Black Holes and Galaxy Evolution

David Merritt

Department of Physics and Astronomy, Rutgers University

Abstract. Supermassive black holes appear to be generic components of galactic nuclei. Following their formation in the early universe, black holes should often find themselves in bound pairs as a consequence of galaxy mergers. The greatest uncertainty in estimating the coalescence time of black hole binaries is the degree to which a binary wanders about the center of the galactic potential. A simple model for binary decay is presented which qualitatively reproduces the evolution observed in the N-body simulations. The model predicts binary coalescence times that are never less than several billion years. Mass ejection by a decaying black hole binary should substantially lower the density of its host nucleus. The weak density cusps of bright ellipticals may be explained in this way if it is assumed that these galaxies formed from nearly gas-free mergers.

1. Introduction

The case for supermassive black holes (BHs) in galactic nuclei is now very strong, and in a few galaxies almost irrefutable. The evidence is most compelling in galaxies where the kinematics of stars or gas can be traced into sub-parsec scales; examples are the Milky Way (Genzel et al. 1997; Ghez et al. 1998), where stellar proper motions can be measured at distances of $\lesssim 0.02$ pc, and NGC 4258 (Miyoshi et al. 1995), where the rotation curve of H_2O maser sources obeys Kepler's law into ~ 0.13 pc. For a larger sample of galaxies, kinematical data extending into ~ 10 or ~ 100 pc provide suggestive though not yet irrefutable evidence of supermassive compact objects (Kormendy & Richstone 1995).

Although the masses of the detected BHs comprise on average only $\sim 0.3\%$ of the mass of their host spheroids (Ho 1998), there is a growing body of work suggesting that the dynamical influence of a supermassive BH can extend far beyond the nucleus. Furthermore the formation and growth of BHs may be intimately connected with the evolution of galaxies on larger scales. For instance, mergers between galaxies containing nuclear BHs would produce supermassive BH binaries which would eventually coalesce via the emission of gravitational radiation (Begelman, Blandford & Rees 1980). The formation and decay of these binaries may be relevant to a wide range of phenomena, from the wiggling of radio jets (Kaastra & Roos 1992) to the destruction of stellar density cusps (Ebisuzaki et al. 1991).

This review discusses the time scale for coalescence of supermassive BH binaries (§2) and the dynamical effect of the BHs on stellar nuclei (§3) and the large-scale structure of galaxies (§4).

2. Supermassive Black Hole Binaries

Supermassive BHs appear to be ubiquitous components of galactic nuclei. Since galaxies often merge, one would expect to form BH binaries at a rate that is roughly equal to the galaxy merger rate. The formation and coalescence of a BH binary is believed to take place via three, fairly distinct stages (Begelman, Blandford & Rees 1980):

1. Two parent galaxies interact; the BHs – surrounded by their dense star clusters – sink to the center of the common potential well via dynamical friction, forming a binary.

2. The BH binary shrinks by ejecting stars from the nucleus via three-body interactions.

3. When the binary separation has fallen to the value at which gravitational radiation becomes efficient, the BHs coalesce.

In the absence of gas, the rates of the first two processes are in principle possible to calculate via N-body simulations. However the problem is a difficult one to simulate due to the wide range of length and time scales; none of the published simulations have succeeded in following the decay to the point where gravitational radiation would become important. Here we derive a simple analytical model for the decay of BH binaries which reproduces the results of the N-body simulations, then use it to estimate the coalescence time.

Consider a binary with total mass $M_{12} = m_1 + m_2$, $m_1 \geq m_2$ and semi-major axis $a(t)$. We assume that the background galaxy is initially a singular isothermal sphere, with density and mass profiles

$$\rho_0(r) = \frac{\sigma^2}{2\pi G r^2}, \qquad M_0(r) = \frac{2\sigma^2 r}{G}; \tag{1}$$

