CSCE 625 Homework 1

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1. a) Following are the literals that we use to solve the tennis ball problem.

- Observations by drawing tennis ball:

Box-1	**Box-2**	**Box-3**
$O1W$	$O2W$	$O3W$
$O1Y$	$O2Y$	$O3Y$

- Correct labels for the boxes:

Box-1	**Box-2**	**Box-3**
$C1W$	$C2W$	$C3W$
$C1Y$	$C2Y$	$C3Y$
$C1B$	$C2B$	$C3B$

- Wrong labels of each box

Box-1	**Box-2**	**Box-3**
$L1W$	$L2W$	$L3W$
$L1Y$	$L2Y$	$L3Y$
$L1B$	$L2B$	$L3B$

- Conditions for observations:

Box-1	**Box-2**	**Box-3**
c1 : $O1Y \implies C1Y \vee C1B$	**c3** : $O2Y \implies C2Y \vee C2B$	**c5** : $O3Y \implies C3Y \vee C3B$
c2 : $O1W \implies C1W \vee C1B$	**c4** : $O2W \implies C2W \vee C2B$	**c6** : $O3W \implies C3W \vee C3B$

- Rules that imply wrong labels:

Box-1	**Box-2**	**Box-3**
c7 : $L1Y \implies \neg C1Y$	**c10** : $L2Y \implies \neg C2Y$	**c13** : $L3Y \implies \neg C3Y$
c8 : $L1W \implies \neg C1W$	**c11** : $L2W \implies \neg C2W$	**c14** : $L3W \implies \neg C3W$
c9 : $L1B \implies \neg C1B$	**c12** : $L2B \implies \neg C2B$	**c15** : $L3B \implies \neg C3B$

- Constraint that there is atleast one box of each kind (W, Y, B)

c16 : $C1Y \vee C1W \vee C1B$
c17 : $C2Y \vee C2W \vee C2B$
c18 : $C3Y \vee C3W \vee C3B$

- Conditions to capture No repetitions in the box type (exactly one of each kind)

Box-1	**Box-2**	**Box-3**
c19 : $C1Y \implies \neg C2Y \wedge \neg C3Y$	**c22** : $C2Y \implies \neg C1Y \wedge \neg C3Y$	**c25** : $C3Y \implies \neg C2Y \wedge \neg C1Y$
c20 : $C1W \implies \neg C2W \wedge \neg C3W$	**c23** : $C2W \implies \neg C1W \wedge \neg C3W$	**c26** : $C3W \implies \neg C2W \wedge \neg C1W$
c21 : $C1B \implies \neg C2B \wedge \neg C3B$	**c24** : $C2B \implies \neg C1B \wedge \neg C3B$	**c27** : $C3B \implies \neg C2B \wedge \neg C1B$

1. b)

Aim: To prove by using the Natural deduction $KB \models C2W$

Consider the initial KB observed from the given scenario:

* Initial observations:

R1 :$O1Y$
R2 :$O2W$
R3 :$O3Y$
R4 :$L1W$
R5 :$L2Y$
R6 :$L3B$

* **R7** :$C3Y \vee C3B$ (Applying Modus Ponens to R3 and c5)
* **R8** :$\neg C3B$ (Applying Modus Ponens to R6 and c)
* **R9** :$C3Y$ (Resolving R7 and R8)
* **R10** :$\neg C1Y \wedge C2Y$ (Applying Modus Ponens to R9 and c25)
* **R11** :$C1Y \vee C1B$ (Applying Modus Ponens to R1 and c1)
* **R12** :$\neg C1Y$ (Applying And-Elimination on R10)
* **R13** :$C1B$ (Resolving R11 and R12)
* **R14** :$C2W \wedge C2B$ (Applying Modus Ponens to R2 and c4)
* **R15** :$\neg C2B \wedge \neg C3B$ (From R13 and c21)
* **R16** :$\neg C2B$ (From R15)
* **R17** :$C2W$ (Resolving R14 and R16)

Hence, we have **KB** $\models$ **C2W**.

