

THE RADIO BACKGROUND: RADIO-LOUD GALAXIES AT HIGH AND LOW REDSHIFTS

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Abstract. This paper is in two unequal halves. After dealing with the possibility of a genuine continuum background at $\lambda \gtrsim 1$ cm, and showing that it is unlikely to arise in interesting circumstances, the remainder of the discussion is devoted to discrete radio sources, and their consequences for cosmology. Three main issues are considered: (i) what makes a galaxy radio loud?; (ii) what do we know about how the population of radio-loud galaxies has changed with epoch?; and (iii) what can observations of high-redshift radio galaxies tell us about general questions of galaxy formation and evolution? The main conclusion is that radio galaxies are remarkably ordinary massive ellipticals. The high-redshift examples are generally old and red and do not make good candidates for primaeval galaxies.

1. INTRODUCTION

The purpose of this review is to see what facts of cosmological interest can be dredged from wavelengths of above a few cm. In order to deal with modern research, rather than ancient history, it will be necessary to cheat a little and concentrate on the discrete-source population, rather than genuine smooth backgrounds – a strategy adopted by many other speakers at this meeting. However, to do duty to the advertised title, we begin with a few comments about what a non-discrete background might actually mean, were it to exist. Following this, the concentration will be on radio galaxies: why are they active, and how has the degree of activity changed with redshift? The final sections attempt to liberate us from the shackle of the radio waveband altogether, and to ask

what general conclusions may be drawn about stellar evolution and galaxy formation from optical/IR data on high-redshift radio galaxies.

Notation: the Hubble constant, where quoted explicitly, is given in the form $h = H_0/100 \text{ kms}^{-1}\text{Mpc}^{-1}$. If not otherwise specified, $\Omega = 1$ and $h = 0.5$ are assumed.

2. THE SMOOTH RADIO BACKGROUND

Malcolm Longair has described how the Cavendish Laboratory spent the 1960s practicing human sacrifice in order to determine the extragalactic radio-source background, with the following approximate result:

$$I_\nu \simeq 6000 \left(\frac{\nu}{1 \text{ GHz}} \right)^{-0.8} \text{ Jy sr}^{-1},$$

to within an uncertainty of about 20% in amplitude and 0.1 in spectral index. This background dominates over the CMB for $\lambda \gtrsim 1 \text{ m}$, and is consistent with the integrated contribution of discrete sources.

On the other hand, it is also not ruled out that a genuine continuum background might exist at up to 10% or so of the above level. What would this mean if it was really so? The hope would be to learn something about diffuse intergalactic gas, and there are two standard emission mechanisms to which we might appeal: synchrotron radiation and bremsstrahlung. The parameters available are the density of the emitting plasma, parameterised by its contribution to Ω (in the case of synchrotron radiation, the electrons would have an assumed power-law energy distribution), plus the local value of either the magnetic field, B or temperature T – both of which should scale as $(1+z)^2$. The resultant background can then be worked out in the standard way (see Longair 1978). For synchrotron radiation, we get

$$I_\nu \simeq 10^{14} \Omega h \left(\frac{B}{\text{nT}} \right)^{1.8} \left(\frac{\nu}{1 \text{ GHz}} \right)^{-0.8} \text{ Jy sr}^{-1}.$$

What is a plausible value for the intergalactic magnetic field? It is worth recalling that magnetic fields are very much a skeleton in the closet of cosmology, since we cannot easily rule out rather large values – which would significantly change our ideas about structure formation, for example. A nice review of the issue is given by Coles (1992); he argues that B could be as large as 10^{-4} nT . This would allow observed magnetic fields in astrophysical sources to be made via compression, rather than dynamo effects, and would greatly alter the progress of galaxy clustering. For such a field, the observed background would be produced with $\Omega h \sim 10^{-3}$. This is an implausibly high density for a plasma with fully relativistic electrons, but it is perhaps surprising that the effect is this close to being interesting.

Turning to bremsstrahlung, one can simply try scaling old solutions for the X-ray background in which a ‘low’-energy flux of around $10^{-3} \text{ Jy sr}^{-1}$ is produced by models

with $T \simeq 10^8$ K and $\Omega h^2 \simeq 0.1$. Since bremsstrahlung emissivity scales as $T^{-1/2}$, this implies

$$I_\nu \simeq 10^3 (\Omega h^2)^2 \left(\frac{T}{1\text{K}} \right)^{-1/2} \text{Jy sr}^{-1}.$$

If we ignore the difficulty in keeping plasma at such temperatures ionized, this seems the closest that the radio background is likely to get to setting constraints on Cold Dark Matter...

3. WHAT MAKES A RADIO-LOUD GALAXY?

Turning now to the infinitely more interesting issue of the population of discrete radio sources, we first review what is known about the causes that lead to enhanced radio emission.

For orientation, it is convenient to give a sketch of the population, ordered according to output. Define $P \equiv \log_{10} L_{21\text{ cm}}/\text{WHz}^{-1}\text{sr}^{-1}$; at $P \gtrsim 24$ we find the classical radio galaxies and quasars, conventionally divided roughly into Fanaroff-Riley (1974) FR II objects like Cygnus A and compact sources such as 3C273. At intermediate powers $23 \lesssim P \lesssim 24$, we find FRI sources: twin-jet objects such as 3C31, often lurking in clusters. At $P \lesssim 24$, we find all the rest of astronomy: ‘radio-quiet’ quasars, starburst galaxies and normal galaxies. We shall be concerned here with the bona-fide radio galaxies having $P \gtrsim 23$.