σ is the one-dimensional stellar velocity dispersion. The binary becomes "hard" when a falls below $\sim a_h = Gm_2/4\sigma^2$ (Quinlan 1996); subsequent evolution is driven by the capture and ejection of stars that interact with the binary. The hardening rate

$$H = \frac{\sigma}{G\rho}\frac{d}{dt}\left(\frac{1}{a}\right) \tag{2}$$

depends only weakly on m_1/m_2 and on a for $a \lesssim a_h$; in the limit $a \ll a_h$, $H \approx 16$ (Hills 1992; Mikkola & Valtonen 1992; Quinlan 1996). Mass ejection occurs at a rate

$$J = \frac{1}{M_{12}}\frac{dM_{ej}}{d\ln(1/a)} \tag{3}$$

where $J \approx 1$ is again nearly independent of $(m_1/m_2,\, a)$ for $a \ll a_h$ (Quinlan 1996). Thus

$$M_{ej} \approx J M_{12} \ln(a_h/a). \tag{4}$$

If the binary remained fixed at the center of a spherical potential, it would quickly eject all stars on orbits with pericenters less than $\sim a$. The local density would fall to zero and the binary would cease to harden. We assume instead that mass removal produces a constant-density core of stars out to some radius $r_c(t)$, where

$$M_{ej} = M_0(r_c) - \frac{4\pi}{3}\rho_0(r_c) r_c^3 = \frac{4}{3}\frac{\sigma^2 r_c}{G}. \tag{5}$$

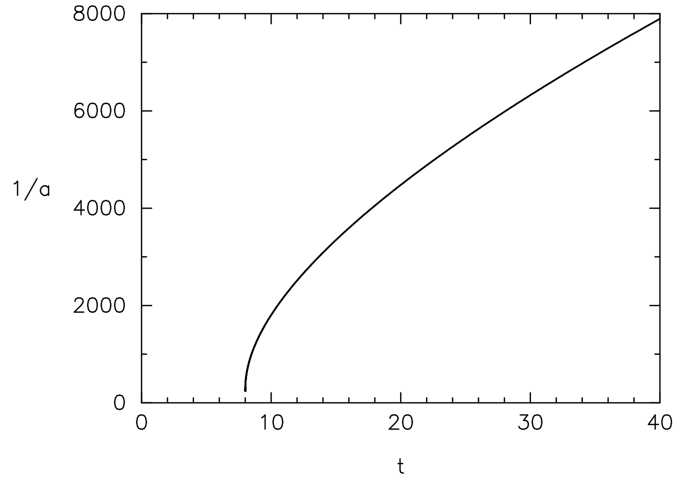

Figure 1. Dependence of binary semimajor axis a on time for the analytic model of eq. (7). Units are discussed in the text.

In order to achieve this, the binary must interact with stars at radii $r \lesssim r_c \gg a$. This may seem an optimistic assumption, but it turns out that something very similar happens in the N-body simulations, as discussed below.

The binary separation rapidly falls below r_c and subsequent evolution occurs against an evolving background density $\rho(t) = \sigma^2/2\pi G r_c^2(t)$. Combining equations (2), (4) and (5), we find for the hardening rate

$$\frac{d(a_h/a)}{d(t/t_0)} = \frac{1}{\ln^2(a_h/a)}, \qquad t_0 = \frac{9\pi J^2}{H}\left(\frac{M_{12}}{2m_2}\right)\left(\frac{GM_{12}}{\sigma^3}\right). \tag{6}$$

The logarithm represents the decrease in the hardening rate due to the declining stellar density. Integrating with respect to time,

$$\frac{t-t_h}{t_0} = \frac{a_h}{a}\left[\ln^2\left(\frac{a_h}{a}\right) - \ln\left(\frac{a_h}{a}\right)^2 + 2\left(1-\frac{a}{a_h}\right)\right] \tag{7}$$

where $a(t_h) = a_h$. The functions $t(a)$ and $a(t)$ may be approximated as

$$\frac{t-t_h}{t_0} \approx 0.6\left(\frac{a_h}{a}\right)\ln^2\left(\frac{a_h}{a}\right), \qquad \frac{a_h}{a} \approx \frac{4(t-t_h)/t_0}{\ln^2\left[(t-t_h)/t_0\right]} \tag{8}$$

for $t - t_h \gtrsim$ a few $\times\, t_0$.

