Dirac particles tunneling from black holes with topological defects

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Abstract

We study Hawking radiation of Dirac particles with spin-1/2 as a tunneling process from Schwarzschild-de Sitter and Reissner-Nordström-de Sitter black holes in background spacetimes with a spinning cosmic string and a global monopole. Solving Dirac's equation by employing the Hamilton-Jacobi method and WKB approximation we find the corresponding tunneling probabilities and the Hawking temperature. Furthermore, we show that the Hawking temperature of black holes remains unchanged in presence of topological defects in both cases.

1 Introduction

After the original Hawking derivation of black holes temperature [1], a number of different approaches were introduced, among others, the Wick rotation method [2, 3], quantum tunneling [4], anomaly method [15], and the technique of dimensional reduction [16]. The tunneling method treated Hawking radiation as a tunneling process using semi classical WKB approximation, where the particle can quantum mechanically tunnel through the horizon and it is observed at infinity as a real particle. The tunneling rate is related to the imaginary part of the action in the classically forbidden region. Generally, there are two methods to obtain the imaginary part of the action. In the first method used by Parikh and Wilczek, the imaginary part of the action is calculated by integrating the radial momentum of the particles. In the second method, [17, 18], the imaginary part of the action is obtained by solving the relativistic Hamilton-Jacobi equation. The quantum tunneling method has been studied in great details for a number of spherically symmetric and stationary spacetimes black holes and also for different particles, including a scalar particles with spin-0, Dirac particles with spin-1/2 and spin-3/2 particles. In particular, the tunneling from the rotating Kerr black hole, Kerr-Newman black hole [7, 10], black hole with topological defects [8, 9, 22], Kerr de Sitter and Kerr-Newman de Sitter black hole [5], black strings [6], black holes with NUT parameter [11] and many others.

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The aim of this paper, is to extend this method for Dirac particles with spin 1/2 to tunneling from the Schwarzschild-de Sitter (SdS) and Reissner-Nordström-de Sitter (RNdS) black holes pierced by an infinitely long spinning cosmic string and a global monopole. The Hawking temperature is shown to be invariant of the nature of particles, including the presence of topological defects. Topological defects may have been produced by the phase transition in the early universe. A spinning cosmic string is characterized by the rational parameter a and the angular parameter J, given by $a = 4J$. The spacetime of cosmic string gives rise to a number of interesting phenomena, cosmic string can act as a gravitational lens [19], it can induce a finite electrostatic self-force on an electric charged particle [20], shifts in the energy levels of a hydrogen atom [21], they were also suggested as an explanation of the anisotropy of the cosmic microwave background radiation.

This paper is organised as follows. In Section 2, we briefly review and introduce the stationary line element near the horizon for SdS black hole in the cosmic string and global monopole background. In Section 3, we calculate the tunneling rate of massive/massless Dirac particles and the corresponding Hawking temperature from this spacetime. In Section 4, similary, we introduce the stationary line element for RNdS black hole in the cosmic string and global monopole background and calculate the tunneling rate and Hawking temperature for charged Dirac particles. In Section 5, we comment on our results.

2 SdS black hole with topological defects

In order to write down the metric of Schwarzschild-de Sitter black hole with positive cosmological factor Λ pierced by an infinitely long spinning cosmic string and a global monopole, one can introduce the rotation of an infinitely long cosmic string by simply doing the transformation $\mathrm{d}t \to \mathrm{d}t + a\,\mathrm{d}\phi$ [23]. Therefore the line element reads

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} - \frac{r^2}{l^2}\right)(\mathrm{d}t + a\,\mathrm{d}\phi)^2 + \left(1 - \frac{2M}{r} - \frac{r^2}{l^2}\right)^{-1} \mathrm{d}r^2 + r^2 p^2 \left(\mathrm{d}\theta^2 + b^2 \sin^2\theta\,\mathrm{d}\phi^2\right) \tag{2.1}$$

where a is the rational parameter of a cosmic string and $l^2 = 3/\Lambda^2$. In this paper, we will consider an idealized cosmic string with a parameter a constant with time, related to the angular parameter J, with $a = 4J$. The presence of a global monopole and a cosmic string is encoded via $p^2 = 1 - 8\pi\eta^2$ and $b^2 = (1 - 4\mu)^2$ respectively. We can sole $r^3 + 2Ml^2 - rl^2 = 0$, and get the black hole event horizon r_H and cosmological horizon r_C, given by

