RUIN PROBABILITIES FOR RISK PROCESSES WITH NON-STATIONARY ARRIVALS AND SUBEXPONENTIAL CLAIMS

LINGJIONG ZHU

ABSTRACT. In this paper, we obtain the finite-horizon and infinite-horizon ruin probability asymptotics for risk processes with claims of subexponential tails for non-stationary arrival processes that satisfy a large deviation principle. As a result, the arrival process can be dependent, non-stationary and non-renewal. We give three examples of non-stationary and non-renewal point processes: Hawkes process, Cox process with shot noise intensity and self-correcting point process. We also show some aggregate claims results for these three examples.

1. INTRODUCTION

Let us consider a classical risk model

$$U_t = u + pt - \sum_{i=1}^{N_t} C_i, \tag{1.1}$$

where C_i are i.i.d. claims distributed as an R^+-valued random variable C, $p > 0$ is the premium rate, $u > 0$ is the initial reserve and N_t is a simple point process.

We are interested in the case when C_i have heavy tails. A distribution function B is subexponential, i.e. $B \in \mathcal{S}$ if

$$\lim_{x\to\infty} \frac{\mathbb{P}(C_1 + C_2 > x)}{\mathbb{P}(C_1 > x)} = 2, \tag{1.2}$$

where C_1, C_2 are i.i.d. random variables with distribution function B. Let us denote $B(x) := \mathbb{P}(C_1 \geq x)$ and let us assume that $\mathbb{E}[C_1] < \infty$ and define $B_0(x) := \frac{1}{\mathbb{E}[C]} \int_0^x \overline{B}(y)dy$, where $\overline{F}(x) = 1 - F(x)$ is the complement of any distribution function $F(x)$. In the paper, the notation $f(x) \sim g(x)$ means $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$.

Goldie and Resnick (1988) showed that if $B \in \mathcal{S}$ and satisfies some smoothness conditions, then B belongs to the maximum domain of attraction of either the Frechet distribution or the Gumbel distribution. In the former case, $\overline{B}$ is regularly varying, i.e. $\overline{B}(x) = L(x)/x^{\alpha+1}$, for some $\alpha > 0$ and we write it as $\overline{B} \in \mathcal{R}(-\alpha - 1)$, $\alpha > 0$, where $L(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is a slowly varying function, i.e. $\lim_{x\to\infty} \frac{L(\gamma x)}{L(x)} = 1$ for any $\gamma > 0$.

We assume that $B_0 \in \mathcal{S}$ and either $\overline{B} \in \mathcal{R}(-\alpha - 1)$ or $B \in \mathcal{G}$, i.e. the maximum domain of attraction of Gumbel distribution $\Lambda(x) = \exp\{-e^{-x}\}$. A distribution

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function F is in the maximal domain of attraction of a distribution with distribution function $H(x)$ if there exist $a_n > 0$, $b_n \in \mathbb{R}$ so that

$$\lim_{n\to\infty} n\overline{F}(a_n x + b_n) = -\log H(x), \quad x \in \mathbb{R}, \tag{1.3}$$

where the limit is interpreted as ∞ when $H(x) = 0$. Therefore, the maximal domain of attraction of Gumbel distribution $\mathcal{G}$ consists of the distribution functions F so that there exist $a_n > 0$, $b_n \in \mathbb{R}$ such that $\lim_{n\to\infty} n\overline{F}(a_n x + b_n) = e^{-x}$, $x \in \mathbb{R}$. $\mathcal{G}$ includes Weibull and lognormal distributions.

$T_i = \tau_i - \tau_{i-1}$ is the length of the time interval between two consecutive arrival times of the point process τ_{i-1} and τ_i. τ_i stands for the ith arrival time of the point process. If T_i are i.i.d., with mean $\mathbb{E}[T_1]$, then N_t is a renewal process and assume the usual net profit condition

$$\rho := \frac{\mathbb{E}[C_1]}{p\mathbb{E}[T_1]} < 1, \tag{1.4}$$

then, it is well known that (see Teugels and Veraverbeke (1973), Veraverbeke (1977) and Embrechts and Veraverbeke (1982)),

$$\lim_{u\to\infty} \frac{\psi(u)}{\overline{B}_0(u)} = \frac{\rho}{1-\rho}, \tag{1.5}$$

where $\psi(u) := \mathbb{P}(\tau_u < \infty)$ is the infinite-horizon ruin probability, where

$$\tau_u := \inf\{t > 0 : U_t \leq 0\}. \tag{1.6}$$

The extensions when N_t is not a renewal process has been studied in Asmussen et al. (1999) when the authors consider a risk process with regenerative structures or a stationary and ergodic process satisfying certain conditions. See also Araman and Glynn (2006), Schlegel (1998) and Zwart et al. (2005).

But in general, for a simple point process N_t, we may not have a regenerative structure and it may not be stationary and ergodic as assumed in Asmussen et al. (1999). For example, none of the examples that we will introduce later in Section 3 are stationary or have a regenerative structure. In this paper, we point out that the classical infinite-horizon ruin probability estimate (1.5) and also finite-horizon ruin probabiliy estimate still hold as long as there exists a large deviation principle for $(N_t/t \in \cdot)$, which is the main result of this paper, i.e. Theorem 3 and Theorem 8 in Section 2.1. The intuition behind it is that if the arrival times deviate away from its mean with an exponentially small probability, it will be dominated by the subexponential distributions of the claim sizes. Our proof is essentially based on checking the conditions proposed in Asmussen et al. (1999).

In Section 2.2, we review some known results about estimates of aggregate claims when N_t is not necessarily renewal and show that a condition is satisfied given the large deviation principle of $(N_t/t \in \cdot)$.

Finally, in Section 3, we give three examples of non-renewal processes, i.e. Hawkes process (which answers a question of Stabile and Torrisi (2010)), Cox process with shot noise intensity (which reproves a result that is known, see Asmussen and Albrecher (2010)), and self-correcting point process for which our results apply.

