Noncommutative information is revealed from Hawking radiation as tunneling

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Abstract

We revisit the tunneling process from a Schwarzschild black hole in the noncommutative spacetime and obtain the non-thermal tunneling probability. In such non-thermal spectrum, the correlations are discovered, which can carry the information about the noncommutativity. Thus this enlightens a way to find the noncommutative information in the Hawking radiation. The entropy is also shown to be conserved in the whole radiation process, which implies that the unitarity is held even for the Hawking radiation from noncommutative black holes.

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I. INTRODUCTION

Hawking's semiclassical analysis [1, 2] of the black hole radiation suggests that the information collapsed into the black hole will lose for ever since the thermal radiation can not carry any information. This means that the unitarity as required by quantum mechanics is violated. However, it is found that the background geometry is considered fixed and the energy conservation is not enforced during the radiation process in Hawking method. Recently, Parikh and Wilczek suggested [3] a method based on energy conservation by calculating the particle flux in Painlevé coordinates from the tunneling picture. Their result recovered the Hawking's original result in leading order and gave the consistent temperature expression and the entropy relation. The method had also been discussed generally in different situations [4–8] and showed the formula was self consistent even when checked by using thermodynamic relation [9–13]. Another important aspect is to give the non-thermal spectrum, as shown in Ref. [14] that there exist information-carrying correlation in the radiation spectrum and the entropy is conserved in the sequential tunneling process. Along this line the extention has been made in Ref. [15–18].

It is noted that the noncommutativity had been introduced into the investigation of Hawking radiation as tunneling [11]. The noncommutativity [19–22] can provide the minimal length scale upon the generalized uncertainty principle since the Heisenberg uncertainty principle may not be satisfied when quantum gravitational effects become important. Moreover, the noncommutativity also provides a totally different black hole and the noncommutative black hole thermodynamics is also investigated in the Parikh-Wilczek tunneling picture [11]. Particularly, the noncommutativity leads to the only remnant result of black hole radiation since it can remove the so-called Hawking paradox where the temperature diverges as the radius of a standard black hole shrinks to zero. This is advantageous over the standard Schwarzschild black hole whose final radiation will lead to divergence of the temperature and over the quantum corrected black hole whose final radiation is dependent on the quantum corrected parameter which may be different in string theory and loop quantum gravity theory [5]. In the paper, we will show there exist correlations among the radiated particles in the situation of noncommutative black holes and these correlations could carry the information about noncommutativity hidden in spacetime. We also check the entropy conservation in the radiation process, which is consistent with the unitarity of quantum mechanics.

The organization of the paper is as follows. In the second section we revisit the tunneling through the noncommutative black hole and discuss its thermodynamics. The third section is devoted to investigation of correlation and entropy conservation for the radiation of noncommutative black hole. Finally, we summarize our results in the fourth section.

In this paper we take the unit convention $k = \hbar = c = G = 1$.

II. TUNNELING IN NONCOMMUTATIVE SPACE AND THERMODYNAMICS

In the section we will recalculate the particles' tunneling probability from the Schwarzschild black hole in noncommutative space, along the line presented in Ref. [11]. In order to include the noncommutative effect in gravity, we can change the mass of gravitating object. The usual definition of mass density in commutative space is expressed in terms of Dirac delta function, but in noncommutative space the form breaks down due to position-position uncertainty relation. It is shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime. The effect of smearing is implemented by redefining the mass density by a Gaussian distribution of minimal width $\sqrt{\theta}$ instead of the Dirac delta function. Here θ is the noncommutative parameter which is considered to be a small (Planck length) positive number and comes from the noncommutator of $[x^\mu, x^\upsilon] = i\theta^{\mu\nu}$ with $\theta^{\mu\nu} = \theta diag\left[\epsilon_1, \cdots, \epsilon_{D/2}\right]$. It is noted the constancy of θ is related to a consistent treatment of Lorentz invariance and unitarity. For the purpose of noncommutativity, the mass density is chosen as

$$\rho_\theta\left(r\right) = \frac{M}{\left(4\pi\theta\right)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right), \tag{1}$$