$$r_H = \frac{2M}{3\Xi} \cos\frac{\pi + \psi}{3}, \tag{2.2}$$

$$r_C = \frac{2M}{3\Xi} \cos\frac{\pi - \psi}{3}, \tag{2.3}$$

where

$$\psi = \cos^{-1}(3\sqrt{3}\Xi). \tag{2.4}$$

Here $\Xi = M^2/l^2$ and belongs to the interval $0 < \Xi < 1/27$. Expanding r_H in terms of M with $\Xi < 1/27$, leads to (see, e.g., [12])

$$r_H = 2M\left(1 + \frac{4M^2}{l^2} + \cdots\right), \tag{2.5}$$

clearly, in the limit $\Xi \to 0$, it follows $r_H \to 2M$. For the sake of convenience, let us write the metric (2.1) near the event horizon. For that purpose, one can define $\Delta = r^2 - 2Mr^2 - r^4/l^2$, so the line element near the event horizon becomes

$$\begin{aligned} ds^2 = & - \frac{\Delta_{,r}(r_H)(r - r_H)}{r_H^2}(dt + a\,d\phi)^2 + \frac{r_H^2}{\Delta_{,r}(r_H)(r - r_H)}dr^2 \\ & + r_H^2 p^2\left(d\theta^2 + b^2\sin^2\theta\,d\phi^2\right), \end{aligned} \tag{2.6}$$

where

$$\Delta_{,r}(r_H) = \left.\frac{d\Delta}{dr}\right|_{r=r_H} = 2\left(r_H - M - 2\frac{r_H^3}{l^2}\right). \tag{2.7}$$

Due to the frame-dragging effect of the coordinate system in the stationary rotating spacetime, we can perform the dragging coordinate transformation $\varphi = \phi - \Omega t$, where

$$\Omega_{b,p}(r) = \frac{a\,\Delta_{,r}(r_H)(r - r_H)}{r_H^4 p^2 b^2 \sin^2\theta - a^2\Delta_{,r}(r_H)(r - r_H)}. \tag{2.8}$$

In this way the metric (2.1) can be written in a more compact form

$$ds^2 = -F(r)dt^2 + \frac{1}{G(r)}dr^2 + K^2(r)d\theta^2 + H^2(r)d\varphi^2, \tag{2.9}$$

where

$$F(r) = \frac{b^2p^2r_H^2\sin^2\theta\Delta_{,r}(r_H)(r - r_H)}{b^2p^2r_H^4\sin^2\theta - a^2\Delta_{,r}(r_H)(r - r_H)}, \tag{2.10}$$

$$G(r) = \frac{\Delta_{,r}(r_H)(r - r_H)}{r_H^2}, \tag{2.11}$$

$$K^2(r) = p^2r_H^2, \tag{2.12}$$

$$R^2(r) = p^2b^2r_H^2\sin^2\theta - a^2\frac{\Delta_{,r}(r_H)(r - r_H)}{r_H^2}. \tag{2.13}$$

In what follows, we will use the metric (2.9), to study the tunneling rate of spin-1/2 particles from the event horizon. The tunneling rate is related to the imaginary part of the action in the classically forbidden region given by

$$\Gamma \sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}\,S\right) \tag{2.14}$$

3 Tunnelig of spin-1/2 particles from SdS black hole

The motion of Dirac particles with mass m in curved spacetime for the spinor field Ψ, is given by Dirac's equation

$$i\gamma^{\mu}\left(D_{\mu}\right)\Psi+\frac{m}{\hbar}\Psi=0, \tag{3.1}$$

where

$$D_{\mu}=\partial_{\mu}+\Omega_{\mu},\quad \Omega_{\mu}=\frac{i}{2}\Gamma_{\mu}^{\alpha\beta}\Sigma_{\alpha\beta},\qquad \Sigma_{\alpha\beta}=\frac{i}{4}\left[\gamma^{\alpha},\gamma^{\beta}\right].$$

The γ^{μ} matrices satisfy the properties $\left[\gamma^{\alpha},\gamma^{\beta}\right]=-\left[\gamma^{\beta},\gamma^{\alpha}\right]$, when $\alpha\neq\beta$ and $\left[\gamma^{\alpha},\gamma^{\beta}\right]=0$, when $\alpha=\beta$. On the other hand, using the values of the Christoffel symbols $\Gamma_{\mu}^{\alpha\beta}$, yields $\Omega_{\mu}=0$. In this way Dirac's equation (3.1) can be written as

$$i\gamma^{t}\partial_{t}\Psi+i\gamma^{r}\partial_{r}\Psi+i\gamma^{\theta}\partial_{\theta}\Psi+i\gamma^{\varphi}\partial_{\varphi}\Psi+\frac{m}{\hbar}\Psi=0. \tag{3.2}$$

