Dirac particles' tunnelling from black rings

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Recent research shows that Hawking radiation can be treated as a quantum tunnelling process, and Hawking temperature of Dirac particles across the horizon of a black hole can be correctly recovered via fermions tunnelling method. In this paper, motivated by fermions tunnelling method, we attempt to apply the analysis to derive Hawking radiation of Dirac particles via tunnelling from black ring solutions of 5-dimensional Einstein-Maxwell-dilaton gravity theory. Finally, it is interesting to find as in black hole cases, fermions tunnelling can also result in correct Hawking temperatures for the rotating neutral, dipole and charged black rings.

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I. INTRODUCTION

Since Hawking had proved that a black hole can radiate particles characterized by the thermal spectrum with the temperature $T = (1/2\pi)\kappa$, where κ is the surface gravity of the black hole, many papers appear to correctly derive Hawking temperature via different methods, such as gravity collapsing method[1], temperature Green function[2], path integral[3], Euclidean action integral[4], second quantum method[5], renormalization energy-momentum tensor[6] and more recently developed technique called generalized tortoise coordinate transformation(GTCT) to deal with Hawking radiation of an evaporating black holes[7, 8], etc. The study of Hawking radiation has long been attracted a lot of attentions of theoretical physicists. The reason is partly due to the fact that a deeper understanding of Hawking radiation may shed some lights on seeking the underlying quantum gravity, and on the other hand, it is the key to make the second law of thermodynamics in spacetimes involving black holes consistent.

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In recent years, a semi-classical quantum tunnelling method, first put forward by Kraus and Wilczek[9] and then elaborated by Parikh and Wilczek[10], has already attracted many people's attentions [11, 12]. Here derivation of Hawking temperature mainly depends on the computation of the imaginary part of the action for the classically forbidden process of s-wave emission across the horizon. Normally, there are two approaches to obtain the imaginary part of the action. One, first used by Parikh and Wilczek[10] and later broadly discussed by many papers[11, 12], is called as the Null Geodesic method, where the contribution to the imaginary part of the action only comes from the integration of the radial momentum p_r for the emitted particles. The other method regards the action of the emitted particles satisfies the relativistic Hamilton-Jacobi equation, and solving it yields the imaginary part of the action[13], which is an extension of the complex path analysis proposed by Padmanabhan et al[14]. In the two tunnelling modes, they use the fact that the tunnelling rate for the classically forbidden trajectory from inside to outside the horizon is given by $\Gamma = \exp\left(-\frac{2}{\hbar}\mathrm{Im}I\right)$, where I is the classical action of the trajectory to leading order in $\hbar$. Where these two methods differ is in how the action is calculated. Ref.[15] has given a detailed comparison between the Hamilton-Jacobi ansatz and the Null Geodesic method.

Although the tunnelling method is shown very robust to successfully derive Hawking radiation of black holes and even black rings, most papers have only considered scalar particle's tunnelling radiation. In fact, a black hole can radiate all types of particles at the Hawking temperature, and the true emission spectrum should contain contributions of both scalar particles and fermions with all spins. Recently, applications of quantum tunnelling methods to fermions case has first been presented in Ref.[16] to correctly describe Hawking radiation of fermions with spin 1/2 via tunnelling from Rindler space-time and that from the uncharged spherically symmetric black holes. Later, to further prove the robustness of fermions tunnelling method, some papers appear to discuss Hawking radiation of fermions via tunnelling from BTZ black hole[17], dynamical black hole[18], Kerr black hole[19], Kerr-Newman black hole[20] and more general and complicated black holes[21]. These involved black holes share in taking $3-$ or $4-$dimensional spacetimes. For spacetimes with different horizon topology and different dimensions, choosing a set of appropriate γ^μ matrices for general covariant Dirac equation is critical for fermions tunnelling method. In 3-dimensional cases, as the Pauli matrices $\sigma^i(i = 1, 2, 3)$ behave independent each other, we can only introduce the matrices σ^i to act as γ^μ functions for the covariant Dirac equation [17].

However for 4-dimensional spacetimes, we need four independent matrices to well describe the matrices γ^μ for the Dirac equation, and a detailed choice for the four matrices γ^μ see Refs.[16, 18, 19, 20, 21]. Then how to choose the γ^μ matrices for 5-dimensional cases? To the best of our knowledge, five independent matrices should be involved in our discussion. On the other hand, the horizon topology also has an important impact on the choice for the matrices γ^μ[20]. In Sec.II and III, we will successfully introduce a set of appropriate matrices γ^μ for the 5-dimensional neutral, dipole and charged black rings with the horizon topology $S^1 \times S^2$ to well describe Dirac particles' tunnelling radiation.

