Evidences of secular dynamical evolution in disk galaxies

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Abstract

After a recall of fundamental concepts used in galactic dynamics, we review observational facts as well as results of orbit theory and numerical simulations which suggest long-term evolution of galaxies. Dynamical interactions between galaxy constituents (bulges, disks, bars, haloes, dark matter) are discussed and effects of external perturbations on internal structures are examined. We report recent developments on the connection between dynamical interaction processes and efficiency of star formation in galaxies of various morphological types. The Hubble sequence as an evolution sequence is revisited.

Contents

1 Introduction

The purpose of the present review is not to extensively discuss all the recent developments on galaxy formation and evolution. As it can be seen below, the literature on the subject is abundant if not redundant. Our attention will be centered on clues of secular dynamical evolution in disk galaxies which can be detected by observations or suggested by numerical simulations or theoretical approaches. We will emphasize some fundamental physical processes at work in this context as resonances, torques, energy dissipation, dynamical friction, loss and gain of angular momentum, accretion, etc.

The most significant progresses in the field come from considerations on facts still neglected or not well understood in the past as the role of gas, the interaction effects, not only between galaxies but also between various components of a given system, the necessity to take account of 3-D structures, the star formation in competition with dynamical processes.

Recent observations as well as numerical experiments or theoretical developments suggest that disk galaxies are the seat of evolutionary processes on time scales of the order of the Hubble time or smaller. We will not deal with galactic formation, since another review in the same journal will be devoted to this phase of evolution.

We assume that the reader has had an introduction course in galactic dynamics and is familiar with classical principles of stellar dynamics as well as hydrodynamics. For those readers who lack this basic knowledge, we warmly recommend "Galactic Dynamics" by Binney and Tremaine (1987) and a general review on hydrodynamics by Monaghan (1992).

We begin (section 2) by recalling the observed global properties of galaxies as well as correlations between typical features along the well known Hubble sequence.

In section 3, we review the fundamental tools of the galactic dynamics and the different approaches such as orbit theory, analytical developments and numerical simulations with their qualities and failures. The use of action-angle variables will be reintroduced in view of applications in other sections.

In section 4, we indicate more specifically what we can learn from stellar orbit theory on galactic evolution, emphasizing concepts newly introduced in the field, in particular in three dimensional problems, such as complex instability or effects of a compact central mass. Chaotic orbital behavior will be mentioned as far as it can play a role in the secular evolution of the systems.

The problem of local and global instabilities of disks taking account of stars and gas will be discussed in section 5. In this context arguments for suggesting cold gas as a main constituent of the dark matter present in galaxies will be given.

In section 6, we explore the interactions between various components of disk galaxies (disks, spheroids, bars) and their consequences on the long term evolution.

The dissipative evolution of disks due to the presence of gas is be discussed in section 7 on the basis of recent simulations using an N-body code coupled either with sticky particle code or with smooth particle hydrodynamic code for gas behavior description. Spiral activity and gas fueling of galactic nuclei driven by bars will be studied.

Galaxies evolve in various environments. Interactions are able to modify their morphologies. In section 8, the consequences of satellite accretions are examined as well as the result of strong encounters of disks. A correct treatment of dynamical friction in numerical simulations is discussed.

In section 9, we examine how the star formation can contribute to modify the ideas on the long-term evolution of galaxies. This part is still exploratory since a rigorous theory of star formation taking account of the implied complex physical processes is not yet available and one is constrained to introduce ad hoc assumptions for this purpose.

By way of conclusion, we give in section 10 some arguments in favor of a possible evolution of galaxies along the Hubble sequence from late to early types, due to the dynamical processes previously described.

Since our purpose is essentially to emphasize the clues of evolutionary dynamical processes during the life of disk galaxies, it is useful to close this introduction by referring the reader to books which deal with the structure and evolution of galaxies in general or which describe in more details some specific subjects which we will not be able to tackle here.

Amongst very numerous books and reviews, we propose the following (not exhaustive) list.

1. In Annual Review of Astronomy and Astrophysics:

    - E. Athanassoula and A. Bosma (1985) "Shells and rings around galaxies", **23**, 147
    - J. Sellwood (1987) "The art of N-body building", **25**, 151
    - C. M. Telesco (1988) "Enhanced star formation and IR emission in the centers of galaxies", **26**, 343
    - J. J. Binney (1992) "Warps", **30**, 51
    - J. E. Barnes and L. E. Hernquist (1992) "Dynamics of interacting galaxies", **30**, 705
    - S. R. Majewski (1993) "Galactic structure surveys and the evolution of the Milky Way", **31**, 575
    - M. S. Roberts and M. P. Haynes (1994) "Physical parameters along the Hubble sequence" **32**, 115

2. In the series of International Astronomical Union Symposia (Reidel Dordrecht, eds.) various reviews can be found. Let us quote in:

    - N° 144 "The interstellar disk-halo connection in galaxies" H. Bloemen ed. (1990)
    - N° 146 "Dynamics and their molecular cloud distribution" F. Combes and F. Casoli eds. (1991)
    - N° 149 "The stellar populations of galaxies" B. Barbuy and A. Renzini eds. (1992)

- N° 153 “Galactic bulges” H. Dejonghe and H. J. Habing” eds. (1993)

3. Amongst the most recent books containing proceedings of other conferences, we mention:

   - “Windows in galaxies” (1990) *Astrophys. Sp. Sc. library.*, Vol. 160, G. Fabbiano, J. S. Gallagher, A. Renzini eds. Kluwer, Dordrecht
   - “Evolution of the Universe of galaxies” (1990) *PASPC*, Vol. 10, R. G. Kron eds.
   - “Galactic models” (1990), Annals of the New York Academy of Sciences Vol. 596 J.R. Buchler ed.
   - “Baryonic dark matter” (1990),*NATO ASI Series C* no 306, D. Lynden-Bell, G. Gilmore eds. Kluwer, Dordrecht
   - “Chemical and dynamical evolution of galaxies” (1990), F. Ferrini, J. Franco, F. Matteucci eds. ETS Editrice Pisa
   - “The interstellar medium in galaxies” (1990), *Astrophys. Sp. Sc. library.*, Vol. 161, H. A. Thronson, J. M. Shull eds. Kluwer, Dordrecht
   - “Dynamics and interactions of galaxies” (1990), R. Wielen (ed.) Springer, Berlin
   - ‘Evolution of interstellar matter and dynamics of galaxies” (1990), J. Palous, W. B. Burton, Lindblad P. O. eds. Cambridge Univ. Press, Cambridge
   - “Dynamics of disk galaxies” (1991), *Proceedings of Varberg Conference*, B. Sundelius eds. Göteborg, Sweden
   - “Physics of nearby galaxies, nature or nurture ?” (1992), T. X. Thuan, C. Balkowski, D. T. T. Van eds. Editions Frontières, Gifs s/ Yvette
   - “Morphological and physical classification of galaxies” (1992), OAC Fifth International Workshop, G. Busarello, M. Capaccioli, G. Longo eds. Kluwer, Dordrecht
   - “Star formation in stellar systems” (1992), G. Tenorio-Tagle, M. Prieto and F. Sanchez eds. Cambridge Univ. Press, Cambridge

4. In the series of Saas-Fee Courses of the Swiss Society of Astrophysics and Astronomy:

   - “The galactic interstellar medium” (1991), D. Pfenniger and P. Bartholdi eds. Springer, Berlin

5. Specific monographies are also recommended

   - “Dynamics of barred galaxies” (1993), J. Sellwood and A. Wilkinson, in *Rep. Prog. Phys.*, **56**, 173
   - “Orbits in barred galaxies” (1989), G. Contopoulos and P. Grosbøl, *Astron. Astrophys. Review*, **1**, 261
   - “Galaxy formation” (1993), J. Silk and R. Wyse, in *Physics Reports*, **231**, 293

2 Global observational properties of spiral galaxies

Whatever the origin of galaxy morphologies may be (initial conditions or (and) result of secular evolution), the structural properties of different Hubble types correlated with physical features have to be explained. It is useful, in our present context, to draw up a list of these properties. One must pay attention to the fact that the basic morphological elements of the Hubble sequence of spirals have been set forth by Sandage for bright galaxies. Moreover the morphological classification highly depends on the used passbands. Recent J, H, K photometry of galaxies confirms this evidence.

1. Systematics of the galactic bulge/disk ratios, carefully studied by Simien and de Vaucouleurs (1986) confirms previous results according to which the mean fractional luminosity of the bulge is a monotonic decreasing function of the Hubble type from early (lenticular) types to late-types, typically from 4 for Sa galaxies to 0.1 for Sc-d galaxies.

2. Kennicutt (1981) gave measurements of shapes and pitch angles $\bar{\imath}$ of spiral arms in Sa-Sc galaxies. A clear dependence of $\bar{\imath}$ on the absolute blue luminosity and V_{rot} appears. The result is in agreement with the increase of theoretical pitch angles from early to late-type spirals inferred from the density wave theory by Roberts et al. (1975).

3. Strong bars (strong in the sense of an important deviation to axisymmetry) are present in ~1/3 of spirals and weak bars (or ovals) represent another third.

Table 1: Percentage of barred galaxies of various Hubble types and different inclination. $(b/a)_{25}$ = isophote axis ratio at R_{25}.

$0 < (b/a)_{25} \le 0.6$	SBa, SBab	23%
	SBb, SBbc	29%
	SBc, SBcd	15%
$0.6 < (b/a)_{25} \le 1$	SBa, SBab	53%
	SBb, SBbc	56%
	SBc, SBcd	30%

Bars are ubiquitous and certainly play an essential role in disk evolution as we will see later. It is interesting to note that these percentages do not vary very much in various environments. A maximum percentage of barred galaxies seems to exist in Sb-Sbc types but we must be very careful with statistics concerning the morphological types. It is obviously easier to detect bars in face-on or nearly face-on galaxies. We separately give in Table 1 the fractions SB/(SA + SB) as a function of the inclination of the galactic disks with respect to the line of sight. The de Vaucouleurs parameter $(b/a)_{25}$ is equal to 1 for face-on objects and 0 for edge-on. The intermediate class SAB (or X in the de Vaucouleurs notation) is excluded from this statistics. We can say that the majority of galaxies must be barred or present an oval in the center if we add the fact that photometric data in IR reveal bars which were not previously observed in bluer band (see for ex. Elmegreen 1981).

Early type galaxies seem to have larger bars relative to the galactic disk size than late type galaxies. The correlation between the size of bulges and the length of bars found by Athanassoula and Martinet (1980) is confirmed by Baumgart and Peterson (1986).

4. Hogg et al. (1993) found a significant tendency to an increase with the Hubble type from Sa's to Sd's of the cool gas (HI, H_2, dust).

   Fig. 10 of an old paper by Roberts (1969) already displayed the same tendency for M_{HI}/M_{TOT} from less than 0.05 for Sa-Sab's to 0.10–0.15 for Sbc–Sd's.

5. From a series of papers by Rubin and collaborators (see a synthesis of results in Rubin et al. (1985)) we have a relation between the optical maximum rotation velocity V_{max}, as well as the blue absolute magnitude, and the Hubble type. This relation is confirmed by using HI rotation curves as shown by Zaritsky (1993). The tendency is a $\langle V_{max} \rangle$ decline from early to late types.

6. The Tully-Fisher direct relation between the absolute luminosity and the maximum rotation velocity (Tully-Fisher, 1977) shown in fig. 5 is another tendency in spiral galaxies. However, this relation seems to be dependent on the pass-band, as confirmed by Gavazzi (1993). In infra-red, there is less type-dependence contrarily to visible bands.

7. The morphology of rotation curves may change according to the luminosity. In the context of the conspiracy problem between optical and dark matter in spirals (see section 5.3), Casertano and van Gorkom (1991) distinguished three kinds of rotation curves as seen in one of their figures reproduced here (fig. 1).

   Spiral galaxies occupy a limited band in the plane (V_{max}, h), where h is the disk exponential scale length. We have suggested that systems with (V_{max}, h) outside this band would be dynamically strongly unstable (see Martinet, 1988).

8. The central velocity dispersion σ_0 is also function of Hubble type (Dressler and Sandage, 1983): $< \sigma_0 >$ varies from $\sim 115\ kms^{-1}$ for Sc + Sbc's galaxies to 170 kms^{-1} for Sa and 220 kms^{-1} for S0.

9. The parameter V_{max}/σ_0 indicates the ratio of systematic rotational motion to central random motions in galaxies. Whereas it is smaller than one for elliptical galaxies,

it is larger than 1 for spirals. But we do not find any significant variation with the spiral Hubble type. Typical values are 1.8 for Sa's, 1.6 for Sb's, 1.7 for Sc's.

10. The distribution of integrated $H_\alpha + [NII]$ equivalent width of normal galaxies, which can be considered as an estimate of the current star formation rate (SFR) scaled to the total red luminosity, shows a strong trend with Hubble type. The total SFR ranges from 0.1 to 1 $M_\odot yr^{-1}$ in S0-a galaxies to 10 $M_\odot yr^{-1}$ in Sc-Irr galaxies (Kennicutt, 1983). The range of variation is larger if we take account of extreme dwarfs (0.001 $M_\odot yr^{-1}$) and starbursts (till more than 100 $M_\odot yr^{-1}$).

11. The history of star formation is globally described by the parameter $b = SFR{\cdot}\tau_d/M_d$, where τ_d is the age of the disk and M_d its total mass. b measures the ratio of the current star formation rate to the average past rate. It increases from 0.05 in Sa-Sab's to 1.0 in Sc's (Kennicutt, 1993).

12. We will come back (section 10) on the problem of dark matter for which some arguments suggest that its quantity increases from Sa's to Sc-d's.

13. Zaritsky et al. (1994) suggest that Sb-Sc galaxies may have steeper metal abundances gradients than either earlier or later spirals. Furthermore, barred galaxies seem to generally have flatter gradients than unbarred galaxies. But these results must be confirmed on larger samples.

Summarizing this section, we retain numerous tendencies of physical features with the Hubble type. The bulge to disk ratio, the maximum rotation velocity, the core velocity dispersion decrease from Sa's to Sc's. But it is important to emphasize a large or even very large scatter at each type or subtype. The mass of cool gas, the pitch angle of spiral arms, the current star formation increase from Sa's to Sc's. Here also, we must account for large scatter inside each type or subtype.

As mentioned above, these tendencies have been observed on samples of giant and supergiant galaxies: the used catalogues are dominated by objects of high luminosity. We must not lose sight of the fact that Sa-Sbc types are much less frequent among dwarfs than for all Shapley-Ames objects. Amongst dwarfs, Scd-Im galaxies represent 29% of the sample compared to only 5% for all Shapley-Ames galaxies. At present time we observe objects from mostly unevolved disks (Malin 1 presents a HI disk size of $0.5h^{-1}$ Mpc !) to disks which have exhausted more than 99% of their gas. In brief, the whole galactic evolution story goes beyond the frame of the giant galaxy correlations.

3 Stellar dynamics in galaxies

For ten to twenty years, studies in galactic dynamics mainly concentrated on the following topics:

1. Dark matter, on which numerous questions remain unsolved: what? how much? where?
2. Triaxiality of elliptical galaxies, bulges of spirals and bars, which is tractable in the frame of dynamical systems with three degrees of freedom.
3. Swing amplification of spiral waves, described by Toomre (1981) which was presented as one possible solution to the problem of persistence of the observed spiral structures.
4. Interaction of galaxies, the most spectacular effect of which being warps, polar rings, shells, bridges, tails and mergers.
5. Dissipative behavior of gas, the importance of which is now recognized in the evolution problems.

With respect to the simplifications generally inherent in classical purely "stellar dynamics" analytical treatments of galactic dynamics, the topics mentioned above introduce various levels of complexity. Dark matter is not necessarily conservative, as suggested in section 5.3, triaxiality introduces new phenomena in orbital behavior (section 4), swing amplification and interactions between galaxies as well as between components of a given galaxy imply non-stationnarity, asymmetries, coupling of dissipative gaseous and conservative stellar components, hence coupling of hydrodynamics and stellar dynamics. It is clear that one can try to succeed in understanding such a complexity only by using numerical calculations, as we will show in the next sections. Nevertheless it is not useless to recall here some of the fundamentals concepts of classical galactic dynamics.

3.1 Fundamentals of classical galactic dynamics

An analytical description of a galaxy can be tried by using a set of relations between the stellar density distribution $\rho(\boldsymbol{x}, t) \geq 0$, the potential $\Phi(\boldsymbol{x}, t)$ and the phase density $F(\boldsymbol{x}, \boldsymbol{v}, t) \geq 0$, where $\boldsymbol{x}$ and $\boldsymbol{v}$ are respectively the position and velocity vectors. These relations are implicitly described by the Boltzmann equation ($\mathcal{B}$) and the Poisson equation ($\mathcal{P}$):

$$(\mathcal{B}) \quad \frac{DF}{Dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial \boldsymbol{x}}\boldsymbol{v} - \boldsymbol{\nabla}\Phi \frac{\partial F}{\partial \boldsymbol{v}} = 0$$

$$(\mathcal{P}) \qquad \nabla^2 \Phi(\boldsymbol{x}, t) = 4\pi G \rho(\boldsymbol{x}, t)$$

Moreover

$$\rho(\boldsymbol{x},t) = \int_{\Gamma_v} F(\boldsymbol{x},\boldsymbol{v},t)d\boldsymbol{v}$$

where Γ_v is the velocity space.

If $\rho(\boldsymbol{x},t)$ is the observed density (or an imposed density ρ_{imp}), $(\mathcal{P})$ gives the corresponding imposed potential, $(\mathcal{B})$ gives in principle the phase density response and the third equation the density response ρ_{resp}. The self-consistency condition implies $\rho_{imp} = \rho_{resp}$.

In $(\mathcal{B})$ such as written, the evolution due to stellar encounters $(\partial F/\partial t)_{enc}$ is ignored. It is generally admitted that in galactic dynamics, the influence of encounters implying stars is negligible, taking account the fact that the 2-body relaxation time $t_{rel} \sim V^3/Nm_1m_2$ (Hénon, 1973) would be much larger than the age of the system (V is the star typical relative velocity, m_1 and m_2 the respective masses of test and field objects). However, encounters of giant molecular clouds with stars could be effective to heat disks of spirals and contribute to $(\partial F/\partial t)_{enc} \neq 0$, so that $(\mathcal{B})$ without second member would be only justified for time scales less than some 10^8 years. Furthermore, even if $(\partial F/\partial t)_{enc}$ is ignored, a system could have a collisionless evolution which is revealed by numerical simulations as well as by the observations. The collective relaxation certainly plays a role in the secular evolution of disks in particular.

Exact analytical solutions of the integro-differential system $(\mathcal{B}) + (\mathcal{P})$ have been found only in particular stationary cases of symmetries such as spherical systems with $F(E)$ or $F(E, J^2)$, or infinitely thin axisymmetric disks with $F(E, J_Z)$. Actually, disks are generally not axisymmetric, they are perturbed by spiral or bar components and they are not very thin. Then

$$\rho = \rho(R,\phi,z,t), \qquad \Phi = \Phi(R,\phi,z,t), \qquad F = F(R,\phi,z,v_R,v_\phi,v_z,t)$$

with a stationary zero order possible solution

$$\rho_0(R,z), \qquad \Phi_0(R,z), \qquad F_0(R,z,v_R,v_\phi,v_z)$$

If the disk is thick or if we look for a solution for a axisymmetric galaxy including a halo, we have the additional problem of the third integral: $F(E, J_z)$, where $E = \frac{1}{2}(v_R^2 + v_z^2 + \frac{J_z^2}{r^2}) + \Phi(R,z)$ and $J_z = rv_\phi$, is not compatible with the velocity distribution in the solar neighborhood where the observations suggest that the velocity dispersion ratio $\sigma_R/\sigma_z \simeq 2$.

In principle, the phase density should be written as a function of a set of isolating distinct integrals (Jeans Theorem). Literature is rich in discussions on the difficulty to realize this end beyond the classical integrals of energy E and angular momentum J_z in the axisymmetric case. Only by having recourse numerically to the Poincaré method of surface of section, it was possible to partially clear up the problem of integrability of specific potentials. An exemplary case was explored by Hénon and Heiles (1964). In general, dynamical models such as those which are able to represent real galaxies are non-integrable, in the sense that there exists always a subspace of the phase space occupied by chaotic orbits in such systems (see also section 4).

For an up-to-date review of the integrability of galaxy models see *i.e.* de Zeeuw (1988).