We are free to choose the γ^{μ} matrices in different ways, let us for simplicity choose γ^{μ} matrices as

$$\gamma^{t}=\frac{1}{\sqrt{F(r)}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix},\quad \gamma^{r}=\sqrt{G(r)}\begin{pmatrix} 0 & \sigma^{3}\\ \sigma^{3} & 0\end{pmatrix},$$

$$\gamma^{\theta}=\frac{1}{K(r)}\begin{pmatrix} 0 & \sigma^{1}\\ \sigma^{1} & 0\end{pmatrix},\quad \gamma^{\varphi}=\frac{1}{H(r)}\begin{pmatrix} 0 & \sigma^{2}\\ \sigma^{2} & 0\end{pmatrix},$$

where $\sigma^{i}\,(i=1,2,3)$ are the Pauli matrices

$$\sigma^{1}=\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix},\ \sigma^{2}=\begin{pmatrix} 0 & -i\\ i & 0\end{pmatrix},\ \sigma^{3}=\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}.$$

As we know, the state of Dirac particles with spin-1/2 is described by two corresponding states, spin-up and spin-down states. In order to solve Dirac's equation (3.2) we can use the following ansatz for Dirac's field Ψ

$$\Psi_{\uparrow}\left(t,r,\theta,\varphi\right)=\begin{pmatrix} A\left(t,r,\theta,\varphi\right)\\ 0\\ B\left(t,r,\theta,\varphi\right)\\ 0\end{pmatrix}\exp\left(\frac{i}{\hbar}S_{\uparrow}\right), \tag{3.3}$$

corresponding to spin up case ($\uparrow$), and

$$\Psi_{\downarrow}\left(t,r,\theta,\varphi\right)=\begin{pmatrix} 0\\ C\left(t,r,\theta,\varphi\right)\\ 0\\ D\left(t,r,\theta,\varphi\right)\end{pmatrix}\exp\left(\frac{i}{\hbar}S_{\downarrow}\right), \tag{3.4}$$

for the spin down case ($\downarrow$). Here, $S_\uparrow$ and $S_\downarrow$ donates the corresponding action of Dirac particles with spin ($\uparrow$) and ($\downarrow$), A and B are two arbitrary functions of the coordinates. Using the symmetries of the metric (2.9), given by Killing vectors, we can choose, therefore, the following ansatz for the action in the spin up case

$$S_\uparrow(t,r,\theta,\varphi) = -\left(E_{b,p} - J_{b,p}\Omega_{b,p}\right)t + R(r) + J_{b,p}\varphi + \Theta(\theta) \tag{3.5}$$

here $E_{b,p}$ is the energy of the emitted particles measured at infinity, and $J_{b,p}$ is the angular quantum number of the particle. However, since a topological defects exits, the Komar's energy $E_{b,p}$ and angular quantum number $J_{b,p}$ of the particles are decreased by a factor of p^2b. Namely, the energy and the angular quantum number are $E_{b,p} = p^2bE$, and $J_{b,p} = p^2bJ$, respectively. Inserting the Eq.(3.3), into Eq.(3.2), and divide by the exponential term and multiply by $\hbar$ we end up with the following four equations

$$-i\frac{A\left(\partial_t S_\uparrow\right)}{\sqrt{F(r)}} - B\sqrt{G(r)}\left(\partial_r S_\uparrow\right) + mA = 0, \tag{3.6}$$

$$-B\left(\frac{\left(\partial_\theta S_\uparrow\right)}{K(r)} + \frac{i}{H(r)}\left(\partial_\varphi S_\uparrow\right)\right) = 0, \tag{3.7}$$

$$i\frac{B\left(\partial_t S_\uparrow\right)}{\sqrt{F(r)}} - A\sqrt{G(r)}\left(\partial_r S_\uparrow\right) + mB = 0, \tag{3.8}$$

$$-A\left(\frac{\left(\partial_\theta S_\uparrow\right)}{K(r)} + \frac{i}{H(r)}\left(\partial_\varphi S_\uparrow\right)\right) = 0. \tag{3.9}$$