which plays the role of a matter source and the mass is smeared around the region $\sqrt{\theta}$ instead of locating at a point. To find a solution of Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ with the noncommutative mass density of the type (1), the energy-momentum tensor is identified as $T^\mu_\nu = diag[-\rho_\theta, q_r, q_t, q_t]$ which provides a self-gravitating droplet of anisotropic fluid with the radial pressure $q_r = -\rho_\theta$ and tangential pressure $q_t = -\rho_\theta - \frac{1}{2}r\partial_r\rho_\theta = \left(\frac{r^2}{4\theta} - 1\right)\frac{M}{(4\pi\theta)^{\frac{3}{2}}}\exp\left(-\frac{r^2}{4\theta}\right)$. It is seen easily that the pressure is anisotropic, but at the large values of r all the components of the energy-momentum tensor tend to zero very quickly and so the pressure is again isotropic and the Schwarzschild vacuum solution is well applicable.

Solving the Einstein equation in the noncommutative space leads to the solution,

$$ds^2 = -\left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right)dt^2 + \left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right)^{-1}dr^2 + r^2 d\Omega^2 \qquad (2)$$

where the lower incomplete gamma function is defined by

$$\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) = \int_0^{\frac{r^2}{4\theta}} t^{\frac{1}{2}} e^{-t} dt \qquad (3)$$

Note that when r goes to infinity, γ approaches to $\sqrt{\pi}/2$. Comparing the the noncommutative coordinates (2) with the commutative one [3], we note that their difference lies in the mass term, that is to say, substituting the mass term of Schwarzschild spacetime by $m_\theta = \int_0^r 4\pi r'^2 \rho_\theta\left(r'\right)dr' = \frac{2M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)$. It is noted that in the $\theta \longrightarrow 0$ limit, the incomplete γ function becomes the usual gamma function and $m_\theta\left(r\right) \longrightarrow M$ that is the commutative limit of the noncommutative mass $m_\theta\left(r\right)$. From the condition of $g_{tt}(r_h) = 0$, the event horizon can be found as $r_h = \frac{4M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r_h^2}{4\theta}\right) \equiv \frac{4M}{\sqrt{\pi}}\gamma_h$. Keeping up to the leading order $\frac{1}{\sqrt{\theta}}e^{-M^2/\theta}$, we find $r_h \simeq 2M(1 - \frac{2M}{\sqrt{\pi\theta}}e^{-M^2/\theta})$.

In what follows, in order to describe cross-horizon phenomena of tunneling particles, we have to change the coordinates (2) to quasi-Painlevé coordinates, which are regular and not singular at the horizon. Doing the time Painlevé coordinate transformation, we obtain the new coordinates as

$$ds^2 = -\left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\right)dt^2 + 2\left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\right)\sqrt{\frac{\frac{4M}{r\sqrt{\pi}}\gamma}{\left(1 - \frac{4M}{r\sqrt{\pi}}\gamma\right)^2}}dtdr + dr^2 + r^2 d\Omega^2 \qquad (4)$$

where $\gamma \equiv \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)$ and the spacetime described by (4) is still stationary. The radial null geodesics are obtained by setting $ds^2 = d\Omega^2 = 0$ in (4),

$$\dot{r} = \pm 1 - \sqrt{\frac{4M}{r\sqrt{\pi}}\gamma} \qquad (5)$$

where the upper (lower) sign can be identified with the outgoing (incoming) radial motion, under the implicit assumption that time t increases towards the future.

Let us consider a positive energy shell to cross the horizon in the outward direction from r_i to r_f. Along the method given by Parikh and Wilczek, the imaginary part of the action for that shell is given by

$$\mathrm{Im}\, I = \mathrm{Im} \int_{r_i}^{r_f} p_r dr = \mathrm{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp_r^{'} dr = \mathrm{Im} \int_0^{H} \int_{r_i}^{r_f} \frac{dr}{\dot{r}} dH^{'} \tag{6}$$

