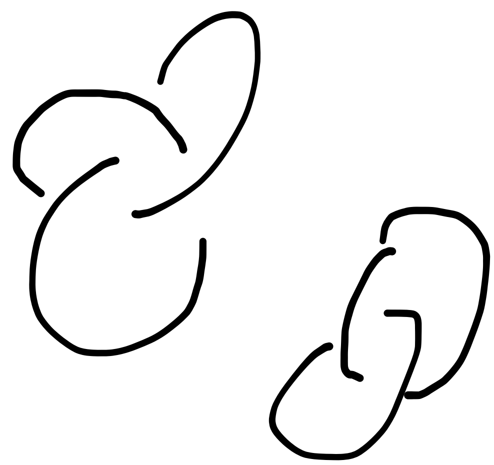
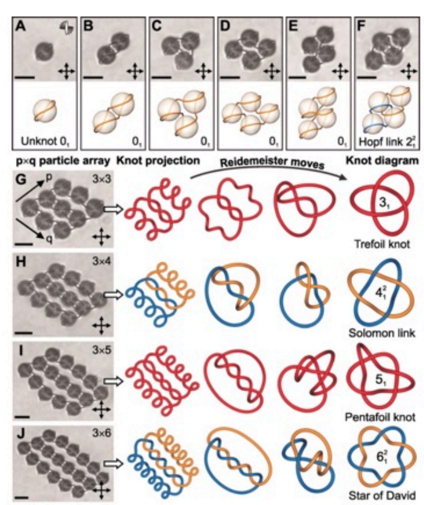
Knots and Links

Can we knot or link disclinations?

Yes (in nematics)





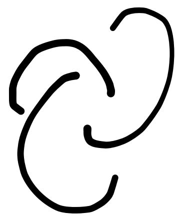
U. Tkalec *et al*. Science **333**, 6265 (2011).

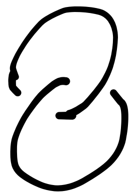
Interaction with π_2 leads to connection with Alexander Polynomial

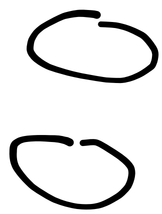
p \q	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	3	2^2	3	1	$1 \times \mathbb{Z}^2$	1	3	2^2	3	1	$1 \times \mathbb{Z}^2$	1	3	2^2	3	1	$1 \times \mathbb{Z}^2$	1	3
4	4	3	$2 \times \mathbb{Z}^2$	5	12	7	$4 \times \mathbb{Z}^2$	9	20	11	$6 \times \mathbb{Z}^2$	13	28	15	$8 \times \mathbb{Z}^2$	17	36	19	$10 \times \mathbb{Z}^2$
5	5	1	5	2^4	5	1	5	1	$1 \times \mathbb{Z}^4$	1	5	1	5	2^4	5	1	5	1	$1 \times \mathbb{Z}^4$
6	6	$1 \times \mathbb{Z}^2$	12	5	$2 \times \mathbb{Z}^4$	7	24	$3 \times \mathbb{Z}^2$	30	11	$4 \times \mathbb{Z}^4$	13	42	$5 \times \mathbb{Z}^2$	48	17	$6 \times \mathbb{Z}^4$	19	60
7	7	1	7	1	7	2^6	7	1	7	1	7	1	$1 \times \mathbb{Z}^6$	1	7	1	7	1	7
8	8	3	$4 \times \mathbb{Z}^2$	5	24	7	$2 \times \mathbb{Z}^6$	9	40	11	$12 \times \mathbb{Z}^2$	13	56	15	$4 \times \mathbb{Z}^6$	17	72	19	$20 \times \mathbb{Z}^2$
9	9	2^2	9	1	$3 \times \mathbb{Z}^2$	1	9	2^8	9	1	$3 \times \mathbb{Z}^2$	1	9	2^2	9	1	$1 \times \mathbb{Z}^8$	1	9
10	10	3	20	$1 \times \mathbb{Z}^4$	30	7	40	9	$2 \times \mathbb{Z}^8$	11	60	13	70	$3 \times \mathbb{Z}^4$	80	17	90	19	$4 \times \mathbb{Z}^8$
11	11	1	11	1	11	1	11	1	11	2^{10}	11	1	11	1	11	1	11	1	11
12	12	$1 \times \mathbb{Z}^2$	$6 \times \mathbb{Z}^2$	5	$4 \times \mathbb{Z}^4$	7	$12 \times \mathbb{Z}^2$	$3 \times \mathbb{Z}^2$	60	11	$2 \times \mathbb{Z}^{10}$	13	84	$5 \times \mathbb{Z}^2$	$24 \times \mathbb{Z}^2$	17	$12 \times \mathbb{Z}^4$	19	$30 \times \mathbb{Z}^2$
13	13	1	13	1	13	1	13	1	13	1	13	2^{12}	13	1	13	1	13	1	13
14	14	3	28	5	42	$1 \times \mathbb{Z}^6$	56	9	70	11	84	13	$2 \times \mathbb{Z}^{12}$	15	112	17	126	19	140
15	15	2^2	15	2^4	$5 \times \mathbb{Z}^2$	1	15	2^2	$3 \times \mathbb{Z}^4$	1	$5 \times \mathbb{Z}^2$	1	15	2^{14}	15	1	$5 \times \mathbb{Z}^2$	1	$3 \times \mathbb{Z}^4$
16	16	3	$8 \times \mathbb{Z}^2$	5	48	7	$4 \times \mathbb{Z}^6$	9	80	11	$24 \times \mathbb{Z}^2$	13	112	15	$2 \times \mathbb{Z}^{14}$	17	144	19	$40 \times \mathbb{Z}^2$
17	17	1	17	2	17	1	17	1	17	1	17	1	17	1	17	2^{16}	17	1	17
18	18	$1 \times \mathbb{Z}^2$	36	5	$6 \times \mathbb{Z}^4$	7	72	$1 \times \mathbb{Z}^8$	90	11	$12 \times \mathbb{Z}^4$	13	126	$5 \times \mathbb{Z}^4$	144	17	$2 \times \mathbb{Z}^{16}$	19	180
19	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	1	19	2^{18}	19
20	20	3	$10 \times \mathbb{Z}^2$	$1 \times \mathbb{Z}^4$	60	7	$20 \times \mathbb{Z}^2$	9	$4 \times \mathbb{Z}^8$	11	$30 \times \mathbb{Z}^2$	13	140	$3 \times \mathbb{Z}^4$	$40 \times \mathbb{Z}^2$	17	180	19	$2 \times \mathbb{Z}^{18}$

Can knotted defects exist in other systems?

$$\pi_1(\mathbb{R}^3 \setminus K) = G_K$$ knot group

e.g. if $K =$  $G_K = B_3$ Braid Group

$K =$  $G_K = \mathbb{Z}^2$

$K =$  $G_K = F_2$ Free group on 2 letters

Now add a GSM X.

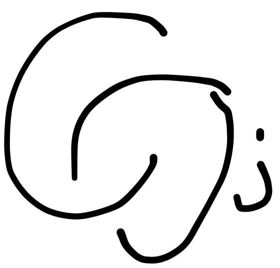
Texture defines a map

$$\phi : G_K \to \pi_1(X)$$

s.t.

$$\phi(ab) = \phi(a)\phi(b)$$

exercise: show that

i  j cannot occur in biaxials