1. c)

- Knowledge Base for the given scenario:

  R1 :$O1Y$
  R2 :$O2W$
  R3 :$O3Y$
  R4 :$L1W$
  R5 :$L2Y$
  R6 :$L3B$

- Converting all constraints to CNF (Conjunctive Normal Form):

  R7 :$\neg O3Y \lor C3Y \lor C3B$ (from c6)

  R8 :$\neg O1Y \lor C1Y \lor C1B$ (from c1)

  R9 :$\neg O2W \lor C2W \lor C2B$ (from c4)

  R10 :$\neg L3B \lor \neg C3B$ (from c15)

  R11a :$\neg C1B \lor \neg C2B$ (from c21)
  R11b :$\neg C1B \lor \neg C3B$

  R12a :$\neg C3Y \lor \neg C2Y$ (from c21)
  R12b :$\neg C3Y \lor \neg C1Y$

- Solving for the query q as C2W, Getting the negation of the query
  R13 :$\neg C2W$

- **R14** :$C3Y \lor C3B$ (Resolution on R3 and R7)

- **R15** :$\neg C3B$ (Resolution on R6 and R10)

- **R16** :$C3Y$ (Resolution on R14 and R15)

- **R17** :$\neg C1Y$ (Resolution on R16 and R12b)

- **R18** :$C1Y \land C1B$ (Resolution on R1 and R8)

- **R19** :$C1B$ (Resolution on R17 and R18)

- **R20** :$\neg C2B$ (Resolution on R19 and R11a)

- **R21** :$C2W \land C2B$ (Resolution on R2 and R9)

- **R22** :$C2W$ (Resolution on R2 and R9)

- **R23** :Φ (Resolution on R22 and R13)

  Since we end up in an empty clause, the solution to the entailment is true. In other words $\mathbf{KB} \models \mathbf{C2W}$

2.
We consider the following 16 literals to construct the propositional logic for 4-Queens problem.

$QA1,\ QA2,\ QA3,\ QA4$
$QB1,\ QB2,\ QB3,\ QB4$
$QC1,\ QC2,\ QC3,\ QC4$
$QD1,\ QD2,\ QD3,\ QD4$

Note: For example, $QB3 = true$, refers to a queen present in position $B3$.

Knowledge Base (KB):

- Constraints to capture at least one Queen in each row.

  z1 : $QA1 \lor QB1 \lor QC1 \lor QD1$
  z2 : $QA2 \lor QB2 \lor QC2 \lor QD2$
  z3 : $QA3 \lor QB3 \lor QC3 \lor QD3$
  z4 : $QA4 \lor QB4 \lor QC4 \lor QD4$

- There are no repeating queens in any row.

$$\begin{aligned}\mathbf{R1}:\ & (\neg QA1 \vee \neg QB1) \wedge (\neg QA1 \vee \neg QC1) \wedge (\neg QA1 \vee \neg QD1) \\ & \wedge (\neg QB1 \vee \neg QC1) \wedge (\neg QB1 \vee \neg QD1) \wedge (\neg QC1 \vee \neg QD1)\end{aligned}$$

$$\begin{aligned}\mathbf{R2}:\ & (\neg QA2 \vee \neg QB2) \wedge (\neg QA2 \vee \neg QC2) \wedge (\neg QA2 \vee \neg QD2) \\ & \wedge (\neg QB2 \vee \neg QC2) \wedge (\neg QB2 \vee \neg QD2) \wedge (\neg QC2 \vee \neg QD2)\end{aligned}$$

$$\begin{aligned}\mathbf{R3}:\ & (\neg QA3 \vee \neg QB3) \wedge (\neg QA3 \vee \neg QC3) \wedge (\neg QA3 \vee \neg QD3) \\ & \wedge (\neg QB3 \vee \neg QC3) \wedge (\neg QB3 \vee \neg QD3) \wedge (\neg QC3 \vee \neg QD3)\end{aligned}$$

$$\begin{aligned}\mathbf{R4}:\ & (\neg QA4 \vee \neg QB4) \wedge (\neg QA4 \vee \neg QC4) \wedge (\neg QA4 \vee \neg QD4) \\ & \wedge (\neg QB4 \vee \neg QC4) \wedge (\neg QB4 \vee \neg QD4) \wedge (\neg QC4 \vee \neg QD4)\end{aligned}$$

- There are no repeating queens in any column.