We may compare the predictions of this simple model to the results of N-body simulations of BH binary decay. Quinlan & Hernquist (1997) (QH) followed the evolution of a BH binary in a galaxy with an initial $\rho \propto r^{-2}$ density cusp. Expressed in their units, the scaling parameters in eq. (7) are $a_h = 1/200$, $t_0 \approx 1/10$ and $t_h \approx 8$. Fig. 1 is a plot of eq. (7) scaled to these units. The predicted time dependence $a(t)$ is qualitatively very similar to that found by QH (their Fig. 2). However, those authors found that the rate of binary decay depended systematically on the number N of particles used to represent the galaxy: larger values for N gave slower evolution. For $N = 6250$, the smallest number considered by them, Fig. 1 overestimates the evolution rate by a factor of only ~ 2, remarkably close given the approximations made. For $N = 2.5\times10^4$ and 10^5, the adjustment factors are ~ 3 and ~ 5 respectively.

QH attributed the N-dependence of the hardening rate to wandering of the binary about the center of the galaxy potential. In a core of density ρ, an rms velocity v_b of the binary's center of mass will produce a wandering with amplitude $r_w \sim v_b/(G\rho)^{1/2}$. If the source of the binary's motion were elastic encounters with stars of mass m_*, equipartition arguments would give $v_b/\sigma \approx (m_*/M_{12})^{1/2} \approx 10^{-4}$. However the binary converts some of its binding energy into bulk motion when it ejects stars, with momenta $\sim m_*(a_h/a)^{1/2}\sigma$. This "super-elastic scattering" allows the binary to sample the stellar density over a much wider range of radii than if it remained stationary; QH in fact verified that fixing the binary to the center of the potential caused the hardening to cease after the binary had ejected most of the stars within $r = a_h$.

If the background density within the wandering radius r_w were to fall, the gravitational restoring force acting on the binary would also drop and the wandering amplitude would increase, roughly in proportion to $\rho^{-1/2}$. This feedback mechanism should tend to inhibit the formation of a central "hole" in the stellar density (Mikkola & Valtonen 1992). The crude model derived above mimics this process by forcing the background density to remain constant for $r < r_c$; the model could be improved by incorporating the wandering radius explicitly.

In the N-body simulations, the perturbations that the binary experiences from the "stars" become less noisy as N is increased, implying a reduced wandering radius and a lower hardening rate. Makino (1997) found that the wandering amplitude varied roughly as $N^{-1/2}$ for $N < 3\times10^5$. However QH found that the hardening rate did not change appreciably when N was increased from 10^5 to 2×10^5. They argued that r_w might reach a limiting amplitude for large N due to the feedback mechanism discussed above. This hypothesis is important to check; however doing so will require simulations with very large particle numbers.

The binary will rapidly coalesce when the time scale for emission of gravitational radiation

$$t_{gr} = \frac{5}{256}\frac{c^5a^4}{G^3m_1m_2M_{12}} \tag{9}$$

(Peters 1964, for a circular binary) equals the hardening time $|a/\dot{a}|$. Using eq. (6), this occurs when $a = a_{gr}$ where

$$\frac{(a_{gr}/a_h)^5}{\ln^2(a_{gr}/a_h)} = \frac{9\pi\times16^5J^2}{20H}\left(\frac{m_1}{m_2}\right)\left(\frac{M_{12}}{2m_2}\right)^2\left(\frac{\sigma}{c}\right)^5 \tag{10}$$

or

$$a_{gr}/a_h \approx A|\ln A|^{0.4}, \qquad A = 9.85\left(\frac{m_1}{m_2}\right)^{0.2}\left(\frac{M_{12}}{2m_2}\right)^{0.4}\left(\frac{\sigma}{c}\right). \tag{11}$$

For $\sigma/c = 300/(3\times10^5)\approx 0.001$, $A\approx 0.01$ and

$$a_{gr}/a_h \approx 0.018, \tag{12}$$

i.e. the binary must shrink by a factor of ~ 50 beyond the hardening radius for coalescence to occur. The simulations of QH followed the decay only over a factor of ~ 12 beyond a_h.

