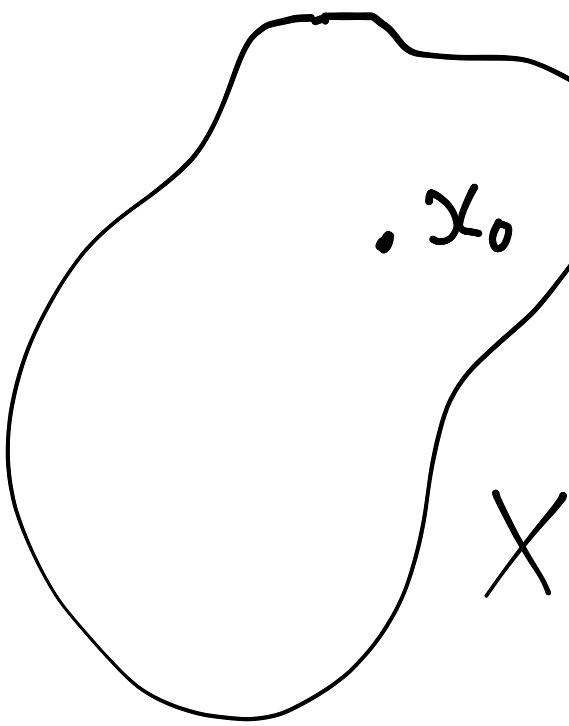
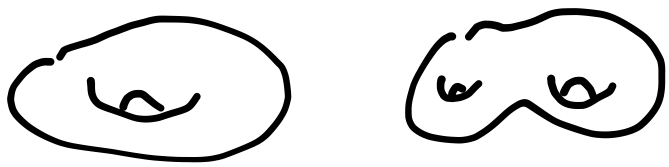
The fundamental group

X is a topological space

x_0 is a point in X.



