New chemical evolution analytical solutions including environment effects

E. Spitoni, 1*

Dipartimento di Fisica, Sezione di Astronomia, Università di Trieste, via G.B. Tiepolo 11, I-34131, Trieste, Italy

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ABSTRACT

In the last years, more and more interest has been devoted to analytical solutions, including inflow and outflow, to study the metallicity enrichment in galaxies. In this framework, we assume a star formation rate which follows a linear Schmidt law, and we present new analytical solutions for the evolution of the metallicity (Z) in galaxies. In particular, we take into account environmental effects including primordial and enriched gas infall, outflow, different star formation efficiencies and galactic fountains. The enriched infall is included to take into account galaxy-galaxy interactions. Our main results can be summarized as: i) when a linear Schmidt law of star formation is assumed, the resulting time evolution of the metallicity Z is the same either for a closed-box model or for an outflow model. ii) The mass-metallicity relation for galaxies which suffer a chemically enriched infall, originating from another evolved galaxy with no pre-enriched gas, is shifted down in parallel at lower Z values, if compared the closed box model. iii) When a galaxy suffers at the same time a primordial infall and a chemically enriched one, the primordial infall always dominates the chemical evolution. iv) We present new solutions for the metallicity evolution in a galaxy which suffers galactic fountains and an enriched infall from another galaxy at the same time. The analytical solutions presented here can be very important to study the metallicity (oxygen), which is measured in high-redshift objects. These solutions can be very useful: a) in the context of cosmological semi-analytical models for galaxy formation and evolution, and b) for the study of compact groups of galaxies.

Key words: galaxies: abundances - galaxies: evolution - galaxies: ISM

INTRODUCTION

The galactic chemical evolution is the study of the transformation of gas into stars and the resulting evolution of the chemical composition of a galaxy. The so called "Simple Model" remains an useful guide for understanding the chemical evolution of galaxies since the pioneering works of Schmidt (1963), Searle & Sargent (1972), Tinsley (1974), Pagel & Patchett (1975). In order to derive analytical solutions for the chemical evolution of galaxies one needs to make several hypotheses: the initial mass function (IMF) should be considered constant, the lifetime of stars should be neglected (instantaneous recycling approximation, IRA), and the complete mixing of chemical elements with the surrounding interstellar medium (ISM). These assumptions, in fact, allow us to have analytical expressions for the metallicity evolution of the galaxies in time and in terms of the gas fraction. One can find analytical solutions even in presence of gas flows (infall, outflow, galactic fountains) as shown

* E-mail: spitoni@oats.inaf.it

by Tinsley (1980), Clayton (1988), Lacey & Fall (1985), Matteucci & Chiosi (1983), Edmunds (1990), Recchi et al. (2008), Spitoni et al. (2010), Peeples & Shankar (2011), Lilly et al. (2013), Pipino et al. (2014), Peng & Maiolino (2014a), Recchi & Kroupa (2014), Kudritzki et al. (2015).

However, in all mentioned works the solutions are obtained only under specific assumptions regarding the infall/outflow rates. However, we note that analytical solutions are not able to give a complete description of the chemistry of a galaxy, since they fail in following the evolution of elements created on long time-scales, such as iron and nitrogen. A satisfactory description of the iron evolution requires detailed numerical models relaxing IRA and including the chemical enrichment from Type Ia SNe (e.g. Matteucci & Greggio 1986) allow one to follow in detail the evolution of single elements. Matteucci & Greggio (1986) showed in detail the effect of the time-delay model, already suggested by Tinsley (1980) and Greggio & Renzini (1983), in particular the effect of a delayed Fe production by Type Ia SNe on abundance ratios involving α -elements (O, Mg, Si). Analytical solution can be, on the other hand, adopted when studying the evolution of oxygen, created on short timescales and tracing the evolution of the global metallicity, Z, of which oxygen is the main component.

In this work we start by using the formalism described by Matteucci (2001), Recchi et al. (2008), and Spitoni et al. (2010) and we show, for the first time, the analytical solution of the evolution of the metallicity of a galaxy in presence of "environment" effects coupled with galactic fountains and primordial infall of gas. In this context for "environment" effect we mean the situation where a galaxy suffers, during its evolution, infall of enriched gas from another evolving galactic system, and this gas represents an enriched infall variable in time.

The dynamics of interacting systems have been the subject of many papers concerning numerical simulations (Toomre & Toomre 1972; Barnes & Hernquist 1992; Berentzen et al. 2003) and spectrophotometric models (Larson & Tinsley 1978; Kennicutt 1990; Temporin et al. 2003a, 2003b). Large amounts of ISM can be removed from the main disks of spiral galaxies by different processes: tides due to the gravitational force of a companion, ram pressure stripping during a near head-on collision between gas-rich galaxies, ram pressure stripping by intracluster gas, and galactic winds driven by supernovae. Smith & Struck (2001) using CO signatures observed 11 extragalactic tails and bridges in nine interacting galaxy systems. Recently, Smith et al. (2010) using Galaxy Evolution Explorer (GALEX) ultraviolet telescope studied star formation morphology and stellar populations and in 42 nearby optically selected pre-merger interacting galaxy pairs. Tails and bridges structures are often more prominent relative to the disks in UV images compared to optical maps. This effect is likely due to enhanced star formation in the tidal features compared to the disks rather than to reduced extinction. We also refer the reader to the review of Boselli & Gavazzi (2006) where a comprehensive description of the environment effects on late-type galaxies in nearby clusters is presented. Among them we recall the tidal interactions among galaxy pairs which act on gas, dust and stars, as well as on dark matter and is depending on the gravitational bounding of the various components; tidal interaction between galaxies and the cluster potential well, and finally the so-called "galaxy harassment": the evolution of cluster galaxies is governed by the combined effect of multiple high speed galaxy-galaxy close ($\sim 50 \text{ kpc}$) encounters with the interaction with the potential well of the cluster as a whole.

As stated by Davies et al. (2010), galaxy-galaxy interactions are particularly important and striking when the speed of the interaction is well matched to the velocities of the stars and gas. Therefore, small galaxy groups can potentially provide the environment for dramatic gravitational disturbances. Beyond the Local Group, the closest example of this is the environment around M81, the M81 group. In fact, in M81 group it is evident that various galaxies are connected by flows. Extended filamentary structures external to the disc of M81, are clearly seen in emission in all of the Herschel bands. These complex interactions cannot be described by the simple tools offered, for example, by Recchi et al. (2008) or similar papers in literature, and with this paper we provide a new set of analytical solutions to take into account this kind of interactions in the framework of analytical chemical evolution models.

Moreover, an enriched infall of gas can be originated from the galaxy itself. In fact, in the galactic fountain models (Shapiro & Field 1976, Houck & Bregman 1990), hot gas is ejected out of the Galactic disk by supernova (SN) explosions, and part of this gas falls back in the form of condensed neutral clouds which move at intermediate and high radial velocities. For example, in the galactic fountain model the ejected gas from SN events falls back ballistically (Bregman 1980). These models are able to explain the vertical motion of the cold and warm gas components observed in several spiral galaxies (e.g. Fraternali et al. 2004; Boosma et al. 2005). Following the analytical implementation of the galactic fountain presented by Recchi et al. (2008), we will consider such an effect in our new analytical solutions.

In the context of cosmological semi-analytical models of galaxy formation and evolution each galaxy is treated as one unresolved object, using integrated properties to describe the mass of stars, cold gas, hot gas and the black hole. Since each component of the galaxy is represented by one number, the dynamics within the galaxy is not resolved, and one needs to assume with laws for star formation, cooling and feedback that are valid on average for the entire galaxy. In this context, our new analytical solutions shall be extremely useful, because they give simple recipes concerning the time evolution of the global metallicity of a galaxy in different situations including inflow and outflow.

The paper is organized as follows: in Sect. 2 we present the main assumptions of the "analytical" chemical evolution models, in Sect. 3 we describe our assumptions and the system of equation we need to solve, and in Sect 4 new analytical solutions are presented. Finally, our conclusions are summarized in Section 5. In Appendix A we draw the complete expressions of some new analytical solutions we presented in this paper, and in Appendix B a list of variables and parameters used throughout the paper is reported.

2 THE CLOSED BOX AND LEAKY-BOX PRESCRIPTIONS

The main assumptions of the Simple Model (Tinsley 1980) are:

- (i) The IMF is constant in time.
- (ii) The gas is well mixed at any time (instantaneous mixing approximation).
- (iii) Stars $\geq 1 \ \mathrm{M}_{\odot}$ die instantaneously; stars smaller than $1 \ \mathrm{M}_{\odot}$ live forever (instantaneous recycling approximation or IRA).

These simplifying assumptions allow us to calculate analytically the chemical evolution of the galaxies once we have defined the fundamental quantities, such as the returned fraction:

$$R = \int_{1}^{\infty} (m - M_R)\phi(m)dm,\tag{1}$$

(where $\phi(m)$ is the IMF and M_R is the mass of the remnant) and the yield per stellar generation:

$$y_Z = \frac{1}{1 - R} \int_1^\infty m p_{Z,m} \phi(m) dm, \qquad (2)$$

(where $p_{Z,m}$ is the fraction of newly produced and ejected metals by a star of mass m).

Recently, Recchi & Kroupa (2015) applied the integrated galactic initial mass function (IGIMF) to the simple model solution, and y_Z and R are not constant, but are functions of time, through the time dependence of the star formation rate (SFR) and the metallicity of the system.

