $M_{BH} = 0$ predicted by the Hawking-temperature formula alone. As long as this feature is not altered but sustained by even higher order corrections (notice that the second term in (14) is of order M_{Pl}^2/M_{BH}^2 as compared to the $1/T_H$ term and therefore shows that it becomes important close to the Planck regime where all the higher order contributions suppressed so far become important as well), it indicates that the chain's free energy $F_c = E_c - T_c\mathcal{S}_c$ actually diverges at this finite M_{BH} value as well, thus signalling a phase transition. This nourishes hope that puzzles like the black hole information puzzle might be completely avoided in this framework if one takes corrections to the standard black hole results into account.

4 The Specific Heat

Let us now similarly explore the black hole's specific heat. It is a characteristic feature of the Schwarzschild black hole to possess a negative specific heat. From the laws of black hole thermodynamics this is known to be

$$C_{BH} = -8\pi G_4 M_{BH}^2 \ . \tag{16}$$

Once more we expect to reproduce this result at leading order as here the chain and the black hole energy and temperature coincide. However again there will be non-trivial corrections to the leading order standard black hole result.

For a microcanonical ensemble of chains one obtains with (12) and (13) the specific heat

$$C_c = \frac{\partial E_c}{\partial T_c} = \frac{dN}{dT_c}\frac{dE_c}{dN} = -2N + (3-4a) + \mathcal{O}\Big(\frac{1}{N}\Big) \ . \tag{17}$$

Using (9) this can be expressed in terms of the black hole's mass as

$$C_c = -8\pi\frac{M_{BH}^2}{M_{Pl}^2} + (3-4a) + \mathcal{O}\Big(\frac{M_{Pl}^2}{M_{BH}^2}\Big) \ . \tag{18}$$

Again the chain's specific heat coincides with the standard black hole result at leading order but deviates from it at rather small black hole masses as the first correction term $3-4a$ is of order M_{Pl}^2/M_{BH}^2 relative to the leading order term. Depending on whether $a > 3/4$ or $a < 3/4$ the specific heat would either be negative for all masses thus pointing towards some unstoppable instability or would become zero already at some finite mass value M_{BH} close to the Planck scale (under the premise that even higher order corrections do not spoil this result). Therefore in contrast to the leading order result (16) which implies an instability down to the last stages of the black hole evaporation process, the inclusion of the correction term indicates in the case of $a < 3/4$ that at some small but finite mass the black hole's evaporation might cease and a stable state be reached as the specific heat becomes positive here. It would therefore be clearly interesting to understand the dynamics of the chain in detail in order to get a rigorous understanding of the final stages of the black hole evaporation.

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