LORENTZ VIOLATION AND HAWKING RADIATION

T. JACOBSON
Department of Physics, University of Maryland, College Park
MD 20742-4111, USA
E-mail: jacobson@physics.umd.edu

Since the event horizon of a black hole is a surface of infinite redshift, it might be thought that Hawking radiation would be highly sensitive to Lorentz violation at high energies. In fact, the opposite is true for subluminal dispersion. For superluminal dispersion, however, the outgoing black hole modes emanate from the singularity in a state determined by unknown quantum gravity processes.

As evidenced by this meeting, there is a growing feeling that Lorentz invariance should be questioned and tested. Tentative results from from quantum gravity and string theory hint that the ground state may not be Lorentz invariant, and new techniques are allowing for experiments with unprecedented precision and astrophysical observations at unprecedented high energies. Moreover, due to the unboundedness of the boost parameter, *exact* Lorentz invariance, while mathematically elegant, is unverifiable and therefore suspect.

Most work exploring possible Lorentz violation has focused on non gravitational physics—i.e. flat spacetime. Much less has been done to investigate Lorentz breaking in curved spacetime, or in gravitational phenomena themselves. The event horizon of a black hole, being a surface of infinite redshift, is a particularly good probe of the limits of boost invariance at high energies. Since Hawking radiation emerges from the vicinity of the horizon, it is therefore interesting to investigate the impact of Lorentz violation on Hawking radiation. To pursue this question, however, one must first address another question: *if Lorentz violation exists, what becomes of general relativity?*

In flat spacetime, Lorentz breaking is described by couplings to constant symmetry breaking tensors V_a, W_{ab}, ... To formulate Lorentz breaking in a curved spacetime without destroying general covariance such tensors must become *dynamical tensor fields* fields that satisfy (effective) field equations. An implementation of this, with a unit timelike vector $u^a(x)$ as the Lorentz breaking field, is the subject of another contribution to these proceedings [1,2]. Such a field preserves rotation invariance and, since it is a *unit* vector, contains no information other than the determination of a preferred rest frame at each point of spacetime. For the present purposes let us just imagine that some such implementation exists.

Hawking radiation is thermal radiation emitted from a black hole. For a

black hole with surface gravity κ, the Hawking temperature is $T_H = \hbar\kappa/2\pi c$. A nonrotating black hole has a surface gravity $c^2/2R_S$, where R_S is the Schwarzschild radius, hence the thermal wavelength is $\lambda_H = 8\pi^2 R_S$. This is the typical wavelength of the Hawking radiation far from the black hole, after it has climbed out of the potential well of the hole. Following the radiation backwards all the way to the horizon, however, it recedes into the vacuum fluctuations and the wavelength goes to zero as measured by an observer falling freely across the event horizon. The Hawking radiation therefore originates from Planck (or trans-Planck) scale vacuum fluctuations, which may be governed by completely unknown physics if Lorentz invariance is broken.

The essence of the Hawking effect is independent of field mass or interactions, hence it is adequate for the present purposes to consider a free massless field. The propagation of the field near the horizon can be inferred in the WKB approximation from the dispersion relation, and Lorentz violation can be parameterized by modifications of the dispersion relation. Consider for example dispersion relations of the form

$$\omega = k + \xi k^{1+n}/k_P^n, \tag{1}$$

where ω is the frequency, k is the magnitude of the wave 3-vector, k_P is the Planck wave vector, and units with $c = 1$ are employed. The dimensionless parameter ξ controls the amount of Lorentz violation and the integer n determines its k dependence. The case $n = 0$ is scale independent and corresponds to the renormalizable Lorentz violation one has in the standard model extension.[3] In the case $n > 0$ the Lorentz violation grows stronger at higher energies. The group velocity $v_g = d\omega/dk$ for the dispersion relation (1) is given by

$$v_g = 1 + \xi(1 + n)(k/k_P)^n, \tag{2}$$

which is superluminal if $\xi > 0$ and subluminal if $\xi < 0$. The frequency and spatial wave 3-vector in (1) are defined with respect to the preferred frame u^a. Explicitly, $\omega = u^a k_a^{(4)}$ and $k_a^{(3)} = (\delta_a{}^b - u_a u^b)k_b^{(4)}$. The dispersion relation is imported into curved spacetime by using the vector field $u^a(x)$.

To investigate the consequences of Lorentz breaking for the Hawking effect it is first of all necessary to specify the profile of the preferred frame in the black hole spacetime. In spherical symmetry a natural frame is the radial free-fall frame which is determined by the geodesics that are asymptotically at rest at spatial infinity and fall freely across the horizon. Whether this particular frame would be dynamically selected by the effective field theory is unknown. However it seems reasonable to assume, as I will here, that the preferred frame is well-behaved at the horizon. This means that it has finite velocity with

respect to the free fall frame and that the invariant 4-acceleration is not large compared to the surface gravity κ.

We can now examine how Lorentz violation might affect the propagation of fields near a black hole horizon. Consider first the $n = 0$ case, for which the group velocity is the k-independent constant $1 + \xi$. For negative ξ this is *less* than the speed of light, so the effective horizon moves out. In fact the dispersion relation is identical to that of a Lorentz invariant massless field coupled to the metric $g'_{ab} = g_{ab} + (2\xi + \xi^2)u_a u_b$. The light cones of g'_{ab} are 'narrower' than those of g_{ab} so the horizon is moved out relative to that of g_{ab}. A matter field with such dispersion would exhibit the usual Hawking effect, with a temperature determined by the surface gravity of the g'_{ab} horizon. For positive ξ the group velocity is instead *greater* than the speed of light, so the g' light cones are 'opened up' and the horizon moves *inward.*

For $n > 0$ the situation is much more interesting.[4] Let us first consider the case where ξ is positive. Then the group velocity is superluminal (as with $n = 0$), and grows with k. This means that as an outgoing wavepacket is followed backwards in time toward the horizon, it speeds up and crosses the horizon, continuing to go faster, until it approaches the curvature singularity inside the black hole. In other words, the outgoing mode emerged from the vicinity of the singularity and propagated superluminally out across the horizon, finally redshifting enough to slow down to the speed of light.