2. Risk Process with Non-Renewal Arrivals and Regularly Varying Claims

2.1. **Ruin Probabilites.** Before we proceed, recall that a sequence $(P_n)_{n\in\mathbb{N}}$ of probability measures on a topological space X satisfies the large deviation principle (LDP) with rate function $I : X \to \mathbb{R}$ if I is non-negative, lower semicontinuous and for any measurable set A, we have

$$-\inf_{x\in A^o} I(x) \leq \liminf_{n\to\infty} \frac{1}{n} \log P_n(A) \leq \limsup_{n\to\infty} \frac{1}{n} \log P_n(A) \leq -\inf_{x\in \overline{A}} I(x). \tag{2.1}$$

Here, A^o is the interior of A and $\overline{A}$ is its closure. See Dembo and Zeitouni (1998) or Varadhan (1984) for general background regarding large deviations and the applications. Also Varadhan (2008) has an excellent survey article on this subject.

The following assumption is the main assumption of this paper.

Assumption 1. *(i) $(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function $I(\cdot)$ such that $I(x) = 0$ if and only if $x = \mu$.*

(ii) $I(x)$ is increasing on $[\mu, \infty)$ and decreasing on $[0, \mu]$.

(iii) The net profit condition is satisfied,

$$\rho := \frac{\mu \mathbb{E}[C_1]}{p} < 1. \tag{2.2}$$

(iv) There exists some $\theta > 0$ such that $\mathbb{E}[e^{\theta \sum_{i=1}^n T_i}] < \infty$ for any $n \in \mathbb{N}$.

Under Assumption 1, the following two lemmas hold.

Lemma 1. *Under Assumption 1, for any fixed $\epsilon, \epsilon' > 0$, there exists a constant $M > 0$ such that*

$$\mathbb{P}\left(\bigcap_{n=1}^{\infty} \left\{ p \sum_{i=1}^{n} T_i \leq n\left(\frac{p}{\mu} + \epsilon\right) + M \right\}\right) > 1 - \epsilon'. \tag{2.3}$$

Proof. Replacing ϵ by $p\epsilon$ and M by pM in the above equation, it is sufficient to prove that

$$\limsup_{M\to\infty} \mathbb{P}\left(\bigcup_{n=1}^{\infty} \left\{ \sum_{i=1}^{n} T_i > n\left(\frac{1}{\mu} + \epsilon\right) + M \right\}\right) = 0 \tag{2.4}$$

Observe that $\{N_t \leq n\} = \{\sum_{i=1}^n T_i > t\}$ for any $n \in \mathbb{N}$ and $t \in \mathbb{R}^+$ and also for any fixed $\mu' < \mu$, there exists some $\delta' > 0$ such that $I(\mu') - \delta' > 0$ and for sufficiently large t,

$$\mathbb{P}(N_t/t < \mu') \leq e^{-t[I(\mu') - \delta']}, \tag{2.5}$$

where we used fact that $I(\mu') > 0$ and $I(\cdot)$ is decreasing on $[0, \mu]$ from Assumption 1.

Also for any $N \in \mathbb{N}$,

$$\limsup_{M\to\infty} \sum_{n<N} \mathbb{P}\left(\sum_{i=1}^{n} T_i > n\left(\frac{1}{\mu} + \epsilon\right) + M\right) = 0. \tag{2.6}$$

Together, take $N \in \mathbb{N}$ sufficiently large,

$$
\begin{aligned}
&\limsup_{M\to\infty}\mathbb{P}\left(\bigcup_{n=1}^{\infty}\left\{\sum_{i=1}^{n}T_i > n\left(\frac{1}{\mu}+\epsilon\right)+M\right\}\right) \\
&\leq \limsup_{M\to\infty}\sum_{n=1}^{\infty}\mathbb{P}\left(\sum_{i=1}^{n}T_i > n\left(\frac{1}{\mu}+\epsilon\right)+M\right) \\
&= \limsup_{M\to\infty}\sum_{n\geq N}\mathbb{P}\left(\sum_{i=1}^{n}T_i > n\left(\frac{1}{\mu}+\epsilon\right)+M\right) \\
&= \limsup_{M\to\infty}\sum_{n\geq N}\mathbb{P}\left(\frac{N_{n(\mu^{-1}+\epsilon)+M}}{n(\mu^{-1}+\epsilon)+M}\leq\frac{n}{n(\mu^{-1}+\epsilon)+M}\right) \\
&\leq \limsup_{M\to\infty}\sum_{n=1}^{\infty}\mathbb{P}\left(\frac{N_{n(\mu^{-1}+\epsilon)+M}}{n(\mu^{-1}+\epsilon)+M}\leq\frac{\mu}{1+\mu\epsilon}\right) \\
&\leq \limsup_{M\to\infty}\sum_{n\geq N}e^{-(n(\mu^{-1}+\epsilon)+M)[I(\frac{\mu}{1+\mu\epsilon})-\delta']} = 0.
\end{aligned}
\tag{2.7}
$$

□

Lemma 2. *Under Assumption 1 and further assume that $B_0 \in \mathcal{S}$,*

$$
\lim_{u\to\infty}\frac{\mathbb{P}(\sup_{n\geq 1}\{n(p\mu^{-1}-\epsilon)-p\sum_{i=1}^{n}T_i\}\geq u)}{\overline{B}_0(u)} = 0,
\tag{2.8}
$$

for any sufficiently small $\epsilon > 0$.

Proof. Notice that

$$
\begin{aligned}
\mathbb{P}\left(\sup_{n\geq 1}\left\{n(p\mu^{-1}-\epsilon)-p\sum_{i=1}^{n}T_i\right\}\geq u\right) &\leq \sum_{n=1}^{\infty}\mathbb{P}\left(\sum_{i=1}^{n}T_i\leq n\left(\frac{1}{\mu}-\frac{\epsilon}{p}\right)-\frac{u}{p}\right) \\
&= \sum_{n>\frac{u}{\frac{p}{\mu}-\epsilon}}\mathbb{P}\left(\sum_{i=1}^{n}T_i\leq n\left(\frac{1}{\mu}-\frac{\epsilon}{p}\right)-\frac{u}{p}\right) \\
&\leq \sum_{n>\frac{u}{\frac{p}{\mu}-\epsilon}}\mathbb{P}\left(\sum_{i=1}^{n}T_i\leq n\left(\frac{1}{\mu}-\frac{\epsilon}{p}\right)\right) \\
&\leq \sum_{n>\frac{u}{\frac{p}{\mu}-\epsilon}}\mathbb{P}\left(N_{n(\frac{1}{\mu}-\frac{\epsilon}{p})}\geq n\right) \\
&\leq \sum_{n>\frac{u}{\frac{p}{\mu}-\epsilon}}e^{-n(\frac{1}{\mu}-\frac{\epsilon}{p})[I((\frac{1}{\mu}-\frac{\epsilon}{p})^{-1})-\delta']},
\end{aligned}
\tag{2.9}
$$

which is exponentially small in u as $u \to \infty$. Since $B_0 \in \mathcal{S}$ is subexponential, we have the desired result. □

Asmussen et al. (1999) proved that (1.5) holds if we have Lemma 1 and Lemma 2. So our main task here is to prove Lemma 1 and Lemma 2 under following assumptions. Notice that Lemma 1 holds if $(T_i)_{i\geq 1}$ is a stationary and ergodic sequence (by using ergodic theorem). And that is the only place Asmussen et al. (1999) used the stationarity and ergodicity assumption. That is why as long as we can prove Lemma 1 we can drop the stationarity and ergodicity assumption. The following is the main assumption for the asymptotic results of ruin probabilities that we are going to establish in this paper.