Black rings in five dimensions have many unusual properties not shared by Myers-Perry black holes with spherical topology, for instance, their event horizon topology is $S^1 \times S^2$, not spherical for the neutral, dipole and charged black rings. (Actually, some topological black holes also have nontrivial topology, see for example, [22]). Therefore, it is very interesting to study Hawking radiation from these black ring solutions. In Ref.[23], scalar particles via tunnelling from black rings has already been discussed by using the so-called Hamilton-Jacobi method. And in [24], following recently hot discussion on anomalous derivation of Hawking radiation, the authors attempt to recover Hawking temperature of black rings via gauge and gravitational anomalies at the horizon. However, when reducing the higher dimensional theory to the effective two dimensional theory, they also only consider scalar field near the horizon. As far as I know, till now, there is no references to report Hawking radiation of Dirac particles across black rings. So it is interesting to see if fermions tunnelling method is still applicable in such exotic spacetime, and how to choose the matrices γ^μ for the covariant Dirac equation of 5-dimensional black rings. In this paper, we shall concentrate ourselves on Dirac particles' tunnelling radiation from 5-dimensional black rings via fermions tunnelling method. We finally find as in black hole cases, fermions tunnelling result in correct Hawking temperatures for the rotating neutral, dipole and charged black rings.

The remainders of this paper is organized as follows. In Sec.II, Hawking radiation of Dirac particles via tunnelling from the 5-dimensional rotating neutral black ring has been studied by improving fermions tunnelling method. Here to make a following analysis on the rotating dipole and charged black rings in a more unified form in Sec.III, we deduce a general 5-dimensional metric from the rotating neutral black ring, and discuss its Hawking radiation of Dirac particles. In fact, the involved 5-dimensional metric is not arbitrarily taken, and after some substitutions has a unified form as the rotating neutral, dipole and charged black

rings(see Ref.[23]). Sec.III is devoted to once again check the validity of fermions tunnelling method for the rotating dipole and charged black rings. Sec.IV contains some conclusions and discussions.

II. DIRAC PARTICLES' TUNNELLING FROM NEUTRAL BLACK RING

In this section, we focus on studying Hawking radiation of Dirac particles via tunnelling from 5-dimensional neutral black ring. In this paper, black rings involved are only special solutions of the Einstein-Maxwell-Dilaton gravity model (EMD) in 5-dimensions, and the corresponding action takes the forms as

$$S = \frac{1}{16\pi} \int d^5x \sqrt{-g} \left(\mathcal{R} - \frac{1}{2} \left(\partial \phi \right)^2 - \frac{1}{4} e^{-\alpha \Phi} F^2 \right), \tag{1}$$

where F is a three-form field strength and Φ is a dilaton. Black ring solutions of the action (1) have special characters: 1) they all have horizon of topology $S^1 \times S^2$; 2) there exist three Killing coordinates to determine their local symmetries; 3) there exists infinitely many different black rings solutions carrying the same mass, angular momentum and electric charge, etc. In this paper, the rotating neutral, dipole and charged black rings accompanied by the action (1) are involved in our discussion. First, we consider the case of the 5-dimensional neutral black ring. The neutral black ring in 5-dimensional EMD theory has been given by[25]

$$
\begin{aligned}
ds^2 &= -\frac{F(y)}{F(x)} \left(dt - C(\nu, \lambda) R \frac{1+y}{F(y)} d\psi \right)^2 \\
&+ \frac{R^2}{(x-y)^2} F(x) \left[-\frac{G(y)}{F(y)} d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\varphi^2 \right],
\end{aligned}
\tag{2}
$$

where

$$
F(\xi) = 1 + \lambda \xi, \quad G(\xi) = (1 - \xi^2)(1 + \nu \xi),
$$
$$
C(\nu, \lambda) = \sqrt{\lambda(\lambda - \nu)\frac{1+\lambda}{1-\lambda}}.
$$

The parameters λ and ν are dimensionless and takes values in the range $(0 < \nu \leq \lambda < 1)$, and to avoid the conical singularity also at $x = 1$, λ and ν must be related to each other via $\lambda = 2\nu/(1 + \nu^2)$. The coordinate ϕ and ψ are two cycles of the black ring, and x and y takes the range as $-1 \leq x \leq 1$ and $-\infty \leq y \leq -1$. The constant R has the dimensional of

length and for thin large rings corresponds roughly to the radius of the ring circle[26]. The horizon is located at $y = y_h = -1/\nu$. The mass of the black ring is $M = 3\pi R^2 \lambda/[4(1-\nu)]$, and its angular momentum takes $J = \pi R^3 \sqrt{\lambda(\lambda-\nu)(1+\lambda)}/[2(1-\nu)^2]$. In addition, the spacetime contains three Killing coordinates t, φ and ψ. Next, we shall study Dirac particles' tunnelling from the above neutral black ring. For simplicity, we take

$$
\begin{aligned}
\mathcal{M}(x,y) &= \frac{F(y)}{F(x)}\left(1 - \frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2}{F^2(x)G(y) + C^2(\nu,\lambda)(1+y)^2(x-y)^2}\right), \\
\mathcal{N}(x,y) &= -\left(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}\right)^{-1}, \\
N^\psi(x,y) &= -\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)}, \\
g_{\psi\psi}(x,y) &= -\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)}{F(x)F(y)(x-y)^2}, \\
g_{xx}(x,y) &= \frac{R^2F(x)}{(x-y)^2G(x)}, \quad g_{\varphi\varphi}(x,y) = \frac{R^2G(x)}{(x-y)^2}.
\end{aligned}
\tag{3}
$$

