

On Hawking/Unruh Process: Where does the Radiation Come from?

Tadas K. Nakamura

February 18, 2013

CFAAS, Fukui Prefectural University, Fukui 910-1195, Japan (tadas@fpu.ac.jp)

Abstract

The energy source of the radiation in Unruh/Hawking process is investigated with emphasis on the particle number definition based on conservation laws. It has been shown that the particle radiation is not the result of pair creation by the gravitational force, but the result of difference in the conservation laws to define the particle number. The origin of the radiated energy in the distant future corresponds to the zero point oscillations with infinitely large wave numbers. This result implies the need of reconsideration on the scenario of black hole evaporation.

1 Introduction

The theory of quantum particle radiation by gravitational force [1, 2] is generally accepted as well established, however, several authors pointed out essential problems that might brow up the whole story [3, 4, 5] (see [6] for a review). What called trans-Plankian problem has been known from very early years [7]. In the last two decades, this problem has been investigated intensively [8, 9].

The derivation of Hawking/Unruh process is based on so called cis-Plankian physics, i.e., the theories of gravitation and quantum field we know at the present. It is believed these theories will break down beyond an extremely small scale, the Plank scale presumably, and we do not know what happens there. The unknown physics in that scale is called trans-Plankian physics. The calculation of Hawking/Unruh effect inevitably requires the wave modes with infinitesimally small wave length (see, e.g., [6]), therefore, we need to know the trans-Plankian physics to understand the radiation mechanism; this is the trans-Plankian problem. There has been attempts [9] to derive the radiation within the cis-Plankian scale, however, they are based on *ad hoc* assumptions yet to be tested experimentally.

Helfer [4] pointed out another issue we have to consider before the trans-Plankian problem. The radiation at later times must have plied up near the horizon at the time of black hole formation, and its backreaction is far from negligible to the black hole metric. This implies the theory of Hawking radiation is intrinsically inconsistent even within the framework of cis-Plankian physics, because such backreaction may completely destroy the black hole formation.

This is a serious problem. Papers on the trans-Plankian problem so far assume that the Hawking radiation is well predicted by the cis-Plankian physics, and discuss what will happen if we have to consider trans-Plankian effects. However, if cis-Plankian physics itself fails to derive the Hawking radiation, then we do not have any reason to believe the existence of radiation.

The purpose of the present paper is to formulate the Helfer's conclusion [4] from a different point of view. We wish to show the theory of black hole evaporation is inconsistent even if the cis-Plankian physics is valid up to infinitesimally small scale. To this end, we need another unknown physics within the cis-Plankian

regime: the effect of the zero-point oscillation to the gravity. We do not know the general theory for this, however, there can be two possibilities for the Hawking radiation.

One is such that the radiation can carry away the black hole energy to cause its evaporation. In this case, the backreaction of the quantum field is so large as to alter the black hole formation completely [4]. We will see in the present paper there can be another possibility that the field has no backreaction to the black hole geometry. In this case, however, there will be no black hole evaporation at all even with the existence of Hawking radiation. This may be plausible because there are considerable amount of observational evidence to believe the existence of black holes.

We take an approach a little different from the conventional quantization with creation/annihilation operators in the present paper; the mathematical structure is equivalent, but its interpretation is not the same. The quantization with creation/annihilation operators takes two different steps, transition from the classic to quantum field and introduction of the particle picture, namely, at the same time. The essential step in the canonical quantization method is to replace a classical Poisson bracket with a quantum commutation relation regarding the field as a collection of quantum operators. If conserved quantities have discrete eigenvalues with equal intervals, then we can construct the particle picture. It is well known the latter is not always possible in a curved spacetime.

It also should be noted these two steps do not have to be done at the same place even when we have the particle picture. The commutation relation must be given on a Cauchy surface on which the Poisson bracket is defined. The particle picture, in contrast, does not have to be on the same surface. It can be on some other spacelike surface, which does not have to be a Cauchy surface as long as there exist some conservation laws on it.

In the present paper, the essential quantization, i.e., definition of the commutation relation, will be done on the surface of constant time in Minkowski/Kruskal coordinates. Then the particle picture is introduced based on that quantization, not only on the same surface but also on the surface of constant time in Rindler/Schwarzschild coordinates.

The particle picture is based on conservation laws in general. What we directly measure is not the particle number itself, but some conserved quantity such as energy or electric charge. We imagine there are n particles, each of which carries a certain amount of conserved quantity, if the total of the quantity has discrete values proportional to an integer n .

This means the concept of “particle number” is defined by conserved quantities. If all the conserved quantities share the same n , then we can define one unique particle number, however, this is not the case. There can be several different definitions of particle numbers because there can be several different Killing vector fields that determine the conservation laws in a relativistic spacetime.