Because of the difficulties to solve the system $(\mathcal{B})$ + $(\mathcal{P})$ or to find explicit form for $F(\boldsymbol{x}, \boldsymbol{v})$ in terms of integrals, one resorted for a long time to the Jeans equations (Stellar hydrodynamic equations) obtained by multiplying $(\mathcal{B})$ by v_i^k with $k = 0$ and 1 and integrating on the velocity space. Some useful relations between the density of a stellar population, the moments of the velocity distribution and the radial or perpendicular forces K_R and K_z have been obtained. For various applications to problems of galactic structure such as the asymmetrical drift, the motions perpendicular to the galactic plane and the determination of the galactic mass from the kinematics of globular clusters in our Galaxy or the relation between the azimuthal systematic motions and the shape of spheroidal systems, cf. Binney and Tremaine (1987). The drawback of the method consists in the impossibility to close the system of equations without introducing arbitrary hypothesis with regard to the moments of F. But see a recent method suggested by Amendt and Cuddeford (1991) to overcome this difficulty.

The introduction of perturbations in the fundamental equations of the self-consistent problem mentioned above very quickly gives cumbersome expressions, even if one makes use of the fact that apparently spiral perturbations, for instance, are small with respect to the axisymmetric field. The linearisation of the equations in this case has been considered as an adequate approximation of the first order.

We have

$$\Phi = \Phi_0 + \Phi_1 + ... \quad , \quad F = F_0 + F_1 + ... \quad , \quad \rho = \rho_0 + \rho_1 + ...$$

where Φ_0 is an imposed axisymmetric potential and Φ_1 equal an imposed spiral or weak bar perturbation $\Phi_1 \ll \Phi_0$, $\rho_1 \ll \rho_0$, $F_1 \ll F_0$.

$(\mathcal{B})$ becomes $D(F_0, \Phi_0) + D(F_1, \Phi_0) + D(F_0, \Phi_1) = 0$

A zero order solution is given by solving $D(F_0, \Phi_0) = 0$. The first order solution is obtained from the equation:

$$D(F_1, \Phi_0) = -D(F_0, \Phi_1)$$

If Φ_1 is known, for ex.

$$\Phi_1(R, \phi, t) = \sum_m \Phi_{1m}(R) exp[i(\omega t - m\phi)]$$

where m represents the symmetry of components (for a spiral, m = number of arms), we will formally have $F_1 = -\int D(F_0, \Phi_1) dt$, where the integration is performed along the unperturbed orbits. $D(F_1, \Phi)$ is non-linear. Non-linear effects in spiral cases are weak in general except for resonance regions which will be described right below.

Calculations lead to denominators in F_1 in the form of

$$\sin(\omega\tau_0 - m\phi_0)$$

where τ_0 = half epicyclic period $= \pi/\kappa_0$, ϕ_0 = angle between the apocenter and pericenter of the orbit, $\omega = m\Omega_p$, where Ω_p is the perturbation pattern angular velocity. This denominator is 0 if

$$\frac{\Omega_p - \Omega}{\kappa} = \pm\frac{n}{m} = \text{rational number}$$

For $m = 2$, if $n = 0$, $\Omega_p = \Omega$ (corotation resonance) and if $n = 1$ $\Omega_p = \Omega - \kappa/2$ or $\Omega + \kappa/2$ respectively corresponding to the Inner (ILR) and Outer (OLR) Lindblad Resonances. They are the most important resonances in the galactic disks. They play a significant role when a perturbation to the axisymmetric case is present. A star at resonance meets the perturbation pattern always at the same point on its orbit so that a cumulative perturbation effect is to expect. It is necessary to stress that these resonances are defined in the frame of a theory of small motions around circular orbits. In the presence of a strong bar, the frequencies $\Omega(R)$ and $\kappa(R)$ as well as the "vertical" frequency $\nu_z(R)$ do not rigorously represent the actual orbit frequencies inside the bar.

Strong bars (axis ratio larger than 1.5 - 2) cannot be treated in the frame of the linearized equations. Non-linear effects are important. Orbits present large deviations with respect to the familiar epicyclic orbits of the axisymmetric case. Fig. 2 shows some typical resonant periodic orbits in perturbed systems, numerically calculated. For the description of non-linear theory at the ILR, the reader is referred to a review by Contopoulos and Grosbøl (1989).

The previous discussion shows that the analytical approach of the self-gravitating problem in disk galaxies is attended by large difficulties. Recently two numerical methods have been proposed to obtain a self-consistent solution for spiral or barred galaxies. The first one (Contopoulos and Grosbøl (1989))is based on the existence and the shape of periodic orbits in spiral galaxies. The other, used by Pfenniger (1984b) for the construction of 2-D self-gravitating barred galaxies, is a variant of the Schwarzschild (1979) linear programming approach to obtain equilibrium model of elliptical galaxies. Both methods consider various kinds of possible orbits and the time they spend in various regions of the system.

A fortiori the treatment of dynamical evolution of the galaxies with time is out of reach by analytical methods. This justifies the use of numerical simulations on computers of galaxy evolution with the help of N-body and hydrodynamical codes. This approach already allowed to treat a lot of problems concerning the equilibrium, the stability and the evolution of disk galaxies such as those described in section 5 and the next ones.

3.2 Angle-action variables

The formalism of angle-action variables was introduced by Born (1927). It appeared in galactic dynamics in papers by Lynden-Bell and Kalnajs (1972) and Lynden-Bell (1973). It has been proved to work in various problems connected to the disk evolution: a) orbital behavior of stars belonging to the spheroidal component of a galaxy during the slow formation of the disk (Binney and May, (1986), b) trapping of orbits in the potential sink of a bar (Lynden-Bell, 1973 and 1979), c) dynamics of resonances in spirals and bars (Contopoulos, 1988), d) evolution of barred galaxies by dynamical friction (Weinberg, 1985), e) the exchange of angular momentum between a bar and a disk (Little and Carlberg, 1991), f) the dynamical interaction between a bar and a spheroid (Hernquist and Weinberg, 1992).

In some cases, it is cumbersome to go beyond a linear treatment but this formalism gives some useful insight on the reality of the mentioned processes. We will come back in next sections on some of the most recent applications of the formalism.

A suitable set of angle-action variables (I_1, w_1, I_2, w_2) for galactic disks is defined as follows:

With R_h = radius of the circular orbit corresponding to the angular momentum h,

$R_1 = R - R_h = asin(w_1)$, w_1 is the radial oscillation phase.

$I_1 = 1/2\kappa a^2 = \left(E_R - E(h)\right)/\kappa$ is a function of the radial amplitude.

w_2 is the galactocentric angle to an epicenter in uniform motion.

$I_2 = h =$ angular momentum.

For a thick disk, a third action is introduced: I_3 is a function of the perpendicular amplitude of motion $= E_z/\nu_z$, where ν_z is the frequency of the perpendicular motion, to which corresponds w_z = latitudinal angle = perpendicular oscillation phase.

For a perturbed system, $H(\boldsymbol{I}, \boldsymbol{w}) = H_0(\boldsymbol{I}) + \epsilon H_1(\boldsymbol{I}, \boldsymbol{w})$, the equations of motion are

$$\left\{ \begin{array}{lcl} \dot{\boldsymbol{I}} & = & -\epsilon \boldsymbol{\nabla}_w H_1(\boldsymbol{I}, \boldsymbol{w}) \\ \\ \dot{\boldsymbol{w}} & = & \boldsymbol{\nabla}_I H_0(\boldsymbol{I}) + \epsilon \boldsymbol{\nabla}_I H_1(\boldsymbol{I}, \boldsymbol{w}) \end{array} \right.$$

the Hamiltonian being explicitly $H = \Phi_0 + \epsilon\Phi_1 - \Omega_p h$ where Ω_p is the angular velocity of the spiral or bar perturbation.

For example $\epsilon\Phi_1 = \sum_m \sum_n [\epsilon_{mn} \cos(mw_1 - nw_2) + \epsilon'_{mn} \sin(mw_1 - nw_2)]$

A useful property of the actions is their adiabatic invariance in cases of slow time variation of potentials.

This consideration has been exploited for instance by Lynden-Bell (1979) in an very elegant treatment of the trapping of quasi-resonant orbits by a bar.

4 What can we learn from studies of stellar orbits

4.1 Introduction

From simplicity to complexity, studies of stellar orbits in galaxies can be tackled according to the three following approaches:

1. Study of orbits in a given potential

   The aim is to obtain a classification of main types of orbits existing in a given system, in particular the periodic orbits. This approach is able to bring constraints on the real existence of such a system and that, in so far as the shape of some orbits is not always compatible with the geometry of the imposed density profile as explained below. In this approach, the collective effects are neglected so that some results need confirmation by N-body simulations, for instance the problem of the disc instabilities perpendicular to the galactic plane (section 6.4).

2. Response of a given population to any perturbation

   This approach implies the calculation of the time spent by the orbits in given regions and allows to study mechanisms which explain some observed structures as rings (resulting from trapping of matter at some resonances) or thickening of disks.

3. If in the frame of the previous approach, we succeed to find a response in density equivalent to the imposed density, for some particular weighting of the various kinds of possible orbits, we can claim to construct a self-gravitating equilibrium model, as already mentioned in section 3.1.

In this section we will show that the study of orbital behaviors in given systems is a complementary approach to other ways of analyzing the structure and the evolution of the galaxies. In particular the question of the importance of chaotic orbits in galactic potentials has been the subject of numerous investigations in the recent past: what is the permissible percentage of such orbits in various morphological types of galaxies if they have to be considered as self-gravitating equilibrium systems?

Let us begin by a question of terminology. Roughly spoken, a system is called ergodic if any trajectory (except for a set of null measures) fills densely its energy surface. The sequence of intersection points in a space of section (Poincaré,1899) corresponding to such an ergodic trajectory would fill densely this space of section. In spite of the widely-held use of the term of "ergodicity" in works dealing with galactic dynamics problems, ergodic orbits strictly as such, are not found in smooth "realistic" as well as in noisy models of galaxies. Apart from very peculiar cases, at the very most we may speak about chaotic behavior in *some* regions of phase space. Some other adjectives are also used in literature: irregular, wild, erratic, semi-ergodic... This behavior implies that in a space of section the sequence of points corresponding to the trajectory jumps more or less randomly in the fraction of space left open by the invariant curves which correspond in contrast to a regular behavior. In fact, as often as not, a chaotic behavior is characterized by the presence of *cantori* which, by any means, are not to be confused with ergodicity.

See Percival (1989) for an introduction to the concept of cantori and Hénon (1981) for a clear description of the surface of section method and applications to various dynamical systems.

4.2 Interaction of resonances. Heteroclinic orbits

Gerhard (1985) looked for the perturbations ϵH_1 of integrable Hamiltonians H_0 which are consistent with observations of early-type galaxies as well as approximately preserving the regular orbital structure of integrable potentials. Limiting his treatment to small perturbations and to homoclinic orbits in order to be able to use the Melnikov integral technique, the author finds that only perturbations such as cos $m\phi(m = 0, 2, 4)$, modest ellipticity gradients or small figure rotation are working in this context. However, it seems to be clear that in realistic systems, a chaotic behavior can be amplified by resonance interactions and the presence of heteroclinic orbits. Such a situation was described in the inner regions of an axisymmetric model of our Galaxy for stars with small angular momentum (Martinet, 1974).

Evidence for chaos triggered by resonance interactions in triaxials models of galaxies, has been given by Martinet & Udry (1990) in connection with the adopted morphology for these systems. Orbits were systematically studied in slowly rotating modified Hubble profile models of various axis ratios ($a : b : c$): I) a nearly spherical one, II) a Schwarzschild model ($1 : 0.625 : 0.5$), III) a strongly triaxial one ($1 : 0.6 : 0.15$) and IV) a bar ($1 : 0.25 : 0.125$). Surfaces of section as well as the rotation number "rot" attached to invariant curves have been obtained for different values of the Hamiltonian. An example is given in Fig. 3 for model II. Rational values of "rot" correspond to resonant periodic orbits. When the orbits are not regular, it is impossible to define "rot" (the invariant curves are dissolved) without ambiguity (bottom figure) and discontinuities in "rot" (x) appear, indicating the range of resonances in interaction responsible of chaotic behavior apparent in the surface of section (top figure).

The main result of this investigation is that only models I and II show moderate chaotic regions (cantori) with not really detected resonance interactions, possibly confined to a very narrow range of rational numbers. On the contrary, models III and IV develop important chaotic regions: for model IV for ex., resonances in the range $1/5 <$ "rot" $< 2/3$ are implied in the set-up of chaos. Figure 4 shows that in an existence diagram of axis ratios, our models I and II are located in the region occupied by real triaxial galaxies or bulges (various symbols) according to predictions inferred from observations. On the contrary, the highly triaxial models III and IV are in a region devoided of such real systems. This could be a sketch of constraints on the possible shape of triaxial galaxies. We must notice that N-body equilibrium figures obtained by gravitational collapse of initial rotating anisotropic bodies have b/a from 0.5 to 1 and $c/a > 0.45$ (Udry, 1992) also in agreement with observational predictions. It is the same for Barnes (1992) products of mergers. Too highly triaxial objects might not exist because real equilibrium systems would not tolerate too much chaos!

4.3 Effects of asymmetries or noise on the orbital behavior in triaxial systems

The gravitational attraction of a co-rotating nearby body is able to deform the shape of orbits in the given systems defined above. In particular, instead of having a main stable orbit x_1 which bifurcates into two stable branches (for ex. Martinet & Zeeuw, 1988), the asymmetry triggered by an eccentric Plummer sphere leads to a continuous deviation of x_1 from the plane, gradually becoming a banana-shaped orbit (Udry 1991). We will come back below to the problems rising from the existence of this kind of centrophobic orbits. The addition of a high frequency sinusoidal function to the given potential to represent some noise locally modifies the isodensity contours. Udry suggested the following form

$$\Phi = \Phi_0 \left[1 + \epsilon \sin \left(\sum_i k_i x_i \right) \right]$$

for a noisy 3-D potential. Φ_0 is the potential defined in the previous section (Hubble profile model). As shown in Fig. 5 displaying surfaces of section for several cases of the amplitude ϵ and the frequencies k_i of the noise in the 2-D case, chaotic behavior is favored by such perturbations. As ϵ and (or) k_i increase, the invariant curves become thicker and are then progressively destroyed. See also Pfenniger and Friedli (1991).

4.4 Complex instability in triaxial systems

This is one of the new orbital behavior which appear in dynamical system with three degrees of freedom (Magnenat and Martinet, 1983). It consists of the fact that the Jacobian matrix associated with the linearized transformation describing the motion close to a periodic orbit has all its eigenvalues complex and outside the unit circle. Real eigenvectors do not exist. This behavior was mentioned by Broucke (1969) in the 3-body problem and emphasized by Magnenat (1982) for the case of cubic potential. Important zones of complex instabilities have been found in rather academic potentials (Contopoulos and Magnenat (1985)). Pfenniger (1985) found complex unstable tridimensional periodic orbits in a large range of energy and parameters in a model of barred galaxy. It is not clear in which situations complex instability would produce a great amount of chaotic motions.

We mention here a case which could be interesting for the equilibrium and the stability of real galaxies. Ordinarily, four main sequences of periodic orbits are considered for a specific triaxial slowly rotating system: the stable anomalous orbits tipped relatively to the equatorial plane and circling the long axis, the unstable anomalous inclined orbits which circle the intermediate axis, the normal stable retrograde orbits in the equatorial plane and the z-axis orbits. The z-axis orbits are stable (s) in the inner part, then become simply unstable (u) against perturbations parallel to the intermediate axis and still further out become doubly unstable (du) against additional perturbations parallel to the long axis. There four sequences are interconnected in a way described by fig. 1 of the paper by Heissler et al. (1982). Martinet and Pfenniger (1987) studied the stability of the z-axis orbit when the figure rotation and the shape of the system change. The sequence of bifurcations mentioned above is modified and the onset of complex instability at high energies on the family of z-axis orbits is fairly general for rotating triaxial stellar systems. For a figure rotation large enough, the direct transition of stability to complex instability prevents the stable anomalous orbits from joining the z-axis. Let us consider a 3-D barred potential (axisymmetric background plus a Ferrer's bar such as used by Pfenniger (1984a), to which a Plummer sphere is added to represent a condensed massive object at the center of the system). It is observed that even a very small mass concentration M_p in the core amplifies the complex instability: the critical value of the Hamiltonian at which the transition stable $\longrightarrow$ complex unstable occurs is lowered. For a value of $M_p/M_{tot} \geq 0.0006$, the z-axis orbit is practically fully complex unstable from $z = 0$. The effect of such an instability is described in Fig. 6: the importance of stochastic diffusion of orbits starting near the z-axis is increasing with the central Plummer sphere mass. The diffusion time by this process could be shorter than the Hubble time if the central mass makes the most of the z-axis unstable. It seems possible that by the association of a central small mass and a rotating bar, which both create a large number of stochastic orbits, bulges may grow secularly through the enhanced diffusion of stochastic orbits. Of course, other stochasticity-producing events, like mergers, may happen in the life of a galaxy. The exact diffusion speed is sensitively dependent on small perturbations. Since real galaxies are much less smooth, symmetric and steady than the potentials above, we can expect that the stochastic diffusion is probably even faster in real galaxies than in the present models. However, in the approach described here as in all calculations where orbits are studied in given fixed potentials, the collective processes are not taken into account so that only N-body simulations are able to give a more specific idea of the mentioned effects.

4.5 Barred galaxies

Detailed numerical calculations of stellar orbits in given barred potentials have been a cornerstone for understanding the dynamics of SB galaxies, which has recently been extensively reviewed by Sellwood & Wilkinson (1993). Here we summarize again some important conclusions inferred from recent works about the relation between the shape of bars and the onset of chaos.

The essential point is that if the axis ratio of the bar in the plane of the disk is larger than 3 to 4, and/or the mass of the bar is larger than 1/4 of the total mass inside corotation, an extended chaotic region can occur in the inner parts of the galaxy and that the corresponding orbits cannot enhance the bar anymore. The stability of the main periodic orbit x_1 along the bar is necessary to maintain the barred structure (for details of the 2-D case see for ex. Athanassoula et al. (1983) or Contopoulos and Papayannopoulos (1980)). These results suggest that models with the properties just mentioned should be excluded for a self-gravitating barred galaxy. As indicated in the introduction, such predictions need to be confirmed taking account of collective effects.

The effect of a compact mass at the center of a galaxy has been reported by Hasan & Norman (1990, and references therein). The percentage of phase space volume occupied by direct orbits along the bar decreases from 50% in absence of such a compact mass or if the axis ratio a/b of the bar is ~ 2 to 10–15%, for instance, if the compact mass is one tenth of the total mass or if $a/b \sim 4$.

At the present time, the only equilibrium model existing for a 2-D barred galaxy is the numerical one constructed by Pfenniger (1984b). In this work it is shown that the percentage of semi-ergotic stars may be as large as 30% but more probably below 10% if the axis ratio is 4 and the mass of the bar is 1/5 of the total mass. Locally, however, around the Lagrangian points at the end of the bar, they can be 100%.

For a 3-D barred galaxy the estimation of the percentage of chaotic orbits is more complicated. From Pfenniger (1984a), it appears that semi-ergodicity is favored by instability strips perpendicular to the galactic plane. The bar growth is limited if axis ratios a/b and a/c are too large. A too thin bar ($a/c \sim 10$) induces a lot of chaos. In a 3-D strongly barred galaxy, the resonant family $4 : 4 : 1$ has been proved to have a complex unstable part (Pfenniger, 1985). The resulting orbital diffusion could be a possibility to populate the inner halo (see 4.6). Much work is necessary to estimate the permissible percentage of chaotic orbits in this case. 3-D equilibrium models do not exist at present time, in particular because vertical density structure of bars is weakly constrained by observations.

4.6 Boxlets

For a wide class of non-rotating triaxial potentials, most of the phase space is occupied by four well known major families: box, inner and outer long axis tubes, short axis tubes. The x-axis periodic family is generic for the boxes provided that it is stable. Boxes appear essential for the construction of equilibrium models of triaxial systems. However, if there is sharp variation of the density profile near the center, x-axis orbits can be unstable. Centrophilic boxes could be replaced by centrophobic boxlets such as bananas (Miralda-Escudé & Schwarzschild 1989).

Two points about boxlets need further investigations: firstly, the fact that this shape is not always compatible with the shape of the imposed density. This may cause a problem for the construction for equilibrium models. Secondly, if the boxlets are unstable, they can participate to diffusion of chaotic behavior. The question concerning what percentage of such orbits is permissible for the construction of self-gravitating systems is still open. This question is still more complicated for rotating systems (Martinet & Udry 1990).

