Black Holes in String Theory

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ABSTRACT

This thesis is devoted to trying to find a microscopic quantum description of black holes. We consider black holes in string theory which is a quantum theory of gravity. We find that the "area law" black hole entropy for extremal and near-extremal charged black holes arises from counting microscopic configurations. We study black holes in five and four spacetime dimensions. We calculate the Hawking temperature and give a physical picture of the Hawking decay process.

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1. INTRODUCTION

1.1. Introduction

It has been a long-standing challege for theoretical physics to construct a theory of quantum gravity. String theory is the leading candidate for a quantum theory of gravity. General Relativity has the seeds of its own destruction in it, since smooth initial data can evolve into singular field configurations [1]. Classically this is not a problem if the singularities are hidden behind event horizons [2], since this means that nothing can come out from the region containing the singularity. However, Hawking showed, under very general assumptions, that quantum mechanics implies that black holes emit particles [3]. In his approximation this radiation is exactly thermal and contains no information about the state of the black hole. This leads to the problem of "information loss", since particles can fall in carrying information but what comes out is featureless thermal radiation [4]. Hawking argued that this would lead to non-unitary evolution, so that one of the basic principles of quantum mechanics would have to be modified.

Black holes are thermal systems that obey the laws of thermodynamics [5]. In fact, they have an entropy proportional to the area of the event horizon. The area of the horizon is just a property of the classical solution, it always increases in classical processes like the collision of two black holes. In most physical systems the thermodynamic entropy has a statistical interpretation in terms of counting microscopic configurations with the same macroscopic properties, and in most cases this counting requires an understanding of the quantum degrees of freedom of the system. For black holes this has been a long-standing puzzle: what are the degrees of freedom that the Hawking-Beckenstein entropy is counting?

String theory, being a theory of quantum gravity [6], should be able to describe black holes. Difficulties were very soon encountered because black holes involve strong coupling and therefore one will have to go beyond simple perturbative string theory to describe them. Recently there has been remarkable progress in understanding some string solitons called "D-branes" [7],[8],[9]. They account for some non-perturbative effects in string theory and they have a very simple description.

Charged black holes in General Relativity are characterized by their mass M and charge Q. The condition that the singularity is hidden behind a horizon

implies that $M \geq Q$. The case of M = Q is called extremal [10]. These black holes have smooth geometries at the horizon and a free-falling observer would not feel anything as he falls through the horizon. The horizon area, and thus the entropy, are nonzero, both for the extremal and non-extremal cases. The Hawking temperature vanishes for the extremal case and it increases as we increase the mass moving away from extremality (keeping Q fixed). For very large mass it decreases again.

We will be considering black holes in a theory, called N=8 supergravity [11], that is not precisely usual General Relativity but that is very similar for the kind of problems we are interested in. The difference with General Relativity is that it contains many extra massless fields: U(1) gauge fields, scalar fields and various fermionic fields. Despite this different field content, there is a charged black hole solution that is exactly like the one in General Relativity: the metric is exactly the same, there is only one gauge field excited (which is a particular combination of the original ones) and the rest of the fields, including the scalars, are all zero. This implies, as in General Relativity, that the geometry at the horizon is smooth. N=8 supergravity in four dimensions is the low energy limit of type II string theory compactified on a six-torus T^6 . String theory contains "D-brane" solitons that are extended membranes of various possible spatial dimensions [8], [9]. When these extended branes are wrapped around the compact directions they appear to the four-dimensional observer as localized objects, as charged particles. There is a symmetry, called U-duality, that interchanges all these objects [12]. Superimposing many of these objects of different dimensions we obtain a string soliton that has many of the properties of a black hole [13],[14],[15],[16],[17],[18],[19],[20],[21]. There is a large degeneracy which gives a statistical interpretation to the thermodynamic entropy. One great virtue of considering this supergravity theory is that the extremal black holes become supersymmetric configurations so that certain quantities can be calculated at weak coupling and are then valid for all values of the coupling. This has been the key to providing a precise calculation of extremal black hole entropy. The entropy calculated using the "D-brane" method agrees precisely, including the numerical coefficient with the classical Hawking-Beckenstein "area law" [13],[15],[19],[20]. The near-extremal black holes can also be considered from this point of view and they correspond to excited states of the solitons. These excited states result from attaching open strings to the D-branes [7][8]. Hawking radiation is described by

open strings colliding and forming a closed string that leaves the soliton [14]. Doing an average over the initial state of the black hole we get thermal Hawking radiation with the correct value for the temperature and the radiation rate is proportional to the area of the black hole [14]. These near-extremal calculations stand on a more shaky ground since one does not have supersymmetry to protect the calculations from strong coupling problems. The successful calculation of the entropy gives evidence in favor of the proposed physical picture. Unfortunately, these uncontrolled approximations for the near-extremal case will prevent us from saying anything about the information loss problem, but deeper analysis of this model might lead to an answer to this elusive problem.

In the rest of this chapter we review some general facts about string theory and we introduce the string solitons called "D-branes". In chapter 2 we describe the classical black hole supergravity solutions. Using some string theory information about the quantization and nature of the various charges, we rewrite the entropy formulas in a very suggestive form in terms of basic constituents. In chapter 3 we will show how to derive these entropy formulas for the extremal case and then consider near-extremal black holes, suggesting a physical picture for black holes in terms of D-branes. We conclude with a discussion on the results.

1.2. Perturbative string theory.

String theory is a quantum theory of interacting relativistic strings. Much of what we can presently do involves treating this interaction in perturbation theory [6]. But before we say anything about interactions let us review some properties of free string theory. We will be considering the theory of closed oriented strings. The free string action is

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[\partial_\alpha X^\mu \partial^\alpha X_\mu + \bar{\psi}^\mu \partial \psi_\mu \right]$$
 (1.1)

where $T = \frac{1}{2\pi\alpha'}$ is the string tension. We also have to impose the additional constraint that the two dimensional supercurrent and stress tensor associated to (1.1) vanish [22]. In this fashion the bosonic part of the action, which involves the ten spacetime coordinates X^{μ} , is just proportional to the area of the worldsheet embedded in ten dimensional space. The string contains fermionic

degrees of freedom living on the worldsheet ψ^{μ} . Depending on the boundary conditions of the fermions when they go around the loop there are four sectors which correspond to whether the left and right moving fermions are periodic or anti periodic as we go around the loop. The spacetime bosons come from the sectors where the boundary conditions for the fermions are the same both for left and right moving strings. They are the (NS,NS) and the (R,R) sectors, NS stands for Neveu-Schwarz and R for Ramond. The (NS,NS) sector contains massless fields corresponding to a graviton, a two form or antisymmetric tensor $B_{\mu\nu}$ and a scalar, the dilaton ϕ . The (R,R) sector contains antisymmetric tensor fields of various number of indices, i.e. p+1 forms A_{p+1} .

Spacetime symmetries correspond to symmetries in the worldsheet conformal field theory. In some cases the symmetry comes from a primary field conserved current. This is the case for translations and for supersymmetry transformations. The translations are associated to the primary fields ∂X^{μ} and the supersymmetries to the fermion vertex operators at zero momentum $V_{\alpha}(z)$. This is the operator that, in CFT, switches between Ramond and NS sectors, as a spacetime supersymmetry should do. The left and right moving spinors on the world sheet can have the same or opposite ten dimensional chiralities, giving the IIB or IIA theory respectively.

It will be interesting to consider strings on compact spaces. We will concentrate on the simplest compactification which is called toroidal and is obtained by identifying one the coordinates as $X^9 \sim X^9 + 2\pi R$ [22]. In this case the momentum P^9 becomes quantized in units of 1/R, $P^9 = n/R$. The string can also wind along this compact direction so that when we go around the string the coordinate has to satisfy the condition $X^9 \to X^9 + 2\pi Rm$. The two integers (n, m) are the momentum and winding numbers of the string. The Virasoro constraints are

$$E^{2} = \vec{P}^{2} + \left(\frac{n}{R} - \frac{mR}{\alpha'}\right)^{2} + \frac{4}{\alpha'}N_{L} ,$$

$$E^{2} = \vec{P}^{2} + \left(\frac{n}{R} + \frac{mR}{\alpha'}\right)^{2} + \frac{4}{\alpha'}N_{R} ,$$

$$(1.2)$$

where \vec{P} is the momentum in the directions 1, .., 8 and $N_{L,R}$ are the total net oscillator level of the string². Combining both equations in (1.2) we get the level

 $^{^2}$ $\,$ We are calculating $N_{L,R}$ it in the light front gauge, so there is no shift in $N_{L,R}$.

matching condition

$$P_{9R}^2 - P_{9L}^2 = 4nm = 4(N_L - N_R) , (1.3)$$

where $P_{9L,R} = \frac{n}{R} \mp \frac{mR}{\alpha'}$ are the left and right moving momenta in the direction 9. Momentum and winding are conserved and appear as charges in the extended dimensions 0, ..., 8. In fact, from the 1+8 dimensional point of view they are central charges because they appear in the supersymmetry algebra. The reason they appear in the supersymmetry algebra is that they appear in the ten dimensional algebra as the left and right moving momenta.

$$\{Q_{\alpha}^{L}, Q_{\beta}^{L}\} = \Gamma_{\alpha\beta}^{\mu} P_{\mu L} , \qquad \{Q_{\alpha}^{R}, Q_{\beta}^{R}\} = \Gamma_{\alpha\beta}^{\mu} P_{\mu R} . \qquad (1.4)$$

The supersymmetry algebra implies that $P^0 \geq |\vec{P}_L|$, $P^0 \geq |\vec{P}_R|$. These are the so called Bogomolny bounds. If any of these bounds is saturated we can see from (1.4) that some supersymmetries annihilate the state. If both bounds are saturated, then we have pure momentum or pure winding, $N_L = N_R = 0$, and half of the supersymmetries are broken. If only one bound, let us say the one involving P_R , is saturated, then $N_R = 0$, N_L is given by (1.3) and only 1/4 of the supersymmetries are left unbroken.

We can see from (1.2) that the spectrum is left invariant under the change of $R \to \alpha'/R$. This turns out to be a symmetry of the whole string theory, also of the interactions, and we expect that it will be valid even non-perturbatively. This very important symmetry of string theory is called T-duality. In fact in order for it to be a symmetry of the interactions also we need to change the coupling constant together with the radius as [23]

$$R \to R' = \frac{\alpha'}{R} , \qquad g \to g' = \frac{g\sqrt{\alpha'}}{R} .$$
 (1.5)

The change in the coupling constant is such that the d dimensional Newton constant stays invariant.

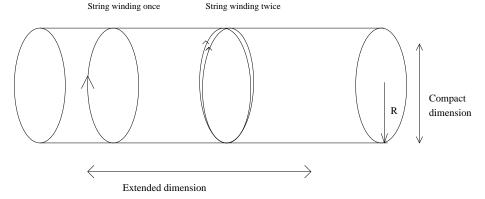


FIGURE 1: Strings winding around a compact direction.

Since string theory contains some massless particles, separated by a large mass gap $1/\sqrt{\alpha'}$ from the massive states of the string, it is natural to study the effective low energy action describing the interacting string theory. It has to have the symmetries of the theory: N=2 local supersymmetry in ten dimensions. To lowest order in string perturbation theory and long distances (we keep only second derivative terms) this Lagrangian is that of type II ten dimensional supergravity. It is called type II because we have two supersymmetries. Depending on the relative chirality of the supersymmetry generators we have the type IIA or IIB theories, which are the limits the IIA or IIB string theories. Let us start with the ten dimensional type IIA supergravity action [24]. This theory contains the fields coming from the (NS,NS) sector which are the metric $G_{\mu\nu}$, a two form $B_{\mu\nu}$ and a scalar ϕ called the dilaton. The fields coming from the (R,R) sector are a one form A_{μ} and a three form $C_{\mu\nu\rho}$. It also has the supersymmetic fermionic partners of all these fields. The bosonic part of the action is

$$S = \frac{1}{16\pi G_N^{10}} \int d^{10}x \sqrt{-G} \left[e^{-2\phi} (R + 4(\nabla\phi)^2 - \frac{1}{3}H^2) - \alpha' G^2 - \frac{\alpha'}{12}F'^2 - \frac{\alpha'}{288} \epsilon^{\mu_1 \dots \mu_{10}} F_{\mu_1 \mu_2 \mu_3 \mu_4} F_{\mu_5 \mu_6 \mu_7 \mu_8} B_{\mu_9 \mu_{10}} \right]$$
(1.6)

where G = dA, H = dB, F = dC and $F' = dC + 2A \wedge H$ are the field strengths associated with each of the differential forms.³ The supersymmetries are generated by two spinors $\epsilon_{L,R}$ of opposite chirality. The gravitational part

³ In components $G_{\mu\nu} = 2\partial_{[\mu}A_{\mu]}$, $H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$, $F'_{\mu\nu\rho\sigma} = 4\partial_{[\mu}C_{\nu\rho\sigma]} + 8A_{[\mu}H_{\nu\rho\sigma]}$.

of the action can be put in the standard form $(S \sim \int \sqrt{g}R)$ by defining a new metric, called the Einstein metric, by $g_E = e^{-\phi/2}G$, where G is the (so called) string metric in (1.6).

The type IIB action contains the same fields coming from the (NS,NS) sector and it contains therefore the first three terms in the action (1.6). The (R,R) fields are a scalar χ (or zero form), a two form $B'_{\mu\nu}$ and and four form $A_{\mu\nu\rho\delta}$ whose field strength is self dual F = dA = *F. Due to this condition it is not possible to write a covariant action for the IIB theory, however the equations of motion and the supersymmetry variations are known. We can also truncate the theory setting F = 0 and then we have a covariant action for the rest of the fields.

Type IIB supergravity [25] has the interesting property that it is S-dual under changing $\phi \to -\phi$ and interchanging the two antisymmetric tensor fields $B \leftrightarrow B'$ [26]. In fact, the classical symmetry is SL(2,R) once one includes shifts in the other scalar χ . In string theory an SL(2,Z) subgroup of this symmetry is expected to survive [12]. We will later make use of this S-duality symmetry to generate solutions and relate them to each other. This S-duality transformation leaves the Einstein metric invariant but it changes the string metric. This means, in particular, that if we have a compactified theory and the radii are measured in string metric as in (1.2) then, under an S-duality transformation they all change as

$$g \to g' = \frac{1}{g} , \qquad \qquad R_i \to R_i = \frac{R_i}{\sqrt{g}} .$$
 (1.7)

We will define throughout this thesis the ten dimensional coupling constant $g = e^{\phi_{\infty}}$ to be such that it transforms as (1.7) under S-duality. We will see in section (2.4) that this fixes the ten dimensional Newton constant in (1.6) to be $G_N^{10} = 8\pi^6 g^2 \alpha'^4$. In compactified theories the S-duality and T-duality groups combine to form a bigger group called U-duality [12].

1.3. String solitons and D-branes.

The low energy supergravity action contains p+1 forms A_{p+1} coming from the RR sector, p is even in IIA, and odd in IIB. There are no objects in perturbative string theory that carry charge under these fields, all vertex operators involve the field strength of these forms. The objects that would carry charge under a p+1 form are extended p branes. The coupling is

$$\mu_p \int_{V_{p+1}} A_{p+1} \tag{1.8}$$

which naturally generalizes the electromagnetic coupling to an electric charge. In addition, if we assume that the spectrum of electric p brane charges is quantized we would expect also "magnetic" 6-p branes that couple to the Dirac dual \tilde{A}_{7-p} form defined through equations of the type $d\tilde{A}_{7-p} = *dA_{p+1}$ (the details are slightly more complicated) [27].

In fact, type II supergravities contain extended black p brane solutions which carry this charge [28]. The extremal limit of these p branes saturates the BPS bound for these charges. These solutions will be presented in chapter 2.

In string theory these solutions appear as some very special solitons [7][8]. They are extended objects with p spatial dimensions and are called "D-branes". Their description is very simple and it amounts to the following definition: D-branes are p-dimensional extended surfaces in spacetime where strings can end. A D-brane is the string theory solution (it is described by a CFT) whose low energy limit is a supergravity extremal p brane. In type II theory we had only closed strings in the vacuum. In the presence of a D-brane there are also open strings which interact with the closed strings by usual splitting and joining interactions [22]. These D-branes have the peculiar property that their mass (tension) goes like 1/g and in fact they would lead to non-perturbative effects of order $e^{-O(1/g)}$. Effects of this magnitude were observed in string theory, specially in matrix models [29]. They also carry RR charges with the values predicted by U-duality.

An open string has a worldsheet that is topologically a strip. One has to specify some boundary conditions on the boundaries of the strip, that is, at the end of the string. The boundary conditions describing an open string attached to a p brane sitting at $x_{p+1} = \cdots = x_9 = 0$ are

$$\partial_{\sigma} X^{\mu} = 0$$
 for $\mu = 0, ..., p$,
 $X^{\mu} = 0$ for $\mu = p + 1, ..., 9$. (1.9)

These are Neumann boundary conditions on the directions parallel to the brane and Dirichlet conditions on the directions perpendicular to the brane. This

is the reason they are called D(irichlet)-branes. These open strings have the characteristic spectrum $P^2=\frac{4}{\alpha'}N_{open}$, with the momentum $P=(P^0,...,P^p)$ being parallel to the brane. These open strings represent excitations of the branes. In general, an excited brane corresponds to having a gas of these open strings on the brane. Of particular interest to us will be the massless bosonic open strings, those for which $N_{open}=0$. The massless open strings have a vector index. If the index lies in the directions parallel to the brane they describe gauge fields living on the brane and if the index is perpendicular to the brane they describe oscillations of the brane in the perpendicular directions. As an example let us take a D-string, consider it winding once around the compact direction $\hat{9}$. Note that S-duality interchanges this D-string with a fundamental string [12][30]. The open strings attached to the D-string can have momentum in the direction $\hat{9}$ which is quantized in units of $1/R_9$. The energy of a D-brane containing a gas of massless open strings is

$$E = \frac{R_9}{\alpha' g} + \sum_{i} \epsilon_i = E_0 + \frac{N_L + N_R}{R_9} \ . \tag{1.10}$$

For each momentum n we have eight bosonic and eight fermionic modes. There can be a number N_n of strings with momentum n and

$$N_R = \sum_{n>0} nN_n , \qquad N_L = \sum_{n<0} nN_n .$$

We see that the spectrum is the same as the one we would obtain for a superstring winding around the 9^{th} direction with tension $T_D = \frac{1}{2\pi g\alpha'}$ if we expand (1.2) in powers of R_9 . Note that the statistics and number of excitations corresponds precisely with that of the fundamental string. In this fashion we can see that the massless open strings describe oscillations of D-branes. Actually, for p > 1 not only oscillations but also fluctuations in the world-brane gauge fields.