We have the following asymptotic estimates for infinite-horizon ruin probabilities.

Theorem 3. *Under Assumption 1 and further assume that $B_0 \in \mathcal{S}$, we have*

$$\lim_{u\to\infty} \frac{\psi(u)}{\overline{B}_0(u)} = \frac{\rho}{1-\rho}. \tag{2.10}$$

Proof. It is a direct result of Lemma 1, Lemma 2 and Theorem 3.1. in Asmussen et al. (1999). □

Remark 4. *In Theorem 3, we can replace the large deviation assumption of $(N_t/t \in \cdot)$ by a large deviation assumption of $(\frac{1}{n}\sum_{i=1}^n T_i \in \cdot)$. But usually, if $(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function $I(x)$ if and only if $(\frac{1}{n}\sum_{i=1}^n T_i \in \cdot)$ satisfies a large deviation principle with rate function $xI(1/x)$. The reason we chose to assume the large deviation for $(N_t/t \in \cdot)$ in Assumption 1 is because when N_t is not renewal, the inter-occurrence times are not i.i.d. and it is usually easier and more natural to establish the large deviation for $(N_t/t \in \cdot)$, which is at least in the case of our three examples, Hawkes process, Cox process with shot noise intensity and self-correcting point process.*

Next, let us consider the finite-horizon ruin probabilities.

Let $e(u) := \mathbb{E}[C_1 - u | C_1 > u]$ be the mean excess function and

$$\psi(u,z) := \mathbb{P}(\tau_u \leq z), \quad z > 0, \tag{2.11}$$

be the finite-horizon ruin probability.

Remark 5. *(i) (Regularly Varying Distributions) If $\overline{B}(u) = \frac{L(u)}{u^{\alpha+1}}$, $\alpha \in (0,\infty)$, i.e. $\overline{B} \in \mathcal{R}(-\alpha-1)$, then, $e(u) \sim \frac{u}{\alpha}$.*

(ii) (Lognormal Distributions) If $B(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{(\log u-\mu)/\sigma} e^{-x^2/2}dx$, then, $B \in \mathcal{G}$ and $B_0 \in \mathcal{S}$ and $e(u) \sim \frac{\sigma^2 u}{\log u - \mu}$.

(iii) (Weibull Distributions) If $B(u) = e^{-u^\alpha}$, where $\alpha \in (0,1)$, then, $B \in \mathcal{G}$ and $B_0 \in \mathcal{S}$ and $e(u) \sim \frac{u^{1-\alpha}}{\alpha}$.

Remark 6. *It is well known that if $B \in \mathcal{G}$, i.e. the maximal domain of attraction of Gumbel distribution, then,*

$$\lim_{u\to\infty} \frac{\overline{B}(u + xe(u))}{\overline{B}(u)} = e^{-x}, \quad x \in \mathbb{R}. \tag{2.12}$$

Lemma 7. *For any $y_0 < \infty$, $\lim_{x\to\infty} \frac{\overline{G}(x+y)}{\overline{G}(x)} = 1$ uniformly for $y \in [0, y_0]$ for any $G \in \mathcal{S}$.*

Lemma 7 can be found in Chapter X of Asmussen and Albrecher (2010).

We have the following asymptotic estimates for finite-horizon ruin probabilities.

Theorem 8. *Under Assumption 1 and further assume that $B_0 \in \mathcal{S}$, we have, for any $T > 0$, (i) If $\overline{B} \in \mathcal{R}(-\alpha - 1)$,*

$$\lim_{u\to\infty} \frac{\psi(u, e(u)T)}{\overline{B}_0(u)} = \frac{\rho}{1-\rho}\left[1 - \left(1 + (1-\rho)\frac{T}{\alpha}\right)^{-\alpha}\right]. \tag{2.13}$$

(ii) If $B \in \mathcal{G}$,

$$\lim_{u\to\infty} \frac{\psi(u, e(u)T)}{\overline{B}_0(u)} = \frac{\rho}{1-\rho}\left[1 - e^{-(1-\rho)T}\right]. \tag{2.14}$$

Proof. The proof is based on the ideas in Asmussen et al. (1999) with some modifications. When N_t is a renewal process, Asmussen and Klüppelberg (1996) proved both (i) and (ii). Now if N_t satisfies Assumption 1, then, by Lemma 1,

$$\begin{aligned}\psi(u, e(u)T) &= \mathbb{P}\left(\sup_{n \le e(u)T}\left\{\sum_{i=1}^n C_i - p\sum_{i=1}^n T_i\right\} > u\right) \\ &\ge (1-\epsilon')\mathbb{P}\left(\sup_{n\le e(u)T}\left\{\sum_{i=1}^n C_i - n\left(\frac{p}{\mu}+\epsilon\right)\right\} > u + M\right).\end{aligned} \tag{2.15}$$

Now, in both cases (i) and (ii), we know that $e(x) \sim \frac{\int_x^\infty \overline{B}(y)dy}{\overline{B}(x)}$. Since both $B(x)$ and B_0 belong to $\mathcal{S}$, Lemma 7 implies that $\lim_{x\to\infty}\frac{e(x+y)}{e(x)} = 1$ uniformly for $y \in [0, y_0]$ for any $y_0 < \infty$. Therefore, for any $\epsilon'' \in (0,1)$, we have $e(u) \ge e(u+M)(1-\epsilon'')$ for any sufficiently large u and thus we get

$$\psi(u, e(u)T) \ge (1-\epsilon')\mathbb{P}\left(\sup_{n\le e(u+M)T(1-\epsilon'')}\left\{\sum_{i=1}^n C_i - n\left(\frac{p}{\mu}+\epsilon\right)\right\} > u+M\right). \tag{2.16}$$