$$\begin{aligned}\mathbf{C1}:\ & (\neg QA1 \vee \neg QA2) \wedge (\neg QA1 \vee \neg QA3) \wedge (\neg QA1 \vee \neg QA4) \\ & \wedge (\neg QA2 \vee \neg QA3) \wedge (\neg QA2 \vee \neg QA4) \wedge (\neg QA3 \vee \neg QA4)\end{aligned}$$

$$\begin{aligned}\mathbf{C2}:\ & (\neg QB1 \vee \neg QB2) \wedge (\neg QB1 \vee \neg QB3) \wedge (\neg QB1 \vee \neg QB4) \\ & \wedge (\neg QB2 \vee \neg QB3) \wedge (\neg QB2 \vee \neg QB4) \wedge (\neg QB3 \vee \neg QB4)\end{aligned}$$

$$\begin{aligned}\mathbf{C3}:\ & (\neg QC1 \vee \neg QC2) \wedge (\neg QC1 \vee \neg QC3) \wedge (\neg QC1 \vee \neg QC4) \\ & \wedge (\neg QC2 \vee \neg QC3) \wedge (\neg QC2 \vee \neg QC4) \wedge (\neg QC3 \vee \neg QC4)\end{aligned}$$

$$\begin{aligned}\mathbf{C4}:\ & (\neg QD1 \vee \neg QD2) \wedge (\neg QD1 \vee \neg QD3) \wedge (\neg QD1 \vee \neg QD4) \\ & \wedge (\neg QD2 \vee \neg QD3) \wedge (\neg QD2 \vee \neg QD4) \wedge (\neg QD3 \vee \neg QD4)\end{aligned}$$

- There are no repeating queens in any diagonal.

$\mathbf{D1}:\ \neg QA1 \vee \neg QB2$
$\mathbf{D2}:\ \neg QA1 \vee \neg QC3$
$\mathbf{D3}:\ \neg QA1 \vee \neg QD4$

$\mathbf{D4}:\ \neg QB1 \vee \neg QA2$
$\mathbf{D5}:\ \neg QB1 \vee \neg QC2$
$\mathbf{D6}:\ \neg QB1 \vee \neg QD3$

$\mathbf{D7}:\ \neg QC1 \vee \neg QB2$
$\mathbf{D8}:\ \neg QC1 \vee \neg QA3$
$\mathbf{D9}:\ \neg QC1 \vee \neg QD2$

$\mathbf{D10}:\ \neg QD1 \vee \neg QC2$
$\mathbf{D11}:\ \neg QD1 \vee \neg QB3$
$\mathbf{D12}:\ \neg QD1 \vee \neg QA4$

$\mathbf{D13}:\ \neg QA2 \vee \neg QB3$
$\mathbf{D14}:\ \neg QA2 \vee \neg QC4$

$\mathbf{D15}:\ \neg QB2 \vee \neg QA3$
$\mathbf{D16}:\ \neg QB2 \vee \neg QC3$
$\mathbf{D17}:\ \neg QB2 \vee \neg QD4$

$\mathbf{D18}:\ \neg QC2 \vee \neg QB3$
$\mathbf{D19}:\ \neg QC2 \vee \neg QA4$
$\mathbf{D20}:\ \neg QC2 \vee \neg QD3$

$\mathbf{D21}:\ \neg QD2 \vee \neg QC3$
$\mathbf{D22}:\ \neg QD2 \vee \neg QB4$

$\mathbf{D23}:\ \neg QA3 \vee \neg QB4$
$\mathbf{D24}:\ \neg QB3 \vee \neg QA4$
$\mathbf{D25}:\ \neg QB3 \vee \neg QC4$

$\mathbf{D26}:\ \neg QC3 \vee \neg QB4$
$\mathbf{D27}:\ \neg QC3 \vee \neg QD4$
$\mathbf{D28}:\ \neg QCD \vee \neg QC4$

Note: I have tried to combine several clauses using AND into a row/column clause for better readability and tractability. If we write them separately, we get a total of 80 different clauses.

2 a. Table. 1 gives detailed calculations of DPLL algorithm with NO heuristics.

2 b. Table. 2 gives detailed calculations of DPLL algorithm with NO heuristics.

Note: Total number of iterations in naive DPLL is 2*41 = 82. Whereas, the DPLL with heuristics takes 37 iterations, which is almost a factor of half reduction in number of iterations.

3.

We consider the following literals to define the propositional logic for the tic-tac-toe problem.

