

Massive particles' Hawking radiation via tunneling from the G.H Dilaton black hole

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In the past, Hawking radiation was viewed as a tunneling process and the barrier was just created by the outgoing particle itself. In this paper, Parikh's recent work is extended to the case of massive particles' tunneling. We investigate the behavior of the tunneling massive particles from a particular black hole solution-G.H Dilaton black hole which is obtained from the string theory, and calculate the emission rate at which massive particles tunnel across the event horizon. We obtain that the result is also consistent with an underlying unitary theory. Furthermore, the result takes the same functional form as that of massless particles.

Key words: Hawking radiation, tunneling process, emission rate.

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I. INTRODUCTION

In 2000, Parikh and Wilczek proposed a new method to calculate the emission rate at which particles tunnel across the event horizon[1, 2, 3, 4]. The key points in their method were that they treated Hawking radiation as a tunneling process and considered that the barrier was just created by the outgoing particle itself, in particular, they found a coordinate system well-behaved at the event horizon to calculate the emission rate[7, 8]. In this way they obtained the corrected emission spectrum of the particles from the spherically symmetric black holes, such as Schwarzschild black hole and Reissner-Norstrom black hole. It's found that their results were consistent with an underlying unitary theory. However, the particles which they have treated all are massless and follow the radial lightlike geodesics when they tunnel across the horizon, while the massive particles don't follow the radial lightlike geodesics. In this paper, we extend Parikh's method to a particular black hole solution-G.H Dilaton black hole[10], and investigate the massive particles' tunneling[5, 6]. Then, we calculate the emission rate at the event horizon of the G.H Dilaton black hole. During the calculation, for the sake of simplicity, we consider the outgoing massive particle as a massive shell (de Broglie s-wave), and the phase velocity and group velocity of the de Broglie wave corresponding to the outgoing particle are respectively obtained.

The rest of the paper is organized as follows. In section 2, we first introduce a Painleve-G.H Dilaton coordinate system[10], and then investigate the behavior of the massive tunneling particles. In section 3, we calculate the emission rate at which massive particles tunnel across the event horizon of G.H Dilaton black hole. Finally, in section 4, we give a brief conclusion and discussion.

II. PAINLEVE COORDINATES AND THE BEHAVIOR OF THE MASSIVE TUNNELING PARTICLES

The G.H Dilaton black hole metric is[9, 10]

$$ds^2 = e^{2U} dt_s^2 - e^{-2U} dr^2 - R^2(r)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

where

$$\begin{aligned} e^{2U} &= \left(1 - \frac{r_H}{r}\right) \left(1 - \frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}}, \\ R &= r \left(1 - \frac{r_-}{r}\right)^{\frac{a^2}{1+a^2}}. \end{aligned} \quad (2)$$

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and r_H , r_- are respectively the event horizon and the inner horizon, constant a is a coupling coefficient. The mass and the charge of the black hole are respectively

$$\begin{aligned} M &= \frac{r_H}{2} + \frac{1-a^2}{1+a^2} \frac{r_-}{2}, \\ Q^2 &= \frac{r_H r_-}{1+a^2}. \end{aligned} \quad (3)$$

It is easy to find that the metric in (1) is singular at the location of the event horizon, $r = r_H$. As mentioned in sec.1, we should first find a coordinate system to calculate the emission rate which is well-behaved at the event horizon. It's found that the Painleve coordinate system is convenient[11]. If let $dt = dt_s + \frac{1}{\Delta} \frac{\sqrt{g}}{1-g} dr$, where $g = \frac{r_-}{r}$, $\Delta = (1 - \frac{r_-}{r})^{\frac{1-a^2}{1+a^2}}$, we can read the Painleve-G.H Dilaton line element[10]

$$\begin{aligned} ds^2 &= (1-g)\Delta dt^2 - 2\sqrt{g}\Delta dt dr - \frac{1}{\Delta} dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) \\ &= g_{00}dt^2 + 2g_{01}dt dr - \frac{1}{\Delta} dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2). \end{aligned} \quad (4)$$

The metric (4) displays the stationary, nonstatic, and nonsingular nature of the space time. Moreover, we find another important feature which we will describe as follows.

As we know, according to Landau's theory of the coordinate clock synchronization, the difference of coordinate times of two events taking place simultaneously in different place is[12]

$$\Delta T = - \int \frac{g_{0i}}{g_{00}} dx^i. \quad (5)$$

where $i=1,2,3$. And we consider the space-time has been decomposed in 3+1. So, if the simultaneity of coordinate clocks can be transmitted from one place to another and have nothing to do with the integration path, the components of the metric should satisfy[15]

$$\frac{\partial}{\partial x^j} \left(\frac{g_{0i}}{g_{00}} \right) = \frac{\partial}{\partial x^i} \left(\frac{g_{0j}}{g_{00}} \right) \quad (i, j = 1, 2, 3). \quad (6)$$

It's easy to find that the line element (4) satisfies condition (6). That is, the coordinate clock synchronization in the Painleve coordinates can be transmitted from one place to another, though the line element is not diagonal. In quantum mechanics, it is an instantaneous process when particle tunnels across a barrier. Thus, this feature is necessary for us to discuss the tunneling process.