Moreover, if the self-gravitation is included, we have to make the replacement $M \longrightarrow M - E$ in Eqs. (4) and (5), where E is the outgoing particle's energy. Thus the expression (6) is modified as

$$\mathrm{Im}\, I = \mathrm{Im} \int_{M}^{M-E} \int_{r_i}^{r_f} \frac{dr d\,(M-E)}{\dot{r}} = -\,\mathrm{Im} \int_0^{E} \int_{r_i}^{r_f} \frac{dr}{\dot{r}} dE^{'} \tag{7}$$

where we have changed the integration variable from $H^{'}$ to $E^{'}$. Then inserting the Eq. (5) into the expression of the imaginary part of the action, we have

$$\mathrm{Im}\, I = -\,\mathrm{Im} \int_0^{E} \int_{r_i}^{r_f} \frac{dr}{1 - \sqrt{\frac{4M}{r\sqrt{\pi}}\gamma}} dE^{'} \tag{8}$$

The r-integration is done by deforming the contour. A detailed calculation gives the tunneling rate,

$$\Gamma_N\,(E) \sim e^{-2\,\mathrm{Im}\, I} = \exp\left(-8\pi E\left(M - \frac{E}{2}\right) + 16\sqrt{\frac{\pi}{\theta}} M^3 e^{\frac{M^2}{\theta}} - 16\sqrt{\frac{\pi}{\theta}}\,(M-E)^3\, e^{\frac{(M-E)^2}{\theta}}\right) \tag{9}$$

where the result is obtained up to the leading order $\frac{1}{\sqrt{\theta}} e^{-M^2/\theta}$.

In what follows we will check the thermodynamics for tunneling from noncommutative black hole. Generally, we can obtain the temperature from the tunneling probability by comparing it with the thermal Boltzmann relation. Here it is difficult to expanding the expression (9) to obtain the leading term. However we could also consider the first law of thermodynamics to check whether the radiation temperature given by thermodynamic relation $\frac{1}{T} = \frac{dS}{dM}$ is equal to $\frac{\kappa}{2\pi}$ where κ is the surface gravity of the black hole. Since the metric (2) is static, the surface gravity can be calculated as $\kappa = \frac{1}{2}\frac{dg_{tt}}{dr}|_{r=r_h} = \frac{1}{2}\left[\frac{1}{r_h} - \frac{r_h^2}{4\theta^{3/2}} \frac{e^{-\frac{r_h^2}{4\theta}}}{\gamma_h}\right]$. Thus one can get the temperature,

$$T_N = \frac{\kappa}{2\pi} = \frac{1}{4\pi r_h}\left[1 - \frac{r_h^3}{4\theta^{3/2}} \frac{e^{-\frac{r_h^2}{4\theta}}}{\gamma_h}\right]. \tag{10}$$

When the noncommutative parameter $\theta \longrightarrow 0$, the temperature decays into the standard form, $T_H = \frac{1}{8\pi M}$. Note that for standard form of black hole temperature, in the limit

$M \longrightarrow 0$, the temperature will be infinite. But the consideration of noncommutative black hole radiation will avoid the divergence problem. From the expression (10), we obtain that the temperature will fall down to zero at some definite value, i.e. $r_h = r_0$. As a result, the noncommutativity restricts evaporation process to a remnant. In the region of $r_h < r_0$, there is no black hole since the temperature can not be defined [20, 22].

On the other hand, we can obtain the entropy of the noncommutative black hole by deforming the Bekenstein-Hawking relation,

$$S_N = \pi r_h^2 = \frac{16M^2}{\pi}\gamma_h^2 \simeq 4\pi M^2 - 16\sqrt{\frac{\pi}{\theta}}M^3 e^{-\frac{M^2}{\theta}}$$

One can check easily that the temperature given by $\frac{1}{T} = \frac{dS}{dM}$ is equivalent to that given by (10). More importantly, we can compare the tunneling probability (9) with the Boltzmann factor $e^{-E/T}$ and find that the tunneling probability (9) gives a non-thermal spectrum. Moreover, a detailed calculation shows that $\Delta S_N = S_{Nf} - S_{Ni} = -2\,\mathrm{Im}\,I$ with $r_i = 2M(1 - \frac{2M}{\sqrt{\pi\theta}}e^{-M^2/\theta})$ and $r_f = 2\,(M-\omega)\,(1 - \frac{2(M-\omega)}{\sqrt{\pi\theta}}e^{-(M-\omega)^2/\theta})$ up to the leading order $\frac{1}{\sqrt{\theta}}e^{-M^2/\theta}$. So we have,

$$\Gamma = e^{\Delta S_N} \tag{11}$$

which shows the tunneling probability is related to the change of noncommutative black hole entropy.