Now the new form of neutral black ring (2) changes as

$$
\begin{aligned}
ds^2 &= -\mathcal{M}(x,y)dt^2 + \frac{1}{\mathcal{N}(x,y)}dy^2 \\
&+ g_{\psi\psi}(x,y)\left(d\psi + N^\psi(x,y)dt\right)^2 + g_{xx}(x,y)dx^2 + g_{\varphi\varphi}(x,y)d\varphi^2.
\end{aligned}
\tag{4}
$$

At the event horizon of the neutral black ring, the coefficients in Eq.(3) obviously obey

$$
\mathcal{M}(x,y_h) = \mathcal{N}(x,y_h) = 0, \quad N^\psi(x,y_h) = -\Omega_h,
\tag{5}
$$

where $y = y_h$ is the event horizon of the neutral black ring and Ω_h is the angular velocity of the black ring at the event horizon. Throughout this paper, the 5-dimensional spacetime coordinates are always chosen as $x^\mu = (t, y, \varphi, x, \psi)$.

Now we focus on studying Dirac particles' tunnelling from the rotating neutral black ring. In curved spacetime, Dirac particles' motion equation satisfies the following covariant Dirac equation

$$
i\gamma^a e_a^\mu D_\mu \Psi - \frac{m}{\hbar}\Psi = 0,
\tag{6}
$$

where D_μ is the spinor covariant derivative defined by $D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{[a}\gamma_{b]}$, and ω_μ^{ab} is the spin connection corresponding to the tetrad e_a^μ. In this paper, we choose the matrices

$\gamma^a = (\gamma^0, \gamma^3, \gamma^4, \gamma^1, \gamma^2)$ for the 5-dimensional rotating neutral black ring, where

$$\gamma^0 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^4 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \tag{7}$$

and the $\sigma^i (i = 1, 2, 3)$ are the Pauli matrices, which are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8}$$

According to the new form of the rotating neutral black ring (4), the tetrad field e_a^μ can be constructed as

$$e_0^\mu = \left(\frac{1}{\sqrt{\mathcal{M}}}, 0, 0, 0, -\frac{N^\psi}{\sqrt{\mathcal{M}}} \right),$$

$$e_1^\mu = \left(0, \sqrt{\mathcal{N}}, 0, 0, 0 \right),$$

$$e_2^\mu = \left(0, 0, \frac{1}{\sqrt{g_{\varphi\varphi}}}, 0, 0 \right),$$

$$e_3^\mu = \left(0, 0, 0, \frac{1}{\sqrt{g_{xx}}}, 0 \right),$$

$$e_4^\mu = \left(0, 0, 0, 0, \frac{1}{\sqrt{g_{\psi\psi}}} \right). \tag{9}$$

As Dirac particles taking spin $1/2$, when measuring spin along y direction, there would be two cases. One is spin up case, which shares the same direction as y, and the other(spin down) case takes the opposite direction. In the Pauli matrix σ^3 representation, they can explicitly expressed by the eigenvectors $\xi_{\uparrow/\downarrow}$, and the corresponding eigenvalues are $1/-1$. In this paper, we only refer to spin field for the upper case $(\xi_\uparrow)$. In fact, after a completely same step for spin down $(\xi_\downarrow)$ case, we can also get the same result. We employ the following ansatz for Dirac field with spin up case as

$$\Psi_\uparrow(t, y, \varphi, x, \psi) = \begin{pmatrix} A(t, y, \varphi, x, \psi)\xi_\uparrow \\ B(t, y, \varphi, x, \psi)\xi_\uparrow \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, y, \varphi, x, \psi) \right]$$

$$= \begin{pmatrix} A(t, y, \varphi, x, \psi) \\ 0 \\ B(t, y, \varphi, x, \psi) \\ 0 \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, y, \varphi, x, \psi) \right]. \tag{10}$$

Substituting the above ansatz (10) for upper-spinning state into the covariant Dirac equation (6), then applying WKB approximation and keeping the prominent terms, we can get the following equations

$$B\left(\frac{1}{\sqrt{\mathcal{M}}}\partial_t I_\uparrow + \sqrt{\mathcal{N}}\partial_y I_\uparrow - \frac{N^\psi}{\sqrt{\mathcal{M}}}\partial_\psi I_\uparrow\right) + A\left(m - \frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_\varphi I_\uparrow\right) = 0, \tag{11}$$

$$B\left(\frac{1}{\sqrt{g_{xx}}}\partial_x I_\uparrow + \frac{i}{\sqrt{g_{\psi\psi}}}\partial_\psi I_\uparrow\right) = 0, \tag{12}$$

$$A\left(\frac{1}{\sqrt{\mathcal{M}}}\partial_t I_\uparrow - \sqrt{\mathcal{N}}\partial_y I_\uparrow - \frac{N^\psi}{\sqrt{\mathcal{M}}}\partial_\psi I_\uparrow\right) - B\left(m + \frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_\varphi I_\uparrow\right) = 0, \tag{13}$$

$$A\left(\frac{1}{\sqrt{g_{xx}}}\partial_x I_\uparrow + \frac{i}{\sqrt{g_{\psi\psi}}}\partial_\psi I_\uparrow\right) = 0. \tag{14}$$