The well known solution of the so called "closed-box" where the system is one-zone and there are no inflows nor outflows with constant mass (gas plus stars) is:

$$Z = y_Z \ln(\mu^{-1}) \tag{3}$$

where μ is the gas fraction M_{gas}/M_{tot} , with $M_{tot} = M_* + M_{gas}$. It is also assumed that the $M_{gas}(0) = M_{tot}(0)$ and the initial metallicity of the system is zero.

Analytical solutions of simple models of chemical evolution including infall or outflow are known since at least 30 years (Pagel & Patchett 1975, Hartwick 1976, Clayton 1988, Twarog 1980, Edmunds 1990). Here, we follow the approach and the terminology of Matteucci (2001), where it was assumed for simplicity linear flows (gas flows are proportional to the SFR). Therefore, the outflow rate W(t) is defined as:

$$W(t) = \lambda(1 - R)\psi(t),\tag{4}$$

where $\psi(t)$ is the SFR, and the infall rate A(t) is given by:

$$A(t) = \Lambda(1 - R)\psi(t). \tag{5}$$

Here λ and Λ are two proportionality constants $\geqslant 0$. The first assumption is justified by the fact that, the larger the SFR is, the more intense are the energetic events associated with it (in particular supernova explosions and stellar winds) and therefore the larger is the chance of having a large-scale outflow (see e.g. Silk 2003). A proportionality between A(t) and $\psi(t)$ has been discussed by Recchi et al. (2008). They tested different prescriptions for the infall of gas showing that their results do not change substantially if a generic exponential infall is assumed. This demonstrates also that the major source of error in the solutions of the simple models is the IRA assumption rather than the assumption of linear flows.

In Lilly et al (2013) and Pipino et al. (2014), they considered the simple chemical evolution model in the cosmological context where the gas accreted is proportional to the dark matter growth. In particular, in Pipino et al. (2014) the infall parameter $\Lambda(t)$ is time dependent and is defined as the ratio between the accretion rate from cosmological simulations and the SFR. In this paper we do not investigate "cosmological" aspects but we study the effects of galaxy-galaxy interactions on the time evolution of galactic metallicity.

The evolution of the metallicity of a system as a function of μ in the case with only outflows, i.e. A(t)=0 and $W(t)\neq 0$ is the following:

$$Z = \frac{yz}{(1+\lambda)} \ln[(1+\lambda)\mu^{-1} - \lambda],\tag{6}$$

with the assumption that at t=0, Z(0)=0, $M_{tot}(0)=M_{gas}(0)$. In the opposite case $(A(t) \neq 0 \text{ and } W(t)=0)$, assuming for the infalling gas a primordial composition (e.g $Z_A=0$), we obtain this solution (Matteucci 2001):

$$Z = \frac{y_Z}{\Lambda} \left\{ 1 - \left[(\Lambda - (\Lambda - 1)\mu^{-1}) \right]^{\frac{\Lambda}{\Lambda - 1}} \right\}. \tag{7}$$

The general solution presented by Matteucci (2001) for a system described by the simple model in the presence of infall of gas with a general metallicity Z_A and outflow is:

$$Z = \frac{\Lambda Z_A + y_Z}{\Lambda} \left\{ 1 - \left[(\Lambda - \lambda) - (\Lambda - \lambda - 1)\mu^{-1} \right]^{\frac{\Lambda}{\Lambda - \lambda - 1}} \right\}.(8)$$

3 THE CHEMICAL EVOLUTION MODEL WITH ENVIRONMENT EFFECTS

Using the formalism introduced by Matteucci (2001), and adopted later by Recchi et al. (2008) and Spitoni et al. (2010), for the first time we intend to study the effects of the environment on the chemical evolution of a galaxy by means of analytical solutions. The environment effects are mimicked by an infall of gas with a time dependent metallicity originated by a nearby galaxy.

In our model we analyze the evolution of the oxygen in a generic galaxy, and assume for the oxygen yield standard values of $y_O = 0.01$ and the returned fraction R = 0.25. As done in Spitoni et al. (2010), those values were obtained by adopting the Salpeter (1955) IMF, and the stellar yields of Woosley & Weaver (1995) for oxygen at solar metallicity.

3.1 Our model for a system formed by two isolated galaxies

We consider the chemical evolution of the galaxy1 with initial mass $M_{tot1}(0) = M_{g1}(0)$. We study here the effects of the interaction with another galaxy on the abundance of oxygen $Z_{O,1}(t) = M_{O,1}(t)/M_{g1}(t)$.

We assume an environment dependent infall, originated by feedback episodes and galaxy-galaxy interactions. We assume that the infall is proportional to the gas outflows from nearby galaxies with wind parameter λ . First we consider the case with the "enriched infall" proportional to the SFR of the galaxy2:

$$A_{\lambda,1}(t) = \epsilon W_{\lambda,2}(t) = \epsilon \lambda (1 - R)\psi_2(t), \tag{9}$$

where $\psi_2(t)$ is the SFR of the galaxy2, and ϵ is the fraction of outflowing gas from galaxy2 which reaches galaxy1. We assume in this work that $\epsilon = 0.5$. In fact, the outflows are generally bipolar and for a lobe approaching galaxy 1, the second lobe moves away from it. The value $\epsilon = 0.5$ is an upper limit for the fraction of outflowing gas from galaxy2 to galaxy1. Only if the outflow is well collimated along a narrow solid angle and only if galaxy1 happens to intercept this galactic wind, ϵ can be equal to 0.5. For example, from GALEX UV images of the starbust galaxy M81 given by Hoopes et al. (2005), it can be seen that the opening angle of the northern side wind is around 55 degrees.

The $Z_{O,2}(t)$ metallicity of the infall is time and depends on the outflows of the galaxy2.

The system of equations we have to solve for our galaxy1 is the following:

$$\begin{cases}
\frac{dM_{tot1}}{dt} = (\epsilon \lambda)(1 - R)\psi_2(t) \\
\frac{dM_{g1}}{dt} = (1 - R)(\epsilon \lambda \psi_2(t) - \psi_1(t)) \\
\frac{dM_{O,1}}{dt} = (1 - R)(\psi_1(t)[y_O - Z_{O,1}(t)] + \psi_2(t)\epsilon \lambda Z_{O,2}(t)),
\end{cases} (10)$$

where $\psi_1(t)$ is the SFR of the galaxy1.

As mentioned above, $Z_{O,2}(t)$ is the metallicity of the galaxy2 which is suffering only gas outflows. We therefore need to solve another system of equations for the evolution of $Z_{O,2}(t)$ in galaxy2.

Recalling Matteucci (2001) the system to be solved for galaxy2 with only outflow of gas is:

$$\begin{cases}
\frac{dM_{tot2}}{dt} = -\lambda(1 - R)\psi_2(t) \\
\frac{dM_{g2}}{dt} = (-\lambda - 1)(1 - R)\psi_2(t) \\
\frac{dM_{O,2}}{dt} = (1 - R)\psi_2(t) \left[-Z_{O,2}(t) + y_O - \lambda Z_{O,2}(t) \right].
\end{cases} (11)$$

We recall that the outflow rate for the galaxy2 is $W_{\lambda,2}(t) = \frac{A_{\lambda,2}(t)}{\epsilon}$. With equations (10) and (11) we study an isolated system formed by two galaxies when $\epsilon = 1$. In fact, in this case we have that $\frac{dM_{tot2}}{dt} + \frac{dM_{tot1}}{dt} = 0$.

On the other hand, imposing $\epsilon \neq 1$, we assume that part of the chemical enriched gas escaping from galaxy2, ends up in the in intergalactic medium (IGM).

3.2 Model including primordial infall of gas and the interactions of 2 galaxies

The majority of detailed chemical evolution models of galactic systems assumes that the galaxies formed by accretion of primordial gas from the IGM. Therefore, we examine here also the more realistic case where both galaxy1 and galaxy2 suffer an inflow of primordial gas. We assume that the infall rate is proportional of the SFR of the galactic system as discussed in Section 2.1. Hence, for the galaxy1 the primordial infall rate is $A_{\Lambda,1}(t) = \Lambda(1-R)\psi_1(t)$, whereas is $A_{\Lambda,2}(t) = \Lambda(1-R)\psi_2(t)$ for the galaxy2. The systems of equation we need to solve are the following ones:

• galaxy1:

$$\begin{cases} \frac{dM_{tot1}}{dt} = (1-R)(\epsilon\lambda\psi_2(t) + \Lambda\psi_1(t)) \\ \frac{dM_{g1}}{dt} = (1-R)(\epsilon\lambda\psi_2(t) + (\Lambda-1)\psi_1(t)) \\ \frac{dM_{O,1}}{dt} = (1-R)(\psi_1(t)[y_O - Z_{O,1}(t)] + \psi_2(t)\epsilon\lambda Z_{O,2}(t)). \end{cases}$$

• galaxy2:

$$\begin{cases}
\frac{dM_{tot2}}{dt} = (-\lambda + \Lambda)(1 - R)\psi_2(t) \\
\frac{dM_{g2}}{dt} = (\Lambda - \lambda - 1)(1 - R)\psi_2(t) \\
\frac{dM_{O,2}}{dt} = (1 - R)\psi_2(t) \left[-Z_{O,2}(t) + y_O - \lambda Z_{O,2}(t) \right].
\end{cases} (13)$$

4 THE NEW ANALYTICAL SOLUTIONS

In this Section we present our new analytical solutions. First, we discuss the case where the galaxy1 suffers an enriched infall from an evolving companion galaxy (galaxy2). In the following we generalize this solution in the case of different SFEs for the galaxy1 and galaxy2. We also present the case where a primordial infall is taken into account. The last result is related to "environment" effects coupled with galactic fountains.

