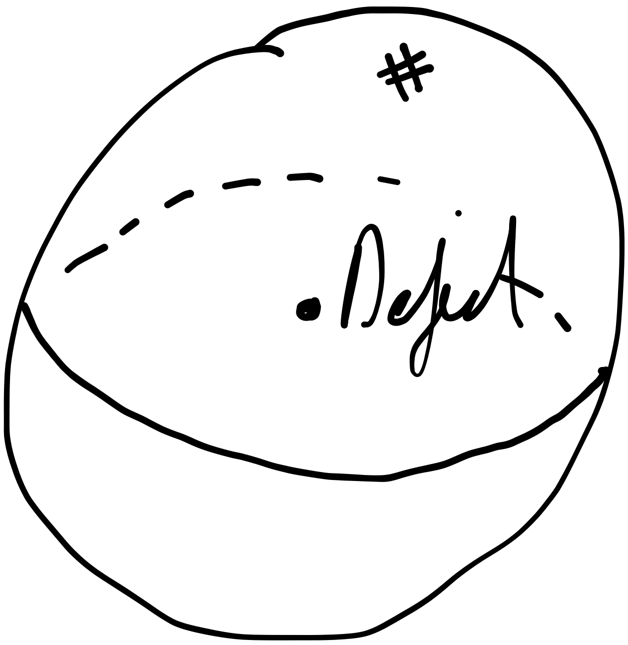
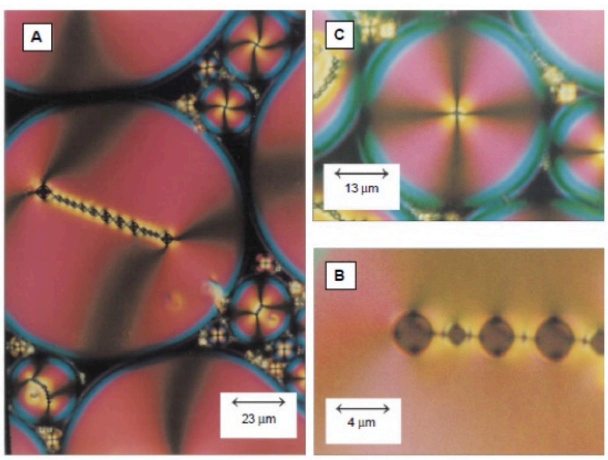
Hedgehogs & Skyrmions

Point defects in 3D



Surround w/ sphere



P. Poulin *et al*. Science **275**, 1770-1773 (1997).

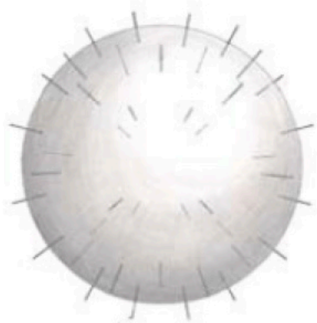
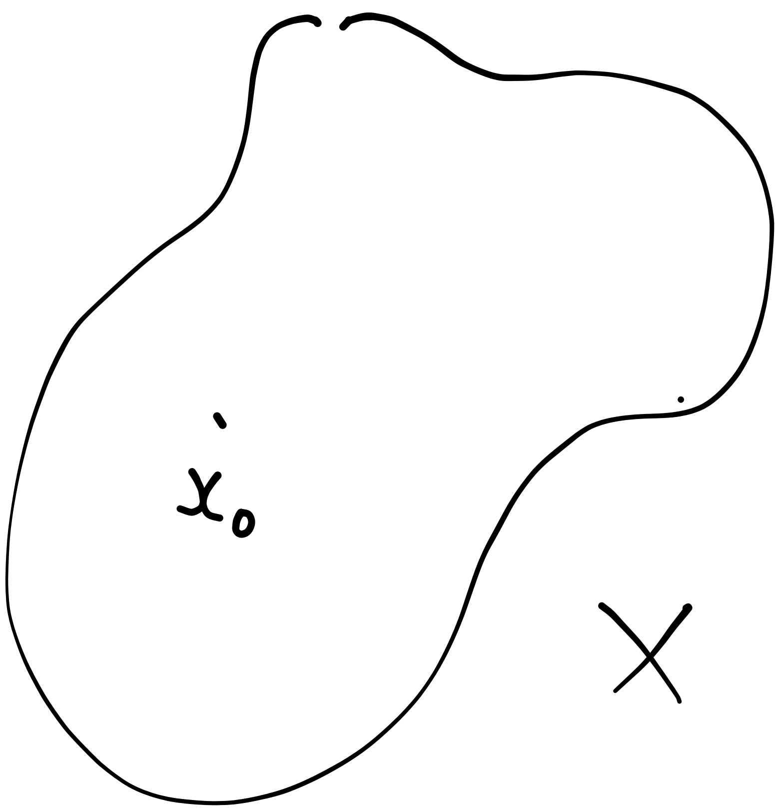


