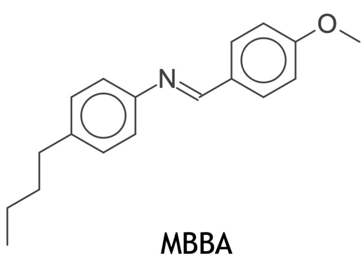
Topology in Liquid Crystals

EUTOPIA SUMMER SCHOOL 22.

Lecturer: Tom Machon (Bristol).

What is a LC?



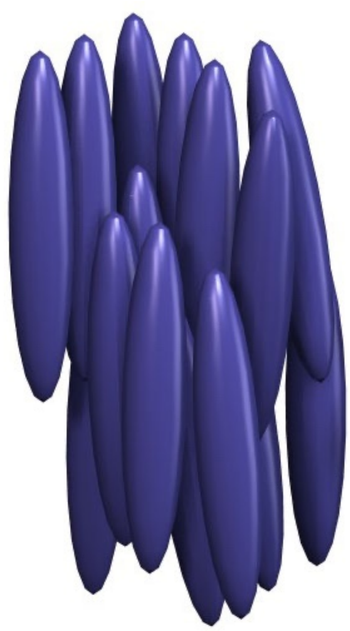
MBBA

High temperature: Liquid

43°C

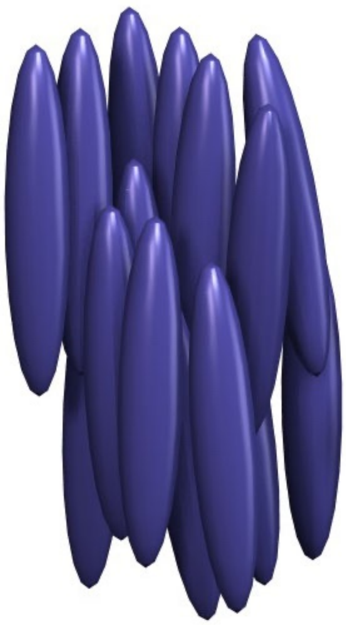
Low temperature: Nematic

Non-zero average (apolar) alignment

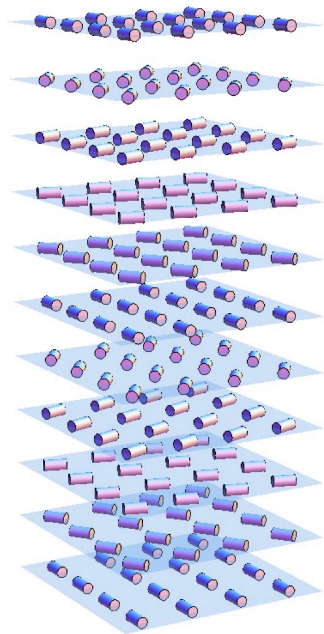


Many types:

nematic



cholsteric (chiral nematic)



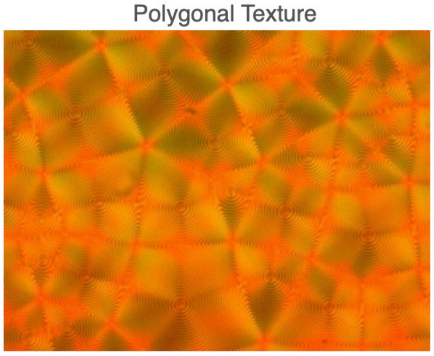
smectic (layers)