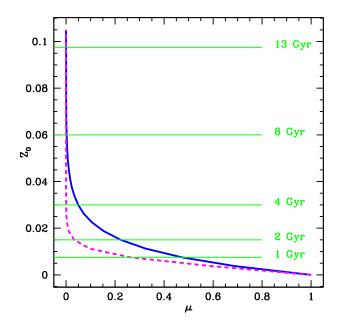


Figure 1. Evolution of the metallicity Z_O in terms of the gas mass fraction μ for the closed-box model (blue solid line) and for a model with only gas outflow with $\lambda=2$ (magenta dotted line). Different horizontal solid green lines refer to different galactic time (1, 2, 4, 8, and 13 Gyr).

4.1 Interactions with a nearby galaxy

From the system of equations (11) we recover $Z_{O,2}(t)$, and following Matteucci (2001) we obtain:

$$Z_{O,2}(t) = \frac{y_O}{1+\lambda} \ln\left[(1+\lambda)\mu_2^{-1} - \lambda \right].$$
 (14)

Assuming a linear Schmidt (1959) law ($\psi = S \times M_g$) (Pipino et al. 2014, Recchi et al, 2008), in the systems (10) and (11), we have the following expressions of the time evolution of the gas for the galaxy1 ($M_{g1}(t)$) and the galaxy2 ($M_{g2}(t)$):

$$(12) M_{g1}(t) = e^{-(1-R)St} (M_{g1}(0) + \epsilon M_{g2}(0) [1 - e^{-\lambda(1-R)St}]), (15)$$

$$M_{q2}(t) = M_{q2}(0)e^{-(\lambda+1)(1-R)St}$$
 (16)

Recalling that the gas fraction is defined as $\mu = M_g(t)/M_{tot}(t)$, we can write for the galaxy2:

$$\mu_2^{-1} = \frac{M_{tot2}}{M_{g2}} = \frac{M_{tot2}}{M_{g2}(0)e^{-(\lambda+1)(1-R)St}}.$$
 (17)

From the system (11) with the initial condition that $M_{tot2}(0) = M_{g2}(0)$, we have that:

$$M_{tot2}(t) = \frac{\lambda}{\lambda + 1} M_{g2}(t) + M_{g2}(0) \left(1 - \frac{\lambda}{\lambda + 1}\right).$$
 (18)

Finally, inserting eq. (18) in eq. (17), we recover the following time dependent expression for $\mu_2^{-1}(t)$:

$$\mu_2^{-1} = \frac{\lambda}{\lambda + 1} + e^{(\lambda + 1)(1 - R)St} \left(\frac{1}{\lambda + 1}\right). \tag{19}$$

Hence, the expression of $Z_{O,2}(t)$ as a function of the galactic time becomes:

$$Z_{O,2}(t) = \frac{y_O}{1+\lambda} \ln\left[\lambda + e^{(\lambda+1)(1-R)St} - \lambda\right] = y_O(1-R)St.(20)$$

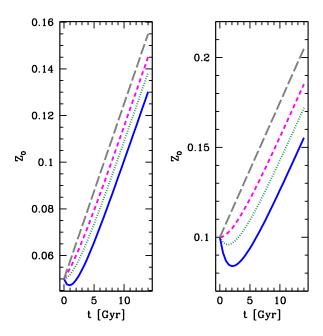


Figure 2. Left panel: The time evolution of the metallicity (oxygen) for the galaxy1 when galaxy-galaxy interactions are taken into account and assuming an initial metallicities fixed at the value of $Z_{O,1}(0){=}0.05$ and $Z_{O,2}(0){=}0$. For all the cases we assume, $\lambda{=}0.4$, $\epsilon{=}0.5$, S=1 Gyr⁻¹. The magenta short dashed line represents the case with $M_{g1}(0)/M_{g2}(0){=}2$, the green dotted line for $M_{g1}(0)/M_{g2}(0){=}1$, the solid blue line represents the case with $M_{g1}(0)/M_{g2}(0){=}0.5$. The long dashed grey line is the closed-box solution with the same initial metallicity. Right panel:: Same of left panel but with an initial metallicity fixed at $Z_{O,1}(0){=}0.1$.

It is worth noting that the evolution in time of $Z_{O,2}(t)$ does not depend on λ in the specific case of the Schmidt (1959) SFR with k=1, and is the same as in the closed-box one. In fact, for a closed-box model with the linear Schmidt (1959) law we have this expression for the gas mass: $M_g(t) = M_g(0)e^{-(1-R)St}$. Following eq. (3) we derive the expression of the metallicity Z_{cb} as a function of time for the closed-box (cb). Therefore, we have that:

$$Z_{cb}(t) = y_O \ln \left(e^{(1-R)St} \right) = y_O(1-R)St.$$
 (21)

In Fig. 1 we draw the evolution of the metallicity Z_O as function of the gas fraction μ for both closed-box model and the one with linear outflows in the case of the linear Schmidt (1959) law, $\psi = SM_g$, with wind parameter $\lambda = 2$ and S = 1 Gyr⁻¹. Horizontal lines indicate different galactic times during the galactic histories.

Even if the closed-box model and the model with only outflow have the same metallicity in time, that same metallicity is reached for different values of the gas fraction μ .

The metallicity $Z_{O,2}(t)$ expressed in eq. (20) can be inserted in the system (10), and recalling that $\frac{dM_{O,1}}{dt} = \frac{d(M_{g1}Z_{O,1}(t))}{dt}$, the third equation of the system (10) can be rewritten as:

$$M_{g1}(t)\frac{dZ_{O,1}(t)}{dt} = (1 - R) \times (\psi_1(t)y_O + \epsilon \psi_2(t) [-\lambda Z_{O,1}(t) + \lambda Z_{O,2}(t)]).$$
(22)

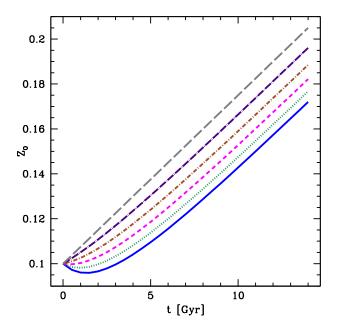


Figure 3. The effect of the fraction of the outflowing gas (ϵ) from galaxy2 to galaxy1 on the time evolution of the metallicity (oxygen) for the galaxy1. We assume $Z_{O,1}(0){=}0.1$, $Z_{O,2}(0){=}0$, $\lambda{=}0.4$, S=1 Gyr⁻¹, and $M_{g1}(0)/M_{g2}(0){=}1$. The blue solid line represents the model with $\epsilon{=}0.5$, the case with $\epsilon{=}0.4$ is reported with the dotted green line, the short dashed magenta line is the case with $\epsilon{=}0.3$. The models with $\epsilon{=}0.2$ and $\epsilon{=}0.1$ are in brown dashed dotted line and violet long dashed dotted line, respectively. The long dashed grey line is the closed-box solution with the same initial metallicity $Z_{O,1}(0){=}0.1$.

Hence, we assume that the SFR follows a Schmidt (1959) law: $\psi_1 = S \times M_{g1}$ and $\psi_2 = S \times M_{g2}$, and the differential equation we need to solve when we include eq. (20) in eq. (22) is:

$$\frac{dZ_{O,1}(t)}{dt} = (1 - R)S \times \left(y_O + \frac{M_{g2}(t)}{M_{g1}(t)} \left[-\epsilon \lambda Z_{O,1}(t) + \epsilon \lambda y_O(1 - R)St \right] \right).$$
(23)

The final expression for $Z_{O,1}(t)$ in a case where the galaxy1 is affected by the interaction with another galactic system (galaxy2), and with $Z_{O,2}(0) = 0$ is the following one:

$$Z_{O,1}(t) = Sy_O(1-R)t + \frac{Z_{O,1}(0)}{1 + \epsilon \frac{M_{g_2}(0)}{M_{g_1}(0)} \left(1 - e^{-\lambda(1-R)St}\right)}.(24)$$

We see that the closed-box solution is still recovered when the initial metallicity $Z_{O,1}(0)$ for the galaxy1 is considered equal to zero. The reason for this behaviour is that the outflow from galaxy2 has the same metallicity of the whole galaxy, i.e. it is expressed by eq. (20). Since the two expressions (eqs. 20 and 21) are identical, and since metallicity is not an additive quantity, the expression $Z_{O,1}(t)$ in eq. (24) is expected. In Fig. 2 we show the time evolution of the metallicity (oxygen) where we assume an initial metallicity different from zero in the galaxy1 and as function of different $M_{g1}(0)/M_{g2}(0)$ ratios: 2, 1 and 0.5. We considering two different initial metallicities $Z_{O,1}(0)=0.05$ (left panel) and

 $Z_{O,1}(0)$ =0.1 (right panel). We are aware that these values are extremely large, but here we only like to show the trends of considering different pre-enriched values. We see that at early times the dilution effect is prominent because the infalling gas has a much lower metallicity of that galaxy1, and as expected the lower the ratio $M_{g1}(0)/M_{g2}(0)$ is, more the dilution effect is important.