- Positions of player X and O and void spaces (?):

$X11,\ X12,\ X13$	$O11,\ O12,\ O13$	?11, ?12, ?13
$X21,\ X22,\ X23$	$O21,\ O22,\ O23$	?21, ?22, ?23
$X31,\ X32,\ X33$	$O31,\ O32,\ O33$	?31, ?32, ?33

- Move positions:

  move$X11$, move$X12$, move$X13$
  move$X21$, move$X22$, move$X23$
  move$X31$, move$X32$, move$X33$

- canWin positions:

canWin$X11$, canWin$X12$, canWin$X13$	canWin$O11$, canWin$O12$, canWin$O13$
canWin$X21$, canWin$X22$, canWin$X23$	canWin$O21$, canWin$O22$, canWin$O23$
canWin$X31$, canWin$X32$, canWin$X33$	canWin$O31$, canWin$O32$, canWin$O33$
canWin$X$	canWin$O$

- forcedMove positions:

  forcedMove$X11$, forcedMove$X12$, forcedMove$X13$
  forcedMove$X21$, forcedMove$X22$, forcedMove$X23$
  forcedMove$X31$, forcedMove$X32$, forcedMove$X33$

Knowledge Base (KB):

- We need at least one move by X:

  c1 :move$X11$ $\vee$ move$X12$ $\vee$ move$X13$ $\vee$ move$X21$ $\vee$ move$X22$ $\vee$ move$X23$ $\vee$ move$X31$ $\vee$ move$X32 \vee$ move$X33$

- canWin moves for X's and O's:

  Completing a row pattern:

c2 : $X11 \wedge X12 \wedge ?13 \implies$ canWin$X13$	**c5** : $O11 \wedge O12 \wedge ?13 \implies$ canWin$O13$
c3 : $X21 \wedge X22 \wedge ?23 \implies$ canWin$X23$	**c6** : $O21 \wedge O22 \wedge ?23 \implies$ canWin$O23$
c4 : $X31 \wedge X32 \wedge ?33 \implies$ canWin$X33$	**c7** : $O31 \wedge O32 \wedge ?33 \implies$ canWin$O33$

Table 1: DPLL with NO Heuristics

Iteration No	QA1	QA2	QA3	QA4	QB1	QB2	QB3	QB4	QC1	QC2	QC3	QC4	QD1	QD2	QD3	QD4	Comments
1	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = T
2	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = F by C1
3	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	-	QA3 = F by C1
4	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	-	QA4 = F by C1
5	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	-	QB2 = F by R1
6	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	-	QB2 = F by D1
7	T	F	F	F	F	F	T	-	-	-	-	-	-	-	-	-	QB3 = T
8	T	F	F	F	F	F	T	F	-	-	-	-	-	-	-	-	QB4 = F by C2
9	T	F	F	F	F	F	T	F	F	-	-	-	-	-	-	-	QC1 = F by R1
10	T	F	F	F	F	F	T	F	F	F	-	-	-	-	-	-	QC2 = F by D18
11	T	F	F	F	F	F	T	F	F	F	F	-	-	-	-	-	QC3 = F by R3
12	T	F	F	F	F	F	T	F	F	F	F	-	-	-	-	-	BackTrack to Iter. 7
13	T	F	F	F	F	F	F	-	-	-	-	-	-	-	-	-	QB3 = F
14	T	F	F	F	F	F	F	T	-	-	-	-	-	-	-	-	QB4 = T
15	T	F	F	F	F	F	F	T	F	-	-	-	-	-	-	-	QC1 = F by R1
16	T	F	F	F	F	F	F	T	F	T	-	-	-	-	-	-	QC2 = T
17	T	F	F	F	F	F	F	T	F	T	F	-	-	-	-	-	QC3 = F by C3
18	T	F	F	F	F	F	F	T	F	T	F	F	-	-	-	-	QC4 = F by C3
19	T	F	F	F	F	F	F	T	F	T	F	F	F	-	-	-	QD1 = F by R1
20	T	F	F	F	F	F	F	T	F	T	F	F	F	F	-	-	QD2 = F by R2
21	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	-	QD3 = F by D20
22	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	F	QD4 BackTrack To Iter. 16
23	T	F	F	F	F	F	F	T	F	F	-	-	-	-	-	-	QC2 = F
24	T	F	F	F	F	F	F	T	F	T	F	-	-	-	-	-	QC3 = F by D26
25	T	F	F	F	F	F	F	T	F	T	F	F	-	-	-	-	BackTrack to Iter. 1
26	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = F
27	F	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = T
28	F	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	QA3 = F by C1
29	F	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	QA4 = F by C1
30	F	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	QB1 = F by D4
31	F	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	QB2 = F by R2
32	F	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	QB3 = F by D13
33	F	T	F	F	F	F	F	T	-	-	-	-	-	-	-	-	QB4 = T
34	F	T	F	F	F	F	F	T	T	-	-	-	-	-	-	-	QC1 = T
35	F	T	F	F	F	F	F	T	T	F	-	-	-	-	-	-	QC2 = F by C3
36	F	T	F	F	F	F	F	T	T	F	F	-	-	-	-	-	QC3 = F by C3
37	F	T	F	F	F	F	F	T	T	F	F	F	-	-	-	-	QC4 = F by C3
38	F	T	F	F	F	F	F	T	T	F	F	F	F	-	-	-	QD1 = F by R1
39	F	T	F	F	F	F	F	T	T	F	F	F	F	F	-	-	QD2 = F by D9
40	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	QD3 = T
41	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	QD4 = F by C4