Two outstanding systematics of such galaxies have long been known: they are virtually without exception associated with elliptical/S0 galaxies, and moreover with the massive members of this class. This strong tendency for the probability of strong radio activity to increase with optical luminosity is illustrated in Figure 1. It is however interesting that this figure conceals a more complex behaviour noted by Owen & White (1991). They showed that the more powerful FR II sources are actually *less* likely in the most massive galaxies – *i.e.* the FR transition shifts to higher radio power at higher optical power. This may indicate an influence of the local density on the ability of a radio jet to remain stable (see Prestage & Peacock 1989). Nevertheless, the increased tendency of more massive galaxies to produce sources of FRI output or above is not in conflict with this interesting discovery.

The rather narrow spread in stellar luminosity for radio galaxies has long been known, and is perhaps best illustrated in the infrared Hubble diagram (Lilly & Longair 1984), which displays an rms of only 0.4 mag. The average absolute magnitude is somewhat brighter than for normal ellipticals. The most direct way of demonstrating this is not to rely on local samples, where powerful radio galaxies are rare, but to turn to a direct comparison at intermediate redshifts. Aragón-Salamanca *et al.* (1991) give K -band data on ellipticals in the A370 cluster at $z = 0.37$, from which a Schechter $K^* = 16.3$ for the ellipticals can be determined. Lilly & Longair give $K = 15.2$ for the mean radio galaxy at this redshift, but this is in a $10''$ diameter aperture, whereas the cluster data are in $4''.8$ apertures. The aperture correction at this radius is well

Figure 1. The differential probability distribution of radio power (for $h = 1$), for different bins in optical luminosity, taken from Sadler, Jenkins & Kotanyi (1989). Above $P = 21 - 22$, the probability of radio emission is a very rapidly rising function of optical luminosity.

modelled by $L \propto r^{0.5}$, which introduces a small (0.4 mag.) correction, and leads to the conclusion

$$\langle L_{\text{RG}} \rangle \simeq 1.9 L_{\text{E}}^*.$$

Both the large size of this mean luminosity and its small dispersion may be understood quantitatively as empirical manifestations of the strong trend of radio activity with optical luminosity. If we say that the probability of a galaxy hosting a strong radio AGN is $P \propto L_{\text{opt}}^{\beta}$, then multiplying this rising power law into the exponential truncation of a Schechter function for the elliptical population as a whole gives roughly the observed mean luminosity and scatter if $\beta \simeq 4 - 5$. This may also go a good deal of the way towards explaining the dominance of elliptical hosts: the L^* values for elliptical galaxies tend to be a few tenths of a magnitude brighter than for spirals, a gap which may be stretched to as much as a magnitude if we allow for typical bulge-to-disk ratios to obtain the L^* ratios between ellipticals and spiral bulges. We would then predict $N_{\text{E}}/N_{\text{S}} \simeq 2.5^{\beta} \sim 10^2$. In other words, massive ellipticals dominate powerful radio sources because only they have the exceptionally deep potential wells needed for the most active radio AGN. This is far from the whole story: first, any possible spiral identifications for powerful radio sources must be more at the $\lesssim 10^{-3}$ level; second, the whole reasoning rests on the strong L^{β} trend which remains unexplained. There is still a major puzzle here.

Are there any other distinguishing features of radio galaxies? Almost all other peculiarities can either be traced directly to the effect of the AGN (such as the strong narrow emission lines), or to the peculiarity of high mass already discussed. It would be

important to know if there were any systematic differences between those galaxies that turn on a radio active nucleus and those that do not – but there is no strong evidence for any such difference. Various suggestions have been made, but these have usually turned out to be small and subtle effects, whose reality generates controversy.

For example, about a decade ago it was suggested that radio-loud galaxies were redder by about 0.03 mag. in $B - V$ (Sparks 1983), rounder (Disney & Sparks 1984), more rapidly rotating (Jenkins 1984) and in denser environments (Sparks *et al.* 1984) than their radio-quiet counterparts of the same optical luminosity. Sparks *et al.* (1984) argued that these trends could be understood within a single picture of fuel gathering in potential wells, with the deeper wells being more successful at generating radio activity. However, in subsequent years the picture has become somewhat more complicated as further data have accumulated. For example, Heckman *et al.* (1985) found that the suggestion of excess rotation was due to a few incorrect measurements in the compilation used by Jenkins. Smith & Heckman (1989) found a normal distribution of axial ratios and claimed that galaxies were bluer – sometimes by as much as 0.2 mag. in $B - V$. Finally, Smith & Heckman (1990) found environments consistent with those of radio-quiet ellipticals. Part of the problem here may be that any peculiarity may be a function of radio power, so that different studies can yield different answers unless they use the same definition of radio-loudness. Also, the range of properties of radio-quiet ellipticals is large and diverse; misleading conclusions may be reached unless there is a large and complete comparison sample. What is needed is a large sample of radio-loud ellipticals whose properties can be compared to a radio-quiet set matched in optical luminosity and redshift.

In the meantime, claimed peculiarities of radio ellipticals need to be treated with caution. Two properties which are presently in this provisional class are the suggestion that radio ellipticals have low-level isophote distortions indicative of merging (Smith & Heckman 1989), and the question of dynamics. Smith, Heckman & Illingworth (1990) found that radio-loud ellipticals lie on the ‘fundamental plane’ in size/luminosity/velocity dispersion space, but there are some suggestions that they may occupy a different region of the plane – being brighter at a given velocity or size (Sansom, Wall & Sparks 1987; Romanishin & Hintzin 1989). It will be interesting to see how these suggestions hold up. We certainly badly need some clear set of clues as to what triggers these objects.