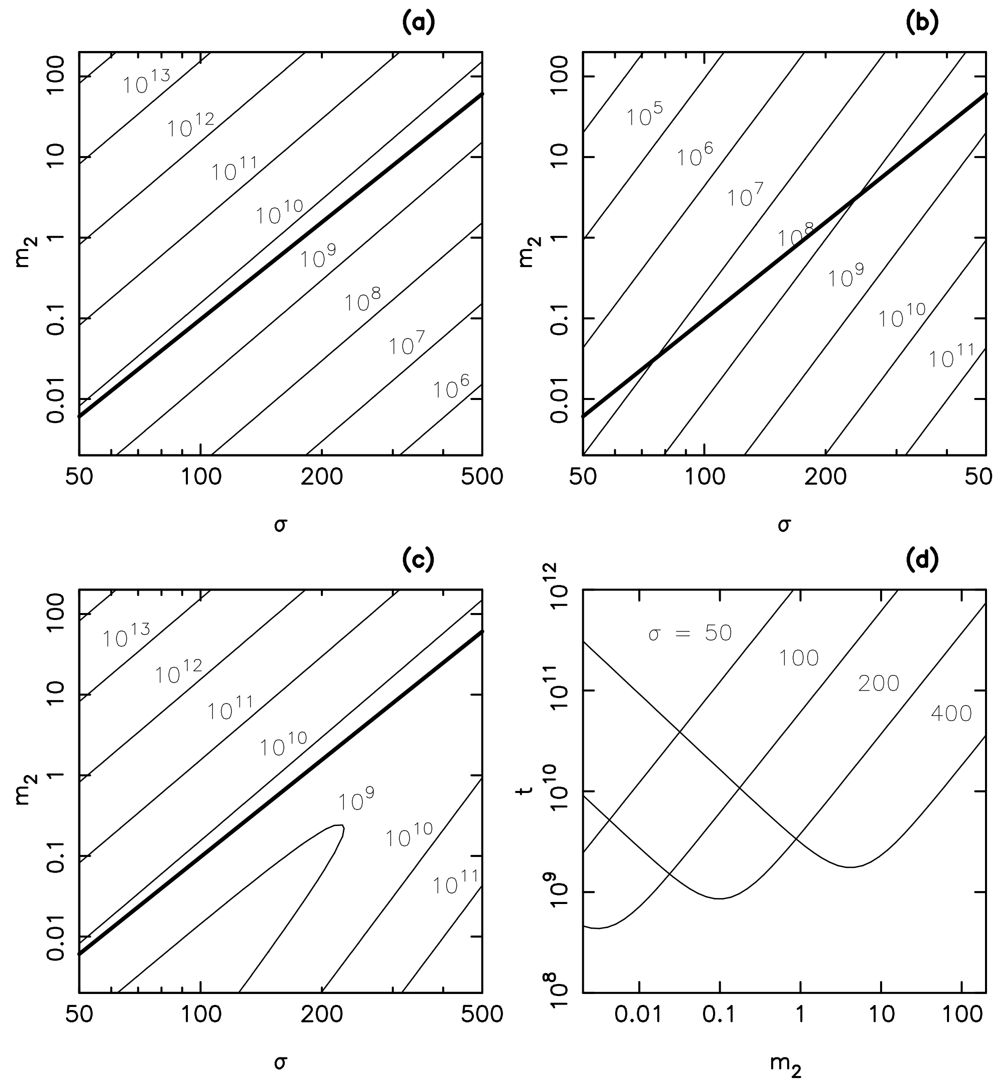

Figure 2. Decay time in years for supermassive BH binaries in singular isothermal sphere galaxies. m_2 is the mass of the smaller BH in $10^8 M_\odot$; σ is the one-dimensional velocity dispersion of the larger galaxy in km s^{-1}. (a) Time $\Delta t_1 = t_{gr} - t_h$ from formation of a hard binary to coalescence by emission of gravitational radiation. The binary mass ratio is assumed to be $m_2/m_1 = 1:2$. The solid line is the "Faber-Jackson relation" for BHs (see text). (b) Orbital decay time Δt_2 of the smaller galaxy (containing the BH of mass m_2) due to dynamical friction against the larger galaxy (of velocity dispersion σ). (c) Total evolution time $T = \Delta t_1 + \Delta t_2$. (d) Evolution time as a function of m_2 for $\sigma = (50, 100, 200, 400)$ km s^{-1}.