Consequently one physical state can have different particle numbers, and this is what is happening in the Unruh/Hawking process. Particles are not created in the literal meaning of “creation”, which means the particle number increases as time goes on. Rather, what takes place is just a difference of particle numbers caused by the difference in their definitions. This is in agreement with the result of Belinski [3] calculated from another viewpoint.

We will see in the present paper the radiation of particles comes from the vacuum state, i.e., zero particle state, of another kind of particle number. This is possible because a vacuum is not a completely empty space but has zero point oscillations. The continuous particle radiation can take place because the zero point oscillations exist up to infinitely large wave numbers, which means infinite amount of energy source.

The present paper is organized in the following. In section 2 we first review some basic concepts to clarify the procedure of quantization used in the later sections. We examine in Section 3 the case of Unruh process in a flat spacetime, since it has the two different types of conservation laws clearly defined; we can understand the problem with this simple analogy. We apply the results obtained in Section 3 to the case of Schwarzschild black holes in Section 4, and a brief summary is given in Section 5.

2 Basics

2.1 Time and Energy

The concept of energy is often used in a sloppy way, which sometimes leads to misconceptions. The integration of the energy-momentum tensor cannot be carried out in the curved spacetime in general, however, there can be well defined “energy” as a globally conserved quantity if there exists a Killing vector field. If a Killing vector ξ_ν is timelike, then the integration $\int_\Sigma \xi_\nu T^{\nu\mu} d\Sigma_\mu$ ($T^{\nu\mu}$: energy-momentum tensor) over an appropriate spacelike surface Σ is conserved with respect to the time evolution in ξ_ν . If there are several different timelike Killing vector fields, there can be the same number of corresponding energies; the energies defined by different Killing vector fields are different physical entities.

Sometimes this difference in energies is not well understood and causes confusion; one good example is an intuitive explanation of the Hawking radiation found in popular science books. It goes like: (1) a vacuum is not an empty space but filled with instantaneous pair production of virtual particles; (2) a pair of virtual particles can exist within a short time period of $\Delta t \sim \hbar/\Delta E$ because of the uncertain principle; (3) when such virtual particles are created near the event horizon, one of the pair may fall into the black hole across the horizon during the time interval of Δt ; (4) once a virtual particle crosses the horizon, its energy becomes negative; (5) then the other particle of the pair can have positive energy without violating the energy conservation law.

This explanation does not specify the Killing vector field with which the time and energy are defined. If the Killing vector is something like the Schwarzschild time, then a particle takes infinitely long time to reach the horizon, and cannot cross the horizon during the period of Δt . If, on the other hand, the Killing vector is such that a particle can cross the horizon within a finite period, then the corresponding energy does not change the sign on the other side of the horizon. The pair production near the horizon is not likely to occur to cause the Hawking radiation in both cases.

2.2 Conservation Laws and Particle Numbers

Usually the quantization process to investigate Hawking/Unruh process is based on the creation/annihilation operators defined by the negative/positive frequency modes. In the present paper we take one step backwards and perform the quantization by replacing the Poisson bracket with the commutation relation. Hereafter, let us use the word “quantization” with the meaning of the transition from the classical to quantum theory, and does not necessarily mean the particle picture.

The particle picture is derived from conserved quantities after the quantization. If the quantum observable of a conserved quantity has the structure of a harmonic oscillator, its eigenvalues are proportional to $n + \frac{1}{2}$ with $n = 0, 1, 2, \dots$. Usually the constant $\frac{1}{2}$ is subtracted out by normal ordering, thus the quantity is proportional to n . When there are several conserved quantities that share the same n for the same state, then we can interpret n as the particle number.

Suppose we establish quantization somehow, and find an observable \hat{a} (hat mark indicates a quantum operator) and its Hermite conjugate \hat{a}^\dagger have the following commutation relation

$$[\hat{a}, \hat{a}^\dagger] = \hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} = 1. \quad (1)$$

We use the unit system with $G = \hbar = c = 1$ throughout the present paper. The general theory of quantum harmonic oscillators tells us (see., e.g., [10]) an observable defined as

$$\hat{A} = \frac{A_0}{2} (\hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{a}) \quad (2)$$

has the eigenstates $|n_A\rangle$ that satisfies

$$\hat{A} |n_A\rangle = A_0 \left(n + \frac{1}{2} \right) |n_A\rangle, (n = 0, 1, 2, \dots), \quad (3)$$

if \hat{a} has the commutation relation of (1).