Table 2: DPLL with Pure Symbol and Unit Clause heuristics

Iteration	QA1	QA2	QA3	QA4	QB1	QB2	QB3	QB4	QC1	QC2	QC3	QC4	QD1	QD2	QD3	QD4	Comments
1	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Q1A = T
2	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QA2 = F by C1
3	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QA3 = F by C1
4	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : Q4A = F by C1
5	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QB1 = F by R1
6	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	-	Unit Clause : QB2 = F by D1
7	T	F	F	F	F	F	-	-	F	-	-	-	-	-	-	-	Unit Clause : QC1 = F by R1
8	T	F	F	F	F	F	-	-	F	-	F	-	-	-	-	-	Unit Clause : QC3 = F by D2
9	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	-	Unit Clause : QD1 = F by R1
10	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	F	Unit Clause : QD4 = F by D3
11	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	F	Q3B = T
12	T	F	F	F	F	F	T	F	F	-	F	-	F	-	-	F	Unit Clause: QB4 = F by C2
13	T	F	F	F	F	F	T	F	F	F	F	-	F	-	-	F	Unit Clause: QC2 = F by D18
14	T	F	F	F	F	F	T	F	F	F	F	-	F	-	F	F	Unit Clause: QD3 = F by R3
15	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	F	Q4C = F Backtrack to Iter. 11
16	T	F	F	F	F	F	F	-	F	-	F	-	F	-	-	F	Q3B = F
17	T	F	F	F	F	F	F	T	F	-	F	-	F	-	-	F	Q4B = T
18	T	F	F	F	F	F	F	T	F	-	F	F	F	-	-	F	Unit Clause: QC4 = F by R4
19	T	F	F	F	F	F	F	T	F	-	F	F	F	F	-	F	Unit Clause: QD2 = F by D22
20	T	F	F	F	F	F	F	T	F	T	F	F	F	F	-	F	Unit Clause: QC2 = F by D18
21	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Q3D = T/F Backtrack to Iter. 1
22	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = F
23	F	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = T
24	F	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA3 = F by C1
25	F	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA4 = F by C1
26	F	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA4 = F by D4
27	F	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	Unit Clause: QB2 = F by R2
28	F	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	Unit Clause: QB3 = F by D13
29	F	T	F	F	F	F	F	-	-	F	-	-	-	-	-	-	Unit Clause: QC2 = F by R2
30	F	T	F	F	F	F	F	-	-	F	-	F	-	-	-	-	Unit Clause: QC4 = F by D14
31	F	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	Unit Clause: QD2 = F by R2
32	F	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	QB4 = T
33	F	T	F	F	F	F	F	T	-	F	F	F	-	F	-	-	Unit Clause: QC3 = F by D26
34	F	T	F	F	F	F	F	T	-	F	F	F	-	F	-	F	Unit Clause: QD4 = F by D27
35	F	T	F	F	F	F	F	T	T	F	F	F	-	F	-	F	QC1 = T
36	F	T	F	F	F	F	F	T	T	F	F	F	F	F	-	F	Pure Symbol: QD1 = F
37	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	Pure Symbol: QD3 = T

$\mathbf{c8}: X11 \ \wedge \ ?12 \ \wedge \ X13 \implies \text{canWin}X12$
$\mathbf{c9}: X21 \ \wedge \ ?22 \ \wedge \ X23 \implies \text{canWin}X22$
$\mathbf{c10}: X31 \ \wedge \ ?32 \ \wedge \ X33 \implies \text{canWin}X32$