4.7 Concluding remarks

In conclusion, we can summarize what we can learn from orbit calculation in imposed galactic potentials

- Existing morphologies of galaxies seem to be incompatible with too high percentages of semi-ergodic orbits. Slowly rotating triaxial dynamical models with a major to minor axis a/c ratio larger than 2.5 or fast rotating barred systems with axis ratio a/b larger than 3 or 4 display important chaotic behaviors. Systems with such morphological features are in fact apparently not observed!

- Association of a compact central mass and a rotating bar, which creates a large number of chaotic orbits, can trigger a secular growth of bulges by enhanced diffusion of there orbits.

- 3-D N-body bars in disks have axis ratios $a/b < 4$. That corresponds to systems for which the orbital calculations predict a moderate percentage of semi-ergodic orbits.

- Quantitative estimations of percentages of semi-ergodic orbits in 3-D systems compatible with observed morphologies of barred galaxies is an open question.

- Response to a given potential is not sufficient for answering the question of the permissible percentage of chaotic orbits in various morphological types. Furthermore galaxies evolve, bars may grow, then dissolve. Ellipticals may accrete. Gas and its various interactions with stars (star formation, gaseous response, stellar winds etc.) contribute to modify the structure of the systems. Does the secular evolution lead such systems to a state close to an integrable potential with modest percentage of chaotic orbits, as suggested by Gerhard (1985)?

5 Stability of disks

In the present section, we will remind some considerations on the local and the global stability of stellar disks. Then we will examine how the stability criteria are altered by the presence of gas. That will serve as a preamble to the description of the effects of gas inflow towards the central regions in section 7.

5.1 Local stability of stellar disks against axisymmetric perturbations in terms of radial distance to the center

The well known criterion introduced by Toomre (1964) indicates that stellar disks are stable against local radial perturbations if

$$Q = \frac{\sigma_R(R)}{(\sigma_R)_{min}} = \frac{\sigma_R(R)\kappa(R)}{kG\Sigma(R)} \geq 1$$

where κ is the epicyclic frequency, $\Sigma(R)$, the projected surface density and k, a constant which is equal to 3.36 for a infinitely thin stellar sheet, 2.6 for a not infinitely thin disk with $\sigma_z/\sigma_R = .6$ (Vandervoort, 1970) and 2.9 if the galaxy contains $\sim$ 10% (in mass) of gas (Toomre, 1974). For some galaxies, k could be higher so that we maintain k as a free parameter for the moment but later we will often refer to the standard value of 3.36.

As soon as $Q = 1$, the Jeans instability modes are suppressed. The criterion has been applied frequently to particular galaxies (i.e. Kormendy, 1984, van der Kruit and Freeman, 1986, Bottema, 1988). The influence of various factors on the radial behavior $Q(R)$ in disks has been discussed by Martinet (1988) and Bottema (1993). We discuss below some of their conclusions. But let us remind without delay that even if $Q \geq 1$, the disk can be violently unstable against non-axisymmetric global perturbations and develop massive bars. It is a result of numerous numerical simulations. In fact, a really satisfactory criterion of global stability against the formation of bars does not exist at the present time in spite of several tentatives (Ostriker and Peebles, 1974; Efstathiou et al., 1982). Although dark halos have often been used to explain the mechanism of stabilization, some more recent numerical experiments by way of a N-body code (Athanassoula and Sellwood, 1986) have shown that halos do not represent the "cure-all" in this context and that a disk which is sufficiently hot in its central parts could prevent the growth of a bar in some cases. The result of their N-body model seems to indicate that $Q \geq 2$ - 2.5 might be a global stability criterion. The $\langle Q \rangle$ values given in recent literature for individual galaxies (see Bottema, 1993, and reference therein) could certainly suffer from more or less uncertainties due to badly determined quantities, such as the mass-to-light ratio in the disc or the central luminosity density $L_{0,0}$. Nevertheless, the parameter Q remains a powerful tool for the estimate of the disc's "temperature" and it is therefore important to understand its behavior under various structural conditions specified below. Furthermore, a better appreciation of the possible values of Q as a function of the distance to the center R can be helpful. Let us consider the following models for the density distribution perpendicular to the galactic plane:

$$\rho(z) = 2^{-2/n} \rho_e \mathrm{sech}^{2/n}(nz/z_e)$$

Two extreme cases are n =1 (isothermal case) and $n = \infty$ (exponential case). For the isothermal case, the surface density at distance R from the center is

$$\begin{aligned}\Sigma(R) &= 2\int_0^\infty \rho(R,0)\mathrm{sech}^2(z/z_0)dz \\ &= 2z_0\rho(R,0)\end{aligned}$$

with $z_0 = 2z_e$ and $\rho_0 = \rho_e/4$.

In such a case the velocity dispersion in the z-direction is given by

$$\begin{aligned}\sigma_z^2(R) &= 2\pi G\rho(R,0)z_0^2 \\ &= \pi G\Sigma(R)z_0\end{aligned}$$

In these formulas, z_0 is an estimate of the constant height of the disk. The distribution of light in the old disk is (van der Kruit and Searle, 1981)

$$L(R,z) = L_{0,0}e^{-R/h}\mathrm{sech}^2(z/z_0)$$

Furthermore, the following expression for $V(R)$ can fit reasonably well many observed rotation curves:

$$V(R) = V_{max}\sqrt{\frac{R^2}{R^2+R_m^2}}$$

In this case

$$\kappa(R) = \sqrt{2}V_{max}\frac{\sqrt{R^2+2R_m^2}}{R^2+R_m^2}$$

Taking account of these relations, we can rewrite the equation for Q in the form

$$Q = .264\frac{\alpha}{k}\cdot\frac{V_{max}}{(L_{0,0})^{1/2}(M/L)^{1/2}}\cdot\frac{e^{R/2h}}{R^2+R_m^2}\sqrt{R^2+2R_m^2}$$

with V_{max} expressed in kms^{-1}, $L_{0,0}$ in $10^{-2}L_{\odot}pc^{-3}$, R in kiloparsecs. $\alpha = \sigma_R/\sigma_z \simeq 2$.

This formula is valid in the region where the model is applicable, i.e. between $R = h$ and $R = 3h$ for late-type spirals (without large bulges).

Figure 7 compares the behavior of $F(R, R_m, h)$, factor Q containing R, for two cases ($R_m = 0$ and $R_m =$ h), assuming α as well as $(M/L)_D$ constant. It is generally admitted that M/L is constant in the whole disc on the basis of some data on the colors. However, we cannot exclude a variation of $(M/L)_D$ with R. If the central parts are redder than the outer ones, M/L could decrease considerably from the center to the edge so that Q could increase more strongly with R. Further, if we suppose Q = const as it has often been assumed, $(M/L)_D$ behaves like

$$\frac{exp(R/h)}{(R^2 + R_m^2)^2}(R^2 + 2R_m^2)$$

One can also write (Bottema, 1993)

$$Q = 0.10 \left(\frac{\langle\sigma_R^2\rangle_{R=0}^{1/2}}{\text{km s}^{-1}}\right) \left(\frac{\Sigma_0}{M_{\odot}\text{pc}^{-2}}\right)^{-1} \left(\frac{V_{max}}{\text{km s}^{-1}}\right) \frac{F(R, R_m, h)}{\text{kpc}^{-1}}$$

with

$$F(R, R_m, h) = e^{R/2h} \left(\frac{\sqrt{R^2 + 2R_m^2}}{R^2 + R_m^2}\right)$$

Adopting a value of 1.4 kpc for R_m considered as independent of the brightness of a galaxy, this author introduces the relation

$$h = 4.95 \times 10^{-6} \times v_{max}^{2.6}$$

inferred from the Tully-Fisher relation and $\log R_{25} = -3.9 - 0.245 M_B$ for Sb and Sc galaxies by Rubin et al. (1985) with $R_{25} \simeq 3.1 \times h$ (Freeman's law).

Replacing Σ_0 by $\mu_0^B(\frac{M}{L})_B$ where $\mu_0^B = 136\ L_{\odot}pc^{-2}$, the expression for Q leads to

$$\sigma_R(R = h) = \frac{816Q(M/L)_B}{V_{max}F(V_{max}, R = h)}$$

The relation between the velocity dispersion at $R = h$ and V_{max} is plotted in fig 8, a) for various M/L, adopting Q =1.7 for all the galaxies which is the value suggested be Sellwood and Carlberg (1984) from their numerical simulations and b) for $Q = 2$ or 3, adopting $M/L = 2.0$ for all the galaxies. This figure reproduced from Fig 11. of Bottema's (1993) paper, shows a very good agreement between the relation adopted above and the observational values for $(M/L)_B \simeq 2.8$ or for Q between 2 and 2.5. Assuming that the central velocity dispersion of disks is not very different from values of σ_0 observed by Whitmore at al. (1985) for the bulges and using values of V_{max} given by Corradi and Capaccioli (1991) for about thirty galaxies, we have constructed a similar diagram which gives the same agreement.

For the model previously defined we can estimate the ratio of the radial (κ) to perpendicular (ν_z) frequencies as a function of R.

The force perpendicular to the galactic plane K_z, is, for a self-gravitating isothermal sheet ($n = 1$),

$$K_z = -\frac{2\sigma_z^2}{z_0} \cdot \mathrm{th}(z/z_0)$$

Then

$$\nu_z^2 = -\left(\frac{\partial K_z}{\partial \dot{z}}\right)_{z=0} = 4\pi G L_{0,0}\left(\frac{M}{L}\right)\exp\left(-\frac{R}{h}\right)$$

and

$$\frac{\kappa}{\nu_z} = \frac{1}{\sqrt{2}\pi} \cdot \frac{k}{\alpha} \cdot Q$$

For example, with $\alpha = 2$ and $k = 3.36$

$$\frac{\kappa}{\nu_z} = .378 \times Q$$

This result is independent of the $(M/L)_B$ ratio as well as of $L_{0,0}$. The behavior of κ/ν_z versus R is easily deduced from that of Q.

In a flattened galaxy we expect that $\frac{\kappa}{\nu_z} < 1$ on a large range of R except for the central parts. Therefore Q ought to be smaller that 2.65 in order to satisfy this condition. If $k = 2.9$ instead of 3.36, the condition becomes $Q < 3$. If $k = 2.6$, $Q < 3.4$.

The case $Q \sim 2.65$ is interesting because the resulting $\kappa/\nu_z = 1/1$ is a case of resonance coupling between the perpendicular and the plane motions for eccentric orbits in an axisymmetric or triaxial galaxy (Binney, 1981) which is able to trigger a secular evolution of the system by way of an excursion of orbits out of the galactic plane. However, the importance of the process can only be tested by N-body simulations which take the collective effects into account.

It is easy to show and interesting to note that if we replace the isothermal model by the exponential one ($n = \infty$) or better by an intermediate one ($n = 2$), the last relation between κ/ν_z and Q is unchanged.

It remains to explain how the stellar velocity dispersion in disks can reach values which lead to $Q > 1$ to 2. Our Galaxy gives the unique opportunity to study in details the kinematic properties of stars (systematic and residual motions), especially in the solar neighborhood: in the external galaxies, kinematical data are restricted to rotation curves, central l.o.s. velocity dispersions and, for several objects (Bottema, 1993), the variation of the radial or perpendicular velocity dispersion in terms of the distance to the center. We have no simultaneous access to the components in the usual preferential directions $\sigma_R, \sigma_\phi, \sigma_z$, as it is the case in our Galaxy. Now these data are important to constrain the possible processes responsible of the disk heating.

5.2 Disk heating in our Galaxy

The relation age-kinematics for stars in the solar neighborhood which is an undisputed evidence for the disk heating has been repeatedly discussed in the past. An extensive review has been published by Lacey (1991). From it, we will retain recent observational data and briefly discuss some of the scenarios suggested to explain the increase of velocity dispersion with age in the galactic disk.

The estimation of stellar ages put difficulties for oldest stars (age higher than 2 billions years). The selection of stellar groups near the main sequence according to the color index or the spectral type gives only very rough results. The chromospheric activity concerns essentially K stars. At present the most effective method is based on the (u, b, v, y) Strömgren photometry added to isochrones of stellar evolution models for B to F spectral types, but the squeezing of the isochrones on a range of color index corresponding to an age range between 3 and 10 billions years induces large uncertainties on the individual ages.

An age-kinematics relation has been obtained by Carlberg et al. (1985) from a sample of 225 F dwarfs and by Strömgren (1987) from 558 population I dwarfs and 1294 stars of any heavy element abundance. Clearly there is a disagreement between these recent relations and the older ones (Wielen, 1977) for ages larger than 2. 10^9 years. A part of this disagreement comes from the choice of the galactic age (15. 10^9 years for recent determination, 10. 10^9 years for the older). The law of variation of the velocity dispersion σ in term of age is

$$\sigma(\tau) = (\sigma_0^{1/p} + C\tau)^p$$

with $p = 0.5$ for the old data and 0.2 - 0.3 for the new ones. As for the velocity dispersion ratios in terms of age, it is not indicated to use them as strong constraints for physical process models because of their high error bars. Average values are $\sigma_R : \sigma_\phi : \sigma_z = 1 : 0.6(\pm 0.03) : .32(\pm 0.03)$. A third group of data should be explained by a correct model of disk heating: it is the radial dependence of σ_R or σ_z. At the present time only the local kinematics has been used to test the various mechanisms suggested, which we will now enumerate. Of course, these mechanisms concern any galactic disk but are only constrained by local observations.

a) <u>Diffusion by massive gas clouds</u>

Analytical investigations and numerical simulations lead to an estimate of orbital diffusion time by 2-body star–cloud encounters

$$\tau_{diff} \div \frac{\sigma_*^3}{G^2 \rho_c M_c \ln\Lambda} \sim 10 \times 10^9 \text{ years}$$

with $\sigma_* \sim 30\ kms^{-1}$, $\rho_c \sim 0.1\ M_\odot pc^{-3}$, M_c = cloud average mass $\approx 10^6 M_\odot$ and Λ, the "Coulomb logarithm". There exists a population of molecular clouds with these properties. Because the diffusion transfers the energy between radial and perpendicular oscillations and increases on average corresponding energies to the detriment of the systematic circular motions, this process implies a velocity isotropisation which contradicts the observed values $\sigma_z/\sigma_R = 0.5$. It also predicts σ (10^{10} years) ~ 20 kms^{-1} whereas the old disk has $\sigma \simeq 50\ kms^{-1}$.

b) Diffusion by spiral density waves

This process, proposed by Toomre (1964) and numerically investigated by various authors (for ex. Sellwood and Carlberg (1984) and references therein), is exposed to a difficulty due to the fact that the stability of disks is very sensitive to the velocity dispersion. If σ_R is too weak, spiral waves (or bars) develop, but because of the field fluctuations, σ increases and Q becomes of the order of 2 after 10^9 years. Then the disk is too hot to maintain the spiral structure. The dissipation in gas or a continuous accretion of matter are able to reduce this effect but one must cope with the problem of maintaining the spiral arms for a long time. Furthermore spiral modes found by analytical or numerical investigation seem to be unable to heat the whole disk. The suggestion of recurrent instabilities by Sellwood and Lin (1989) could be a solution. However, Carlberg (1987) found that vertical heating by the process invoked does not work by lack of conspiration between the radial and the perpendicular frequencies. In general $\mid m(\Omega - \Omega_p) \mid \leq \kappa$ and $\nu_z/\kappa \sim$ 2-3. But see sect. 6 for barred galaxies.

c) Combined effect of spirals and clouds

The respective failures of the previous processes led Carlberg (1987) and Jenkins and Binney (1990) to consider the combined effect of both. A stochastic acceleration, described by Fokker - Planck diffusion equation, is produced. For a clear setting-up of this equation, cf. Hénon (1973). The ratio σ_z/σ_R depends on a parameter β which represents the ratio of the heating actions by spiral wave and cloud diffusion ($\beta =$ 0 when the later acts alone). A good agreement with observations is obtained for $\beta = 90$, which represents a spiral perturbation of the potential $\Delta\Phi \approx 11\ (kms^{-1})^2$. Then $\sigma_R \div t^\alpha$ with $\alpha \sim 0.5$ and $\sigma_z \simeq t^\alpha$ with $\alpha \simeq 0.3$.

The resulting $F(v_z)$ is Gaussian. However better observational data with more accurate age calibration are necessary, in particular to specify the value of α. Let us note that in fact, molecular clouds are a small scale limit of a chaotic spiral structure. Finally, the self-gravitation of the disk should be taken into account: Heating induces the disk thickening, then density decreases what induces an adiabatic cooling whereas matter accretion on the disk can trigger the formation of new stars which increases density, inducing an adiabatic heating.

d) Heating of disks by satellite accretions

This process will be examined in section 8, devoted to environmental effects on the internal dynamical evolution of disks.

e) Heating by vertical resonances

This process will be described in section 6.4.

5.3 Stability of a two-fluid system against axisymmetric and non–axisymmetric perturbations.

A real galaxy consisting of stars and gas, it is important to study the physical effects of gas inclusion on the stability.

Local gravitational instabilities in a two-component system (gas and stars) obviously depends on the parameters of both gas and stellar fluids. A condition of neutral stability has been given by Jog and Solomon (1984) in the form of the dispersion relation

$$\frac{2\pi G k \Sigma_*}{\kappa^2 + k^2\sigma_*^2} + \frac{2\pi G k \Sigma_g}{\kappa^2 + k^2\sigma_g^2} - 1 = 0$$

the first and the second terms corresponding respectively to stars and gas with k = wave number, σ_g = sound speed in the gas, σ_* = velocity dispersion of stars. Even a small gaseous fraction (less than 10 - 20%) of the total disk density significantly decreases the stability of disks against axisymmetric perturbations. An important extension of this work has been recently published by Jog (1992) who studied the growth of non-axisymmetric perturbations in disks represented as two-fluid systems. The underlying phenomenon previously studied for one-component cases and called "swing amplification" by Toomre (1981 and references therein) results from a conspiration of three effects: shearing, due do the differential rotation, shaking due to the epicyclic motions and self-gravity of the matter involved. Tagger et al. (1994) add a fourth factor, the thickness of the disk.

In her treatment, Jog assumes that the stellar (s) and gaseous (g) components are two isothermal fluids with surface density μ_i ($i = s$ or g) and sound velocity or 1-D velocity dispersion c_i ($c_g \ll c_s$). μ_0 and Φ_0 are the unperturbed density and potential in an infinitely thin disk supported by the differential rotation and residual motions of both fluids. The fluid representation is considered as relevant for local analysis far from resonances if the shearing is not too important. The mathematical treatment is simplified. However in a more rigorous approach, the isothermal assumption for the stellar velocity distribution has to be dropped and the viscosity of the gas has to be taken into account.

We quote here the main points of the formalism used by Jog. The Euler equation in an uniformly rotating frame are

$$\frac{\partial V_i}{\partial t} + (V_i \cdot \nabla)V_i = -\frac{c_i^2}{\mu_{0i}}\nabla\mu_i - \nabla(\Phi_s + \Phi_g) - 2\Omega \times V_i + \Omega^2 r$$

where V_i is the 2-D fluid velocity with respect to rotating axes.

It is a matter of studying the dynamics of a local region around a point (r_0, ϕ_0) corotating in the defined frame, when a perturbation $\delta\mu, \delta\phi$ is introduced. The Euler, continuity and Poisson equations can successively be written, by using sheared comoving axes (coordinates x', y', z', t'), as

$$\frac{\partial v_{xi}}{\partial \tau} - \frac{\Omega}{A} v_{yi} = -i\frac{k_y}{2A}\tau\left[-(\delta\Phi_s + \delta\Phi_g) - \frac{c_i^2}{\mu_{0i}}(\delta\mu_i)\right]$$

$$\frac{\partial v_{yi}}{\partial \tau} + \frac{B}{A} v_{xi} = i\frac{k_y}{2A}\left[-(\delta\Phi_s + \delta\Phi_g) - \frac{c_i^2}{\mu_{0i}}(\delta\mu_i)\right]$$

$$\frac{\partial}{\partial \tau}(\delta\mu_i) - i\frac{k_y}{2A}\tau\mu_{0i}v_{xi} + i\frac{k_y}{2A}\mu_{0i}v_{yi} = 0$$

$$\left[-k_y^2(1+\tau^2) + \frac{\partial'^2}{\partial z^2}\right](\delta\Phi_s + \delta\Phi_g) = 4\pi G(\delta\mu_s + \delta\mu_g)\delta(z')$$

$$(\delta\Phi_s + \delta\Phi_g) = -\left[\frac{2\pi G}{k_y(1+\tau^2)^{1/2}}\right](\delta\mu_s + \delta\mu_g)$$

Here $\tau = 2At - \frac{k_x}{k_y}$ is a dimensionless measure of time, k_x and k_y the wave numbers in x, y directions, x being along the initial outward direction. A and B are the Oort constants. A trial solution has been introduced, proportional to $\exp[i(k_x x' + k_y y')]$. The equation must be solved to obtain $\delta\mu_s$ and $\delta\mu_g$ as functions of time, given their initial values.