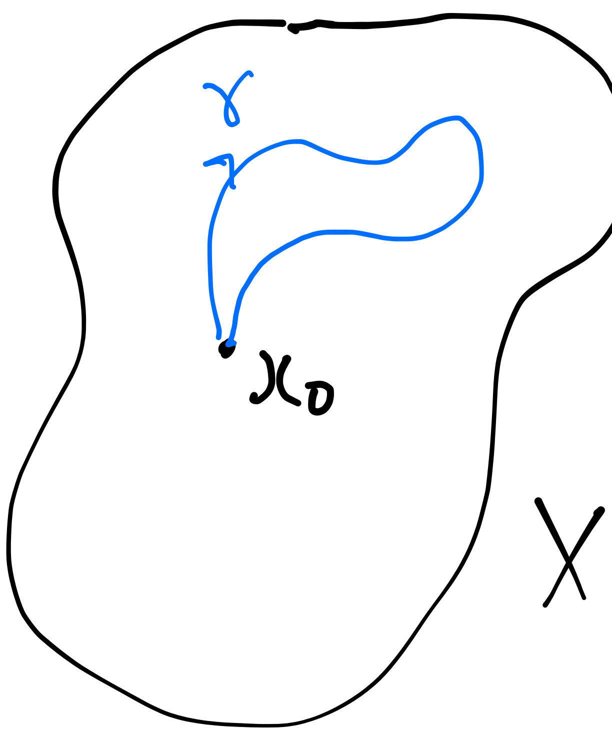
e.g. $X = \mathbb{R}\mathbb{P}^1$

$X =$ 

a based loop is a map
pointed

$$\gamma : [0,1] \longrightarrow X$$

s.t. $\gamma(0) = \gamma(1) = x_0$

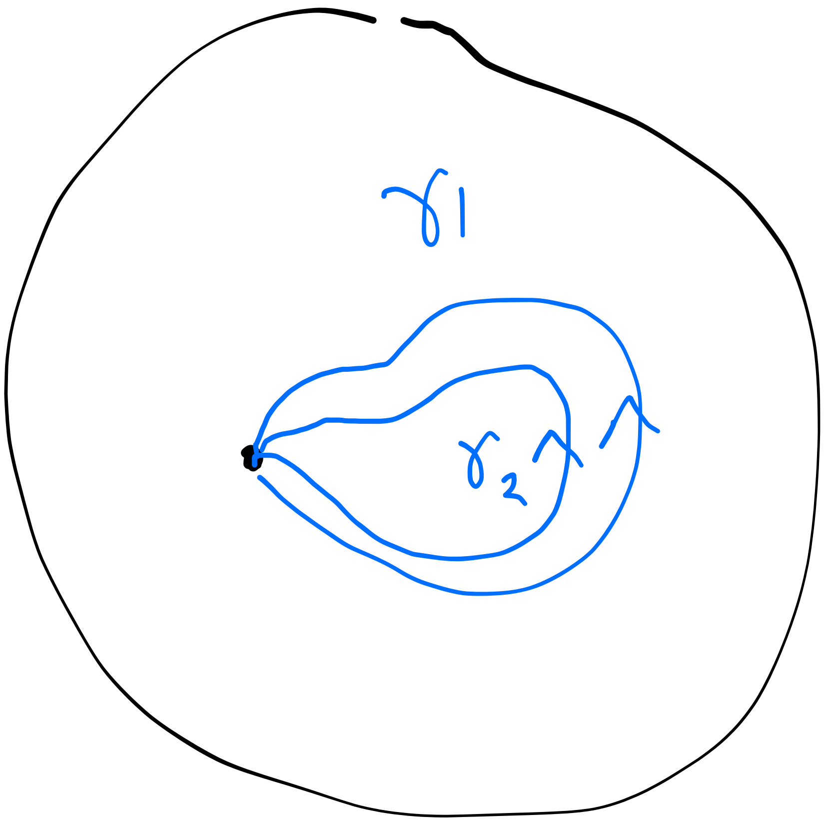


two loops are homotopic if there is a 1-parameter family interpolating

$\gamma_t \quad t \in [0,1]$

each γ_t is a loop

↳ homotopy between γ_1 and γ_2

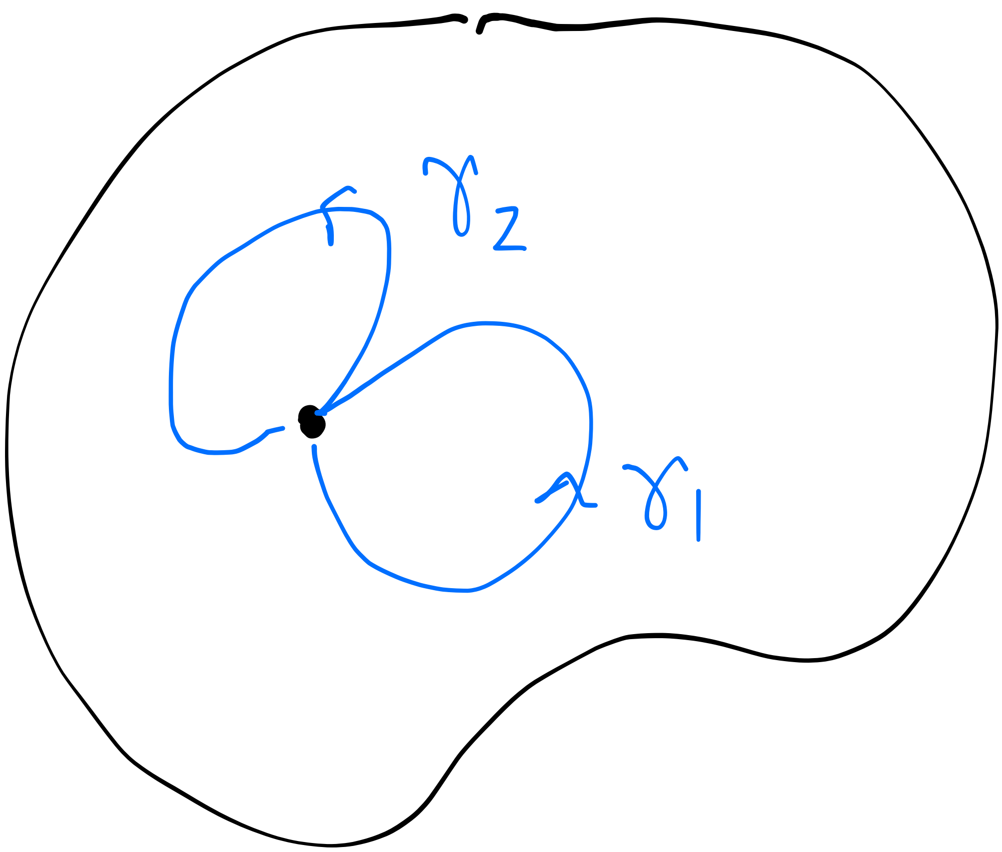


γ_1 is homotopic to γ_2.

Loops can be added

$\gamma_2 \cdot \gamma_1$ means go around

γ_1, then γ_2



there is an inverse:

γ_1^{-1} is go around γ_1 backwards

there is an identity:

stay still

exercise: show

$\gamma^{-1} \cdot \gamma$ is homotopic

to identity.

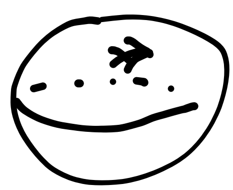
1

Define $\pi_1(X, x_0)$

= homotopy classes of loops w/ addition operation.