At first, it seems that Eqs.(3.7) and (3.9), suggest that there should be a contribution to the imaginary part of the action coming from $\Theta(\theta)$, however, one can show that the contribution of $\Theta(\theta)$ to the imaginary part of the action is cancelled out, since the contribution from $\Theta(\theta)$ is completely same for both the outgoing and ingoing solutions [5]. Hence, only the first and the third equation remains to be discussed. The radial part $R(r)$ of the action $S_\uparrow$ can be calculated from the following equations

$$ip^2b\,A\left(E - J\Omega_{b,p}\right) - B\sqrt{F(r)\,G(r)}\,\partial_r R + mA\sqrt{F(r)} = 0, \tag{3.10}$$

$$ip^2b\,B\left(E - J\Omega_{b,p}\right) + A\sqrt{F(r)\,G(r)}\,\partial_r R - mB\sqrt{F(r)} = 0. \tag{3.11}$$

Solving first for the massless case, $m = 0$, we get two solutions $A = \pm iB$. Therefore, the radial part of the action reads

$$R_\pm(r) = \pm\int \frac{p^2\,b\,\left(E - J\Omega_{b,p}\right)}{\sqrt{F(r)\,G(r)}}\mathrm{d}r \tag{3.12}$$

where $+/-$ correspond to the outgoing/ingoing solutions. Solving the last equation by integrating around the pole $r = r_H$, we find

$$R_\pm(r_H) = \pm\frac{\pi i r_H^2 p^2 b\left(E - J\Omega_{b,p}(r_H)\right)}{\Delta_{,r}(r_H)}. \tag{3.13}$$

We can now express the imaginary part of this result near the event horizon using $r_H = 2M(1+4M^2/l^2+...)$, where the angular velocity vanishes i.e. $\Omega_{b,p}(r_H)=0$, this leads to

$$\mathrm{Im}R_+(r_H) = 2\pi M p^2 b\left(1+\frac{16M^2}{l^2}+\cdots\right)E. \tag{3.14}$$

On the other hand, the probabilities of crossing the horizon for Dirac particles in each direction are given by

$$P_{emission} \sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}S_\uparrow\right) = \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}R_+\right), \tag{3.15}$$

$$P_{absorption} \sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}S_\downarrow\right) = \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}R_-\right). \tag{3.16}$$

Since we are interested in computing the probability of Dirac particles tunneling from inside to outside the horizon we write

$$\Gamma = \frac{P_{emission}}{P_{absorption}} = \frac{\exp\left(-2\,\mathrm{Im}R_+\right)}{\exp\left(-2\,\mathrm{Im}R_-\right)} = \exp\left(-4\,\mathrm{Im}R_+\right), \tag{3.17}$$

in the last equation we have set Planck's reduced constant equal to unity. Taking the imagionary part of $R_+(r)$ near the horizon the tunneling rate becomes

$$\Gamma = \exp\left[-8\pi M p^2 b\left(1+\frac{16M^2}{l^2}+\cdots\right)E\right]. \tag{3.18}$$

In order to find the Hawking temperature of the black hole we have to compere the last equation with the Boltzmann factor $\Gamma = \exp\left(-\beta\left(E_{b,p} - J_{b,p}\Omega_{b,p}(r_H)\right)\right)$, where $\beta = 1/T_H$. The Hawking temperature at the event horizon from SdS black holes with topological defects reads

$$T_H = \frac{1}{8\pi M}\left(1-\frac{16M^2}{l^2}\right). \tag{3.19}$$

From the last two equations it's clear that Hawking radiation deviates from pure thermality, as a consequence, there is a correction to the Hawking temperature of SdS black hole. However, this result shows that Hawking temperature is unchanged in the presence of topological defects. In the particular case, setting $l \to \infty$, i.e., $\Lambda = 0$, the Hawking temperature reduces to Schwarzschild black hole temperature. For the massive case, $m \neq 0$, using Eqs.(3.10) and (3.11) we get

$$\left(\frac{A}{B}\right)^2 = -\frac{ip^2 b\left(E - J\Omega_{b,p}\right)\sqrt{F(r)\,G(r)} - mF(r)\sqrt{G(r)}}{ip^2 b\left(E - J\Omega_{b,p}\right)\sqrt{F(r)\,G(r)} + mG(r)\sqrt{F(r)}}. \tag{3.20}$$

However, near the horizon, $r_H = 2M(1+4M^2/l^2+\cdots)$, we get $A^2 = -B^2$, since $F(r_H) = G(r_H) = 0$, yielding the same Hawking temperature as in the massless case. In other words, the mass m of the particle plays no relevant role in the process of Hawking radiation.