4. LUMINOSITY FUNCTIONS

Now consider what we know empirically about the abundance of radio AGN at high redshift, and what constraints this information may set on models of structure formation.

4.1 Observational results

No significant new datasets relevant to the luminosity function of powerful radio sources have been published since the study of the RLF published by James Dunlop and myself

in 1990. This was based on nearly-complete redshift data on roughly 500 sources down to a limit of 100 mJy at 2.7 GHz, plus fainter number-count data and partial identification statistics.

The main conclusions of this study were firstly to affirm long-standing results (Longair 1966; Wall, Pearson & Longair 1980) that the RLF undergoes differential evolution: the highest luminosity sources change their comoving densities fastest. Nevertheless, because the RLF curves, the results can be described by a model of pure luminosity evolution for the high-power population, in close analogy with the situation for optically-selected quasars (Boyle *et al.* 1987). The characteristic luminosity in this case increases by a factor $\simeq 20$ between the present and a redshift of 2. Similar behaviour applies for both steep-spectrum and flat-spectrum sources, which provides some comfort for those wedded to unified models for the AGN population. There is a remarkable similarity here to the evolution of ‘starburst’ galaxies, distinguished by blue optical-UV continua and strong emission from dust which make them very bright in the IRAS 60- μ m band. It has been increasingly clear since the work of Windhorst (1984) that such galaxies make up a substantial part of the radio-source population below $S \simeq 1$ mJy. The evolution of these objects at radio wavelengths and at 60 μ m is directly related because there exists an excellent correlation between output at these two wavebands. Rowan-Robinson *et al.* (1993) have exploited this to investigate the implications of IRAS evolution for the faint radio counts. They find good consistency with the luminosity evolution $L \propto (1+z)^3$ reported for the complete ‘QDOT’ sample of IRAS galaxies by Saunders *et al.* (1990).

Were it not for the fact that some populations of objects show little evolution (*e.g.* normal galaxies in the near-infrared: Glazebrook 1991), one might be tempted to suggest an incorrect cosmological model as the source of this near-universal behaviour. The alternative is to look for an explanation which owes more to global changes in the Universe than in the detailed functioning of AGN. One obvious candidate, long suspected of playing a role in AGN, is galaxy mergers; Carlberg (1990) suggested that this mechanism could provide evolution at about the right rate (although see Lacey & Cole 1993). Why the evolution does not look like density evolution is still a major stumbling block, but it seems that we should be looking at this area quite intensively, given that mergers have been implicated in both AGN and starbursts, and that there may be some evidence for their operation from the general galaxy population (Broadhurst, Ellis & Glazebrook 1992).

However, it is unclear how much emphasis should be placed on this apparent universality; particularly, limited statistics make it uncertain just how well luminosity evolution is obeyed. For example, Goldschmidt *et al.* (1992) have produced evidence that the PG survey is very seriously incomplete at $z \lesssim 1$; if confirmed, this would imply that the evolution of quasars of the very highest luminosities is *less* than for those a few magnitudes weaker. Furthermore, the QDOT database was afflicted by an error in which 10% of the galaxies were assigned incorrectly high redshifts (Lawrence, private communication); this will probably weaken the IRAS degree of evolution. It may well be that the degree of unanimity described above will prove spurious, and that we will be left with the unsurprising situation that a complex phenomenon like AGN evolution can only be described simply when the samples are too small to show much of the detail.

4.2 Redshift cutoff and interpretation

At higher redshifts, the uncertainties increase as the data thin out, but there is evidence that the luminosity function cannot stay at its $z = 2$ value at all higher redshifts. The form of this ‘redshift cutoff’ is uncertain: we cannot at present distinguish between possibilities such as a gradual decline for $z > 2$, or a constant RLF up to some critical redshift, followed by a more precipitous decline. We therefore present a ‘straw man’ model designed to concentrate the minds of observers, in which the luminosity evolution goes into reverse at $z \simeq 2$ and the characteristic luminosity retreats by a factor $\simeq 3$ by $z = 4$ (Figure 2).

Figure 2. The evolving RLF, according to the pure luminosity evolution model of Dunlop & Peacock (1990). The main features are a break which moves to higher powers at high redshift, but which declines slightly at $z \gtrsim 2$. the strength of the break and the rate of evolution are comparable for both radio spectral classes.

This model predicts the following fraction of objects at $z > 3.5$ as a function of 1.4-GHz flux-density limit: 0.5% at 100 mJy; 3% at 1 mJy. Without some form of cutoff, these numbers would be about a factor of 5 higher. The reason for the increased ease of detecting a cutoff at low flux density is that the RLF is rather flat at low powers; for $\rho \propto P^{-\beta}$ and $S \propto \nu^{-\alpha}$, we expect $dN/dz \propto (1+z)^{-\beta(2+\alpha)-1/2}$. Steep spectra and a steep RLF thus discriminate against high redshifts, but at low powers the flatter RLF

helps us to see whatever high- z objects there are more easily. It should be relatively easy to test for the presence of a cutoff on the basis of these predictions. This is especially true at low flux densities (see Figure 3). Here, we still sample the flat portion of the RLF even at high redshift, and so the predicted numbers of high-redshift sources is large without a cutoff – around 15% at $z > 4$ for a sample at 1 mJy.