Needless to say, this is very different from the usual Hawking effect. In the usual case, the outgoing mode is stuck just outside the horizon, exponentially blueshifting backwards in time. The Hawking effect is deduced from a vacuum condition imposed on the quantum field near the horizon, namely that outgoing modes with frequency $\omega \gg \kappa$ are in their ground state as seen by a free-fall observer. This physically reasonable condition need not refer to any Planck or trans-Planck scale quantities. It merely expresses the hypothesis that after the black hole formed, there is no process going on that could excite these modes. (However, due to the continued blueshift backwards in time, it cannot be *deduced* from the initial conditions before the collapse that formed the black hole without reference to trans-Planckian quantities.) In this sense the usual Hawking effect is a robust prediction of Lorentz invariant field theory.

In the presence of $n > 0$ superluminal dispersion, however, the outgoing modes emerge from the vicinity of the singularity inside the black hole where unknown physics takes place. If they emerge in their ground state then the usual Hawking effect would occur. But something entirely different might happen, and indeed seems more plausible, since the modes propagate through a region of diverging spacetime curvature.

Finally, let us consider the case of negative ξ. The group velocity is then

subluminal, and decreases with increasing k. In this case, as first shown by Unruh [5], an outgoing wavepacket originates as an ingoing wavepacket, containing typical wavevectors of order k_P, which is outgoing with respect to the preferred frame but not fast enough to compensate for the infalling of that frame. Thus the wavepacket is dragged towards the horizon. As this happens the wavepacket redshifts, and near the horizon it finally redshifts enough for its group velocity to exceed the infall speed of the preferred frame. At this point it turns around and finally climbs away from the horizon, continuing to redshift on the way out. This sort of continuous evolution of a mode from one type to another, with a change of group velocity, in response to propagation through an inhomogeneous medium, is well known is various other areas of physics. It is called "mode conversion" in the plasma physics literature.

Now what does this mode conversion imply for the Hawking effect? Actually, it makes no difference! The same physical reasoning as in the Lorentz invariant case supports the hypothesis that the outgoing modes with $\omega \gg \kappa$ near the horizon are in their ground state as seen by free fall observers. The only difference is in the *ancestry* of the modes. In the Lorentz invariant case the modes have trans-Planckian ancestry due to the continued exponential blueshifting at the horizon, and they originate as ingoing modes that arrive at the horizon just before it forms from the collapse that produced the black hole. In the dispersive case on the other hand, the ancestor is only Planckian, and it originates as an ingoing (or rather, dragged-in outgoing) mode that arrives at the horizon *after* the black hole forms. It was shown numerically in Ref. [5] (using a dispersion relation for which the group velocity vanishes at large k) that if these ingoing modes are in their ground state far from the black hole then the usual Hawking flux emerges as long as $\kappa \ll k_P$.

A weakness in the above account is that the model breaks down if one attempts to trace the ingoing mode all the way back out to spatial infinity, since the wavelength gets arbitrarily blueshifted in the process. An improved model would start from some sort of discreteness for spacetime with a physically sensible short distance cutoff. The Hawking effect has been studied in such a model, in which space is treated as a lattice that falls freely into a black hole in 1+1 dimensions.[6]

Linear fields propagating on the lattice naturally have subluminal dispersion at high wavevectors. For a lattice with spacing a the dispersion relation is $\omega = (2/a)|\sin(ka/2)|$. Wavevectors differing by $2\pi/a$ are identified, so the independent modes are labelled by the so-called Brillouin zone $(-\pi/a, \pi/a)$. The group velocity vanishes for $k = \pi/a$, and it is negative for yet higher wavevectors, which are identified with negative wavevectors. The wavepacket propagation picture discussed above suggests that mode conversion would take

place outside a black hole horizon, with an outgoing wavepacket of small positive k components arising from an ingoing packet of k components greater than π/a. (This is analogous to a Bloch oscillation, which occurs when a charged particle in a periodic potential is subjected to a uniform electric field.) This behavior was confirmed by numerical propagation of a scalar field wavepacket on a falling lattice using the lattice field equation. Moreover, assuming the ingoing modes are in their quantum ground states, the Hawking radiation thermal occupation numbers for the outgoing modes were found as long as the lattice spacing was small compared with the length scale κ^{-1} of the black hole.

In this falling lattice model the slow time-dependent spreading of the lattice points is critical to producing the net frequency change of the mode that we know must occur if the Hawking effect is to take place. Such rarefaction of the lattice is an artificial feature of this simple 1+1 dimensional model however. Moreover, in any process involving modes with Planckian wavevectors, the linear model of the test field is surely unjustified. The back-reaction on the quantum gravity vacuum must be essential. It seems unavoidable to suppose that the fluctuations of the actual quantum gravity vacuum obviate the need for the explicit time dependence of the falling lattice, and that the dynamics of this vacuum provides a non-linear way of partially converting ingoing fluctuations to outgoing ones.

In conclusion, Lorentz violation is entirely compatible with the usual understanding of black holes in general relativity, including the Hawking effect, except in the case of superluminal dispersion with a group velocity that grows without bound at high wavevectors. In that case, modes emanating from the vicinity of the singularity would emerge from the black hole, their state having been determined by unknown quantum gravity processes.

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References

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