Image: Elisabetta Matsumoto

Can be calculated with an integral:

$$d = \frac{1}{4\pi}\int_{S^2} d\theta d\varphi\, \boldsymbol{n} \cdot (\partial_\theta \boldsymbol{n} \times \partial_\varphi \boldsymbol{n})$$

How to formalise:

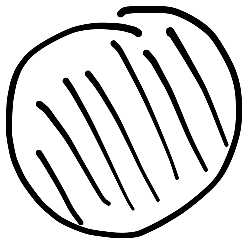


X is a space

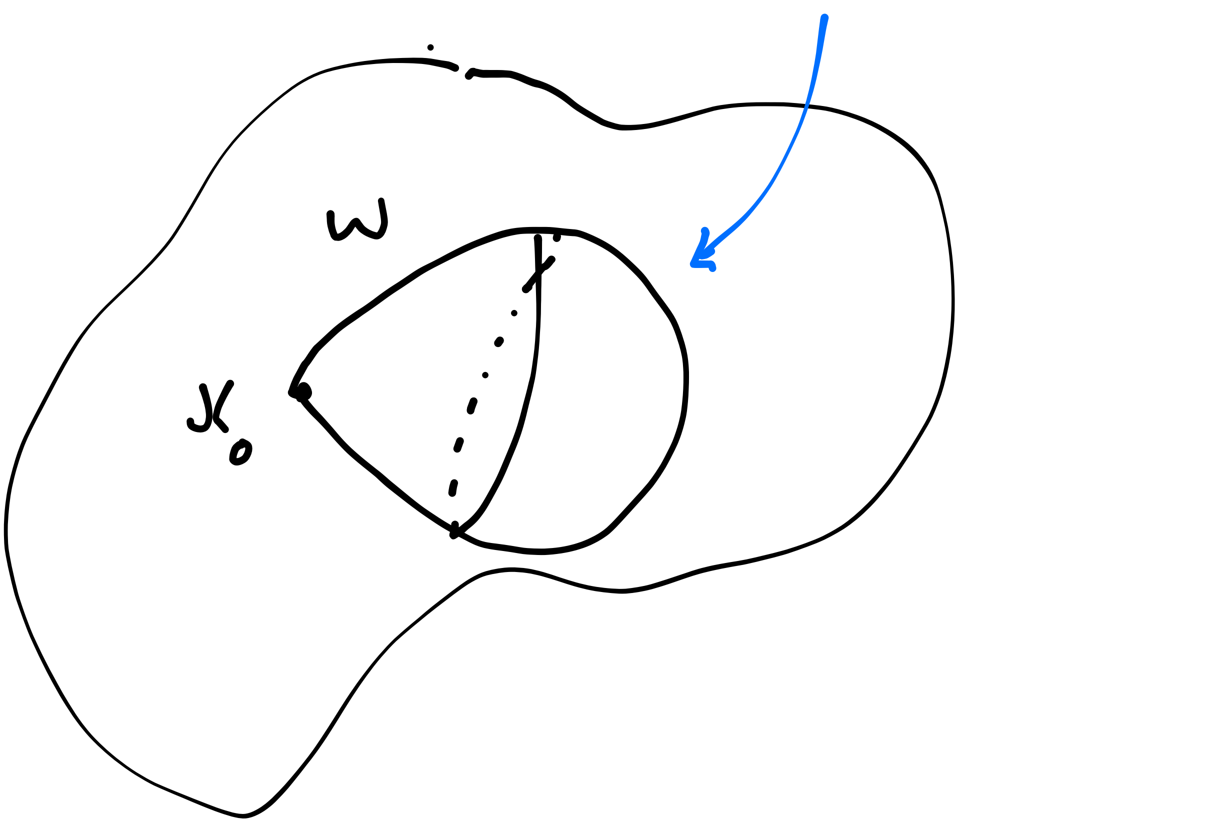