III. CORRELATION AND ENTROPY CONSERVATION IN THE RADIATION PROCESS

In the previous section it has been shown that the tunneling probability satisfied the relation $\Gamma = e^{\Delta S}$ and gave a non-thermal spectrum for the noncommutative black hole. Such a spectrum is intriguing and could give some suggestions for the black hole information loss paradox. In the section, along the line outlined by us earlier [14], we will show that there exists correlation between the tunneling particles and the entropy is conserved in the tunneling process.

Considering two emissions with energies E_1 and E_2 and using the expression (9), we have

$$\Gamma_N\left(E_1\right)=\exp\left(-8\pi E_1\left(M-\frac{E_1}{2}\right)+16\sqrt{\frac{\pi}{\theta}}M^3e^{\frac{M^2}{\theta}}-16\sqrt{\frac{\pi}{\theta}}\left(M-E_1\right)^3e^{\frac{(M-E_1)^2}{\theta}}\right)$$

$$\begin{aligned}\Gamma_N\left(E_2|E_1\right)=&\exp(-8\pi E_2\left(M-E_1-\frac{E_2}{2}\right)\\&+16\sqrt{\frac{\pi}{\theta}}\left(M-E_1\right)^3e^{\frac{(M-E_1)^2}{\theta}}-16\sqrt{\frac{\pi}{\theta}}\left(M-E_1-E_2\right)^3e^{\frac{(M-E_1-E_2)^2}{\theta}})\end{aligned}$$

where the tunneling probability for the second radiation is a conditional probability given the occurrence of tunneling of the particle with energy E_1. According to the definition of joint probability in statistical theory [23], we get

$$\begin{aligned}\Gamma_N\left(E_1,E_2\right)&=\Gamma_N\left(E_1\right)\Gamma_N\left(E_2|E_1\right)\\&=\exp(-8\pi\left(E_1+E_2\right)\left(M-\frac{E_1+E_2}{2}\right)\\&+16\sqrt{\frac{\pi}{\theta}}M^3e^{\frac{M^2}{\theta}}-16\sqrt{\frac{\pi}{\theta}}\left(M-E_1-E_2\right)^3e^{\frac{(M-E_1-E_2)^2}{\theta}})\end{aligned}$$

We can check that $\Gamma_N\left(E_1,E_2\right)=\Gamma_N\ \left(E_1+E_2\right)$ which is the emission of particle with the energy E_1+E_2.

In order to evaluate the statistical correlation which says that two events are correlated if the probability of the two events arising simultaneously is not equal to the product probabilities of each event occurring independently, we have to integrate the E_1 variable in $\Gamma_N\left(E_1,E_2\right)$ to attain the independent probability $\Gamma_N\left(E_2\right)$,

$$\begin{aligned}\Gamma_N\left(E_2\right)&=\Lambda\int_0^{M-E_2}\Gamma_N(E_1,E_2)dE_1\\&=\exp\left(-8\pi E_2\left(M-\frac{E_2}{2}\right)+16\sqrt{\frac{\pi}{\theta}}M^3e^{\frac{M^2}{\theta}}-16\sqrt{\frac{\pi}{\theta}}\left(M-E_2\right)^3e^{\frac{(M-E_2)^2}{\theta}}\right)\end{aligned}$$

where Λ is the normalized factor which is the function of the black hole mass M, stemmed from the normalization of tunneling probability $\Lambda\int\Gamma_N\left(E\right)dE=1$. Now we can calculate the statistical correlation between the two radiations

$$C\left(E_1,E_2,\theta\right)=\ln\Gamma(E_1+E_2)-\ln\left[(\Gamma(E_1)\ \Gamma(E_2)\right]\neq0. \tag{12}$$

Thus the adoption of a non-commutative spacetime does not change our statement that a non-thermal spectrum affirms the existence of correlation, as is illustrated for a Schwarzschild black hole. We find that the information associated with noncommutativity is factored out in the correlation even in the early stage of Hawking radiation. Thus even though non-commutativity only exists at the small scale, we can still test its effect through correlations contained in the non-thermal spectrum of Hawking radiation. Especially, the LHC experiment could produce the micro black hole [24] and the analogous black hole radiation experiment has been realized through the laser [25] or BEC [26], so one can observe such radiations to check whether there is the information about noncommutativity in them.

