Hopf Solitons

Suppose we have a nematic in $\mathbb{R}^3$

with $\lim_{|\vec{x}| \to \infty} n(\vec{x}) = (0,0,1)$

and no defects

the 1-point compactification

gives us a map

$$\vec{n} : S^3 \to \mathbb{RP}^2$$

3-sphere, unit sphere in four dimensions

Topological Classes Given by

$$\pi_3(\mathbb{RP}^2) = \pi_3(S^2) = \mathbb{Z}$$

Hopf charge

Formula: $A_i = e_1^a \partial_i e_2^a$

where $\vec{e}_1, \vec{e}_2$ are v. fields w/

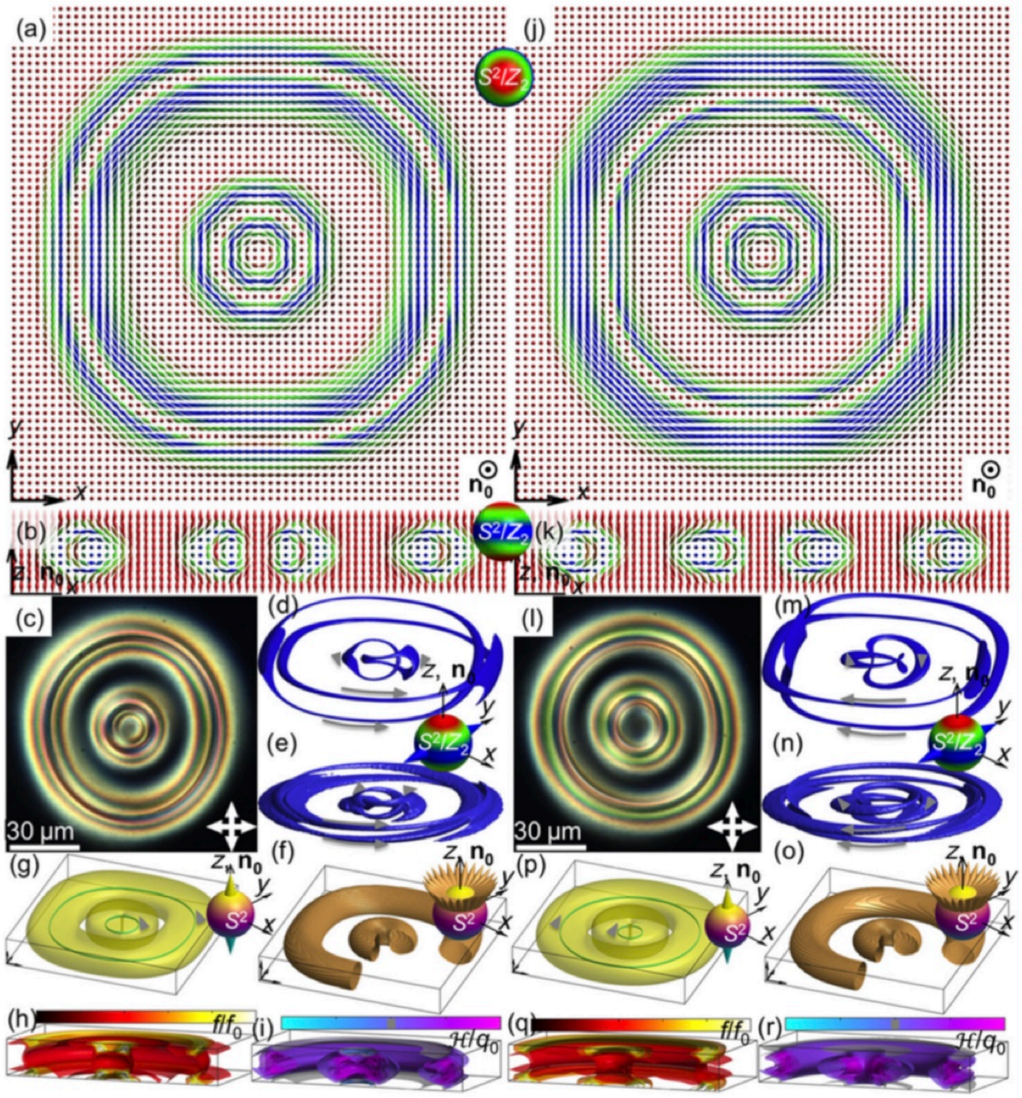
$$\vec{e}_1 \times \vec{e}_2 = \vec{n}$$