x_0 is a point

Map

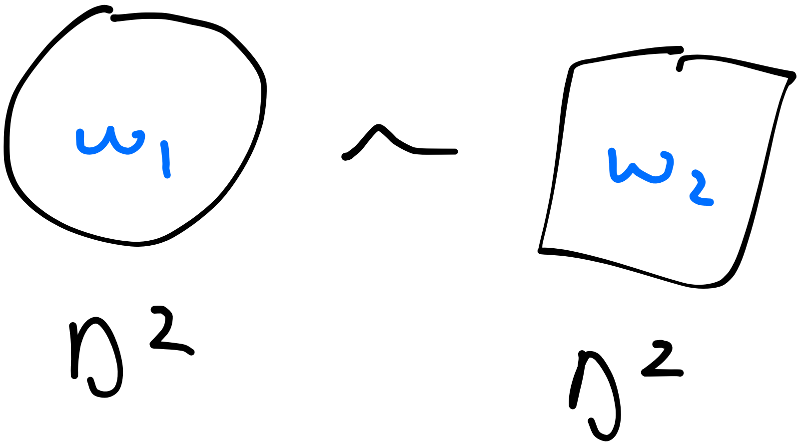
$\omega : D^2 \to X$

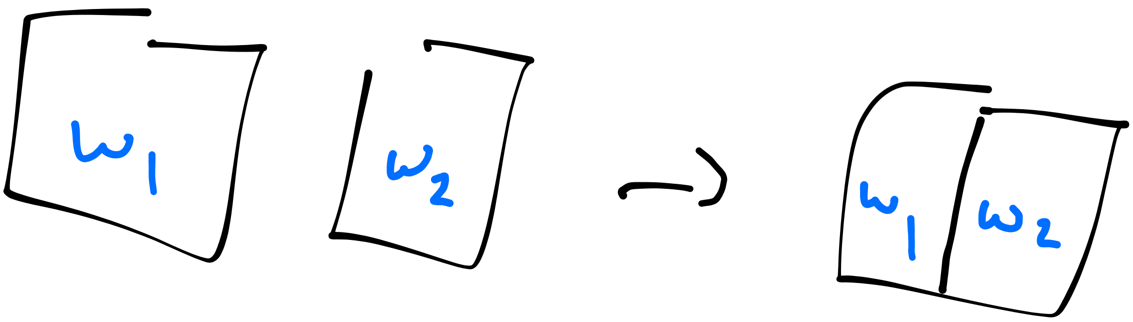
$D^2 =$  unit disk

s.t. $\partial D^2 \to x_0$



composition rule:





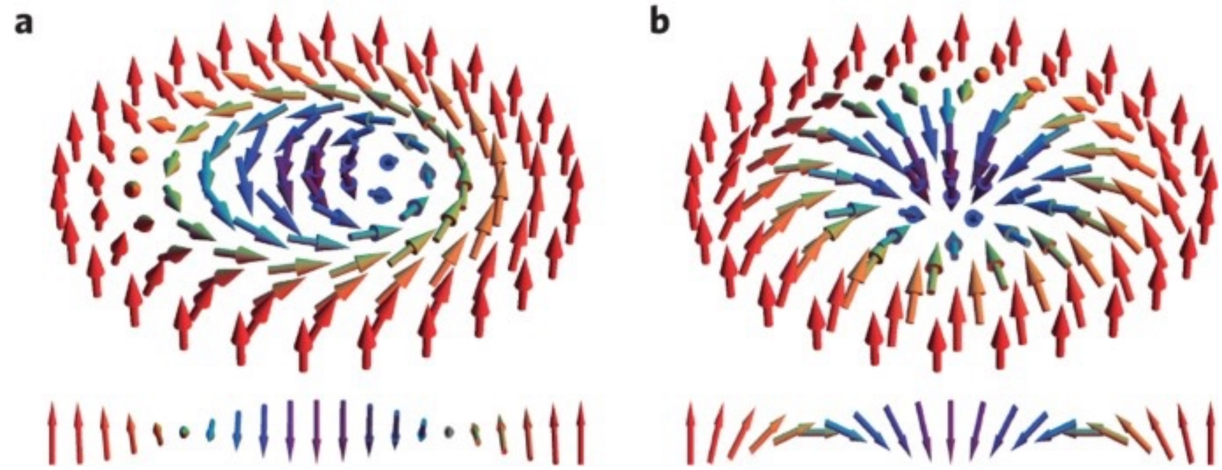
exercise: show this commutes (up to homotopy) inverse : reflect horizontally