4 Tunneling from RNdS black holes

The line element of the Reissner-Nordström black hole with positive Λ in the background spacetime with a spinning cosmic string and a global monopole is given by

$$\begin{aligned} \mathrm{d}s^2 &= -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{r^2}{l^2}\right)(\mathrm{d}t+a\,\mathrm{d}\phi)^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{r^2}{l^2}\right)^{-1}\mathrm{d}r^2 \\ &+ r^2p^2\left(\mathrm{d}\theta^2+b^2\sin^2\theta\,\mathrm{d}\phi^2\right). \end{aligned} \tag{4.1}$$

Solving for $r^4-l^2r^2+2Ml^2r-l^2Q^2=0$, we can get the event horizon r_H and the cosmological horizon r_C location. Without getting into details (see, e.g., [13]), after expanding r_H in terms of M, Q and l, with $\Xi<1/27$, it was found

$$r_H=\frac{1}{\alpha}\left(1+\frac{4M^2}{l^2\alpha^2}+\cdots\right)\left(M+\sqrt{M^2-Q^2\alpha}\right) \tag{4.2}$$

where $\alpha=\sqrt{1+4Q^2/l^2}$. By following the same arguments that have been used in the last section, we can define $\tilde{\Delta}=r^2+Q^2-2Mr^2-r^4/l^2$, in this way the metric (4.1) near the horizon takes a similar form as (2.9), since

$$\Delta_{,r}(r_H)=\left.\frac{\mathrm{d}\tilde{\Delta}}{\mathrm{d}r}\right|_{r=r_H}=2\left(r_H-M-2\frac{r_H^2}{l^2}\right). \tag{4.3}$$

In what follows, we will use this result to study the Dirac equation and calculate the tunneling rate of spin-1/2 particles from RNdS black holes in the cosmic string and global monopole background. The equation which has to be solved is the charged Dirac equation for a particle with mass m, and charge q, given by

$$i\gamma^\mu\left(\partial_\mu+\Omega_\mu-\frac{iq}{\hbar}A_\mu\right)\Psi+\frac{m}{\hbar}\Psi=0 \tag{4.4}$$

where A_μ is the electromagnetic four-potential given by $A_\mu=(A_t,0,0,0)$. Choosing the γ^μ matrices as before, applying WKB approximation and divide by the exponential term and multiply by $\hbar$ we end up with four equations

$$-i\frac{A\left((\partial_t S_\uparrow)-qA_t\right)}{\sqrt{F(r)}}-B\sqrt{G(r)}\,(\partial_r S_\uparrow)+mA=0, \tag{4.5}$$

$$-B\left(\frac{(\partial_\theta S_\uparrow)}{K(r)}+\frac{i}{H(r)}(\partial_\varphi S_\uparrow)\right)=0, \tag{4.6}$$

$$i\frac{B((\partial_t S_\uparrow)-qA_t)}{\sqrt{F(r)}}-A\sqrt{G(r)}\,(\partial_r S_\uparrow)+mB=0, \tag{4.7}$$

$$-A\left(\frac{(\partial_\theta S_\uparrow)}{K(r)}+\frac{i}{H(r)}(\partial_\varphi S_\uparrow)\right)=0, \tag{4.8}$$

Following the same arguments that have been used in the last section, we will focus only on the first and third equation. It should also be stressed that due to the presence of topological defects, the charge of the Dirac particles also shifts $q \to p^2 b\, q$. For massless Dirac particles $m=0$, we can find the radial part $R(r)$ of the action $S_\uparrow$ using Eq.(4.5) and Eq. (4.7), yielding

$$\begin{aligned} i\frac{p^2 bA}{\sqrt{F(r)}}\left(E - J\Omega_{b,p} + qA_t\right) - B\sqrt{G(r)}\left(\partial_r R(r)\right) &= 0, && (4.9)\\ -i\frac{p^2 bB}{\sqrt{F(r)}}\left(E - J\Omega_{b,p} + qA_t\right) + A\sqrt{G(r)}\left(\partial_r R(r)\right) &= 0. && (4.10)\end{aligned}$$

