Landauer transport model for Hawking radiation from a Reissner-Nordstrom black hole

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The recent work of Nation et al in which Hawking radiation energy and entropy flow from a black hole can be regarded as a one-dimensional (1D) Landauer transport process is extended to the case of a Reissner-Nordstrom (RN) black hole. It is found that the flow of charge current can also be transported via a 1D quantum channel except the current of Hawking radiation. The maximum entropy current, which is shown to be particle statistics independence, is also obtained.
Keywords: Hawking radiation; entropy; Landauer transport model; Reissner-Nordstrom black hole

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I. INTRODUCTION

Hawking radiation from a black hole is an eternal topic in theoretical physics because it provides not only a clue to detect black holes but also a platform to research the quantum gravity. Since the first proposal of Hawking [1, 2] that a black hole can emit radiation in the curved space time background, there are many methods to derive it [3–9]. Now it is believed that Hawking radiation arises from the production of virtual particle pairs spontaneously near the inner of horizon due to the vacuum fluctuation. When the negative energy virtual particle tunnels inwards, the positive energy virtual particle materializes as a real particle and escapes to infinity [10–21]. But, how does the positive energy particle run? Recently, Nation et al [22] gave a model where the positive energy particle escapes to infinity via a 1D quantum channel. The key idea there was that the black hole and vacuum can be viewed as two thermal reservoirs connected by a single 1D quantum channel. As the scattering effect was ignored, Hawking radiation flow rate was obtained using Landauer transport model, which is shown to be equal to the energy-momentum tensor expectation value of an infinite observer as the left and right are regarded as the black hole and the thermal environment with absolute temperature zero surrounding the black hole. They also found that the upper limits of entropy current were same for both bosons or fermions.

In this letter, we would like to extend this idea to study Hawking radiation of charged particles from a RN black hole. When the near horizon conformal symmetry is considered, the expectation value of electric current flow must be investigated except the flow of energy-momentum tensor due to the existence of electromagnetic field. In Landauer transport model, the flow of charge current must also be considered besides the energy and entropy flows. In the letter, we use the units $G = \hbar = c = k_B = 1$.

II. CONFORMAL SYMMETRY AND HAWKING RADIATION FLUX

The line element of a RN black hole is

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2, \tag{1}$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r}, \tag{2}$$

in which M, Q are the mass and charge of the black hole. The electromagnetic four-vector is

$$A_\mu = (-\frac{Q}{r}, 0, 0, 0). \tag{3}$$

From Eq.(1), one can get the event horizon $r_h = M + \sqrt{M^2 - Q^2}$ immediately.

According to the dimensional reduction technique, Eq.(1) can be expressed as the two-dimensional form effectively

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2. \tag{4}$$

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Under the tortoise coordinate transformation defined as $dr^* = \frac{1}{f(r)}dr$ and the null coordinates $u = t + r^*$, $v = t - r^*$, we can construct the Kruskal coordinates as $U = -\frac{1}{\kappa}e^{-\kappa u}$, $V = \frac{1}{\kappa}e^{\kappa V}$, so we get the corresponding conformal form

$$ds^2 = -f(r)dudv, \tag{5}$$

$$ds^2 = -f(r)e^{-2\kappa r^*}dUdV, \tag{6}$$

in which $\kappa = \frac{r_h - M}{r_h^2}$ is the surface gravity.

In the following, we intend to find the expectation values of energy-momentum tensor and gauge current. It is well known that the classical Einstein field equation can be derived from the classical action by the minimal variational principle. In the semiclassical theory, this principle is still valid and thus we have

$$\frac{2}{\sqrt{-g}}\frac{\delta\Gamma}{\delta g^{\mu\nu}} = \langle T_{\mu\nu}\rangle, \tag{7}$$

in which Γ is the effective action with central charge $c = 1$. In the gravitational field with electric field background, the action consists the contributions of gravitational part [23–28]

$$\Gamma_{grav} = \frac{1}{96\pi}\int d^2xd^2y\sqrt{-g}R(x)\frac{1}{\Delta_g}(x,y)\sqrt{-g}R(y), \tag{8}$$

and the gauge part

$$\Gamma_{U(1)} = \frac{e^2}{2\pi}\int d^2xd^2y\epsilon^{\mu\nu}\partial_\mu A_\nu(x)\frac{1}{\Delta_g}(x,y)\epsilon^{\rho\sigma}\partial_\rho A_\sigma(y), \tag{9}$$