For the above argument the observable \hat{A} does not have to be related to the Hamiltonian explicitly (note: the Poisson bracket has something to do with the Hamiltonian implicitly), and there can be several choices for such observables. For example if we define a new observable \hat{b} as

$$\hat{b} = \alpha \hat{a} + \beta \hat{a}^\dagger \quad (4)$$

with $\alpha^2 - \beta^2 = 1$ then it also satisfies the commutation relation like (1) and thus $\hat{B} = \frac{1}{2}B_0(\hat{b}\hat{b}^\dagger + \hat{b}^\dagger\hat{b})$ has the eigenvalues $B_0(n + \frac{1}{2})$. It is easy to confirm its eigenstates $|n_B\rangle$ are not the eigenstates of \hat{A} , i.e., $\hat{A}|n_B\rangle \neq (n + \frac{1}{2})|n_B\rangle$, and vice versa.

Both pairs \hat{a}, \hat{a}^\dagger and \hat{b}, \hat{b}^\dagger have the structure of annihilation/creation operators, however, it is not enough for the particle picture. To construct the particle picture with \hat{a} and \hat{a}^\dagger , \hat{A} must obey a conservation law in time, i.e., $\partial\hat{A}/\partial t = 0$ (here we employ the Heisenberg picture) at least approximately. If \hat{A} rapidly changes even without interactions, so does n , and it is not appropriate to regard n as a particle number.

When the Hamiltonian does not depend on time explicitly, the condition of $\partial\hat{A}/\partial t = 0$ is equivalent to the following commutation relation:

$$[\hat{A}, \hat{H}] = 0. \quad (5)$$

Obviously the Hamiltonian itself satisfies this condition, therefore, the Hamiltonian is usually used to introduce the particle picture. Then the operators \hat{a} and \hat{a}^\dagger become the amplitudes of wave modes with positive and negative frequencies respectively, which are usually used in the procedure of quantization as the annihilation and creation operators. Therefore, the mathematical structure in the present paper is equivalent to the one in the conventional method.

If there are other conserved observables with respect to the time t , then they share the same set of eigenstates with the Hamiltonian \hat{H} because of the commutation relation (5). Therefore n can be regarded as the particle number without specifying the conserved quantities, as long as the conservation laws are on the same time evolution of t . Especially, the ground state of the observables is uniquely determined, and we call it “vacuum”. We can define the number operator as

$$\hat{N} = \hat{a}^\dagger \hat{a}, \quad (6)$$

whose eigenvalue is the particle number n , and the vacuum means the eigenstate with $n = 0$.

However, in relativistic spacetimes there can be several different types of time evolution with different sets of conservation laws because several different Killing vector fields can exist; the Minkowski and Rindler times in a flat spacetime are a good example.

If two different types of time evolution have their own conservation laws, then the conserved quantities that belong to different time evolution may have different sets of eigenstates. Consequently the ground state in one time evolution is not the ground state in another, in other words, they have different vacuum states. This is what causes Hawking/Unruh radiation as we will see in the next section.

3 Unruh process

3.1 Minkowski Coordinates

Suppose the following real valued Klein-Gordon equation in a two dimensional Minkowski spacetime where t and x are the time and space coordinates:

$$\phi_{,tt} - \phi_{,xx} = 0. \quad (7)$$

We write $\partial\phi/\partial t = \phi_{,t}$ etc. in shorthand. We take the Cauchy surface for the canonical dynamics as the one defined with $t = \text{constant}$, then the Hamiltonian may be written as

$$H = \int_{-\infty}^{\infty} \frac{1}{2} [\phi_{,t}^2(t, x) + \phi_{,x}^2(t, x)] dx. \quad (8)$$

We expand the field as

$$\phi(x, t) = \int [a(k) u(k; x, t) + a^*(k) u^*(k; x, t)] dk, \quad (9)$$

with mode functions

$$u(k; x, t) = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega t + ikx}, \quad (10)$$

where $\omega = |k|$ and the asterisk indicates complex conjugate. Precisely speaking, a_k diverges to infinity as we usually encounter in the Fourier transform; we assume some appropriate prescription, such as the distribution/hyperfunction formulation, has been applied to avoid this difficulty in this paper.

The essential transition from the classical to the quantum field is done by replacing $a(k)$ and $a^*(k)$ with the operators satisfying the commutation relation of

$$[\hat{a}(k), \hat{a}(k')^\dagger] = \hat{a}(k) \hat{a}^\dagger(k') - \hat{a}^\dagger(k) \hat{a}(k') = \delta(k - k'). \quad (11)$$

The above quantization is based on the Cauchy surface of $t = \text{constant}$. It should be noted that the quantization in this paper takes place only once at this point. Later we introduce the particle picture on the surface of constant Rindler time, but it is expressed by a linear superposition of \hat{a} and \hat{a}^\dagger and based on the commutation relation defined here.