In Fig. 3 the effects of the fraction of outflowing gas ϵ on the time evolution of the galaxy1 oxygen abundance obtained by eq. (24) are shown. Assuming $Z_{O,1}(t)=0.1$, $M_{g1}(0)/M_{g2}(0)=1$, $\lambda=0.4$, and S=1 Gyr⁻¹, we present different models varying ϵ : $\epsilon=0.5, 0.4, 0.3, 0.2, 0.1$. As expected, the dilution effect is larger assuming higher ϵ values.

We discuss now the case where the initial metallicity for galaxy1 and galaxy2 are $Z_{O,1}(0)=0$, $Z_{O,2}(0)\neq 0$, respectively. In this case the differential equation we need to solve is:

$$\frac{dZ_{O,1}(t)}{dt} = (1 - R)S \times$$

$$\bigg(y_O + \frac{M_{g2}(t)}{M_{g1}(t)} \bigg[-\epsilon \lambda Z_{O,1}(t) + \epsilon \lambda \bigg(Z_{O,2}(0) + y_O(1-R)St \bigg) \bigg] \bigg). (25)$$

The new solution is:

$$Z_{O,1}(t) = Sy_O(1-R)t + \frac{Z_{O,2}(0)\epsilon \frac{M_{g2}(0)}{M_{g1}(0)} \left(1 - e^{-\lambda(1-R)St}\right)}{1 + \epsilon \frac{M_{g2}(0)}{M_{g1}(0)} \left(1 - e^{-\lambda(1-R)St}\right)}.(26)$$

In Fig. 4 we show the evolution of the oxygen abundance of galaxy1 using the new analytical solution presented in eq. (26) with λ =0.4, ϵ =0.5, and $Z_{O,1}(0)$ =0, considering two different initial metallicities $Z_{O,2}(0)$ =0.05 and $Z_{O,2}(0)$ =0.1. In this case we use such high values for the pre-enrichment in order to better visualize the general trends and the effects of different $M_{g1}(0)/M_{g2}(0)$ ratios. As expected, at variance with Fig. 2, the smaller is the $M_{g1}(0)/M_{g2}(0)$ ratios, the less efficient the chemical evolution is for galaxy1.

Overall, the general solution for the evolution of the oxygen abundance for the galaxy1 with both initial metallicities $Z_{O,1}(0)$, $Z_{O,2}(0)$ different from zero, is given by the following expression:

$$Z_{O,1}(t) = Sy_O(1-R)t + (27)$$

$$+\frac{Z_{O,1}(0)+Z_{O,2}(0)\epsilon\frac{M_{g_2}(0)}{M_{g_1}(0)}\bigg(1-e^{-\lambda(1-R)St}\bigg)}{1+\epsilon\frac{M_{g_2}(0)}{M_{g_1}(0)}\bigg(1-e^{-\lambda(1-R)St}\bigg)}.$$

The general solution reported in eq. (27) describes the more realistic scenario where galaxies exhibit different chemical enrichment rates and therefore they posses different metallicity when the interaction starts. It seems plausible that galaxy2 presents a faster chemical enrichment in systems as the M81 group, where large and metal rich galaxies (M81, M82, and NGC3077) interact and eject gas. In other environments, it can be more reasonable that only dwarf galaxies would show galactic winds, and therefore in this situation galaxy2 is less chemically enriched than galaxy1. In conclusion, both cases $Z_{O,2}(0) > Z_{O,1}(0)$, and $Z_{O,2}(0) < Z_{O,1}(0)$ have physical meaning. Recently, in Recchi et al. (2015) it was shown that the values of initial metallicities depend on the level of the interaction between galaxies.

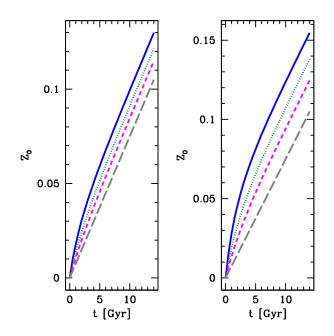


Figure 4. Left panel: The time evolution of the metallicity (oxygen) for the galaxy1 when galaxy-galaxy interactions are taken into account and assuming an initial metallicities fixed at the values of $Z_{O,1}(0){=}0$ and $Z_{O,2}(0){=}0.05$. For all the cases we assume, $\lambda{=}0.4$, $\epsilon{=}0.5$, S=1 Gyr⁻¹. The magenta short dashed line represents the case with $M_{g1}(0)/M_{g2}(0){=}2$, the green dotted line for $M_{g1}(0)/M_{g2}(0){=}1$, the solid blue line represents the case with $M_{g1}(0)/M_{g2}(0){=}0.5$. The long dashed grey line is the closed-box solution with the same initial metallicity. Right panel:: Same of left panel but with the initial metallicity fo galaxy2 fixed at $Z_{O,2}(t){=}0.1$.

4.2 Some applications to real cases

The evolution of the stellar mass content in the galaxy1 $M_{*,1}(t)$, can be simply inferred by the relation $M_{*,1}(t) = M_{tot1}(t) - M_{g1}(t)$. The time evolution of the total mass of galaxy1 $M_{tot1}(t)$, is given by the first equation of system (10), and with the initial condition $M_{tot1}(0) = M_{g1}(0)$ we have that:

$$M_{tot1}(t) = M_{g1}(0) + \frac{\lambda \epsilon M_{g2}(0)}{1+\lambda} \left(1 - e^{-\lambda(1-R)St}\right).$$
 (28)

In Fig. 5 we compare the time evolution of $\log(O/H)+12$ as function of stellar mass for the closed-box model with a model where environment effects are taken into account. The $\log(O/H)+12$ quantity is recovered from $Z_{O,1}(t)$ using the following expression: $\log\Big(Z_{O,1}(t)/(16\times 0.75)\Big)+12$, where 0.75 is the assumed fraction of hydrogen. The assumed initial mass in each model is $10^{10}~\rm M_{\odot}$. In the last case, at a fixed stellar mass value the system shows a smaller oxygen abundance compared to the closed-box evolution.

In Fig. 6 we study the effects of the environment on the mass-metallicity (MZ) relation by comparing with the observed relation of Kewley & Ellison (2008) for Sloan Digit Sky Survey (SDSS) star-forming galaxies. To estimate the amount of gas that resides in each star-forming galaxy, and

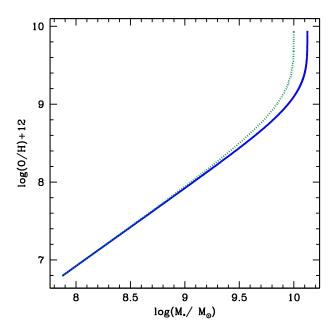


Figure 5. The evolution in the framework of the closed-box model of the abundance $\log(O/H)+12$ as a function of the stellar mass M_* for the closed-box model for a galaxy with total mass equal to $10^{10}~M_{\odot}$ is drawn with the dotted green line. With the solid blue line we show the results for the galaxy1 with a initial mass of $10^{10}~M_{\odot}$, and when we take into account the enriched infall from a companion galaxy with the same initial mass and the wind parameter equal to $\lambda=2$, and $\epsilon=0.5$. The initial oxygen abundances are $Z_{O,1}(0)=Z_{O,2}(0)=0$.

the gas fraction μ as a function of the galactic stellar mass, we use the method described in Spitoni et al. (2010).

We determine the cold gas mass of each galaxy on the basis of its SFR, using the following inverted Kennicutt (1998) relation, which links the gas surface density to the SFR per unit area:

$$\Sigma_{gas} = \left(\frac{\dot{\Sigma}_*}{2.5 \times 10^{-4}}\right)^{0.714} \quad M_{\odot} \, pc^{-2}. \tag{29}$$

where the gas density Σ_{gas} is expressed in $M_{\odot} \, \mathrm{pc}^{-2}$, and the SFR surface density $\dot{\Sigma}_*$ in $M_{\odot} \, \mathrm{yr}^{-1} \, \mathrm{kpc}^{-2}$. The gas mass M_{gas} (in M_{\odot}) is given by:

$$M_{gas} = \Sigma_{gas} \times 2\pi R_d^2. \tag{30}$$

where R_d is the scaling radius calculated as in Mo et al. (1998). At this point we have a relation between μ and the stellar mass for each considered galaxy.

First, we follow the evolution of the closed box models with zero initial metallicity for different initial masses $M_{tot}(0){=}M_g(0){=}M_{tot}(t){=}8.8{\times}10^8,~5{\times}10^9,~10^{10},~5{\times}10^{10},$ and $10^{11}~{\rm M}_{\odot}$, respectively. For each model we compute the time evolution of $Z_O(t),~\mu(t),$ and $M_*(t)$. Thus, we consider the time t_{MZ} where the stellar mass $M_*(t_{MZ})$ and the corresponding μ belong to the fit reported in Fig. 2 of Spitoni et al. (2010). At this point the metallicity at the time t_{MZ} is computed using eq. (21). In Fig. 6 we show the closed-box model results for systems with initial masses $M_{tot}(0) = M_g(0) = M_{tot}(t) = 8.8{\times}10^8, 5{\times}10^9, 10^{10}, 5{\times}10^{10},$ and $10^{11}~{\rm M}_{\odot}$, respectively.