$\pi_2(X)$ classifies point defects in 3D

$\pi_2(\mathbb{RP}^2) = \mathbb{Z}$ ← hedgehogs!

one wrinkle (orientability)

Skyrmions

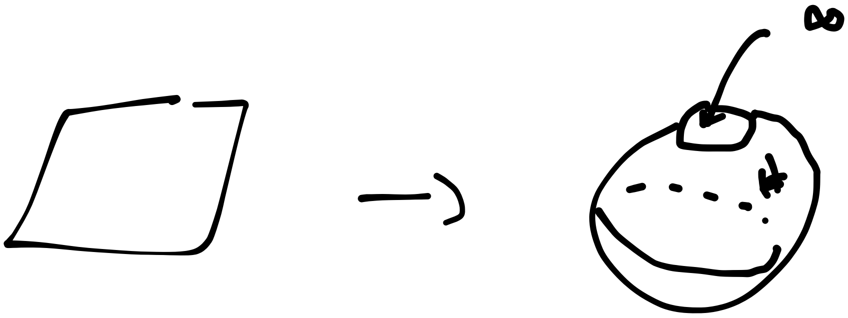


Say we have a 3d director in a 2D region $\sim \mathbb{R}^2$

Then $n : \mathbb{R}^2 \to S^2$

Suppose $\lim_{|x| \to \infty} n(\vec{x}) = (0,0,1)$

Then we can compactify



$\mathbb{R}^2$

add a point $\mathbb{R}^2 \cup \{pt\} = S^2$

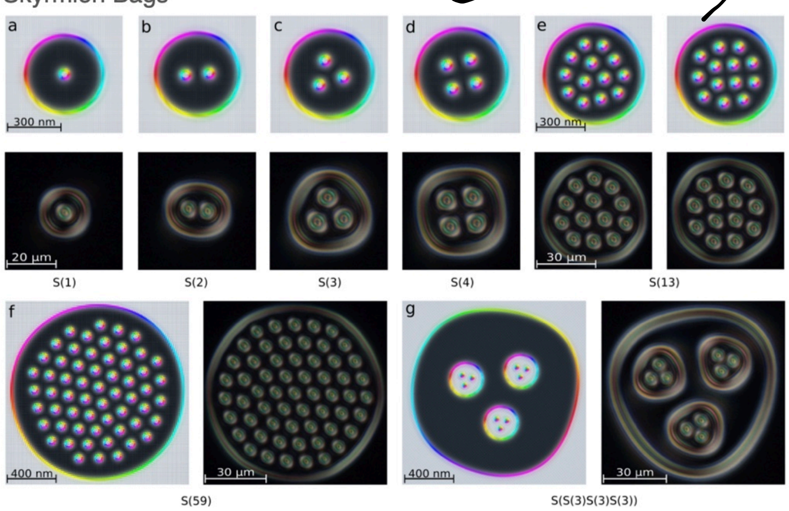
only possible because $n = (0,0,1)$ at ∞.

↳ we then get a map

$\check{n} : S^2 \to \mathbb{RP}^2$

↳ an element $\in \pi_2(\mathbb{RP}^2)$ Skyrmions.

Skyrmion Bags



D. Foster et al. Nature Physics (2019).

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}
S^0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S^1	Z	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S^2	0	Z	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	Z_2^2	$Z_{12}×Z_2$	$Z_{84}×Z_2^2$	Z_2^2
S^3	0	0	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	Z_2^2	$Z_{12}×Z_2$	$Z_{84}×Z_2^2$	Z_2^2
S^4	0	0	0	Z	Z_2	Z_2	$Z×Z_{12}$	Z_2^2	Z_2^2	$Z_{24}×Z_3$	Z_{15}	Z_2	Z_2^3	$Z_{120}×Z_{12}×Z_2$	$Z_{84}×Z_2^5$
S^5	0	0	0	0	Z	Z_2	Z_2	Z_{24}	Z_2	Z_2	Z_2	Z_{30}	Z_2	Z_2^3	$Z_{72}×Z_2$
S^6	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	Z	Z_2	Z_{60}	$Z_{24}×Z_2$	Z_2^3
S^7	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	0	Z_2	Z_{120}	Z_2^3
S^8	0	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	0	Z_2	$Z×Z_{120}$