in which R is the two-dimensional scalar curvature, Δ_g is the Laplacian, $\epsilon^{\mu\nu}$ represents the two-dimensional Levi-Civita tensor. From these actions, the expectation values of energy-momentum tensor and gauge current can be solved as [25–28]

$$\langle T^{grav}_{\mu\nu}\rangle = \frac{1}{48\pi}(2g_{\mu\nu}R - 2\nabla_\mu\nabla_\nu S + \nabla_\mu S\nabla_\nu S - \frac{1}{2}g_{\mu\nu}\nabla^\rho S\nabla_\rho S), \tag{10}$$

$$\langle T^{U(1)}_{\mu\nu}\rangle = \frac{e^2}{\pi}(\nabla_\mu B\nabla_\nu B - \frac{1}{2}g_{\mu\nu}\nabla^\rho B\nabla_\rho B), \tag{11}$$

$$\langle J^\mu\rangle = \frac{1}{\sqrt{-g}}\frac{\delta\Gamma}{\delta A_\mu} = \frac{e^2}{\pi}\frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}\partial_\nu B, \tag{12}$$

where

$$S(x) = \int d^2y\frac{1}{\Delta_g}(x,y)\sqrt{-g}R(y), \tag{13}$$

$$B(x) = \int d^2y\frac{1}{\Delta_g}(x,y)\epsilon^{\mu\nu}\partial_\mu A_\nu(y). \tag{14}$$

The concrete values of above equations have been given in Refs.[25, 26], where the boundary conditions are invoked. In order to get Hawking radiation flux, we must use boundary conditions as Unruh effect in the Unruh vacuum. In the advanced Eddington coordinate, there is a finite mount of flux at the horizon while no flux at infinity. Meanwhile, in the retarded Eddington coordinate, there is no flux at the horizon while a finite mount of flux at infinity. For observers in the Kruskal coordinate, the relation $J_U = -\frac{J_u}{\kappa U}$ and $T_{UU} = (\frac{1}{\kappa U})^2 T_{uu}$ also should be imposed [23, 24]. In this case, we find the gauge fluxes

$$\langle J_u\rangle = \frac{e^2}{2\pi}(A_t(r) - A_t(r_h)), \tag{15}$$

$$\langle J_v\rangle = \frac{e^2}{2\pi}A_t(r), \tag{16}$$

and the energy-momentum tensor fluxes

$$\langle T_{uu}\rangle = \frac{1}{24\pi}(\frac{\kappa_h^2}{2} - \frac{M}{r^3} + \frac{3M^2}{2r^4} + \frac{3e^2}{2r^4} - \frac{3Me^2}{r^5} + \frac{e^4}{r^6}) + \frac{e^2}{4\pi}(A_t(r) - A_t(r_h))^2, \tag{17}$$

$$\langle T_{vv}\rangle = \frac{1}{24\pi}(-\frac{M}{r^3} + \frac{3M^2}{2r^4} + \frac{3e^2}{2r^4} - \frac{3Me^2}{r^5} + \frac{e^4}{r^6}) + \frac{e^2}{4\pi}A_t^2(r). \tag{18}$$

Obviously, from Eq.(17) we can get that the infinite observers will see a bunch of flow $\frac{\pi T_h^2}{12} + \frac{e^2}{4\pi}A_t^2(r_h)$, in which $T_h = \frac{\kappa}{2\pi}$ is Hawking temperature. Correspondingly, Eq.(18) indicates that a near horizon observer may find that a bunch of flow $-\frac{\pi T_h^2}{12} + \frac{e^2}{4\pi}A_t^2(r_h)$ will drop into event horizon. From Eq.(15) and Eq.(16), we also know that a negative flow $\frac{e^2}{2\pi}A_t(r_h)$ at the event horizon is responsible for the flow of charge current observed by an infinite observer.

III. HAWKING RADIATION FROM LANDAUER TRANSPORT MODEL

The Landauer transport model was first proposed to study electrical transport in mesoscopic circuits, and subsequently used to describe thermal transport. In 2000, the phononic quantized thermal conductance counterpart was measured for the first time [29]. For the 1D quantum channel of thermal conductance, it is supposed that there are two thermal reservoirs characterized by the temperatures T_L, T_R and chemical potentials u_L, u_R, where subscripts L and R denote the left and right thermal reservoirs respectively. They are coupled adiabatically through an effective 1D connection. Because of the temperature difference, the thermal transportation will happen. Typically, a wire will provide several available parallel channels for the given values of chemical potential and temperature. For the sake of simplicity in the present investigation, we restrict only to ballistic transport, which means the channel currents do not interfere with each other [22]. Universally, there are several distribution functions and here we adopt the Haldane's statistics [30]

$$f_g(E) = \{\omega[\frac{(E-u)}{T}] + g\}^{-1}, \tag{19}$$

where the function $\omega(x)$ satisfies the relation

$$\omega(x)^g[1+\omega(x)]^{1-g} = e^x, \tag{20}$$

in which $g = 0$, $g = 1$ correspond bosons and fermions respectively.