Figure 3. A plot of the integral redshift distributions predicted for two samples limited at 1.4-GHz flux densities of 100 mJy and 1 mJy. The upper line shows a prediction for a luminosity function which is held constant for $z \gtrsim 2$; the lower line shows the prediction of the ‘negative luminosity evolution’ model of Dunlop & Peacock (1990).

Whether or not the redshift cutoff is real, we seem to have direct evidence that the characteristic comoving density of radio galaxies has not altered greatly between $z \simeq 4$ and the present. Integrating to 1 power of 10 below the break in the RLF, we find

$$\rho \simeq 10^{-6} h^3 \text{ Mpc}^{-3}.$$

Is this a surprising number? In models involving hierarchical collapse, the characteristic mass of bound objects is an increasing function of time. At high mass, the abundance of objects falls exponentially if the statistics of the density field are Gaussian. Clearly, a model such as CDM (which falls in this class) will be embarrassed if the density of massive objects stays high to indefinite redshifts. The analysis of this problem, using

the Press-Schechter mass-function formalism (Press & Schechter 1974) was first given by Efstathiou & Rees (1988) for optically-selected quasars.

There are two degrees of freedom in the analysis: what mass of object is under study, and what are the parameters of the fluctuation power spectrum? For the first, Efstathiou & Rees had to construct a long and uncertain chain of inference leading from quasar energy output, to black-hole mass, to baryonic galaxy mass, to total halo mass. For radio galaxies, things are much simpler, because we can see the galaxy directly. Infrared observations imply that, certainly up to $z = 2$, the stellar mass of radio galaxies has not changed significantly. At low redshift, there is direct evidence that the mass of radio galaxies exceeds $10^{12} M_{\odot}$, so it seems reasonable to adopt this value at higher redshift. Figure 4 shows the Press-Schechter predictions for two COBE-normalized CDM models. The low- h model which fits the shape of the galaxy-clustering power spectrum (Peacock 1991) intersects the observed number density at low-ish redshifts (7 – 8), whereas the ‘standard’ $h = 0.5$ model with its higher degree of small-scale power predicts many more objects. This is clearly only a suggestive coincidence at present, but it is clearly interesting that the model which most nearly describes large-scale structure also predicts that the formation of massive objects should occur near the point at which we infer a lack of high- z AGN.

4.3 Black-Hole abundances

In the spirit of this meeting, it is probably important to concentrate on integrated properties of the radio-source population. One important feature of this sort is the relic density of black holes deposited by the work of past AGN. This is something which has been discussed extensively for radio-quiet quasars, but which has not been given so much attention in the radio waveband alone. The advantage of doing this is that, as discussed above, we have a rather good idea of which galaxies host radio-loud AGN, and therefore we know where to look for any debris from burned-out AGN. The basic analysis of this problem goes back to Soltan (1982). He showed that the relic black-hole density may be deduced observationally in a model-independent manner, as follows.

The mass deposited into black holes in time dt by an AGN of luminosity L_{ν} is

$$d[M_{\bullet}c^2] = \epsilon^{-1} g [\nu L_{\nu}] dt,$$

where ϵ is an efficiency, and g is a bolometric correction. To obtain the total mass density in black holes, we have to multiply the above equation by the luminosity function (which already gives the comoving density, as required) and integrate over luminosity. The integral can be converted to one over redshift and flux density, and the integrand depends of the observable distribution of redshifts and flux densities, so the answer is model dependent. Doing this for the Radio LF gives a much lower answer than for optically-selected QSOs, which have a much higher density:

$$\begin{aligned} \rho_{\bullet} &= 10^{11.7} \epsilon^{-1} g M_{\odot} \text{Gpc}^{-3} \quad (\text{QSO}) \\ \rho_{\bullet} &= 10^{9.0} \epsilon^{-1} g M_{\odot} \text{Gpc}^{-3} \quad (\text{Radio}) \end{aligned}$$

Figure 4. The epoch dependence of the integral mass function in CDM, calculated using the Press-Schechter formalism as in Efstathiou & Rees (1988). The normalization is to the COBE detection of CMB fluctuations. Results are shown for two Hubble constants: the ‘standard’ $\Omega h = 0.5$ (upper panel) and $\Omega h = 0.3$ (lower panel). Here, Ωh is merely a fitting parameter used to describe the shape of the power spectrum, and it does not presuppose a true value of the Hubble constant. The vertical scaling of density with h is given explicitly, and the mass values assume $h = 0.5$. The extra small-scale power in the former case means that many more massive hosts than the observed radio-galaxy number (horizontal line) are predicted, even at $z \gtrsim 10$.

Since we know rather well the present density of massive elliptical galaxies (*e.g.* Loveday *et al.* 1992), we may distribute half the above radio mass into ellipticals above the median radio-galaxy luminosity, with the following result for the mean hole mass:

$$\langle M_{\bullet} \rangle \simeq 2000 \epsilon^{-1} g h^{-3} M_{\odot}.$$

What is the bolometric correction for radio galaxies? We know that the total output generally peaks in the IRAS wavelength regime, with an effective $g \sim 100$ (Heckman, Chambers & Postman 1992); this gives

$$\langle M_{\bullet} \rangle \simeq 2 \times 10^5 \epsilon^{-1} h^{-3} M_{\odot},$$

which paints a rather less optimistic prospect for detection than studies based on the output of QSOs. This is because, even with such a large g , the actual energy radiated