In fact, the derivatives of A and B, and the components $\frac{1}{4}\omega_\mu^{ab}\gamma_{[a}\gamma_{b]}$ are all of order $\mathcal{O}(\hbar)$, and according to WKB approximation have already been neglected for the above equations. Considering the symmetries of the rotating neutral black ring, we employ the following ansatz

$$I_\uparrow = -\mathcal{E}t + \mathcal{J}\psi + \mathcal{L}\varphi + \mathcal{W}(x,y) + \mathcal{K}, \tag{15}$$

where $\mathcal{E}$, $\mathcal{J}$ and $\mathcal{L}$ are all real constants which respectively represent the emitted particle's energy and angular momentum corresponding to the angles ψ and φ , and $\mathcal{K}$ is a complex constant(where we consider only the positive frequency contributions without loss of generality). Inserting the ansatz (15) into Eqs.(11)(12),(13),(14), and expanding the resulting equations near the event horizon of the black ring, we have

$$B\left(\frac{-\mathcal{E} + \Omega_h \mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}} + \sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y \mathcal{W}(x,y)\right)$$
$$+ A\left(m - \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right) = 0, \tag{16}$$

$$B\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x \mathcal{W}(x,y) + \frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}\mathcal{J}\right) = 0, \tag{17}$$

$$A\left(\frac{-\mathcal{E} + \Omega_h \mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}} - \sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y \mathcal{W}(x,y)\right)$$
$$- B\left(m + \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right) = 0, \tag{18}$$

$$A\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x \mathcal{W}(x,y) + \frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}\mathcal{J}\right) = 0. \tag{19}$$

Here $\mathcal{M}_{,y}(x,y_h) = \partial_y \mathcal{M}(x,y)|_{y=y_h}$ and $\mathcal{N}_{,y}(x,y_h) = \partial_y \mathcal{N}(x,y)|_{y=y_h}$. Now we carry on an

explicit analysis on the above equations. From Eqs.(17) and (19) can we obtain

$$\partial_x \mathcal{W}(x,y) = -i\sqrt{\frac{g_{xx}(x,y_h)}{g_{\psi\psi}(x,y_h)}}\mathcal{J}.$$

(20)

And from Eqs.(16) and (18), one can easily see the two equations have a non-trivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, so we have

$$\partial_y \mathcal{W}(x,y) = \pm\frac{\sqrt{\left(\mathcal{E} - \Omega_h \mathcal{J}\right)^2 + \mathcal{M}_{,y}(x,y_h)(y - y_h)\left(m^2 - \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)}}{\sqrt{\mathcal{M}_{,y}(x,y_h)\mathcal{N}_{,y}(x,y_h)}(y - y_h)}.$$

(21)

It should be noted that Eq.(20) implies near the horizon of the black ring $\partial_x \mathcal{W}(x,y)$ has no explicit y dependence. On the other hand, in Eq.(21) $\mathcal{M}_{,y}(x,y_h)$ and $\mathcal{N}_{,y}(x,y_h)$ are both related to the coordinate x, but their product $\mathcal{M}_{,y}(x,y_h) \cdot \mathcal{N}_{,y}(x,y_h)$ is independent of x. So, near the horizon ($y \simeq y_h$), $\partial_y \mathcal{W}(x,y)$ is independent of x. Now the function $\mathcal{W}(x,y)$ can be separated as $\mathcal{W}(x,y) = \mathcal{W}(x) + \mathcal{W}(y)$, which means near the horizon of the black ring $\partial_x \mathcal{W}(x,y) = \partial_x \mathcal{W}(x)$ and $\partial_y \mathcal{W}(x,y) = \partial_y \mathcal{W}(y)$.

The WKB approximation tells us the tunnelling rate for the classically forbidden trajectory from inside to outside the horizon is related to the imaginary part of the emitted particle's action across the event horizon. Now our first job is to find the imaginary part of the action. From Eq.(15), we find only $\mathcal{W}(x,y)$ and $\mathcal{K}$ yield contributions to the imaginary part of the action. As $\mathcal{K}$ is a complex constant, all is focus on computing $\mathcal{W}(x)$ and $\mathcal{W}(y)$. In fact, after an integration on Eq.(20), $\mathcal{W}(x)$ must be given by a complex constant, so will yield a contribution to the imaginary part of the action. From Eq.(21) yields

$$\mathcal{W}_{\pm}(y) = \pm i\pi\frac{\mathcal{E} - \Omega_h \mathcal{J}}{\sqrt{\mathcal{M}_{,y}(x,y_h)\mathcal{N}_{,y}(x,y_h)}},$$