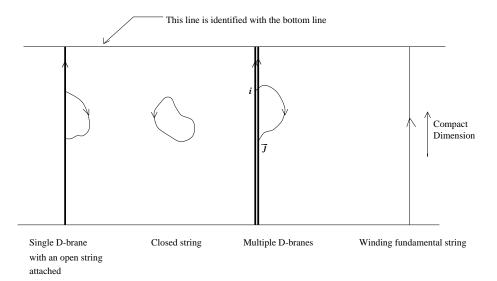


FIGURE 2: D-branes winding around a compact direction with open strings attached. Only closed strings exist between widely separated D-branes. Open strings carry U(N) Chan Paton factors when we have several D-branes.

If one considers many D-branes of the same type sitting on top of each other, the open strings carry Chan-Paton indices (i, \bar{j}) specifying the starting and ending point of the string [9][31]. The interactions of these massless open strings can be described by a U(N) Yang-Mills action. Since T-duality transformations change the dimensionality of D-branes the simplest way to obtain this action is to do a T-duality transformation into 9-branes filling the space and we have an N=1 ten dimensional YM Lagrangian

$$S = \frac{1}{4g} \int d^{10}x Tr[F_{\mu\nu}F^{\mu\nu}] + \text{fermions} .$$
 (1.11)

This Lagrangian describes the low energy limit of open string amplitudes. If we perform T-duality transformations the amplitudes will not change. The massless vertex operators change a little, the vertex operators for coordinates with Neumann boundary conditions involve the derivative along the boundary $\partial_t X$ while the ones for coordinates with Dirichlet conditions involve the normal derivative $\partial_n X$. T-duality interchanges the normal and tangential derivative. Another difference is that the momentum perpendicular to the branes vanishes. Otherwise the amplitudes are exactly the same. So we conclude that the low energy action

describing the interaction of the massless modes on a D-brane is just the dimensional reduction of (1.11) to p+1 dimensions. So we replace $dx^{10} \to d^{p+1}x$, the fields A_{α} , $\alpha = 0, ..., p$, are gauge fields on the D-brane and A_{I} , I = p+1, ..., 10, are related to the motion of the D-brane in the transverse dimensions. Separating the branes corresponds to breaking the symmetry down to $\mathrm{U}(1)^N$ by giving an expectation value to the fields A_I , I = p+1, ..., 10. These expectation values have to be commuting diagonal matrices (up to gauge transformations), the elements on the diagonal represent the position of the branes [7][31]. In the case of a fundamental string we can have many different configurations depending on how the string is wound in the compact direction. We could have a single string wound N times or N strings each winding only once. For D-branes we have a similar situation. Different windings correspond to different boundary conditions along the compact direction. The physics will be different depending on how they are wound. For example, if we have a single D-string winding Q times all the fields will satisfy the boundary condition $A_{\mu}(x_9 + 2\pi R_9) = UA_{\mu}(x_9)U^{-1}$ where U is the transformation that cyclically permutes the Chan-Paton indices $i \to i+1$. Now we are interested in finding states of the system corresponding to oscillating D-strings. Naively we might think that Q D-strings have a set of Q^2 independent massless excitations, corresponding to the different components of the gauge field. However we should be more careful because there are interactions, so if we consider, for example, a configuration with waves along the diagonal Q directions corresponding to separating the D-strings, then the other components of the gauge field become massive. In other words, in the worldbrane gauge theory there are 8 scalars in the adjoint A_I and there is a potential for these scalars, coming from the commutator terms in the YM action, $V = \sum_{I,J} Tr[A_I, A_J]^2$. In order to see this more explicitly let us take diagonal matrices

$$A_{I} = \begin{pmatrix} f_{I}^{1}(u, v) & & & \\ & \cdot & & \\ & & \cdot & \\ & & f_{I}^{Q}(u, v) \end{pmatrix} , \qquad (1.12)$$

where $v, u = x^9 \pm x^0$. If we insert this ansatz in the equations of motion we find that f_I^m obey the massless wave equation. Now consider, on this background, a small off diagonal component $(\delta A_I)_{mn} \neq 0$, where $m \neq n$ are some fixed indices,

and all other components of δA are zero. The equation of motion will be of the form

$$4\partial_u \partial_v (\delta A_I)_{mn} - (f_J^n - f_J^m)^2 (\delta A_I)_{mn} = 0.$$
(1.13)

We see that the oscillating background acts like a mass term for this off diagonal component. The effect of this mass term is more clear if we consider purely left moving excitations. Then we see that the maximum number of independent oscillations is 8Q, corresponding to diagonal matrices A_I , since the equation (1.13) cannot be solved with purely left moving excitations if f_J^n are arbitrary. In the case that the f_J^n contain both left and right moving waves it is reasonable to assume that for generic f's we are not going to have any resonances and that off diagonal excitations will be effectively massive.

In the case the D-string is multiply wound these diagonal elements f_I^n are cyclically permuted in going around the compact direction $f_I^n(x_9 + 2\pi R_9) = f_I^{n+1}(x_9)$ so that we could think that the momentum is quantized in units of $1/QR_9$. This correctly reproduces the energy levels of a multiply wound string [32]

$$E = \frac{R_9 Q}{g \alpha'} + \frac{N_L' + N_R'}{Q R_9} \ . \tag{1.14}$$

The total physical momentum still has to be quantized in units of 1/R so $P = (N'_L - N'_R)/QR_9 = N/R_9$. This is the condition analogous to (1.3). Here we have assumed that R_9 is very big so that we can neglect interactions and massive open strings.

The states with $N_R = 0$ are BPS and supersymmetry ensures that (1.14) is precisely right. This configuration is related by S duality to a fundamental string of winding number Q carrying left moving oscillations. We can see that the degeneracies are precisely the same since we have eight bosonic and fermionic excitations with momenta quantized in units of 1/RQ. It was crucial to obtain the reduction of the independent degrees of freedom from $Q^2 \to Q$. We will see this mechanism working again for the black hole case.

It is quite straightforward to compute the interactions of these open strings [33], the interactions of closed strings and open strings [34] and the scattering of closed strings from the D-brane [35][36][37]. To lowest order in string perturbation theory they reduce to calculations on the disc with vertex insertions at the boundary associated with open strings and insertions in the interior of the disc

associated to closed string states. In this way we can compute the scattering of closed strings from the D-brane and we indeed find that in the low energy limit the stringy amplitudes agree with those calculated purely in the supergravity p-brane solutions [36][34].

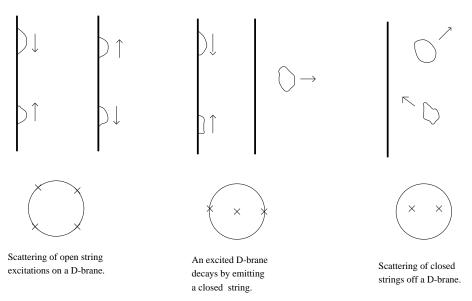


FIGURE 3: String theory diagrams appearing in various scattering processes.

The second process is the relevant one for Hawking radiation.

In the presence of a D-brane it is easy to see how supersymmetries are broken. We said before that the right and left ten dimensional supersymmetries are generated by the right and left moving spinors on the worldsheet. The presence of a boundary in the world sheet relates the left and right moving spinors through a boundary condition. This is something familiar from open string theories, which have only one supersymmetry in ten dimensions (Type I). As argued in [36][9] the boundary condition for the spinors is

$$S_R(z) = \pm \Gamma^0 \cdots \Gamma^p S_L(\bar{z}) \Big|_{z=\bar{z}} . \tag{1.15}$$

The two choices of sign in (1.15) corresponds to opposite D-brane orientations and therefore opposite D-brane charges. Note that in the type IIA theory we have p even and therefore different chiralities for the worldsheet spinors, while

for IIB theory we have odd p and the same chirality for both spinors. This in turn translates into the following condition for the parameters that generate the unbroken supersymmetries in the presence of a p brane

$$\epsilon_R = \Gamma^0 \cdots \Gamma^p \epsilon_L \ . \tag{1.16}$$

Since the BPS p-brane solution is the extremal limit of a black p-brane we would expect that D-branes provide a quantum description for these black branes. This naive expectation is not quite so because the Schwarschild radius of a D-brane is of order $r_s^{7-p} \sim g$ which is much smaller than $\sqrt{\alpha'}$ for small g. So the strings are typically much larger than the black hole radius [9]. We might try to solve this by considering many D-branes, in that case the Schwarschild radius would grow like $r_s^{7-p} = Qg$. However in any process we consider there will be open string loop corrections which will be of order qQ, the extra factor of Qcomes from the sum over the Chan Paton index. If we compactify the D-brane to make a black hole we see that the supergravity solution already shows that there are scalar fields that are blowing up as we approach the horizon, this also indicates that near the brane the strings are not free any more and also that these black holes are very different than the ones we are used to in General Relativity. Of course the size of loop corrections depends on where the Q D-branes are, if they are sitting on top of each other the corrections are big but if they are separated in space the corrections are small. From the point of view of string theory, separating the branes in space means giving an expectation value to the translational zero modes of the brane, which means putting many open strings on the D-brane.

We will show in what follows that there are some properties of black holes that are correctly described by D-branes. But in order to describe those black holes we need configurations with more than one type of D-branes.

If we introduce another type of D-brane we have even more types of open strings. We would like to choose p-brane and p'-brane superpositions in such a way that some supersymmetries are still preserved. The additional boundary will introduce a new condition on the spinors of the type (1.16) with $p \to p'$. We can see that if p - p' = 4,8 we can have a supersymmetric configuration preserving 1/4 of the supersymmetries if the p'-brane is parallel to the p-brane [38]. Other configurations with non-parallel D-branes can be obtained from this

one by applying T-duality transformations. The different branes need not be on top of each other and wherever the branes are, we have a supersymmetric configuration that saturates the BPS bound $M = c_p Q_p + c_{p-4} Q_{p-4}$, where c's are some fixed coefficients. If p - p' = 2,6 then two conditions of the type (1.16) seem to be in conflict because they impose chirality conditions that cannot be satisfied for real spinors. Nevertheless BPS configurations carrying p - 2 and p brane charges are predicted by U-duality, this basically comes from the fact that fundamental strings can be bound to D-strings [31]. But the BPS formula for this case [31] has a different structure, $M \sim \sqrt{c_p' Q_p^2 + c_{p-2}' Q_{p-2}^2}$, with nonzero binding energy and suggests that we indeed should not be able to see this configuration as two separate D-branes in equilibrium at weak coupling.

If we have Q coinciding D-p-branes, there are instanton solutions of the $\mathrm{U}(Q)$ world-brane-volume gauge theory with dimension p-4 which carry RR p-4 brane charge. In fact the D-(p-4)-brane corresponds to the zero size limit of these instantons [38].

Intersecting D-brane configurations with (p, p') = (1, 5) and (2, 6) will appear when we describe five and four dimensional black holes. In these cases the low energy worldbrane field theory describing the interactions of the massless modes is the dimensional reduction of an N = 1 theory in six dimensions, corresponding to the case (p, p') = (9, 5) [38]. In chapter 3 we will study this case in more detail.

2. CLASSICAL BLACK HOLE SOLUTIONS

In general relativity plus electromagnetism there are charged black hole solutions. They are the most general spherically symmetric, stationary solutions and are characterized by the charge Q and the mass M. The cosmic censorship hypothesis [2] which says that gravitational collapse does not lead to naked singularities implies that in physical situations only $M \geq Q$ black holes will form, since the solution would otherwise contain a naked singularity. The case M = Q is called extremal, since it has the minimum possible mass for a given charge. This charged black holes are given by the Reissner-Nordström solution [10]

$$ds^{2} = -\Delta dt^{2} + \Delta^{-1}dr^{2} + r^{2}d\Omega_{2}^{2} , \qquad (2.1)$$

$$\Delta = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right) \; .$$

The outer horizon is at $r = r_{+}$ and the mass and charge are

$$M = \frac{1}{2G_N^4}(r_+ + r_-) , \qquad Q = \frac{1}{G_N^4}\sqrt{r_+ r_-} .$$
 (2.2)

In this chapter we will find black hole solutions to type II supergravity compactified down to d=4,5 dimensions. For d=4 this leads to N=8 supergravity. The familiar solution (2.1) will be indeed one particular case of the black holes we consider. Of course, the theory in which it is embedded is different but the metric is the same and the gauge field will be a particular linear combination of the ones appearing in N=8 supergravity. These black holes can be thought of as extended membranes wrapping around internal dimensions. We will therefore start by studying the extended brane solutions in ten dimensions. In the following section we will show how to construct oscillating BPS solutions, this section could be skiped in a quick reading. Then we show how lower dimensional black holes are obtained from the ten dimensional solutions. We discuss the role of U-duality and Dirac duality for quantizing the charges. We finally consider extremal and non-extremal black hole solutions in five and four dimensions. We will define new variables identified with the number of some hypothetical non-interacting "constituents" in terms of which the entropy takes a surprisingly simple form.

2.1. Extended p-brane solutions

We will now consider solutions to type II supergravity theories in ten dimensions. We will concentrate first with solutions that preserve some supersymmetries, the so called BPS solutions. We start with one of the simplest, which is the solution corresponding to the fields outside a long fundamental string [39]. It only has fields in the first three terms in (1.6) excited and it is a solution in both type II theories and also in the heterotic string theory. It carries charge under the NSNS $B_{\mu\nu}$ field, this charge appears as a central charge in the supersymmetry algebra. The solution with the minimum mass for a given charge will then be BPS. The simplest way to find this BPS solution is the following. Start with a $SO(1,1)\times SO(8)$ symmetric ansatz for the metric, in string frame,

$$ds^{2} = h \left[f_{s}^{-1} (-dt^{2} + dx_{9}^{2}) + dx_{1}^{2} + \dots + dx_{8}^{2} \right] . \tag{2.3}$$

We also allow the dilaton ϕ and the component B_{09} of the antisymmetric tensor to be nonzero and we set all other fields to zero. Now we try to find Killing spinors, which generate infinitesimal local supersymmetry transformations that leave the solution invariant. In order to be definite we consider the type IIA theory, a similar treatment goes through for the IIB and heterotic theories. The existence of unbroken supersymmetry implies that the gravitino and dilatino variations

$$\delta\lambda = \left[\partial_{\mu}\phi\gamma^{\mu}\Gamma_{11} + \frac{1}{6}H_{\mu\nu\rho}\gamma^{\mu\nu\rho}\right]\eta ,$$

$$\delta\psi_{\mu} = \left[\partial_{\mu} + \frac{1}{4}\left(\omega_{\mu}^{ab} + H_{\mu}^{ab}\Gamma_{11}\right)\Gamma_{ab}\right]\eta ,$$
(2.4)

should vanish for appropriate values of the spinor η , where $\eta = \epsilon_R + \epsilon_L$ is the sum the a possitive and negative chirality spinor. Greek letters label coordinate indices, and latin letters label tangent space indices. Coordinate and tangent indices are related by the zehnbeins e^a_μ and ω^{ab}_μ is the corresponding spin connection. Γ_a are the flat space gamma matrices satisfying $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}, \gamma^\mu = e^\mu_a \Gamma^a$ and $\gamma^{\mu_1 \cdots \mu_n}$ is the antisymmetrized product with unit weight (i.e. dividing by the number of terms). In order for the equations (2.4) to have solutions, the

dilaton, the antisymmetric tensor field and the metric have to be related to each other and take the form [39]

$$ds^{2} = f_{f}^{-1}(-dt^{2} + dx_{9}^{2}) + dx_{1}^{2} + \dots + dx_{8}^{2} ,$$

$$B_{09} = \frac{1}{2}(f_{f}^{-1} - 1) ,$$

$$e^{-2(\phi - \phi_{\infty})} = f_{f} ,$$
(2.5)

where f_f is a function of the transverse coordinates $x_1, ..., x_8$ and the rest of the fields are zero. With this ansatz (2.5) the supersymmetry variations (2.4) vanish if the spinors satisfy the conditions

$$\epsilon_{R,L} = f_f^{-1/4} \epsilon_{R,L}^0 , \qquad \Gamma^0 \Gamma^9 \epsilon_R^0 = \epsilon_R^0 , \qquad \Gamma^0 \Gamma^9 \epsilon_L^0 = -\epsilon_L^0 , \qquad (2.6)$$

where the spinors $\epsilon_{R,L}^0$ are independent of position and are the asymptotic values of the Killing spinors. So we see that the solution preserves half of the supersymmetries for any function f_f . Actually, the equations of motion of the theory (related to the closure of the supersymmetry algebra) imply that f_f is a harmonic function $\partial^2 f_f = 0$ where ∂^2 is the flat Laplacian in the directions $1, \dots, 8$. Taking

$$f_f = 1 + \frac{Q_f}{r^6} \,\,\,\,(2.7)$$

we get a solution that looks like a long string. It is singular at r=0 but in fact one can see from the metric that it is a so called null singularity, there is a horizon at the singularity and we do not have a naked singularity. In this classical solution the constant Q_f is arbitrary. However, this long string solution carries a charge under the B field, this charge is carried in string theory by the fundamental strings. The charge that the fundamental strings carry is their winding number and it is not continuous, it is a multiple of some minimum value. An easy way to see this is to consider this theory compactified on a circle by periodically identifying the direction $\hat{9}$ by $x_9 \sim x_9 + 2\pi R$. In that case the $B_{\mu 9}$ components of the antisymmetric tensor field become a gauge field in the extended dimensions. The "electric" charge associated with this gauge field is the winding number along the direction $\hat{9}$ which counts how many strings are wound along this circle. In string theory this number is an integer, there is a geometric quantization condition. This is why we say that the fundamental strings can carry

only integer multiples of this charge. We conclude that $Q_f = c_f^{10} m$, with m an integer representing the winding number. One can determine c_f^{10} by comparing the charge of (2.5) with that of a fundamental string with winding number m. This is equivalent, due to the fact that both are BPS solutions, to comparing the masses. The ADM mass is determined from (2.5) from the g_{00} component of the Einstein metric of the extended 1 + 8 dimensional theory. This gives

$$c_f^{10} = \frac{8G_N^{10}}{\alpha'6\omega_7} \,\,\,(2.8)$$

where $\omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$ is the volume of the sphere in d dimensions S_{d-1} .

Since this supergravity solution carries the same charge and mass as the fundamental string and has the same supersymmetry properties, it is natural to regard (2.5)(2.7)(2.8) as describing the long range fields produced by a long fundamental string. This is analogous to saying, in quantum electrodynamics, that the electric field of a point charge describes the fields far from an electron. Actually, in [39] this coefficient (2.8) was determined by matching the solution (2.5) to a fundamental string source of the form (1.1).