We can use the analytic expressions derived here to extend the N-body simulations until a time corresponding to coalescence; the correction factor of 5 mentioned above is applied to (a/a_h) in order to reproduce the hardening rate observed in the largest-N simulations. Fig. 2a shows the time from formation of a hard binary until coalescence, Δt_1, as a function of σ and m_2 for a binary with $m_2/m_1 = 1/2$. An approximate expression is

$$\Delta t_1 \approx 8 t_0 A^{-1} |\ln A|^{8/5}; \tag{13}$$

if we further approximate the logarithm as a constant, $|\ln A| \approx |\ln 0.01| \approx 4.61$, then

$$\Delta t_1 \approx 1.4 \times 10^{10} \mathrm{yr} \left(\frac{m_2}{m_1}\right)^{0.2} \left(\frac{M_{12}}{2m_2}\right)^{0.6} \left(\frac{M_{12}}{10^9 M_\odot}\right) \left(\frac{\sigma}{200 \text{ km s}^{-1}}\right)^{-4}. \tag{14}$$

While eq. (14) is an accurate extrapolation of the evolution seen in the N-body experiments, it should probably be interpreted as a lower limit to the coalescence time expected in a real spherical galaxy due to the likely overestimate of the wandering radius in the simulations. However the dependence of Δt_1 on m_2 and σ is probably more robust. Since for real galaxies $L \sim \sigma^4$ and $M_{12} \sim L$, eq. (14) predicts a coalescence time that is almost independent of σ and M_{12}. The solid line in Fig. 2a is the "Faber-Jackson law" for BHs,

$$\frac{M_\bullet}{10^8 M_\odot} \approx 3.1 \left(\frac{\sigma}{200 \text{ km s}^{-1}}\right)^4; \tag{15}$$

this relation is consistent with three galaxies whose BH masses are well-determined: M84 (Bower et al. 1998), M87 (Macchetto et al. 1997) and NGC 4258 (Miyoshi et al. 1995). Fig. 2a suggests a typical coalescence time of several billion years.

Very low mass binaries would evolve more rapidly (eq. 14), but another factor would delay their coalescence: the necessity of the two BHs finding their way to the potential center following a galaxy merger. Governato et al. (1994) carried out merger simulations of galaxies containing BHs; they concluded that inspiral times could be very long if the infalling galaxy had a sufficiently low mean density to be tidally disrupted. A more likely scenario is that the infalling BH retains some of the mass of its host galaxy. We can estimate the inspiral time by assuming that the smaller galaxy is tidally limited by the larger galaxy as its orbit decays. Following Merritt (1984), the tidal radius is located at the point where the effective potential (gravitational plus centrifugal) has a saddle point. Approximating both galaxies as singular isothermal spheres and assuming that the smaller galaxy (of mass m_g) is on a circular orbit of radius r, its tidal radius and mass become

$$r_g^3 \approx \frac{G m_g r^2}{4\sigma^2}, \qquad m_g \approx \frac{\sigma_g^3 r}{2G\sigma} \tag{16}$$

with σ and σ_g the velocity dispersions of the larger and smaller galaxies respectively. Chandrasekhar's formula then gives for the orbital decay rate and infall time

$$\frac{dr}{dt} = -0.30 \frac{G m_g}{\sigma r} \ln \Lambda \approx -0.151 \frac{\sigma_g^3}{\sigma^2} \ln \Lambda, \qquad \Delta t_2 \approx 0.30 \frac{r_e \sigma^2}{\sigma_g^3} \tag{17}$$

(Tremaine 1990), where $\ln \Lambda \approx \ln(r/0.5 r_g) \approx \ln(4\sigma/\sigma_g) \approx 2$. Taking $r(0) = r_e \approx 2.6$ kpc$(\sigma/200 \text{ km s}^{-1})^3$ (Valluri & Merritt 1998), the effective radius of the larger galaxy, the inspiral time becomes

$$\Delta t_2 \approx 9.6 \times 10^7 \text{yr} \left(\frac{\sigma}{200 \text{ kms}^{-1}} \right)^5 \left(\frac{m_2}{10^8 M_\odot} \right)^{-3/4} \tag{18}$$

where eq. (15) has been used to relate σ_g to m_2. This relation is plotted in Fig. 2b and the total time $t_{tot} = \Delta t_1 + \Delta t_2$ in Fig. 2c. Remarkably, Fig. 2c suggests that real BHs have roughly the mass that would be required to minimize their total coalescence time. Nevertheless, this time appears to be very long.