Now assume $\overline{B} \in \mathcal{R}(-\alpha-1)$. We have by the corresponding result for renewal N_t in Asmussen and Klüppelberg (1996) and Lemma 7,

$$\begin{aligned}\liminf_{u\to\infty}\frac{\psi(u, e(u)T)}{\overline{B}_0(u)} &= \liminf_{u\to\infty}\frac{\psi(u, e(u)T)}{\overline{B}_0(u+M)} \\ &\ge (1-\epsilon')\frac{\rho_\epsilon}{1-\rho_\epsilon}\left[1 - (1 + (1-\rho_\epsilon)T(1-\epsilon'')/\alpha)^{-\alpha}\right],\end{aligned} \tag{2.17}$$

where $\rho_\epsilon := \frac{\mathbb{E}[C_1]}{\frac{p}{\mu}+\epsilon}$. Since it holds for any $\epsilon, \epsilon', \epsilon'' > 0$, we proved the lower bound. The case for $B \in \mathcal{G}$ is similar.

Now, let us prove the upper bound. Choose $\epsilon > 0$ small enough that $\frac{p}{\mu} - \epsilon > \mathbb{E}[C_1]$,

$$\begin{aligned}&\limsup_{u\to\infty}\frac{\psi(u, e(u)T)}{\overline{B}_0(u)} \\ &= \limsup_{u\to\infty}\frac{\mathbb{P}(\sup_{n\le e(u)T}\{\sum_{i=1}^n C_i - n(p\mu^{-1}-\epsilon) + n(p\mu^{-1}-\epsilon) - \sum_{i=1}^n T_i\} > u)}{\overline{B}_0(u)} \\ &\le \limsup_{u\to\infty}\frac{\mathbb{P}(X_\epsilon(u) + Y_\epsilon > u)}{\overline{B}_0(u)},\end{aligned} \tag{2.18}$$

where $X_\epsilon(u) := \sup_{n\leq e(u)}\{\sum_{i=1}^n C_i - n(p\mu^{-1}-\epsilon)\}$ and $Y_\epsilon := \sup_{n\geq 1}\{n(p\mu^{-1}-\epsilon) - \sum_{i=1}^n T_i\}$. By Lemma 2, we have $\lim_{u\to\infty}\frac{\mathbb{P}(Y_\epsilon>u)}{\overline{B}_0(u)} = 0$ and by the results for the renewal case (Asmussen and Klüppelberg (1996)), for $\overline{B}\in\mathcal{R}(-\alpha-1)$,

$$\mathbb{P}(X_\epsilon(u)>u) \sim \frac{\rho_\epsilon}{1-\rho_\epsilon}\left[1-(1+(1-\rho_\epsilon)T/\alpha)^{-\alpha}\right]\overline{B}_0(u), \tag{2.19}$$

where $\rho_\epsilon := \frac{\mathbb{E}[C_1]}{p\mu^{-1}-\epsilon}$. Let us recall the Proposition 1.9. of Chapter X in Asmussen and Albrecher (2010) which says that for any distributions A_1, A_2 on $\mathbb{R}^+$, if we have $\overline{A}_i(x) \sim a_i\overline{G}(x)$ for some $G\in\mathcal{S}$ and some constants $a_1+a_2>0$, then, $\overline{A_1 * A_2}(x) \sim (a_1+a_2)\overline{G}(x)$. In our case $G(x) = B_0(x)\in\mathcal{S}$ and A_1, A_2 are the distributions of $X_\epsilon(u)$ and Y_ϵ with $a_1>0$ and $a_2=0$. Notice that $X_\epsilon(u)$ and Y_ϵ may be negative. To save the argument, we can simply use the fact that $X_\epsilon(u)\leq\max\{X_\epsilon(u),1\}$ and $Y_\epsilon\leq\max\{Y_\epsilon,1\}$ then apply it to $\max\{X_\epsilon(u),1\}$ and $\max\{Y_\epsilon,1\}$ instead. Also, in our case, $X_\epsilon(u)$ depends on u, but the proof of Proposition 1.9. Chapter X in Asmussen and Albrecher (2010) still works. Hence, we get

$$\limsup_{u\to\infty}\frac{\psi(u,e(u)T)}{\overline{B}_0(u)} \leq \frac{\rho_\epsilon}{1-\rho_\epsilon}\left[1-(1+(1-\rho_\epsilon)T/\alpha)^{-\alpha}\right]. \tag{2.20}$$

Since it holds for any ϵ, we proved the upper bound. The case for $B\in\mathcal{G}$ is similar. □

2.2. **Aggregate Claims.** Let $A_t := \sum_{i=1}^{N_{t-}} C_i$ be the aggregate claims up to time t, where as before we assume here that C_i are i.i.d. positive random variables

Consider the following assumptions.

Assumption 2. *(i)* $\mathbb{E}[N_t]<\infty$ *for any* t *and* $\mathbb{E}[N_t]\to\infty$ *as* $t\to\infty$.

(ii) $\frac{N_t}{\mathbb{E}[N_t]}\to 1$, *as* $t\to\infty$.

(iii) There exist $\epsilon,\delta>0$ *such that*

$$\sum_{k>(1+\delta)\mathbb{E}[N_t]}\mathbb{P}(N_t>k)(1+\epsilon)^k\to 0, \tag{2.21}$$

as $t\to\infty$.

Klüppelberg and Mikosch (1997) proved that under Assumption 2, for fixed time t, we have

$$\mathbb{P}(A_t-\mathbb{E}[A_t]>x)\sim\mathbb{E}[N_t]\mathbb{P}(C_1\geq x), \tag{2.22}$$

uniformly for $x\geq\gamma\mathbb{E}[N_t]$ for any $\gamma>0$.