It is interesting that the equations of motion just demand that f_f in (2.5) is a harmonic function. Taking it to be $f_f = \sum_i c_f^{10}/(\vec{r} - \vec{r_i})^6$ we describe a collection of strings sitting at positions $\vec{r_i}$ in static equilibrium. The gravitational attraction and the dilation force cancel against the electric repulsion. This "superposition principle" is a generic property of BPS solutions and will appear several times in the construction of BPS black holes. We indeed expect to have no force since the energy of a BPS configuration with charge m, as given by the BPS formula, does not depend on the position of the charges.

Now we turn to other ten dimensional solutions that preserve 1/2 of the supersymmetries. The fundamental string solutions carried "electric" charge under the B field. The corresponding field strength H=dB is dual to a seven index field strength $F_7 \sim *H$ and can be written in terms of a six form $F_7 = d\tilde{B}_6$. This six form couples naturally to a five-brane. The supergravity solution, called solitonic (symmetric) fivebrane, is again determined in terms of a single harmonic function [40]. In string frame it reads

$$ds^{2} = -dt^{2} + f_{s5}(dx_{1}^{2} + \dots + dx_{4}^{2}) + dx_{5}^{2} + \dots + dx_{9}^{2} ,$$

$$e^{-2(\phi - \phi_{\infty})} = f_{s5}^{-1} ,$$

$$H_{ijk} = (dB)_{ijk} = \frac{1}{2} \epsilon_{ijkl} \partial_{l} f_{s5} , \qquad i, j, k, l = 1, 2, 3, 4 ,$$

$$(2.9)$$

and all other fields are zero. ϵ_{ijkl} is just the flat space epsilon tensor. The harmonic function f_{s5} depends on the coordinates transverse to the fivebrane $(x_1 \cdots x_4)$ and for a single fivebrane takes the form $f_{s5} = 1 + \frac{c_{s5}}{(x_1^2 + \ldots + x_4^2)}$. The constant c_{s5} is determined from the Dirac quantization condition. That is to say, the B field that results from (2.9) cannot be defined over all space and will have have some discontinuities. These discontinuities will be invisible to fundamental strings if the fivebrane charge obeys the condition analogous to the Dirac quantization condition for electric and magnetic charges. This condition implies that $c_5^s = \alpha'$ [40], so that the mass of the fivebrane goes as $1/g^2$ showing a typical solitonic behavior, what is more, the string metric (2.9) shows a geometry with a long throat at r = 0 so that it has some "size". The Killing spinors that generate the unbroken supersymmetries are determined, as in the case of the fundamental string (2.6), by some constant spinors at infinity which satisfy the conditions

$$\epsilon_L^0 = \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \epsilon_L^0 , \qquad \epsilon_R^0 = -\Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \epsilon_R^0 . \qquad (2.10)$$

Even though we have presented these solutions just as supergravity solutions it is possible to show that they define conformal field theories, which implies that they are solutions to the full string classical action, and not just to the low energy supergravity.

In type II theories it is natural to look for supergravity solutions describing the long range fields away from a D-brane. They will be extended branes of p spatial dimensions, carrying "electric" charge under the A_{p+1} forms, or "magnetic" under the A_{7-p} forms.

These solutions have the form, in string frame [28],

$$ds^{2} = f_{p}^{-1/2}(-dt^{2} + dx_{1}^{2} + \dots + dx_{p}^{2}) + f_{p}^{1/2}(dx_{p+1}^{2} + \dots + dx_{9}^{2}) ,$$

$$e^{-2\phi} = f_{p}^{\frac{p-3}{2}} ,$$

$$A_{0\cdots p} = -\frac{1}{2}(f_{p}^{-1} - 1) ,$$

$$(2.11)$$

where f_p is again a harmonic function of the transverse coordinates $x_{p+1}, ..., x_9$. All these solutions are BPS and break half of the supersymmetries through the conditions (1.16). They correspond to the extremal limit of charged black p-branes when the harmonic function is $f_p = 1 + nc_p^{10}/r^{7-p}$, where n is an integer and c_p^{10} is related to the minimum charge of a D-brane and will be calculated later using U-duality. In the type IIA we will have only solutions like (2.11) for p even and in the type IIB only for p odd. In type IIB theory there are two kinds of strings: the fundamental strings and the D-strings. Similarly there are two kinds of fivebranes, the solitonic fivebrane and the D-fivebrane, the difference between them is whether they carry charge under the antisymmetric tensor field $B_{\mu\nu}$ or $B'_{\mu\nu}$. The dilaton and the string metric are also different in both solutions, but they transform into each other under S duality. The three brane is self dual under S-duality.

Note that all these extremal solutions are boost invariant for boosts along the brane, in that sense they are relativistic branes like the fundamental string. This property is related to the fact that they preserve some supersymmetries. The extremal branes therefore cannot carry momentum in the longitudinal directions by just moving in a rigid fashion but, of course, they can carry transverse momentum. In order to carry longitudinal momentum they have to oscillate in some way, that is the topic of the next section. These oscillations propagate at the velocity of light since the tension is equal to the mass per unit brane-volume.

2.2. Oscillating strings and branes.

This section is aimed at providing a more direct correspondence between BPS oscillating strings and fundamental string states. It can be skipped in a first quick reading.

As discussed in section 1.2 a fundamental string containing only left moving oscillations is a BPS state breaking 1/4 of the supersymmetries. It is natural to look for supergravity solutions that describe the long distance behaviour of these oscillating strings. We can take R_9 to be large and we can make coherent states with the string oscillators, leading to macroscopic classical oscillations. Therefore, we expect the supergravity solutions to exhibit these oscillations which describe traveling waves on a fundamental string. The general method to construct these solutions was developed by [41]. In the case of fundamental strings the oscillating solutions take the form [42]

$$ds^{2} = f_{f}^{-1} du [dv + \tilde{k}(r) du + 2F'^{i}(u) dy^{i}] + dy^{i} dy^{i} ,$$

$$B_{uv} = -\frac{1}{4} (f_{f}^{-1} - 1) ,$$

$$B_{ui} = f_{f}^{-1} F'^{i}(u) ,$$

$$e^{-2\phi} = f_{f} ,$$
(2.12)

where $u = x_9 - t$, $v = x_9 + t$ and $F^i(u)$ are arbitrary functions describing a traveling wave on the string. f_f and \tilde{k} are harmonic functions. The solution (2.12) arises from the chiral null models studied in [43]. Since this metric is not manifestly asymptotically flat, we prefer to make the simple change of coordinates

$$y^{i} = x^{i} - F^{i}(u)$$
, $v = \tilde{v} + \int^{u} \left[F^{\prime i}(u_{0}) \right]^{2} du_{0}$. (2.13)

which puts the fields in the form

$$ds^{2} = f_{f}^{-1}(\vec{r}, u)du \left[d\tilde{v} - 2(f_{f}(\vec{r}, u) - 1)F'^{i}(u)dx^{i} + k(\vec{r}, u)du\right] + dx^{i}dx^{i} ,$$

$$k(\vec{r}, u) = \tilde{k}(\vec{r}, u) + (f_{f} - 1)(F'(u))^{2} ,$$

$$B_{uv} = -\frac{1}{4}(f_{f}^{-1}(\vec{r}, u) - 1) ,$$

$$B_{ui} = \left(f_{f}^{-1}(\vec{r}, u) - 1\right)F'^{i}(u) ,$$

$$(2.14)$$

where $f_f(\vec{r}, u) = f_f(\vec{r} - \vec{F}(u))$ and $k(\vec{r}, u) = k(\vec{r} - \vec{F}(u))$. Here $f_f(r)$ is as in (2.7)(2.8) with winding number m and $k(r) = P(u)2\pi\alpha'c_f^{10}/r^6$, with P(u) being the physical momentum per unit length carried by the string. The metric is now manifestly asymptotically flat, and, in the limit $F^i(u) \to 0$, it reduces to the static solution (2.5).

This oscillating string solution (2.12) preserves 1/4 of the supersymmetries. The spinors that generate the unbroken supersymmetries satisfy

$$\epsilon_R^0 = \Gamma^0 \Gamma^9 \epsilon_R^0 , \qquad \epsilon_L^0 = 0 . \qquad (2.15)$$

As a check on our understanding of the physics of these solutions, we should verify that the excited strings do indeed transport physical momentum and angular momentum. Since we have written the metric in a gauge where it approaches the Minkowski metric at spatial infinity, we can use standard ADM or Bondi mass techniques to read off kinetic quantities from surface integrals over the deviations of the metric from Minkowski form. Following [39][44], we pass to the physical (Einstein) metric $g_E = e^{-\phi/2}G_{string}$, expand it at infinity as $g_{E\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and use standard methods to construct conserved quantities from surface integrals linear in $h_{\mu\nu}$. We find that the transverse momentum per unit length on a slice of constant u is

$$P_i = \frac{m}{2\pi\alpha'} F^{\prime i}(u) \tag{2.16}$$

in precise accord with "violin string" intuition about the kinematics of disturbances on strings. Similarly, the net longitudinal/time energy-momentum per unit length $\Theta_{\alpha\beta}$, $\alpha, \beta = 0, 9$, in a constant u slice is

$$(\Theta_{\alpha\beta}) = \begin{pmatrix} \frac{m}{2\pi\alpha'} + P(u) & -P(u) \\ -P(u) & -\frac{m}{2\pi\alpha'} + P(u) \end{pmatrix}.$$

Finally, we consider angular momenta. For the string in ten dimensions there are four independent (spatial) planes and thus four independent angular momenta M^{ij} per unit length. Evaluating as an example M^{12} we obtain [42]

$$M^{12} \sim (f'^1 f^2 - f'^2 f^1)(u)$$
 (2.17)

There are no surprises here, just a useful consistency check.

Note that a single fundamental string satisfies the level matching condition (1.3) so we might wonder if there is an analogous condition in the supergravity solution. One way to find this condition is to demand that the solution matches to a fundamental string source [45]. Another way is to demand that the singularity, when we approach the string is not naked but null [42]. This amounts to demanding that the function \tilde{k} in (2.12) vanishes, which leads to

$$\frac{P(u)m}{2\pi\alpha} = \frac{m^2}{(2\pi\alpha')^2} F^{i}(u)^2 \ . \tag{2.18}$$

There are also BPS multiple string solutions where the different strings are oscillating independently. They are described in [42] and they involve new conformal field theories which are a generalization of the chiral null models considered by [43]. If we have such a superposition the condition (2.18) need not be satisfied. Actually one has to effectively average over functions $F^{i}(u)$ [42]. For a general ensemble of functions $F^{i}(u)$ will be uncorrelated with $F^{j}(u)$ and the $g_{\mu i}$, $B_{\mu i}$ components of the metric and antisymmetric tensor will vanish, leaving just the function k in (2.12). Note that this is not the case if they are carrying some net angular momentum (2.17).

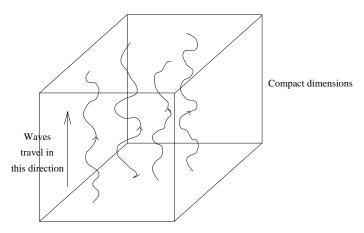


FIGURE 4: Ensemble of many oscillating strings carrying traveling waves.

In a very similar fashion it is possible to construct oscillating p-branes. In fact, if we just average over the oscillations we simply get one more harmonic function $K = \frac{c_P N}{r^{7-p}}$ in the solution. The coefficient $c_P = \frac{\alpha'}{R_9^2} c_f$ is calculated using U-duality (see section 2.4) and N is the momentum, measured in units of the minimum allowed. In conclusion, when momentum is carried in a direction parallel to the brane (call it $\hat{9}$), then the solution can be found by replacing $-dt^2 + dx_9^2 \rightarrow -dt^2 + dx_9^2 + k(dt - dx_9)^2$ in the metric in (2.11) or (2.9) . Adding momentum leads to BPS solutions preserving 1/4 of the supersymmetries by imposing the additional constraints on the spinors at infinity, due to the momentum,

$$\epsilon_R^0 = \Gamma^0 \Gamma^9 \epsilon_R^0 , \qquad \epsilon_L^0 = \Gamma^0 \Gamma^9 \epsilon_L^0 .$$
 (2.19)

2.3. $d \le 9$ Black Holes From d = 10 strings or branes.

Since all the BPS solutions treated in the previous section depend on some harmonic function f one can make multiple brane solutions by taking $f = 1 + \sum_{i} c_p/(\vec{r} - \vec{r_i})^{7-p}$ which describes a set of branes at positions $\vec{r_i}$ in static equilibrium. The gravitational attraction is balanced by the repulsion due to their charges.

In this section the word "brane" will indicate any of the BPS solutions discussed above. We will now consider the type IIB theory compactified to d dimensions on a torus T^{10-d} , identifying the coordinates by $x_i \sim x_i + 2\pi R_i$, choosing periodic boundary conditions on this 10-d dimensional "box". Fields that vary over the box will acquire masses of the order $m \sim 1/R$ where R is

the typical compactification size. The easiest way to see this is by expanding the fields in Fourier components along the internal dimensions. So if we are interested in the low energy physics in d extended dimensions the fields will be independent of the internal coordinates of the torus. If we want to find solutions to this d dimensional supergravity theory, does it help us to know the solutions in ten dimensions? Yes, it does. The key point to observe is that if we have any solution in ten dimensions which is periodic under $x_i \to x_i + 2\pi R_i$, then it will also be a solution of the compactified theory. For any p-brane, the solution is automatically translational invariant in the directions parallel to the brane. In order to produce a periodic solution we superimpose BPS solutions forming a lattice in the transverse directions, producing a harmonic function

$$f = 1 + \sum_{\vec{n} \in Lattice} \frac{c}{(\vec{r} - 2\pi R_i \vec{n})^{7-p}}$$
 (2.20)

We can view this as solving the Laplace equation with the method of images in a periodic box. It is this nice superposition principle for BPS solutions that enables us to find a very direct correspondence between ten dimensional objects and the ddimensional ones. We will be interested in solutions where the brane is completely wrapped along the internal directions so that from the point of view of the observer in d dimensions one has a localized, "spherically" symmetric solution. These solutions will correspond to extremal limits of charged black hole solutions. The first point to notice that if the brane wraps p of the torus dimensions then the sum in (2.20) runs over a 10-d-p dimensional lattice. If we are looking at the solutions at distances much bigger than the compactification scale, then we are allowed to replace the sum in (2.20) by an integral. This integral would naturally appear also if we average over the position of the brane on the internal torus. The net effect of the integral will be to give the function $f = 1 + c_P^{(d)}/r^{d-3}$, where r is the distance in the extended d dimensional coordinates. Note that the power of r is independent of p and is the appropriate one to be the spherically symmetric solution to Laplace's equation in d-1 spatial dimensions. So when we are in the d dimensional theory, the only way we have to tell that the black hole contains a particular type of p brane is by looking at the gauge fields that it excites. The final result is that the d dimensional solutions are given again by (2.5), (2.9) and (2.11) but now in terms of d dimensional harmonic functions.

As a particular example we will consider the black holes resulting from compactifying the oscillating strings treated in the previous section. The oscillation will be along a compact direction, and we average over them. We could think that we are looking at distances larger than the compactification radius, or that we do an average over the phase of the oscillation. It is important that this average is done at the level of the harmonic function that specifies the solution, and not on the individual components of the fields, which are non-linear in terms of the harmonic functions. This procedure produces a solution of the d dimensional supergravity theory. To be more precise, we build a periodic (9-d) dimensional array of strings by taking the harmonic function as in (2.20). For large distances ρ in the extended dimensions we can ignore the dependence on the internal dimensions and find.

$$f_f^{(d)} = 1 + \frac{c_f^{(d)} m}{\rho^{d-3}}$$
, where $c_f^{(d)} = \frac{16\pi G_N^d R_9}{\alpha'(d-3)\omega_{d-2}}$, (2.21)

and ω_d is the area of the d dimensional unit sphere and m is the total winding number. We could have taken directly $f_f^{(d)}$ as a solution of the Laplace equation in the uncompactified dimensions, but we obtained it from superimposing solutions to clarify the connection to underlying string states. As we will now show, the result of this procedure can be interpreted as a lower-dimensional extremal black hole. The general idea that ten-dimensional string solutions can be used to generate four-dimensional black holes is not new and has been explored in [46], [47], [48], [49], [42].

We now look in more detail at the d-dimensional fields generated by this compactification. Using the dimensional reduction procedure of [50], we find that the d-dimensional fields obtained from wrapping a string with oscillations are, in d-dimensional Einstein metric,

$$e^{-2\phi_d} = e^{-2\phi_{10}} \sqrt{G_{99}} = \sqrt{f_f^{(d)}(1+k^d)}$$

$$ds_E^2 = -\frac{1}{\left[f_f^{(d)}(1+k^d)\right]^{\frac{d-3}{d-2}}} dt^2 + \left[f_f^{(d)}(1+k^d)\right]^{\frac{1}{d-2}} d\vec{x}^2 . \tag{2.22}$$

This and all the other fields obtained by dimensional reduction turn out to be the type II analogs of Sen's four-dimensional black holes and their higher-dimensional

generalizations [51], [52]. The Einstein metric of the d dimensional solution has the same form if we consider any other oscillating brane completely wrapped around the internal torus since the Einstein metric is invariant under U-duality. We can check that for these black holes (2.22) the area of the horizon, which is at $\rho = 0$, is zero, so that the classical entropy is zero. It is possible to define a nonzero "classical" entropy at the "stretched" horizon which agrees up to a numerical constant with the counting of states [53].