It is illuminating to have Euler's equations rewritten for $\Theta_i = \delta\mu_i/\mu_{0i}$ and after substitution of Poisson equation into them,

$$\left(\frac{d^2\Theta_i}{d\tau^2}\right) - \left(\frac{d\Theta_i}{d\tau}\right)\left(\frac{2\tau}{1+\tau^2}\right) + \Theta_i\left[\frac{\kappa^2}{4A^2} + \frac{2B/A}{1+\tau^2} + \frac{k_y^2}{4A^2}(1+\tau^2)c_i^2\right]$$
$$= (\mu_{0s}\Theta_s + \mu_{0g}\Theta_g)\left(\frac{\pi G k_y}{2A^2}\right)(1+\tau^2)^{1/2}$$

We clearly see in the brackets the terms respectively corresponding to the epicyclic motion, the shear and the fluid pressure whereas the right hand side corresponds to the self-gravity of the $s - g$ two-fluid system.

From this equation, following a mode evolution from a leading feature ($\tau < 0$) to a trailing one ($\tau > 0$), it is easy to infer: 1) At large $|\tau|$, the pressure term dominates, Θ_i

is oscillatory with constant amplitude and a frequency proportional to $c_i\tau$ ($c_s > c_g$) and the s and g equations are weakly coupled. 2) For $\tau \to 0$, epicyclic and shearing terms become important, eventually with cancellation for a flat rotation curve. If the self-gravity dominates, we have swing amplification, which is temporary because for $\tau \gg 0$, we find back an oscillatory solution. 3) The coupling between both fluids is higher at low $\mid \tau \mid$. It is noted that if $k_y \to 0$ (purely radial perturbation), the last equation, after Fourier analysis, is reduced to the dispersion relation for the axisymmetric case obtained by Jog and Solomon (1984)

$$(\omega^2 - \kappa^2 - k_x^2 c_s^2 + 2\pi G k_x \mu_{0s})(\omega^2 - \kappa^2 - k_x^2 c_g^2 + 2\pi G k_x \mu_{0g})$$

$$-(2\pi G k_x \mu_{0s})(2\pi G k_x \mu_{0g}) = 0$$

ω being the perturbation frequency in the trial solution $\Theta_i = \Theta_{i0} exp(i\omega t)$

The next step is to look for dependence of the solution on various crucial parameters such as Q_s and Q_g ("Toomre's parameters"), the fraction of mass ϵ in form of gas, the rate of shearing $\eta = 2A/\Omega_0$ and X, the wavelength of perturbation in terms of the critical wavelength for growth of instabilities

$$X = \frac{\lambda_y}{\lambda_{crit}} = \frac{2\pi r}{m} \cdot \frac{\kappa^2}{4\pi^2 G \mu_0^2}$$

where m is the arm number, $\mu_0 = \mu_{0s} + \mu_{0g}$.

It is to note that in the present context (the gas is the colder component)

$$\frac{Q_g \epsilon}{Q_s(1-\epsilon)} < 1$$

Jog gives the new form of the differential equation for Θ_i when these parameters are introduced ($\xi^2 = \kappa^2/4A^2 = 2(2-\eta)/\eta^2$)

$$\left(\frac{d^2\Theta_s}{d\tau^2}\right) - \left(\frac{d\Theta_s}{d\tau}\right)\left(\frac{2\tau}{1+\tau^2}\right)$$

$$+ \Theta_s \left[\xi^2 + \frac{2(\eta-2)}{\eta(1+\tau^2)} + \frac{(1+\tau^2)Q_s^2(1-\epsilon)^2\xi^2}{4X^2}\right]$$

$$= \frac{\xi^2}{X}(1+\tau^2)^{1/2}[\Theta_s(1-\epsilon) + \Theta_g \epsilon]$$

$$\left(\frac{d^2\Theta_g}{d\tau^2}\right) - \left(\frac{d\Theta_g}{d\tau}\right)\left(\frac{2\tau}{1+\tau^2}\right)$$

$$+ \quad \Theta_g \left[\xi^2 + \frac{2(\eta-2)}{\eta(1+\tau^2)} + \frac{(1+\tau^2)Q_g^2\epsilon^2\xi^2}{4X^2} \right]$$

$$= \frac{\xi^2}{X}(1+\tau^2)^{1/2}[\Theta_s(1-\epsilon)+\Theta_g\epsilon]$$

A typical case is illustrated in fig. 9. The choice of parameters is Q_s =1.5, Q_g =1.5, $\epsilon = 0.1$, X =1, η =1. The two-fluid system is stable to axisymmetric perturbations. The amplification is higher in the gas with a more highly wound spiral feature.

Other cases, shown in Jog's paper, indicate the dependence of the solution on the mentioned parameters and lead to the following important conclusion: Growth of non axisymmetric perturbations in a real galaxy may occur even if the system is stable against axisymmetric perturbations and/or if either fluid component is stable against non-axisymmetric component.

We add that, according to the fact that the process is more effective when the gas fraction is high, it must be more important in late-type galaxies.

5.4 Global stability of disks and the dark matter problem

The essential of what we know on the large scale instabilities (or stability!) of disks has been obtained through N-body simulations. In the early seventies, two kinds of related results appeared simultaneously: whereas the first flat rotation curves were published suggesting the presence of hidden forms of matter in isolated galaxies, seminal papers by Miller and Prendergast (1968) and Hohl and Hockney (1969) presented N-body simulations of disk galaxies in view of explaining the formation and the maintenance of spiral structures in the disks. At that time, the galactic astronomers were focused on the linear density wave theory (Lin and Shu, 1964; Goldreich and Lynden-Bell, 1965) concerning tightly wound spirals.

The central problem for people working on N-body numerical experiments was to suppress these strong bars which form spontaneously in the simulations, in order to obtain the well defined grand design spiral structure, "so often observed" and "well explained by the density wave theory". In fact, in the seventies, bars were not much studied and practically not observed in details with the noticeable exception of de Vaucouleurs who insisted many times on their importance in the spiral classification. A further motivation to ignore them was that bars could hardly be included in the density wave theory.

Essentially, two solutions were often considered to prevent the formation of bars: hot disks or massive halos (Ostriker and Peebles 1973; Einasto et al. 1974). But in the seventies no data could indicate that the inner parts of the galactic disks were hot enough. Discs were considered as consisting of cold populations by extrapolation of the solar neighborhood data. Now, more recent observations indicate that the velocity dispersions in inner parts of typical Sbc galaxies can be of the order of 100 kms^{-1} (e.g. Lewis et Freeman 1989; Bottema 1993), showing that disks can be hot there.

In the last decade, theoretical arguments, as well as new numerical experiments and deductions from the observations of the luminosity profile of disks, have contributed to modify the point of view about the role played by halos on global stability. The stabilization of disks against non-axisymmetric perturbations, if ensured by halos, should only depend on the halo mass located in the inner region of the disc, where a bar usually develops; it should not depend on the halo mass in the outer regions, where the rotation curve remains flat. Kalnajs (1987) disturbed minds by claiming 1) that the "luminous " rotation curves, calculated from the observed exponential stellar disk luminosity profiles, would fit very well the observed rotation curves in the optical region of galaxies, and 2) that, as a consequence, dark halos would be useless to stabilize against a bar.

Today, the global stability in the disks against bar-like perturbations is no longer the acute problem that it used to be in the seventies. As mentioned in section 2, at least two thirds of observed spirals are recognized to be barred or to present an oval structure in the inner regions (de Vaucouleurs 1963). The remaining third includes edge-on and dusty galaxies in which a bar can hardly be detected. In many cases, IR photometry, less affected by dust, reveals barred structures invisible in the B band. Further on, extensive numerical simulations by Athanassoula and Sellwood (1986) have shown that the growth of a bar may be prevented if the central part of the disk is hot enough. $Q \simeq 2.5$ at all radii seems to be a sufficient criterion for global stability against non-axisymmetric perturbations in all disks. However, this result concerns pure stellar disks.

Coming back to the kinematic observations, we point out that spiral HI rotation curves, which extend typically up to 2 - 3 R_{25}, are well known to stay in general sufficiently flat over this range for requiring an important presence of mass in the outer parts of spirals. Some authors tried to solve the question of the contribution of various components (disc, bulge halo) to the observed rotation curves produced by HI data (see e.g. Sancisi and van Albada 1987). Among the proposed solutions, the "maximum disc" solution consists in maximizing the contribution of the luminous matter to the observed rotation curve. This yields to a conservative lower limit for the amount of dark matter and an estimate of the M/L ratio for the disk. The maximum disk solution received recently a strong support from an extensive work on kinematical data of ~ 500 galaxies; it reproduces well the fine wiggles of the light, confirming that the fraction of dark halo within the stellar optical parts of the galaxies must not be important (Freeman 1993). Athanassoula et al. (1987) have had recourse to the swing amplification theory to better constrain the contribution of the halo and the disk to the rotation curve. Considering that a lot of galaxies display a 2-arm spiral structure, they were able to estimate the disk mass required to kill the mode 1 and to preserve the mode 2. This interesting approach is worth to be pursued, based on more extensive data.

From the present discussion, we can conclude that the only really obvious dark matter problem subsisting in spirals concerns the outer regions beyond the optical disk. The fact that the disk plus the bulge on one hand, and the dark halo on the other hand, bring an essential and about equal contribution to the flat rotation curve in two distinct regions, namely in the inner luminous region and in the outer 21 cm emitting HI region, has suggested a physical coupling between these components, also called disc-halo conspiracy

(Bahcall and Casertano 1985; van Albada and Sancisi 1986). No convincing explanation of it has yet been given.

Furthermore new HI observations have restricted the range of the conspiracy: Casertano and van Gorkom (1991) published HI rotation curves characterized by a large decrease between 1 and 3 R_{25} and have found a clear correlation between the peak circular velocity, its central brightness and the slope of the rotation curve in the outer parts of the disks (see section 2, remark 7). This result leads these authors to suggest that the ratio of the dark to luminous matter might be the critical parameter controlling the Hubble sequence. It is crucial to determine if really late-type galaxies contain more dark matter than early-type ones, or if the importance of dark matter decreases with luminosity. These questions introduced by Tinsley (1981) have been recently discussed again (see for example Salucci et al. 1991) and references therein).

A curious coincidence (Bosma 1981), until now unexplained, has been recently recalled by Freeman (1993, and references therein) and better documented by Broeils (1992): the surface density ratio of dark matter and HI gas in a sample of galaxies remains constant outside the optical disc, around 10 - 30. If dark matter tends to follow HI with a constant ratio of about 20, we have the very curious situation that dark matter is concomitant to HI radially, yet dark matter does not prevent HI to flare out the plane just beyond the optical disk. The flaring shows at least that dark matter cannot be much flatter than HI.

It can be assumed that if the dark matter is radially proportional to HI, it should be also proportional perpendicularly. We quote here the arguments given by Pfenniger et al. (1994) and Pfenniger and Combes (1994) to suggest that dark matter in spiral galaxies is essentially made of cold hydrogen in a disc supported mostly by rotation.

Different observational constraints can be advanced on the presumed form of hydrogen. A massive amount of hot or warm form of hydrogen can be ruled out, essentially because hot and warm gas already fills most of the interstellar volume at a much too low density to contribute to the mass in an appreciable amount. On the contrary, although cold gas fills a small part of the volume, its density is large enough to encompass most of the ISM mass. Moreover, this condensed structure explains in part the invisible character of the medium, its low cross-section for absorption studies, and its transparency to external radiation. Even if the medium is essentially molecular, the CO molecule can hardly serve as a tracer, because of the low metallicity of this quasi-primordial gas. A fraction could also be in atomic form. but HI emission cannot be detected in a medium at 3 K. Absorption studies of the outer parts of galaxies could probe this medium. Cold gas is observed to be fractal over several decades of length and density. Details are given in Pfenniger and Combes (1994). It turns out that both the problem of mass underestimate in HI disks and the problem of star non-formation in outer disks is closely linked to the fractal structure. The physical state of this gas must be high density and cold temperature. Since no significant heating sources in the outer disks presumably exist, it can be assumed that the gas is bathing in the cosmological background, and that its temperature is about 3 K. In these nearly isothermal conditions, clouds can fragment until they reach small clump units, where the cooling time becomes comparable to the free-fall time. The average typical density of these elementary clouds lets, called "clumpuscules" is 10^{10} cm^{-3} , column density 10^{24} cm^{-2}, size 30 AU, and mass 10^{-3}M$_\odot$. These small units are the building blocks of a fractal structure, that ranges upwards over 4 to 6 orders of magnitude in scales. They are gravitationally bound, and the corresponding thermal width along the line of sight for molecular hydrogen at $T = 3K$ is about 0.1 kms^{-1}.

We will come back later (section 10) to this suggestion since it could certainly have consequences on the secular evolution of disks and particularly on the morphological changes that galaxies can suffer with time.

If the dark matter in disk galaxies is assumed to be made of cold gas, the question of stability of such disks must be discussed. In fact, HI disks are far from being perfectly axisymmetric, smooth and thin: large asymmetries, warps, spiral arms, massive HI complexes are observed. The gas is unsteady. Q could be subcritical. But the classical Toomre formula is less obvious in this context. An extensive discussion in Pfenniger et al. (1994) shows that the stability and self-consistency of cold gaseous disks is a less severe problem than commonly believed.

6 Interaction between components

6.1 Disk-halo interactions

a) Adiabatic invariance of actions for spheroid stars

When the actions $\boldsymbol{J}(J_r, L_z, J_\theta)$ characterizing stellar orbits in a galactic potential are well defined (for stars on regular orbits), they can be considered as adiabatic invariants if the potential is varying slowly (see section 3). Such is the case of the actions of stars belonging to a spheroid in the frame of the scenario (Fall and Efstathiou, 1980) according to which the disk gradually forms in the potential well of the spheroid. Information on the original orbits of these stars can be inferred owing to this property. The distribution function $F(\boldsymbol{J})$ is also an invariant.

In axisymmetric models of our Galaxy, the most important resonance in the meridian plan (R, z) is 1/1. The corresponding resonant orbits occupy a small fraction of phase space ($\leq 8\%$). Chaotic orbits only concern stars with very small angular momentum ($\leq$ 200 $kms^{-1}kpc$) (Cretton and Martinet, 1994). It is shown that actions could be defined for many halo stars in the solar neighborhood even if the z-velocity is relatively large ($\approx$ 100 kms^{-1}).

Binney and May (1986) have studied the response of a galactic spheroid to the slow accumulation of a massive disk. It appears that models which start from a spherical distribution cannot reproduce the presently observed solar neighborhood because the final state in this case would have $\sigma_\theta > \sigma_r$ or $\sigma_\theta > \sigma_\phi$ or both, which are in contradiction with the kinematical properties of halo stars. Models starting from flattened initial conditions (axis ratio $q_0 = 0.7$) can yield velocity dispersions σ_ϕ/σ_r or $\sigma_\phi/\sigma_\theta$ in agreement with observations and the final axis ratio is $q_F = 0.33$. The slow accretion of a disk must progressively flatten the spheroid.

b) Transfer of angular momentum from disk to the halo in N-body simulations

In many N-body simulations of disk galaxy evolution, a current approximation is to consider the halo as a rigid body. It is necessary to know whether simulations in which disk and halo stars are treated self-consistently give the same results. Sellwood (1980) has shown that rigid approximation is sufficient for studies of global stability. However it is expected that the disk loses angular momentum to the halo. In fact a significant transfer of angular momentum from the disk to the halo is observed only after the formation of a strong bar, which appears in fact as a tool of this transfer. In this work, the evolution was followed for at most 1200 Myr and the long term consequences of the interaction were not evaluated. The problem has been recently restarted by Little and Carlberg (1991) and Hernquist and Weinberg (1992) (see the next subsections).

6.2 Dynamical friction and bar-disk interactions

Let us consider a set of stars (field stars) of individual mass m having a velocity distribution $F(v_m)$, and a test body of mass M moving through the stellar background with an initial velocity V_o relatively to the center of mass of the field stars. As a result, M

undergoes a purely gravitational retarding force due to the density enhancement, behind it, of stars deviated by its passage. The net effect, termed "dynamical friction", is a motion deceleration of M, which has been evaluated, fifty years ago, by Chandrasekhar (1943) for the case of a rigid body passing through an infinite homogeneous medium (see also Hénon, 1973).

For instance if $F(v_m)$ is Maxwellian, the deceleration is

$$\frac{d\boldsymbol{V}_M}{dt} = -\frac{4\pi \ln\Lambda G^2(M+m)}{v_M^3}\rho\left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}}e^{-X^2}\right]\boldsymbol{V}_M$$

where $X = v_m/2\sigma$, $\ln\Lambda$ is the Coulomb logarithm with $\Lambda = b_{max}/b_{min}$ = ratio of the maximum to the minimum impact parameter, ρ is the field stellar density. So, the deceleration is proportional to $M \cdot \rho/V_M^2$ and only stars with velocity $v_m < V_M$ contribute to it.

Dynamical friction is potentially important in various problems of galactic dynamics: the motion of globular clusters or dwarf galaxies around parent galaxies, the motion of bars within galactic disks or the interaction of disk galaxies. A number of questions arise about the application of the Chandrasekhar formula in such contexts: Is the self-gravity of the field population negligible? Is the local treatment adequate? What about the hypothesis of rigidity? Is the dynamical friction dominant with regard to resonance effects occurring in disk galaxy interactions? Furthermore, we must recall shortcomings such as the arbitrary choice of b_{max} or the use of keplerian hyperbolae for orbits in 2-body encounters (on which the calculation of dynamical friction is based). Especially a controversy recently arised concerning the importance of the self-gravity in dynamical friction (see for instance Combes (1992) and references therein). In our context of interactions between components of disk galaxies, we will here be concerned by the self-gravity problem in the frame of the bar-disk angular momentum exchange. We will come back in section 8 on the dynamical friction in galaxy-galaxy interactions.

A 2-D analytic calculation of the friction between a bar and a disk has been made in the angle-action formalism by Little and Carlberg (1991) in the absence of self gravity. The chosen unperturbed potential Φ_0 is that of a Kuzmin-Toomre disc

$$\Phi_0 = -\frac{GM_{tot}}{\sqrt{r^2 + r_h^2}} = -\frac{1}{\sqrt{r^2+1}}$$

In presence of a bar, the Hamiltonian of a disk star is

$$H' = H_0 + \epsilon H_1$$

where H_1 includes a barlike perturbing potential

$$\Phi_1(r, \varphi, t) = \Psi(r)\cos[2(\varphi - \Omega_b t)]\sin^2 \nu t \quad , \quad 0 \leq \nu t \leq \pi$$

with

$$\Psi(r) = Q_{bar}\frac{(r/b)^2}{1+(r/b)^5}$$

with Q_{bar} = constant controlling the strength of the bar, b measuring the radial size of the bar. The cosine term indicates the bipolar nature of Φ_1, Ω_b is the rotation frequency of the bar, the $\sin^2 \nu t$ term is there to gradually turn the bar on at $t = 0$ and off at $t = t_f$.

Introducing the angle-action variables corresponding to the perturbed Hamiltonian:

$$J'_i = J_i + \sum \epsilon^n \Delta_n J \quad , \quad w'_i = w_i + \sum \epsilon^n \Delta w$$

the authors use the classical theory of perturbations to evaluate the first and second order change $\Delta_1 J_2$ and $\Delta_2 J_2$ in the star's angular momentum J_2. Φ_1 being periodic in w_1, w_2 is expanded as usually in Fourier series

$$\Phi_1(\boldsymbol{J}, \boldsymbol{w}, t) = \frac{1}{4\pi^2} \sum \Psi_{lm}(\boldsymbol{J}, t) e^{i(lw_1 + mw_2)}$$

In view of comparing the analytic treatment with $N-$body simulations, it is necessary to average $\Delta_1 J_2$ and $\Delta_2 J_2$ over (w_1, w_2) for many stars. The contributions to the leading term $\langle \Delta_2 J_2 \rangle$ can be calculated in the epicyclic approximation ($\langle \Delta_1 J_2 \rangle = 0$). They are essentially resonant ($l = 0$ for the corotation, $l = \pm 1$ for the ILR and OLR). Their sum approximately gives the true phase average angular momentum change per star by unit mass. It is proportional to $(Q_{bar})^2$.

