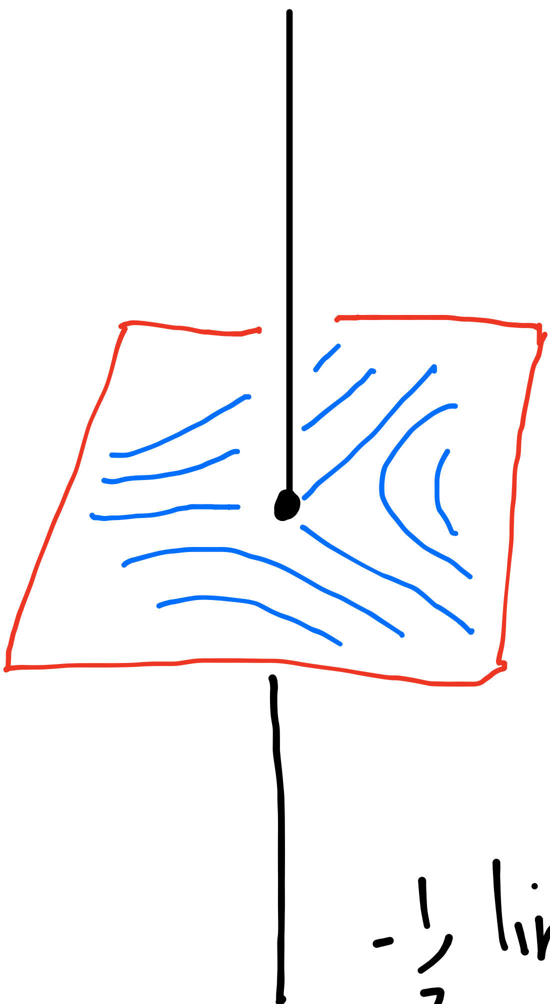
S. Čopar and S. Žumer, Proc. R. Soc. A, 469, 1471-2946 (2013).