Remark 9. *Indeed, Klüppelberg and Mikosch (1997) proved a slightly stronger result which says* (2.22) *holds assuming that the claim sizes* C_i *are i.i.d. with a distribution function* $\overline{F}\in ERV(-\alpha,-\beta)$ *for some* $1<\alpha\leq\beta<\infty$, *where* ERV *denotes the space of extended regular varying functions.*

It is usually easy to check (i) and also under the assumptions in Theorem 3, $\frac{N_t}{t}\to\mu$ and $(N_t/t\in\cdot)$ satisfies a large deviation principle with rate function $I(x)$ which is nonzero if and only if $x\neq\mu$. Therefore, if we assume we could prove that $\frac{\mathbb{E}[N_t]}{t}\to\mu$ as $t\to\infty$, then (ii) is satisfied. Moreover (iii) can be replaced by

(iii') For any $\mu'>0$, $c_{\mu'} := \inf_{x\geq\mu'}\frac{I(x)}{x}>0$.

Assume (iii'), we can find some $0 < \delta' < \delta$ such that for any t sufficiently large,

$$\begin{aligned} \sum_{k>(1+\delta)\mathbb{E}[N_t]} \mathbb{P}(N_t > k)(1+\epsilon)^k &\leq \sum_{k>(1+\delta')\mu t} \mathbb{P}(N_t > k)(1+\epsilon)^k \\ &\leq \sum_{k>(1+\delta')\mu t} e^{-(I(k/t)-\epsilon')t}(1+\epsilon)^k \\ &\leq \sum_{k>(1+\delta')\mu t} e^{-(I(k/t)\frac{t}{k}-\epsilon'\frac{1}{(1+\delta')\mu})k}(1+\epsilon)^k \\ &\leq \sum_{k>(1+\delta')\mu t} e^{-(c_{(1+\delta')\mu}-\epsilon'\frac{1}{(1+\delta')\mu})k}(1+\epsilon)^k. \end{aligned} \tag{2.23}$$

If we pick up $\epsilon' > 0$ small enough such that $\epsilon' \frac{1}{(1+\delta')\mu} < c_{(1+\delta')\mu}$, then, we can pick up $\epsilon > 0$ small enough so that $c_{(1+\delta')\mu} - \epsilon' \frac{1}{(1+\delta')\mu} > \log(1+\epsilon)$ and therefore by letting $t \to \infty$, (iii) is satisfied.

3. Examples of Non-Renewal Arrival Processes

3.1. **Example 1: Hawkes Process.** Hawkes process is a simple point process that has self-exciting property, clustering effect and long memory. It was first introduced by Hawkes (1971) and has been widely applied in finance, seismology, neuroscience, DNA modelling and many other fields. A simple point process N_t is a linear Hawkes process if it has intensity

$$\lambda_t = \nu + \sum_{\tau < t} h(t - \tau), \tag{3.1}$$

$h(\cdot) : [0, \infty) \to (0, \infty)$ is integrable and $\|h\|_{L^1} < 1$. We also assume that N_t starts with empty past history, i.e. $N(-\infty, 0] = 0$. By our definition, the Hawkes process is non-stationary and is in general even non-Markovian (unless $h(\cdot)$ is an exponential function). Also, it does not have a regenerative structure. Thus, the conditions in Asmussen and Albrecher (2010) do not apply here.

Notice that it is well known that, (see for example Daley and Vere-Jones (2003))

$$\lim_{t\to\infty} \frac{N_t}{t} = \mu := \frac{\nu}{1 - \|h\|_{L^1}}, \tag{3.2}$$

and Bordenave and Torrisi (2007) proved the a large deviation principle for $(N_t/t \in \cdot)$, i.e. Lemma 10. Therefore, it is natural that we can apply the results in our paper to study the ruin probabilities with subexponential claims when the arrival process is a non-stationary linear Hawkes process.

Lemma 10 (Bordenave and Torrisi (2007))**.** *$(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function,*

$$I(x) = \begin{cases} x \log\left(\frac{x}{\nu + x\|h\|_{L^1}}\right) - x + x\|h\|_{L^1} + \nu & \text{if } x \in [0, \infty) \\ +\infty & \text{otherwise} \end{cases}. \tag{3.3}$$

Remark 11. *Indeed, in Bordenave and Torrisi (2007), they expressed the rate function $I(\cdot)$ in an alternative way, which is less explicit. The expression of the rate function in Lemma 10 was first pointed out in Zhu (2011a).*

Lemma 12. *$\frac{\mathbb{E}[N_t]}{t} \to \frac{\nu}{1-\|h\|_{L^1}}$ as $t \to \infty$.*

Proof. Taking expectation of the indentity $\lambda_t = \nu + \int_0^t h(t-s)N(ds)$, we get

$$\mathbb{E}[\lambda_t] = \nu + \int_0^t h(t-s)\mathbb{E}[\lambda_s]ds \leq \nu + \|h\|_{L^1} \sup_{0\leq s\leq t} \mathbb{E}[\lambda_s]ds, \tag{3.4}$$

which implies that for any t, $\sup_{0\leq s\leq t}\mathbb{E}[\lambda_s] \leq \frac{\nu}{1-\|h\|_{L^1}}$ and therefore $\mathbb{E}[\lambda_t] \leq \frac{\nu}{1-\|h\|_{L^1}}$ uniformly in t. Next, let $H(t) := \int_t^\infty h(s)ds$ and

$$\begin{aligned}\mathbb{E}[N_t] &= \mathbb{E}\left[\int_0^t \lambda_s ds\right] \\ &= \nu t + \int_0^t \int_0^s h(s-u)d\mathbb{E}[N_u]ds \\ &= \nu t + \int_0^t \int_u^t h(s-u)dsd\mathbb{E}[N_u] \\ &= \nu t + \mathbb{E}[N_t]\|h\|_{L^1} - \int_0^t H(t-u)d\mathbb{E}[N_u],\end{aligned} \tag{3.5}$$

which implies that

$$\mathbb{E}[N_t] = \frac{\nu t}{1-\|h\|_{L^1}} - \int_0^t H(t-u)\mathbb{E}[\lambda_u]du, \tag{3.6}$$

and

$$\begin{aligned}\limsup_{t\to\infty}\frac{1}{t}\int_0^t H(t-u)\mathbb{E}[\lambda_u]du &\leq \frac{\nu}{1-\|h\|_{L^1}}\limsup_{t\to\infty}\frac{1}{t}\int_0^t H(t-u)du \\ &= \frac{\nu}{1-\|h\|_{L^1}}\limsup_{t\to\infty}\frac{1}{t}\int_0^t H(u)du = 0,\end{aligned} \tag{3.7}$$

since $H(t) = \int_t^\infty h(s)ds \to 0$ as $t\to\infty$. $\square$

Assume the net profit condition $p > \mathbb{E}[C]\frac{\nu}{1-\|h\|_{L^1}}$.