(22)

where $+/-$ sign corresponds to outgoing/incoming solutions. As we all know, the tunnelling probabilities is proportional to the imaginary part of the action. So when particles tunnel across the horizon each way, the outgoing and ingoing rates are respectively given by

$$P_{out} = \exp\left[-\frac{2}{\hbar}\mathrm{Im}I_\uparrow\right] = \exp\left[-\frac{2}{\hbar}\Big(\mathrm{Im}\mathcal{W}_+(y) + \mathrm{Im}\mathcal{W}(x) + \mathrm{Im}\mathcal{K}\Big)\right],$$

$$P_{in} = \exp\left[-\frac{2}{\hbar}\mathrm{Im}I_\uparrow\right] = \exp\left[-\frac{2}{\hbar}\Big(\mathrm{Im}\mathcal{W}_-(y) + \mathrm{Im}\mathcal{W}(x) + \mathrm{Im}\mathcal{K}\Big)\right].$$

(23)

Noted that any particles classically enter the horizon with no barrier, that means the tunnelling rate should be unity for incoming particles crossing the horizon. In our case, that

implies $\text{Im}\mathcal{W}_-(y) = -\text{Im}\mathcal{W}(x) - \text{Im}\mathcal{K}$. Set $\hbar$ to unity, and the tunnelling probability of Dirac particles crossing from inside to outside horizon is naturally written as

$$\begin{aligned} \Gamma &= \exp\left[-4\text{Im}\mathcal{W}_+(y) \right] \\ &= \exp\left[-\frac{4\pi}{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}\left(\mathcal{E} - \Omega_h \mathcal{J}\right) \right], \end{aligned} \tag{24}$$

which results in the expected temperature of the rotating neutral black ring

$$T = \frac{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}{4\pi} = \frac{1}{4\pi R}\frac{1+\nu}{\sqrt{\nu}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \tag{25}$$

This result is exactly consistent with that in Refs.[23, 24, 25], where respectively present the correct Hawking temperature of the rotating neutral black ring by using the so-called Hamilton-Jacobi method, anomalous cancellation method and the original definition of the surface gravity. Noted that the resulting temperature (25) is only for Dirac particles with spin up. For spin down case, taking a manner fully analogous to the spin up case will result the same result, which means both spin up and spin down particles are emitted at the same rate. So such treatment does not loss the generality of fermions tunnelling method. In addition, the tunnelling rate (24) is derived by neglecting the higher terms about $\mathcal{E}$ and $\mathcal{J}$, and the resulting spectrum is purely thermal. If we consider energy and angular momentum conservation when particles tunnelling out from the horizon, the higher terms will be present in the tunnelling rate, and the radiation spectrum is not thermal, and related to the change of Bekenstein-Hawking entropy, which discussed a lot in Refs.[10, 11, 12, 23]. In the next section, to further verify the validity of application of fermions tunnelling method to black rings, we additionally take dipole and charged black ring as an example to discuss their Hawking radiation of Dirac particles.

III.   DIRAC PARTICLES' TUNNELLING FROM DIPOLE AND CHARGED BLACK RINGS

In the section, we will discuss Hawking radiation of Dirac particles via tunnelling from dipole and charged black rings, and expect to result in correct Hawking temperatures.

A. Dipole black ring

Dipole black ring shares the same action (1) as neutral black ring, so they physically take many similar characters. The five dimensional dipole black rings was first constructed in [25], its metric takes the form as

$$ds^2 = -\frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{N/3}\left(dt - C(\nu,\lambda)R\frac{1+y}{F(y)}d\psi\right)^2$$

$$+\frac{R^2}{(x-y)^2}F(x)\left(H(x)H^2(y)\right)^{N/3}$$

$$\times\left[-\frac{G(y)}{F(y)H^N(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H^N(x)}d\varphi^2\right], \tag{26}$$

where $F(\xi), G(\xi)$ and $C(\nu,\lambda)$ are of the same form as neutral black ring, and $H(\xi) = 1 - \mu\xi$ $(0 \leq \mu < 1)$. The dilaton coupling constant is related to the dimensionless constant N as $\alpha^2 = (\frac{4}{N} - \frac{4}{3})(0 < N \leq 3)$. The horizon is also located at $y = y_H = -1/\nu$. Taking the limit of $\mu = 0$ in Eq.(26), this solution degenerates into neutral black ring[25]. In suitable limits, dipole black ring also contains Myers-Perry black hole[27]. This metric (26) takes the same form as (2), so we can apply the same procedure in Sec.(II) to correctly recover Hawking temperature of dipole black ring. Before that, we take

$$\mathcal{M}(x,y) = \frac{F(y)}{F(x)}\left(\frac{H(x)}{H(y)}\right)^{N/3}\left(1 - \frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2}{F^2(x)G(y) + C^2(\nu,\lambda)(1+y)^2(x-y)^2}\right),$$

$$\mathcal{N}(x,y) = -\left(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}\left(H(x)H^2(y)\right)^{N/3}\right)^{-1},$$

$$N^\psi(x,y) = -\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)},$$

$$g_{\psi\psi}(x,y) = -\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2 + R^2F^2(x)G(y)}{F(x)F(y)(x-y)^2}\left(\frac{H(x)}{H(y)}\right)^{N/3},$$