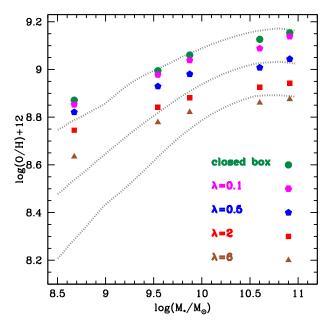


Figure 6. The observed mass-metallicity relation and related standard deviation of Kewley & Ellison (2008) are indicated with the dotted grey lines. With the green circles we show the closed-box results for galaxies with total masses: 8.8×10^8 , 5×10^9 , 10^{10} , 5×10^{10} , $10^{11}~M_{\odot}$, respectively. We compare them with model results where we consider the interactions with a companion galaxy with the same initial masses. For all the models it is assumed that the initial oxygen abundances are $Z_{O,1}(0)=Z_{O,2}(0)=0$. With the magenta hexagon models with the wind parameter fixed at the value of $\lambda=0.1$ are presented. The cases with $\lambda=0.5$, $\lambda=2$, and $\lambda=6$ are represented with blue pentagons, red squares, and brown triangles respectively.

The closed box model is not able to reproduce the observed MZ relation, and in Spitoni et al. (2010) we concluded that a galactic wind rate increasing with decreasing galactic mass or a variable IMF are both viable solutions for reproducing the MZ relation. We consider also galactic systems with the same initial masses of the closed-box models, but taking into account environment effects: an enriched infall from a galaxy with the same mass and with the same SFE with zero initial metallicities for galaxy1 and galaxy2: $Z_{O,1}(0)=Z_{O,2}(0)=0$.

In Fig. 6 we show the results when we consider four different inflow parameters: λ =0.1, 0.5, 2, 6. We see that at a fixed stellar mass the metallicity of the galactic system drops down in presence of inflow. We are in agreement with the work of Torrey et al. (2012), where the effects of dynamical interaction of companion galaxies were studied. In Spitoni et al. (2010) we have already proved that a combination of primordial infall and outflows with variable wind parameters as functions of the stellar mass is required to reproduce it. Here, we show the effects of this interaction compared to the closed-box case.

In Torrey et al. (2012) galaxy-galaxy interactions do not change the slope of the MZ, but the MZ is just shifted down in parallel at smaller metallicities. In our case, if we consider a constant λ (in Fig. 6 connecting the same type and colour points) for all the galaxy masses we have a flatten-

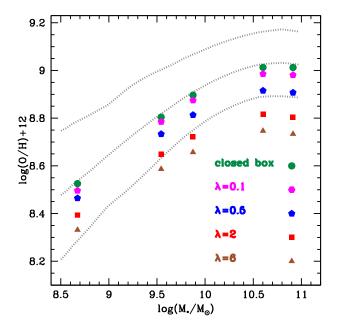


Figure 7. The observed mass-metallicity relation and related standard deviation of Kewley & Ellison (2008) are indicated with the dotted grey lines. With the green circles we show the closed-box with the variable y_O values of Spitoni et al. (2010) for galaxies with total masses as in Fig. 6. We compare them with model results where we consider the interactions with a companion galaxy with the same initial masses. For all the models it is assumed that the initial oxygen abundances are $Z_{O,1}(0) = Z_{O,2}(0) = 0$. Models with different wind parameters λ are indicated with symbols as Fig. 6.

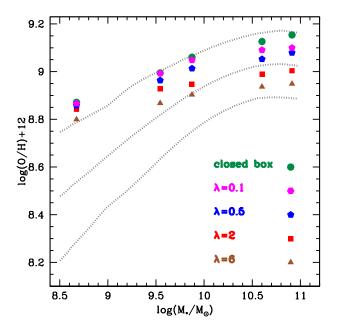


Figure 8. The observed mass-metallicity relation and related standard deviation of Kewley & Ellison (2008) are indicated with the dotted grey lines. Models are indicated with symbols as Fig. 6 assuming that the initial oxygen abundances are for galaxy 1 $Z_{O,1}(0)$ =0, and for galaxy 2 $Z_{O,2}(0)$ = 5 × 10⁻³, respectively.

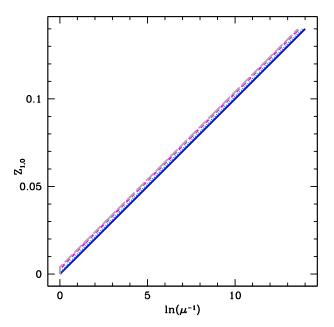


Figure 9. We show the metallicity $Z_{O,1}$ in the case of enriched infall from the evolution of galaxy2 as a function of $\ln(\mu^{-1})$. We fix ϵ =0.5, λ =2, and consider different ratio $M_{g1}(0)/M_{g2}(0)$ values: 1 (green dotted line), 0.1 (magenta short dashed line), 0.01 (grey long dashed line). We compare our results with closed model case (solid blue line). The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0)$ = $Z_{O,2}(0)$ =0.

ing of the MZ slope compared to the closed-box. However, from Spitoni et al. (2010) we know that more massive galactic systems are associated to smaller outflow episodes and therefore smaller wind parameters. Hence, if we consider a variable λ (decreasing towards higher mass galaxies) we preserve the slope of the MZ relation.

Our results are also in agreement with the observed depression of 0.05 -0.10 dex found by Ellison et al. (2008) for the MZ relation in interacting galaxies, if we consider a variable wind parameter λ with a maximum value smaller than $\lambda=6$.

In Fig. 7 we show the MZ relation obtained by the closed-box models with the same prescriptions adopted in Fig. 6 but with the variable yields y_O^1 of Spitoni et al. (2010) which is able to perfectly reproduce the MZ relation. Again, the fact of considering "environment" effects leads to a drop in the metallicity of the galaxy1 at a fixed stellar mass compared to the closed-box models. The fit with the observations deviates for $\lambda > 0.1$, and this might be a hint that the kind of interactions treated in this paper cannot be very general, but it surely applies for specific systems (such as the M81 group).

Finally, in Fig. 8 we show the effects of different initial metallicities $Z_{O,2}(0)$ of galaxy2 on the MZ relation. Assuming the same model parameters of Fig. 6 but with $Z_{O,2}(0) = 5 \times 10^{-3}$, we see that at a fixed stellar mass, the

¹ We remind that varying the yield for stellar generation corresponds to vary either the stellar nucleosynthesis or the IMF.

decrease of the metallicity is less prominent compared Fig. 6 especially for small stellar masses.

Edmunds (1990) studied the effects of gas flows on the chemical evolution of galaxies showing that it does exist a forbidden area in the plane $\log(\mu^{-1})$ versus metallicity above the simple model solution, for models with inflow of unenriched gas or outflows. Here, we want to test if this result is still valid in the case of a time dependent enriched infall from a companion galaxy with initial metallicities for galaxy1 and galaxy2 set to zero: $Z_{O,2}(0) = Z_{O,1}(0) = 0$. The expression of $\mu_1(t)$ as a function of time is:

$$\mu_1(t) = \frac{e^{-(1-R)St} \left(1 + \epsilon \frac{M_{g2}(0)}{M_{g1}(0)} \left[1 - e^{-\lambda(1-R)St} \right] \right)}{1 + \frac{\lambda \epsilon}{1+\lambda} \frac{M_{g2}(0)}{M_{g1}(0)} \left(1 - e^{-\lambda(1-R)St} \right)}.$$
 (31)

We need an expression for $\mu_1(t)$ as a function of the metallicity Z_O . We have seen that the solution $Z_{O,1}(t)$ in the case of zero initial metallicity is $(1-R)Sy_Ot$. Therefore, we can rewrite eq. (31) as:

$$\mu_1(Z_1) = \frac{e^{-Z_1/y_O} \left(1 + \epsilon \frac{M_{g2}(0)}{M_{g1}(0)} \left[1 - e^{-\lambda Z_1/y_O} \right] \right)}{1 + \frac{\lambda \epsilon}{1 + \lambda} \frac{M_{g2}(0)}{M_{g1}(0)} \left(1 - e^{-\lambda Z_1/y_O} \right)}.$$
 (32)

In Fig. 9 we show the metallicity $Z_{O,1}$ in case of enriched infall from the evolution of galaxy2 as the function of $\ln(\mu^{-1})$. We fix ϵ =0.5, λ =2, and consider different ratio $M_{g1}(0)/M_{g2}(0)$ values:1, 0.1, 0.01. We see that all the studied cases are in the forbidden area defined by Edmunds (1990), and they evolve similarly to the closed box model with small differences. This result, at variance with Edmunds (1990) is due to the fact that here is the first time in which these particular cases are studied and the theorems T(2) and T(3) of Edmunds (1990) cannot be applied because of the presence of enriched infall.

4.3 The analytical solution including both galactic fountain and environment effects

In Recchi et al. (2008), new analytical solutions in the framework of differential winds were presented, namely galactic winds in which the metals are ejected out of the parent galaxy more efficiently than the other elements. The existence of differential winds has been first introduced in the context of chemical evolution of galaxies by Pilyugin (1993) and Marconi et al. (1994). A realistic case of variable infall metallicity is represented by the situation in which the metallicity of the infalling gas is set to be always equal to the one of the galactic wind. This condition implies that the very same gas that has been driven out of the galaxy by energetic events can subsequently fall back to the galaxy, due to the gravitational potential well.

This kind of duty cycle is called galactic fountain (Shapiro & Field 1976, Bregman 1980). Spitoni et al. (2008, 2009) showed the effect of galactic fountains on a detailed chemical evolution model (where the instantaneous recycling and mixing approximation were relaxed) of the Milky Way. They discussed the delay in the chemical enrichment due to the fact that the gas takes a finite time to orbit around the Galaxy an fall back into the disk. In Recchi et al. (2008) an analytical solution was presented in the case for galactic fountains.