Little and Carlberg undertaken numerical simulations of the bar-disc system described above to evaluate the role of the self-gravity which is parameterized by $s_g = M_{disc}/M_{tot}$ ($0 \leq s_g \leq 1$). The axisymmetric galaxy model consists of a N-body disk and a rigid halo. An artifice permits to vary the relative masses of the disk and halo without changing the overall potential set equivalent to Kuzmin disk in the plane (with the object of comparing with the theoretical results). As an example, fig. 10 shows the angular momentum change $\langle \Delta J \rangle$ for a case of sufficiently weak bar having a high pattern speed ($s_g = 0$). The agreement between theoretical prediction and numerical simulation is very good. The angular momentum exchange occurs at the corotation ($r_0 \sim 2.3$) and at the OLR ($r_0 \sim 3.2$). In another example (low pattern speed of the bar) an absorption of angular momentum near the corotation located just beyond the edge of the disk is observed as well as an emission near the outer ILR. These behaviors are consistent with the detailed and very clear discussion by Lynden-Bell and Kalnajs (1972).

As for the self-gravity effect, studied by varying s_g in the simulation, it appears that it tends to increase the absolute angular momentum transfer for high Ω_b and inversely to decrease it for low Ω_b. In fact the present situation is quite similar to the familiar problem of an harmonic oscillator driven at steady frequency $2(\Omega - \Omega_b)$ lower or higher than its natural frequency (κ) as underlined by Little and Carlberg. As the case may be, this oscillator responds in phase or in antiphase with the imposed perturbation. Self-gravity amplifies the process significantly but by less than 60%.

The authors mention that the present results on the role of the self-gravity cannot be compared with others connected with the decay of a satellite around a parent galaxy for which case the physics is very different. Therefore it is not surprising that the respective conclusions be different.

6.3 Bar-spheroid interactions

Bars also can interact with spheroidal components of galaxies such as bulges, luminous or dark halos. By using the angle-action formalism introduced in section 3 and by restricting oneself to a linear approximation of the perturbation theory, it is possible to estimate the torque exerted by a bar on a spheroid. The usual starting point is the collisionless Boltzmann equation formally written

$$\frac{\partial F}{\partial t} + \frac{\partial F}{\partial \boldsymbol{w}} \cdot \frac{\partial H}{\partial \boldsymbol{I}} - \frac{\partial F}{\partial \boldsymbol{I}} \cdot \frac{\partial \Phi}{\partial \boldsymbol{w}} = 0$$

where H is the Hamiltonian and Φ, the potential. Introducing a small potential perturbation Φ_1 uniformly rotating with a rotation frequency ω, and with $F = F_0 + F_1 + ...$, $\Phi = \Phi_0 + \Phi_1 + ...$, the Fourier-transformed linearized Boltzmann equation gives a response

$$\tilde{F}_n = \frac{\boldsymbol{n} \cdot \partial F_0 / \partial \boldsymbol{I}}{\boldsymbol{n} \cdot \boldsymbol{\Omega} - \omega} \, \tilde{\Phi}_{1n}$$

where $\Omega = \boldsymbol{\nabla}_I H_0(\boldsymbol{I})$ and $\boldsymbol{n} = (n_1, n_2, n_3)$. This classical result has been reproduced by Hernquist and Weinberg (1992) in the context of the interaction between a rigid bar with an initially non-rotating spheroid. It shows that large variations of density occur when $\boldsymbol{n} \cdot \boldsymbol{\Omega} - \omega \simeq 0$ that is to say at and near to resonances. In the limited frame of the non-self-gravitating problem and if the bar rotation axis is parallel to the z-axis of the spheroid, Weinberg (1985) had calculated that the torque exerted by the bar, given by the time derivative of the angular momentum J_z, depends on the square of the perturbed amplitude

$$\frac{dJ_z}{dt} = 4\pi^4 \int\int\int d\boldsymbol{I} \sum_n n_3 \Omega_b \boldsymbol{n} \frac{\partial F_0}{\partial I} \mid \Phi_{1n} \mid^2 \delta(\boldsymbol{n} \cdot \boldsymbol{\Omega} - n_3 \Omega_b)$$

where $\Omega_b = \omega/2$ is now the angular velocity of the bar.

Numerical simulations allow to study more thoroughly the effects of the interaction, in particular how the angular momentum transfer depends on various parameters implied in the problem as Ω_b, the bar to spheroid mass ratio, or the choice of the model for both.

For their simulations, Hernquist and Weinberg (1992) chosen for the bar the Ferrer's density profile, often used to describe the barred component in galaxies (see for ex. Athanassoula and al. (1983) for details)

$$\rho(\mu^2) = \rho_0 (1 - \mu^2)^2 \;\text{ if }\; \mu^2 \leq 1 \;\text{ with }\; \mu^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

and for the spheroid, various models, the more realistic of which having a density profile

$$\rho(r) = \frac{M}{2\pi} \cdot \frac{a_s}{r} \cdot \frac{1}{(r + a_s)^3}$$

associated to the potential

$$\Phi_s = -\frac{GM}{r + a_s}$$

A general result is that most of angular momentum exchange is resonant confirming the prediction of the linear theory. The quantity of angular momentum ΔJ transferred to the spheroid depends on the presence of low order resonances which are the most important and on the location and the number of resonances. In a typical simulation, with $M_b/M_s = .3$ inside corotation and a bar rotation period of 10^8 years, the bar loses its angular momentum in less than 10^9 years. Furthermore, a change of structure is observed in the spheroid: the density near the center decreases with time, the density in the outer parts increases with time. The spheroid becomes rotationally flattened. A pic value of the ratio of the systematic to the random velocity in the azimuthal direction is 0.2 - 0.25 which corresponds for an oblate spheroid to an ellipticity ~ 0.9. The physical process which triggers the angular momentum transfer is qualitatively well described by using the quadrupole component of the bar potential only.

It seems that we have here a new indication that bars are responsible of secular dynamical evolution in galaxies. However, in the mentioned experiments, the spheroid is initially non-rotating, the bar is rigid and does not respond to the spheroid. Now we know that bars can evolve for various reasons as seen in the others subsections. These limitation can restrict the bearing of the results mentioned above, at least quantitatively.

6.4 Stellar motions perpendicular to a disk perturbed by a bar: Resonant excitations.

Resonant excitation of motion perpendicular to the galactic plane (z-direction) can concern stars on quasi-circular orbits in a flattened slightly non-axisymmetric rotating potential. Theoretical prediction about it have been given by Binney (1981) and the results are clearly confirmed by 3-D numerical simulations which will be described in the next subsection.

Binney starts from the equation of z-motion for a star in a rotating potential $\Phi(R, \phi, z) = \Phi_0(R, z) + \Phi_1(R, z)\cos(2\phi) + \Phi_2(R, z)\cos(4\phi) + ...$

$$\ddot{z} + \{\nu_o^2 + 2q'_A \cos(\kappa t + \phi_0) + 2q'_B \cos[2t(\Omega - \Omega_p)]\}z = 0$$

with

$$\phi = (\Omega - \Omega_p)\ , \quad q'_A = \frac{1}{2}A\frac{\partial \nu_0^2}{\partial R}\ , \quad \nu_0^2 = \frac{\partial^2 \Phi_0}{\partial z^2}$$

and

$$q'_B = \frac{1}{2}\left\{\nu_1^2 - \frac{2\Omega\Phi_1 \partial\nu_0^2/\partial R}{R_0(\Omega - \Omega_p)[\kappa^2 - 4(\Omega - \Omega_p)^2]}\right\}$$

with $\nu_1^2 = \frac{\partial^2 \Phi_1}{\partial z^2}$.

In the case $q'_A = 0$, this equation of motion is reduced to the Mathieu equation

$$\frac{d^2 z}{d\tau^2} + [a + 2q\cos 2\tau]z = 0$$

with

$$a = \frac{\nu_0^2}{(\Omega - \Omega_p)^2} = n^2$$

and

$$q = \frac{1}{2}\left[\nu_1^2 - \frac{2\Omega\Phi_1\partial\nu_0^2/\partial R}{R_0(\Omega - \Omega_p)[\kappa^2 - 4(\Omega - \Omega_p)^2]}\right] / (\Omega - \Omega_p)^2$$

Instability strips able to trigger important perpendicular motions are characterized (for the most important) by

$$-q - \frac{1}{8}q^2 < a - 1 < q - \frac{1}{8}q^2$$

$$-\frac{1}{12}q^2 < a - 4 < \frac{5}{12}q^2$$

Between $R = 0$ and $R = R_{corotation}$, a increases from $-\nu_0^2/\Omega^2$ to ∞. ν_0^2/Ω^2 is in principle larger than 1 for a flattened galaxy. For $R > R_{corotation}$, a decreases as $\sim R^{-2}$ at large R for a flat rotation curve.

Amongst the resonance conditions, $n = 2$ is the most important for direct orbits. If the potential is not strongly barred, the first term of q in the bracket is negligible and q would be large only if $\kappa \sim 2(\Omega - \Omega_p)$ or $\Omega \sim \Omega_p$ corresponding to the classical resonance regions. In fact, only the Lindblad resonance case has to be considered, because if $\Omega = \Omega_p$, $a \gg 4$. For a strongly barred potential, ν_1/Ω is high and q can be large. Outside the outer Lindblad resonance, a may go through 1.

For a retrograde orbit, $\Omega_p < 0$, a decreases from $\sim (\nu_0/\Omega)^2$ at $R_0 \approx 0$ to zero at large R. Then if $\nu_0 > \Omega$, $\nu_0/\Omega - \Omega_p$ can be < 1 or go through 1. Then $n = 1$ is the most interesting resonance in this case and large z-motions can also be developed.

In such an investigation, the potential is given. If numerous stars develop such z-oscillations, the potential will be modified. Consequently rigorous treatment of the problem requires to take collective effects into consideration. 3-D N-body simulations such as those reported by Combes at al. (1990) confirm the importance of perpendicular resonances in the secular evolution of barred galaxies (see next subsection).

6.5 3-D simulations of bar-disk interactions: box and peanut-shapes

Bars are often strong non-axisymmetric perturbations in regions of the disks where the coupling of motions in the plane of the disk and perpendicular to it could be important. Instabilities perpendicular to the plane (often called “vertical” instabilities) must be not neglected a priori. Therefore a realistic treatment of barred galaxies needs a full 3-D dynamics. As we are going to see, many vertical instability strips occur in bars even if they have no important thickness in the z-direction and peanut-shaped bulges are formed in 3D N-body bars as a consequence of resonant coupling between relative plane motions and perpendicular oscillations. That confirms the predictions of the previous subsection.

A detailed analysis of the 3-D problem can be found in Combes et al. (1990) and Pfenniger and Friedli (1991) where the authors use the "purely stellar dynamics" part of the Geneva 3-D fully consistent numerical code with gas and stars (PMSPH). As the aim is to study the structure of the bar as well as the orbital behavior of stars in the perturbed potential, the initial conditions are chosen so that the disc be initially globally unstable. For a representative model characterized by bulge scale length 0.14 and disk scale length 3, with a mass ratio $M_B/M_D = .18$, a bar is formed in 3 - 4 dynamical time $\simeq$ 3 to $4{\cdot}10^8$ years. The angular momentum is ejected by strong two armed spiral patterns. A secular evolution of the bar is observed in z and a box or a peanut-shaped structure is reached after 2×10^9 years, through a short phase of symmetry breaking in z (Fig. 11). The bar is quasi stationary after 2.5×10^9 years. Its pattern speed slowly decreases ($\Omega_p(15\tau_{dyn}) = 0.035$ and $\Omega_p(50\tau_{dyn}) = 0.028$ in the units of the code).

The surface density profile of the disk outside the bar region is exponential along the major axis of the bar and follows a $R^{1/4}$ law along its minor axis. It results from some relaxation process produced by the formation and the evolution of the bar. The velocity ellipsoid is characterized by $\sigma_R : \sigma_\phi : \sigma_z = 1. : 0.85 : 0.75$ in the bar and $1. : 0.55 : 0.45$ in the disk. The radial behavior of the velocity dispersions is shown in fig. 12, displaying in particular a significant vertical heating.

Repeatedly, the bar is a very efficient engine for transferring the angular momentum outwards: the total angular momentum inside the corotation radius decreases by more than 50% between the initial time and 5×10^9 years.

To the first order, N-body bars are similar to observed stellar bars. The morphology and the cylindrical rotation agree with observations by Jarvis (1990) in NGC 128 for instance, which is the prototype of peanut galaxies. The actual fraction of box-peanut bulges is $\sim$ 20%. Taking account of the time scale for forming this structure and the selection effects due to the difficulty of their detection according to the inclination of the galaxies, Combes and al. (1990) suggested that any observed box- or peanut-shaped bulge could be the signature of a strong bar inside the galaxy.

To the second order, the central parts of N-body and observed bars are different: N-body bars show similar ellipticities in the isophotes whereas most of observed SB0's for instance have round isophotes. An explanation of the disagreement could be found in the possibility of gas accretion towards the central parts. In fact an essential ingredient is missing in the simulations described above: the gaseous component, and consequently the star formation, which imply energy and angular momentum dissipation, as well as self-regulating processes which control the balance between cooling and heating. This problem will be pointed out in sections 7 and 9.

Let us come to the connection between the box- or peanut-shape and resonances. It can be understood by studying orbits in the N-body bars, which have the advantage on existing analytical models to be self-consistent, to have a measurable Ω_b and to permit to follow the time evolution. The 2-D orbital structure of N-body bars has been examined by Sparke and Sellwood (1987). For the extension to 3-D structure, Pfenniger and Friedli (1991) used the Geneva code already mentioned. The position of the resonances can be estimated by averaging on azimuthal angles the generalized frequencies defined by Pfenniger (1990)

$$\Omega' = \frac{1}{R}\frac{\partial\Phi}{\partial R} \ , \quad \kappa'^2 = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + 2\Omega'^2 \ , \quad \nu_z^2 = \frac{\partial^2\Phi}{\partial z^2}$$

at any point (x, y, $z = 0$).

The main new result of the 3-D orbital structure is the existence of vertical bifurcations, in particular:

- The direct x_1 family of periodic orbits which essentially sustains the bar has a vertical instability strip (2/1 resonance) responsible for the box-peanut shape. Examples of periodic and quasi-periodic orbits supporting the peanut-shape are elongated orbits along the bar with $\Omega/\kappa/\nu_z = 1/2/2$ or eventually 1/4/4 (Fig 13).

- The usual retrograde family x_4 has a vertical instability strip (1/1 resonance). From the boundaries of the strip, anomalous inclined orbits bifurcate. They were previously mentioned (section 4) as one of the important class of orbits in systems with 3 degrees of freedom.

We note that the x_1 and x_4 families are connected by two resonant families 3/1 and 4/1.

7 Dissipation in disks

Until recently, apart from studies of the gas response to perturbations (bar-like for ex.), the dynamical importance of gas on large galactic scale has not drawn the attention very much, partially because it was considered as a minor contributor to the total mass of galaxies. However, because of its dissipative nature, its behavior is fundamentally different from the stellar one. In the disk evolution, the role of this tracer of spiral patterns as coupling element with stars and as fuel for nuclear activity is essential.

In the present section, we begin with a description of various methods used to introduce dissipation in evolution calculations. Then we will examine some of the key numerical experiments which allow to understand a) the spiral structure evolution and the various Hubble types of spirals, b) the effect of gas adjunction on the disk stability against the bar formation, c) the gas inflow towards the central region of galaxies and its consequences. Recent 3-D fully consistent simulations represent decisive progress about these questions.

One of the key physical processes in disk evolution is the angular momentum redistribution between the components (see also section 6). Probably the most effective mechanisms responsible of the loss of angular momentum by the gas are a) the collisional viscosity due to highly dissipative cloud-cloud collisions and supersonic velocity, b) the dynamical friction on massive clouds produced by the background stars, c) the gravitational torques exerted by non-axisymmetric perturbations in the stellar component (bars or spiral arms) on the gas. Shlosman and Noguchi (1994) recently have evaluated the relative importance of these mechanisms and concluded that the dynamical friction can cause a rapid inflow of gas if much of it is in massive clumps. The efficiency of the bar driven inflow depends on the bar strength and on the gas mass whereas the collisional viscosity seems to be unimportant in general. The development of bar-like structures and their interaction with the gas has been extensively studied by several groups in recent years and we will report on some of works in this section.

7.1 Modelisation of the gas behavior

Until now, essentially two different types of approaches have been proposed to follow the gas behavior in numerical simulations of disc galaxies in evolution:

1) Ballistic particles

Cold dense clouds have been considered as ballistic particles with a finite cross section (Schwarz, 1981 and 1984). The basic principle is to take a rectangular grid with specified box size overlaid on the disk. Two gas particles (clouds) collide if they are in the same box and have a component of relative velocity towards each other. The collision reverses the relative velocities of the gas particles and reduces them by some "restitution coefficient". Typically the size of the box is of the order of 0.5% of the disk radius. The "particle graininess" is a non negligible source of small amplitude initial perturbation: cold disk are strong amplifiers of small disturbances. But in a real galaxy, a significant fraction of gas is considered into a large number of molecular clouds which display a similar level of graininess. Therefore this approach has been often considered as sufficiently realistic in spite of the rather ad hoc estimation of energy dissipation in clouds. It has been used for instance by Carlberg and Freedman (1985), Combes and Elmegreen (1993) amongst others.

2) Smooth particle hydrodynamics

A collection of clouds is considered as a fluid with a sound speed of the order of the velocity dispersion of the clouds (typically 5 - 10 kms^{-1}) At present time, for simulations of long-time evolution, the smooth particle hydrodynamics (SPH) method is often used: a set of smoothed-out quasi-particles is considered so that the fluid density at any point is obtained by summation on all particles at that point. SPH is based on the Lagrangian description. For a detailed review on this technique, see for ex. Monaghan (1992) and Benz (1990). Here a stumbling-block is the perhaps excessive shear viscosity derived from an artificial viscosity term introduced in the hydrodynamical equations to simulate the dissipation in shocks. Various solutions have been proposed to try to control this effect (Friedli and Benz, 1993) by reducing the artificial viscosity in largely shear flows and leaving it unchanged in strong shocks.

7.2 Spiral activity in disks

A first example will be given here to underline the importance of gas relatively to the large frequency of observed spiral structures. A purely stellar disk displays a strong spiral activity which quickly raises the velocity dispersion of stars and the disk becomes less responsive to further disturbances. It is a general result of purely N-body simulations. Dissipation and (or) accretion can act as gas cooling factors. That could be a way to maintain some fraction of the disk mass at a low velocity dispersion resulting in a quasi-permanent sensitivity to the growth of spiral disturbances.

This process has been investigated by Carlberg and Freedman (1985) by adding the Schwarz dissipative collision scheme to the Sellwood N-body disk + halo code (Sellwood, 1981). $N = 40000$ and a 3:1 stellar to gas mass ratio were adopted. The initial values of Q_{star} and Q_{gas} were respectively 1.33 and 0.

After 5 revolutions ($\sim 10^9$ years), it clearly appears that the spiral structure is much more prominent in the gas than in the star component because of the low velocity dispersion of the gas. The morphology depends on the relative mass of the disk in the sense that the number of arms is a strong function of the disc-halo mass ratio, as expected from the swing amplification theory (Toomre, 1981). For a shearing disc, the critical wavelength λ_c for long wave instabilities is

$$\lambda_c = \frac{4\pi^2 G\Sigma(R)}{\kappa^2}$$

Let us assume the rotation velocity $V_0 = cte$, then $\kappa = \frac{\sqrt{2}V_0}{R}$, $\Sigma(R) = \frac{fV_0^2}{2\pi GR}$ where f is the mass fraction in the disk. The number of arms m in the spiral pattern is connected to the azimuthal wavelength λ_ϕ by

$$\lambda_\phi = \frac{2\pi R}{m}$$

The peak amplification corresponds to $\lambda_\phi \sim 2\lambda_c$ (Toomre, 1981).
From these relations, it comes

$$m = \frac{1}{f}$$

In a pure stellar disc, the trailing spirals transfer angular momentum outwards, according to Lynden-Bell and Kalnajs (1972). In the simulations here described, the gas systematically loses angular momentum to the stars and a radial inflow is settling with a time scale of the order of an Hubble time. We will find back later this key problem of angular momentum transfer in experiments dealing with bars.