$$g_{xx}(x,y) = \frac{R^2F(x)}{(x-y)^2G(x)}\left(H(x)H^2(y)\right)^{N/3},$$

$$g_{\varphi\varphi}(x,y) = \frac{R^2G(x)}{(x-y)^2}\frac{(H(x)H^2(y))^{N/3}}{H^N(x)}, \tag{27}$$

which results in the metric (26) taking the same form as (4). At the horizon, the functions $\mathcal{M}(x,y), \mathcal{N}(x,y)$ and $N^\psi(x,y)$ still satisfy Eq.(5). Now substituting the matrices γ^a (7) and the tetrad e_a^μ (9) into the covariant Dirac equation (6), and then adopting the same procedure present in Sec.(II), one can reads out Hawking temperature of Dirac particles via

tunnelling from dipole black ring

$$T = \frac{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}{4\pi} = \frac{1}{4\pi R}\frac{\nu^{(N-1)/2}(1+\nu)}{(\mu+\nu)^{N/2}}\sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \tag{28}$$

This result has been identically derived by using Hamilton-Jacobi method [23] and anomalous cancellation method [24], where particles across the horizon are only for scalar cases. Note that dipole black ring actually contains a gauge field. Here we do not consider its effect because it is magnetic, and its electric dual are two-form fields that do not couple to point particles(see Chen and He's paper in [24]). In the next subsection, we will further study Hawking radiation of a rotating black ring with a single electric charge by using fermions tunnelling method.

B. Charged black ring

In this subsection, we consider Hawking radiation from black ring with only one single electric charge [28]. For black rings with two or three charges [29], we can take the similar procedure to get the correct results. The metric of black ring with a single electric charge can be written in the form consistent with neutral and dipole cases as

$$
\begin{aligned}
ds^2 &= -\frac{F(y)}{F(x)K^2(x, y)}\left(dt - C(\nu, \lambda)R\frac{1+y}{F(y)}\cosh^2\alpha d\psi\right)^2 \\
&+ \frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right],
\end{aligned} \tag{29}
$$

where some tricks are needed to reduce the original metric of black ring with a single electric charge to the form as (29) (refer to Chen and He's paper in [24]). Here $F(\xi)$ and $G(\xi)$ are defined as before, and $K(x, y) = 1 + \lambda(x - y)\sinh^2\alpha/F(x)$, where α is the parameter representing the electric charge. The metric also has a killing horizon at $y = y_h = -1/\nu$. The dilaton field is $e^{-\Phi} = K(x, y)$, and the gauge fields accompanied by the metric are

$$\mathcal{A}_t = \frac{\lambda(x - y)\sinh\alpha\cosh\alpha}{F(x)K(x, y)}, \quad \mathcal{A}_\psi = \frac{C(\nu, \lambda)R(1 + y)\sinh\alpha\cosh\alpha}{F(x)K(x, y)}, \tag{30}$$

with the electric charge $Q = 2M \sinh 2\alpha / \left(3\left(1 + \frac{4}{3}\sinh^2 \alpha\right)\right)$. To do an explicit computation on Hawking radiation of the black ring, we first introduce the following substitution

$$\mathcal{M}(x,y) = \frac{F(y)}{F(x)K^2(x,y)}\left(1 - \frac{C^2(\nu,\lambda)(1+y)^2(x-y)^2\cosh^4\alpha}{F^2(x)G(y)K^2(x,y) + C^2(\nu,\lambda)(1+y)^2(x-y)^2\cosh^4\alpha}\right),$$

$$\mathcal{N}(x,y) = -\left(\frac{R^2}{(x-y)^2}\frac{F(x)}{G(y)}\right)^{-1},$$

$$N^\psi(x,y) = -\frac{C(\nu,\lambda)R(1+y)F(y)(x-y)^2\cosh^2\alpha}{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2\cosh^4\alpha + R^2F^2(x)G(y)K^2(x,y)},$$

$$g_{\psi\psi}(x,y) = -\frac{C^2(\nu,\lambda)(x-y)^2R^2(1+y)^2\cosh^4\alpha + R^2F^2(x)G(y)K^2(x,y)}{F(x)F(y)(x-y)^2K^2(x,y)},$$

$$g_{xx}(x,y) = \frac{R^2F(x)}{(x-y)^2G(x)}, \quad g_{\varphi\varphi}(x,y) = \frac{R^2G(x)}{(x-y)^2}, \tag{31}$$

where at the event horizon $\mathcal{M}(x,y), \mathcal{N}(x,y)$ and $N^\psi(x,y)$ take the values in Eq.(5). Now the metric (29) has the same form as (4). In the spacetime, gauge fields (30) couple to Dirac particles, so we should introduce the following covariant Dirac equation

$$i\gamma^a e_a^\mu \left(D_\mu + \frac{ie}{\hbar}\mathcal{A}_\mu\right)\Psi - \frac{m}{\hbar}\Psi = 0. \tag{32}$$

