

CSCE 625 Homework 1

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1. a) Following are the literals that we use to solve the tennis ball problem.

- Observations by drawing tennis ball:

Box-1	Box-2	Box-3
$O1W$	$O2W$	$O3W$
$O1Y$	$O2Y$	$O3Y$

- Correct labels for the boxes:

Box-1	Box-2	Box-3
$C1W$	$C2W$	$C3W$
$C1Y$	$C2Y$	$C3Y$
$C1B$	$C2B$	$C3B$

- Wrong labels of each box

Box-1	Box-2	Box-3
$L1W$	$L2W$	$L3W$
$L1Y$	$L2Y$	$L3Y$
$L1B$	$L2B$	$L3B$

- Conditions for observations:

Box-1	Box-2	Box-3
$c1 : O1Y \implies C1Y \vee C1B$	$c3 : O2Y \implies C2Y \vee C2B$	$c5 : O3Y \implies C3Y \vee C3B$
$c2 : O1W \implies C1W \vee C1B$	$c4 : O2W \implies C2W \vee C2B$	$c6 : O3W \implies C3W \vee C3B$

- Rules that imply wrong labels:

Box-1	Box-2	Box-3
$c7 : L1Y \implies \neg C1Y$	$c10 : L2Y \implies \neg C2Y$	$c13 : L3Y \implies \neg C3Y$
$c8 : L1W \implies \neg C1W$	$c11 : L2W \implies \neg C2W$	$c14 : L3W \implies \neg C3W$
$c9 : L1B \implies \neg C1B$	$c12 : L2B \implies \neg C2B$	$c15 : L3B \implies \neg C3B$

- Constraint that there is atleast one box of each kind (W, Y, B)

$c16 : C1Y \vee C1W \vee C1B$
 $c17 : C2Y \vee C2W \vee C2B$
 $c18 : C3Y \vee C3W \vee C3B$

- Conditions to capture No repetitions in the box type (exactly one of each kind)

Box-1

c19 : $C1Y \implies \neg C2Y \wedge \neg C3Y$
c20 : $C1W \implies \neg C2W \wedge \neg C3W$
c21 : $C1B \implies \neg C2B \wedge \neg C3B$

Box-2

c22 : $C2Y \implies \neg C1Y \wedge \neg C3Y$
c23 : $C2W \implies \neg C1W \wedge \neg C3W$
c24 : $C2B \implies \neg C1B \wedge \neg C3B$

Box-3

c25 : $C3Y \implies \neg C2Y \wedge \neg C1Y$
c26 : $C3W \implies \neg C2W \wedge \neg C1W$
c27 : $C3B \implies \neg C2B \wedge \neg C1B$

1. b)

Aim: To prove by using the Natural deduction $KB \models C2W$

Consider the initial KB observed from the given scenario:

* Initial observations:

R1 : $O1Y$
R2 : $O2W$
R3 : $O3Y$
R4 : $L1W$
R5 : $L2Y$
R6 : $L3B$

- * **R7** : $C3Y \vee C3B$ (Applying Modus Ponens to R3 and c5)
- * **R8** : $\neg C3B$ (Applying Modus Ponens to R6 and c)
- * **R9** : $C3Y$ (Resolving R7 and R8)
- * **R10** : $\neg C1Y \wedge C2Y$ (Applying Modus Ponens to R9 and c25)
- * **R11** : $C1Y \vee C1B$ (Applying Modus Ponens to R1 and c1)
- * **R12** : $\neg C1Y$ (Applying And-Elimination on R10)
- * **R13** : $C1B$ (Resolving R11 and R12)
- * **R14** : $C2W \wedge C2B$ (Applying Modus Ponens to R2 and c4)
- * **R15** : $\neg C2B \wedge \neg C3B$ (From R13 and c21)
- * **R16** : $\neg C2B$ (From R15)
- * **R17** : $C2W$ (Resolving R14 and R16)

Hence, we have $KB \models C2W$.