plus more

we focus on nematics

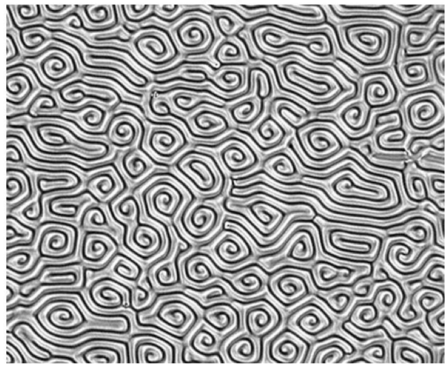
Cholesteric Textures

Polygonal Texture



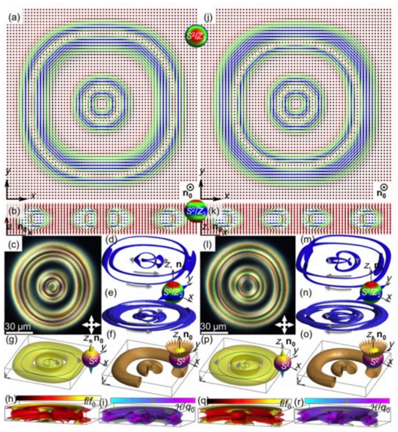
http://www.personal.kent.edu/~bisenyuk/

Fingerprint



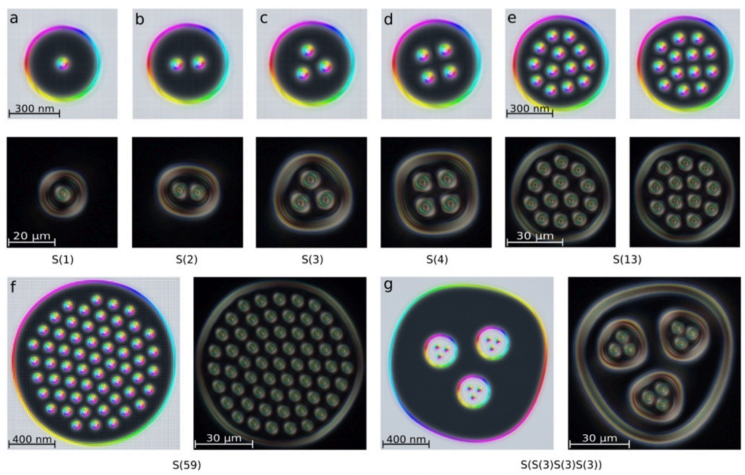
http://www.personal.kent.edu/~bisenyuk/

Hopf Solitons

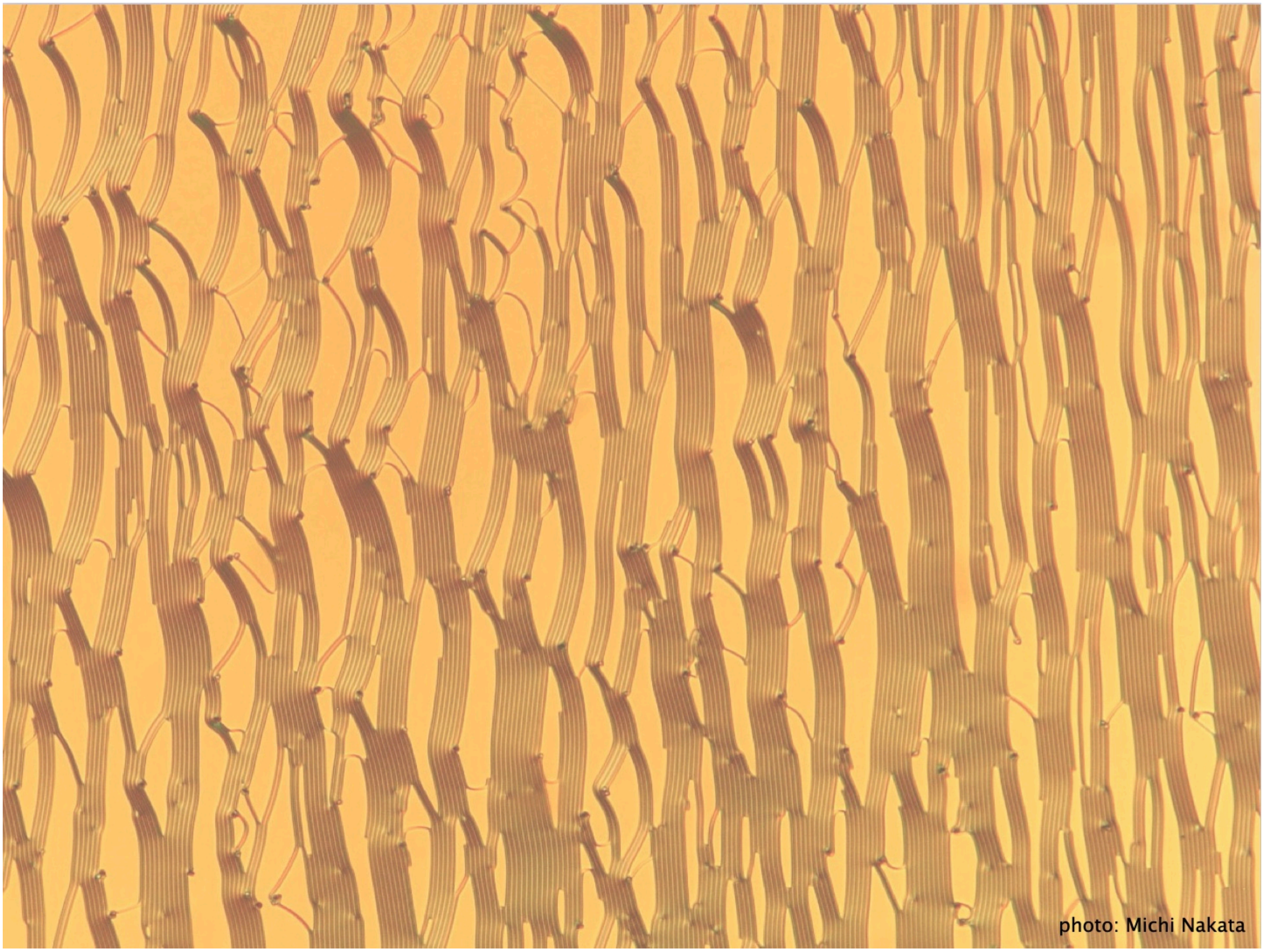


P.J. Ackerman and I. I. Smalyukh Phys. Rev. X **7**, 011006 (2017)

“Skyrmion Bags”