7.3 Bar-disk evolution in early and late-type spirals

In the past, N-body simulations have been essentially devoted to studies of disk stability and growth of modes without consideration for the observed distinct morphology of Sa to Sd galaxies. However, one of the aims of galactic studies is to explain the different Hubble types and eventually the evolution from one type to another. For example differences between early and late-type barred spirals, such as emphasized by Elmegreen and Elmegreen (1985, 1989) will have to be taken into account. Some properties of bars in these different types are summarized in the table 2. These are general tendencies that nevertheless admit exceptions.

Table 2: Properties of bars in early- and late-type galaxies

	Early-type	Late-type
Bar perturbation	strong	weak
Length of the bar/R_{25}	long	short
Extension of bars	corotation ?	well inside corotation(ILR?)
Density of bars	exponential profile	exponential profile
	flatter near the center	
Spiral morphology	grand design, 2 arms	multiarms or flocculent structure
Spiral amplitude	radially decreasing	radially increasing

In order to understand the origin of the differences mentioned in the table, Combes and Elmegreen (1993) studied by numerical simulations the formation and the evolution of bars in N-body systems having characteristics of early and late-type galaxies (essentially large and small bulges respectively). They also examined the gas behavior. The stars are initially in a Toomre disk ($Q = const.$) and the gas distributed in an exponential disk.

The bulge-spheroid component is a Plummer sphere. The mass ratio $M_B/M_D = 2$ for early-type system and 1/10 to 1/5 in late-type ones. The main evolution phases observed in the case of purely stellar system are the following:

For late-type systems, at the beginning, development of a small-amplitude m =2 wave with $\Omega_p \gg \Omega - \frac{\kappa}{2}$ which transfers angular momentum from the stars in the inner disk to the stars in the outer disk; then growth of a bar from the center which traps particles (stars) at larger and larger radii. After $\sim 10^9$ years, the bar ceases from growing because corotation overtakes regions with not enough particles to receive angular momentum. The length of the bar is constrained by the scale length of the disk. The value of Ω_p (~ 10 $kms^{-1}kpc^{-1}$) is of the order of $\Omega - \frac{\kappa}{2}$. In this case, the bar seems to end inside corotation. This limitation comes from the fact that the bar is unable to transfer angular momentum in low density region.

For early-type systems we expect that the disk is more stable. The evolution begins similarly to the late-type case, but Ω_p is initially much larger ($\Omega - \frac{\kappa}{2}$ is larger for galaxies with a large bulge). The corotation radius is sufficiently short so that angular momentum from the inner disk finds a reservoir for transfer up to corotation during the slow growth in length and strength of the bar, for over a Hubble time. An other difference obtained between late and early type systems is the density distribution in the bars: the density profiles are exponential but, for early types, they are flatter near the center.

The gas is expected to be important in the evolution of spiral waves and bars: it dissipates the wave energy and amplifies the spirals through gaseous self-gravity. Particularly in late-type galaxies, the mass as well as the angular momentum fraction of gas is relatively high. This gas is a reservoir for the angular momentum transferred out by the wave. Combes and Elmegreen (1993) suggest that this angular momentum absorption is responsible for a prolonged bar evolution by comparison with the case of a purely stellar disk. The growth phase for the bar is also larger.

The main result concerning the late-type simulation is the appearance of a very thin bar in the gas with the same length as the fat stellar bar. More structure is present in the gas than in the stars, with transient waves. The gas is strongly driven inwards by the gravitational torque from the bar. The end of the bar coincides with one disk exponential scale length (Fig. 14). In the case of early-type simulation, the stellar bar extends nearly up to corotation and a thin bar develops in the gas as well as a nuclear ring at the ILR.

An important difference between simulations with and without gas appears in the time scale for the setting-up of the bar: $2 \cdot 10^8$ years for a late-type galaxy and $5 \cdot 10^8$ years for an early type one. That is much faster than for purely stellar cases which can be stable with $Q = 1.5$ for a Hubble time. This result is in agreement with predictions by Jog and Solomon (1984) mentioned in sect. 5, according to which a gas-star system can be unstable even if one of both components is stable by itself. The fact that the dissipation maintains $Q = 1$ for the gas explains the evolution described above.

7.4 Bar driven gas fueling of galactic nuclei

As indicated in 7.1, another effective method to describe the behavior of gas in galaxies is based, via the classical hydrodynamic equations, on smooth particle hydrodynamics (SPH) codes. Such a code has been used by Friedli and Benz (1993) (see also Friedli et al. (1991)). These authors investigated the ability of bars to transport gas towards the central parts of galaxies. In particular, they looked for what parameters at work are the most important in this context. A fully consistent 3-D simulation of stars/gas systems consists in 3 main steps:

1. Given the mass distribution for gas and stars, to compute the gravitational potential Φ by using the part of the code consisting in a particle-mesh (PM) algorithm in Fourier space on a polar grid. This fast method allows to include a large number of particles (1 to $5 \cdot 10^5$), a fraction of them being gaseous ($\sim 10\%$).

2. Using the SPH approach, to compute the hydrodynamical forces acting on the gaseous component only. At the present time, the modelisation of this component does neither take account of the variety of the interstellar matter composition (diffuse, atomic, warm and cloudy, molecular, cold components) nor of its fractal structure. It would be illusory to introduce complexity at this level as far as the permitted numerical resolution prevents to describe the ISM in fine details.

3. To move all the particles by using for instance a second order Runge-Kutta-Fehlberg integrator.

The main results from this fully 3-D self-consistent evolution confirm the fact previously suggested by Shlosman et al. (1989) that a bar acting on the gaseous component of an isolated galaxy allows a transport of mass up to several $10^9 M_\odot$ towards the center with a mean rate of some $M_\odot yr^{-1}$ but reaching roughly 200 $M_\odot yr^{-1}$ in extreme situations. On average more than 1% of the total mass of the galaxy in the form of gas could be brought near the nucleus. The mass accretion is increased 1) if the bulge/disk mass ratio is lowered and 2) if the axis ratio of the stellar bar a/b is larger. The accretion is smaller by a factor 4 for gas in retrograde motion compared with direct motion. Finally the gas self-gravitation reinforces the accretion only when the gas density becomes high enough.

Different experiments by Friedli and Benz (1993) lead to the conclusion that the gaseous bars are always induced by barred stellar potentials. No gaseous bar is formed spontaneously in models with fixed axisymmetric stellar potential. Moreover the gaseous bars are always shorter than their stellar counterparts and their axis ratio b/a is smaller by a factor 2 - 3. Fig. 15 indicates the time evolution of the central mass concentration inside the radius $R_L = 0.2$, 0.5 and 1 kpc for the gas and the stars, for one of their models.

The mass accreted can represent a small but non-negligible part of the galactic mass (1 - 2%). Consequently a strong ILR can settle, which causes the main elongated plane orbits which support the bar to be replaced by orbits perpendicular to the bar which is no longer sustained. In a few Gyr, the initial bar can be transformed into a triaxial bulge such as observed in many spirals. These bulges could be the relics of old bars (see also section 8).

The dynamical effects of star formation are not included in this scenario. It is a new scaling in complexity. At present time, star formation cannot be rigorously modeled as an additional equation in the present context. We will come back in section 9 on recipes which allow to mimic this part of the global evolution scenario.

7.5 Bars within bars and their effects

The reality of the peculiar phenomenon of bars within bars has emerged from the following three complementary approaches:

1) Secondary nuclear bars or triaxial bulges have been observed in visible light by different authors in several barred spiral or lenticular galaxies, for example NGC 1291 (de Vaucouleurs 1974), or NGC 3945 (Kormendy 1979, 1981, 1982); in addition, recent CO observations have also revealed central molecular bars or rings not aligned with the stellar bars (Kenney 1991; Devereux et al. 1992, see also Lo et al. 1984). Galaxies with a nuclear ring eventually crossed by a secondary bar were studied by Buta (1990). The prototype could be NGC 3081.

2) Theoreticians have proposed bars within bars as a possible mechanism of gas fueling in active galactic nuclei (Shlosman et al. 1989, 1990). The possible existence of self-consistent models of periodically time-dependent stellar systems has been confirmed by Louis and Gerhard (1988) and Sridhar (1989).

3) Self-consistent 3D N-body simulations with gas and stars (Friedli, 1992) can lead to the formation of systems with two bars rotating with two different pattern speeds provided that the initial gas mass amounts to about 10% of the total mass (Friedli and Martinet, 1993). This emphasizes the key role of dissipation. The secondary bar results from a decoupling between the nearly self-gravitating central part and the outer part of the primary bar. The decoupling seems to be sensitive to mass accumulation onto the x_2 orbit families of the primary bar resulting from the presence of a moderate ILR. Either the primary bar first appears followed by the secondary one, or the two bars almost simultaneously develop. In the latter case, a nuclear gaseous ring is in general formed near the secondary bar end.

The best model has shown that these double-barred systems are stable over more than 5 turns of the secondary bar. It has the following characteristics: $\alpha = \Omega_s/\Omega_p \approx 79.0/25.5$ $kms^{-1}kpc^{-1} \approx 3.1$, $\beta = l_p/l_s \approx 9.5/2.5$ $kpc \approx 3.8$, and the axis-ratios are $a_s \approx 0.6$ and $a_p \approx 0.7$ (Fig. 16).

Bar-driven gas fueling can be significant and can compress gas inside the bar to about 10 - 20% of the bar length. The system of nested bars thus transports amounts of gas much closer to the galactic center and could be invoked as a possible mechanism to fuel AGN. The gas accretion rate can be very high and could be related to some starburst mechanism. However, the efficiency of star formation must be explored before definite conclusions can be drawn.

The two bar phase is followed by the dissolution of the secondary bar (or even the two bars depending on the model) which is induced by a broad and strong ILR. The final central galaxy shape resulting from the two bar destruction is similar to triaxial observed bulges, suggesting that some of them are relics of destroyed bars.

The above scenario is different from that proposed by Shaw and al. (1993). Both scenarios need the presence of a massive and strongly dissipative component but in the one described above it is supposed to initiate the central dynamical decoupling while in Shaw et al., it is only supposed to gravitationally deform the stellar component. As a major consequence, in our scenario, the two stellar bars have different pattern speeds whereas in theirs they have the same. Since theoretical, numerical as well as observational studies seem to indicate that single and multiple pattern speeds can occur in galaxy dynamics, it is difficult to dismiss either of these scenarios, and consequently to determine if only one (and which one?) or both are in action in real double-barred galaxies. However, the existence of secondary trailing stellar bars seems difficult to explain with Shaw and al.'s scenario. A mixing up can also eventually follow from the existence of both central twisted isophotes (resulting either from projection effects onto triaxial bodies or from torques initiated by strong spiral arms) and two distinct bars. Shaw et al.'s scenario may eventually better correspond to the case of twists than the one of two distinct and persisting bars. Clearly, more studies are necessary to assess the respective merit of these two scenarios as well as to determine the essential parameters which initiate the models to be channeled or not into the way of the multiple pattern speeds.

8 Environment effects. Accretion of satellite galaxies

The present review essentially turns on the internal dynamical evolution of galaxies. The previous section displayed important effects able to produce morphological modifications of galaxy structure only due to interactions between components of the system considered, including the gas. In the discussion of the processes at work, it was implicitly admitted that the systems were isolated. However, at present time, nobody contests that most galaxies must suffer more or less strong interactions with their neighbours during their lifetimes. The proceedings edited by Wielen (1990) reviewed various problems concerning these phenomena. Visible effects of interactions such as bridges, tails, warps, shells, ripples, dust rings, multiple nuclei, more or less advanced stages of merging are the subject of continuous attention. The interest for these questions is strengthened by recent qualitative evidences of a connection between interactions and bursts of star formation (see section 9).

In our own context, we will be here interested in the morphology transformations of disks and bulges resulting from accretion of small satellite galaxies. Beforehand, we will briefly mention the case of strong destroying collisions between two similar disk galaxies, the result of which could account for starbursts which will be discussed in section 9.

8.1 An example of strong interaction between galaxies

Strong interactions can be defined as those in which galaxy number is not conserved. They have been recently discussed especially by Barnes and Hernquist (1991, 1992) and Barnes (1992). In particular these authors presented a simulation of merger of two self-consistent bulge/disk/halo galaxies of equal mass including 10% of gas. The mass ratio of the components is 1:3:16 in both galaxies. Two fundamental questions about such simulated parabolic collision are: 1) What is the morphology of the remnant? 2) What is the fate of gas?

Using a SPH algorithm to describe the gas behavior, Barnes and Hernquist (1991) concluded the following:

- Mergers such as considered in the frame of the chosen peculiar initial conditions produce a triaxial slowly rotating remnant with a $r^{1/4}$ luminosity profile.
- The interaction transfers a large fraction of the energy of the relative motion to the internal degrees of freedom. The "passage" leads to a more tightly bound orbit, then to a complete merger in ~ 750 millions years.
- Gas and disk stars have a similar behavior on large scale but not on small scale: the strong tidal field triggers a strong stellar bar. The gas response is similar but dissipative shocks appear. The gaseous bar is narrower and shorter.
- Half the gas initially in the disk is transferred toward the center of the merger by purely gravitational torque ($5 \cdot 10^9 M_\odot$ within ~ 200 pcs).

- Strong shocks and radiative cooling are a necessary condition for such a process.

We refer the reader to the review mentioned above for the details. An analogous mechanism has been described in section 7 in connection with the bar-driven gas fueling of nucleus in an isolated galaxy. In both cases the final evolution is uncertain due to the fact that star formation and supernova heating have been ignored in the experiments.

Let us add that some peculiar initial conditions in other simulations (Pfenniger, 1994) are able to produce counter-rotating disks similar to that observed in NGC 4550 for ex. (Rix et al., 1992)

8.2 Coming back to dynamical friction

Dynamical friction mechanism has been described in section 6. Orbits of satellites around massives galaxies decay due to such an effect so that galaxies could have accreted some mass in the form of dwarf galaxies or globular clusters over a Hubble time. Even if the implied mass quantities are not so important as in collisions between giant galaxies, the resulting deformation of the target can be not negligible as we will see in the next subsection.

Three fundamental physical aspects of the problem must be mentioned to recall that the well known simplifying Chandrasekhar formula cannot claim to give a quantitatively correct estimate of the deceleration due to dynamical friction in the present context:

1) The self-gravity of the target.

2) The global nature of the target tidal deformations.

3) The non-rigidity of the satellite.

Combes (1992) draws attention to the dissociation of two effects connected with the self-gravity: the barycenter displacement of the "target" galaxy and its true deformation. Conflicting results as regards to the role of the self-gravity in the satellite braking come from the different way of calculating or interpreting it. If the true deformation, which is the only deceleration factor, is computed in the center of mass frame, it appears that the self-gravity is negligible for a satellite-target mass ratio ~ 0.1 (Prugniel and Combes, 1992). If the target is fixed in an inertial frame (Weinberg, 1989), taking the center of mass shift as a self-gravity factor, it is found that self-gravity slowers the decay! The Chandrasekhar formula is also based on a fixed target and also overestimates the friction: its local nature is unrealistic in the present context since the tidal deformation of the target which induces the dynamical friction may be considered as global.

Moreover, the tidal deformation of the satellite which is able to take away some orbital energy and angular momentum of the relative motion must not be neglected: the decay time scale is appreciably reduced with respect to the case of a rigid satellite as seen in Prugniel and Combes (1992).

These considerations have been inferred from experiments on an elliptical galaxy and its companion. Combes (1992) also mentioned a possible complication with spiral galaxies coming from resonance effects which could remove the satellite instead of braking it. In fact simulations show that the friction remains dominant in this case but self-gravity typically reduces the decay time by a factor of two.

8.3 Heating of the disks by satellite mergers. An explanation for thick disk?

This mechanism has already been mentioned in section 5 amongst those able to heat the disk in our Galaxy. Certainly it is quite general and we have discussed it in more details by basing ourselves on the deepest treatment of the problem nowadays (Quinn et al. 1993), in spite of some limitation given below.

The used N-body models consists of self-consistent bulge-less exponential disks having constant vertical scale length h_z.

These disks are imbedded in rigid modified isothermal halos, the density of which is

$$\rho(r) = \frac{\rho_0}{1 + (r/\gamma)^2}$$

The density distribution in satellite galaxies is given by the spherical Jaffé models

$$\rho(r) = \frac{M}{4\pi r_0^3} \left(\frac{r}{r_0}\right)^{-2} \left(1 + \frac{r}{r_0}\right)^{-2}$$

with the mass ratio $M_{sat}/M_{disk} = 4$ to 20%.

The satellites are initially placed on circular prograde orbits at a radius $6h_R$ (h_R = exponential disk length scale), with various inclinations relative to the disk plane. Friction due to the halo is ignored.

The main limitations of such experiments, which could lessen the bearing of the results are the following: 1) the rigidity of the halo does not allow to give quantitatively reliable sinking rates, 2) only prograde encounters are considered, 3) the gas effects are not included, 4) only 32768 particles in the disk and 4096 in the satellite are used.

The different mergers obtained are characterized by various common effects:

- The dynamical coupling between disks and satellites leads to satellite orbital decay, at first by sinking into the plane of the disk, then radially into the center.
- A strong two-armed spiral pattern is generally induced in the disk which transports angular momentum outwards.
- The disk radially spreads and perpendicularly inflates.
- As the satellite sinks into the central region of the galaxy, the disk is able to find back a new axisymmetric equilibrium.
- The disk is thickened, warped and flared by the merger. The quantitative effect depends on the initial conditions, in particular the mass ratio and the initial inclination of the satellite orbit. Typically for a mass ratio of 1/10 and an inclination of 30°, the vertical scale height h_z inside $h_R = 3.5$ *kpc* has increased by 50%.
- Moreover, the mergers perturb the velocity distribution of stars in the disks due to three effects: a) the deposit of kinetic energy and angular momentum from satellites, b) the action of spiral pattern and c) the accretion of satellite debris.

In the typical case mentioned above, the ratio of azimuthal to radial velocity dispersion σ_ϕ/σ_R varies from 1 in the center to 0.7 between 10 and 20 *kpc*. σ_z/σ_R varies from 0.75 in the center to values larger than 1 for $R > 15$ *kpc*. The initial range of Q is $1 < Q < 2$ for $0 < R < 15$ *kpc* which allows a response to the mode $m = 2$ (spiral arms). At the end of the simulation ($T = 5.4 \cdot 10^9$ years), $Q(R < 2h) \sim 1.5$ and, $Q(2h < R < 4h) \sim$ 2-4 which is able to stabilize the disk against the mode $m = 2$.

The recurrence of mergers shows less spectacular effects: the disk is hotter and more diffuse than the original thin disk! The evolution of the satellite when it is represented by a realistic self-consistent N-body model is characterized by a stripping due to the tidal field and an impulsive heating.

There is competition between internal heating of the satellite and external heating of the disk. It appears that if the satellite is denser than the disk, the disk is heated impulsively whereas if the satellite is less dense, it is heated by the disk. Of course the stripping of a self-consistent satellite and the disk tidal field control the efficiency of the disk heating by the satellite. Typically, in the model considered as standard by Quinn and al. (mass ratio $= 1/10$, $30°$ inclination) a significant z-heating is obtained with 60% of the satellite kinetic energy transferred into z motions of disk stars.

On the other hand, in the same experiment, the kinetic energy of the disk increases by 30 - 40% of the satellite orbital energy through the spiral response of the disk to the satellite perturbation.

The flare in the disk is formed by satellite debris and stars from the disk. It seems to be due to the fact that the velocity impulse is especially in the vertical direction at large radii, resulting there in a more efficient absorbtion of vertical kinetic energy than of azimuthal kinetic energy by the disk.

In brief, a thin disk having suffered satellite impacts must present the observed features already mentioned: radial spreading, thickening, vertical flaring and warping. Many galaxies display these features. See, for instances Shaw and Gilmore (1989) for thickening, Bosma and Freeman (1991) for flaring, Sanchez-Saavedra and al. (1990) for warping.

Although the final structures obtained by the mechanism here described are similar to those observed in numerous galaxies, we must remember that other mechanisms of disk heating could be efficient, in particular the bars. The relative quantitative contribution of all mechanisms put forward here and in section 6 to the evolution towards the observed structures of disk is difficult to estimate: the simulations which reveal the dynamical processes at work suffer from limitations mentioned repeatedly and sufficiently detailed observations of external galaxies concerning particularly kinematics and multi-band photometry are still lacking.