If C_i have light tails, then Stabile and Torrisi (2010) obtained the asymptotics for the infinite-horizon ruin probability $\psi(u)$ and the finite-horizon ruin probability $\phi(u, uz)$ for any $z > 0$. As pointed out in Stabile and Torrisi (2010) the case when C_i are heavy-tailed is open and now we have the tools to handle the case.

Proposition 13. *Assume the net profit condition $p > \mathbb{E}[C]\frac{\nu}{1-\|h\|_{L^1}}$.*

(i) (Infinite-Horizon)

$$\lim_{u\to\infty}\frac{\psi(u)}{\overline{B}_0(u)} = \frac{\nu\mathbb{E}[C_1]}{p(1-\|h\|_{L^1}) - \nu\mathbb{E}[C_1]}. \tag{3.8}$$

(ii) (Finite-Horizon) For any $T > 0$,

(3.9)

$$\lim_{u\to\infty}\frac{\psi(u,uz)}{\overline{B}_0(u)} = \begin{cases}\frac{\nu\mathbb{E}[C_1]}{p(1-\|h\|_{L^1})-\nu\mathbb{E}[C_1]}\left[1-\left(1+\left(\frac{p(1-\|h\|_{L^1})-\nu\mathbb{E}[C_1]}{p(1-\|h\|_{L^1})}\right)\frac{T}{\alpha}\right)^{-\alpha}\right] & \text{if } \overline{B}\in\mathcal{R}(-\alpha-1) \\ \frac{\nu\mathbb{E}[C_1]}{p(1-\|h\|_{L^1})-\nu\mathbb{E}[C_1]}\left[1-e^{-\frac{p(1-\|h\|_{L^1})-\nu\mathbb{E}[C_1]}{p(1-\|h\|_{L^1})}T}\right] & \text{if } B\in\mathcal{G}\end{cases}.$$

(iii) (Aggregate Claims) For fixed time t,

$$\mathbb{P}(A_t - \mathbb{E}[A_t] > x) \sim \mathbb{E}[N_t]\mathbb{P}(C_1 \geq x), \tag{3.10}$$

uniformly for $x \geq \gamma\mathbb{E}[N_t]$ for any $\gamma > 0$.

Proof. To prove (i) and (ii), by Theorem 3 and Theorem 8, it is enough to check the conditions in Assumption 1. (i) and (ii) of Assumption 2 can be verified by the large deviations result in Lemma 10 and the properties of the rate function. (iii) of Assumption 1 is the assumption of the Proposition 13. To check (iv) of Assumption 1, notice that by the definition of Hawkes process, N_t stochastically dominates N_t^ν, a homogenous Poisson process with parameter $\nu > 0$. But T_i^ν corresponding to N_t^ν are i.i.d. exponentially distributed with parameter ν and they stochastically dominate T_i, the length of time interval between two consecutive arrivals of a Hawkes process. But we know that exponentially distribution has exponential tails and thus for $\theta > 0$ small enough, $\mathbb{E}[e^{\theta\sum_{i=1}^n T_i}] \leq \mathbb{E}[e^{\theta\sum_{i=1}^n T_i^\nu}] = \mathbb{E}[e^{\theta T_1^\nu}]^n < \infty$ for any $n \in \mathbb{N}$. Thus (iv) of Assumption 1 holds. Now, to prove (iii), it is enough to check (i), (ii) and (iii') of Assumption 2. In the proof of Lemma 12, we showed that $\mathbb{E}[\lambda_t] \leq \frac{\nu}{1-\|h\|_{L^1}}$ uniformly in t and thus $\mathbb{E}[N_t] = \mathbb{E}\left[\int_0^t \lambda_s ds\right] \leq \frac{\nu t}{1-\|h\|_{L^1}} < \infty$ and (i) of Assumption 2 is verified. (ii) of Assumption 2 is a result of Lemma 12 and law of large numbers of N_t/t and finally (iii') of Assumption 2 can be verified by easily checking the rate function in Lemma 10. □

Remark 14. *Indeed, the large deviations for nonlinear Hawkes processes have been established in Zhu (2011a) and Zhu (2011b). Unlike linear Hawkes processes, the rate function for the large deviations for nonlinear Hawkes processes are less explicit and it is therefore more difficult to check if it satisfies the conditions in this paper. This has to be left for future investigations.*

3.2. **Example 2: Cox Process with Shot Noise Intensity.** We consider a Cox process N_t with intensity λ_t that follows a shot noise process

$$\lambda_t = \nu(t) + \sum_{\tau^{(1)}<t} g(t - \tau^{(1)}), \tag{3.11}$$

where $\tau^{(1)}$ are the arrival times of an external homogenous Poisson process with intensity γ. Here, $g(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is integrable, i.e. $\int_0^\infty g(t)dt < \infty$ and $\nu(t)$ is a positive, continuous, deterministic function such that $\nu(t) \to \nu$ as $t \to \infty$.

The ruin probabilities for heavy-tailed claims with arrival process being a shot noise Cox process is known in the literature, e.g. see the book by Asmussen and Albrecher (2010). But the techniques in the literature use the very specific features of shot noise Cox process and the proofs are much longer. Our proof essentially only needs the large deviation result for $(N_t/t \in \cdot)$ which is very easy to establish.

Since $N^{(1)}$ is a Poisson process with intensity γ, by the definition of λ_t, it is easy to see that

$$\frac{N_t}{t} \to \nu + \gamma\|g\|_{L^1}, \quad \text{as } t \to \infty. \tag{3.12}$$

It is not clear to the author if the large deviation result for $(N_t/t \in \cdot)$ is known in the literature. For the sake of completeness, let us establish the large deviation principle here.