Similarly, we get two solutions, $A = \pm iB$, corresponding to the outgoing/ingoing solutions. The radial part of the action reads

$$R_\pm(r) = \pm \int \frac{p^2 b\left(E - J\Omega_{b,p} + qA_t\right)}{\sqrt{F(r)\,G(r)}}\mathrm{d}r. \tag{4.11}$$

We can solve the last equation by integrating around the pole at the event horizon $r = r_H$, the dragged angular velocity vanishes at the horizon, i.e., $\Omega_{b,p}(r_H) = 0$, yielding

$$R_\pm(r_H) = \pm\frac{\pi i r_H^2 p^2 b(E + qA_t)}{\Delta_{,r}(r_H)}. \tag{4.12}$$

Neglecting M^3 terms and its higher order terms near the event horizon and using the electromagnetic potential of the black hole $A_t = Q/r_H$, the imaginary part of the last equation reads

$$\mathrm{Im}R_+(r_H) = \frac{\pi}{2\alpha}\frac{\left(M + \sqrt{M^2 - Q^2\alpha}\right)^2 p^2 b\left(E + qA_t\right)}{M(1-\alpha) + \sqrt{M^2 - Q^2\alpha}}. \tag{4.13}$$

We therefore conclude that the tunnelling rate of the charged Dirac particles at the event horizon is

$$\Gamma = \exp\left\{-\frac{2\pi}{\alpha}\frac{\left(M + \sqrt{M^2 - Q^2\alpha}\right)^2 p^2 bE}{M\left(1-\alpha\right) + \sqrt{M^2 - Q^2\alpha}}\left[1 - \frac{\alpha e Q}{M + \sqrt{M^2 - Q^2\alpha}}\left(1 - \frac{4M^2}{l^2\alpha^2} + ...\right)\right]\right\}. \tag{4.14}$$

The Hawking temperature of Dirac particles for Reissner-Nordström black holes in spacetime of topological defects can be found by comparing the last equation with the Boltzmann factor $\Gamma = \exp(-\beta\left(E_{b,p} - J_{b,p}\Omega_{b,p}(r_H)\right))$, it follows

$$T_H = \frac{\alpha}{2\pi}\frac{M(1-\alpha) + \sqrt{M^2 - Q^2\alpha}}{\left(M + \sqrt{M^2 - Q^2\alpha}\right)^2} \tag{4.15}$$

As expected, there are corrections to the Hawking radiation, but the Hawking temperature remains unaltered in presence of topological defects. In the particular case, setting $l \to \infty$, i.e., $\Lambda = 0$, and $\alpha = 1$, we recover the Hawking temperature of charged Dirac particles for RNdS black hole without topological defects. Clearly, for uncharged particles $q = 0$, the last equation reduces to Perkih's result. In the case of massive Dirac particles $m \neq 0$, we can use Eqs. (4.5) and (4.7) and get the following result

$$\left(\frac{A}{B}\right)^2 = -\frac{i\,p^2 b\,(E - J\Omega + qA_t)\,\sqrt{F\,(r)\,G\,(r)} - mF\,(r)\,\sqrt{G\,(r)}}{i\,p^2 b\,(E - J\Omega + qA_t)\,\sqrt{F\,(r)\,G\,(r)} + mG\,(r)\,\sqrt{F\,(r)}}. \tag{4.16}$$

At the event horizon $r_H = (1/\alpha)\,(1 + 4M^2/l^2\alpha^2 + \cdots)\,(M + \sqrt{M^2 - Q^2\alpha})$, we get $A^2 = -B^2$, since $F(r_H) = G(r_H) = 0$, those, we have shown that the mass of the particles is irrelevant in this process.

5 Conclusion

In this paper, we have extended the quantum tunneling of Dirac particles with spin 1/2, from Schwarzschild-de Sitter and Reissner-Nordström-de Sitter black holes in the spacetimes background with a spinning cosmic string and a global monopole. The Dirac's equation has been solved via WKB approximation and using the Hamilton-Jacobi equation. Taking into account the change of the Komer's energy, angular quantum number, and charge of the particles in spacetimes with topological defects, we have calculated the tunneling rate and the correspondin black holes Hawking temperature in both cases. As a result, it is shown that Hawking temperature remains unchanged for massive as well as for massless Dirac particles and unaffected by the presence of topological defects in both cases. The results agree in full with Parikh's conclusion [4].

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