It is interesting to note that the accretion of satellites by spiral galaxies is a process similar to that suggested by Searle and Zinn (1978) for the halo formation from subsystems such as dwarf spheroidal galaxies.

8.4 Bars induced by companion passages and dissolved by satellite mergers

a) Influence of interaction on stellar bar evolution

In section 5, the conditions of disk instabilities to bar formation were examined in the case of isolated galaxies. Since the seminal paper by Toomre and Toomre (1972), there are numerous evidences that tidal interactions can have an effect on the developments of substructures inside the galaxies. Therefore it is important to bring out the dominating features in the interplay of bar formation and interactions. To date, the most complete study was undertaken by Gerin and al. (1990) who simulated close 2-D and 3-D encounters between a disk galaxy formed of a Toomre disk and a rigid Plummer halo and a perturber (Plummer sphere) on direct or retrograde orbit (in the sense of the galaxy rotation or in the opposite sense). Various perturbing masses, pericenter distance and mass distribution of the galaxy were tried as determinant parameters. Two different situations were considered: a) the "target" galaxy is initially axisymmetric and b) it is initially already strongly barred.

Amongst the results, we point out the following facts:

1. A direct tidal interaction destabilizes a disk which otherwise does not develop a bar for more than 1 Gyr. An interaction reduces the time scale of bar formation. Retrograde encounters are not so effective. The process obviously depends on the mass ratio M_{halo}/M_{disk}.

2. In the perturbed case (compared to the isolated case), the interaction triggers a spiral wave in the outer parts of the disk, which propagates towards the central regions, strengthening the bar formation.

3. In the case of an already initially barred galaxy, the phase of the perturber acts on the strength and the pattern speed of the bar, practically without change of the axisymmetric part of the galaxy potential. Consequently an interaction may shift the Lindblad Resonances in a stellar disk.

4. Bending and fattening of the disk are essentially produced by interactions with a companion perpendicular to the plane of the galaxy. The increase of the bar strength is less pronounced than in the other geometries due to the shorter time during which the axisymmetric perturbing force acts.

b) Destruction of bars by satellite mergers

The influence of the mass concentration growth on the dynamics of disks in isolated galaxies has been discussed in section 4 (effect of a blackhole on orbits plunging towards the center) and in section 7 (gas inflow towards central regions induced by a bar). In both cases mentioned, the process implies the creation of an ILR and the enlargement of the influence zone of the resonance with the growth of the compact mass. The response of stellar orbits in the plane is perpendicular to the bar. Consequently the bar shape is no longer supported. Starting from the N-body simulation of a robust bar, Friedli and

Pfenniger (1991) and Friedli (1994) confirmed the effect mentioned. Artificially increasing slowly the central mass of the disk, they observed that the addition of 1-2% of the total mass in the region inside the ILR is enough to destroy the strong bar.

But a bar can also be dissolved after being induced as a consequence of an environment effect in the form of satellite merger. The experiment described below is complementary to the Quinn et al. (1993) study presented in the previous subsection, which did not imply a bar evolution.

Pfenniger (1991a) showed the dynamical evolution of a disk perturbed by a compact satellite, initially on an inclined nearly circular direct orbit, with a mass equal to 10% of the system total mass. Induced by the tidal action of the perturber, a bar is formed in 100 Myr, and produces spiral arms and disk heating. Simultaneously the satellite spirals by resonance and dynamical friction coupling effect. It reaches the bar Lagrangian points respectively at $T = 1200$ Myr, then at $T \simeq 1300$ Myr. Afterwards it plunges toward the center in 50 Myr and the bar is destroyed according to the process above mentioned (see fig 1. in Pfenniger (1991a)). The final structure is a small spheroidal bulge. During the spiraling of the satellite towards the center, there is transfer of its orbital angular momentum to the disk which gets warped.

Inasmuch as the bar formation is easy and their observed frequency is high, it is not impossible that the barred state be a compelled step in the galaxy life. There is apparently no problem to explain that 1/3 of disks are unbarred since efficient mechanisms seem to exist to destroy the bars!

8.5 Bulge growth induced by satellite

The bulge-to-disk ratio is a qualitative as well as a quantitative classification criterion of galaxies along the Hubble sequence (cf. Simien and de Vaucouleurs, 1985). In spite of a large dispersion in the relation between this ratio and the Hubble type, early-type galaxies generally have the largest bulges. Bulge-less galaxies are also observed. Is this diversity a signature of initial conditions or the clue of secular evolution? The simulations presented elsewhere in this review (6.5, 8.4) supply evidences in favor of the latter possibility. Small bulges could result from disk material accretion through the bar dissolution process. There we present an additional argument for secular evolution, which concerns the galaxies with big bulges, the well known prototype of which is the Sombrero galaxy.

A possible explanation for the existence of these bulges has been recently proposed by Pfenniger (1993). Considering a model which consists in an initial disk with a scale length $h_R = 4\ kpc$ and a scale height $h_z = 1 kpc$, as already described in 6.5. The disk was initially slightly unstable to bar formation and a peanut-shaped bulge-bar structure was formed. A 3-D simulation has been started in which ten point mass satellites initially at the outskirts of the galaxy are slowly accreted by dynamical friction. When the ratio of satellite mass to galaxy mass M_s/M_G is of the order of 4%, the satellites modify the structure of the bar which becomes more oval. If this ratio is increased to 10%, the bar is destroyed and the disk is so much inflated by heating that the remaining structure is dominated by a big bulge where all mass comes from the disk. If $M_s/M_G > 10\%$, the disk is destroyed, the remnant is a purely spheroidal system.

We must note that these simulations as well as those of the previous subsection suppose that the satellites are rigid (point masses). It has been mentioned in 8.3 that there could exist a competition between internal heating of the satellites and external heating of the disk, when the satellites are presented by realistic self-consisting models. Therefore the mechanisms described above and in 8.4 can be really effective only if the satellites are denser than the disk.

9 Coupling of dynamics and star formation

As seen in section 7, effective numerical codes now exist which allow to follow the dynamical evolution of galaxies consisting of stars and gas. However an essential ingredient is still lacking: star formation. Theoretical as well observational primordial questions about it are: Where does it occur? What is its rate? What are the main parameters which govern the star formation efficiency?

At present time, only partial answers can be given, inasmuch as the complexity and the diversity of processes at work is high. Silk (1992) draws up an impressive list of mechanisms at work in star formation which ought to be ideally taken into account. I quote: " Ionization and heating by X-rays, cosmic rays and UV-radiation, growth and destruction of dust grains, absorption and photo-desorption of molecules from grain surfaces, gas phase and surface molecular chemistry, cooling transitions by molecular and atomic species, molecular photodissociation, recombination of electrons and ions, radiative transfer of molecular lines important for cooling, heat input via external shocks, turbulence dissipation, Alfven wave dissipation, internally and externally generated radiation fields and feedback from proto-stars via bipolar flows, winds and flares, dynamics of cloud contraction, collapse and fragmentation, role of magnetic fields in cloud support and ambipolar diffusion, without speaking of initial mass function origin".

In view of the extremely complicated nature of local star formation problems, global approaches involving a not too large number of parameters must be adopted. A minimum task should be: 1) to find a collapse criterion in order to determine where the stars are formed, 2) to introduce a parameter connected with the star formation efficiency and 3) to introduce other parameters which allow to take account of thermal and mechanical energies injected in the medium through supernovae explosions and stellar winds. The present discussion will focus on some recipes recently introduced into simulations of galactic evolution for lack of a detailed theory of star formation. Simple models will allow to tackle the problem of the gas fate in central galactic regions and the connection between galaxy interactions and observed big starbursts, for instance. Proceeding step by step, it will be possible in a near future to introduce in the models more detailed physics specifying the conditions transforming gas into stars inside molecular clouds.

9.1 Observational tracers of star formation

The observational tracers generally used to infer star formation rates (SFR) are H_α, far-UV and far-IR fluxes. The calculation of SFR results from the luminosity L_i in the band i by

$$L_i = SFR \int t_i(m) L(m) \phi(m) dm / \int m \phi(m) dm$$

where $L(m)$ is the luminosity of a star of mass m, $\phi(m)$ is the initial mass function (IMF) in the range of mass 0.1 - 100 $M_\odot$ and $t_i(m)$ is the characteristic time scale over which a star of mass m emits radiation in the band i.

The main source of systematic errors in the interpretation of the H_α emission is extinction (Kennicutt, 1989). Typical extinctions of HII regions in galaxies are of the order of 0.5 to 2 magnitudes. In so far as corrections for this effect can be applied, the fluxes can be converted to current star formation rates. Uncertainties on the Initial Mass Function are another origin of errors. According to Kennicutt (1983)

$$SFR_{H_\alpha}(\gtrsim 10 M_\odot) = 1.42 \times 10^{-42} L(H_\alpha) M_\odot\, yr^{-1}$$

and

$$SFR_{H_\alpha}(\text{total}) = 8.9 \times 10^{-42} L(H_\alpha) M_\odot\, yr^{-1}$$

where $L(H_\alpha)$ is the H_α luminosity obtained from the H_α flux $F(H_\alpha)$ $[erg\ cm^{-2} s^{-1}]$ by

$$L(H_\alpha) = 3 \times 10^{16} F(H_\alpha) D^2 [L_\odot]$$

D being the galaxy distance in unit of Mpc. The total SFR is a strong function of Hubble type as mentioned in section 2. Integrated over all stellar masses it was estimated by Kennicutt (1983) and Caldwell et al. (1991) from H_α data: From 0.1 - 1 $M_\odot\ yr^{-1}$ in SO/a galaxies to $\sim$ 10 $M_\odot\ yr^{-1}$ in Sc-Irr galaxies. Overall it ranges from less than 0.001 $M_\odot\ yr^{-1}$ in extreme dwarf galaxies to more than 100 $M_\odot\ yr^{-1}$ in luminous starbursts.

The same kind of difficulty connected with the extinction is present in tentatives to estimate SFR from UV datas (Donas et al. (1987), Bersier et al. (1994)). The current star formation rate inferred from UV data (2000 Å), obtained with the SCAP balloon experiment, is

$$SFR = 2.3 \times 10^{-40} L(\lambda 2000) M_\odot\, yr^{-1}$$

which represents a recent star formation rate on a period of time ($\sim$ 100 Myr), about 50 times longer than SFR based on H_α.

From the appendix of Catalogued Galaxies and Quasars in the IRAS Survey (1985), the IRAS 40 - 120 μm luminosities are, according to Devereux and Young (1990)

$$L(40 - 120 \mu m) = 3.65 \times 10^5 (2.58 S_{60} + S_{100}) D^2 [L_\odot]$$

S_{60} and S_{100} are the IRAS 60 and 100 μm flux density in unit of J_y. The resulting star formation rates given by these authors are

$$SFR_{IR}(\geq 10 M_\odot) = 9.2 \times 10^{-11} L_{IR} M_\odot\, yr^{-1}$$

$$SFR_{IR}(\geq 0.1 M_\odot) = 6.3 \times 10^{-10} L_{IR} M_\odot\, yr^{-1}$$

However Sauvage and Thuan (1994) discuss the ambiguity of the information on star formation inferred from far-IR measurements in particular through the 4 IRAS pass-bands

(12, 25, 60 and 100μm). The problem rests on the fact that the far-IR emission can have a multiple origin: 1) Dust associated to molecular clouds and star-forming regions heated by the radiation of newly formed OB stars, 2) dust in the interstellar medium heated by an interstellar radiation field (cirrus), 3) photospheres or circumstellar envelopes of evolved stars, 4) dust heated by a compact source inside an active galactic nucleus. The present and past SFR, the chemical composition and the size distribution with respect to the heating sources, the optical depth of the dust play a significant role in the emission mechanisms mentioned above.

In the extensively used FIR color-color diagram ($\log(S_{60}/S_{100})$ versus $\log(S_{12}/S_{25})$ where S_i represents the flux in the 4 IRAS bands), galaxies populate a relatively narrow sequence from the high S_{12}/S_{25} and low S_{60}/S_{100} end to the low S_{12}/S_{25} and high S_{60}/S_{100} end. Typical representatives of quiescent spiral disks and starbursts would be respectively in the lower right end and in the upper left end of the sequence. According to Sauvage and Thuan (1994), the FIR colors of galaxies along the Hubble sequence are a combined effect of the star formation efficiency (SFE) and variations in dust distribution and composition. Only for Hubble types from Sbc to Sdm-Im, the FIR colors seem to be essentially controlled by an increasing SFE from quiescent to starburst objects. If the major part of the 60 - 100 μ emission is associated with regions of current star formation for these types, the IR-to-blue flux ratio can be used as an indicator of relative present (IR) to past (measured by B passband) star formation rate. According to Keel (1993),

$$L_{FIR}/L_B = 4.16 \times 10^{-7} \cdot 10^{0.4B}(2.58S_{60} + S_{100})$$

for blue B magnitude.

9.2 Star formation rate and gas density

The Schmidt law (Schmidt, 1959) $\dot{M}_s \div \rho_g^n$, where M_s is the stellar mass and ρ_g the gas density, has abundantly been used in the past to modelize the average star formation rate in galaxies. The observational fits to this law have displayed a large dispersion (0< n <4), also present in the applications of the law to different regions of a same galaxy. Clearly, a part of the physics associated to the star formation process is neglected. Taking account of the numerous processes at work, one could have formalized SFR as $SFR(\rho_g, c_s, w_s, V_T/c_s, \Omega, A, |\boldsymbol{B}|, Z, \rho_s)$ (Lynden-Bell, 1977) where ρ_g = gas density, c_s = gas sound speed, w_s =shock frequency, V_T/c_s = shock strength, Ω = gas rotation, A = shearing rate, $|\boldsymbol{B}|$ magnetic field strength, Z = gas metal abundance, ρ_s = star density. Unfortunately, it is difficult to make this dependence quantitatively explicit!

The tracers introduced in the previous subsection can be used with recent data on HI and H_2 galaxy contents to refine the relation between SFR and the gas density. However, we must pay attention to the fact that larger and more luminous galaxies may have more of everything: more blue stars, more gas, more dust. For instance, if H_α is used as a star formation tracer, SFR being proportional to $L(H_\alpha)$, a linear relation between total SFR and total gas mass, such as found by several authors could be due to purely scaling effect and not to a true physical dependence. The conclusion by Devereux and Young (1991) that the $L_{FIR}/M(H_2)$ ratio is constant along the Hubble sequence could mean that L_{FIR} is simply proportional to molecular hydrogen mass without necessarily implying the constancy of the star formation efficiency (SFE) from Sa to Sc. Sauvage and Thuan (1992) showed that in fact L_{FIR}/L_{H_α} decreases towards later types. Since L_{H_α} is proportional to SFR (M $\gtrsim$ 10 $M_\odot$), L_{FIR}/L_{H_α} is proportional to the ratio of gas mass to SFR $\div$(SFE)$^{-1}$: The star formation efficiency increases by a factor of ~ 7 from Sa to Sd.

From H_α and UV data, it clearly appears that the SFR indicators are not well correlated with surface density of molecular gas. Average SFR is most strongly correlated either with HI or with HI + H_2, (Kennicutt, 1989). The important scatter must be attributed to the extinction problem as seen above but also to the uncertainty on H_2 masses and densities extrapolated from CO measurements. Large variations in the conversion factor between CO intensity and H_2 density from galaxy to galaxy have not to be excluded.

Other informations on the star formation law can be inferred from comparison of SFR and gas density within individual galaxies. The relationship between H_α surface brightness and the total gas density within field and Virgo spirals which is shown in fig. 17 is due to Kennicutt (1993). The figure suggests three distinct regimes for the star formation law: At high gas densities, the SFR follows a Schmidt law with $n \sim 1.3$. Below a critical density (between 1 and 10 $M_\odot$ pc^{-2}) the relation is much steeper. Well below this critical threshold, SFR is very low if not zero. A more detailed study shows that, for early type galaxies, the upper right quasi-linear part of the law is absent contrary to late types.

9.3 Dynamical indicators of star formation

Cloud complexes and new stars are thought to be produced by the conjugate action of gravity, stirring from massive stars, energy dissipation and magnetic forces. Spontaneous cloud formation results from combination of three instability types: gravitational, Parker and thermal instabilities as described in details by Elmegreen (1992). In the calculations reported by this author, the rate at which mass is processed through the instability is proportional to $\rho^{1.3} - \rho^{1.5}$. The characteristic wavelength of the instability is ~ 2.5 kpc and the mass of the condensation $\sim 10^7$ $M_\odot$. The formation of molecular clouds ($\sim 10^6$ $M_\odot$) can occur through condensation and dissipation inside the core of these complexes or fragmentation of giant shells. Nevertheless, the details of the scenario as well as the connection between the instabilities responsible of cloud formation and the efficiency of star formation are still too speculative to be taken into account.

Twenty years ago, Quirk (1972) had suggested that the star formation threshold was associated with gravitational instabilities in the gas. Guiderdoni (1987) has shown that the observed threshold corresponds to the critical density for stability given by the Toomre criterion

$$\Sigma_c = \alpha \frac{\kappa c}{3.36G}$$

where c is the velocity dispersion of the gas (see also Kennicutt, 1989). So star formation onset could be characterized by $Q < 1$. Recently, several models of large-scale star formation resulting from gravitational instability in disks have been proposed with the least number of free parameters (Silk (1992), Wyse and Silk (1989), Wang and Silk (1994)).

The starting point is the equation for the star formation rate

$$SFR = \epsilon \frac{\Sigma_g}{t_s}$$

where ϵ =SFE, Σ_g is the gas surface density and t_s is the time-scale of star formation. Wang and Silk (1994) assume that $1/t_s$ is related to the growth rate of the gravitational instability of the gas disk. Identifying $1/t_s$ with the maximum growth rate of the instability inferred from a local linear analysis, they show that

$$1/t_s \simeq \frac{\kappa(1 - Q^2)^{1/2}}{Q}$$

Then

$$SFR = \frac{\epsilon\kappa\Sigma_g(1 - Q^2)^{1/2}}{Q} \qquad (Q < 1)$$

This result indicates that 1) star formation occurs when $Q < 1$, 2) t_s is related to κ^{-1} or Ω^{-1}, 3) the smaller Q, the more rapidly stars formed, 4) SFR depends not only on Σ_g but also on the galaxy rotation rate. If Q and c are approximatively constant, $\kappa \div \Sigma_g$ so that SFR $\div \Sigma_g^2$. Point 1) is equivalent to Quirk's proposition mentioned above. Let us note with Wyse and Silk (1989) that a SFR proportional to $\Sigma_g \Omega$ is able to reproduce color and metallicity gradients in galaxy disks.

If the energy dissipation is rapid and the gas gravitational instability triggers star formation, a regulation process could be started: when Q is below the threshold, SFR increases, the formed stars stir up the interstellar medium, Q increases, star formation is stopped, Q decreases etc...

Friedli and Benz (1993) tested different prescriptions relative to SFR in view of using the most suitable in global evolution simulation of disks. It is a question of comparing the predicted SFR sites in their star + gas evolution models (described in section 7) with the usually observed ones. Four criteria for star formation were considered a priori: 1) $\Sigma_g > 4$ $M_\odot\ pc^{-2}$, 2) $J_g < 1$ (Jeans criterion), 3) $Q_g < 1.4$) $\Delta S < 0$ (S=entropy).

As seen in fig. 18, this preliminary approach seems to favour the third criterion (or perhaps the fourth one) which at best restores the observed zones of strong shocks obtained in the simulations (where star formation is effectively expected). Other experiments based on more elaborate physics are planned by the same authors.

9.4 Star formation and dynamical perturbations in galaxies

Enhanced rates of star formation (starbursts) have been often associated in recent past with perturbations occurring in the host systems such as bars and interactions. In fact both are able, in principle and with more or less high efficiency, to trigger inflows of gas and accumulation of matter towards the central regions as previously explained.

Enhanced star formation in the cores of SB galaxies must depend on the strength of the bar (mass and (or) axis ratio) and on the available gas quantity. The fact that for instance only 40% of early-type SB galaxies exhibit excess 10μm central emission (Devereux, 1987) indicates that the *presence* of the bar is only a necessary condition to create a starburst.

Using IRAS galaxies of Hubble type Sbc to Sdm-Im from samples selected by Devereux (1987) and Young et al. (1989), we observe that SA galaxies rather lie in the lower right part of the color-color sequence defined in subsection 9.1 whereas SB galaxies are found everywhere along the sequence: the starburst region is essentially populated by SB galaxies but some SB's seem quiescent (see Martinet, 1994 for details). It would be important to quantitatively estimate the relation between the starburst intensity and the structural parameters of bars concerned. Work in progress including star formation processes into the evolution scheme of barred systems was planned to this effect (Friedli and Benz, in preparation).