The left (right) components of the single channel energy and entropy currents are

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi}\int_{x_{L(R)}^0}^{\infty} dx(x + \frac{u_{L(R)}}{T_{L(R)}})f_g(x), \tag{21}$$

$$\begin{aligned}\dot{S}_{L(R)} &= \frac{T_{L(R)}}{2\pi}\int_{x_{L(R)}^0}^{\infty} dx\{f_g ln f_g + (1-gf_g)ln(1-gf_g)\\ &\quad -[1+(1-gf_g)]ln[1+(1-g)f_g)]\},\end{aligned} \tag{22}$$

in which $x_{L(R)}^0 = -\frac{u_{L(R)}}{T_{L(R)}}$. As done in Ref.[22], we define the zero level of energy with respect to the longitudinal component of the kinetic energy. The total energy and entropy currents are then just $\dot{E} = \dot{E}_L - \dot{E}_R$, $\dot{S} = \dot{S}_L - \dot{S}_R$. We first consider the case of fermions, namely $g = 1$. According to the viewpoint of Landauer, the maximum energy flow rate of fermions can be treated as the combination of fermionic particle and antiparticle single channel currents. As $x = \frac{(E-u)}{T}$, $y = \frac{(E+u)}{T}$ are defined, the flow of energy therefore can be expressed as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi}\Big[\int_{\frac{-u_{L(R)}}{T_{L(R)}}}^{\infty} dx(x + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^x+1} + \int_{\frac{u_{L(R)}}{T_{L(R)}}}^{\infty} dy(y + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^y+1}\Big]. \tag{23}$$

To get the final value, we first vary the lower limit of integral, that is

$$\begin{aligned}\dot{E}_{L(R)} &= \frac{T_{L(R)}^2}{2\pi}\Big[\int_0^{\infty} dx(x + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^x+1} + \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} dx(-x + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^{-x}+1} + \\ &\quad \int_0^{\infty} dy(y + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^y+1} - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} dy(y + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^y+1}\Big].\end{aligned} \tag{24}$$

As $\frac{1}{e^{-x}+1} = 1 - \frac{1}{e^{x}+1}$ is replaced, Eq.(24) takes the form as

$$\begin{aligned}\dot{E}_{L(R)} &= \frac{T_{L(R)}^2}{2\pi}\Big[\int_0^\infty \frac{x}{e^x+1}dx + \int_0^\infty \frac{y}{e^y+1}dy + 2\frac{u_{L(R)}}{T_{L(R)}}(\int_0^\infty \frac{1}{e^x+1}dx \\ &\quad - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} \frac{1}{e^x+1}dx) + \frac{u_{L(R)}^2}{2T_{L(R)}^2}\Big]. \end{aligned} \tag{25}$$

According to the technique of Landau [31], the convergence rate of $\frac{1}{e^x+1}$ is very fast, the upper limit of integral $\frac{u_{L(R)}}{T_{L(R)}}$ hence can be changed as infinity. So we get the final value of the energy current

$$\dot{E} = \dot{E}_L - \dot{E}_R = \frac{\pi}{12}(T_L^2 - T_R^2) + \frac{1}{4\pi}(u_L^2 - u_R^2). \tag{26}$$

As the left and right are regarded as the RN black hole and the thermal environment with absolute temperature zero surrounding the black hole respectively, we find

$$\dot{E} = \frac{\pi T_h^2}{12} + \frac{e^2}{4\pi}A_t^2(r_h), \tag{27}$$

in which $u_R = 0$ and $u_L = \frac{eQ}{r_h}$ are used. Obviously, this result agrees with the energy-momentum tensor flux obtained by conformal symmetry.