Lemma 15. *$(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function,*

$$I(x) = \begin{cases} \sup_{\theta\in\mathbb{R}} \left\{\theta x - (e^{\theta}-1)\nu - \gamma(e^{(e^{\theta}-1)\|g\|_{L^1}}-1)\right\} & \text{if } x \in [0,\infty) \\ +\infty & \text{otherwise} \end{cases}. \tag{3.13}$$

Proof. For any $\theta \in \mathbb{R}$, we have

$$\begin{aligned} \mathbb{E}[e^{\theta N_t}] &= \mathbb{E}\left[e^{(e^{\theta}-1)\int_0^t \lambda_s ds}\right] \\ &= e^{(e^{\theta}-1)\int_0^t \nu(s)ds}\mathbb{E}\left[e^{(e^{\theta}-1)\int_0^t\int_0^s g(s-u)N^{(1)}(du)ds}\right] \\ &= e^{(e^{\theta}-1)\int_0^t \nu(s)ds}\mathbb{E}\left[e^{\int_0^t\left[\int_u^t(e^{\theta}-1)g(s-u)ds\right]N^{(1)}(du)}\right] \\ &= e^{(e^{\theta}-1)\int_0^t \nu(s)ds}e^{\gamma\int_0^t(e^{\int_u^t(e^{\theta}-1)g(s-u)ds}-1)du} \\ &= e^{(e^{\theta}-1)\int_0^t \nu(s)ds}e^{\gamma\int_0^t(e^{\int_0^{t-u}(e^{\theta}-1)g(s)ds}-1)du} \\ &= e^{(e^{\theta}-1)\int_0^t \nu(s)ds}e^{\gamma\int_0^t(e^{\int_0^{u}(e^{\theta}-1)g(s)ds}-1)du}. \end{aligned} \tag{3.14}$$

Therefore, we have

$$\lim_{t\to\infty}\frac{1}{t}\log\mathbb{E}[e^{\theta N_t}] = (e^{\theta}-1)\nu + \gamma(e^{(e^{\theta}-1)\|g\|_{L^1}}-1). \tag{3.15}$$

By Gärtner-Ellis theorem, we conclude that $(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function

$$I(x) = \sup_{\theta\in\mathbb{R}}\left\{\theta x - (e^{\theta}-1)\nu - \gamma(e^{(e^{\theta}-1)\|g\|_{L^1}}-1)\right\}. \tag{3.16}$$

Now, if $x < 0$, then for any $\theta < 0$, $\theta x - (e^{\theta}-1)\nu - \gamma(e^{(e^{\theta}-1)\|g\|_{L^1}}-1) \geq \theta x \to \infty$ if we let $\theta \to -\infty$. Hence, $I(x) = +\infty$ for $x < 0$. □

Lemma 16. *$\frac{\mathbb{E}[N_t]}{t} \to \nu + \gamma\|g\|_{L^1}$ as $t \to \infty$.*

Proof. Observe that

$$\begin{aligned} \mathbb{E}[N_t] &= \mathbb{E}\left[\int_0^t \lambda_s ds\right] \\ &= \int_0^t \nu(s)ds + \mathbb{E}\left[\int_0^t\int_0^s g(s-u)N^{(1)}(du)ds\right] \\ &= \int_0^t \nu(s)ds + \gamma\int_0^t\int_0^s g(s-u)duds \\ &= \int_0^t \nu(s)ds + \gamma\int_0^t\int_0^s g(u)duds, \end{aligned} \tag{3.17}$$

which implies that $\frac{\mathbb{E}[N_t]}{t} \to \nu + \gamma\|g\|_{L^1}$ as $t \to \infty$. □

Proposition 17. *Assume the net profit condition $p > \mathbb{E}[C](\nu + \gamma\|g\|_{L^1})$.*
(i) (Infinite-Horizon)

$$\lim_{u\to\infty}\frac{\psi(u)}{\overline{B}_0(u)} = \frac{(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}{p-(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}. \tag{3.18}$$

(ii) (Finite-Horizon) For any $T>0$,

(3.19)

$$\lim_{u\to\infty}\frac{\psi(u,uz)}{\overline{B}_0(u)}$$
$$=\begin{cases}\frac{(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}{p-(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}\left[1-\left(1+\left(1-\frac{(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}{p}\right)\frac{T}{\alpha}\right)^{-\alpha}\right] & \text{if } \overline{B}\in\mathcal{R}(-\alpha-1)\\ \frac{(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}{p-(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1]}\left[1-e^{-(p-(\nu+\gamma\|g\|_{L^1})\mathbb{E}[C_1])T/p}\right] & \text{if } B\in\mathcal{G}\end{cases}.$$

(iii) (Aggregate Claims) For fixed time t,

$$\mathbb{P}(A_t-\mathbb{E}[A_t]>x)\sim\mathbb{E}[N_t]\mathbb{P}(C_1\geq x), \tag{3.20}$$

uniformly for $x\geq\gamma\mathbb{E}[N_t]$ for any $\gamma>0$.

Proof. To prove (i) and (ii), by Theorem 3 and Theorem 8, it is enough to check the conditions in Assumption 1. (i) and (ii) of Assumption 2 can be verified by the large deviations result in Lemma 15 and the properties of the rate function. (iii) of Assumption 1 is the assumption of the Proposition 17. To check (iv) of Assumption 1, notice that by the definition of Hawkes process, N_t stochastically dominates $N_t^{\nu^*}$, an homogenous Poisson process with parameter $\nu^*:=\max_{t\geq 0}\nu(t)$. But $T_i^{\nu^*}$ corresponding to $N_t^{\nu^*}$ are i.i.d. exponentially distributed with parameter ν^* and they stochastically dominate T_i, the length of time interval between two consecutive arrivals of a Hawkes process. But we know that exponentially distribution has exponential tails and thus for $\theta>0$ small enough, $\mathbb{E}[e^{\theta\sum_{i=1}^n T_i}]\leq\mathbb{E}[e^{\theta\sum_{i=1}^n T_i^{\nu^*}}]=\mathbb{E}[e^{\theta T_1^{\nu^*}}]^n<\infty$ for any $n\in\mathbb{N}$. Thus (iv) of Assumption 1 holds. Now, to prove (iii), it is enough to check (i), (ii) and (iii') of Assumption 2. It is easy to see that that $\mathbb{E}[\lambda_t]=\nu(t)+\gamma\int_0^t g(s)ds<\infty$ for any $t>0$ and thus $\mathbb{E}[N_t]=\mathbb{E}\left[\int_0^t\lambda_s ds\right]<\infty$ and (i) of Assumption 2 is verified. (ii) of Assumption 2 is a result of Lemma 16 and law of large numbers of N_t/t and finally (iii') of Assumption 2 can be verified by easily checking the rate function in Lemma 15. ☐

3.3. **Example 3: Self-Correcting Point Process.** A self-correcting point process, also known as the stress-release model, is a simple point process N with empty history, i.e. $N(-\infty,0]=0$ such that it admits the $\mathcal{F}_t$-intensity

$$\lambda_t:=\lambda(Z_t),\quad\text{and}\quad Z_t:=t-N_{t-}. \tag{3.21}$$

The rate function $\lambda(\cdot):\mathbb{R}\to\mathbb{R}^+$ is continuous and increasing such that

$$0<\lambda^-=\lim_{z\to-\infty}\lambda(z)<1<\lim_{z\to+\infty}\lambda(z)=\lambda^+<\infty. \tag{3.22}$$

Notice that in the definition of intensity in (3.21), we used N_{t-} instead of N_t. That is crucial to guarantee that the intensity λ_t for the self-correcting point process is $\mathcal{F}_t$-predictable.