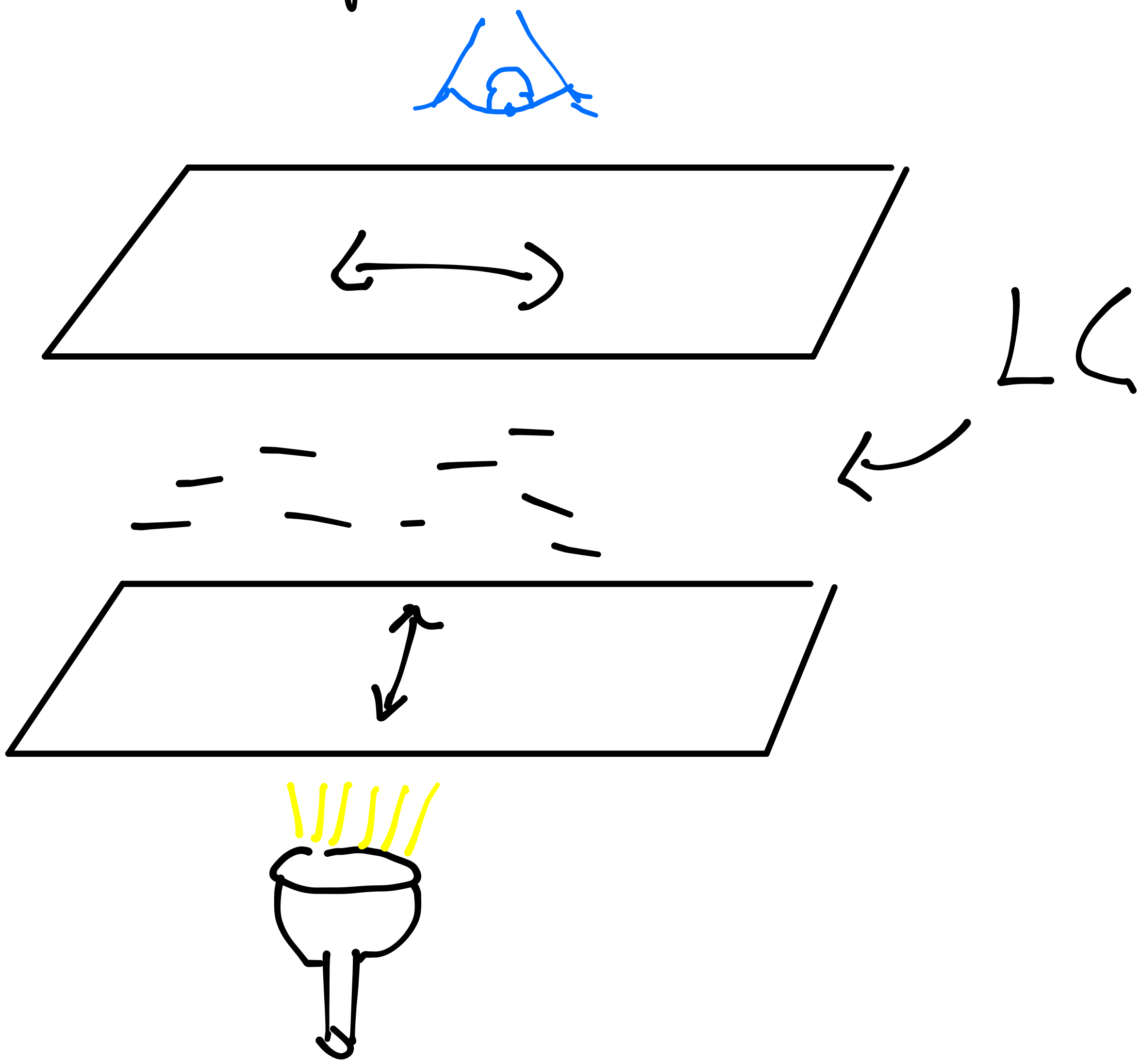


D. Foster et al. Nature Physics (2019).



+ active. (videos)