1. c)

- Knowledge Base for the given scenario:

R1 : $O1Y$
R2 : $O2W$
R3 : $O3Y$
R4 : $L1W$
R5 : $L2Y$
R6 : $L3B$

- Converting all constraints to CNF (Conjunctive Normal Form):

R7 : $\neg O3Y \vee C3Y \vee C3B$ (from c6)

R8 : $\neg O1Y \vee C1Y \vee C1B$ (from c1)

R9 : $\neg O2W \vee C2W \vee C2B$ (from c4)

R10 : $\neg L3B \vee \neg C3B$ (from c15)

R11a : $\neg C1B \vee \neg C2B$ (from c21)

R11b : $\neg C1B \vee \neg C3B$

R12a : $\neg C3Y \vee \neg C2Y$ (from c21)

R12b : $\neg C3Y \vee \neg C1Y$

- Solving for the query q as C2W, Getting the negation of the query

R13 : $\neg C2W$

- **R14** : $C3Y \vee C3B$ (Resolution on R3 and R7)

- **R15** : $\neg C3B$ (Resolution on R6 and R10)

- **R16** : $C3Y$ (Resolution on R14 and R15)

- **R17** : $\neg C1Y$ (Resolution on R16 and R12b)

- **R18** : $C1Y \wedge C1B$ (Resolution on R1 and R8)

- **R19** : $C1B$ (Resolution on R17 and R18)

- **R20** : $\neg C2B$ (Resolution on R19 and R11a)

- **R21** : $C2W \wedge C2B$ (Resolution on R2 and R9)

- **R22** : $C2W$ (Resolution on R2 and R9)

- **R23** : Φ (Resolution on R22 and R13)

Since we end up in an empty clause, the solution to the entailment is true. In other words $\mathbf{KB} \models \mathbf{C2W}$

2.

We consider the following 16 literals to construct the propositional logic for 4-Queens problem.

$QA1, QA2, QA3, QA4$

$QB1, QB2, QB3, QB4$

$QC1, QC2, QC3, QC4$

$QD1, QD2, QD3, QD4$

Note: For example, $QB3 = true$, refers to a queen present in position $B3$.

Knowledge Base (KB):

- Constraints to capture at least one Queen in each row.

z1 : $QA1 \vee QB1 \vee QC1 \vee QD1$

z2 : $QA2 \vee QB2 \vee QC2 \vee QD2$

z3 : $QA3 \vee QB3 \vee QC3 \vee QD3$

z4 : $QA4 \vee QB4 \vee QC4 \vee QD4$

- There are no repeating queens in any row.

$$\begin{aligned} \mathbf{R1} : & (\neg QA1 \vee \neg QB1) \wedge (\neg QA1 \vee \neg QC1) \wedge (\neg QA1 \vee \neg QD1) \\ & \wedge (\neg QB1 \vee \neg QC1) \wedge (\neg QB1 \vee \neg QD1) \wedge (\neg QC1 \vee \neg QD1) \end{aligned}$$

$$\begin{aligned} \mathbf{R2} : & (\neg QA2 \vee \neg QB2) \wedge (\neg QA2 \vee \neg QC2) \wedge (\neg QA2 \vee \neg QD2) \\ & \wedge (\neg QB2 \vee \neg QC2) \wedge (\neg QB2 \vee \neg QD2) \wedge (\neg QC2 \vee \neg QD2) \end{aligned}$$

$$\begin{aligned} \mathbf{R3} : & (\neg QA3 \vee \neg QB3) \wedge (\neg QA3 \vee \neg QC3) \wedge (\neg QA3 \vee \neg QD3) \\ & \wedge (\neg QB3 \vee \neg QC3) \wedge (\neg QB3 \vee \neg QD3) \wedge (\neg QC3 \vee \neg QD3) \end{aligned}$$

$$\begin{aligned} \mathbf{R4} : & (\neg QA4 \vee \neg QB4) \wedge (\neg QA4 \vee \neg QC4) \wedge (\neg QA4 \vee \neg QD4) \\ & \wedge (\neg QB4 \vee \neg QC4) \wedge (\neg QB4 \vee \neg QD4) \wedge (\neg QC4 \vee \neg QD4) \end{aligned}$$

- There are no repeating queens in any column.

$$\begin{aligned} \mathbf{C1} : & (\neg QA1 \vee \neg QA2) \wedge (\neg QA1 \vee \neg QA3) \wedge (\neg QA1 \vee \neg QA4) \\ & \wedge (\neg QA2 \vee \neg QA3) \wedge (\neg QA2 \vee \neg QA4) \wedge (\neg QA3 \vee \neg QA4) \end{aligned}$$

$$\begin{aligned} \mathbf{C2} : & (\neg QB1 \vee \neg QB2) \wedge (\neg QB1 \vee \neg QB3) \wedge (\neg QB1 \vee \neg QB4) \\ & \wedge (\neg QB2 \vee \neg QB3) \wedge (\neg QB2 \vee \neg QB4) \wedge (\neg QB3 \vee \neg QB4) \end{aligned}$$