$\mathbf{c11}: O11 \ \wedge \ ?12 \ \wedge \ O13 \implies \text{canWin}O12$
$\mathbf{c12}: O21 \ \wedge \ ?22 \ \wedge \ O23 \implies \text{canWin}O22$
$\mathbf{c13}: O31 \ \wedge \ ?32 \ \wedge \ O33 \implies \text{canWin}O32$

$\mathbf{c14}: ?11 \ \wedge \ X12 \ \wedge \ X13 \implies \text{canWin}X11$
$\mathbf{c15}: ?21 \ \wedge \ X22 \ \wedge \ X23 \implies \text{canWin}X21$
$\mathbf{c16}: ?31 \ \wedge \ X32 \ \wedge \ X33 \implies \text{canWin}X31$

$\mathbf{c17}: ?11 \ \wedge \ O12 \ \wedge \ O13 \implies \text{canWin}O11$
$\mathbf{c18}: ?21 \ \wedge \ O22 \ \wedge \ O23 \implies \text{canWin}O21$
$\mathbf{c19}: ?31 \ \wedge \ O32 \ \wedge \ O33 \implies \text{canWin}O31$

Completing a column pattern:

$\mathbf{c20}: X11 \ \wedge \ X21 \ \wedge \ ?31 \implies canWinX31$
$\mathbf{c21}: X12 \ \wedge \ X22 \ \wedge \ ?23 \implies canWinX23$
$\mathbf{c22}: X13 \ \wedge \ X23 \ \wedge \ ?33 \implies canWinX33$

$\mathbf{c23}: O11 \ \wedge \ O21 \ \wedge \ ?31 \implies canWinO31$
$\mathbf{c24}: O12 \ \wedge \ O22 \ \wedge \ ?23 \implies canWinO23$
$\mathbf{c25}: O13 \ \wedge \ O23 \ \wedge \ ?33 \implies canWinO33$

$\mathbf{c26}: X11 \ \wedge \ ?21 \ \wedge \ X31 \implies canWinX21$
$\mathbf{c27}: X12 \ \wedge \ ?22 \ \wedge \ X23 \implies canWinX22$
$\mathbf{c28}: X13 \ \wedge \ ?23 \ \wedge \ X33 \implies canWinX23$

$\mathbf{c29}: O11 \ \wedge \ ?21 \ \wedge \ O31 \implies canWinO21$
$\mathbf{c30}: O12 \ \wedge \ ?22 \ \wedge \ O23 \implies canWinO22$
$\mathbf{c31}: O13 \ \wedge \ ?23 \ \wedge \ O33 \implies canWinO23$

$\mathbf{c32}: ?11 \ \wedge \ X21 \ \wedge \ X31 \implies canWinX11$
$\mathbf{c33}: ?12 \ \wedge \ X22 \ \wedge \ X23 \implies canWinX12$
$\mathbf{c34}: ?13 \ \wedge \ X23 \ \wedge \ X33 \implies canWinX13$

$\mathbf{c35}: ?11 \ \wedge \ O21 \ \wedge \ O31 \implies canWinO11$
$\mathbf{c36}: ?12 \ \wedge \ O22 \ \wedge \ O23 \implies canWinO12$
$\mathbf{c37}: ?13 \ \wedge \ O23 \ \wedge \ O33 \implies canWinO13$

Completing a diagonal pattern:

$\mathbf{c38}: X11 \ \wedge \ X22 \ \wedge \ ?33 \implies canWinX33$
$\mathbf{c39}: X13 \ \wedge \ X22 \ \wedge \ ?31 \implies canWinX31$

$\mathbf{c40}: O11 \ \wedge \ O22 \ \wedge \ ?33 \implies canWinO33$
$\mathbf{c41}: O13 \ \wedge \ O22 \ \wedge \ ?31 \implies canWinO31$

$\mathbf{c42}: X11 \ \wedge \ ?22 \ \wedge \ X33 \implies canWinX22$
$\mathbf{c43}: X13 \ \wedge \ ?22 \ \wedge \ X31 \implies canWinX22$

$\mathbf{c44}: O11 \ \wedge \ ?22 \ \wedge \ O33 \implies canWinO22$
$\mathbf{c45}: O13 \ \wedge \ ?22 \ \wedge \ O31 \implies canWinO22$