by radio galaxies is rather low, and this is not compensated for fully by the relative rareness of the host galaxies. The above figure is not easy to reconcile with large black-hole masses suggested for some radio AGN. For example, Lauer *et al.* (1992) suggest a central mass of $M_{\bullet} \simeq 3 \times 10^9 M_{\odot}$ for M87. Without suggesting that M87 is greatly atypical, this can always be made consistent by assuming a low enough efficiency. However, this would not fit well with the view that radio galaxies are powered via electrodynamic extraction of black-hole rotational energy (*e.g.* Blandford 1990); here the efficiency can be up to $\epsilon = 1 - 2^{-1/2}$. If masses of order $10^9 M_{\odot}$ are substantiated in several radio galaxies or radio-quiet massive ellipticals, this would be quite a puzzle. Probably the simplest solution would be to suggest that the total energy was higher than suggested by the above sum – perhaps because radio ellipticals spend part of their lives as QSOs, where the total energy output would be considerably higher for a given radio power.

5. HI SEARCHES FOR HIGH-REDSHIFT GALAXIES

We now turn to the question of what the radio waveband has to say about the properties of galaxies seen at high redshifts. One unique capability of radio astronomy for cosmology is the detection of neutral hydrogen via the 21 cm line. This tends to receive most attention at low redshifts via the Tully-Fisher relation and the studies of the distance scale and peculiar velocities. However, it also gives a unique way of detecting neutral gas at high redshift – even beyond the limit of $z \simeq 5$ where quasar absorption-line studies can probe. Particularly motivated by early ‘pancake’ theories of galaxy formation in which purely baryonic models give a supercluster-scale coherence length to the mass distribution, there have been a number of attempts over the years to use low-frequency observations to detect neutral hydrogen at high redshifts (*e.g.* Davies, Pedlar & Mirabel 1978 [$z = 3.3$ & 4.9]; Bebbington 1986 [$z = 8.4$]; Uson, Bagri & Cornwell 1991 [$z = 3.3$]; Wieringa, de Bruyn & Katgert 1992 [$z = 3.3$]). These are sensitive only to rather large structures: for a Gaussian velocity dispersion σ_v , the expected flux density is

$$\frac{S}{\text{mJy}} = 19.9 \left(\frac{M_{\text{HI}}}{10^{14} M_{\odot}} \right) \left(\frac{\sigma_v}{10^3 \text{ km s}^{-1}} \right)^{-1} \frac{h^2}{D^2 (1+z)},$$

where D is comoving distance divided by c/H_0 – *e.g.* $D = 2(1 - [1+z]^{-1/2})$ in an $\Omega = 1$ model. Since sensitivities of a few mJy are typically attained, the experiment is sensitive to masses in the range $10^{14} - 10^{15} M_{\odot}$.

Most such experiments have yielded only upper limits, but the VLA experiment of Uson, Bagri & Cornwell (1991) claimed the detection of a resolved object with a peak flux density of 10 mJy. The inferred parameters of their object were

$$\begin{aligned} z &= 3.397 \\ M_{\text{HI}} &\simeq 10^{14} h^{-2} M_{\odot} \\ \theta &\simeq 5' \simeq 1 h^{-1} \text{ proper Mpc} \\ \sigma_v &\simeq 77 \text{ km s}^{-1}. \end{aligned}$$

This experiment caused much debate, particularly the authors' claim that this was an example of a Zeldovich pancake. The characteristics of the emission are certainly hard to understand in any other way. The gas mass and size of object, together with the effective volume of space surveyed, are about right for a rich cluster of galaxies. However, in addition to the minute velocity dispersion, one would also not expect to find intracluster gas in a neutral state. In hierarchical models, it is continually shock heated by new infalling clumps of mass as structure grows. The only neutral gas would be associated with individual galaxies, producing much less massive neutral condensations (*e.g.* Subrahmanian & Padmanabhan 1993). The only possibility might be a group of unusually neutral-rich galaxies resembling the damped Lyman- α absorption systems seen in quasar spectra; in this context, it is worth noting that Wolfe (1993) has shown these to lie in regions of high density (at least in terms of cross-correlation with weaker Lyman- α emitters). In any case, it would still be necessary to appeal to the coincidence of seeing a cluster close to its turn-round time to explain why the velocity dispersion is so small (and even this does not solve things completely, since there will be a dispersion associated with substructure).

Only in models with an initial coherence length does the gas have time to cool and regain its neutrality following heating at the initial collapse of the cluster. Without attempting to turn history back to a time before dark matter, perhaps the least radical modification would involve warm dark matter with a coherence length of a few Mpc. This would in any case lead to the usual 'top-down' chain of events for galaxy formation. Since we believe that objects of cluster mass in fact mainly formed relatively recently (Lacey & Cole 1993; see also the contribution to this volume by S. White), this would have important implications for the ages of galaxies. For this reason, it is vital that the Uson *et al.* object be either confirmed or shown not to exist. Van der Kruit (private communication) suggests that the Westerbork group have indeed failed to detect it, which may cause some relief to those distressed by the above discussion. Whatever the eventual outcome, such observations will continue with increasing sensitivity and will be capable of setting interesting constraints on conditions at high redshift.