From (4) we can easily obtain the radial null geodesics[10]

$$\dot{r} \equiv \frac{dr}{dt} = -\Delta\sqrt{g} \pm \Delta. \quad (7)$$

where the upper(lower) sign corresponds the outgoing(ingoing) geodesic.

But in this paper we consider the tunneling of massive particles. When the massive particles tunnel across the horizon, they do not follow the radial lightlike geodesics in (7). In order to obtain the \dot{r} of massive particles, and for the sake of simplicity, we consider the outgoing massive particle as a massive shell (nonrelativistic de Broglie s-wave). According to the WKB formula, the approximative wave equation is[5, 6]

$$\Psi(r, t) = C e^{i(\int_{r_i-\varepsilon}^r p_r dr - \omega t)}. \quad (8)$$

where $r_i - \varepsilon$ represents the initial location of the particle. Using the non-relativistic quantum mechanics, and if we let

$$\int_{r_i-\varepsilon}^r p_r dr - \omega t = \phi_0, \quad (9)$$

then, we have

$$\frac{dr}{dt} = \dot{r} = \frac{\omega}{k}. \quad (10)$$

where k is the de Broglie wave number. Comparing the definition of the phase velocity, we find that \dot{r} is just the phase velocity of the de Broglie wave. For the nonrelativistic de Broglie wave, the definitions of the group velocity v_g and the phase velocity v_p , and the relationship between them are[16]

$$v_p = \frac{dr}{dt} = \dot{r} = \frac{\omega}{k}, \quad (11)$$

$$v_g = \frac{dr_c}{dt} = \frac{d\omega}{dk}, \quad (12)$$

$$v_p = \frac{1}{2}v_g. \quad (13)$$

where r_c is the location of the particle. In order to obtain the formula of the phase velocity \dot{r} , let us first investigate the behavior of a massive particle tunneling across the horizon[5, 6].

Since tunneling across the barrier is an instantaneous process, there are two simultaneous events during the process of emission. One event is particle tunneling into the barrier, and the other is particle tunneling out the barrier. In terms of Landau's theory of the coordinate clock synchronization, the difference of coordinate times of these two simultaneous events is

$$dt = -\frac{g_{0i}}{g_{00}}dx^i = -\frac{g_{01}}{g_{00}}dr_c \quad (d\theta = d\varphi = 0), \quad (14)$$

so the group velocity is

$$v_g = \frac{dr_c}{dt} = -\frac{g_{00}}{g_{01}}, \quad (15)$$

and therefore the phase velocity is

$$v_p = \dot{r} = \frac{1}{2}v_g = -\frac{1}{2}\frac{g_{00}}{g_{01}}. \quad (16)$$

Substituting g_{00} and g_{01} into (16), we obtain the expression of \dot{r}

$$\dot{r} = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \frac{1}{2}\frac{(1-g)\Delta}{\sqrt{g}}. \quad (17)$$

Here (17) is corresponding to the outgoing motion of the massive particles, and we don't consider the effect of self-gravitation. If the self-gravitation is included, (17) should be modified by replacing M with $M - \omega$, where ω is the particle's energy.

III. MASSIVE PARTICLES' TUNNELING AND THE EMISSION RATE

For a positive-energy s-wave, the rate of tunneling Γ could take the form

$$\Gamma \sim \exp(-2\text{Im}I). \quad (18)$$

where $I(r)$ is the action. According to the WKB approximation, the action has been found to have a conveniently simple form[1]

$$I = \int_{r_i}^{r_f} p_r dr = \int_{r_i}^{r_f} \int_0^{p_r} dp_r' dr. \quad (19)$$

where p_r is the radial momentum. And r_i is the initial radius corresponding the site of pair-creation, which should be slightly inside the event horizon r_H , while r_f is the final radius, which is slightly outside the final position of the horizon.

Using the Hamilton's equation $\frac{dH}{dp_r} = \dot{r}$, and substituting (17) into (19), the action will be

$$I = \int_{r_i}^{r_f} p_r dr = \int_{r_i}^{r_f} \int_{M_i}^{M_f} \frac{dM}{\dot{r}} dr = \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{2\sqrt{rr_H} dr dM}{(r - r_H)(1 - \frac{r_-}{r})^{\frac{1-a^2}{1+a^2}}}. \quad (20)$$

where the Hamiltonian $H = M$, M_i and M_f are respectively the initial mass and the final mass of the black hole.