The Hamiltonian (8) can be expressed as a collection of quantum harmonic oscillators:

$$\hat{H} = \int \frac{\omega}{2} [\hat{a}(k) \hat{a}(k)^\dagger + \hat{a}^\dagger(k) \hat{a}(k)] dk. \quad (12)$$

Therefore, we can define the particle number as explained in the previous section, and ground state of \hat{H} is called “vacuum”.

Now that we have the quantized operators $\hat{a}(k)$ and $\hat{a}^\dagger(k)$, we can calculate the field $\hat{\phi}(x, t)$ at any point of the spacetime as a quantum observable. Any classical quantity defined from the classical field ϕ can be quantized by replacing

$$\phi \rightarrow \hat{\phi} = \int [\hat{a}(k) u(k; x, t) + \hat{a}^\dagger(k) u^*(k; x, t)] dk. \quad (13)$$

3.2 Rindler Coordinates

The Hamiltonian H in (8) is the energy with the conservation law based the Killing vector field of Minkowski time ∂_t . We examine in the following another conservation law resulting from another Killing vector field $\kappa x \partial_t - \kappa t \partial_x$, where κ is a real constant that corresponds to the relativistic acceleration. The energy M for this Killing vector field is written in the classical field theory as

$$M = \int_{\Sigma(\eta)} \frac{1}{\kappa(t^2 - x^2)} (t\phi_x^2 + x\phi_t^2) d\Sigma, \quad (14)$$

where $\Sigma(\eta)$ is a surface specified by $t/x = \tanh(\kappa\eta)$ and $x > 0$. Clearly M is not the same quantity as H , therefore, let us distinguish M and H by calling them “Rindler energy” and “Minkowski energy” respectively.

The density of the Rindler energy is conserved locally, and there is no Rindler energy flow across the left and right Rindler wedges, thus we have $\partial M / \partial \eta = 0$; once we calculate M on a surface $\Sigma(\eta)$ with a given

η , then the result holds for all η . When we choose $\eta = 0$, we can express M with the coefficients $a(k)$ of the Minkowski modes. Since M is quadratic in ϕ we can write

$$M = \iint [A(k, k') a(k) a(k') + B(k, k') a(k) a^*(k') + C(k, k') a^*(k) a^*(k')] dk dk', \quad (15)$$

with coefficients A , B , and C that do not depend on η or t .

The above quantity is quantized by replacing $a(k) \rightarrow \hat{a}(k)$ and $a^*(k) \rightarrow \hat{a}^\dagger(k)$ as

$$\hat{M} = \iint \left[A \hat{a}(k) \hat{a}(k') + \frac{B}{2} (\hat{a}(k) \hat{a}^\dagger(k') + \hat{a}^\dagger(k) \hat{a}(k')) + C \hat{a}^\dagger(k) \hat{a}^\dagger(k') \right] dk dk'. \quad (16)$$

As noted before, this quantization is based on the Cauchy surface of $t = \text{constant}$, not $\Sigma(\eta)$. Therefore, $\Sigma(\eta)$ does not have to be a Cauchy surface.

What we wish to show in the following is that the particle numbers defined by \hat{M} and \hat{H} are not the same. Before that, we have to show that \hat{M} surely can define the particle number. Suppose an operator \hat{b} is defined as a linear superposition of \hat{a} and \hat{a}^\dagger as

$$\hat{b}(p) = \int [\alpha(p, k) \hat{a}(k) + \beta(p, k) \hat{a}^\dagger(k)] dk. \quad (17)$$

If \hat{b} satisfies the commutation relation

$$[\hat{b}(p), \hat{b}^\dagger(p')] = \delta(p - p'), \quad (18)$$

and \hat{M} can be expressed with \hat{b} as

$$\hat{M} = \frac{1}{2} \int \sigma [\hat{b}(p) \hat{b}^\dagger(p) + \hat{b}^\dagger(p) \hat{b}(p)] dp, \quad (19)$$

then we can define the particle number with \hat{M} in the similar way as done in the previous subsection with \hat{H} .