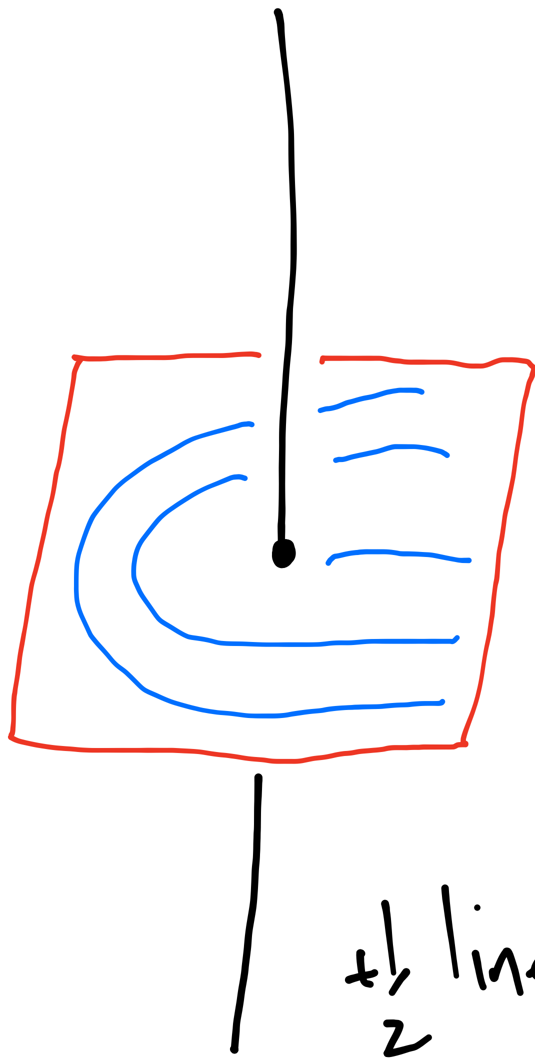
3D Nematics

Points → Lines

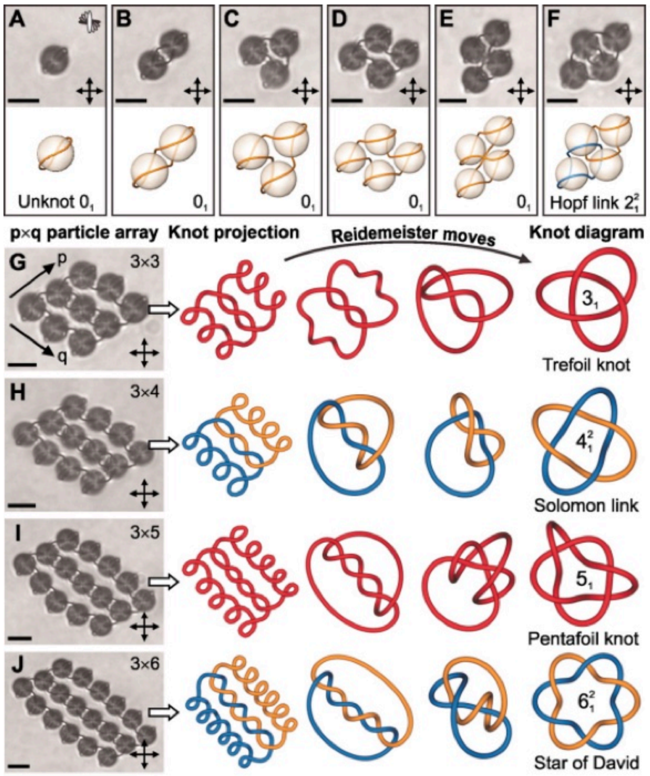
Disclinations

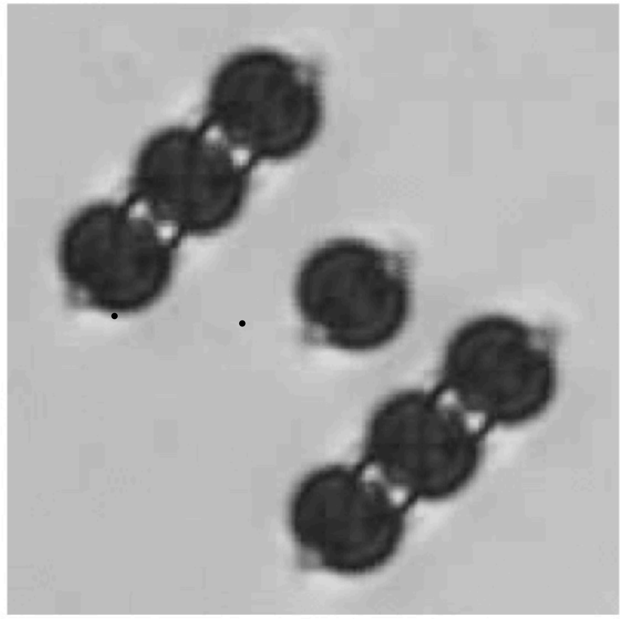


$-\frac{1}{2}$ line



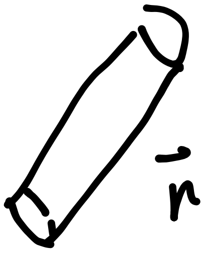
$+\frac{1}{2}$ line

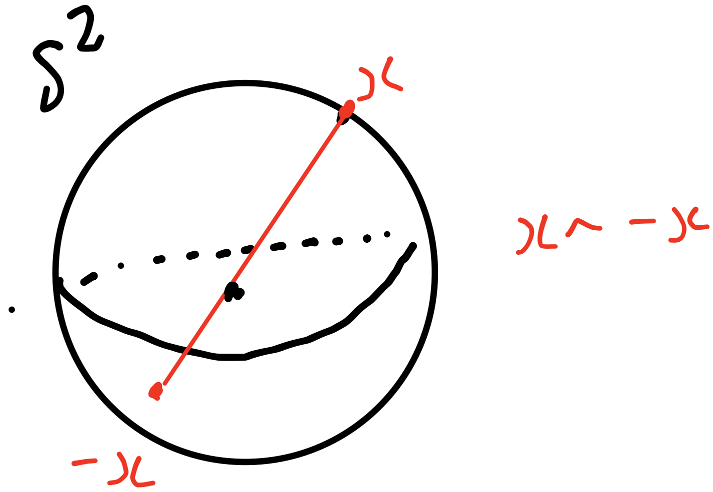




U. Tkalec *et al*. Science 333, 6265 (2011).

What is GSM for 3D nematics?