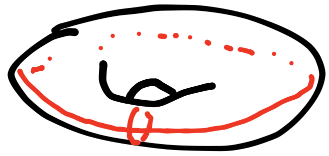
e.g. $X = \mathbb{RP}^1$

$\pi_1(X, x_0) = \frac{1}{2}\mathbb{Z}$ (Defects)

e.g. $X = S^2$ 

$\pi_1(X, x_0) = 0$ (simply connected)

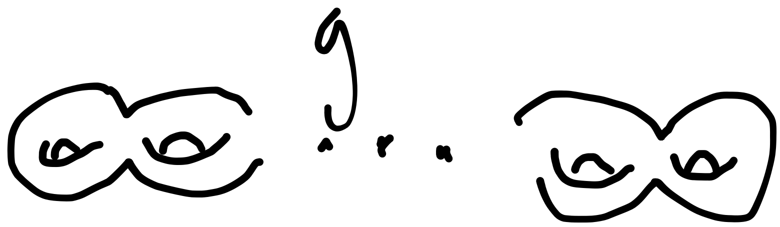
e.g. $X = T^2$



$$\pi_1(X, x_0) = \mathbb{Z}^2$$

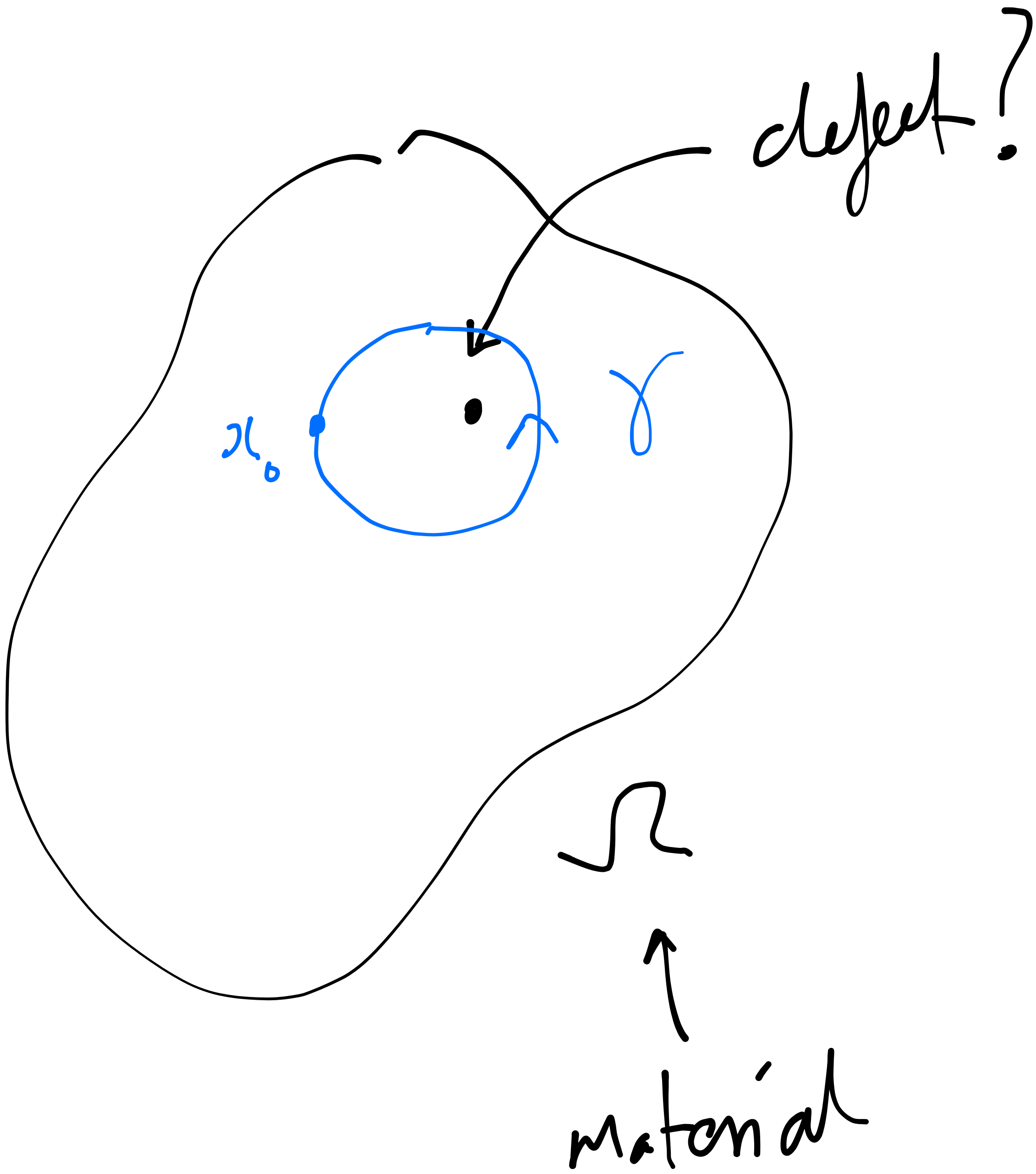
e.g.

$X = S_g$



$$\pi_1(X, x_0) = \mathbb{Z}^{2g}$$

Back to our system



$n|_\gamma$ defines an element of $\pi_1(\mathbb{RP}^1, n(x_0)) \leftrightarrow$ winding no.

Iff this element is trivial,
then we can fill in the loop.
$\hookrightarrow$ no defect
If it is not trivial
there is a defect !

Point defects in 2D (Almost)
classified by $\pi_1(X)$
GSM.