Other mechanisms might accelerate the coalescence. If the potential of the stellar nucleus is non-axisymmetric, orbits will not conserve angular momentum and stars with a much wider range of energies may interact with the binary. However it is not clear whether self-consistent triaxiality can be maintained in a nucleus where the potential is dominated by a BH (Sridhar & Touma 1999; Merritt & Valluri 1999). Gas if present would also exert a drag force on the binary. "Loss-cone refilling" by two-body interactions between stars (Frank & Rees 1976) is almost certainly unimportant due to the long relaxation times in nuclei (Valtonen 1996).

If coalescence times are comparable to the mean time between galaxy mergers, a third BH would often be introduced into a nucleus containing an uncoalesced pair. The most likely outcome is ejection of two BHs in opposite directions with the third remaining at the center (Valtonen 1996). This process could substantially reduce the mean ratio of BH mass to galaxy mass over time.

3. Formation and Destruction of Nuclei

Combining eqs. (4) and (12), the total mass ejected by the BH binary between t_h and t_{gr} is $\sim 4M_{12}$. The structure of the pre-existing nucleus should therefore be disturbed out to a radius where the enclosed stellar mass was a few times M_{12}. A reasonable guess for the initial density profile is $\rho \approx r^{-\gamma}, \gamma \approx 2$, as assumed in the model above; adiabatic growth of a BH in a nucleus with a shallower profile generically leads to a power-law cusp with index near 2 (Quinlan et al. 1995; Merritt & Quinlan 1998), and this is also the slope observed in the faintest ellipticals (Kormendy et al. 1995) which are the least likely to have experienced the sort of core disruption discussed above.

Simply removing all of the stars with energies below some minimum E_{min} produces a density profile near the BH of

$$\rho(r) = 4\pi \int_{E_{min}}^{0} f(E) \sqrt{2\left[E - \Phi(r)\right]} dE \approx C \times f(E_{min}) \sqrt{\frac{GM_{12}}{r}} \propto r^{-1/2} \tag{19}$$

assuming an isotropic velocity distribution $f(E)$. Nakano & Makino (1999a,b) found this to be a good description of what happens in N-body simulations of a single BH spiralling into a pre-existing core. In the case of a decaying BH binary, stars on eccentric orbits are most likely to interact with the binary and be removed, leaving behind a nucleus in which the stellar motions are strongly

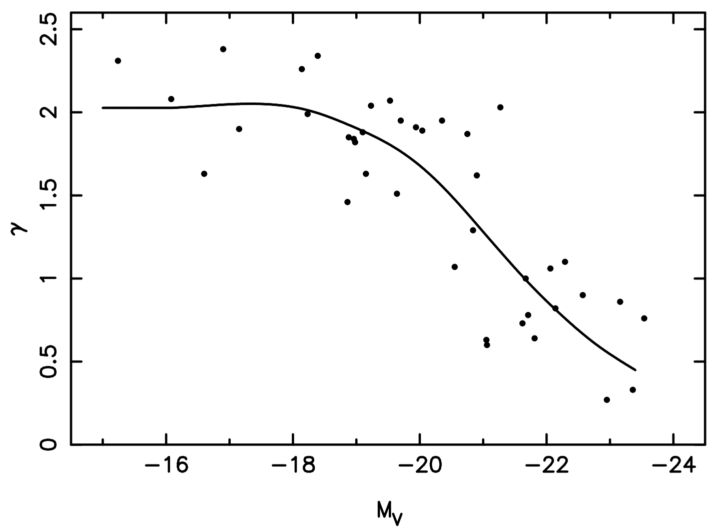

Figure 3. Luminosty density slopes, $\rho \sim r^{-\gamma}$, for elliptical galaxies vs. absolute magnitude (adapted from Gebhardt et al. 1996). The solid line is a nonparametric regression fit to $\gamma(M_v)$.

biased toward circular. The resulting density profile is difficult to calculate. The N-body simulations discussed above (Makino 1997; QH) yield central profiles that are crudely describable as power laws, with indices $0 \lesssim \gamma \lesssim 1$; however these results are probably dependent on the degree of BH wandering and hence on N.