2.4. U-duality and quantization of the charges

We will show in this section how to quantize the charges using U-duality [12]. There has been some disagreement in the literature concerning the precise quantization condition so we have decided, for completeness, to explain it in detail. Since the quantum of charge will depend on the normalization chosen for the gauge field we find it more convenient to find the "quantum of mass". This quantity has a well defined meaning since the solution is BPS and the mass is proportional to the charge and protected from quantum corrections so that it can be calculated using the weakly coupled theory. When we perform S-duality transformations we should remember that the mass measured in the Einstein metric $g_E = e^{-\phi/2}G$ (which includes a power of g) stays invariant. This is not how we normally measure masses, we normally leave a power of g in the Newton constant. The masses we are going to calculate are defined in terms of a modified Einstein metric which is $\tilde{g}_E = e^{-\frac{(\phi - \phi_{\infty})}{2}} G = g^{1/2} g_E$ which agrees with the string metric at infinity. All we are saying is that we keep the factor of g^2 in the Newton constant. Masses measured in the two metrics differ by $M_E = g^{1/4}M$, where M is the mass measured in the metric \tilde{g}_E which is the one we are going to use here. The d dimensional Newton constant is $G_N^d = G_N^{10}/V_{10-d}$ where V_{10-d} is the volume of the internal torus. We start with the minimum mass of a winding string which is (1.2)

$$M_f = \frac{R_9}{\alpha'} \ . \tag{2.23}$$

Similarly the minimum mass for momentum states is M=1/R. Now we want to calculate the mass of a D-string with unit winding using ten dimensional S-duality. We know that the Einstein metric is invariant under S-duality so that M_E is invariant, this implies

$$g'^{1/4}M' = M_E' = M_E = g^{1/4} \frac{R_9}{\alpha'}$$
 (2.24)

so that the mass of the D-string is

$$M^{1D} = \frac{R_9}{g\alpha'} \,, \tag{2.25}$$

where we took into account the change in R_9 as in (1.7). Applying T-duality transformations (1.5) along a direction perpendicular to the D-string we turn it into a D-twobrane with mass

$$M = \frac{R_9}{g\alpha'} = \frac{R_9 R_8'}{g'\alpha'^{3/2}} \ . \tag{2.26}$$

Proceeding in this fashion we find the minimum mass for any D-brane

$$M^{pD} = \frac{R_{10-p} \cdots R_9}{g\alpha'^{(p+1)/2}} \ . \tag{2.27}$$

Doing now an S-duality transformation on the D-fivebrane, as in (2.24) we get the mass of the solitonic fivebrane

$$M^{s5} = \frac{R_5 \cdots R_9}{q^2 \alpha'^3} \ . \tag{2.28}$$

Our objective is to determine the coefficients that appear in the harmonic functions specifying the solutions (2.5)(2.9)(2.11). Since we will be mainly interested in four and five dimensional black holes we are interested in the coefficient that appears in the d dimensional harmonic functions as in (2.21)(2.22). Actually from (2.22) by setting $k^{(d)} = 0$ we can read off the mass of these objects in terms of the coefficients in the harmonic function $f^{(d)}$. The mass is calculated from the behaviour of $\tilde{g}_{E\,00}$ of the metric at infinity [54]

$$\tilde{g}_{E\,00} \sim \frac{16\pi G_N^d M}{(d-2)\omega_{d-2}} \frac{1}{r^{d-3}} = \frac{d-3}{d-2} \frac{c^{(d)}}{r^{d-3}}$$
 (2.29)

where ω_n is the volume of the unit sphere S_n , $\omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$. This determines the coefficients for all excitations. We still have to express G_N in terms of g, remember that we defined g to be such that it goes to 1/g under S-duality (1.7). In order to do that, we use Dirac duality of the fundamental string and the solitonic fivebrane. The fundamental string carries electric charge under the NSNS $B_{\mu\nu}$ field while the solitonic fivebrane carries magnetic charge. It is not possible to

define globally the $B_{\mu\nu}$ field of the fivebrane (2.9). This field will contain a singularity, analogous to the Dirac string for a monopole in electrodynamics. The condition that this singularity is invisible for the fundamental strings fixes the coefficient of the fivebrane harmonic function as $c_{s5}^{(5)} = \alpha'$ [40]⁴. Comparing this value with the one resulting from (2.29) and (2.28) we find the ten dimensional Newton constant

$$G_N^{10} = 8\pi^6 g^2 \alpha'^4 \ . \tag{2.30}$$

In string theory one can independently calculate the mass of D-branes from virtual closed string exchange diagrams, in a similar fashion as one calculates the force between two charges in quantum electrodynamics. The string "miracle" [8] is that this string theory calculation of masses of D-branes agrees with the masses predicted by U-duality as above.

Now for later convenience let us quote the results, which are obtained from (2.27)(2.29)(2.30) for the D-onebrane, D-fivebranes and momentum in five extended dimensions, which we will need for the five dimensional black holes,

$$c_1^{(5)} = \frac{4G_N^5 R_9}{\pi \alpha' q} , \qquad c_5^{(5)} = g \alpha' , \qquad c_P^{(5)} = \frac{4G_N^5}{\pi R_9} .$$
 (2.31)

We will also need the corresponding coefficients for D-twobranes, D-sixbranes, solitonic fivebranes and momentum in four extended dimensions

$$c_{2}^{(4)} = \frac{4G_{N}^{4}R_{4}R_{9}}{g\alpha'^{3/2}} , \qquad c_{5}^{(4)} = \frac{\alpha'}{2R_{4}} ,$$

$$c_{6}^{(4)} = \frac{g\alpha'^{1/2}}{2} , \qquad c_{P}^{(4)} = \frac{4G_{N}^{4}}{R_{9}} ,$$

$$(2.32)$$

where we have used the value of the Newton constant (2.30).

⁴ Note that in comparing with [40] we only have to check that they used the same definition of the string tension as in (1.1).

2.5. Black hole solutions in five dimensions.

In section 2.3 we considered black holes coming from wrapping just one type of branes on the torus, or at most one type of branes with oscillations. All those black holes have zero horizon area and are singular at the horizon, since there are scalar fields diverging at the horizon. By looking at the classical solutions we see that in almost all of them the dilaton is going to plus or minus infinity. Also the physical longitudinal size goes to zero, measured in Einstein metric, for all the branes with no oscillations. Adding momentum in the internal directions does not help, we still have some diverging scalar. The three brane (2.11) has constant dilaton but suffers of this problem about the physical size.

Our goal is to construct solutions with well defined geometries at the horizon, like the ones appearing in General Relativity. The key principle is that we need to balance the scalars at the horizon. Different branes have different scalar charges, which can be interpreted as pressures or tensions in the compact direction. Note that even the dilaton falls in this category when we think of it as the size of the 11^{th} dimension in M-theory. If a scalar diverges when we approach the horizon the d dimensional character of the solution is lost. This forces us to consider more than one type of branes. We need three different types for black holes in five dimensions and four different types for black holes in d = 4, here two non-parallel p-branes count as being of different type.

2.5.1. Extremal black holes in five dimensions.

We construct the five dimensional black hole with nonzero area by superposing a number Q_5 of D-fivebranes, Q_1 D-onebranes and Kaluza-Klein momentum. We consider type IIB compactified on T^5 . We wrap a number Q_5 of D-fivebranes on T^5 . Then we wrap Q_1 D-strings along one of the directions of the torus, let us pick the 9^{th} direction. In addition we put some momentum $P_9 = N/R_9$ along the string, i.e. in the direction $\hat{9}$. The solution is given by three harmonic functions f_5 , f_1 and f_2 . We start writing the solution in terms of the ten dimensional string

metric, so that the relation to (2.11) becomes more apparent [55][14]

$$ds_{str}^{2} = f_{1}^{-\frac{1}{2}} f_{5}^{-\frac{1}{2}} \left(-dt^{2} + dx_{9}^{2} + k(dt - dx_{9})^{2} \right) +$$

$$+ f_{1}^{\frac{1}{2}} f_{5}^{\frac{1}{2}} (dx_{1}^{2} + \dots + dx_{4}^{2}) + f_{1}^{\frac{1}{2}} f_{5}^{-\frac{1}{2}} (dx_{5}^{2} + \dots + dx_{8}^{2}) ,$$

$$e^{-2(\phi_{10} - \phi_{\infty})} = f_{5} f_{1}^{-1} ,$$

$$B'_{09} = \frac{1}{2} (f_{1}^{-1} - 1) ,$$

$$H'_{ijk} = (dB')_{ijk} = \frac{1}{2} \epsilon_{ijkl} \partial_{l} f_{5} , \qquad i, j, k, l = 1, 2, 3, 4$$

$$(2.33)$$

where ϵ_{ijkl} is again the flat space epsilon tensor. The three harmonic functions are

$$f_1 = 1 + \frac{c_1^{(5)}Q_1}{x^2}$$
, $f_5 = 1 + \frac{c_5^{(5)}Q_5}{x^2}$, $k = \frac{c_P^{(5)}N}{x^2}$ (2.34)

with $x^2 = x_1^2 + \cdots + x_4^2$ and the coefficients are given in (2.31). The components of the Ramond-Ramond antisymmetric tensor field, $B'^{RR}_{\mu\nu}$, that are excited behave as gauge fields when we dimensionally reduce to five dimensions. The three independent charges arise as follows: Q_1 is a RR electric charge, coming from B'^{RR}_{09} and counts the 1D-branes. Q_5 is a magnetic charge for the three form field strength $H'^{RR}_{3} = dB'^{RR}_{2}$, which is dual in five dimensions to a gauge field, $F_2 = *_5 H'^{RR}_{3}$. Q_5 is thus an electric charge for the gauge field F_2 and it counts the number of 5D-branes. The third charge, N, corresponds to the total momentum along the branes in the direction $\hat{9}$, and it is associated to the five dimensional Kaluza-Klein gauge field coming from the G_{09} component of the metric.

Let us understand what happens to the supersymmetries. In the ten dimensional type IIB theory the supersymmetries are generated by two independent chiral spinors ϵ_R and ϵ_L ($\Gamma^{11}\epsilon_{R,L} = \epsilon_{R,L}$). The presence of the D-strings and the D-fivebranes imposes additional conditions on the surviving supersymmetries

$$\epsilon_R = \Gamma^0 \Gamma^9 \epsilon_L , \qquad \epsilon_R = \Gamma^0 \Gamma^5 \Gamma^6 \Gamma^7 \Gamma^8 \Gamma^9 \epsilon_L , \qquad (2.35)$$

where the first condition is due to the presence of the string and the second to the presence of the fivebrane (1.16). When we put momentum we break additional supersymmetries through the conditions

$$\Gamma^0 \Gamma^9 \epsilon_R = \epsilon_R , \qquad \Gamma^0 \Gamma^9 \epsilon_L = \epsilon_L.$$
(2.36)

Taken together with (2.35) we get the following decomposition of the spinor under the group $SO(1,1)\times SO(4)_E\times SO(4)_I$, which is the subgroup of Lorentz transformations that leaves the ten dimensional solution (2.33) invariant,

$$\epsilon_L = \epsilon_R = \epsilon_{SO(1,1)}^+ \epsilon_{SO(4)}^+ \epsilon_{SO(4)}^+$$
 (2.37)

The positive chirality SO(4) spinor is pseudoreal and has two independent components so that 1/8 (4 out of the original 32) supersymmetries are preserved by this configuration. The first $SO(4)_E$ corresponds to spatial rotations in 4+1 dimensions. $SO(4)_I$ corresponds to rotations in the internal directions 5, 6, 7, 8 and is broken by the compactification. The solution is supersymmetric, and has the same energy, independent on whether all the branes are sitting at the same point or not, so in principle we can separate the different constituents of the black hole. The resulting black hole will have lower entropy so this process violates the second law of thermodynamics.

Now we dimensionally reduce (2.33) to five dimensions in order to read off black hole properties. The standard method of [56] yields a five-dimensional Einstein metric, $g_E^5 = e^{-4\phi_5/3}G_{string}^5$,

$$ds_E^2 = -\frac{1}{(f_1 f_5 (1+k))^{\frac{2}{3}}} dt^2 + (f_1 f_5 (1+k))^{\frac{1}{3}} (dx_1^2 + \dots + dx_4^2) , \qquad (2.38)$$

which describes a five dimensional extremal, charged, supersymmetric black hole with nonzero horizon area. Calculating the horizon area in this metric (2.38) we get the entropy

$$S_e = \frac{A_H}{4G_N^5} = 2\pi\sqrt{NQ_1Q_5} \ . \tag{2.39}$$

In this form the entropy does not depend on any of the continuous parameters like the coupling constant or the sizes of the internal circles, etc. This "topological" character of the entropy was emphasized in [57], [58], [55]. It is also symmetric under interchange of N, Q_1, Q_5 . In fact, U duality [12], [59], [60] interchanges the three charges. To show it in a more specific fashion, let us define T_i to be the usual T-duality that inverts the compactification radius in the direction i and S the ten dimensional S duality of type IIB theory. Then a transformation that sends (N, Q_1, Q_5) to (Q_1, Q_5, N) is $U = T_8T_7T_6T_5ST_6T_9$. Note however that this transformation changes the coupling constant and the sizes of the T^5 .

The standard five-dimensional extremal Reissner-Nordström solution [54] is recovered when the charges are chosen such that

$$c_P N = c_1 Q_1 = c_5 Q_5 = r_e^2 . (2.40)$$

The crucial point is that, for this ratio of charges, the dilaton field and the internal compactification geometry are independent of position and the distinction between the ten-dimensional and five-dimensional geometries evaporates. What is at issue is not so much the charges as the different types of energy-momentum densities with which they are associated. An intuitive picture of what goes on is this [14]: a p-brane produces a dilaton field of the form $e^{-2\phi_{10}} = f_p^{\frac{p-3}{2}}$, with f_p a harmonic function [28]. A superposition of branes produces a product of such functions and one sees how 1-branes can cancel 5-branes in their effect on the dilaton. A similar thing is true for the compactification volume: For any p-brane, the string metric is such that as we get closer to the brane the volume parallel to the brane shrinks, due to the brane tension, and the volume perpendicular to it expands, due to the pressure of the electric field lines. It is easy to see how superposing 1-branes and 5-branes can stabilize the volume in the directions $\hat{6}$, $\hat{7}$, $\hat{8}$, $\hat{9}$, since they are perpendicular to the 1-brane and parallel to the 5-brane. The volume in the direction $\hat{5}$ would still seem to shrink, due to the tension of the branes. This is indeed why we put momentum along the 1-branes, to balance the tension and produce a stable radius in the 5 direction. If we balance the charges precisely (2.40) (we can always do this for large charges) the moduli scalar fields associated with the compactified dimensions are not excited at all, which is what we need to get the Reissner-Nordström black hole. Of course, if we do not balance them precisely we still have a black hole with nonzero area, as long as the three charges are nonzero.

2.5.2 Non-extremal black holes in five dimensions.

The five dimensional Reissner-Nordström black hole is a solution of the five dimensional Einstein plus Maxwell action. The metric reads [54]

$$ds^{2} = -\lambda dt^{2} + \lambda^{-1} dr^{2} + r^{2} d\Omega_{3}^{2} , \qquad (2.41)$$

$$\lambda = \left(1 - \frac{r_{+}^{2}}{r^{2}}\right) \left(1 - \frac{r_{-}^{2}}{r^{2}}\right) .$$

There is a horizon at $r = r_+$, mass and charge are given by

$$M = \frac{3\pi}{8G_N^5} (r_+^2 + r_-^2) , \qquad Q = \frac{3\pi}{4G_N^5} r_+ r_- . \qquad (2.42)$$

The extremal solution is obtained by taking $r_{+} = r_{-} \equiv r_{e}$ and reduces to (2.38), with the charges related by (2.40), after doing the coordinate transformation $r^{2} = x^{2} + r_{e}^{2}$.

Now we would like to construct the non-extremal five dimensional black holes with arbitrary values of the charges. The method is very simple [18][17]. First we start with the non-extremal Reissner-Nordström (2.41) which has some constraints on the charges (2.40), then we lift up this configuration to ten dimensions. That is done by inverting the standard dimensional reduction procedure [56], and we find the ten dimensional form of the various fields. This gives a non-extremal configuration where the charges are related by (2.40). We will apply some transformations which remove the constraints of (2.40). We start by boosting the solution along the direction of the onebranes (we called it 9). This introduces some extra momentum, so that now the RR charges are constrained but the momentum is arbitrary. The result is a solution which can be viewed as a black string in six dimensions [18]. Now we need to remove the constraint on the RR charges. To that effect we do a U duality transformation that interchanges the three different charges. More precisely we perform the transformation $U=T_8T_7T_6T_5ST_6T_9$ that sends (N,Q_1,Q_5) to (Q_1,Q_5,N) . This transformed one RR charge into momentum, so that we can boost the solution to produce a solution with arbitrary value of this RR charge. After doing all these transformations, and choosing some appropriate coordinates, the resulting ten dimensional solution is, in string metric,

$$e^{-2(\phi-\phi_{\infty})} = \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right) \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{-1}, \qquad (2.43)$$

$$ds_{str}^2 = \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{-1/2} \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right)^{-1/2} \left[-dt^2 + dx_9^2 + \frac{r_0^2}{r^2}(\cosh \sigma dt + \sinh \sigma dx_9)^2 + \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right) (dx_5^2 + \dots + dx_8^2)\right]$$

$$+ \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right)^{1/2} \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right)^{1/2} \left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2\right]. \qquad (2.44)$$

This solution is parameterized by the six independent quantities $\alpha, \gamma, \sigma, r_0, R_9 \equiv R$ and V. The last two parameters are the radius of the 9^{th} dimension and the product of the radii in the other four compact directions $V = R_5 R_6 R_7 R_8$. They appear in the charge quantization conditions, indeed the three charges are

$$Q_{1} = \frac{V}{4\pi^{2}g} \int e^{\phi_{6}} * H' = \frac{Vr_{0}^{2}}{2g} \sinh 2\alpha,$$

$$Q_{5} = \frac{1}{4\pi^{2}g} \int H' = \frac{r_{0}^{2}}{2g} \sinh 2\gamma,$$

$$N = \frac{R^{2}Vr_{0}^{2}}{2g^{2}} \sinh 2\sigma,$$
(2.45)

where * is the Hodge dual in the six dimensions $x^0, ..., x^5$. For simplicity we set from now on $\alpha' = 1$. The last charge N is related to the momentum around the S^1 by $P_9 = N/R_9$. All charges are normalized to be integers.

Reducing (2.44) to five dimensions using [56], the solution takes the remarkably simple and symmetric form:

$$ds_5^2 = -\lambda^{-2/3} \left(1 - \frac{r_0^2}{r^2} \right) dt^2 + \lambda^{1/3} \left[\left(1 - \frac{r_0^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3^2 \right] , \qquad (2.46)$$

where

$$\lambda = \left(1 + \frac{r_0^2 \sinh^2 \alpha}{r^2}\right) \left(1 + \frac{r_0^2 \sinh^2 \gamma}{r^2}\right) \left(1 + \frac{r_0^2 \sinh^2 \sigma}{r^2}\right) . \tag{2.47}$$

This is just the five-dimensional Schwarzschild metric with the time and space components rescaled by different powers of λ . The factored form of λ was known to hold for extremal solutions (2.38) [49]. It is surprising that it continues to hold even in the non-extremal case. The solution is manifestly invariant under permutations of the three boost parameters as required by U-duality. The event horizon is clearly at $r = r_0$. The coordinates we have used present the solution in a simple and symmetric form, but they do not always cover the entire spacetime. When all three charges are nonzero, the surface r = 0 is a smooth inner horizon. This is analogous to the situation in four dimensions with four charges [61]. When at least one of the charges is zero, the surface r = 0 becomes singular. Several thermodynamic quantities can be associated to this solution. They can

be computed in either the ten dimensional or five dimensional metrics and yield the same answer. For example, the ADM energy is

$$E = \frac{RVr_0^2}{2a^2}(\cosh 2\alpha + \cosh 2\gamma + \cosh 2\sigma) . \qquad (2.48)$$

The Bekenstein-Hawking entropy is

$$S = \frac{A_{10}}{4G_N^{10}} = \frac{A_5}{4G_N^5} = \frac{2\pi RV r_0^3}{g^2} \cosh \alpha \cosh \gamma \cosh \sigma.$$
 (2.49)

where A is the area of the horizon and we have used the value (2.30) for the Newton constant. The Hawking temperature is

$$T = \frac{1}{2\pi r_0 \cosh \alpha \cosh \gamma \cosh \sigma}.$$
 (2.50)

In ten dimensions, the black hole is characterized by pressures which describe how the energy changes for isentropic variations in R and V. In five dimensions, these are 'charges' associated with the two scalar fields coming from the components G_{99} and G_{55} in (2.44), which can be interpreted as the pressures in the directions 9 and 5 respectively, and they read

$$P_{1} = \frac{RVr_{0}^{2}}{2g^{2}} \left[\cosh 2\sigma - \frac{1}{2} (\cosh 2\alpha + \cosh 2\gamma) \right] ,$$

$$P_{2} = \frac{RVr_{0}^{2}}{2g^{2}} (\cosh 2\alpha - \cosh 2\gamma) .$$

$$(2.51)$$

The extremal limit corresponds to the limit $r_0 \to 0$ with at least one of the boost parameters $\alpha, \gamma, \sigma \to \pm \infty$ keeping R, V and the associated charges (2.45) fixed. If we keep all three charges nonzero in this limit, one obtains

$$E_{ext} = \frac{R|Q_{1}|}{g} + \frac{RV|Q_{5}|}{g} + \frac{|N|}{R} ,$$

$$S_{ext} = 2\pi \sqrt{|Q_{1}Q_{5}N|} ,$$

$$T_{ext} = 0 ,$$

$$P_{1ext} = \frac{|N|}{R} - \frac{R|Q_{1}|}{2g} - \frac{RV|Q_{5}|}{2g} ,$$

$$P_{2ext} = \frac{R|Q_{1}|}{g} - \frac{RV|Q_{5}|}{g} .$$
(2.52)

The first equation is the saturated Bogomolnyi bound for this theory.