Taking the same matrices γ^a and tetrad fields e_a^μ as those in Eqs.(7) and (9) for the black ring, and employing the ansatz (10) for the spin-up Dirac particles and then expanding the resulting equation near the horizon yields

$$B\left(\frac{-\mathcal{E} + \Omega_h\mathcal{J} + e\Phi_h}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}} + \sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y\mathcal{W}(x,y)\right)$$

$$+A\left(m - \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right) = 0, \tag{33}$$

$$B\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x\mathcal{W}(x,y) + \frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}(\mathcal{J} + \mathcal{A}_\psi(x,y_h))\right) = 0, \tag{34}$$

$$A\left(\frac{-\mathcal{E} + \Omega_h\mathcal{J} + e\Phi_h}{\sqrt{\mathcal{M}_{,y}(x,y_h)(y-y_h)}} - \sqrt{\mathcal{N}_{,y}(x,y_h)(y-y_h)}\partial_y\mathcal{W}(x,y)\right)$$

$$-B\left(m + \frac{\mathcal{L}}{\sqrt{g_{\varphi\varphi}(x,y_h)}}\right) = 0, \tag{35}$$

$$A\left(\frac{1}{\sqrt{g_{xx}(x,y_h)}}\partial_x\mathcal{W}(x,y) + \frac{i}{\sqrt{g_{\psi\psi}(x,y_h)}}(\mathcal{J} + \mathcal{A}_\psi(x,y_h))\right) = 0, \tag{36}$$

where $\Phi_h = \mathcal{A}_t(x,y_h) + \Omega_h\mathcal{A}_\psi(x,y_h)$ is the electric chemical potential at the horizon and Ω_h is the angular velocity at the horizon. Carrying on a similar analysis of the neutral black ring, we easily find the tunnelling rate of charged Dirac particles across the horizon of the

charged black ring taking the form as

$$\Gamma = \exp\left[-\frac{4\pi}{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}\Big(\mathcal{E} - \Omega_h \mathcal{J} - e\Phi_h\Big)\right]. \tag{37}$$

The Hawking temperature of the charged black ring is then given by

$$T = \frac{\sqrt{\mathcal{M}_{,y}(x, y_h)\mathcal{N}_{,y}(x, y_h)}}{4\pi} = \frac{1}{4\pi R \cosh^2 \alpha} \frac{1+\nu}{\sqrt{\nu}} \sqrt{\frac{1-\lambda}{\lambda(1+\lambda)}}. \tag{38}$$

This result is exactly consistent with Hawking temperature derived by cancelling gauge and gravitational anomalies at the horizon of the charged black ring (Chen and He's paper in [24]). Here to reduce the higher dimensional theory to the effective two dimensional theory, a dimensional reduction technique is carried out by using scalar field near the horizon of the charged black ring. So the resulting Hawking temperature is only for scalar particles across the horizon. Now we can also conclude that scalar and Dirac particles can tunnel across the horizon of black rings at the same Hawking temperature.

IV. CONCLUSIONS AND DISCUSSIONS

Hawking radiation of scalar particles across black holes or black rings have been discussed a lot via different methods, such as recently hot discussing tunnelling method, and anomalous cancellation method, etc. And Hawking radiation of Dirac particles across 3- or 4-dimensional black holes have also been presented in recent papers via fermions tunnelling method. In this paper, choosing a set of appropriate matrices γ^μ for the the 5-dimensional neutral, diploe and charged black rings, we successfully recover Hawking temperatures of these black rings via fermions tunnelling method.

Fermions tunnelling method has already been successfully applied to derive Hawking radiation of Dirac particles across stationary back holes[16, 17, 19, 20, 21] and black rings (appeared in this paper). For a non-stationary black hole, although [18] has discussed fermions tunnelling from Barddeen-Vaidya and cosmological black holes, there is no coupling effect between the spin of Dirac particles and the angular momentum of the black hole in the tunnelling rate. That is because the involved non-stationary black holes in [18] is of spherical symmetry, and have no angular momentum for themselves. So we expect when Dirac particles tunnelling from non-stationary black holes with one or more angular momentum, the spin coupling effect will be present. This is our next job. In addition, noted that

choosing a set of appropriate matrices γ^μ is a important technique for fermions tunnelling method, or we can not correctly recover Hawking temperature we expected. Finally, it is need to say that, in Sec.(II) and (III), we only consider the case of Dirac particles with spin up. In fact, adopting a similar procedure, we will find the same result for Dirac particles with spin down. That meas both spin up and spin down Dirac particles tunnel across the horizon at the same Hawking temperature[20], and such handling does not loss its generality.

[1] S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[2] G. W. Gibbons and M.J. Perry, Proc. R. Soc. Lond. **A358** (1978) 467.

[3] J.B. Hartle and S.W. Hawking, Phys. Rev. **D13** (1976) 2188.

[4] G. W. Gibbons and S. W. Hawking, Phys. Rev. **D15** (1977) 2738.

[5] D. Boulware, Phys. Rev. **D11** (1975) 1404.

[6] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, (Cambridge University Press1982).

[7] Z. H. Li and Z. Zhao, Chin. Phys. Lett. **10** (1993) 126.