Evidences that galaxy interactions can trigger starbursts have been given by various authors (table 3).

Table 3: Observational evidences of enhanced star formation in interacting galaxies

Pass-band	Reference
UV	Larson and Tinsley (1978)
Near-IR	Joseph and Wright (1985)
Optical emission line strength	Kennicutt and Keel (1984)
Radio-emission	Hummel (1981)
IRAS pass-bands	Sanders et al. (1988)

Starbursts in interacting galaxies must depend on the encounter orbital parameters and also on the gas available in the pre-encounter galaxies. Advanced mergers for instance seem to be also only a necessary condition for an enhanced IR luminosity. Whereas bright normal isolated spirals have a range of 10 μm luminosity between 10^5 and $7 \times 10^8 L_\odot$, for interacting galaxies, the range is 4×10^7 to $7 \times 10^9 L_\odot$ and for mergers 4×10^9 to $5 \times 10^{10} L_\odot$. These typical values correspond to a conversion of gas into early type stars in the central few arc seconds at rates of 1 to 100 $M_\odot\ yr^{-1}$. By comparison, for our Galaxy, SFR in a region of 1 kpc diameter is $\sim 0.003\ M_\odot\ yr^{-1}$. Large starbursts are expected in stellar systems that readily form molecular gas out of atomic gas (Mirabel, 1992). The most luminous IR galaxies have larger $L_{FIR}/(M_{H_2} + M_{HI})$ ratio than isolated galaxies. For Arp 220, this ratio is $\sim$ 20 times the Milky Way value (1-3 $L_\odot {M_\odot}^{-1}$) and $\sim$ 5 times the M82-typical starburst value (5-10 $L_\odot {M_\odot}^{-1}$). However some strongly interacting and merging systems are not ultra-luminous in infra-red. Here also it will be important to quantitatively estimate the relation between some features of the encounter and the intensity of starburst which could be produced. The recent approach of the problem by Mihos et al. (1992, 1993) can be used as an example:

The authors carried out 3 types of merger simulations: 1) Prograde-prograde (PP) halo-disk galaxy interaction (the disks rotate in the same sense as their initially parabolic orbital motion, 2) retrograde-retrograde (RR) interaction (disk rotate in the opposite sense to the orbital motion, 3) each disk plane is orthogonal to the orbital plane and to the companion galaxy (O). They used Hernquist's tree code (Hernquist, 1987) to calculate the gravitational forces acting on stars and gas, modifying it to model star formation and interactions between gas clouds. The gas consists of discrete clouds interacting with one another through the merging of colliding clouds and fragmentation of massive clouds into smaller ones: if two clouds are at a distance less than the size of the larger cloud, they collide and merge. A continued coalescence leading to gas locked into a too small number of too massive clouds (unobserved!) is avoided by introduction of an exponential lifetime $\tau_i \div (\log M_i)^{-1}$. In fact the real process explaining the lack of too massive clouds (heating process by star formation and supernovae) is here explicitly burked.

Star formation rate in each cloud is given by

$$\dot{M}_i = C M_i \rho_i$$

where M_i is the cloud mass, ρ_i the local density in a spherical region surrounding the cloud.

Let us note that this law corresponds to a Schmidt law index $n \approx 1.8$. The initial gas depletion time is chosen so that $\mathrm{SFR}_0 \sim$ 1-5 $M_\odot\ yr^{-1}$. In order to appraise the only interaction effect, the considered initial conditions imply neither diffusion of gas toward the center nor starburst development.

Essential of a typical PP merger is the quick increase of SFR to 5 times the pre-interaction rate followed by a decline as tidal tails develop. Because of angular momentum loss, the galaxies fall back upon one another and merge by $T = 165\ Myr$. A dramatic increase of star formation (factor > 20) is due to a large quantity of gas falling into the center of the remnant ($\sim 50\%$ into the inner kpc) (Fig. 19). The lifetime of the starburst is $\sim 200\ Myrs$.

Actually a competition settles between the gravitational forces which draw the gas towards the center and the energy input from the starburst which tends to disperse the gas. A very important question is: what level of concentration could be reached? It is impossible to answer as long as one has not a detailed understanding of star formation. The central density able to trigger a starburst in the simulation described here is far from that suspected in Arp 220 for instance (5 $M_\odot\ pc^{-3}$ instead of 60 $M_\odot\ pc^{-3}$!). Is the used star formation law correct in so extreme conditions?

The result of RR merger is qualitatively similar to the PP case, but the massive tails are prevented by the lack of resonances between the inner rotational and orbital motions. More gas remain for a starburst which lasts 300 Myr, consuming 70% of the gas.

The orthogonal merger (O) takes longer time to reach completion (400 Myr) due to the drag minimisation on the galaxies. Less gas is channelled into the central region since the clouds suffer a perturbation out of the disk plane. SFR is 10 times that of an isolated disk. Only 25% of gas was consumed by the starburst.

Fly-by interactions have been also studied by Mihos et al. (PP, RR, O, PR). The only encounters which produce a significant increase of SFR (factor of 5 typically) are PP. The formation of a bar triggers a flow of gas towards the center. Enhanced star formation is not only found in the central region but also in the outer disk. However 70% of the star formation activity occurs in the central *kpc*. The "RR" and "O" fly-by interactions are very ineffective. Finally, in the PR interaction, 12% of gas from the prograde disk is transferred to the retrograde one. Accreted disk matter is captured on counterrotating orbits. Very effective collisions in the retrograde disk lead to flows of gas towards the center developing a strong starburst (SFR three time greater than in the isolated galaxies whereas the SFR increase in the prograde disk is only 50%.

These examples suggest the tight dependence of the starburst intensity on the various orbital features of the encounter.

The results can be compared to observational data suggesting starbursts in various types of interactions. Using information on the star formation efficiency (SFE = L_{IR}/L_{CO}) given by Solomon and Sage (1998), Tinney et al. (1990) and Sanders et al.(1991), we can observe that only some galaxies in strong interactions (in process of merger) show a spectacular SFE of the order of 100 with respect to those with companions without morphological disturbances or with disturbances but no tidal tails or bridges (flybies!), for which L_{IR}/L_{CO} is of the order of 10-30. Star formation rates from FIR data for the same galaxies display a large range of values from several $M_\odot\, yr^{-1}$ to more than 200 $M_\odot\, yr^{-1}$. The upper values are obtained only for galaxies in strong interaction. But it is not easy to establish one-to-one correspondance between morphological features and starburst activity. Experiments similar to those described above including a realistic treatment of star formation will certainly allow to better understand the strong dispersion observed in the SFR's of weakly and strongly interacting galaxies, confirming the dependence of enhanced star formation rate on the parameters characterizing the encounters. To go thoroughly into the relation between enhancements of star formation and galaxy interaction, Keel (1993) analysed kinematical data concerning direct and retrograde pairs of galaxies. Disturbances in rotation curves can be indicators of interactions with companions. Effectively in the mentioned work, high rates of star formation inferred from H_α data are associated with large relative amplitudes of velocity disturbances for nuclei and integrated measures. But statistically the level of star formation is found not to be dependent on orbital direction whereas models described before predict that fly-by encounters must be direct to produce star formation enhancements.

Independently of questions of orbits and inclinations, detailed studies of physical conditions in the gaseous medium can also bring some enlightments on the fact that all interacting galaxies do not show evidence for starburst. Jog and Das (1992) suggest that if the central molecular pressure is less than the pressure in a giant molecular cloud (GMC), an incoming GMC does not suffer the necessary overpressure and a starburst will not be triggered despite gas infall due to the interaction (or to a bar!) (see also Jog and Solomon, 1992).

Let us note in addition that the Mihos merger models seem to be unable to produce starbursts which could power the ultra-luminous infrared galaxies with $L_{FIR} > 10^{12}L_{\odot}$. The implied star formation rates would be one order greater than those predicted by the models. There exists at least three possible sources for this disagreement: 1) the protogalaxies are actually more gas rich than assumed 2) the star formation criterion based on the Schmidt law is not correct (see the previous subsection) and 3) the cloud-collision model is too crude.

An other divergence between the present models and the observations concerns the sites of star formation. Mihos et al. (1993) point out various cases of pairs, for instance NGC 6872 / IC 4970 the H_{α} maps of which show star formation along the tidal arms whereas the model indicates the bar and the nuclear region as the only privileged sites. Probably a more rigourous approach of gas dynamics, for instance concerning the viscosity, could clear up the question.

10 Does a spiral galaxy change its Hubble type during evolution?

At length of the sections 2 and 4 to 9 of this review, a set of processes have been discussed which are so many clues of galaxy evolution *on time scales equal or smaller than the Hubble time*. Let us summarize the essential points

1. From studies on orbital behavior, we learn for instance that too highly triaxial spheroids might not exist because real equilibrium systems would not tolerate too much chaos. The conjugate effects of a bar and a compact central mass cause bulges to secularly grow through enhanced stochastic orbital diffusion. The width of instability strips on the main periodic orbits, which trigger radial or vertical diffusion, depends on the geometry of the system which can be modified in time.

2. In systems consisting of stars and gas, recurrent growth of non-axisymmetric perturbations must be the rule to explain the high frequency of spirals observed.

3. The shape of the spheroid (halo) can become flatter during the slow setting up of a disk in the potential well of the spheroid.

4. The friction effect between a bar and a disk or a bulge produces a transfer of angular momentum from the bar to the disk or to the bulge. Bars evolve in response to these interactions.

5. The structure of a barred galaxy perpendicularly to the disk is changed by resonant excitations: observed box or peanut shapes can be the signature of a bar.

6. Satellites which merge into a disk cause thickening, flattening and warping of the disk.

7. Bulges can grow by satellite accretions.

8. Bars easily form in a disk, they are robust, unless a strong ILR be created by satellite accretion, compact mass or gas inflow towards the center, what contributes to destroy the bars.

9. The dissipative nature of gas plays an important role in spiral activity development and in fueling the galactic nuclei, the gas systematically losing angular momentum to the stars.

10. Differences in the structure of early and late-type galaxies cannot be explained without reference to the gaseous component the mass of which can vary in time due to either accretion or star formation.

11. Some observed phenomena such as bars within bars are episodic.

12. Weak interactions between galaxies trigger spiral or bar modes. Strong interactions generate more drastic transformations. Mergers of spirals can produce ellipticals.

13. It is possible that the local star formation be self-regulated by the global dynamics, then the galaxy morphology must change over time-scales of the order of gas consumption time scale which is shorter than the Hubble time.

14. Under certain conditions, galaxy interactions enhance star formation which can in its turn influence the subsequent dynamical evolution.

Jeans (1929) postulated that galaxies evolve from elliptical to spiral, and then to irregular galaxies: this idea, which influenced Hubble himself, has been hard to kill. In the seventies, the initial conditions were considered as the key factor for creating the various observed morphologies (elliptical and spirals). Larson (1976) and Gott and Thuan (1976) introduced the ratio of the star formation to the collapse time scales τ_{sf}/τ_{coll} as the parameter which determines the initial evolution: early-types (ellipticals) would result if $\tau_{sf}/\tau_{coll} < 1$, late-types if $\tau_{sf}/\tau_{coll} > 1$.

In view of the processes mentioned above, it is difficult to still claim that only the initial conditions are responsible of the diversity of the observed Hubble types. Here we present a quintessence of arguments recently put forward in favor of secular evolution through the Hubble sequence for spirals from Sd to Sa. Some ideas in this connection had been already expressed by Kormendy (1982), particularly concerning the fate of bars.
Larson (1993) recalled some theoretical predictions useful for the discussion:

- The gas depletion time scale τ is expected to increase by a factor of 3 along the Hubble sequence.

- The rotation period could be the basic clock for the evolution (if $Q =$ ct., $\tau \div \kappa^{-1}$).

- The spacing of arms depends on $\lambda_{crit} = 4\pi^2 G\mu/\kappa^2$. A more open spiral pattern is expected if a galaxy has either μ higher or κ lower.

Thus the variation of the gas content as well as the star formation and the winding of arms must be mainly influenced by the mass distribution and the rotation rate.

Now the Hubble sequence from Sd to Sa is a sequence of increasing bulge-to-disk ratio, rotation, arm winding, metallicity, ratio of current to average past star formation rate. Which regard to the list of processes drawn up at the beginning of this section, something irreversible appears in this sequence: a possible scenario could be the following (Pfenniger et al. (1994) : Cooling in a mostly gaseous disk $\longrightarrow$ gravitational instability $\longrightarrow$ formation of a bar $\longrightarrow$ secular formation of a small bulge $\longrightarrow$ accumulation of mass towards the center $\longrightarrow$ dissolution of the bar into a spheroidal component. Growth of bulge can be produced by satellite mergers. Moreover referring to the virial theorem and taking account of dissipation (Pfenniger, 1991b), it becomes clear that a galaxy could be at first a slowly rotating weakly concentrated object, then dynamically evolves into a rapidly rotating and more condensed one. The outer parts are progressively symmetrized and arms tightly wind.

As already emphasized, the role of bars seems to be here essential. A possible evolution sequence resulting from our discussion could be (Friedli and Martinet, 1993): Sd $\longrightarrow$ Sc $\longrightarrow$ Sbc $\longrightarrow$ SBb $\longrightarrow$ Sb .... or $\longrightarrow$ Sc $\longrightarrow$ SBb $\longrightarrow$ SBa $\longrightarrow$ SBO $\longrightarrow$ SO, perhaps refined in S $\longrightarrow$ SB $\longrightarrow$ S2B $\longrightarrow$ SB $\longrightarrow$ S by the "bars within bars" episode. Such scenarios could be either strengthened or modified by the star formation contribution for which quantitative estimates are urgently requested in the frame of evolution simulations. Furthermore, the scenario proposed above concerns essentially isolated galaxies. Interactions can perturb it. It could be possible that mass inflows in interacting systems change a late unbarred galaxy into an early barred one as suggested by Elmegreen et al. (1990). Their statistics indicates a simultaneous change of Hubble type and structure (SA $\longrightarrow$ SB). But the size of the sample does not allow a really decisive conclusion.

Including Ellipticals, SO and early-type spirals in the discussion, Schweizer (1993) presented some statistical arguments according to which many of these galaxies could be the product of collisions and mergers . He suggested that "the Hubble sequence may rank galaxies mainly by the number and the vehemence of mergers in their past history". Progressive mergers depend on the density. In the context of hierarchy formation of galaxies, Larson (1993) distinguished Ellipticals and SO as formed from subunits of stars and spirals as formed from subunits of gas.

Study by Combes and Elmegreen (1993) already mentioned in section 7.3 is a starting point to compare in details the evolution inside the early and late-type spirals. Whereas their early-type models evolve slowly, late-type models evolve quickly with gas inflow towards the center which leads to a more condensed inner region. Consequently, these late-type models evolve towards a early-type. What is not under control is the possible reduction of gas fraction by star bursts. But with continuous accretion process from various origins (tidal interactions for instance), evolution towards earlier type can continue. In fact as we know, without infall, a Sc galaxy would exhaust its gas in a small fraction of the Hubble time.

Kennicutt (1990) claimed that more than one evolutionary path could lead to a particular type and that many scenarios could lead identical protogalaxies to evolve into very different types. Taking account of the internal processes described in this review, the interaction effects and the changeable star formation efficiency, still badly understood, we must obviously not exclude these possibilities.

Finally, Pfenniger et al. (1994) have discussed the implications of the proposed evolution on the nature of dark matter. We must take into consideration the following facts:

1) The ratio of dark matter to stellar mass decreases from Sd's to Sa's by a factor of 10^2. In comparison, the ratio of dark to HI mass is nearly constant ($\sim$ 10-30).

2) The early types, gas depleted, have a declining star formation rate (SFR). Sb's and Sc's have a SFR = ct. The latest and irregular types are the site of starbursts. But average time scale for gas consuming is of the order of 4 Gyr! Either gas infall representing some percents of total mass and (or) delayed gas recycling or stellar winds can solve this gas consumption problem.

The solution suggested by Pfenniger et al. (1994) in order to collect these facts in agreement with the evolution scenario from Sd to Sa is to admit that dark matter be in a form able to create stars, i.e. fresh diluted hydrogen. A model of fractal cold gas, clumpy down to very small scales has been proposed by Pfenniger and Combes (1994).

The problem of dark matter in disk galaxies could be solved to a large extent as far as it can be proved that gas mass given by present data is underestimated by a factor of 5 to 10.

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Figure captions

Fig. 1 Position of galaxies with different shape of HI rotation velocity curve in the plane V_{max} versus h = radial optical scale length. Thick part of curve corresponds to the region outside two-thirds of optical radius. (From Casertano and van Gorkom, 1991).

Fig. 2 Resonant periodic orbits inside corotation in a barred galaxy. (From Contopoulos and Grosbøl, 1989).

Fig. 3 Example of surface of section and variation of the rotation number $Rot(x)$ in a triaxial model with axis ratio $(1 : 0.625 : 0.5)$. (From Martinet and Udry, 1990).

Fig. 4 Prediction of axis ratios of real triaxial systems (elliptical and bulges) inferred from observations, compared with analytical models used by Martinet and Udry (1990) (I to IV). (From Udry and Martinet, 1994).

Fig. 5 Surface of section in a noisy triaxial potential $\Phi = \Phi_0 + \epsilon\Phi_1(k_i x_i)$ for different values of the amplitude ϵ and frequency k_i of noise. (From Udry, 1991).

Fig. 6 Diffusion of stochastic orbits close to the z-axis orbit in galaxies with a central compact Plummer mass M_p: a) $GM_p = 0$, b) $GM_p = 10^{-3}$. (From Martinet and Pfenniger, 1987).

Fig. 7 Behaviour of the part of Q containing R for two theoretical values of R_m, radius from which the rotation curve becomes approximately flat. h is the radial scale length. (From Martinet, 1988).

Fig. 8 Theoretical relation between radial velocity dispersion taken at $R = h$ and the maximum rotational velocity a) for different $(M/L)_B$, b) for different Q's (see the text), in comparison with observed values. (From Bottema, 1993).

Fig. 9 Variation in the ratio of the perturbation surface density to the unperturbed surface density with time for star (S) and gas (G) showing the swing amplification at $\tau \geq 0$. (From Jog, 1992).

Fig. 10 Theoretical and experimental predictions of the phase-averaged angular momentum change $\langle \Delta J \rangle$ between a bar and a disk described in the text. Case of a weak fast rotating bar. Disk without self-gravity. (From Little and Carlberg, 1991).

Fig. 11 Evolution of the projected density in a 3-D N-body model as seen disk face-on, bar end-on and bar edge-on, at T = 5 Gyr compared with the initial configuration. (From Pfenniger and Friedli,1991).

Fig. 12 Velocity dispersions in a 3-D N-body simulation at T = 5 Gyr compared with the initial isotropic velocity dispersion (From Pfenniger and Friedli, 1991).

- Fig. 13 Projections of 3-D 4/4/1 resonant orbits in a strongly barred galactic potential in the (y, x) (upper left), (z, x) (upper right) and (y, z) (lower left) planes. (From Pfenniger, 1985).
- Fig. 14 Evolution of star+gas model simulating a) a late-type galaxy after 2.4 (top) and $4.8 \times 10^8 yr$ (bottom) and b) an early-type one, after 8.4 (top) and $10.8 \times 10^8 yr$ (bottom). Stars (left) and gas (right) particles are represented. (From Combes and Elmegreen, 1993).
- Fig. 15 Time evolution of the relative central mass concentration within 0.5, 1.0, 1.5 kpc from the center for a barred galaxy model consisting of gas (solid lines) and stars (dotted lines). (From Friedli and Benz, 1993).
- Fig. 16 Time evolution of the stellar surface density during the two-bar phase for a galaxy model consisting of gas and stars. t is the time in Myr and θ the approximate angle in degree between the primary and the secondary bar. (From Friedli and Martinet, 1993).
- Fig. 17 Distinct physical regimes in field and Virgo galaxies from H_α surface brightness/gas surface density. (From Kennicutt, 1993).
- Fig. 18 Sites of star formation deduced from various criteria described in the text for a galaxy model consisting of gas and stars (From Friedli and Benz, 1993).
- Fig. 19 Star formation evolution in a prograde-prograde merger model. Unit time is ~ 50 Myr. (From Mihos et al., 1992).