It is possible show the above two equations with direct calculation, however, it is easier to use the following Rindler coordinates (η, ρ) for the right Rindler wedge i.e., region of $x > 0$, $|x| > |t|$:

$$t = \rho \sinh(\kappa\eta), \quad x = \rho \cosh(\kappa\eta). \quad (20)$$

Then the Rindler energy \hat{M} may be written as

$$\hat{M} = \int_0^\infty \frac{1}{\kappa\rho} (\hat{\phi}_{,\eta}^2 + \kappa^2 \rho^2 \hat{\phi}_{,\rho}^2) d\rho. \quad (21)$$

We introduce the eigenfunctions

$$v(p; \eta, \rho) = \frac{1}{\sqrt{4\pi\sigma}} \exp(-i\sigma\eta + i\kappa^{-1}p \ln(\kappa\rho)), \quad (22)$$

with $\sigma = |p|$. The wave function $\hat{\phi}$ can be expanded in the right Rindler wedge as

$$\hat{\phi} = \int [\hat{b}(p) v(p) + \hat{b}^\dagger(p) v^*(p)] dp, \quad (23)$$

then \hat{M} in (21) can be cast into (19). In this context α and β in (17) are equivalent to the Bogolubov coefficients that satisfy

$$\int [\alpha(p_1, k)\alpha^*(p_2, k) - \beta(p_1, k)\beta^*(p_2, k)] dk = \delta(p_1 - p_2), \quad (24)$$

The above property combined with (11) yields the commutation relation of (18), therefore we can define particle numbers with \hat{M} .

What we do next is to compare the eigenstates of \hat{M} and \hat{H} . From (16) it is clear that \hat{M} and \hat{H} do not share the same set of eigenstates unless A and C vanishes. We can see from (17) and (15) that A and C vanished when $\beta = 0$, however, β can be evaluated by a straightforward integration (see, e.g., [11]), resulting

$$|\beta(p, k)|^2 = \frac{1}{2\pi\omega} \frac{1}{e^{2\pi\sigma/\kappa} - 1} \neq 0. \quad (25)$$

Therefore, the eigenstates of \hat{M} are not the eigenstates of \hat{H} . This means that the ground state of \hat{H} is not the ground state of \hat{M} , which can be stated in other words as “a vacuum defined by the Minkowski energy is not a vacuum defined by the Rindler energy”. The expected value of the particle number defined by Rindler energy (we call Rindler particle number hereafter) in the Minkowski vacuum can be calculated using (25), resulting the well known Plankian distribution [11].

What we have seen above is not surprising since different operators may have different sets of eigenstates. However, it contradicts with the picture of the pair production by the gravitational force often found in intuitive explanations like the one in Section 2. The particle number defined by Minkowski energy (Minkowski particle number hereafter) is zero for all t , and the Rindler particle number has the fixed Plankian distribution for all η . This is consistent with the time symmetry; the vacuum in the flat spacetime must be symmetric in time, both in t and η , but the particle creation process is not symmetric.

3.3 Origin of the Radiation

The Rindler particle number in the Unruh process is calculated by the expected value of the Rindler energy for the ground state of the Minkowski energy, i.e., $\langle 0_H | \hat{M} | 0_H \rangle$. If the “vacuum” were a completely empty space, i.e., $\hat{a} | 0_H \rangle = \hat{a}^\dagger | 0_H \rangle = 0$, then $\langle 0_H | \hat{M} | 0_H \rangle$ would vanish. However, this is not true since the quantum ground state has zero point oscillation, and hence $\langle 0_H | \hat{M} | 0_H \rangle \neq 0$. This means the “particles” found in the Unruh process comes from the zero point oscillation of the Minkowski modes.

Then what we wish to know is the properties of zero point oscillations that contribute to the continuous radiation of Rindler energy. In the present paper we concentrate on right moving waves, i.e., $\omega k < 0$ or $\sigma p < 0$, since their analog in the Schwarzschild spacetime play the key role in the black hole evaporation. It should be noted, however, left moving waves are also problematic and should be examined in the next step. We first examine the properties of waves in the classical limit, and then apply the result to the quantum vacua.

To begin with, we observe that the a eigenmode (22) has infinite Rindler energy in a finite region of $0 \leq \rho < \rho_c$ with arbitrary position ρ_c in the right Rindler wedge; this can be confirmed by the following direct integration:

$$\lim_{\epsilon \rightarrow 0} \int_{\epsilon}^{\rho_c} \frac{1}{\kappa \rho} (\hat{\phi}_{,\eta}^2 + \kappa^2 \rho^2 \hat{\phi}_{,\rho}^2) d\rho \rightarrow \infty. \quad (26)$$

Also it is easy to confirm there is a constant rightward outflux of the Rindler energy at $\rho = \rho_c$ by direct calculation. This outflux comes from the region of $0 \leq \rho < \rho_c$, but the total Rindler energy can be conserved because the amount of the Rindler energy in that region is infinitely large. Belinski [3] considered this fact as physically unacceptable and concluded the radiation results from $v(p)$ is just a mathematical illusion.

The present paper takes a different interpretation. The infinite Rindler energy can be physically real as long as we believe zero point oscillations exist for any high frequency modes, because the collection of such oscillations has infinitely large Rindler energy even in a finite volume.