Classic LC object: Schlieren Texture

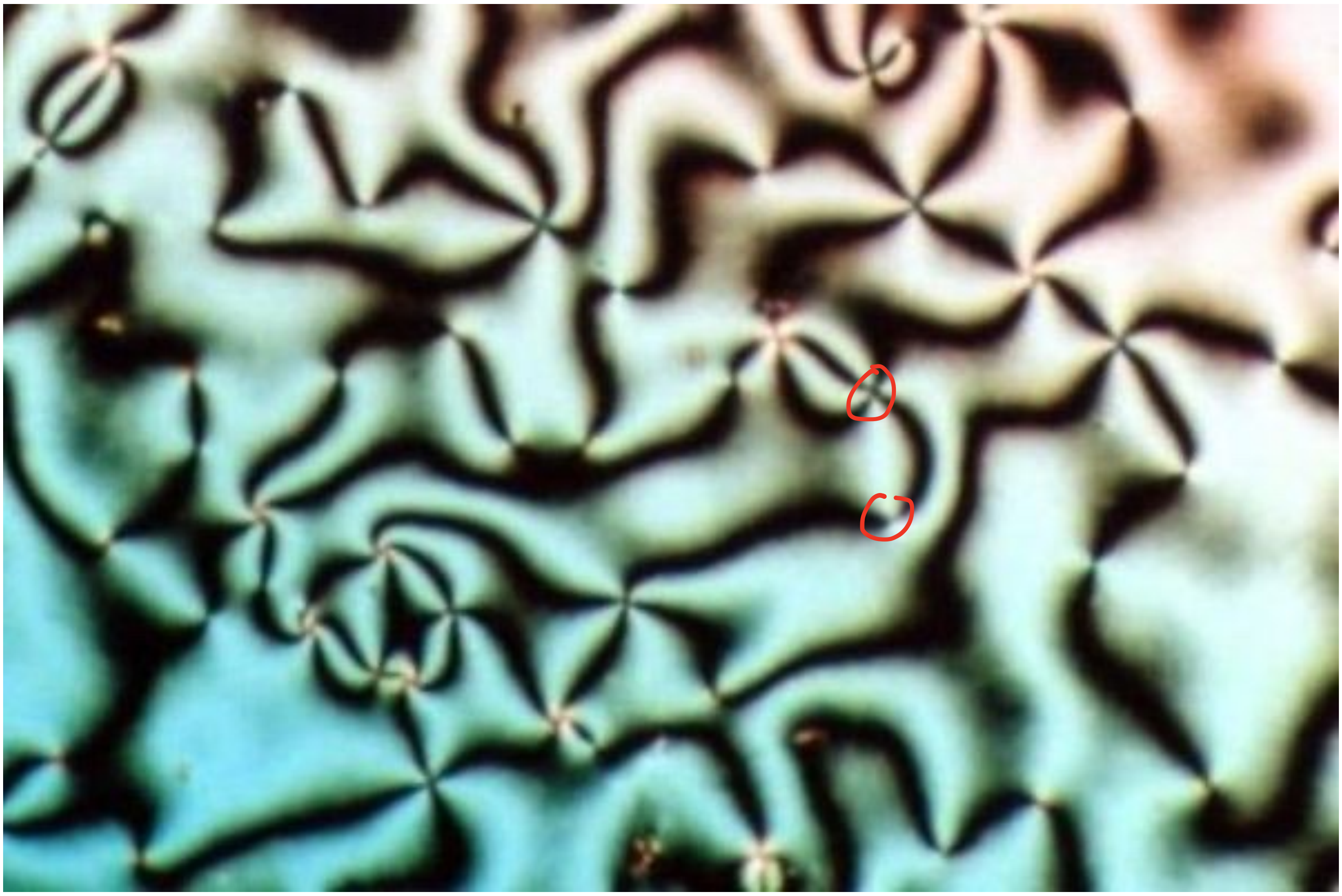


No LC $\rightarrow$ no light

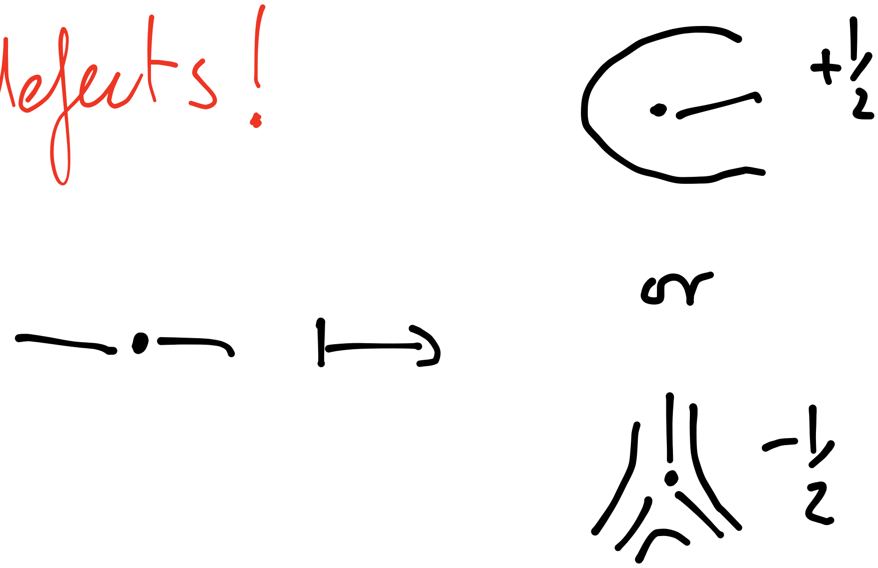
LC $\updownarrow$ $\rightarrow$ no light

LC $\leftrightarrow$ $\rightarrow$ no light

LC ⤢ or → Light!