6. STELLAR POPULATIONS AT HIGH REDSHIFT

6.1 The golden age

Finally, for a line of argument that turns out to lead in completely the opposite direction – *i.e.* to galaxy formation at rather high redshift – we turn to the stellar populations in high-redshift radio galaxies. Most of the 1980s constitute a vanished age of innocence for the radio cosmologists: at this time, they were the only ones able to find galaxies at $z \gtrsim 1$ in any sort of numbers. A series of investigations established several interesting properties for these objects, in particular

- (i) The well-defined K - z relation for 3CR radio galaxies, consistent with purely passive evolution of their stellar populations, and producing 1 mag. of brightening by $z \simeq 1$ (Lilly & Longair 1984).

- (ii) The large scatter in the optical-IR (Lilly & Longair 1984) and optical (*e.g.* Spinrad & Djorgovski 1987) colours of 3CR galaxies, which was interpreted as reflecting the occurrence of bursts of star formation in otherwise passively evolving objects.
- (iii) Lilly (1989) argued for a two-component model in which a bluer component was superimposed onto a rather red underlying galaxy. To reproduce the spectral energy distribution of the red component (and thus of the reddest radio galaxies) required ages greater than 1 Gyr, pointing to high formation redshifts (Lilly 1989, Dunlop *et al.* 1989b, Windhorst, Koo & Spinrad 1986), although there was some controversy over the model dependence of the exact ages (Chambers & Charlot 1990).
- (iv) Perhaps the high-water mark of this period was the discovery by Lilly (1988) of 0902+34 at $z = 3.4$ (at a time when the galaxy redshift record was 1.8). The apparently red colours of this object argued for a large enough age that the bulk of the stars must have formed at $z \gtrsim 6$ – an inference of enormous importance for models of galaxy formation.

However, over the last few years a revisionist tendency has appeared – leading to all the above achievements being questioned. Even at the time, there was some doubt whether we could be sure that the above behaviour was representative of all galaxies. Fears of a radio-induced bias appeared well founded with the discovery of what has become known as the ‘alignment effect’: the realisation that at large redshifts ($z \gtrsim 0.8$) the optical and radio axes of many of the most powerful radio galaxies are aligned (McCarthy *et al.* 1987; Chambers, Miley & van Breugel 1987). Near-IR images of 3CR galaxies appeared to confirm that the infrared morphologies of these objects were in general just as peculiar as their optical morphologies (Chambers, Miley & Joyce 1988; Eisenhardt & Chokshi 1990; Eales & Rawlings 1990). These discoveries provide direct evidence of radio-induced ‘pollution’ of the UV-optical light of radio galaxies, and this led some authors to suggest that these sources are thus useless as probes of galaxy evolution in general (*e.g.* Eisenhardt & Chokshi 1990).

Furthermore, it has become apparent that Lilly’s galaxy 0902+34 does not have the properties initially claimed. The K flux is rather lower than Lilly’s measurement, and a large fraction of this smaller total is contributed by the [OII] 3727Å line, which is redshifted into the K window. The result is that the galaxy in fact looks very young: nearly flat-spectrum with no evidence for the presence of an old component. On this basis, and considering other similar objects at extreme redshifts, Eales *et al.* (1993) have argued that radio galaxies at $z \gtrsim 2$ are in effect protogalaxies observed in the process of formation.

Before accepting this remarkable reversal of conventional wisdom, however, it is worth bearing in mind that the galaxies under discussion are among the most luminous few dozen radio AGN in the entire universe (inevitably: they are the high-redshift members of bright samples with $S \sim 1 - 10$ Jy). In order to draw any general conclusions about galaxy formation, it is necessary to understand the effect the AGN has on the optical/IR properties of the galaxy within which it is embedded.

6.2 Alignments as a function of power

What is required is to be able to study the properties of galaxies with a wide range of radio powers, and this is what James Dunlop & I have attempted in some recent work (Dunlop & Peacock 1993). In order to eliminate possible confusion with any epoch dependence, we worked with a redshift band around $z \simeq 1$. At this redshift, it is relatively easy to select samples unbiased by optical selection, and the objects are bright enough that high-quality data can be obtained. We considered galaxies from two catalogues: 19 high-power 3CR galaxies; 14 low-power comparison galaxies with $S_{2.7 \text{ GHz}} > 0.1 \text{ Jy}$ from the Parkes Selected Regions (PSR) (Downes *et al.* 1986; Dunlop *et al.* 1989a). The PSR galaxies are a factor $\simeq 20$ less radio luminous than their 3CR counterparts. Radio luminosity is the only significant difference between the radio properties of the two samples.

Our principal dataset on these galaxies is deep infrared images, taken with the 62×58 pixel InSb array camera IRCAM, on the 3.9m United Kingdom Infrared Telescope (UKIRT), with the camera operating in the 0.62-arcsec/pixel mode. From these images, we investigated the extent of the the alignment effect at $z \simeq 1$. To avoid subjective factors, the infrared position angles were determined automatically by using the moments of the sky-subtracted flux within some circular aperture. We decided to vary the diameter of the aperture to adapt to the size of the radio source, because there are virtually no examples of optical or IR emission extending beyond the radio lobes. If the diameter of the radio source lay between 5 and 8 arcsec, an aperture equal in diameter to the radio source was used. If the radio source was greater than 8 arcsec in diameter, an 8 arcsec diameter was used (larger apertures generally contain foreground objects). If the radio source was smaller than 5 arcsec in diameter, a 5-arcsec diameter was used.

Figure 5. Histograms of (IR – Radio) position angle differences for the 3CR and PSR samples. The clear difference seen here is completely robust to different methods for determining position angles. It is related to the fact that the PSR galaxies are also rounder, and generally lacking in an extended aligned component of blue light.