The model was first introduced by Isham and Westcott (1979) as an example of a process that automatically corrects a deviation from its mean. Later, it was studied as a model in seismology. The stress builds up at the linear rate 1 in our model and releases by the amount 1 at ith jump. Vere-Jones (1988) discussed an insurance interpretation.

Under these assumptions, it is well known that $\frac{N_t}{t} \to 1$ as $t \to \infty$ (See for example Proposition 4.3 in Zheng (1991)). Recently, Sen and Zhu (2013) proved the following large deviation result.

Lemma 18 (Sen and Zhu (2013))**.** *$(N_t/t \in \cdot)$ satisfies a large deviation principle with rate function*

$$I(x) = \begin{cases} \Lambda^-(x) & \text{if } x > 1 \\ 0 & \text{if } x = 1 \\ \Lambda^+(x) & \text{if } 0 \le x < 1 \\ +\infty & \text{otherwise} \end{cases}, \tag{3.23}$$

where

$$\Lambda^{\pm}(x) = \log\left(\frac{x}{\lambda^{\pm}}\right) x + \lambda^{\pm} - x, \quad x \ge 0. \tag{3.24}$$

Lemma 19. $\frac{\mathbb{E}[N_t]}{t} \to 1$ *as* $t \to \infty$*.*

Proof. $\mathbb{E}[N_t] = \mathbb{E}\left[\int_0^t \lambda(Z_s)ds\right]$. Zheng Zheng (1991) proved that there exists a unique invariant measure $\pi(dz)$ for the Markov process Z_t. By ergodic theorem, we have

$$\frac{1}{t}\int_0^t \lambda(Z_s)ds \to \int \lambda(z)\pi(dz), \tag{3.25}$$

as $t \to \infty$. We know that $Z_t = t - N_t$ has the generator

$$\mathcal{A}f(z) = \frac{\partial f}{\partial z} + \lambda(z)(f(z-1) - f(z)), \tag{3.26}$$

and we have $\mathcal{A}z\pi = 0$ which implies that $\int \lambda(z)\pi(dz) = 1$ and thus $\frac{1}{t}\int_0^t \lambda(Z_s)ds \to 1$ a.s. as $t \to \infty$. Since $\lambda^- \le \lambda(\cdot) \le \lambda^+$, by bounded convergence theorem, we conclude that $\frac{\mathbb{E}[N_t]}{t} \to 1$ as $t \to \infty$. $\square$

Proposition 20. *Assume the net profit condition $p > \mathbb{E}[C]$.*

(i) (Infinite-Horizon)

$$\lim_{u\to\infty} \frac{\psi(u)}{\overline{B}_0(u)} = \frac{\mathbb{E}[C_1]}{p - \mathbb{E}[C_1]}. \tag{3.27}$$

(ii) (Finite-Horizon) For any $T > 0$,

$$\lim_{u\to\infty} \frac{\psi(u,uz)}{\overline{B}_0(u)} = \begin{cases} \frac{\mathbb{E}[C_1]}{p-\mathbb{E}[C_1]}\left[1 - \left(1 + \left(1 - \frac{\mathbb{E}[C_1]}{p}\right)\frac{T}{\alpha}\right)^{-\alpha}\right] & \text{if } \overline{B} \in \mathcal{R}(-\alpha-1) \\ \frac{\mathbb{E}[C_1]}{p-\mathbb{E}[C_1]}\left[1 - e^{-(p-\mathbb{E}[C_1])T/p}\right] & \text{if } B \in \mathcal{G} \end{cases}. \tag{3.28}$$

(iii) (Aggregate Claims) For fixed time t,

$$\mathbb{P}(A_t - \mathbb{E}[A_t] > x) \sim \mathbb{E}[N_t]\mathbb{P}(C_1 \ge x), \tag{3.29}$$

uniformly for $x \ge \gamma\mathbb{E}[N_t]$ for any $\gamma > 0$.

Proof. To prove (i) and (ii), by Theorem 3 and Theorem 8, it is enough to check the conditions in Assumption 1. (i) and (ii) of Assumption 2 can be verified by the large deviations result in Lemma 18 and the properties of the rate function. (iii) of Assumption 1 is the assumption of the Proposition 20. To check (iv) of Assumption 1, notice that by the definition of Hawkes process, N_t stochastically dominates $N_t^{\lambda^-}$, an homogenous Poisson process with parameter λ^-. But $T_i^{\lambda^-}$ corresponding to $N_t^{\lambda^-}$ are i.i.d. exponentially distributed with parameter λ^- and they stochastically dominate T_i, the length of time interval between two consecutive arrivals of a Hawkes process. But we know that exponentially distribution has exponential tails and thus for $\theta > 0$ small enough, $\mathbb{E}[e^{\theta \sum_{i=1}^n T_i}] \leq \mathbb{E}[e^{\theta \sum_{i=1}^n T_i^{\lambda^-}}] = \mathbb{E}[e^{\theta T_1^{\lambda^-}}]^n < \infty$ for any $n \in \mathbb{N}$. Thus (iv) of Assumption 1 holds. Now, to prove (iii), it is enough to check (i), (ii) and (iii') of Assumption 2. It is easy to see that that $\lambda_t \leq \lambda^+ < \infty$ for any $t > 0$ and thus $\mathbb{E}[N_t] = \mathbb{E}\left[\int_0^t \lambda_s ds\right] \leq \lambda^+ t < \infty$ and (i) of Assumption 2 is verified. (ii) of Assumption 2 is a result of Lemma 19 and law of large numbers of N_t/t and finally (iii') of Assumption 2 can be verified by easily checking the rate function in Lemma 18. □

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Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York, NY-10012
United States of America
E-mail address: ling@cims.nyu.edu