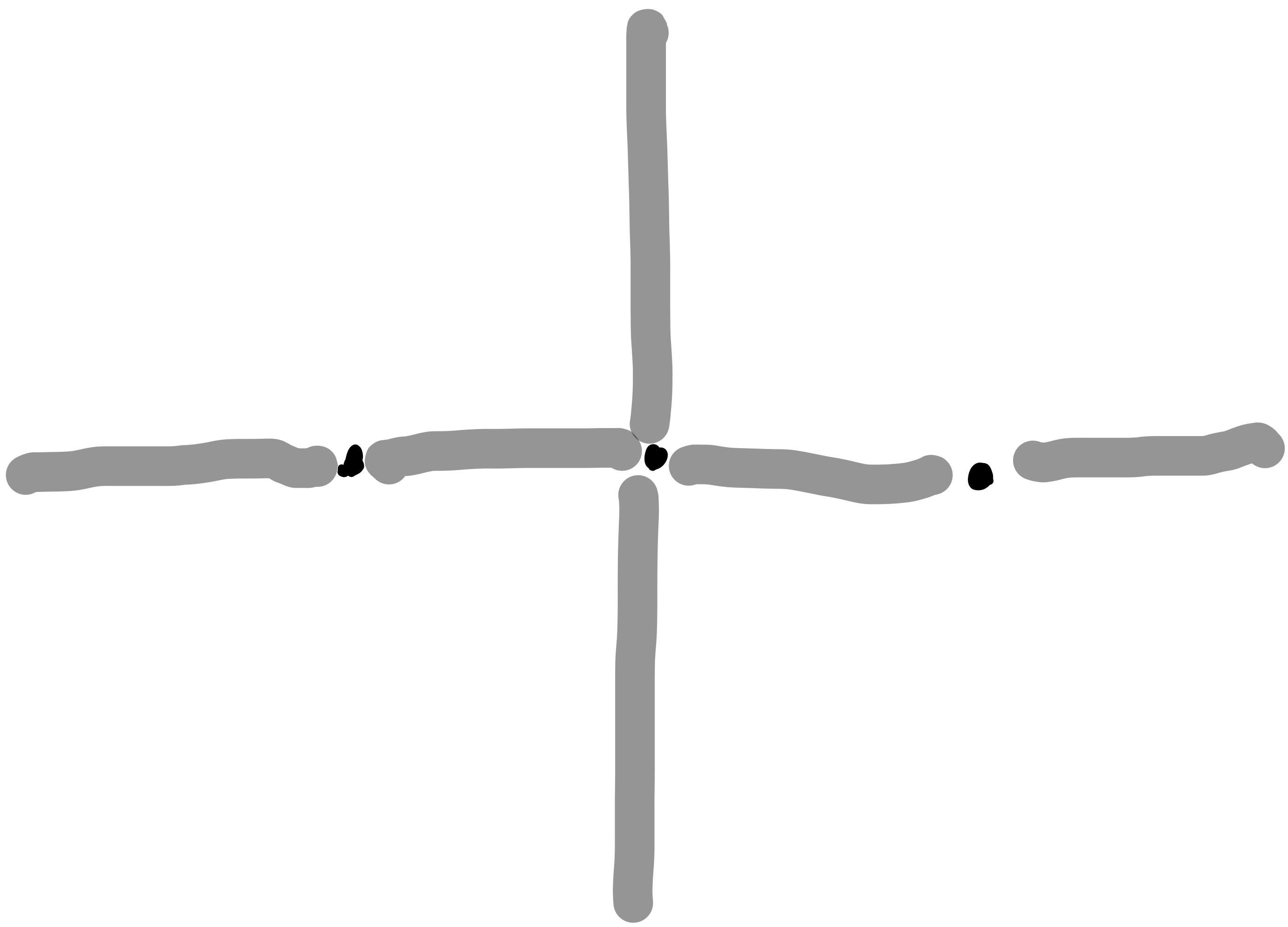
○ — defects!