Figure 5 shows the resulting IR–radio alignment histogram for the 3CR and PSR subsamples. The infrared alignment effect is extremely obvious in our data for the 3CR

galaxies, which appears to contrast with the conclusions of Rigler *et al.* (1992). Much of the apparent discrepancy arises from the fact that we have a larger sample. Position angles for objects in common generally agree well, but with some exceptions which are due to different methods of analysis; Rigler *et al.* (1992) sometimes use a large aperture where their position angle is affected by companion objects. In contrast to the 3CR subsample, there is no evidence of any significant alignment between the infrared and radio morphologies of the PSR galaxies. This result is very robust and quite obvious given the images: the PSR galaxies are rounder, with generally little sign of the disturbance evident in many of the 3CR images.

This argues in favour of the two-component model advanced by Lilly (1989) and Rigler *et al.* (1992). In this, the underlying galaxy is round, but there is a component of variable amplitude which is elongated along the radio axis, and it is this which leads to the alignment. Our data demonstrate that the strength of this component correlates well with radio power, as is perhaps not so surprising in retrospect. Certainly, several models for the production of this light exist that predict a correlation with radio power (scattering, induced star formation, inverse Compton emission – see *e.g.* Daly 1992 for a review). We shall not be concerned here with having to plump for a specific model, but it is worth noting that evidence is starting to mount in favour of the explanation in terms of scattering from a hidden blazar. The main argument in this direction is the measurement of polarization with E-vector perpendicular to the radio axis. The first measurements of this effect gave very low percentage polarizations, implying that this could not be the dominant mechanism. However, with better resolution, imaging polarimetry is now producing polarized fractions of $\gtrsim 20\%$ in the outer parts of strongly aligned galaxies (Jannuzi & Elston 1991; Tadhunter *et al.* 1992; Cimatti *et al.* 1993). Given geometrical dilution, it now seems plausible that the aligned component results from scattering in at least some objects.

6.3 Colours and ages of radio galaxies

Having seen that the extent of the aligned component scales so dramatically with radio power, we now look for other optical/infrared properties which correlate with power. Given that the aligned component is often bluer than the nucleus of the galaxy, we should certainly expect to see some correlation between colour and power. A useful way of quantifying the degree of UV activity was introduced by Lilly (1989). He assumes that the observed spectrum of a radio galaxy arises from a combination of two distinct components – an ‘old’ population with a well-developed 4000\AA break, and a ‘young’ flat-spectrum component. This simple model can be fitted to the observed colours by varying one parameter. This is f_{5000} : the fractional contribution of the flat-spectrum component to the galaxy light at a rest wavelength of 5000\AA . This method can also be used with some success to estimate the redshift for objects which lack spectroscopy (see Lilly 1989; Dunlop & Peacock 1993). Some of the PSR objects had their redshifts estimated in exactly this way: the redder objects with low f_{5000} also have low levels of emission-line activity and so are of course the hardest spectroscopic targets.

This procedure is illustrated in Figure 6. For the ‘old’ or ‘red’ component we chose to adopt a spectrum capable of producing the reddest colours seen in radio galaxies

Figure 6. Two examples of the spectral fitting used to determine estimated redshifts and f_{5000} , the relative contribution of the flat-spectrum component. The ‘red’ component is the spectrum produced by a 1-Gyr ‘Burst’ model of galaxy evolution at an age of 10 Gyr. The blue component is a power-law with spectral index $\alpha = 0.2$ ($f_\nu \propto \nu^{-\alpha}$), the mean optical spectral index found for quasars by Barvainis (1990). 2355–010 is a red radio galaxy with only a very small value of f_{5000} , while 0059+027 is one of the bluer galaxies in the PSR sample.

at $z \simeq 1$ (*e.g.* 3C65); in practice this was achieved using the spectrum produced by a stellar population of age 10 Gyr in an updated version of the models of Guiderdoni & Rocca-Volmerange (1987). For the ‘young’ or ‘blue’ component, we decided to adopt a power-law spectrum ($f_\nu \propto \nu^{-\alpha}$) with a spectral index $\alpha = 0.2$. This choice of spectrum can be justified at two different levels. First, the exact value of α was chosen in the spirit of scattered quasar light; Barvainis (1990) concluded that the mean value for the optical spectral index in high luminosity quasars (*i.e.* those whose optical spectra are essentially uncontaminated by a host galaxy contribution) is $\alpha = 0.2$. Second, empirically, this form of spectrum is an excellent representation of the approximately flat f_ν optical-UV continuum actually observed in high-redshift radio galaxies.

In Figure 7 we show the quantitative relation between this definition of UV activity and radio power. Radio power and f_{5000} appear to be strongly correlated (no PSR galaxy has $f_{5000} > 0.19$ whereas more than half the 3CR galaxies have $f_{5000} > 0.20$). This result contrasts sharply with that of Lilly (1989), who reported that in his combined 3CR and 1-Jy sample there was no significant correlation between f_{5000} and $P_{408 \text{ MHz}}$. The origin of the difference appears to be an error in Lilly’s calculation of radio luminosity. An interesting aspect of the relation with power is that all sub-samples appear to possess a range of f_{5000} values, but with power apparently setting the upper limit in f_{5000} . This suggests the existence of a second parameter which determines the actual level of UV light – see Dunlop & Peacock (1993) for further discussion.