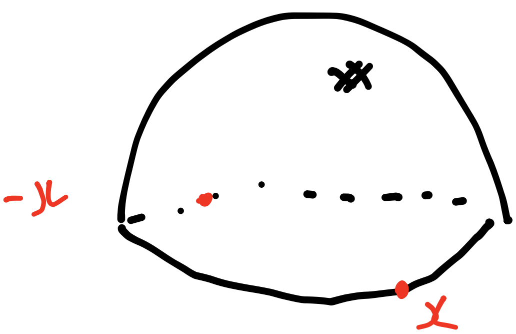




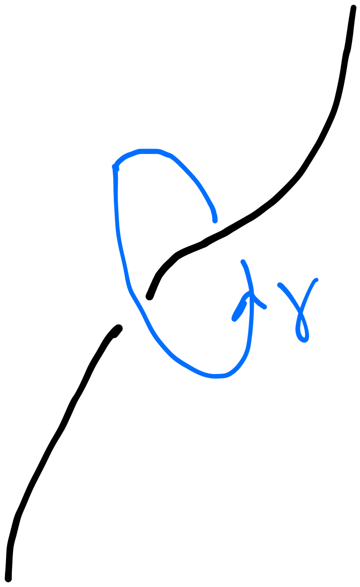
$$S^2/x \sim -x = \mathbb{RP}^2$$

real projective plane

View $\mathbb{RP}^2$ as a hemisphere



A line defect in a nematic

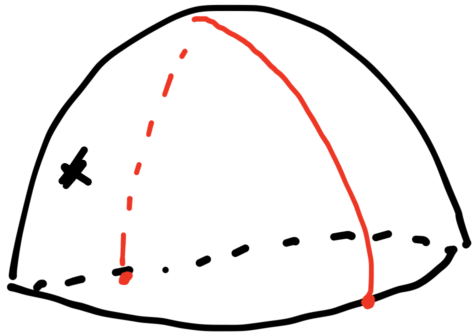


$n|_\gamma$ defines an element of $\pi_1(\mathbb{RP}^2)$

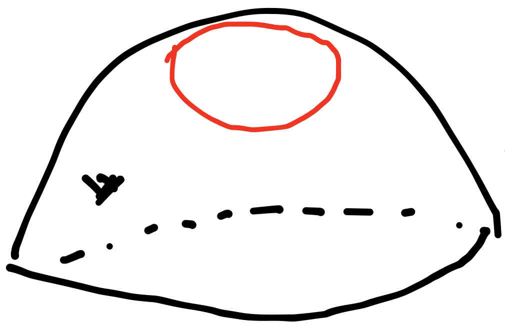
$\pi_1(\mathbb{RP}^2)$ classifies profiles of line defects

$$\pi_1(\mathbb{RP}^2) = \mathbb{Z}_2$$

$\mathbb{Z}_2$ is $0, 1$ w/ addition mod 2.



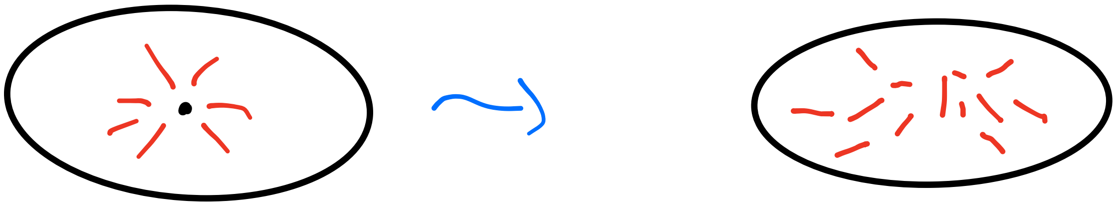
non-contractible loop $1 \in \mathbb{Z}_2$