Here, we want to generalize the results of Recchi et al.

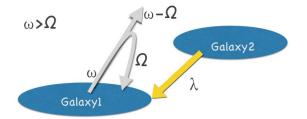


Figure 10. In this sketch the gas flows related the galaxy1 are represented when galactic fountain are considered. The enriched infall with the parameter λ is drawn with the yellow arrow. The outflowing gas from galaxy1 with parameter ω and the fraction which fall back (Ω) are in gray arrows. Our model is in the framework of differential wind theory when metals are more easily channeled out.

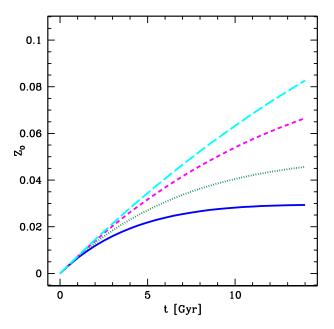


Figure 11. We show the time evolution of the metallicity $Z_{O,1}$ in presence of galactic fountains when mass ratios $M_{g1}(0)/M_{g2}(0)=1$ and SFE S=1 Gyr $^{-1}$, using the new analytical solution presented in eq. (A1) with $\epsilon=0.5$, and $\lambda=0.4$, $\Omega=0.1$ and $\omega=0.2$. The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0)=Z_{O,2}(0)=0$. The model with $\alpha=4.5$ is drawn with the blue long solid line, the model with $\alpha=3$ is represented by dotted green line solid line, the case with $\alpha=2$ is drawn with with the short dashed magenta line. With the cyan long dashed line the model with $\alpha=1.5$ is represented.

(2008), and find a new analytical solution for the metallicity (oxygen) in the galaxy1 when the galactic fountain effect is included together with the the enriched infall from the galaxy2. To consider galactic fountains we have to take into account an outflow episode and a new infall one caused by SN explosion events in the galaxy, with parameters ω and Ω . The metallicities of the outflows an fountains are:

$$Z_{\omega} = Z_{O,1} \alpha \omega (1 - R) \psi_1, \tag{33}$$

$$Z_{\Omega} = Z_{O,1} \alpha \Omega (1 - R) \psi_1, \tag{34}$$

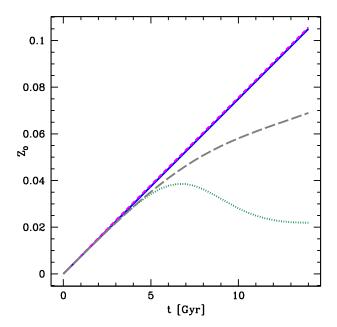


Figure 12. We show the time evolution of the metallicity $Z_{O,1}$ when we consider different SFEs S_2 for the galaxy2 following the new analytical solution presented in eq. (40). We consider the case with $M_{g1}(0)/M_{g2}(0){=}1$, $\epsilon{=}0.5$, and $\lambda{=}0.1$. The SFE for the galaxy1 is fixed at the value of $S_1{=}1~{\rm Gyr}^{-1}$. The model with $S_2{=}1~{\rm Gyr}^{-1}$ is drawn with the blue solid line, the model with $S_2{=}2~{\rm Gyr}^{-1}$ is represented by the magenta short dashed line, With the long dashed grey line the model with $S_2{=}0.5~{\rm Gyr}^{-1}$ is represented. Finally, the model with $S_2{=}0.1~{\rm Gyr}^{-1}$ is drawn with the dotted green line. The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0){=}Z_{O,2}(0){=}0$.

We are studying galactic fountain effects in the framework of differential winds as done in Recchi et al. (2008) where it was introduced the parameter $\alpha>1$, in order to take into account for the fact that metals are more easily channeled out from the parent galaxy compared to the unprocessed gas.

In Fig. 10 we present a sketch of the gas flow patterns for the galaxy1 when galactic fountain are considered. It is shown the gas inflow from the galaxy 2 with the associated wind parameter λ , the outflow from galaxy1 (ω), and the fraction which comes back (Ω).

Melioli et al. (2015) using three-dimensional hydrodynamical simulations investigated the impact of SN feedback in gas rich dwarf galaxies and the formation of galactic fountain and outflows. They found a similar circulation flow of the one presented in Fig. 10: galactic fountains is generally established and the metal-rich SN ejecta is instead more significant fraction (25-80 percent) is vented in the intergalactic medium, even.

It must be underlined that the treatment of the galactic fountains is actually analogous to what is done in Recchi et al. (2008) with ω replacing λ and Ω replacing Λ . The difference is in the role of galaxy2.

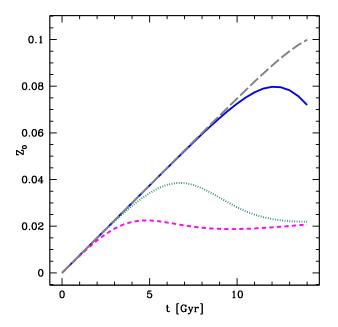


Figure 13. We show the time evolution of the metallicity $Z_{O,1}$ when we consider different mass ratios $M_{g1}(0)/M_{g2}(0)$, whereas SFEs S_1 and S_2 have been fixed at the values of 1 and 0.1 Gyr⁻¹, respectively. We use the new analytical solution presented in eq. (40) with ϵ =0.5, and λ =0.1. The model with $M_{g1}(0)/M_{g2}(0)=10^3$ is drawn with the grey long solid line, the model with $M_{g1}(0)/M_{g2}(0)=10^2$ is represented by the blue solid line, the case with $M_{g1}(0)/M_{g2}(0)=1$ is drawn with with the dotted green line. With the magenta short dashed line the model with $M_{g1}(0)/M_{g2}(0)=10^{-1}$ is represented. The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0)=Z_{O,2}(0)=0$.

$$\begin{cases}
\frac{dM_{tot1}}{dt} = (1 - R) \left(\epsilon \lambda \psi_2(t) + (\Omega - \omega) \psi_1(t) \right) \\
\frac{dM_{g1}}{dt} = (1 - R) \left(\epsilon \lambda \psi_2(t) + (\Omega - \omega - 1) \psi_1(t) \right) \\
\frac{dM_{O,1}}{dt} = (1 - R) \left(\psi_1(t) \left[y_O + \Theta Z_{O,1}(t) \right] + \psi_2(t) \epsilon \lambda Z_{O,2}(t) \right).
\end{cases} (35)$$

with $\Theta = -1 + \alpha \Omega - \alpha \omega$.

The differential equation of the metallicity in terms of oxygen for the system is:

$$\frac{dZ_{O,1}(t)}{dt} = (1 - R)S \times$$

$$\bigg(y_O + Z_{O,1}(t)\chi + \frac{M_{g2}(t)}{M_{g1}(t)} \Big[-\epsilon \lambda Z_{O,1}(t) + \epsilon \lambda y_O(1-R)St \Big] \bigg), (36)$$

with $\chi = (\alpha - 1)(\Omega - \omega)$. It can be easily proved that the evolution in time of the mass of gas of the galaxy1 is given by:

$$M_{g1}(t) = \frac{e^{-(1-\Omega+\omega)(1-R)St}}{1+\Omega-\omega} \times \left(\lambda(1-e^{-(\lambda+\Omega-\omega)(1-R)St})\epsilon M_{g2}(0) + M_{g1}(0)(\lambda+\Omega-\omega)\right). (37)$$

In eq. (A1) of the Appendix A we show the complete expression of the analytical solution in presence of galactic fountains.

In Fig. 11 the effects of different values of the parameter α are tested. The higher α is, the less the galaxy gets enriched. This is because we assumed that $\Omega < \omega$ and con-

sequently the metals (in this case oxygen) which escape, are larger than the ones which rain back into the galaxy.

4.4 Model with different star formation efficiencies

Finally, we test the case where the interacting galaxies have different SFEs. It is well known that different galactic systems can be characterized by different SFEs (Matteucci 2001), and generally, higher SFEs can be associated to more massive systems. Here, we show the new analytical solution for the galaxy1 with a SFE S_1 receiving enriched outflow of gas from another galaxy associated with a SFE S_2 . Therefore, with the new SFRs $\psi_1 = S_1 \times M_{g1}$ and $\psi_2 = S_2 \times M_{g2}$ the eq. (15) becomes:

$$M_{g1}(t) = \frac{e^{-(1-R)S_1t}}{S_1 - (\lambda+1)S_2} \times \left\{ M_{g1}(0) \left(S_1 - (\lambda+1)S_2 \right) + \epsilon \lambda M_{g2}(0) S_2 \left(e^{\delta t} - 1 \right) \right\}, \quad (38)$$

with $\delta = (1 - R)(S_1 - (\lambda + 1)S_2)$. It is trivial to see that when $S_1 = S_2$ eq. (38) is identical to eq. (15). The differential equation we have to solve to have the metallicity evolution of the galaxy1 in presence of different SFEs is:

$$\frac{dZ_{O,1}(t)}{dt} = (1 - R) \times \left(y_O S_1 + S_2 \frac{M_{g2}(t)}{M_{c1}(t)} \left[-\epsilon \lambda Z_{O,1}(t) + \epsilon \lambda y_O (1 - R) S_2 t \right] \right). (39)$$