$\mathbf{c46}: ?11 \ \wedge \ X22 \ \wedge \ X33 \implies canWinX11$
$\mathbf{c47}: ?13 \ \wedge \ X22 \ \wedge \ X31 \implies canWinX13$

$\mathbf{c48}: ?11 \ \wedge \ X22 \ \wedge \ X33 \implies canWinO11$
$\mathbf{c49}: ?13 \ \wedge \ X22 \ \wedge \ X31 \implies canWinO13$

$$\begin{aligned}\mathbf{c50}: & \ canWinX11 \ \vee \ canWinX12 \ \vee \ canWinX13 \\ & \vee \ canWinX21 \ \vee \ canWinX22 \ \vee \ canWinX23 \\ & \vee \ canWinX31 \ \vee \ canWinX32 \ \vee \ canWinX33 \implies canWinX\end{aligned}$$

$$\begin{aligned}\mathbf{c51}: & \ \text{canWin}O11 \ \vee \ \text{canWin}O12 \ \vee \ \text{canWin}O13 \\ & \vee \ \text{canWin}O21 \ \vee \ \text{canWin}O22 \ \vee \ \text{canWin}O23 \\ & \vee \ \text{canWin}O31 \ \vee \ \text{canWin}O32 \ \vee \ \text{canWin}O33 \implies \text{canWin}O\end{aligned}$$

- Conditions for winning moves by X :

$$\begin{array}{llll} \textbf{c52}: \text{canWin}X11 & \implies & \text{move}X11 & \textbf{c55}: \text{canWin}X21 \implies \text{move}X21 \quad \textbf{c58}: \text{canWin}X31 \implies \text{move}X31 \\ \textbf{c53}: \text{canWin}X12 & \implies & \text{move}X12 & \textbf{c56}: \text{canWin}X22 \implies \text{move}X22 \quad \textbf{c59}: \text{canWin}X32 \implies \text{move}X32 \\ \textbf{c54}: \text{canWin}X13 & \implies & \text{move}X13 & \textbf{c57}: \text{canWin}X23 \implies \text{move}X23 \quad \textbf{c60}: \text{canWin}X33 \implies \text{move}X33 \end{array}$$

- Conditions for forcedMoves by X :

$$\begin{array}{lll} \textbf{c61}: \text{canWin}O11 \implies \text{forcedMove}X11 & & \textbf{c64}: \text{canWin}O21 \implies \text{forcedMove}X21 \\ \textbf{c62}: \text{canWin}O12 \implies \text{forcedMove}X12 & & \textbf{c65}: \text{canWin}O22 \implies \text{forcedMove}X22 \\ \textbf{c63}: \text{canWin}O13 \implies \text{forcedMove}X13 & & \textbf{c66}: \text{canWin}O23 \implies \text{forcedMove}X23 \end{array}$$

$$\begin{array}{lll} \textbf{c67}: \text{canWin}O31 & \implies & \text{forcedMove}X31 \\ \textbf{c68}: \text{canWin}O32 & \implies & \text{forcedMove}X32 \\ \textbf{c69}: \text{canWin}O33 & \implies & \text{forcedMove}X33 \end{array}$$

$$\begin{array}{ll} \textbf{c70}: \neg\text{canWinX} \wedge \text{forcedMove}X11 \implies \text{move}X11 & \textbf{c73}: \neg\text{canWinX} \wedge \text{forcedMove}X21 \implies \text{move}X21 \\ \textbf{c71}: \neg\text{canWinX} \wedge \text{forcedMove}X12 \implies \text{move}X12 & \textbf{c74}: \neg\text{canWinX} \wedge \text{forcedMove}X22 \implies \text{move}X22 \\ \textbf{c72}: \neg\text{canWinX} \wedge \text{forcedMove}X13 \implies \text{move}X13 & \textbf{c75}: \neg\text{canWinX} \wedge \text{forcedMove}X23 \implies \text{move}X23 \end{array}$$

$$\begin{array}{lll} \textbf{c76}: \neg\text{canWinX} \wedge \text{forcedMove}X31 & \implies & \text{move}X31 \\ \textbf{c77}: \neg\text{canWinX} \wedge \text{forcedMove}X32 & \implies & \text{move}X32 \\ \textbf{c78}: \neg\text{canWinX} \wedge \text{forcedMove}X33 & \implies & \text{move}X33 \end{array}$$