Calculating R_0 - SIR example

We wish to calculate R_0 which is defined as the expected number of secondary infections per generation given one infected individual is introduced to an entirely susceptible population.

Mathematically, we examine our system equations for the **infectious compartments only** at **disease free equilibrium** and see how they **change** as each infectious variable changes.

We divide this into two situations:

Transmission events where a new infectious variable is created eg. a susceptible person becomes infected

and

Transition events where an infectious variable is lost eg. an infected person recovers.

SIR example

$$\frac{dS}{dt} = \mu H - \beta SI - \mu S$$
$$\frac{dI}{dt} = \beta SI - \mu I - \gamma I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

We only have one infectious variable: I.

So our system of infectious compartments is only the equation for the change in the infected population over time.

$$\frac{dI}{dt} = \beta SI - \mu I - \gamma I$$

Next we wish to know how this equation varies as the infectious variable, I, varies.

To do this we ________________________ with respect to I to get:

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We are interested in our system at disease free equilibrium, so wherever we see a variable, we replace it with its **equilibrium value.**

At disease free equilibrium: S*=H, I*=0, R*=0.

Now we can divide our equation into **transmission** events and **transition** events.

Transmission terms:

Transition terms:

Finally, R_0 is calculated by dividing the **Transmission** terms by (- **Transition** terms).

The intuition is that **1/Transition** is equal to the **generation time.**

Thus, we arrive at the **transmission events per generation**.

R_0 =