We now show that there is a formal sense in which the entire family of solutions discussed above can be viewed as "built up" of branes, anti-branes, and momentum. The extremal limits with only one type of excitation are obtained by letting r_0 go to zero and taking a boost parameter go to infinity keeping only one charge fixed. These extremal metrics represent a D-onebrane wrapping the S^1 , or a D-fivebrane wrapping the T^5 , or the momentum modes around the S^1 . From (2.48) and (2.51) we see that a single onebrane or anti-onebrane has mass and pressures

$$M = \frac{R}{g}, \qquad P_1 = -\frac{R}{2g}, \qquad P_2 = \frac{R}{g}.$$
 (2.53)

Of course a one-brane has $Q_1 = 1$, while an anti-one-brane has $Q_1 = -1$. A single five-brane or anti-five-brane has

$$M = \frac{RV}{g}, \qquad P_1 = -\frac{RV}{2g}, \qquad P_2 = -\frac{RV}{g}.$$
 (2.54)

For left- or right-moving momentum

$$M = \frac{1}{R}, \qquad P_1 = \frac{1}{R}, \qquad P_2 = 0$$
 (2.55)

Given (2.53) - (2.55), and the relations (2.45), (2.48), and (2.51), it is possible to trade the six parameters of the general solution for the six quantities $(N_1, N_{\bar{1}}, N_5, N_{\bar{5}}, N_R, N_L)$ which are the "numbers" of onebranes, anti-onebranes, fivebranes, anti-fivebranes, right-moving momentum and left-moving momentum respectively. This is accomplished by equating the total mass, pressures and charges of the black hole with those of a collection of $(N_1, N_{\bar{1}}, N_5, N_{\bar{5}}, N_R, N_L)$ non-interacting "constituent" branes, antibranes and momentum. By non-interacting we mean that the masses and pressures are simply the sums of the masses and pressures of the constituents. The resulting

expression for the N's are

$$N_{1} = \frac{Vr_{0}^{2}}{4g}e^{2\alpha},$$

$$N_{\bar{1}} = \frac{Vr_{0}^{2}}{4g}e^{-2\alpha},$$

$$N_{5} = \frac{r_{0}^{2}}{4g}e^{2\gamma},$$

$$N_{\bar{5}} = \frac{r_{0}^{2}}{4g}e^{-2\gamma},$$

$$N_{R} = \frac{r_{0}^{2}R^{2}V}{4g^{2}}e^{2\sigma},$$

$$N_{L} = \frac{r_{0}^{2}R^{2}V}{4g^{2}}e^{-2\sigma}.$$
(2.56)

(2.56) is the *definition* of the N's, but we will refer to them as the numbers of branes, antibranes and momentum because (as will be seen) they reduce to those numbers in certain limits where these concepts are well defined.

In terms of the numbers (2.56), the charges are simply $Q_1 = N_1 - N_{\bar{1}}$, $Q_5 = N_5 - N_{\bar{5}}$, $N = N_R - N_L$, the total energy is

$$E = \frac{R}{g}(N_1 + N_{\bar{1}}) + \frac{RV}{g}(N_5 + N_{\bar{5}}) + \frac{1}{R}(N_R + N_L) , \qquad (2.57)$$

and the volume and radius are

$$V = \left(\frac{N_1 N_{\bar{1}}}{N_5 N_{\bar{5}}}\right)^{1/2} , \qquad (2.58)$$

$$R = \left(\frac{g^2 N_R N_L}{N_1 N_{\bar{1}}}\right)^{1/4} . {(2.59)}$$

From (2.52) we see that the extremal solutions correspond to including either branes or anti-branes, but not both. Notice that for the general Reissner-Nordstrom solutions ($\alpha = \gamma = \sigma$) the contribution to the total energy from onebranes, fivebranes, and momentum are all equal:

$$\frac{R}{g}(N_1 + N_{\bar{1}}) = \frac{RV}{g}(N_5 + N_{\bar{5}}) = \frac{1}{R}(N_R + N_L). \tag{2.60}$$

The actual number of branes of each type depends on R and V and can be very different.

Of course there seems to be no reason for neglecting interactions between collections of branes and momentum modes composing a highly non-extremal black hole at strong or intermediate coupling. Hence the definitions (2.56) would seem to be inappropriate for describing a generic black hole. However, the utility of these definitions can be seen when we reexpress the black hole entropy (2.49) in terms of the N's. It takes the remarkably simple form

$$S = 2\pi(\sqrt{N_1} + \sqrt{N_{\bar{1}}})(\sqrt{N_5} + \sqrt{N_{\bar{5}}})(\sqrt{N_L} + \sqrt{N_R}). \qquad (2.61)$$

In the next chapter we will compute this formula in string theory in some special limits. An interesting property of this entropy formula (2.61) is that if one takes the brane-antibrane numbers to be free variables and then one maximases the entropy (2.61) subject to the constraints that the charges and the total energy (2.57) are fixed, then one gets the relations (2.59)(2.58) and hence (2.56) for the brane-antibrane numbers. So, in this very specific sense, the black hole solution represents a system of branes and antibranes in thermodynamic equilibrium.

A puzzling feature of (2.61) is that it only involves one branes, fivebranes, and momentum. This is understandable for extremal solutions with these charges, but when one moves away from extremality, one might expect pairs of threebranes and anti-threebranes or fundamental string winding modes to contribute to the entropy. To understand the roles of these other objects, one should start with the full Type II string theory compactified on T^5 . The low energy limit of this theory is N=8 supergravity in five dimensions (we measure N in four d terms, i.e. by the amount of supersymmetry that it has reduced to d=4). This theory has 27 gauge fields, 42 scalars and a global E_6 symmetry. Since only the scalar fields which couple to the gauge fields are nontrivial in a black hole background, we expect the general solution to be characterized by 27 scalars in addition to the 27 charges. One can interpret the 27 scalar parameters as 26 scalars plus the ADM energy. Each charge corresponds to a type of soliton or string. Thus we expect the solution to again be characterized by the number of solitons and anti-solitons. For an extremal black hole, the entropy can be written in the E_6 invariant form |62|, |63|

$$S = 2\pi |T_{ABC}V^A V^B V^C|^{1/2} , (2.62)$$

where V^A is the 27 dimensional charge vector and T_{ABC} is a symmetric cubic invariant in E_6 . For the non-extremal black holes, the above argument suggests that one can introduce two vectors V_i^A i=1,2 which represent the number of solitons and anti-solitons. Although we have not done the calculation, the general black hole entropy might take the E_6 invariant form

$$S = 2\pi \sum_{i,j,k} |T_{ABC}V_i^A V_j^B V_k^C|^{1/2} , \qquad (2.63)$$

where $i, j, k = 1, 2, V_1^A$ indicates the number of charges and V_2^A the number of anticharges, each is a vector in the 27 of E_6 . The entropy of non-extremal black holes can be represented in terms of charges and anti-charges in many different (equivalent) ways which are related by E_6 transformations. Now we see that our choice of D-onebranes, D-fivebranes and momentum was like a choice of basis and other configurations are related by $E_6(Z)$ U-duality transformations.

One can similarly construct rotating black holes in five dimensions. The spatial rotation group is $SO(4)_E \sim SU(2)_R \times SU(2)_L$. We we can view the angular momentum as a 4×4 antisymmetric matrix. We can choose a basis such that it reduces to 2×2 blocks, each block corresponds to a rotation on a plane and there are two orthogonal planes. The angular momentum is characterized by the angular momentum eigenvalues J_1, J_2 on these two planes. Also the angular momenta are characterized by the U(1) charges $F_R/2, F_L/2$ which are two eigenvalues of the SU(2)'s (we define $F_{R,L}$ to have integer eigenvalues). We have

$$J_1 = \frac{1}{2}(F_R + F_L), \qquad J_2 = \frac{1}{2}(F_R - F_L).$$
 (2.64)

The solution with angular momentum can be found in [15], [16], [64], we will be just interested in the entropy of that solution in the extremal limit, for which the mass is the minimum consistent with a given angular momentum and charges. The entropy is then

$$S_{ext} = 2\pi \sqrt{NQ_1Q_5 - J_1J_2} \ . \tag{2.65}$$

For $J_1 = J_2$ the solution is also BPS [15].

2.6. Black hole solutions in four dimensions

Now we turn to the more realistic case of four dimensional black holes. It is still not totally realistic since the compactification we will consider is on T^6 which is not the one that describes our four dimensional world. The supergravity theory however contains black hole solutions which are exactly those of General Relativity. The difference between the two theories is that the N=8 supergravity theory one obtains by compactifying on T^6 has many more gauge fields (28 of them) and massless scalars (70 of them). Black hole solutions are characterized by 56 charges, 28 electric and 28 magnetic. One hopes that the general features of black hole physics will not depend too much on the content of the theory, as long as it includes gravity and one is studying black holes with the same metric as the ones appearing in General Relativity. In fact some of the solutions we study are also solutions in general relativity.

2.6.1 Extremal black holes in four dimensions

Let us start with the extremal black holes [19], [20]. Taking the configuration of 1D-branes, 5D-branes and momentum that we had in d=5 and putting it on T^6 we obtain a black hole solution that preserves 1/8 of the supersymmetries. In order to put it on T^6 one has to form a lattice of the extremal five dimensional black holes (2.33) and define new harmonic functions as in (2.20). This makes all harmonic functions to depend on 1/r where now r is the spatial distance in 1+3 dimensions. The unbroken supersymmetries are given by (2.37). Now we do a T duality transformation (along the direction 4) to the IIA theory and we get a system of 2D-branes, 6D-branes and momentum. In addition we flip the chirality of the ten dimensional spinor ϵ_R . We will have $\epsilon_R^{IIA} = \Gamma^4 \epsilon_R^{IIB}$, so that the chirality that is flipped in ϵ_R is that of the "external" $SO(4)_E$. Of course, only the SO(3) subgroup corresponding to spatial rotations in the directions 1, 2, 3 is a symmetry of the solution. However, this black hole has zero area and has a singular geometry at the horizon. The reason is that some of the scalar fields are unbalanced, for example, we can see from (2.11) that the dilaton field will not go to a constant as we approach the horizon, $e^{-2\phi} = f_2^{-1/2} f_6^{3/2}$. It is interesting that one can put an additional type of charge without breaking any additional supersymmetry. This charge has to be a solitonic fivebrane, it is the only one allowed by supersymmetry that is not just a U-duality tranformation of the others.

This, in a sense, is analogous to putting left moving oscillations on a macroscopic heterotic string [42] which does not break any additional supersymmetry. It also has the virtue of balancing all the scalars, for example the dilaton now behaves as $e^{-2\phi} = f_2^{-1/2} f_6^{3/2} f_{s5}^{-1}$. In order to be more precise let us say that our torus is $T^6 = T^4 \times S_1' \times S_1$ and we have the 6D-branes wrapping all T^6 , the 2D-branes wrap $S_1' \times S_1$ (directions 4,9), the solitonic fivebranes wrap $T^4 \times S_1$ (directions 5,6,7,8,9) and the momentum is flowing along S_1 (direction 9). Notice that the momentum flows parallel to the fivebranes and the two D-branes.

We can see from (1.16),(2.19) and (2.10) that the fivebrane does not break any additional supersymmetry. The final configuration still preserves 1/8 of the original supersymmetries. Decomposing the surviving spinor in terms of $SO(1,1)\times SO(4)_E\times SO(4)_I$ we find

$$\Gamma^4 \epsilon_R = \epsilon_L = \epsilon_{SO(1,1)}^+ \epsilon_{SO(4)}^+ \epsilon_{SO(4)}^+ . \tag{2.66}$$

The extremal four dimensional black hole, constructed this way, written in ten dimensional string metric, has the form [55]

$$ds_{str}^{2} = f_{2}^{-\frac{1}{2}} f_{6}^{-\frac{1}{2}} \left(-dt^{2} + dx_{9}^{2} + k(dt - dx_{9})^{2} \right) + f_{s5} f_{2}^{-\frac{1}{2}} f_{6}^{-\frac{1}{2}} dx_{4}^{2} + f_{2}^{\frac{1}{2}} f_{6}^{-\frac{1}{2}} \left(dx_{5}^{2} + \dots + dx_{8}^{2} \right) + f_{s5} f_{2}^{\frac{1}{2}} f_{6}^{\frac{1}{2}} \left(dx_{1}^{2} + \dots + dx_{3}^{2} \right) ,$$

$$e^{-2(\phi_{10} - \phi_{\infty})} = f_{s5}^{-1} f_{2}^{-\frac{1}{2}} f_{6}^{\frac{3}{2}} ,$$

$$H_{ij4} = \frac{1}{2} \epsilon_{ijk} \partial_{k} f_{s5} \qquad i, j, k = 1, 2, 3 ,$$

$$C_{049} = \frac{1}{2} (f_{2}^{-1} - 1) ,$$

$$(dA)_{ij} = \frac{1}{2} \epsilon_{ijk} \partial_{k} f_{6} \qquad i, j, k = 1, 2, 3 ,$$

$$(2.67)$$

where ϵ_{ijk} is the flat space epsilon tensor. The harmonic functions are

$$f_2 = 1 + \frac{c_2^{(4)}Q_2}{r}, \quad f_5 = 1 + \frac{c_{s5}^{(4)}Q_5}{r}, \quad f_6 = 1 + \frac{c_6^{(4)}Q_6}{r}, \quad k = \frac{c_P^{(4)}N}{r}, \quad (2.68)$$

where the coefficients $c^{(4)}$'s are given in (2.32) and the charges Q_2, Q_5, Q_6, N are integers. Calculating the entropy we find

$$S = \frac{A_4}{4G_N^4} = 2\pi\sqrt{Q_2Q_5Q_6N} , \qquad (2.69)$$

which is, as (2.39), U-dual and independent of the moduli.

2.6.2 Non-extremal black holes in four dimensions

In a similar way as we did for five dimensions one can construct the non-extremal four dimensional solution. After doing the dimensional reduction to four dimensions the Einstein metric reads [61]⁵

$$ds^{2} = -\chi^{-1/2}(r) \left(1 - \frac{r_{0}}{r}\right) dt^{2} + \chi^{1/2}(r) \left[\left(1 - \frac{r_{0}}{r}\right)^{-1} dr^{2} + r^{2} (d\theta^{2} + \sin^{2}\theta d\phi^{2})\right] ,$$

$$\chi(r) = \left(1 + \frac{r_{0} \sinh^{2}\alpha_{2}}{r}\right) \left(1 + \frac{r_{0} \sinh^{2}\alpha_{5}}{r}\right) \left(1 + \frac{r_{0} \sinh^{2}\alpha_{6}}{r}\right) \left(1 + \frac{r_{0} \sinh^{2}\alpha_{p}}{r}\right) .$$

$$(2.70)$$

This metric is parameterized by the five independent quantities α_2 , α_5 , α_6 , α_p and r_0 . The event horizon lies at $r=r_0$. The special case $\alpha_2=\alpha_5=\alpha_6=\alpha_p$ corresponds to the Reissner-Nordström metric (2.1), so that we see that, as we promised, the General Relativity solution is among the cases studied. The overall solution contains three additional parameters which are related to the asymptotic values of the three scalars. From the ten-dimensional viewpoint, these are the product of the radii of T^4 , $V = R_5 R_6 R_7 R_8$, and the radii of S^1 and $S^{\prime 1}$, R_9 and R_4 , and they appear in the quantization condition for the charges. There are, in addition, U(1) gauge fields excited, corresponding to the four physical charges. One is the Kaluza Klein gauge field coming from the component G_{09} of the metric, which carries the momentum charge, N. Then we have a RR gauge field coming from the component C_{049} of the three form RR potential which carries the 2D-brane charge, Q_2 . The 6D brane charge, Q_6 , appears as magnetic charge for the one form RR potential A_{μ} , and finally the fivebrane charge, Q_5 also appears as magnetic charge for the gauge field coming from the NS antisymmetric tensor with one index along the direction 4, $B_{\mu 4}$.

⁵ We use the classical solution from [61] but with our quantization condition for the charges derived in section (2.4).

The physical charges are expressed in terms of these quantities as [21]

$$\begin{split} Q_2 &= \frac{r_0 V}{g} \sinh 2\alpha_2 \ , \\ Q_5 &= r_0 R_4 \sinh 2\alpha_5 \ , \\ Q_6 &= \frac{r_0}{g} \sinh 2\alpha_6 \ , \\ N &= \frac{r_0 V R_9^2 R_4}{g^2} \sinh 2\alpha_p \ , \end{split} \tag{2.71}$$

where we have again set $\alpha' = 1$ and from (2.30) the four-dimensional Newton constant becomes $G_N^4 = g^2/(8VR_4R_9)$.