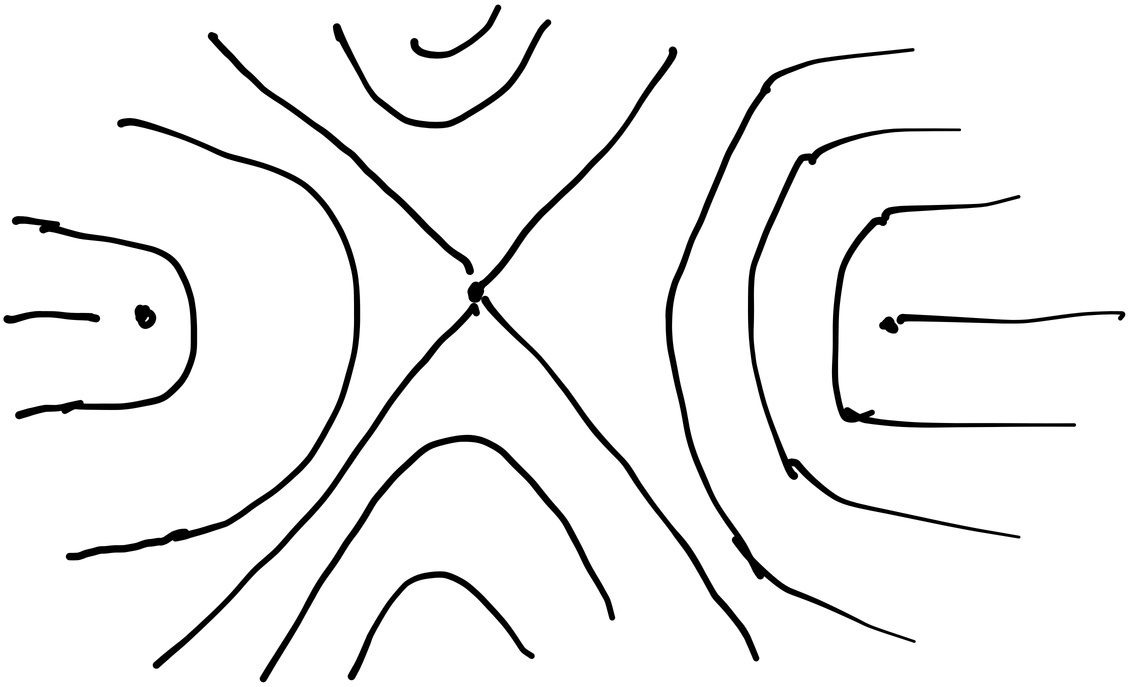


<u>Exercise</u>

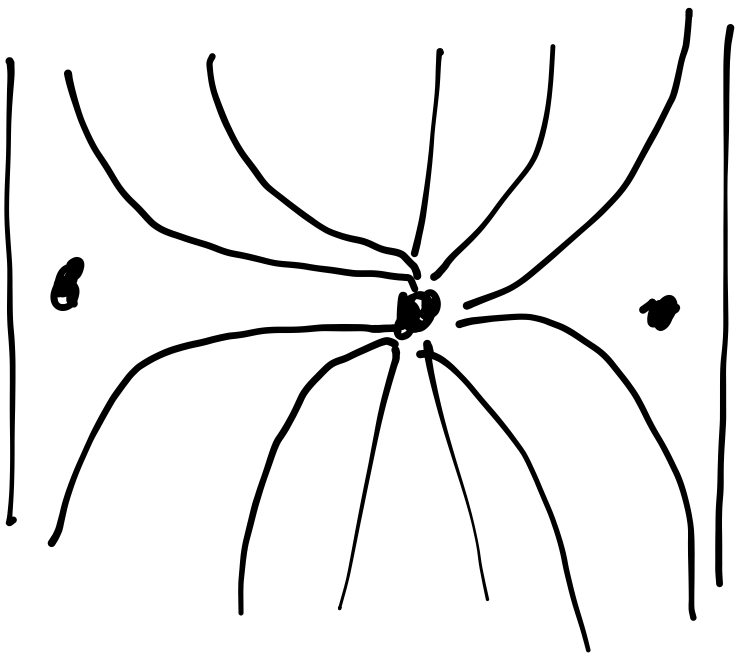
Reconstruct nematic orientation

for:

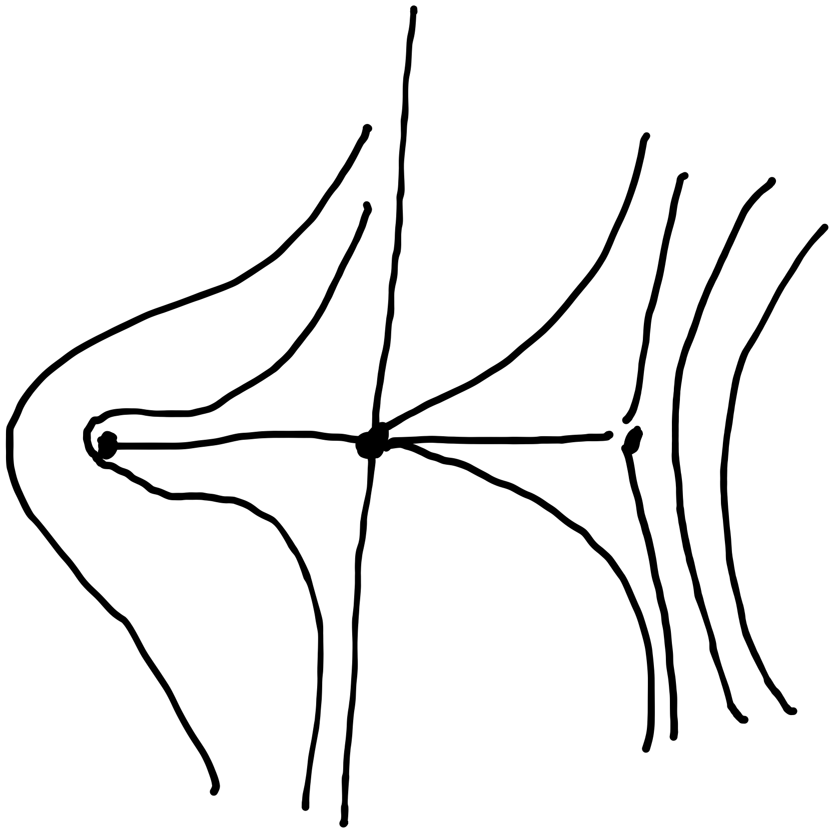




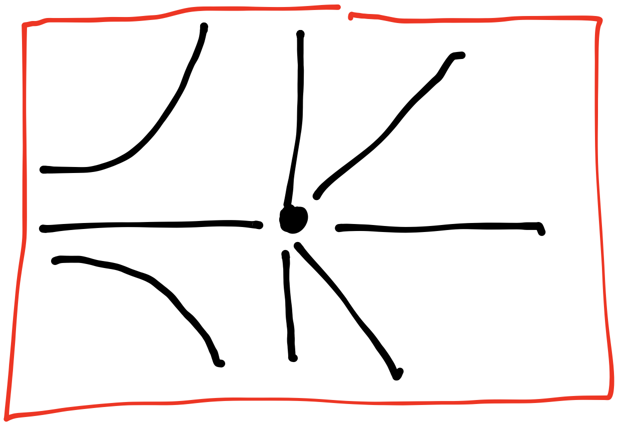
or

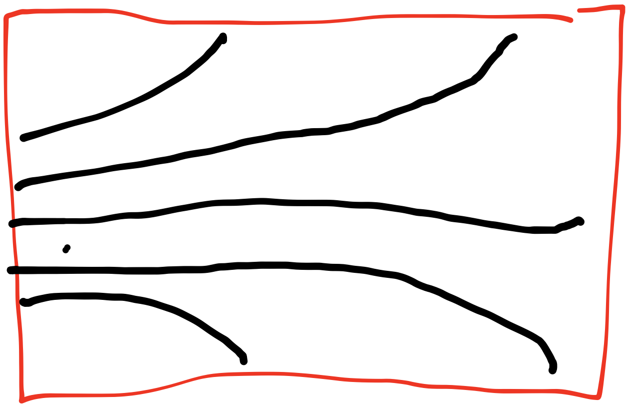


or



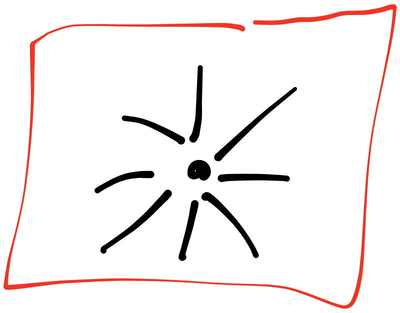
what's wrong?



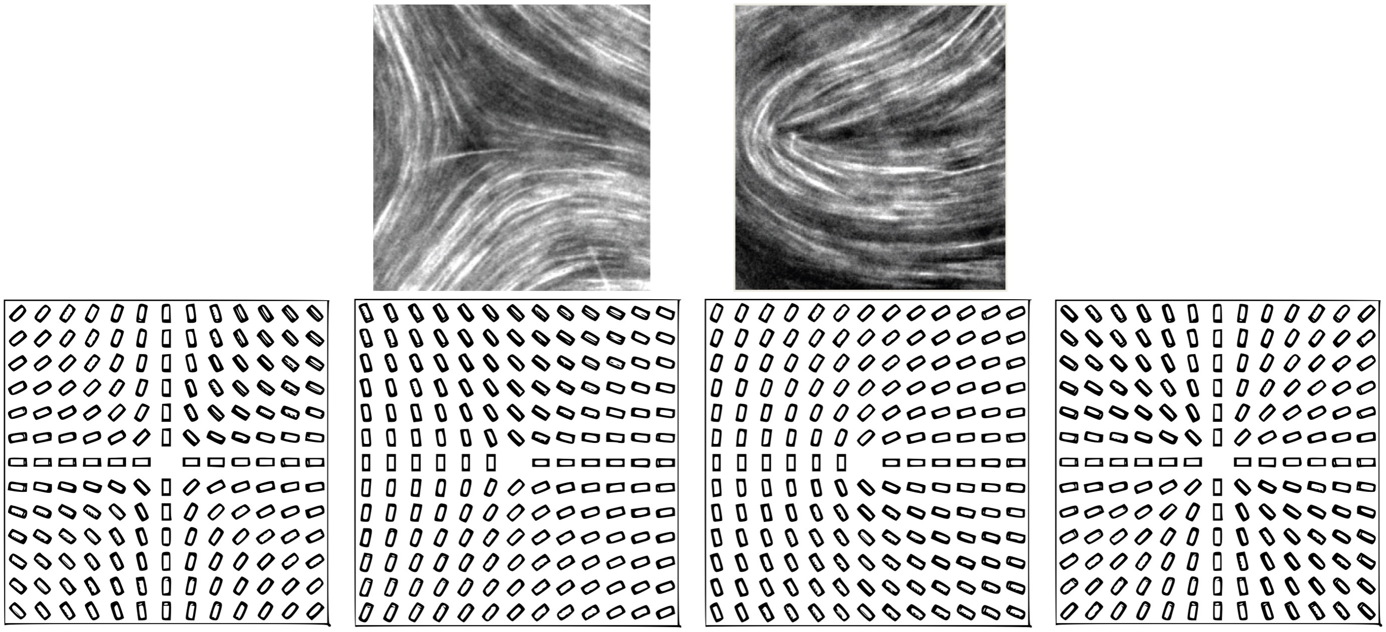


can be removed by local change

not topological!