To see this, we examine the behavior of the a wave packet in the following form (“*c.c.*” means complex conjugate):

$$\phi(\eta, \rho) = \exp\left(\frac{-(\rho - e^{\kappa\eta} \rho_0)^2}{(e^{\kappa\eta} s_0)^2}\right) \exp[-i\sigma\eta + i\kappa^{-1}p \ln(\kappa\rho)] + c.c. . \quad (27)$$

This wave packet was initially localized around $\rho = \rho_0$ with width s_0 at $\eta = 0$, and propagates rightward. The width of the packet becomes larger and the wave number becomes smaller as a result of wave propagation.

The wave packet can be expanded by the Minkowski modes $u(k; t, x)$ as

$$\phi(t, x) = \int \left[(\phi, u(k)) u(k; t, x) + (\phi, u^*(k)) u^*(k; t, x) \right] dk \quad (28)$$

with the Klein-Gordon inner products (ϕ_1, ϕ_2) , which can be calculated at $t = \eta = 0$ as

$$(\phi_1, \phi_2) = \int \left[\phi_{1,t} \phi_2^* - \phi_1 \phi_{2,t}^* \right]_{t=0} dx . \quad (29)$$

When $s \ll \rho_0 \ln(\kappa\rho_0)$ then we can approximate

$$\phi(\eta, \rho) \simeq \exp\left(\frac{-(\rho - \rho_1)^2}{s_1^2} - ip\eta + \frac{ip}{\kappa} \left[\ln(\kappa\rho_1) + \frac{1}{\rho_1} (\rho - \rho_1) \right] \right) + c.c. , \quad (30)$$

where $s_1 = s_0 e^\eta$ and $\rho_1 = \rho_0 e^\eta$ are the width and center of the wave packet at a time η . Using the above approximation we obtain

$$\phi(t, x) = \frac{2}{s_0 \sqrt{\pi}} \exp\left[i\kappa^{-1}p \ln(\kappa\rho_0)\right] \int e^{-ik\rho_0} \exp\left[-s_0^2(k - p/\kappa\rho_0)^2\right] e^{-i\omega t + ikx} dk + c.c. \quad (31)$$

from (28) with (29). The above expression means that the wave packet comes from the Minkowski modes with wave number around $k_0 = p/\kappa\rho_0$ when $s_0 \gg \kappa\rho_0/p$.

Suppose we find a wave packet around ρ_1 at a given time $\eta = \eta_1 (> 0)$ in the Rindler space then its position at $\eta = 0$ was $\rho_0 = \rho_1 e^{-\kappa\eta_1}$, therefore, the packet consists of the Minkowski modes with $k \sim k_0 = p e^{\kappa\eta_1}/\kappa\rho_0$. When we regard the wave field at $\eta = \eta_1$ as a superposition of such wave packets, we see that the waves in the region of $0 < \rho < \rho_1$ at $\eta = \eta_1$ consists of Minkowski modes with wave numbers larger than $k_0 = p e^{\kappa\eta_1}/\kappa\rho_0$.

Since $k_0 \rightarrow \infty$ in the limit of $\eta_1 \rightarrow \infty$, we understand the Rindler energy radiation at the distant future in η comes from the Minkowski modes with infinitely large wave numbers. The Rindler coordinates represent an observer with constant acceleration, and the relative velocity of the accelerating observer to the rest frame becomes infinitely large in the limit of $\eta \rightarrow \infty$. Waves with finite wave numbers in this limit are infinitely red shifted, therefore its original wave number must have been infinitely large.

Now let us apply the above observation to quantum vacua to see the origin of the Rindler particles. Suppose the quantum state is Minkowski vacuum, i.e., the ground state of the Minkowski energy. Then the state has zero point oscillation up to infinitely large wave numbers. Usually the energy of these zero point oscillation is subtracted out by normal ordering, and we regard there is no particle in the ground state. However, the ground state of the Minkowski energy is not the ground state of the Rindler energy, which means the existence of the Rindler particles. The continuous radiation of Rindler particles is possible for any large η because the zero point Minkowski energy exist for modes with any large wave numbers. The radiated Rindler energy must have been piled up near $\rho = 0$ at the initial time of $\eta = 0$, since there is no particle creation as we have seen in the previous subsection.

4 Hawking Radiation

Let us move on to the Hawking radiation from a Schwarzschild black hole in this section. We introduce the Schwarzschild coordinates (t, r, θ, φ) whose metric is

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \cos^2 \theta d\varphi. \quad (32)$$

The Kruscal coordinates (u, v, θ, φ) are related to (t, r, θ, φ) as

$$u^2 - v^2 = 2M(2M - r) \exp\left(\frac{r}{2M}\right), \quad (33)$$

$$\left|\frac{u - v}{u + v}\right| = \exp\left(\frac{t}{2M}\right). \quad (34)$$

The rest of the coordinates, θ and φ , are unchanged.