Finally, the new analytical solution in this case is the following:

$$Z_{O,1}(t) = \frac{1}{(S_1 - (1+\lambda)S_2)} \times \frac{\left[\epsilon \lambda S_2 y_o\right] A(t) - \frac{M_{g_1}(0)}{M_{g_2}(0)} \left[B(t)\right]}{(-1 + e^{\delta t})\epsilon \lambda S_2 + \frac{M_{g_1}(0)}{M_{g_2}(0)} \left(S_1 - (1+\lambda)S_2\right)},$$
(40)

where A(t) and B(t) are the following time dependent terms:

$$A(t) = S_2 + e^{\delta t} \left\{ S_1 - S_2 + (-1 + R)S_2 \left(-S_1 + S_2(1 + \lambda) \right) t \right\} +$$

$$+ S_1 \left\{ -1 + (-1 + R) \left(S_1 - (1 + \lambda) S_2 \right) t \right\}, \tag{41}$$

$$B(t) = (-S_1 + S_2 + \lambda S_2)^2 \Big((-1 + R)S_1 t y_o - Z_{O,1}(0) \Big). (42)$$

We note that the result depends on the ratio between the initial masses of galaxy1 and galaxy2. Moreover, as expected, when we impose $S_1 = S_2$ in eq. (40), the resulting $Z_{O,1}(t)$ is identical to the one obtained with eq. (24). In Fig. 12 we test the effect of different SFEs for the galaxy1 and galaxy2 when we consider the same initial gas mass ratio $(M_{g1}(0)/M_{g2}(0)=1)$. We tested different values for S_2 : 2, 1, 0.5, 0.1 Gyr⁻¹. For the galaxy1 we assume S_1 =1 Gyr⁻¹ in all models

We obtain, as expected, that smaller values of S_2 lead to a smaller chemical enrichment of the system. In the case of $S_2 = 0.1 \text{ Gyr}^{-1}$ the metallicity of the system even decreases at later galactic times due to the strong dilution effect of

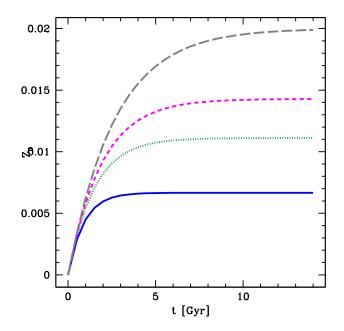


Figure 14. We report the time evolution of the metallicity $Z_{O,1}$ when we take into account also a primordial infall with wind parameter Λ . We consider the case when $M_{g1}(0)/M_{g2}(0)=1$, $\epsilon=0.5$, $\lambda=0.3$, and S=1 Gyr $^{-1}$. The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0)=Z_{O,2}(0)=0$. The model with $\Lambda=0.5$ is drawn with the long dashed grey line, the model with $\Lambda=0.7$ is represented by the magenta short dashed line, the case with $\Lambda=0.9$ is drawn with the dotted green line. With the solid blue line the model with $\Lambda=1.5$ is represented.

the infalling gas. In fact, we can consider it like pristine gas because of, as expected, the small SFE value of galaxy2 compared to galaxy1.

In Fig. 13, fixing S_1 and S_2 at the constant values of 1 and 0.1 Gyr⁻¹, respectively, we show the effect of different initial gas mass ratios for the galaxy1 and galaxy2. We consider four cases: $M_{g1}(0)/M_{g2}(0)=10^3$, 10^2 , 1, and 10^{-1} . Because of the choice of an extremely low SFE for galaxy2 we expect that the most important deviation from the closed-box solution is obtained with larger $M_{g2}(0)$ values, hence smaller $M_{g1}(0)/M_{g2}(0)$ ratios. In Fig. 13 we confirm it, and the model with $M_{g1}(0)/M_{g2}(0)=10^3$, is almost identical to the closed-box evolution model.

4.5 The effects of the primordial infall

We present here the results in the presence of a primordial infall of gas for both galaxy1 and galaxy2. First, we need to compute the time evolution for the metallicity $Z_{O,2}(t)$ when a primordial infall of gas is considered. To this aim we need the expression of the mass fraction $\mu_2(t)$ as a function of time. From the system (13), with the condition that the total initial mass is equal to the gas content $M_{tot2}(0) = M_{g2}(0)$ we have that:

$$M_{tot2}(t) = \frac{\Lambda - \lambda}{\Lambda - \lambda - 1} M_{g2}(t) - M_{g2}(0) \frac{1}{\Lambda - \lambda - 1}.$$
 (43)

The mass fraction $\mu_2 = M_{g2}(t)/M_{tot2}(t)$ can be written as:

$$\mu_2^{-1} = \left(\frac{\Lambda - \lambda}{\Lambda - \lambda - 1} - \frac{1}{\Lambda - \lambda - 1}e^{-(\Lambda - \lambda - 1)(1 - R)St}\right).$$
 (44)

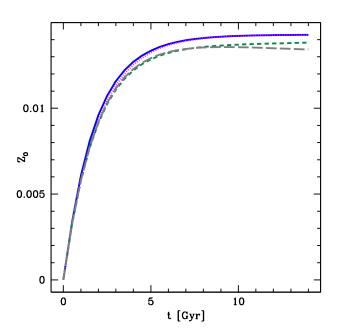


Figure 15. We show the time evolution of the metallicity $Z_{O,1}$ when we consider different SFEs S_2 for the galaxy2 coupled with a primordial infall of gas following the new analytical solution presented in eq. (A2) of Appendix A. We consider the case when $M_{g1}(0)/M_{g2}(0)=1$, and $\epsilon=0.5$, and the SFE fro the galaxy1 is fixed at the value of $S_1=1~{\rm Gyr}^{-1}$. All the models assume $\lambda=0.3$ and $\Lambda=0.7$. The initial oxygen abundances for the galaxy1 and galaxy2 are $Z_{O,1}(0)=Z_{O,2}(0)=0$. The model with $S_2=2~{\rm Gyr}^{-1}$ is represented by the magenta dotted line, the model with $S_2=1~{\rm Gyr}^{-1}$ is drawn with the blue solid line, the model with $S_2=0.2~{\rm Gyr}^{-1}$ is drawn with the green short dashed line. Finally, the model with $S_2=0.1~{\rm Gyr}^{-1}$ is drawn with the green short dashed line.

We recall that the general solution as function of the gas fraction for a system with primordial infall and outflow is given by eq. (8) with $Z_A=0$. Finally, if we insert eq. (44) in eq. (8), we obtain that

$$Z_{O,2} = \frac{y_O}{\Lambda} \left(1 - e^{-\Lambda(1-R)St} \right). \tag{45}$$

The new expressions for the time evolution of $M_{g1}(t)$ and $M_{g2}(t)$ are, respectively:

$$M_{g1}(t) = e^{(\Lambda - 1)(1 - R)St} \left\{ M_{g1}(0) + \epsilon M_{g2}(0) \left[1 - e^{-\lambda(1 - R)St} \right] \right\}, (46)$$

$$M_{g2}(t) = M_{g2}(0)e^{(\Lambda - \lambda - 1)(1 - R)St}. \tag{47}$$

The differential equation we should to solve in presence of a primordial infall, is the following one:

$$\frac{dZ_{O,1}(t)}{dt} = (1 - R)S(y_O - \Lambda Z_{O,1}(t)) +$$

$$(1-R)S\frac{M_{g2}(t)}{M_{g1}(t)} \left\{ \lambda \epsilon \frac{y_O}{\Lambda} \left[1 - e^{-\Lambda(1-R)St} \right] - \epsilon Z_{O,1}(t) \lambda \right\}. (48)$$

The solution of eq. (48) for the time evolution of the oxygen abundance for the galaxy1 is then given by:

$$Z_{O,1}(t) = \frac{1}{\Lambda \left(M_{g1}(0) + (1 - e^{\lambda(-1+R)St})\epsilon M_{g2}(0) \right)} \times$$

$$(A2(t) + [M_{g1}(0) + \epsilon M_{g2}(0)]y_o - B2(t) + C2(t)),$$
 (49)

with

$$A2(t) = (-1 + e^{\Lambda(-1+R)St})e^{\lambda(-1+R)St} \epsilon M_{q2}(0)yo,$$

$$B2(t) = e^{(\Lambda)(-1+R)St} \epsilon M_{g2}(0) y_o,$$

and

$$C2(t) = -e^{(\Lambda)(-1+R)St} M_{g1}(0) [y_o - \Lambda Z_{O,1}(0)].$$

It is easy to show that it is possible to recover the solution of eq. (24) presented in Section 4.1 for $\lim_{\Lambda\to 0} Z_{O,1}(t)$. In Fig. 14 we show the time evolution of the metallicity $Z_{O,1}$ when $M_{g1}(0)/M_{g2}(0)=1$, $\epsilon=0.5$, and the SFE for the galaxy1 and galaxy2 are fixed at the value S=1 Gyr⁻¹. Assuming for all the models $\lambda=0.3$, we test different values for the the infall parameter Λ which is associated with the primordial infall: 0.5, 0.7, 0.9, 1.5.

We note that the effect of dilution of a primordial gas infall overwhelms the enriched infall from the companion galaxy, and the time evolution of the metallicity for the galaxy1 deviates substantially from the closed-box solution (we have proved in previous Sections that the evolution in time of a system with only enriched gas from a companion galaxy follows the closed-box solution) even with a primordial infall parameter comparable to the one of enriched infall.