For the entropy flow in Eq.(22), the value is independent on the chemical potential and it depends only on the lower limit of integral. We can get its maximum value after calculating, $\dot{S} = \frac{\pi T_h}{6}$, under the degenerate limit $\frac{u_L}{T_L} \to \infty$. As the maximum energy and entropy current expressions are combined by eliminating T_L, the relation $\dot{S}^2 \leq \frac{\pi \dot{E}}{3}$, which was proved to be universal for 1D quantum channels with arbitrary reservoir temperatures, chemical potentials and particle statistics [22, 32, 33], also can be reproduced.

Thinking of the transportation of charge via the 1D quantum channel, the current flow from left (with higher chemical potential u) to right without scattering can be expressed as [32]

$$\dot{I} = \frac{e}{2\pi}\int_0^\infty f(\omega)d\omega. \tag{28}$$

For the case of fermions, the contribution of antiparticle also should be considered, we have

$$\dot{I} = \frac{e}{2\pi}\int_0^\infty (\frac{1}{e^{\frac{\omega-u_h}{T_h}}+1} - \frac{1}{e^{\frac{\omega+u_h}{T_h}}+1})d\omega, \tag{29}$$

in which $f(\omega) = 1/(e^{\frac{\omega-u_h}{T_h}}+1)$ is the radiation spectrum of fermions at the event horizon. Finishing the integral, we obtain

$$\dot{I} = -\frac{e^2}{2\pi}A_t(r_h), \tag{30}$$

which is consistent with the value of charge current flow at the event horizon in Eq.(15). Namely the gauge flux with respect to electric charge of the RN space time also can be transported via the 1D quantum channel.

Now, we check whether the Landauer transport model is valid for the bosons. Putting $g = 0$ into Eq.(19) and Eq.(20), the energy current can be expressed as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi}\int_{\frac{-u_{L(R)}}{T_{L(R)}}}^\infty dx(x + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^x-1}. \tag{31}$$

After varying the lower limit of integral and adopting $\frac{1}{e^{-x}-1} = -1 - \frac{1}{e^x-1}$, the above equation can be rewritten as

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi}\Big[\int_0^\infty (x + \frac{u_{L(R)}}{T_{L(R)}})\frac{1}{e^x-1}dx - \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} (\frac{u_{L(R)}}{T_{L(R)}} - x)\frac{1}{e^x-1}dx - \frac{u_{L(R)}^2}{2T_{L(R)}^2}\Big]. \tag{32}$$

Finishing the integration, we find

$$\dot{E}_{L(R)} = \frac{T_{L(R)}^2}{2\pi}\Big[\frac{\pi^2}{6} + \int_0^{\frac{u_{L(R)}}{T_{L(R)}}} ln(e^x-1)dx\Big]. \tag{33}$$

Adopting the similar technique as done in the case of fermions, where $\frac{u_{L(R)}}{T_{L(R)}} \gg 0$ can be produced, the second term of above equation can be integrated easily. When the the left and right are treated as the RN black hole and thermal environment with absolute temperature zero, we find

$$\dot{E} = \frac{\pi T_h^2}{12} + \frac{e^2}{4\pi} A_t^2(r_h). \tag{34}$$

This result also agrees with the energy-momentum tensor flux observed by the infinite observers in Eq.(17). For a boson, the maximum value of entropy flow, $\dot{S} = \frac{\pi T_h}{6}$, and the charge current flow can also be reproduced by adopting similar skills as the case of fermions.

IV. DISCUSSION AND CONCLUSION

Flows of Hawking radiation energy and charge current from a RN black hole are investigated using a 1D quantum transport model, which are shown to be consistent with the vacuum expectation values of energy-momentum tensor and gauge flux. The maximum value of entropy flow in different degeneration is obtained and the result is independent on the statistics. For flows of energy and charge current of fermions, we get the same result as that of bosons when the contribution of antiparticle is considered, which confirms the viewpoint of Davies [33] that the fermionic field describing a massless particle plus its antiparticle is equivalent to a single massless bosonic field in a (1+1)-dimensional curved spacetime.

Note that when we investigate the flow of fermions and bosons using Landauer transport model, the condition $\frac{u_{L(R)}}{T_{L(R)}} \gg 0$ should be imposed in both of them. In statistics physics, this condition should be satisfied in order to avoid the degeneration of boson distribution and fermion distribution to the Boltzmann distribution.

As the first law of black hole thermodynamics and energy conservation are considered, one also can get the net entropy production rate defined as $R = \frac{dS}{dS_{BH}}$ [34] in the two dimensional space time. Because of the existence of electromagnetic field, we find the production rate of RN black hole is smaller than the case of Schwarzschild black hole with $R = 2$.

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