It is generally accepted that the quantum properties of vacuum near the Schwarzschild event horizon is essentially the same as those in the Rindler spacetime [2, 12], therefore, the results we obtained in the previous section are basically valid by replacing Minkowski/Rindler coordinates with Kruscal/Schwarzschild coordinates (note: t in the Schwarzschild spacetime corresponds to η , not t , in the flat spacetime). There are, however, two fundamental differences. One is the definition of the energy in Kruscal coordinates, and the other is the backreaction of the quantum fields to the black hole metric. The latter causes the essential problem in the scenario of the black hole evaporation.

The first difference is about the energy that corresponds to the Minkowski energy. The Kruscal time u is not a global Killing time, and thus there is no global energy conservation law like for the Minkowski energy. However, u can be approximately regarded as a Killing time near the horizons. As we have seen in the previous section, the radiation in later time comes from the infinitely high frequency modes infinitesimally near the horizon, therefore the energy is conserved approximately for these waves.

This is in parallel to the approximation of geometrical optics used by Hawking in his original paper [1]; geometrical optics assumes locally constant frequency, which means locally constant energy. Hereafter we assume the energy corresponds to the Kruscal time u is approximately conserved, and treat it in the same way as for the Minkowski energy in the previous subsection. We call it Hartle-Hawking energy since its ground state is often called Hartle-Hawking vacuum. The energy defined with the Schwarzschild time is called Boulware energy hereafter for the same reason.

We have another problem in the definition of the Hartle-Hawking energy in a Schwarzschild spacetime. Minkowski energy is defined as an integration over a surface of $t = \text{constant}$ in a flat spacetime. If we introduce a similar definition for the Hartle-Hawking energy with Kruscal time $u = \text{constant}$, the energy would include the part of the white hole in the extended Schwarzschild spacetime. This difficulty may be avoided by analyzing the black hole formation process by a star collapse, or the analytical continuation method proposed by Hartle and Hawking [13]. A detailed analysis on this point will be given in a forthcoming paper of the author.

The second difference is far more serious; the energy of zero point oscillations may change the metric. Usually the zero point energy is subtracted out by normal ordering in the source term of the Einstein equation, and the vacuum does not have effect on the metric. This means only the excited state of the energy can cause gravitation. However, as we have seen in the previous section, the ground state of Hartle-Hawking energy is not the ground state of the Boulware energy, and vice versa.

In a flat spacetime we consider the ground state of the Minkowski energy is the state of no gravitation, because the Minkowski coordinates are the “natural” coordinate system. We have seen there is infinite accumulation of the Rindler energy near $\rho = 0$ for a Rindler mode $v(p; \eta, \rho)$. There must be an infinitely

strong source of gravitational force at $\rho = 0$ if we assume the ground state of the Rindler energy is the state of no gravitation, since the vacuum state defined by the Minkowski energy is the excited state of the Rindler energy. This is not plausible, and we can conclude the ground state of the Minkowski energy has no effect on the metric.

In contrast, we do not know which coordinate system is “natural” to calculate the energy (stress-energy tensor) for a curved spacetime in general (see, e.g., [14]). The Schwarzschild coordinates are implicitly assumed to be “natural” in the scenario of the black hole evaporation, in other words, excited states of Boulware energy causes the gravity. The black hole evaporation is believed to be the result of the Hawking radiation that carries the energy away from the black hole, and the energy in this context is the Boulware energy; this means the Boulware energy can have effects on the black hole metric somehow.

If this is true, however, the Rindler energy radiated at later times must have been exist just outside of the horizon from the beginning [3]. The energy does not come from inside the black hole, but comes from the Minkowski modes with extremely high wave numbers. This means the backreaction of the quantum field is far from negligible to the black hole metric [4].

On the contrary, we can imagine the gravity is caused by the excited state of Hartle-Hawking energy and its ground state has no effect on the metric. The black hole can exist in this case, however, it cannot evaporate. There exists a constant outflow of Boulware energy, but it is the ground state of the Hartle-Hawking energy and does not have a backreaction on the metric. Consequently the black hole metric is unchanged at all, just like the Unruh process does not change the flat metric. There can be other possibilities for the effect of the zero point energy to the metric, however, it is hard to imagine there is an extremely convenient case which is favorable for the scenario of black hole evaporation.