Hopf charge

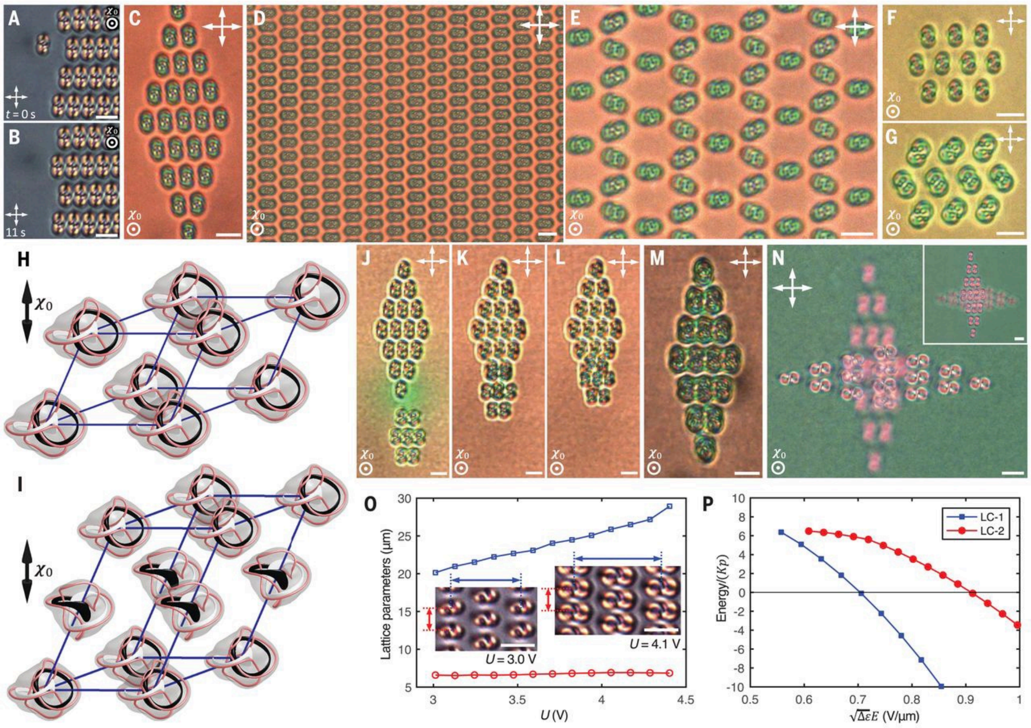
Then

$$\frac{1}{4\pi^2} \int_{\mathbb{R}^3} A \cdot \nabla \times A \, dV = Q$$

What do Hopf solitons look like?



P.J. Ackerman and I. I. Smalyukh Phys. Rev. X 7, 011006 (2017)



JS Tai, II Smalyukh, Science (2019)

(video)

How can we compute Hopf charge?

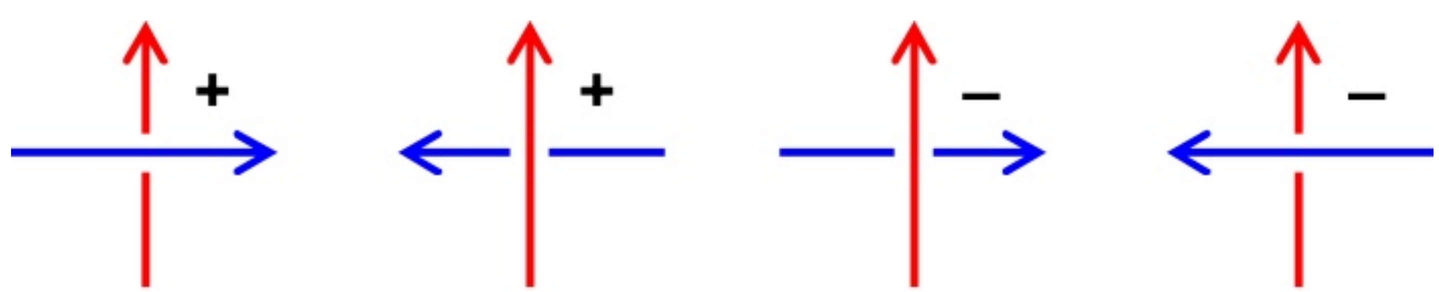
Linking of pre-images

$n^{-1}(a, b, c)$ ← curve in sample

e.g. where $n = (1, 0, 0)$

($(0, 0, 1)$ is a bad choice)

Then compute linking no.



exercise : use poraview