The ADM mass of the solution is

$$M = \frac{r_0 V R_4 R_9}{q^2} \left(\cosh 2\alpha_2 + \cosh 2\alpha_5 + \cosh 2\alpha_6 + \cosh 2\alpha_p\right) \tag{2.72}$$

and the Bekenstein-Hawking entropy is

$$S = \frac{A_4}{4G_N^4} = \frac{8\pi r_0^2 V R_4 R_9}{g^2} \cosh \alpha_2 \cosh \alpha_5 \cosh \alpha_6 \cosh \alpha_p . \qquad (2.73)$$

There are three nontrivial scalar fields present in the solution and associated with these scalar fields are three pressures (scalar charges)

$$P_{1} = \frac{r_{0}VR_{4}R_{9}}{g^{2}}(\cosh 2\alpha_{2} + \cosh 2\alpha_{6} - \cosh 2\alpha_{5} - \cosh 2\alpha_{p}) ,$$

$$P_{2} = \frac{r_{0}VR_{4}R_{9}}{g^{2}}(\cosh 2\alpha_{2} - \cosh 2\alpha_{6}) ,$$

$$P_{3} = \frac{r_{0}VR_{4}R_{9}}{g^{2}}(\cosh 2\alpha_{5} - \cosh 2\alpha_{p}) .$$
(2.74)

As we did for the five dimensional black hole in section (2.5) we calculate the values for the mass and scalar charges of each type of brane or string. This can be calculated from the solution we have presented by taking the four extremal limits: $r_0 \to 0$, $\alpha_i \to \pm \infty$ with Q_i and α_j $(j \neq i)$ fixed. We find that D-twobranes have mass and pressures [21]

$$M = P_1 = P_2 = \frac{R_4 R_9}{g} , \qquad P_3 = 0 ,$$
 (2.75)

while for the D-sixbranes we have

$$M = P_1 = -P_2 = \frac{VR_4R_9}{g} , \qquad P_3 = 0 .$$
 (2.76)

For the solitonic fivebrane we have

$$M = -P_1 = P_3 = \frac{VR_9}{g^2} , P_2 = 0 , (2.77)$$

and for the momentum we find

$$M = -P_1 = -P_3 = \frac{1}{R_9}$$
, $P_2 = 0$. (2.78)

Using these relations plus the charges (2.71) we trade in the eight parameters of the solution for the eight quantities $(N_R, N_L, N_2, N_{\bar{2}}, N_5, N_{\bar{5}}, N_6, N_{\bar{6}})$ which are the numbers of right(left)-moving momentum modes, twobranes, anti-twobranes, fivebranes, anti-fivebranes, sixbranes and anti-sixbranes. We do this by matching the mass (2.72), pressures (2.74), and gauge charges (2.71) with those of a collection of noninteracting branes. This leads to

$$N_{R} = \frac{r_{0}VR_{9}^{2}R_{4}}{2g^{2}}e^{2\alpha_{p}} , \qquad N_{L} = \frac{r_{0}VR_{9}^{2}R_{4}}{2g^{2}}e^{-2\alpha_{p}} ,$$

$$N_{2} = \frac{r_{0}V}{2g}e^{2\alpha_{2}} , \qquad N_{\bar{2}} = \frac{r_{0}V}{2g}e^{-2\alpha_{2}} ,$$

$$N_{5} = \frac{r_{0}R_{4}}{2}e^{2\alpha_{5}} , \qquad N_{\bar{5}} = \frac{r_{0}R_{4}}{2}e^{-2\alpha_{5}} ,$$

$$N_{6} = \frac{r_{0}}{2g}e^{2\alpha_{6}} , \qquad N_{\bar{6}} = \frac{r_{0}}{2g}e^{-2\alpha_{6}} .$$

$$(2.79)$$

In terms of the brane numbers, the ADM mass is reexpressed as

$$M = \frac{1}{R_1}(N_R + N_L) + \frac{R_9 R_4}{g}(N_2 + N_{\bar{2}}) + \frac{V R_9}{g^2}(N_5 + N_{\bar{5}}) + \frac{V R_9 R_4}{g}(N_6 + N_{\bar{6}}) ,$$
(2.80)

the gauge charges are simply differences of the brane numbers, and the other parameters are

$$V = \sqrt{\frac{N_2 N_{\bar{2}}}{N_6 N_{\bar{6}}}} , \qquad R_4 = \sqrt{\frac{N_5 N_{\bar{5}}}{g^2 N_6 N_{\bar{6}}}} , \qquad R_9^2 R_4 = \sqrt{\frac{g^2 N_R N_L}{N_2 N_{\bar{2}}}} . \tag{2.81}$$

The entropy (2.73) then takes the surprisingly simple form

$$S = 2\pi(\sqrt{N_R} + \sqrt{N_L})(\sqrt{N_2} + \sqrt{N_{\bar{2}}})(\sqrt{N_5} + \sqrt{N_{\bar{5}}})(\sqrt{N_6} + \sqrt{N_{\bar{6}}}) . \quad (2.82)$$

This is the analog of (2.61) for four-dimensional black holes. When one term in each factor vanishes, the black hole is extremal (2.69). Although we cannot derive the general formula from counting string states, we will do so in certain limits corresponding to near-extremal black holes.

Since the full theory should be E_7 invariant we should be able to write the general entropy formula in an invariant way. If we denote by V_1^A the 56dimensional vector giving the number of solitons and by V_2^A the number of antisolitons, the formula for the entropy takes the form [65][66]

$$S = 2\pi \sum_{i,j,k,l} \sqrt{T_{ABCD} V_i^A V_j^B V_k^C V_l^D} , \qquad (2.83)$$

where T_{ABCD} is the E_7 quartic invariant.

We now consider adding rotation to the black holes discussed above. Since the rotation dependent terms in the solution fall off faster at infinity than the charges, the definition of the brane numbers (2.79) is unchanged. If we take nearly extremal black holes with $N_{\bar{2}} \sim N_{\bar{5}} \sim N_{\bar{6}} \sim 0$, and R_1 large, the Bekenstein-Hawking entropy takes the form [64]

$$S = 2\pi \left(\sqrt{N_R N_2 N_5 N_6} + \sqrt{N_L N_2 N_5 N_6 - J^2} \right) . \tag{2.84}$$

where J is the angular momentum of the black hole.

3. D-BRANE DESCRIPTION OF BLACK HOLES

In this chapter we will describe some properties of the black holes that we studied in chap. 2 in terms of the D-brane string solitons described in chap. 1. The black holes we study have non-zero horizon area and contain, as special cases, the charged black holes of general relativity. The description that will emerge depends on some detailed properties of the string theory D-brane solitons which are obvious from just the supergravity theory. The key property that will be used is that when many D-branes sit at the same point there is a large number of massless states coming from open strings with ends attached to different branes [9].

We will start with the five dimensional black hole which is simpler because it involves only three different kinds of charge. Then we treat the four dimensional case. In both cases we will start with the BPS extremal black holes where the calculation of the entropy can be justified on the basis of supersymmetry and then we will explore the near-extremal limits.

4. Extremal Five dimensional black holes.

We consider the type IIB theory on $T^5 = T^4 \times S^1$. We consider a configuration of Q_5 D-fivebranes wrapping the whole T^5 , Q_1 D-strings wrapping the S^1 and momentum N/R_9 along the S^1 , choosing this S^1 to be in the direction $\hat{9}$. All charges N, Q_1, Q_5 are integers (2.38).

Since extremal D-branes are boost invariant along the directions parallel to the branes they cannot carry momentum along S_1 by just moving rigidly. Our first task will be to identify the D-brane excitations that carry the momentum. In the discussion of the oscillating fundamental string in sec. (2.2) the momentum was carried by the oscillations. However, just oscillations of the branes do not have enough entropy to match the classical result. As we saw in the D-brane section, oscillations of the branes are described by massless open strings with both ends attached to the same brane. There are many types of open strings to consider: those that go from one 1-brane to another 1-brane, which we denote as (1,1) strings, as well as the corresponding (5,5), (1,5) and (5,1) strings (the last two being different because the strings are oriented). We want to excite these strings

and put some momentum on them. As it was shown for oscillating D-branes in section 1.3 exciting some of them makes others massive so we have to see what is the way to excite the stings so that a maximum number remains massless, since this configuration will have the highest entropy. Let us work out the properties of (1,5) and (5,1) strings. The string is described by the action (1.1) where two of the coordinates have Neumann-Neumann boundary conditions (X^0, X^9) , four coordinates have Dirichlet-Dirichlet boundary conditions (X^1,X^2,X^3,X^4) and the other four have Neumann-Dirichlet conditions (X^5, X^6, X^7, X^8) . The vacuum energy of the worldsheet bosons is E = 4(-1/24 + 1/48). Consider the NS sector for the worldsheet fermions, the 4 that are in the ND directions will end up having R-type quantization conditions. The net fermionic vacuum energy is E = 4(1/24 - 1/48) and exactly cancels the bosonic one. This vacuum is a spinor under $SO(4)_I$, is acted on by $\Gamma^5, \Gamma^6, \Gamma^7, \Gamma^8$, and obeys the GSO chirality condition $\Gamma^5\Gamma^6\Gamma^7\Gamma^8\chi=\chi$. What remains is a two dimensional representation. There are two possible orientations and they can be attached to any of the different branes of each type. This gives a total of $4Q_1Q_5$ different possible states for these strings. Now consider the Ramond sector, the four internal fermions transverse to the string will have NS type boundary conditions. The vacuum again has zero energy and is an SO(1,5) spinor and a spacetime fermion. Again the GSO condition implies that only the positive chirality representation of SO(1,5) survives. When it is also left moving only the 2^+_+ under $SO(1,1)\times SO(4)_E$ survives. This gives the same number of states as for the bosons. Note that the fermionic (1,5) (or (5,1)) strings carry angular momentum under the spatial rotation group $SO(4)_E$ but the bosonic (1,5) (or (5,1)) strings do not carry angular momentum.

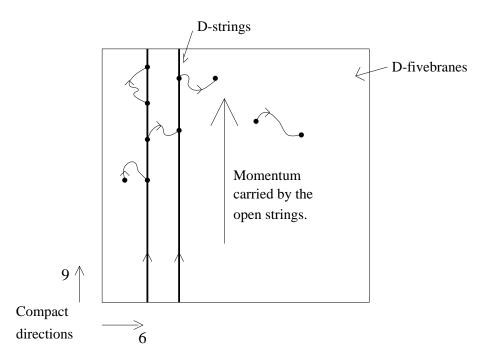


FIGURE 5: Configuration of intersecting D-branes. We show two of the internal dimensions and several types of open strings. The open strings going between 1 and 5 branes are the most relevant for the black holes that we analyze.

In order to be more definite about the number of massless degrees of freedom it is necessary to know something about the interactions of these open strings. It was shown in [38] that this interaction Lagrangian is determined by supersymmetry and gauge symmetry. The 1+1 dimensional field theory is a (4,4)supersymmetric theory, which has the same amount of supersymmetry as N=2in four dimensions or N=1 in six dimensions. For simplicity we will discuss just the bosonic part of the Lagrangian. We saw in chapter 1 that the Lagrangian for the (1,1) or (5,5) strings is the dimensional reduction of the d=10 Yang Mills Lagrangian and a way to see this was to do T-duality transformations so that the p-branes became 9-branes. In the same spirit we do T-duality transformations that map our 1D and 5D branes into 5D and 9D branes. Now we have a six dimensional theory on the fivebrane. Let us denote by $\alpha, \beta = 0, 1, 2, 3, 4, 9$ the indices along the fivebrane, by I, J = 5, ...8 the indices perpedicular to the fivebranes and by $\mu, \nu = 0, ..., 9$ the full ten dimensional indices. For simplicity we will concentrate only on the bosonic part of the Lagrangian, and we will give only the bosonic part of the supermultiplets. We have N=1 supersymmetry in

six dimensions. There are two possible supermultiplets, the vector multiplet and the hypermultiplet. The vector contains a six dimensional vector field A_{α} and its spin 1/2 superpartner. The hypermultiplet contains four scalar fields, and the spin 1/2 superpartners. The ten dimensional vector A_{μ} describing (5,5) strings decomposes into a six dimensional vector A_{α} plus a hypermultiplet containing the four scalars A_I representing the transverse motion of the fivebranes. Both of these fields are in the adjoint of the $U(Q_1)$ gauge group⁶. The YM Lagrangian (1.11) has N=2 supersymmetry in six dimensions and it can, of course, be thought of as an N=1 Lagrangian. The (5,9) and (9,5) strings together form a hypermultiplet with fields which transform as the product of the fundamental of $\mathrm{U}(Q_1)$ and the anti-fundamental of $U(Q_5)$ and their complex conjugate. These are the fields χ_a^B discussed above, χ was a 2⁺ spinor under the SO(4)_I and a and B represent $U(Q_1) \times \bar{U}(Q_5)$ Chan Paton factors. The other two components of this hypermultiplet are their complex conjugate $\chi_B^{\dagger a}$. The interaction Lagrangian is determined largely by supersymmetry. The only allowed coupling between vector and hypermultiplets is the gauge coupling. The hypermultiplets could, in principle, have a metric in the kinetic term but to lowest order in the string coupling this metric is flat. Supersymmetry requires, however, some potential for the hypermultiplets, this is given by the so called "D-terms" (no relation to "D"-branes). They arise when we have gauge symmetries, there are three D-terms for each gauge generator. One way to think about them is through the $SO(4)_I$ language as self dual antisymmetric tensors $D_{IJ} = \frac{1}{2} \epsilon_{IJKL} D_{KL}$, such a tensor has three independent components. The potential is given by $V \sim \sum_a D_{12}^{a2} + D_{13}^{a2} + D_{14}^{a2}$. D-terms have the structure $D^a \sim \phi^{\dagger} T^a \phi'$ where ϕ are components of the hypermultiplets and T^a are the gauge group generators. As an example of D-terms let us consider the pure YM Lagrangian dimensionally reduced to six dimensions. The potential term for the hypermultiplets comes simply form the commutator terms in the YM Lagrangian

$$V = TrF_{IJ}F^{IJ} = \sum_{IJ} Tr[A_I, A_J]^2 = \sum_a D_{12}^{a2} + D_{13}^{a2} + D_{14}^{a2}$$
 (4.1)

⁶ The index of Q_1 reminds us that in the aplication of interest these fivebranes are D-strings, hopefully this will not cause confusion.

where D_{IJ}^a is defined through

$$D_{IJ}^{a}T^{a} = [A_{I}, A_{J}] + \frac{1}{2}\epsilon_{IJKL}[A_{K}, A_{L}]$$
(4.2)

In checking (4.1) with (4.2) one should use the Jacobi identity for the commutators. We would have obtained the same result if we had taken the antiself dual part of $[A_I, A_J]$ in (4.2), this a reflection that the YM Lagrangian (1.11) really has N=2 supersymmetry as a six dimensional theory.

Now let us consider the D-terms for the full theory, they have the form

$$D_{IJ}^{a} = f_{bc}^{a} (A_{I}^{b} A_{J}^{c} + \frac{1}{2} \epsilon_{IJKL} A_{K}^{b} A_{L}^{c}) + \chi^{\dagger} T^{a} \Gamma_{IJ} \chi$$

$$V = \sum_{aIJ} D_{IJ}^{a2}$$
(4.3)

where the index a runs over the gauge group generators, first of $U(Q_1)$ and then of $U(Q_5)$. Note that the first term involves (5,5) or (9,9) fields depending on which generator we consider. The second term is automatically self dual due to the chirality condition of the spinor χ . We have not been very careful with the precise numerical normalization of these two terms because we will not need it in what follows. The full action has the form, up to numerical normalizations,

$$S = \frac{1}{g} \int Tr(F_{\alpha\beta}F^{\alpha\beta}) + Tr(F'_{\alpha\beta}F'^{\alpha\beta}) + Tr[(\partial_{\alpha}A_{I} + [A_{\alpha}, A_{I}])^{2}] + + Tr[(\partial_{\alpha}A'_{I} + [A'_{\alpha}, A'_{I}])^{2}] + |(\partial_{\alpha} + A^{a}_{\alpha}T^{a} + A'^{a}_{\alpha}T^{a})\chi|^{2} + \sum_{aLI} D^{a2}_{IJ},$$
(4.4)

where we denoted by A_{α} , A'_{α} the gauge fields of $U(Q_1)$ and $U(Q_5)$ respectively. The index a in the D-terms runs over both gauge groups.

Now that we understand better the Lagrangian let us do a T-duality transformation back to the 1D and 5D branes. Now we have an N=4 theory in two dimensions, the supermultiplets are just the dimensional reduction of the six dimensional ones and therefore recieve the same name. Some of the components of the six dimensional vector multiplet become scalars and they represent the motion of the D-string or the D-fivebrane transverse to the fivebrane. The motion of the D-string on the fivebrane, as well as the fivebrane gauge fields transverse to the string are hypermultiplets in this language.

The BPS states that we are considering have only left moving excitations. One can view these states classically as traveling waves propagating along S_1 . In order to have traveling wave solutions the mass terms have to vanish exactly. If we set all the fields in the Lagrangian to zero then we can have traveling waves for any field. However once we have a wave for one field the potential terms in (4.4) in the case of hypermultiplets or gauge couplings in the case of vector multiplets imply that there will be mass terms for other waves as in (1.13). One should find which is the way to excite the system in such a way that a maximum number of particles remains massless. For determining the mass terms, as in (1.13), it is the same to consider configurations that are $u = x^9 - t$ dependent or not. The problem is analogous to finding the sector of the moduli space of vacua with the largest number of massless particles. If we give some expectation value to the scalars coming from the six dimensional vector fields, then we see that we are effectively separating the strings and fivebranes and we expect a small number of massless particles (proportional to $Q_1 + Q_5$). Indeed there are mass terms for the hypermultiplets in the fundamental due to the gauge couplings $\chi^{\dagger} A_{\alpha} A^{\alpha} \chi$ (4.4). This mass term also implies that if the (1,5) strings condense then there is a mass term for the transverse motion. In fact a configuration with a large number of massless particles is achieved by exciting the hypermultiplets, both the ones in the fundamental and the ones in the adjoint. This gives mass to the scalars describing the transverse motion of the branes which means that we have a bound state. The total number of components of the hypermultiplets is $4Q_1^2+4Q_5^2+4Q_1Q_5$. In order to preserve supersymmetry we must set the potential to zero, which also minimizes the energy. This implies that the D-terms (4.3)should vanish, imposing $3Q_1^2 + 3Q_5^2$ constraints. In addition we should identify gauge equivalent configurations. The number of possible gauge transformations is $Q_1^2 + Q_5^2$. This implies that the remaining number of massless degrees of freedom is $4Q_1Q_5$. The counting, as we have done it here, is correct for large charges up to possible subleading corrections. One can think that what we did was to determine the dimension of the classical moduli space of vacua of this theory and then considered oscillations around a given vacuum.

Let us remark for later use that in this picture the momentum is carried by the hypermultiplets. The bosonic components of the hypermultiplets do not carry angular momentum under $SO(4)_E$ of the external rotations, while the fermions do indeed carry it. In fact the $SO(4)_E \sim SU(2)_L \times SU(2)_R$ symmetry appears as an R-symmetry of the theory and left or right moving fermions carry spin under $SU(2)_{L,R}$ respectively [15].