← Topological



-1 $\qquad -\frac{1}{2}$ $\qquad +\frac{1}{2}$ $\qquad +1$

S J. DeCamp, *et al.*, Nature **491**, 431-434 (2012).

How to measure winding?

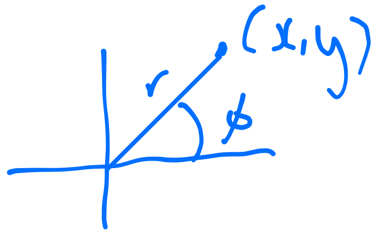
Describe nematic with unit v. field $\vec{n} = (n_x, n_y)$

$$\vec{n}\cdot\vec{n} = n_i n_i = n_x^2 + n_y^2 = 1 \quad \text{(unit magnitude)}$$

$\vec{n} \sim -\vec{n}$ (apolar)

called director

Defect of charge q



$\phi = \arctan \frac{y}{x}$

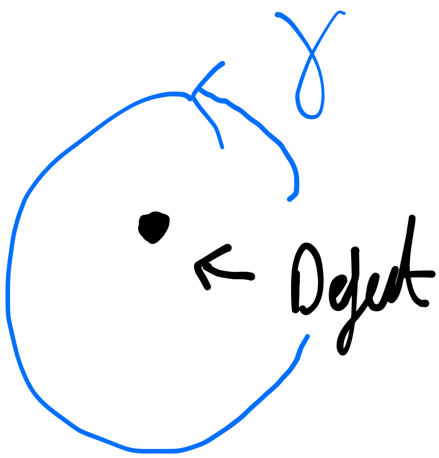
$$\vec{n} = \left(\cos(q\phi + \phi_0), \sin(q\phi + \phi_0)\right)$$

$q \in \mathbb{Z}/2$ is charge.

in general $\vec{n} = \left(\cos\theta(x,y), \sin\theta(x,y)\right)$

$$\varepsilon^{ab} n_a \partial_i n_b = \partial_i \Theta \quad \leftarrow \text{local rotation}$$

$$\oint_\gamma \partial_i \Theta \cdot dl = 2\pi q$$



exercise: compute charge for

$$\vec{n} = \left(\cos(q\phi+\phi_0), \sin(q\phi+\phi_0)\right)$$

sol:

$$\partial_i \Theta \text{ or } \vec{\nabla}\Theta = q\,\frac{(-y,x)}{x^2+y^2} = \frac{q}{r}\,\hat{e}_\theta$$

$$\vec{\nabla}\Theta = q\,\vec{\nabla}\phi$$

$$\vec{\nabla}\phi = \frac{1}{r}\,\hat{e}_\theta$$

$$\oint \nabla\Theta\cdot d\ell = \oint \frac{q}{r}\, r\, d\theta = q\oint d\theta = q\,2\pi$$

Think about $\vec{n}|_\gamma$

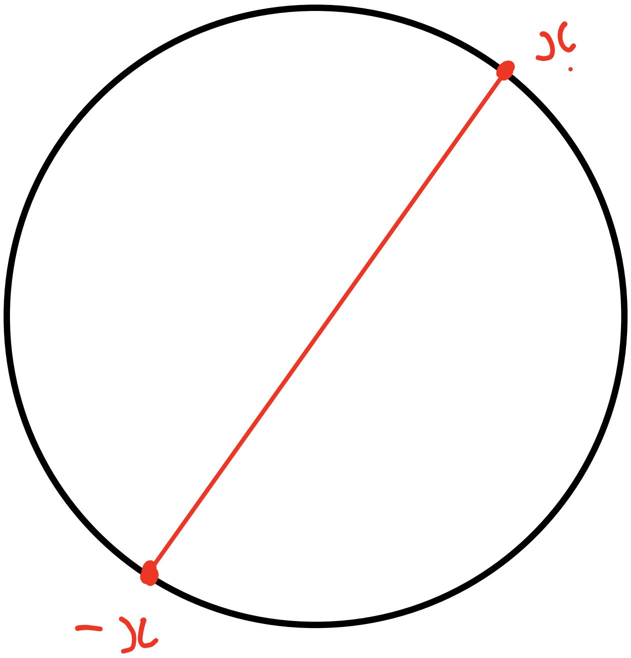
$\vec{n}_1|_\gamma$ can be deformed into $\vec{n}_2|_\gamma$

homotopic

if and only if winding no. of n_1 = winding no. of n_2.

Let's formalise this

where does $\vec{n}$ live?



$x, -x$ define same nematic orientation

So space of nematic orientations is

$S^1/_{x \sim -x}$ $\qquad S^1 \leftarrow$ unit circle.

$\simeq \mathbb{RP}^1 \leftarrow$ real projective line

$\mathbb{RP}^1$ is the ground state manifold (GSM) of 2D nematics.

a nematic defines a map

$$\vec{n} : (\Omega \setminus D) \to \mathbb{RP}^1$$

Ω: nematic domain (e.g. cell)

D: defect set

we need to analyse topology of this map.