Which galaxies in the current universe would be expected to contain these low-density cores? Kauffmann & Haehnelt (1999) present a semi-analytic model for the formation of galaxies in a cold-dark-matter universe; they assume that supermassive BHs form from gas that is driven into the centers of galaxies during mergers. The predicted ratio of gas mass to stellar mass during the last major merger is a steep function of galaxy luminosity in their model; this ratio drops from ~ 3 for $M_v = -18$ to ~ 0.3 for $M_v = -21$, albeit with considerable scatter (their Fig. 14). Gas-rich mergers would be expected to form dense nuclei while gas-free mergers should produce low-density cores after the BHs coalesce. Figure 3 shows that the variation of γ with galaxy luminosity is roughly consistent with Kauffmann & Haehnelt's model. Faint ellipticals, $M_v \gtrsim -19$, have $\gamma \approx 2$, while for brighter galaxies γ decreases with increasing luminosity. This trend is often described as a dichotomy although the data of Fig. 3 suggest a continuous distribution, as expected in a model like Kauffmann & Haehnelt's.

4. Large-Scale Evolution

The gravitational influence of a supermassive BH can extend far beyond the nucleus in a galaxy that is not axisymmetric (Gerhard & Binney 1985). The mechanism is dynamical chaos induced in the stellar orbits by close passages to the BH (Merritt & Valluri 1996). Although relatively little N-body work has been done on this problem, one study (Merritt & Quinlan 1998) found that nuclear point masses can cause initially triaxial galaxies to evolve to globally axisymmetric shapes in little more than a crossing time when the ratio of BH mass to galaxy mass exceeds $\sim 2\%$. This mass ratio is consistent with the value that induces a transition to global stochasticity and rapid mixing in the phase space of box-like orbits (Valluri & Merritt 1998).

Mergers of stellar disks produce generically triaxial objects (Barnes 1996); adding a dissipative component to the simulations produces end states that are much more nearly axisymmetric (Barnes & Hernquist 1996). The evolution in shape occurs rapidly once a few percent of the total mass has accumulated in the center (Barnes 1999). The stars in these simulations respond to the "gas" only insofar as the latter affects the potential; thus the change in shape is a purely stellar-dynamical phenomenon, and we would expect to see similar evolution even in gas-free mergers if the BHs are sufficiently massive.

The typical ratio of BH mass to galaxy mass is ~ 0.003 (Ho 1998), somewhat less than the value required to induce a rapid transition to axisymmetry. For $M_\bullet/M_{gal} = 0.003$, Merritt & Quinlan (1998) found a time scale of $\sim 10^2$ crossing times for the evolution to axisymmetry. This exceeds a Hubble time for galaxies with luminosities above $M_v \approx -19$ (Valluri & Merritt 1998); hence we might expect bright ellipticals to often retain their merger-induced triaxial shapes. In fact there is evidence for a systematic change in the shape of ellipticals at about this magnitude (Tremblay & Merritt 1996); bright ellipticals as a class show evidence for triaxiality, while the axis-ratio distribution of faint ellipticals is consistent with axisymmetry.

The same orbital evolution that destroys triaxiality also tends to produce a smoother phase-space density. One consequence is that "boxiness" in the isodensity contours – a natural consequence of a non-smooth phase space distribution (Binney & Petrou 1985) – should be destroyed along with triaxiality. This too has been observed in simulations of gaseous mergers; the effect is sometimes attributed to "dissipation" (e.g. Bekki & Shioya 1997) but it is again almost certainly a purely stellar dynamical effect. A prediction is that triaxiality should correlate with boxiness, since a central mass concentration or BH should tend to destroy both following a merger. Evidence for such a correlation has been noted by Kormendy & Bender (1996).

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