The state counting is the same as that of the left moving oscillator modes of $4Q_1Q_5$ superconformal fields. These modes will be carrying purely left moving momentum. In order to calculate the entropy we notice that we have a gas of left moving particles with $N_{B,F} = 4Q_1Q_5$ bosonic and fermionic species with energy $E = N/R_9$ on a compact one dimensional space of length $L = 2\pi R_9$. The standard entropy formula gives [13][14]

$$S_e = \sqrt{\pi (2N_B + N_F)EL/6} = 2\pi \sqrt{Q_1 Q_5 N}$$
, (4.5)

in perfect agreement, including the numerical coefficient, with (2.39).

It is interesting to understand the relation of this description in terms of open strings and the approach taken in the original derivation [13]. In [13] the D-strings were viewed as intantons on the $U(Q_5)$ fivebrane gauge theory [38][67]. Due to some Chern Simons couplings these instantons carry RR 1D-brane charge [38]. The moduli space of these instantons is given by a (4,4) superconformal field theory with central charge $c = 6Q_1Q_5$. The Q_5 factor comes basically from all the possible orientations of the instanton, and this is where the entropy comes from (for large Q_5). These instantons live on the fivebrane and it is only when they shrink to zero size that they become a D-string and are allowed to leave the fivebrane. Note that the steps followed in determining the number of massless open string modes above is very similar to calculations of instanton moduli spaces in [68].

In our previous argument we implicitly took the D-strings and the fivebranes to be singly wound. For large $N, N \gg Q_1Q_5$, the entropy (4.5) is the same no matter how we the branes are wound. However for $N \sim Q_1$ the winding starts to matter. The reason is that in order for the asymptotic formula to be correct for low N we need to have enough states with small energies [50]. Let us study the effect of different wrappings. We begin with an analogy from elementary quantum mechanics. Consider a circular loop of wire of unit radius whose center is at the origin of the r, θ plane. A bead of unit mass moves on the wire and for obvious reasons the angular momentum of the bead is quantized in integer multiples of \hbar . The energy spectrum is given by

$$E = \frac{l^2}{2} \tag{4.6}$$

for all integer l. Now consider a wire which is wrapped n times around the same circle. Eq. (4.6) still gives the energy levels if we allow l to be an integer multiple of 1/n. The system simulates fractional angular momentum. The real physical system of wire plus bead must, of course, have integer angular momentum but the energy spectrum may be expressed in terms of a "psuedo-angular momentum" which is not the true generator of spatial rotations but rather the generator of rotations of the bead relative to the physical wire.

Next let us consider a set of Q_1 1-branes wrapped on S^1 , ignoring for the time being, the 5-branes. We may distinguish the various ways the branes interconnect. For example, they may connect up so as to form one long brane of total length $R' = RQ_1$. At the opposite extreme they might form Q_1 disconnected loops. The spectra of open strings is different in each case. For the latter case the open strings behave like Q_1 species of 1 dimensional particles, each with energy spectrum given by integer multiples of 1/R. In the former case they behave more like a single species of 1 dimensional particle living on a space of length Q_1R . The result [32] is a spectrum of single particle energies given by integer multiples of $\frac{1}{Q_1R}$. In other words the system simulates a spectrum of fractional charges. For consistency the total charge must add up to an integer multiple of 1/R but it can do so by adding up fractional charges. Note that in this case, as opposed to the bead and wire example, the branes by themselves cannot carry any momentum since they are invariant under boosts along directions parallel to the branes.

Now let us return to the case of both 1 and 5 branes. By suppressing reference to the four compact directions orthogonal to x^9 we may think of the 5 branes as another kind of 1 brane wrapped on S^1 . The 5-branes may also be connected to form a single multiply wound brane or several singly wound branes. Let us consider the spectrum of (1,5) type strings (strings which connect a 1-brane to a five-brane) when both the 1 and 5 branes each form a single long brane. The 1-brane has total length Q_1R and the 5-brane has length Q_5R . A given open string can be indexed by a pair of indices $[i,\bar{j}]$ labelling which loop of 1-brane and 5-brane it ends on. As a simple example choose $Q_1 = 2$ and $Q_5 = 3$. Now start with the [1,1] string which connects the first loop of 1-brane to the first loop of 5-brane. Let us transport this string around the S^1 . When it comes back to the starting point it is a [2,2] string. Transport it again and it becomes a [1,3] string. It must be cycled 6 times before returning to the [1,1] configuration.

It follows that such a string has a spectrum of a single species living on a circle of size 6R. More generally, if Q_1 and Q_5 are relatively prime the system simulates a single species on a circle of size Q_1Q_5R . If the Q's are not relatively prime the situation is slightly more complicated but the result is the same. For example suppose $Q_1 = Q_5 = Q$, again assume the 5 and 1-branes each form a single long brane, then a string will return to its original configuration after cycling around Q times. This time the system simulates Q species living on a circle of length Q. But it is also possible to remove one loop from either the 1 or 5 brane and allow it to form a separate disconnected loop. In this case we have a system consisting of a brane of length QR, one of length QR and a short loop of length R. Since Q and Q-1 are relatively prime the open strings which connect them live on an effective brane of length Q(Q-1)R. Thus there is always a way of hooking up branes so that the effective length is of order Q_1Q_5R . In fact we will argue that this type of configurations give the largest entropy, and will therefore be dominant [50].

It can also be seen from the original derivation of the black hole entropy by Vafa and Strominger [13], that the system should have low energy modes with energy of order $1/RQ_1Q_5$. In this derivation the degrees of freedom that carry the momentum were described by a superconformal field theory on the orbifold $(T^4)^{Q_1Q_5}/S(Q_1Q_5)$. A careful analysis of this theory shows that low energy modes are present. For example, excitations with angular momentum are associated to energies $\delta E \sim J^2/RQ_1Q_5$ [15][16] so that for small angular momentum we are having a gap of the correct magnitude.

We can easily see that this way of wrapping the branes gives the correct value for the extremal entropy. Let us consider the case where Q_1 and Q_5 are relatively prime. As in [14] the open strings have 4 bosonic and 4 fermionic degrees of freedom and carry total momentum N/R. This time the quantization length is $R' = Q_1Q_5R$ and the momentum is quantized in units of $(Q_1Q_5R)^{-1}$. Thus instead of being at level N the system is at level $N' = NQ_1Q_5$. In place of the original Q_1Q_5 species we now have a single species. The result is

$$S = 2\pi\sqrt{N'} = 2\pi\sqrt{NQ_1Q_5} \tag{4.7}$$

The picture is very reminiscent of that proposed in [58] although the details differ.

The case we have been considering so far corresponds to black holes in N=8 supergravity. With these methods we can also calculate the entropy for black holes in N=4 supergravity. This theory is the low energy limit of the heterotic string compactified on T^5 . These are the dyonic black holes considered by various authors [58], [55], [69], [70]. In that case, the analogous D-brane description takes place in the type I theory, which is S-dual to the heterotic theory. Type I string theory indeed contains D-strings and D-fivebranes (but no other D-branes). These two D-branes correspond to the fundamental string and the solitonic fivebrane on the heterotic side [71]. The D-brane counting can also be done, and it is interesting to notice that to get the correct result one must include the 5D-brane SU(2) degrees of freedom found in [68].

Something remarkable has happened here. We started with some configuration of D-branes sitting at r=0, a point in 5-dimensional space. To start with, this is a "point with nothing inside it." However, having put all these open strings on the branes we find that the configuration matches a solution of the classical low energy action such that r=0 is really a 3-sphere with non-zero area! What happened? Well, the ten dimensional classical solutions for D-branes show that as we get closer to the D-brane the transverse space expands and the longitudinal space shrinks. This configuration has expanded the transverse space in such great way that what previously was a point is now a sphere. The most exciting aspect of this is that the classical solution continues beyond the horizon, into the black hole singularity, whereas, according to the D-brane picture space simply stops at r=0. States inside the horizon would have to be described by the massless modes on the D-brane. The basic horizon degrees of freedom are denumerated by three integers: the momentum, the index labeling the 1-brane and the index labeling the 5-brane. When a string "falls" into the black hole and crosses the horizon, it turns into open strings traveling on the D-branes (see figure 2). There should be a mapping between the closed string degrees of freedom, like the angle on S_3 where the infalling particle hit the horizon and the open string states. Of course, the transformation of "physical" space coordinates and open strings on the D-brane could be very complicated. All of this is reminiscent of the "holographic" principle [72], as well as the membrane paradigm [73], [74], in that dynamics occurring inside the black hole would be described as occurring on the horizon.

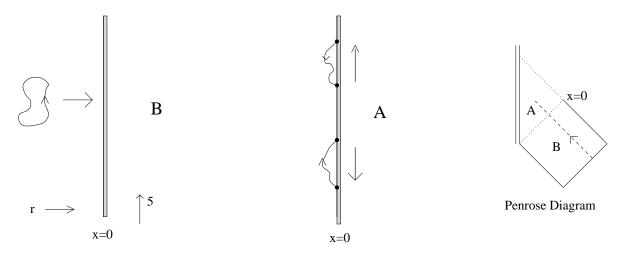


FIGURE 6: D-brane description of a string before and after falling through the horizon.

4.1. Near-Extremal 5 d Black Holes and Hawking Radiation.

We now turn to a discussion of nearly extremal five-dimensional black holes (2.46). Since the coupling constant (2.43) in these solutions is bounded in space we can choose it to be weak everywhere. This should be a favorable case for examining the non-BPS states of the D-brane, doing perturbative computations of their interactions and comparing to the canonical expectations for the nonextremal black holes (2.46). We will see that for the near-extremal case the agreement between the two approaches is just as impressive as in the extremal case. There is, however, a hitch: the presence of a large number of D-branes $(Q_1, Q_5 \gg 1)$ amplifies the effective open string coupling constant and, in principle, renders any perturbative analysis of horizon dynamics unreliable [13], [18], [9]. We think the situation may not be so desperate and will present a (non-rigorous!) argument that open string loop corrections might not, in fact, change the essential physics. The near-extremal entropy was also calculated for D-branes that do not excite the dilaton, for example the three brane, but the factors do not quite agree [75][76]. In those cases the corresponding black holes have scalar fields blowing up as we approach the horizon. For the cases we consider the agreement is precise. It will be interesting to understand the origin of the numerical disagreement for the calculations in [75][76].

We perturb away from the BPS limit in a macroscopically small but microscopically large fashion ($M \gg \delta M \gg$ mass of typical excitations). There are many ways to do this by adding stringy excitations to the basic D-brane

configuration. We are interested in those excitations which cause the entropy to increase most rapidly with added energy. One could add fundamental string modes traveling on the torus, but they have too small a central charge to be relevant. Massive open or closed string excitations also give a subleading contribution since the entropy of a gas of these excitations increases at most as $\delta M^{2/3}$, and we will find a leading contribution going as $\delta M^{1/2}$. One could have five brane excitations in any direction, but that entropy increases as $\delta M^{\frac{p}{p+1}}$ for a membrane of p dimensions. So we conclude that the most important will be the ones along the string. There are various modes associated to the open strings going between the various branes. Some of these are massive due to the presence of a large number of left movers. The ones that are massless will give the dominant contribution and correspond, as in the case of left movers, to oscillations on the moduli space of vacua. We are saying that the first right moving strings to be excited will be the ones that continue keeping the D-terms (4.3) zero. Note the very important fact that if the branes are multiply wrapped, as we argued they had to be, these excitations will be very light, with masses of order $1/RQ_1Q_5$. This alone will favor these excitations over the ones we listed above. If we perturb away from a purely left-moving extremal background by adding δN_R right-moving oscillations, we also have to add $\delta N_L = \delta N_R$ left-moving oscillations to keep the total $N = N_L - N_R$ charge fixed. These oscillations have 4 bosons and 4 fermions, so the central charge is the same as it was before. The change in left-moving entropy is proportional to $\sqrt{N+\delta N_R}-\sqrt{N}$ and is of order $\delta N_R/N_L$. The change in right-moving entropy is, however, of order $\sqrt{\delta N_R/N_L}$ and dominates. More specifically, we find that

$$\frac{\Delta S}{S_e}\bigg|_{oscill} = \sqrt{\frac{\delta N_R}{N}} \ . \tag{4.8}$$

This result agrees with (2.61) when the number of antibranes is zero, $N_{\bar{1}} = N_{\bar{5}} = 0$. We can make these numbers to be zero by taking R_9 to be very large (2.56). In that case we have the long string limit considered in [18]. Notice that we are using here the number of branes and antibranes obtained in (2.56).

If we want to consider more general near-extremal black holes, we need to find the contribution to the entropy due to the addition of a small amount of antibranes. We have already calculated the increase in entropy due to a small amount of rightmovers (4.8) we need to find the corresponding increments due to $\delta N_{\bar{1}}, \delta N_{\bar{5}}$. They are presumably independent and should be added to get the total entropy increment. We have already calculated the increase due to the right and left movers, but it is not obvious what the entropy increase due to the anti-branes will be. There is however a U-duality transformation [12],[59],[67],[60] that, for example, turns anti-1-branes into right moving momentum states at the price of some transformation of coupling and compactification radii. Since the entropy increase is independent of the coupling constant and the compactification radii, we will take the duality argument as telling us that the counting of the brane-antibrane states is just the same as the counting of the left- and right-moving oscillator states. The net result for the entropy increment is

$$\frac{\Delta S}{S} \bigg|_{anti-1-branes} = \sqrt{\frac{\delta N_{\bar{1}}}{Q_1}}.$$
 (4.9)

Since the same argument applies to δN_5 . These increments were calculated explicitly in limit in which they are dominant, for small R_9 or small V, in [17]. The nontrivial, not completely justified, assumption is that we can extrapolate those results to a regime there the three contributions are comparable. Making this naive extrapolation we find that the total entropy of the non-extremal solution is

$$\frac{\Delta S}{S}\Big|_{total} = \sqrt{\frac{\delta N_R}{N}} + \sqrt{\frac{\delta N_{\bar{1}}}{Q_1}} + \sqrt{\frac{\delta N_{\bar{5}}}{Q_5}}$$
 (4.10)

which agrees with the classical formula (2.61) when the antibrane numbers are all small. In the Reissner-Nordström case we can see from (2.40) and (2.56) that the three terms in (4.10) are equal and in terms of the mass above extremality we get

$$\left. \frac{\delta S_e}{S} \right|_{string} = \frac{3}{\sqrt{2}} \sqrt{\frac{\delta M}{M_e}} \tag{4.11}$$

This is the standard Bekenstein-Hawking result for the strict five-dimensional Reissner-Nordström black hole, with the correct normalization and functional dependence on mass. Although the arguments that led us here are less than rigorous (especially the duality argument for entropies associated with branes and antibranes), the simple end result gives us some confidence in the intermediate steps.

These non-BPS states will decay. The simplest decay process is a collision of a right moving string excitation with a left moving one to give a closed string that leaves the brane. We will calculate the emission rate for chargeless particles, so that the basic process is a right moving open string with momentum $p_9 = n/R_9Q_1Q_5$ colliding with a left moving one of momentum $p_9 = -n/R_9Q_1Q_5$ to give a closed string of energy $k_0 = 2n/RQ_1Q_5$. Notice that we are considering the branes to be multiply wound since that is the configuration that had the highest entropy. If the momenta are not exactly opposite the outgoing string carries some momentum in the 9^{th} direction and we get a charged particle. Notice that the momentum in the 9^{th} direction of the outgoing particle has to be quantized in units of $1/R_9$. This means that outgoing charged particles have a very large mass, and we see that they will be thermally suppressed. All charged particles will have a masses of at least the compactification scale.

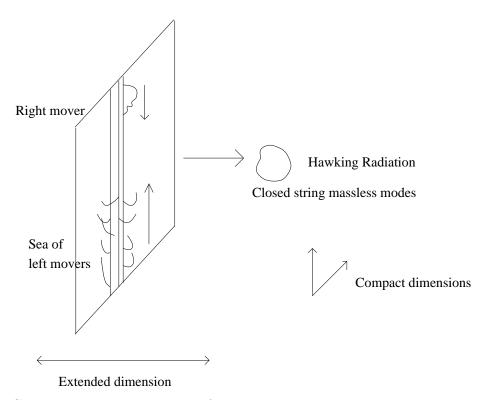


FIGURE 7: D-brane picture of the Hawking radiation emision process.

We will calculate the rate for this process according to the usual rules of relativistic quantum mechanics and show that the radiation has a thermal spectrum if we do not know the initial microscopic state of the black hole.