For the present, the point to emphasise is that this diagram provides a quantitative definition of a radio-quiet galaxy. At least at $z \simeq 1$, any galaxy with $P_{2.7} \lesssim 10^{25.5} \text{ WHz}^{-1} \text{sr}^{-1}$ (for $h = 1/2$) has a negligibly small level of UV activity. There have been some suggestions that UV activity and alignments are functions specifically of redshift, but there is little evidence that this is anything other than a reflection of the above

Figure 7. Radio power, $P_{2.7 \text{ GHz}}$, versus f_{5000} for the combined 46-source 3CR/1-Jy/PSR sample. 3CR sources are shown as squares, 1-Jy sources as triangles, and the PSR sources as circles. Notice that the correlation is mainly in the sense of setting an upper limit to f_{5000} at given power.

trend in a flux-limited sample. Until proven otherwise, the natural null hypothesis is that galaxies below this power level at higher redshifts also reflect the properties of the general population of massive ellipticals.

In Figure 8 we compare the $R - K$ colours of the PSR and 3CR galaxies. Several other objects which are not part of our PSR and 3CR subsamples have been included here for comparison purposes. These are (i) the very red 3CR galaxy 3C65, (ii) the five 1-Jy galaxies with measured redshifts for which $r - K$ colours are given by Lilly (1989), and (iii) all spectroscopically confirmed quasars with $0.5 < z < 2.0$ in the Parkes Selected Regions sample for which $R - K$ colours exist (Dunlop *et al.* 1989a).

This diagram displays a number of important features. First, with the obvious exception of 3C65, the PSR galaxies are consistently redder than the 3CR galaxies; moreover, the PSR galaxies display remarkably little dispersion in their optical-infrared colours. This is well consistent with the findings of Rixon, Wall & Benn (1991) at lower redshift: they found the rest-frame colours of radio ellipticals at $z < 0.3$ to be constant to within a few hundredths of a magnitude. In contrast, the 3CR galaxies scatter downwards from the well-defined PSR locus towards the region of colour space occupied by the PSR quasars (the very red galaxies 3C65 and 1129+37 appear to be exceptional). Of the six 3CR galaxies with $R - K \leq 4.0$, all but one (3C252) have K -band morphologies clearly aligned with their radio axes.

The homogeneity of the PSR galaxies, along with the lack of any dramatic alignment effect in the redder galaxies, suggests instead that the true optical-infrared colour of a radio-quiet elliptical at $z \simeq 1$ is actually $R - K \simeq 4.8$. Values of $f_{5000} \simeq 0.05$ might be a feature of most elliptical galaxies at $z \simeq 1$. This is certainly consistent

Figure 8. Comparison of the $R - K$ colours of the PSR galaxies (solid squares and triangles) and the 3CR galaxies in the subsample (open circles and diamonds). PSR galaxies with measured redshifts are denoted by solid triangles, those with estimated redshifts by solid squares. 3CR galaxies whose K -band morphologies are aligned with 15° of the radio axis are denoted by diamonds, and the remainder by open circles. Also shown are five 1-Jy galaxies (from Lilly 1989) (asterisks), and all spectroscopically confirmed quasars with $0.5 < z < 2.0$ in the PSR sample (stars). The dashed line shows the effect of simply k-correcting the spectrum. The solid line shows a very old ($z_f = 50$, $\Omega_0 = 0$, $H_0 = 50 \text{ kms}^{-1} \text{Mpc}^{-1}$) UV-hot model of elliptical galaxy evolution (Rocca-Volmerange 1989).

with the results of Aragón-Salamanca *et al.* (1993). From optical/IR photometry of clusters of galaxies up to $z = 0.8$, they conclude that ellipticals (mainly radio-quiet) in the highest-redshift clusters are slightly bluer than present-day ellipticals. On the assumption that these galaxies formed in a single burst, their data allow the epoch of formation to be as low as $z = 2$. However, the radio-selected samples extend the range still further. Although the above discussion has concentrated on the situation at $z \simeq 1$, the PSR sample contains a number of galaxies inferred from colour-estimated redshifts and from the K - z relation to have $z \simeq 2$. These also are apparently old and red, with $R - K \simeq 4 - 5$. If this is taken to imply a minimum age of $1h^{-1}$ Gyr, the formation redshift is pushed out to between 3.3 and 7.2, depending on Ω . Note that this is the epoch at which the whole galaxy must be assembled: ellipticals cannot have been assembled from many small clumps after star formation had ceased (Bower, Lucey & Ellis 1992). It will be fascinating to pursue this line of argument in mJy samples, where we may hope to find ‘normal’ radio galaxies at $z > 3$. If these are still red, the consequences for galaxy formation models will be radical indeed.

7. CONCLUSIONS

This review has given a brief summary of the properties of galaxies as viewed in the radio background. In conclusion, it is worth emphasising three points:

- (i) Although some factors such as galaxy mass and Hubble type strongly dispose a galaxy to host a radio-loud AGN, we still have no definite understanding of why this should be so. Other ‘distinguishing marks’ of radio galaxies might be helpful in this process, but few if any are clearly established.
- (ii) With certain exceptions (such as the situation at $z \gtrsim 2$), we have a good statistical description of how the abundance of radio AGN evolves. Again, though, we are very far from understanding why active nuclei found it so much easier to function at high redshift.
- (iii) High-redshift radio galaxies should probably not be thought of as in any way primaeval. If we ignore the few dozen most luminous sources in the universe, then the optical/IR properties of high-redshift radio galaxies are consistent with those of radio-quiet ellipticals. They appear to be red and old: theories in which most massive galaxies complete their star formation at $z \gtrsim 4$ are required.

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