[8] S. Q. Wu and X. Cai, Gen. Rel. Grav. **33** (2001) 1181; Gen. Rel. Grav. **34** (2002) 1207; Q. Q. Jiang and S. Z. Yang, Chin. Phys. Lett **23** (2006) 3169.

[9] P. Kraus and F. Wilczek, Nucl. Phys. **B433** (1995) 403; Nucl. Phys. **B437** (1995) 231.

[10] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85** (2000) 5042; M. K. Parikh, Phys. Lett. **B546** (2002) 189; Int. J. Mod. Phys. **D13** (2004) 2351.

[11] S. Hemming and E. Keski-Vakkuri, Phys. Rev. **D64**, 044006 (2001); A. J. M. Medved, Phys. Rev. **D66** (2002) 124009; Class. Quant. Grav. **19** (2002) 589; E. C. Vagenas, Phys. Lett. **B503** (2001) 399; Phys. Lett. **B559** (2003) 65; Mod. Phys. Lett. **A17** (2002) 609.

[12] A. J. M. Medved and E. C. Vagenas, Mod. Phys. Lett. **A20** (2005) 1723; Mod. Phys. Lett.

A20 (2005) 2449; M. Arzano, A. J. M. Medved and E. C. Vagenas, JHEP. **0509** (2005) 037; J. Y. Zhang and Z. Zhao, Phys. Lett. **B618** (2005) 14; JHEP **0505** (2005) 055; Q. Q. Jiang and S. Q. Wu, Phys. Lett. **B635** (2006) 151; Q.Q. Jiang, S.Q. Wu and X. Cai, Phys. Rev. **D73** (2006) 064003; S. Q. Wu and Q. Q. Jiang, JHEP. **0603** (2006) 079; Y. P. Hu, J. J. Zhang and Z. Zhao, Mod. Phys. Lett. **A21** (2006) 2143; S. P. Kim, JHEP. **0711** (2007) 048; S. Sarkar and D. Kothawala, Phys. Lett. **B659** (2008) 683; T. Pilling, Phys. Lett. **B660** (2008) 402; M. H. Ali, Class. Quant. Grav. **24** (2007) 5849; R. Kerner and R. B. Mann, Phys. Rev. **D75** (2007) 084022; C. Z. Liu, J. Y. Zhang and Z. Zhao, Phys. Lett. **B639** (2006) 670; W. B. Liu, Phys. Lett. **B634** (2006) 541; P. Mitra, Phys. Lett. **B648** (2007) 240.

[13] M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, J. High Energy Phys. **05** (2005) 014; M. Nadalini, L. Vanzo and S. Zerbini, J. Phys. A **39** (2006) 6601; R. di. Criscienzo, M. Nadalini, L. Vanzo, S. Zerbini and G. Zoccatelli, Phys. Lett. B **657** (2007) 107.

[14] K. Srinivasan and T. Padmanabhan, Phys. Rev. **D60** (1999) 024007; S. Shankaranarayanan, K. Srinivasan, and T. Padmanabhan, Mod. Phys. Lett. **A16** (2001) 571; S. Shankaranarayanan, T. Padmanabhan, and K. Srinivasan, Class. Quant. Grav. **19** (2002) 2671.

[15] R. Kerner and R. B. Mann, Phys. Rev. **D73** (2006) 104010.

[16] R. Kerner and R. B. Mann, Class. Quant. Grav. **25** (2008) 095014.

[17] R. Li and J. R. Ren, Phys. Lett. **B661** (2008) 370.

[18] R. di. Criscienzo and L. Vanzo, Europhys. Lett. **82** (2008) 60001.

[19] R. Li, J. R. Ren and S. W. Wei, Class. Quant. Grav. **25** (2008) 125016.

[20] R. Kerner and R. B. Mann, Phys. Lett. **B** (2008) (in press), arXiv: 0803.2246 [hep-th].

[21] D.Y. Chen, Q.Q. Jiang, and X. T. Zu, Phys. Lett. **B665** (2008) 106; D.Y. Chen, Q.Q. Jiang, S.Z. Yang, and X.T. Zu, arXiv: 0803.3248 [hep-th].

[22] D. Brill, Helv. Phys. Acta. **69** (1996) 249; R. B. Mann, Class. Quant. Grav. **14** (1997) L109; L. Vanzo, Phys. Rev. **D56** (1997) 6475.

[23] L. Zhao, arXiv: hep-th/0602065.

[24] U. Miyamoto and K. Murata, Phys. Rev. **D77** (2008) 024020; B. Chen and W. He, Classical Quantum Grav. **25**(2008) 135011.

[25] R. Emparan, JHEP. **03** (2004) 064.

[26] H. Elvang and B. Zumino, JHEP. **11** (2003) 035.

[27] R.C. Myers and M.J. Perry, Ann. Phys. (N.Y.) **172**, 304 (1986).

[28] H. Elvang, Phys. Rev. **D68** (2003) 124016.

[29] H. Elvang and R. Emparan, JHEP. **11** (2003) 035.