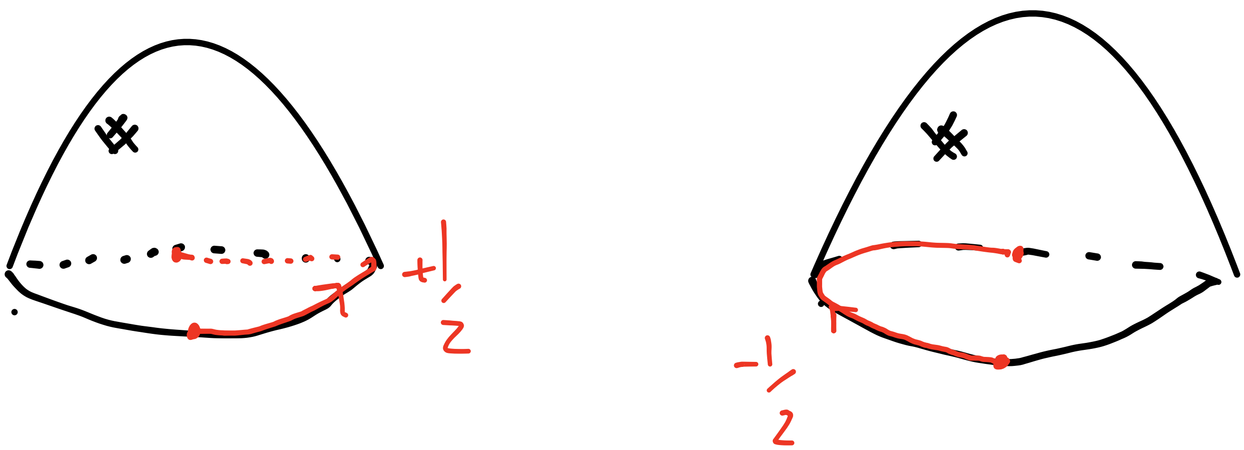
contractible loop $0 \in \mathbb{Z}_2$

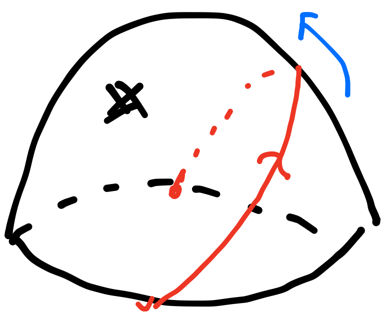
Exercise: turn a +1 line into
a non singular line
(escape in 3D)

turn a $+\frac{1}{2}$ line into $-\frac{1}{2}$ line

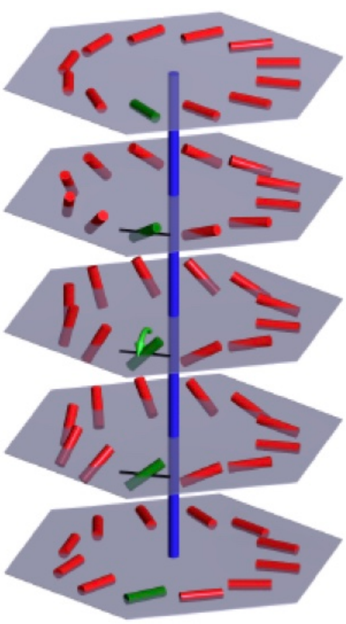


escape in 3D





twist disclination



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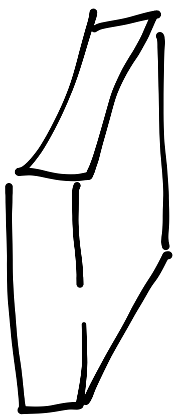
Biaxial nematics

 ← Brick

symmetry of π rotation about each axis.

GSM : $SO(3) \Big/ \text{symmetries}$

3D rotation group

in this case $\pi_1(GSM) = Q_8$

Q_8 is $(1, i, j, k)$

w/ $i^2 = j^2 = k^2 = ijk = -1$ $\quad ij = k$

$ji = -k.$

Table:

	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

4 kinds of line defect:

$(i, -i)$ $\quad (j, -j)$ $\quad (k, -k)$ $\quad -1.$

This group is not commutative
abelian

$$ij \neq ji$$

$\Rightarrow$ has interesting consequences for knots and links