We also consider the case with different SFEs for the galaxy1 and galaxy2 in presence of primordial infall. We do not enter into details about the procedures and the relations used to recover the new analytical solution $Z_{O,1}(t)$, and we only show the solution in eq.(A2) of Appendix A.

In Fig. 15 we show the effects of different values for SFEs of the galaxy2 on the chemical evolution of galaxy1 in presence of primordial infall. We consider the case when the mass ratio is $M_{g1}(0)/M_{g2}(0)=1$, and $\epsilon=0.5$, and the SFE for the galaxy1 is fixed at the value of $S_1=1~{\rm Gyr}^{-1}$. All the models assume $\lambda=0.3$ and $\Lambda=0.7$. We consider different models with different S_2 values: 2,1, 0.2, 0.1 ${\rm Gyr}^{-1}$. In Fig. 15 it is clearly shown that all the models with different SFEs show more or less the same time evolution for the $Z_{O,1}$.

Comparing the models with S_2 =0.2 Gyr⁻¹ and S_2 =0.1 Gyr⁻¹ we note that at late times, as expected, the chemical evolution of galaxy1, which suffers the enriched infall from galaxy2 with S_2 =0.2, shows a higher metallicity. This behavior is inverted at early times. This is due to the way in which we consider the primordial infall. In fact as shown in eq. (5), the primordial infall is assumed to be proportional to the SFR of the galaxy, therefore at early times the galaxy2 with S_2 =0.2 Gyr⁻¹ suffers a larger dilution effect than the same system with S_2 =0.1 Gyr⁻¹.

We conclude that in presence of primordial infall the chemically enriched gas coming from galaxy2 has not a big effect on the chemical evolution of the galaxy1.

In this work we consider only primordial infall from the IGM, whereas in Peng & Maiolino (2014b) it was studied how global environment properties (overdensity of galaxies) can modify the metallicity of the IGM. The observed strong correlation between over-density and metallicity for star-forming satellites suggests that the gas infall is getting progressively more metal-enriched in dense regions.

5 CONCLUSIONS

In this paper we presented new analytical solutions for the evolution of the metallicity (oxygen) of a galaxy in presence of "environment" effects coupled with galactic fountains, and primordial infall of gas. The main results of our study are the following ones:

- If we consider a linear Schmidt (1959) law for the star formation rate we have the same time evolution of the metallicity for both the closed-box model and the one with only outflow. Therefore, in the last case the result does not depend on the wind parameter λ . This result is holding when the outflow is not differential. It is worth to note that even if at the same time those two systems show the same content of metals they show different gas fractions as shown in Fig. 1.
- The new analytical solution for the evolution of the galaxy1 where we consider the enriched inflow of gas from the galaxy2 with initial metallicities $Z_{O,1}(0)$ (galaxy1) and $Z_{O,2}(0)$ (galaxy2) is:

$$Z_{O,1}(t) = Sy_O(1 - R)t +$$

$$+\frac{Z_{O,1}(0)+Z_{O,2}(0)\epsilon\frac{M_{g2}(0)}{M_{g1}(0)}\bigg(1-e^{-\lambda(1-R)St}\bigg)}{1+\epsilon\frac{M_{g2}(0)}{M_{g1}(0)}\bigg(1-e^{-\lambda(1-R)St}\bigg)}.$$

where: λ is the wind parameter, ϵ is the fraction of the outflowing gas from galaxy2 to galaxy1, S is the star formation efficiency, and $\frac{M_{g2}(0)}{M_{g1}(0)}$ is the ratio between the initial gas masses of the two galaxies.

- When we consider the time evolution of a galaxy including an enriched infall due to the interactions with a nearby galaxy assuming no pre-enriched gas for both galaxies, the chemical enrichemnt is less efficient than in the closed-box solution at a fixed stellar mass. Moreover, the mass-metallicity relation for galaxies which suffer a gas infall from an evolving galaxy, is shifted down at lower Z values if compared to the closed-box model results for isolated galaxies.
- We show a new analytical solution where we consider galactic fountain effects in the framework of differential winds coupled with the interaction with a nearby galaxy.
- We presented the new solution in case of different SFEs. If the infall gas originated by a nearby galaxy has a smaller SFE than the galaxy suffering the infall, the recipient galaxy will show a less efficient chemical enrichment. In this case we showed that the smaller is the ratio between the initial gas masses of the recipient and donor galaxy, the less efficient the chemical evolution is.
- A new analytical solution when a primordial infall is coupled with interactions with a nearby galaxy is presented. The effect of dilution of a primordial gas infall overwhelms the enriched infall from the chemical evolution of a companion galaxy even with a primordial infall parameter comparable to the one of enriched infall. We have also shown how different SFEs for the companion galaxy does not affect the chemical evolution of galaxy1 in presence of primordial infall.

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APPENDIX A: NEW ANALYTICAL SOLUTIONS WITH GALACTIC FOUNTAINS AND PRIMORDIAL INFALL

• We report here the analytical solution for the time evolution of the oxygen abundance $Z_{O,1}$ when galactic fountain and the enriched infall of gas from a companion galaxy are taken into account. This solution is obtained when the initial metallicity of the galaxy1 is assumed equal to zero.

$$Z_{O,1} = \frac{e^{-(-1+\alpha)(\Omega-\omega)(-1+R)St}y_{O}}{(-1+\alpha)(\lambda+\alpha(\Omega-\omega))^{2}A3(t)(\Omega-\omega)} \times$$

$$\left(\alpha^{2}(1-e^{(-1+\alpha)(\Omega-\omega)(-1+R)St})M_{g1}(0)(\Omega-\omega)^{3} + \lambda^{3}B3(t) + \lambda^{2}(\Omega-\omega)C3(t) + \lambda(\Omega-\omega)^{2}D3(t)\right)$$
with
$$A3(t) = \lambda(M_{g1}(0) + (1-e^{(\lambda+\Omega-\omega)(-1+R)St})\epsilon M_{g2}(0)) + M_{g1}(0)(\Omega-\omega)$$

$$B3(t) = \left(1-e^{(-1+\alpha)(\Omega-\omega)(-1+R)St}\right)(M_{g1}(0) + \epsilon M_{g2}(0)) + (-1+\alpha)e^{(\lambda+\alpha(\Omega-\omega))(-1+R)St}\epsilon M_{g2}(0)(\Omega-\omega)(-1+R)St$$

$$C3(t) = \left(1-e^{(-1+\alpha)(\Omega-\omega)(-1+R)St}\right)\left(M_{g1}(0) + 2\alpha M_{g1}(0) + 2\alpha\epsilon M_{g2}(0)\right) + (-1+\alpha^{2})e^{(\lambda+\alpha(\Omega-\omega))(-1+R)St}\epsilon M_{g2}(0)(\Omega-\omega)(-1+R)St,$$

$$D3(t) = -\epsilon M_{g2}(0) + \alpha E3(t) + (-1+\alpha)e^{(\lambda+\alpha(\Omega-\omega))(-1+R)St}\epsilon M_{g2}(0)\left(-1+\alpha+\alpha(\Omega-\omega)(-1+R)St\right),$$

$$E3(t) = (2+\alpha)M_{g1}(0) + 2\epsilon M_{g2}(0) - e^{(-1+\alpha)(\Omega-\omega)(-1+R)St}\left((2+\alpha)M_{g1}(0) + \alpha\epsilon M_{g2}(0)\right).$$

• The following solution is related to the case with a primordial infall coupled with an enriched infall from a companion galaxy, considering different SFEs:

$$Z_{O,1}(t) = \frac{A4(t) + B4(t)}{\Lambda(S_1 - (1+\lambda)S_2)(1 - e^{qt})\epsilon\lambda M_{g2}(0)S_2 + M_{g1}(0)((-1+\lambda)S_1 + (1-\Lambda+\lambda)S_2)}$$
(A2)

with

$$q = (-1 + R) \Big((-1 + \Lambda)S_1 + (1 - \Lambda + \lambda)S_2 \Big),$$

$$q_2 = q/(-1+R),$$

$$\Lambda_* = \Lambda(-1+R)S_1,$$

 $\epsilon \lambda M_{g2}(0)S_2$,

$$A4(t) = \epsilon \lambda M_{g2}(0) S_2 \Big([1 - e^{qt} + (e^{\Lambda * t} - e^{qt})(-1 + \Lambda)] S_1 - [1 + (e^{\Lambda * t} - e^{qt})(-1 + \Lambda - \lambda) + \lambda - e^{qt}(1 + \lambda)] S_2 \Big) yo,$$

$$B4(t) = M_{g1}(0) \Big(S_1 - (1 + \lambda) S_2 \Big) q_2 \Big(yo + e^{\Lambda * t} [-yo + \Lambda Z_{O,1}(0)] \Big).$$

APPENDIX B: LIST OF VARIABLES AND PARAMETERS

 $Z_{O,i}$: oxygen abundance of galaxy i;

 M_{qi} : mass of gas in galaxy i;

 ψ_i : star formation rate fo galaxy i;

 S_i : star formation efficiency of galaxy i;

 μ_i : gas fraction of galaxy i;

 y_O : oxygen yield;

R: returned fraction;

 λ : outflow parameter;

 Λ : infall parameter;

 ϵ : fraction of the outflowing gas from galaxy2 to galaxy1;

 ω : outflow parameter connected to galactic fountains;

 Ω : infall parameter connected to galactic fountains;

 α : differential wind parameter.