5 Summary

What we have seen in the present paper are:

1. The ground state of Minkowski/Hartle-Hawking energy is not the ground states of Rindler/Boulware energy, and this is what causes the Hawking/Unruh process.
2. The quantum state is unchanged and particles pairs are not created in any coordinate system; the number of Minkowski/Hartle-Hawking particles is zero and number of Rindler/Boulware particles has time stationary Plankian distribution all through the time, where “time” means the Rindler/Schwarzschild time .
3. The radiation of Rindler/Boulware energy in the distant future of Rindler/Schwarzschild time comes from the zero point oscillation of Minkowski/Hartle-Hawking energy with infinitely large wave frequencies.
4. The effect of zero point energy to the metric is not known, however, we have the following two possibilities for a Schwarzschild spacetime. The scenario of black hole evaporation is inconsistent in both cases.
 - (a) If the Hawking radiation causes the black hole evaporation, it means the excitation in the Boulware energy can cause the metric change. The Boulware energy radiated later time was accumulated near the horizon at the initial time, whose existence essentially alter the Schwarzschild metric from the beginning.
 - (b) If, on the contrary, the Boulware energy of the Hawking radiation does not affect the metric, then the Schwarzschild metric can exist as we expect, but exists forever. There is no evaporation of the black hole.

We started the present study by assuming that the cis-Planckian physics is valid for any small scale phenomena, and end up with the inconsistency of black hole evaporation. This fact means we have no reason to believe the black hole evaporation. A new theory of physics in trans-Planckian scale may save the evaporation, but may not; we can imagine anything, but cannot believe. What we can say for sure is that the physics we know at the present is not able to predict the black hole evaporation, if the calculations in the present paper are correct.

We see the scenario of black hole evaporation is inconsistent, however, we do not know what is the consistent theory even within the cis-Planckian regime. The problem deeply depends on the renormalization procedure in curved spacetimes, to which we do not know the answer yet. It is often said Hawking process can be a touchstone for the theory of quantum gravity. The author of the present paper would like to say it also can be a touchstone for the renormalization theory, or theory on what is avoided by renormalization at the present, in curved spacetimes.

References

- [1] Hawking, S. W., *Comm. Math. Phys.* **43**, 199 (1975).
- [2] Unruh, W. G., *Phys. Rev.* **D14**, 870 (1976).
- [3] Belinski, V. A., *Phys. Lett.* **A209**, 13 (1995); *Phys Lett.* **A354**, 249 (2007).
- [4] Helfer, A. D., [gr-qc/0008016](#).
- [5] Helfer, A. D., *Int. J. Mod. Phys.* **D13**, 2299 (2004), [gr-qc/0503052](#).
- [6] Helfer, A. D., *Rept.Prog.Phys.* **66**, 943 (2003), [gr-qc/0304042](#).
- [7] Gibbons, G. W., in *Proc. First Marcel Grossman Meeting on General Relativity*, ed. R. Ruffini, 499 North-Holland (1977).
- [8] Jacobson, T., *Phys. Rev.* **D44**, 1731 (1991); *Phys. Rev.* **D48**, 728 (1993); *Prog The Phys Suppl* **136**, 1 (1999), [hep-th/0001085](#).
- [9] Unruh, W. G., *Phys. Rev.*, **D51**, 2827 (1995); Brout, R., Massar, S., Parentani, R., and Spindel, Ph., *Phys. Rev.* **D52**, 4559 (1995), [hep-th/9606121](#); Corely, S. and Jacobson, T., *Phys. Rev.*, **D53**, 6720, [hep-th/9601973](#); Himemoto, Y. and Tanaka, T., *Phys. Rev.* **D61**, 064004, [gr-qc/9904076](#); Saida, H. and Sakagami, M, *Phys. Rev.* **D61**, 084023, [gr-qc/9905034](#).
- [10] Messiah, A., *Quantum Mechanics*, North Holland, (1961).
- [11] Wipf, A., in *Black Holes: Theory and Observation, Proc. 179th W. E. Heraeus Seminar*, ed. F. W. Hehl, C. Kiefer, and R. J. K. Metzler, 385, Springer (1998), [hep-th/9801025](#); DeWitt, B., *The Global Approach to Quantum Field Theory*, Oxford (2003).
- [12] Fulling, S. A., *J. Phys.* **A10**, 917 (1977); Wald, R. M., *Quantum Field in Curved Spacetime and Black Hole Thermodynamics*, U. Chicago Press (1994).
- [13] Hartle, J. B., and Hawking, S. W., *Phys. Rev.* **D13**, 2188 (1976).
- [14] Birrell, N. D., and Davies, P. C. W., *Quantum Fields in Curved Space*, Cambridge Univ. Press (1982); Fulling, S. A., *Aspects of Quantum Field Theory in Curved Space-Time*, Cambridge Univ. Press (1989).