The state of the black hole is specified by giving the left and right moving occupation numbers of each of the bosonic and fermionic oscillators. In fact, the nearly extremal black holes live in a subsector of the total Hilbert space that is isomorphic to the Hilbert space of a one dimensional gas of massless particles of 4 different types, either bosons or fermions. This state $|\Psi_i\rangle$ can then emit a closed string and become $|\Psi_f\rangle$. The rate, averaged over initial states and summed over final states, as one would do for calculating the decay rate of an unpolarized atom, is

$$d\Gamma \sim \frac{d^4k}{k_0} \frac{1}{p_0^R p_0^L V R} \delta(k_0 - (p_0^R + p_0^L)) \sum_{i,f} |\langle \Psi_f | H_{int} | \Psi_i \rangle|^2$$
 (4.12)

We have included the factor due to the compactified volume RV. The relevant string amplitude for this process is given by a correlation function on the disc with two insertions on the boundary, corresponding to the two open string states and an insertion in the interior, corresponding to the closed string state (see figure 3). The boundary vertex operators change boundary conditions for four of the coordinates when we are dealing with a (1,5) or (5,1) string. We consider the case when the outgoing closed string is a spin zero boson in five dimensions, so that it corresponds to the dilaton, the internal metric, internal $B_{\mu\nu}$ fields, or internal components of RR gauge fields. This disc amplitude, call it A, is proportional to the string coupling constant g and to k_0^2 [34]. The reason for this last fact is that it has to vanish when we go to zero momentum, otherwise it would indicate that there is a mass term for the open strings (since one can vary the vacuum expectation value of the corresponding closed string fields continuosly). It cannot be linear either since the amplitude is symmetric under $k_0 \rightarrow -k_0$ and $X_0 \rightarrow -X_0$, at least for these polarizations of the vertex operators. In conclusion, up to numerical factors,

$$A \sim gk_0^2. \tag{4.13}$$

Note that performing the average over initial and sum over final states will just produce a factor of the form $\rho_L(n)\rho_R(n)$ with

$$\rho_R(n) = \frac{1}{N_i} \sum_i \langle \Psi_i | a_n^{R\dagger} a_n^R | \Psi_i \rangle \tag{4.14}$$

where N_i is the total number of initial states and a_n^R is the creation operator for one of the 4 bosonic open string states. The factor $\rho_L(n)$ is similar. Since we are just averaging over all possible initial states with given value of δN_R , this corresponds to taking the expectation value of $a_n^{\dagger}a_n$ in the microcanonical ensemble with total energy $E_R = \delta N_R/R_9 = \delta N_R'/R_9 Q_1 Q_5$ of a one dimensional gas. Because $\delta N_R'$ is large compared to one (but still much smaller than N_L'), we can calculate (4.14) in the canonical ensemble. Writing the partition function as

$$Z = \sum_{N'} q^{N'} d(N') = \sum_{N'} q^{N'} e^{2\pi\sqrt{N'}},$$

doing a saddle point evaluation and then determining q from

$$\delta N_R Q_1 Q_5 = \delta N_R' = q \frac{\partial}{\partial q} \log Z,$$

we find $\log q = -\pi \sqrt{1/Q_1 Q_5 \delta N_R}$. Then we can calculate the occupation number of that level as

$$\rho_R(k_0) = \frac{q^n}{1 - q^n} = \frac{e^{\frac{-k_0}{2T_R}}}{1 - e^{\frac{-k_0}{2T_R}}} .$$

We can read off the "right moving" temperature

$$T_R = \frac{1}{\pi} \frac{1}{R} \sqrt{\frac{\delta N_R}{Q_1 Q_5}} \ .$$

Now using (2.56) we find

$$T_R = \frac{r_0^2 R_9 V}{2g S_e} \tag{4.15}$$

in the near-extremal case, when r_0 very small. There is a similar factor for the left movers ρ_L with a similar looking temperature

$$T_L = \frac{1}{\pi} \frac{1}{R} \sqrt{\frac{N}{Q_1 Q_5}} \ . \tag{4.16}$$

The two temperatures $T_{L,R}$ can be thought of as the effective temperature of the gas of left movers and the gas of right movers. They are different because the gas

carries some net momentum. Since $T_R \ll T_L$ the typical energy of the outgoing string will be $k_0 \sim T_R$ and $k_0/T_L \sim T_R/T_L \ll 1$ so that we could approximate

$$\rho_L \sim \frac{2T_L}{k_0} = \frac{2}{\pi k_0 R} \sqrt{\frac{N}{Q_1 Q_5}} \ . \tag{4.17}$$

The expression for the rate then is, up to a numerical constant,

$$d\Gamma \sim \frac{d^4k}{k_0} \frac{1}{p_0^R p_0^L RV} |\mathcal{A}|^2 Q_1 Q_5 R \rho_R(k_0) \rho_L(k_0)$$
 (4.18)

where \mathcal{A} is the disc diagram result. The factor Q_1Q_5R is a volume factor, which arises from the delta funtion of momenta in (4.12) $\sum_n \delta(k_0 - 2n/RQ_1Q_5) \sim RQ_1Q_5$. The final expression for the rate is then, using (4.13), (4.17) in (4.18), up to a numerical constant,

$$d\Gamma \sim \frac{g^2}{RV} \sqrt{Q_1 Q_5 N} \frac{e^{\frac{-k_0}{2T_R}}}{1 - e^{\frac{-k_0}{2T_R}}} d^4 k \sim (\text{Area}) \frac{e^{\frac{-k_0}{2T_R}}}{1 - e^{\frac{-k_0}{2T_R}}} d^4 k \tag{4.19}$$

Note that the powers of k_0 have cancelled. We conclude that the emission is thermal, with a physical Hawking temperature

$$T_H = 2T_R \tag{4.20}$$

which exactly matches the classical result (2.50). It is an interesting result that the area appeared correctly in (4.19). Actually, the coupling constant coming from the string amplitude \mathcal{A} is hidden in the expression for the area (area = $4G_N^5S$). Of course, it will be very interesting to calculate the coefficient in (4.19) to see whether it exactly matches the absorbtion coefficient of a large classical black hole. However it is easy to see that the absorbtion coefficient is proportional to the area for energies higher than the inverse of the Schwarschild radius where one can calculate the cross section just from the behaviour of classical geodesics. The coefficient in (4.19) involves the cross section at energies much lower than the inverse of the Schwarschild radius, which is basically related to the temperature of the left movers. This means that calculating the absorbtion coefficient for this energy requires solving the Klein Gordon equation

on the black hole background. In fact the result should not depend on the internal polarizations of the outgoing particles⁷ The string theory calculation for (1,5) strings could be done using techniques similar to those in [80]. Note that the gas of antibranes is also at the temperature (4.20) and they also seem to contribute to Hawking radiation. Notice that if we were emitting a spacetime fermion then the left moving string could be a boson and the right moving string a fermion, this produces the correct thermal factor for a spacetime fermion. The opposite possibility gives a much lower rate, since we do not have the enhancement due to the large ρ_L (4.17). Notice also that when separation from extremality is very small, then the number of right movers is small and the statistical arguments used to derive (4.19) fail. Classically this should happen when the temperature is so low that the emission of one quantum at temperature T causes the temperature to change by a large amount. This occurs when the specific heat is of order one corresponding to a mass difference from extremality [81]

$$\delta M_{min} \sim \frac{G_N^5}{r_e^4} \tag{4.21}$$

for a Reissner-Nordström black hole, with r_e as in (2.40). The D-brane approach suggests the existence of a mass gap of order

$$\delta M_{min} \sim \frac{1}{Q_1 Q_5 R} \tag{4.22}$$

which using (2.31)(2.40) scales like (4.21). This is an extremely small energy for a macroscopic extremal black hole. In fact, it is of the order of the kinetic energy that the black hole would have, due to the uncertainty principle, if we want to measure its position with an accuracy of the order of its typical gravitational radius r_e : $\delta M \sim (\Delta p)^2/M$ with $\Delta p \sim 1/r_e$.

We now examine the range of validity of these approximations. For the purposes of this argument, we take the compactification radii to be of order α' and set $\alpha' = 1$. In this case (2.40) implies $Q_1 \sim Q_5 \equiv Q$ and $Q \sim gN$. Then, by (2.39), we find that the area of the horizon is $A \sim (g^2N)^{3/2}$. In order for

⁷ Recently this question was addressed in [77][78] [79]. They calculated [78] the precise numerical coefficient in (4.19) and they found that the two calculations agree when R is very large.

the classical black hole interpretation to hold, this area has to be much larger than α' , so $g^2N \gg 1$. Since we want g to be small, N is very large. This seems to invalidate the perturbative D-brane picture since open string loop corrections are of order $gQ = g^2N$, and, due to the large number Q of D-branes, they are likely to be large [13], [18], [9]. We will try to argue that open string corrections might in fact be suppressed. We note that the loop will be in a nontrivial background of open strings. In fact, this background was crucial to obtain a small five dimensional coupling constant and non-zero area, which implies that somehow the D-branes might be "separated" from each other. We suspect that this background of open strings suppresses open string loops, enabling us to get results off extremality. This is, of course, something to be checked in detail. It is clear, however, that there are some circumstances where open string backgrounds suppress loop contributions. For example, compare loop contributions of n Dbranes on top of each other and n widely separated D-branes. The difference is just a background of open string translational zero modes. It could be that the modes related to the entropy and Hawking radiation are weakly interacting while higher energy modes might indeed recieve large corrections. In this respect we might recall an entirely different, but apparently analogous, physical situation: electrons and nuclei in a metal. The low energy thermodynamics can be fairly well reproduced by considering the electrons to be free, though there are lots of charges present. In that case we have a better understanding as to which physical questions can be answered by regarding the electrons as free and which require taking into account the interactions. Hopefully, further studies of these models will provide a similar understanding for the black hole case.

Finally, the fact that the perturbative D-brane treatment of non-extremal physics gives the right results strongly suggests that there is more than a grain of truth here. We think it quite possible that open string quantum corrections are not as large as suggested by naive estimates. The skeptic is entitled to disagree!

4.2. Extremal and Near-Extremal Four Dimensional Black Holes.

The extremal black hole solution of type IIA compactified on $T^6 = T^4 \times S_1' \times S_1$ is constructed by wrapping Q_6 D-sixbranes on the whole T^6 , Q_2 D-twobranes on $S_1' \times S_1$, Q_5 solitonic fivebranes along $T^4 \times S_1$ and momentum N/R flowing along S_1 . All charges Q_2, Q_5, Q_6, N are integers [19]. The supergravity solution

was given in (2.67). Note that if we momentarily set $Q_5 = 0$ we just simply have a configuration which is T-dual to the one we had for the five dimensional black hole, so that the entropy is $\sqrt{Q_2Q_6N}$ and is calculated as before in terms of open strings going between the D-twobranes and the D-sixbranes. Alternatively, as in [13] we could view the D-twobranes as instantons on the D-sixbrane and then the entropy comes from putting some left moving momentum along one direction in the 2+1 dimensional field theory whose target space is the moduli space of these instantons $(T^4)^{Q_2Q_6}/S(Q_2Q_6)$ where S(q) is the permutation group of q elements [67].

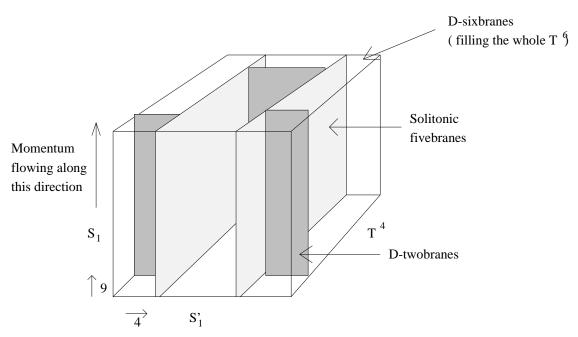


FIGURE 8: Brane picture of a four dimensional black hole showing how the branes are wrapped in the compact dimensions.

Let us see what happens when we introduce the fivebranes. The fivebranes intersect the two branes along the S_1 . Different fivebranes will be at different positions along the S'_1 . The two branes can break and the ends separate in T^4 when it crosses the fivebrane. This was derived in [82],[83] and it follows, using U-duality, from the fact that fundamental strings can end on D-branes. Hence the Q_2 toroidal twobranes break up into Q_5Q_2 cylindrical twobranes, each of

which is bounded by a pair of fivebranes. The momentum-carrying open strings now carry an extra label describing which pair of fivebranes they lie in between. The number of species becomes $N_B = N_F = 4Q_2Q_5Q_6$. The number of BPS-saturated states of this system as a function of Q_2, Q_5, Q_6 and N follows from the standard (1+1)-dimensional entropy formula for a gas of massless particles

$$S = 2\pi \sqrt{\frac{(2N_B + N_F)ER_9}{12}},\tag{4.23}$$

where N_B (N_F) is the number of species of right-moving bosons (fermions), E is the total energy and $2\pi R_9$ is the size of the box. Using $N_B = N_F = 4Q_2Q_5Q_6$ and $E = N/R_9$, we find the R_9 -independent result for the large N thermodynamic limit

$$S_{stat} = 2\pi \sqrt{Q_2 Q_5 Q_6 N}. (4.24)$$

which indeed reproduces the classical result (2.69). This formula can be justified using the usual BPS arguments.

This result can also be extended to black hole solutions in N=4 supergravity by replacing T^4 in the previous argument by K3 [50]. The entropy is the same as for the N=8 case (4.24).

Again, as in the five dimensional case, the calculation leading to (4.24) implicitly assumed that the branes were singly wound and it is valid when N is very large, $N \gg Q_2Q_5Q_6$. For not so big values of N the entropy comes from configurations where the branes are multiply wound. In a fashion analogous to the five dimensional case this leads to just 4 species of fermions and bosons propagating on a circle of radius $RQ_2Q_5Q_6$ with the same result (4.24) for the entropy. Indeed, the classical energy gap for an extremal black hole is [81]

$$\delta M \sim \frac{G_N^4}{r_e^3} \sim \frac{1}{RQ_2Q_5Q_6} \tag{4.25}$$

which agrees with the multiply wound D-brane result.

Now we consider near-extremal four dimensional black holes [21]. The simplest case to consider is when the size R_9 is much bigger than the rest of the compact dimensions. This corresponds to taking $N_{\bar{2}} \sim N_{\bar{5}} \sim N_{\bar{6}} \sim 0$. Note that

these antibrane excitations are very massive when R_9 is large. When we have both left and right movers, using the above arguments we find the entropy

$$S = 2\pi \sqrt{N_2 N_5 N_6} (\sqrt{N_R} + \sqrt{N_L}) \tag{4.26}$$

which is the classical result (2.82) in the limit we are considering.

Now we consider the case with angular momentum, again in the limit of large R_9 . With just twobranes and sixbranes present, the D-brane excitations of this system are described by a 1+1-dimensional field theory which turns out to be a (4,4) superconformal sigma model [15]. The fivebrane breaks the right-moving supersymmetry [84], leaving us with (0,4) superconformal symmetry. The N=4 superconformal algebra gives rise to a left-moving $SU(2)_L$ symmetry. Since fermionic states in the sigma model become spinors in spacetime, the action of O(3) spatial rotations has a natural action on this $SU(2)_L$. The charge F_L under one U(1) subgroup of this $SU(2)_L$ will then be related to the four-dimensional angular momentum (along one of the three axes) carried by the left movers by $J=F_L/2$. Due to the presence of the fivebrane the right-moving $SU(2)_R$ symmetry of the original (4,4) superconformal field theory is broken and the right movers cannot carry macroscopic angular momentum. The number of states with fixed N_L , N_R , $F_L \gg 1$ may be computed as in [15],[16] to yield the entropy

$$S = 2\pi \sqrt{\frac{c}{6}} (\sqrt{N_R} + \sqrt{\tilde{N}_L}) , \qquad (4.27)$$

where $\tilde{n}_L = n_L - 6J^2/c$ is the effective number of left movers that one is free to change once one has demanded that we have a given macroscopic angular momentum. For our problem the central charge is $c = 6N_2N_5N_6$ [19], thus the entropy (2.84) agrees with the D-brane formula (4.27).

It is interesting to take the extremal limit of these rotating black holes, when the mass takes the minimum value consistent with given angular momentum and charges. This happens when $\tilde{n}_L = 0$, so the left movers are constrained to just carry the angular momentum and do not contribute to the entropy. When the angular momentum is nonzero, even the extremal black hole is not supersymmetric. Using (4.27) and writing the result in terms of the charge $N = N_R - N_L$ we find

$$S = 2\pi\sqrt{J^2 + NQ_2Q_5Q_6} , \qquad (4.28)$$

which indeed agrees with the entropy of an extremal charged rotating black hole [64]. Notice the surprising fact that although we derived this formula in the large R_1 regime (and $J/M^2 \ll 1$), it continues to be valid for arbitrary values of the parameters. Since this is far from the BPS state, we had no reason to expect the weak-coupling counting to continue to agree with the black hole entropy.

All the worries we had about possible strong coupling effects in five dimensions are also a source of concern for these four dimensional black holes, but the successful calculation for the entropies encourages us to take this picture more seriously. The discussion on Hawking radiation also carries over with almost no modification, giving the correct Hawking temperature.

5. DISCUSSION

We have studied black holes in type II supergravity compactified to five and four dimensions. These theories have several U(1) charges and we considered black holes that carry several of these charges. The extremal black holes have a direct correspondence with a superposition of string solitons and they preserve some of the supersymmetries. For extremal black holes the entropy does not depend on any of the continuous parameters and it is given just in terms of the integer values of the quantized charges. In addition we constructed the nonextremal versions of these black holes. This construction can be done by applying U-duality symmetries and boosts to the standard Reissner-Nordström solution in four or five dimensions. We have shown how U-duality relates the different quantization conditions on the charges. Black holes have some scalar charges which are determined, due to the no hair theorem, by the U(1) charges and the mass. If we compare the U(1) charges, the scalar charges, and the mass with the corresponding values for a non-interacting collection of branes and antibranes we can calculate the number of hypothetical "non-interacting" constituents of the black hole. The entropy formula has a very suggestive form when it is written in terms of these numbers.

We then viewed the same collection of branes and strings from the point of view of string theory. Applying the rules for quantizing D-brane solitons we were able to count the number of microscopic states of such configurations. In the extremal case this counting can be justified rigorously by using the standard BPS arguments. In the near-extremal case one can certainly do the counting on the string theory side when the coupling is very weak. However, in order to compare to the black hole answer we need to make the coupling bigger. It turns out that the necessary size of the coupling is such that one might expect large corrections. However, the near extremal entropy is precisely accounted for by this weak coupling argument. D-branes account for some non-perturbative effects, so the question is whether they account for all the necessary ones to describe black holes. The answer is that they seem to be doing that, at least as far as entropy calculations is concerned. These near-extremal black holes

⁸ See also [77][78][79] regarding the comparison of scattering amplitudes between the classical approach and the D-brane approach.

have many free parameters, in fact one can also consider solutions with angular momentum. For all these cases it is possible to account for the entropy, indicating that the understanding of the black hole degrees of freedom seems to be not too far from reality. The energy gap for excitations of an extremal black hole also agrees using both methods (classical and D-brane).

Hawking radiation is viewed in this approach as the collision of two oppositely moving open strings attached to the branes that decay into a closed string that leaves the brane. As expected from thermodynamics arguments, the Hawking temperature is precisely the classical result. Furthermore, the Hawking radiation rate shows that the D-brane calculation "knows" about the geometry, since the rate is proportional to the area of the black hole. The overall coefficient in this rate is proportional to the absorbtion cross section of the black hole for that particular mode. It will be interesting to study these more dynamical questions to understand better whether this object really represents a black hole or not.

One might worry that we are not considering a realistic conpactification since our world is not described by N=8 supergravity, at least at small energies. It will be indeed very interesting to extend these results for the case of N=2,1,0 and see how much of this description carries over. From a purely theoretical point of view, the problem of black hole entropy is as puzzling in N=8 supergravity as it is in General Relativity, but it is easier to analyse in N=8 supergravity. All our results carry over to the N=4 theory, both in their type II version (type II on K3) as in the heterotic theory (type I) compactified on a torus.

On a more speculative note we would say that if this picture were qualitatively right, there would be no information loss, the information would stay on the open strings that live on the branes which sit at the horizon (for an extremal black hole). In this picture, the classical region inside the horizon would be an effective description of the dynamics of these open strings. What would happen is that an observer, made of closed strings, that falls through the horizon is turned into open strings together with all his measuring apparatus, so it seems plausible that he would not notice the difference. There should be a way to describe the subsequent dynamics in terms of some effective closed strings that falled through the horizon. Note that the problem of information loss could be analysed in terms of near-extremal black holes. Because of our lack of control on the strong coupling problem we cannot say anything definite about information loss. This

D-brane description is the description of the physics as seen by the asymptotic observer. It is for this observer that evolution should be unitary since he sees the black hole formation and evaporation process.

Let us end by saying that black holes are an excellent theoretical laboratory for understanding some features of quantum gravity. One could say that they are the "Hydrogen atom" of quantum gravity. It will be interesting to see what string theory will say about this in the future.

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