For tunneling of two particles with energies E_1 and E_2, we find the entropy

$$S_N(E_1) = -\ln\Gamma_N(E_1) = 8\pi E_1\left(M - \frac{E_1}{2}\right) - 16\sqrt{\frac{\pi}{\theta}}M^3 e^{-\frac{M^2}{\theta}} + 16\sqrt{\frac{\pi}{\theta}}\left(M - E_1\right)^3 e^{-\frac{(M-E_1)^2}{\theta}}, \tag{13}$$

$$\begin{aligned} S_N(E_2|E_1) = -\ln\Gamma_N(E_2|E_1) &= 8\pi E_2\left(M - E_1 - \frac{E_2}{2}\right) \\ &- 16\sqrt{\frac{\pi}{\theta}}\left(M - E_1\right)^3 e^{-\frac{(M-E_1)^2}{\theta}} + 16\sqrt{\frac{\pi}{\theta}}\left(M - E_1 - E_2\right)^3 e^{-\frac{(M-E_1-E_2)^2}{\theta}}. \end{aligned} \tag{14}$$

It is seen easily that they satisfy the definition for conditional entropy $S(E_1, E_2) = -\ln\Gamma(E_1 + E_2) = S(E_1) + S(E_2|E_1)$. A detailed calculation confirms that the amount of correlation is exactly equal to the mutual information described in Ref. [14], and this shows that it is the correlation that carries away the information. If we count the total entropy carried away by the outgoing particles, we find

$$S_N(E_1, E_2, \cdots, E_n) = \sum_{i=1}^{n} S_N(E_i|E_1, E_2, \cdots, E_{i-1}). \tag{15}$$

Thus we show that entropy is conserved in Hawking radiation for a Schwarzschild black hole in a noncommutative spacetime. Note that the temperature will be zero before black hole vanishes, that is, the black hole will evolve into a remnant which is in a high-entropy state with entropy $S_{NC} = 4\pi E_{NC}^2$. So finally the black hole entropy can be found as

$$S_{NB} = S_N(E_1, E_2, \cdots, E_n) + S_{NC}$$

Especially if the reaction is included, the tunneling probability is

$$\Gamma_{NR}(E) \sim \left[1 - \frac{2E\left(M - \frac{E}{2}\right)}{M^2 + \alpha}\right]^{-4\pi\alpha} \exp\left[-8\pi E\left(M - \frac{E}{2}\right)\right] \times$$
$$\exp\left[16\sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{\theta}} - 16\sqrt{\frac{\pi}{\theta}}\left(M - E\right)^3 e^{-\frac{(M-E)^2}{\theta}} + \text{const.}\,(\text{independent of } M)\right]. \tag{16}$$

Using the same method as above, we can also show that there exists correlations among radiations and the entropy is conserved in the tunneling radiation process. Our method for this case, however, does not solve the remaining problem of whether the black hole will evaporate to exhaustion or will halt at some value of a critical mass because of the reaction effect or the use of a noncommutative spacetime. As we discussed before, when the quantum reaction parameter α is negative, a black hole will leave behind a remnant instead of radiating into exhaustion [27]. If noncommutative spacetime is introduced, however, the parameter α cannot be negative, in order to avoid a divergent temperature at the end of radiation. That is to say, when the non-commutative parameter $\theta \neq 0$, a reasonable value for α would be positive, and thus when the mass of the black hole is reduced to a certain value, the temperature will drastically decrease to zero to form an extreme black hole [11]. Despite these subtleties, we have shown in this section that the entropy is conserved in the tunneling process and so the information could not be lost even for the noncommutative Schwarzschild black hole by taking into account information carried away by correlations in emitted particles.

IV. CONCLUSION

Using the tunneling method and considering the noncommutative effect in Schwarzschild spacetime, the modified tunneling probability is derived. Based on this probability, we have shown that the adoption of a noncommutative spacetime supports that there exist correlations in non-thermal spectrum and the correlation can carry all information, even including the information about the noncommutativity, which may be observed in the future LHC experiment or simulating experiment in laboratory. The entropy conservation is also investigated, which implies that the radiation process of black hole is unitary in the background of noncommutative spacetime.

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