From (18), we get that we can just consider the imaginary part of the action. In (20) if we do the r integral first, and then we can obtain the imaginary part of the action

$$\text{Im}I = -\pi \int_{M_i}^{M_f} \frac{2r_H}{(1 - \frac{r_-}{r_H})^{\frac{1-a^2}{1+a^2}}} dM = -\pi \int_{M_i}^{M_f} \frac{2r_H^{\frac{2}{1+a^2}}}{(r_H - r_-)^{\frac{1-a^2}{1+a^2}}} dM. \quad (21)$$

In order to calculate (21) conveniently, we can change the variant M into r_H [10, 13], which we can get from (2) (3)

$$dM = \frac{r_H^2 - (1 - a^2)Q^2}{2r_H^2} dr_H. \quad (22)$$

Substituting (22) into (21) yields

$$\text{Im}I = \int_{r_i}^{r_f} k(r_H) dr_H, \quad (23)$$

where

$$k(r_H) = -\pi r_H^{\frac{1-3a^2}{1+a^2}} \frac{r_H^2 - Q^2(1 - a^2)}{[r_H^2 - Q^2(1 + a^2)]^{\frac{1-a^2}{1+a^2}}}. \quad (24)$$

The integral in (23) is not easy to calculate directly. However, what we need is not the direct result but the comparison between $\text{Im}I$ and ΔS . So first we can expand $k(r_H)$ at the near field of r_i as a Taylor series[10, 13]

$$k(r_H) = k(r_i) + k'(r_i)(r_H - r_i) + \frac{k^{(2)}(r_i)}{2}(r_H - r_i)^2 + \dots \quad (25)$$

Substituting (25) into (23), we obtain

$$\text{Im}I = k(r_i)\Delta r_H + \frac{1}{2}k'(r_i)(\Delta r_H)^2 + \frac{k^{(2)}(r_i)}{3!}(\Delta r_H)^3 + \dots \quad (26)$$

where $\Delta r_H = r_f - r_i$.

For the G.H Dilaton black hole, the entropy is[10, 14]

$$\begin{aligned} S &= \frac{1}{4}A = \pi R^2(r_H) = \pi r_H^2(1 - \frac{r_-}{r_H})^{\frac{2a^2}{1+a^2}} \\ &= \pi[r_H^2 - Q^2(1 - a^2)]^{\frac{2a^2}{1+a^2}} r_H^{\frac{2(1-a^2)}{1+a^2}} \end{aligned} \quad (27)$$

The difference of the entropies of the black hole before and after the emission is

$$\Delta S = S(r_f) - S(r_i) = \frac{dS}{dr_H} \Delta r_H + \frac{1}{2!} \frac{d^2 S}{dr_H^2} (\Delta r_H)^2 + \frac{1}{3!} \frac{d^3 S}{dr_H^3} (\Delta r_H)^3 + \dots \quad (28)$$

Using (27), we get

$$\frac{dS}{dr_H} = 2\pi r_H^{\frac{1-3a^2}{1+a^2}} \frac{r_H^2 - Q^2(1-a^2)}{[r_H^2 - Q^2(1+a^2)]^{\frac{1-a^2}{1+a^2}}}. \quad (29)$$

In principle, if we substitute (29) into (28), we can obtain ΔS , and then we can compare it with (26). But fortunately, comparing (29) with (24), we find

$$\frac{dS}{dr_H} = -2k(r_H) \quad (30)$$

Now, we can easily obtain the following equation by comparing (26) with (28) and using (30)

$$\Delta S = -2\text{Im}I. \quad (31)$$

which expresses that the emission rate is also consistent with an underlying unitary theory[10].

IV. CONCLUSION AND DISCUSSION

In this paper, the space-time is a spherically symmetric charged dilaton black hole. Not as usual is that it is obtained from the string theory[9]. However, the result is also consistent with the underlying unitary theory. Comparing with the Schwarzschild-desitter space-time in Ref[5], We can find that the conclusion at the end of Sec.3 is more general. Moreover, the particles which we discuss are massive, but the results take the same functional form as that of massless particles[10], which expresses that the tunneling effect is an intrinsic property of the black hole. As a further discussion and viewed from the calculation, we find that though \dot{r} of massive particles in (17) is different from that of massless particles in (7), the results are the same after we do the r integral first in (20). They are all equal to

$$\text{Im}I = - \int_{M_i}^{M_f} f(r_H) dM, \text{ here } f(r_H) = \frac{2\pi r_H^{\frac{2}{1+a^2}}}{(r_H - r_-)^{\frac{1-a^2}{1+a^2}}}. \quad (32)$$

For the G.H Dilaton black hole, the expression of the inverse temperature, β , is[10]

$$\beta \equiv \frac{1}{T} = \frac{4\pi r_H^{\frac{2}{1+a^2}}}{(r_H - r_-)^{\frac{1-a^2}{1+a^2}}}. \quad (33)$$

Comparing (32) with (33), we can easily obtain

$$f(r_H) = \frac{1}{2}\beta. \quad (34)$$

which implicates more clearly that the tunneling effect is an intrinsic property of the black holes.

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