$$\begin{aligned} \mathbf{C3} : & (\neg QC1 \vee \neg QC2) \wedge (\neg QC1 \vee \neg QC3) \wedge (\neg QC1 \vee \neg QC4) \\ & \wedge (\neg QC2 \vee \neg QC3) \wedge (\neg QC2 \vee \neg QC4) \wedge (\neg QC3 \vee \neg QC4) \end{aligned}$$

$$\begin{aligned} \mathbf{C4} : & (\neg QD1 \vee \neg QD2) \wedge (\neg QD1 \vee \neg QD3) \wedge (\neg QD1 \vee \neg QD4) \\ & \wedge (\neg QD2 \vee \neg QD3) \wedge (\neg QD2 \vee \neg QD4) \wedge (\neg QD3 \vee \neg QD4) \end{aligned}$$

- There are no repeating queens in any diagonal.

$$\begin{aligned} \mathbf{D1} : & \neg QA1 \vee \neg QB2 \\ \mathbf{D2} : & \neg QA1 \vee \neg QC3 \\ \mathbf{D3} : & \neg QA1 \vee \neg QD4 \end{aligned}$$

$$\begin{aligned} \mathbf{D15} : & \neg QB2 \vee \neg QA3 \\ \mathbf{D16} : & \neg QB2 \vee \neg QC3 \\ \mathbf{D17} : & \neg QB2 \vee \neg QD4 \end{aligned}$$

$$\begin{aligned} \mathbf{D4} : & \neg QB1 \vee \neg QA2 \\ \mathbf{D5} : & \neg QB1 \vee \neg QC2 \\ \mathbf{D6} : & \neg QB1 \vee \neg QD3 \end{aligned}$$

$$\begin{aligned} \mathbf{D18} : & \neg QC2 \vee \neg QB3 \\ \mathbf{D19} : & \neg QC2 \vee \neg QA4 \\ \mathbf{D20} : & \neg QC2 \vee \neg QD3 \end{aligned}$$

$$\begin{aligned} \mathbf{D7} : & \neg QC1 \vee \neg QB2 \\ \mathbf{D8} : & \neg QC1 \vee \neg QA3 \\ \mathbf{D9} : & \neg QC1 \vee \neg QD2 \end{aligned}$$

$$\begin{aligned} \mathbf{D21} : & \neg QD2 \vee \neg QC3 \\ \mathbf{D22} : & \neg QD2 \vee \neg QB4 \end{aligned}$$

$$\begin{aligned} \mathbf{D10} : & \neg QD1 \vee \neg QC2 \\ \mathbf{D11} : & \neg QD1 \vee \neg QB3 \\ \mathbf{D12} : & \neg QD1 \vee \neg QA4 \end{aligned}$$

$$\begin{aligned} \mathbf{D23} : & \neg QA3 \vee \neg QB4 \\ \mathbf{D24} : & \neg QB3 \vee \neg QA4 \\ \mathbf{D25} : & \neg QB3 \vee \neg QC4 \end{aligned}$$

$$\begin{aligned} \mathbf{D13} : & \neg QA2 \vee \neg QB3 \\ \mathbf{D14} : & \neg QA2 \vee \neg QC4 \end{aligned}$$

$$\begin{aligned} \mathbf{D26} : & \neg QC3 \vee \neg QB4 \\ \mathbf{D27} : & \neg QC3 \vee \neg QD4 \\ \mathbf{D28} : & \neg QCD \vee \neg QC4 \end{aligned}$$

Note: I have tried to combine several clauses using AND into a row/column clause for better readability and tractability. If we write them separately, we get a total of 80 different clauses.

2 a. Table. 1 gives detailed calculations of DPLL algorithm with NO heuristics.

2 b. Table. 2 gives detailed calculations of DPLL algorithm with Pure Symbol and Unit Clause heuristics.

Note: Total number of iterations in naive DPLL is $2 \cdot 41 = 82$. Whereas, the DPLL with heuristics takes 37 iterations, which is almost a factor of half reduction in number of iterations.

3.

We consider the following literals to define the propositional logic for the tic-tac-toe problem.

- Positions of player X and O and void spaces (?):

$X11, X12, X13$	$O11, O12, O13$	$?11, ?12, ?13$
$X21, X22, X23$	$O21, O22, O23$	$?21, ?22, ?23$
$X31, X32, X33$	$O31, O32, O33$	$?31, ?32, ?33$

- Move positions:

moveX11, moveX12, moveX13
 moveX21, moveX22, moveX23
 moveX31, moveX32, moveX33

- canWin positions:

canWinX11, canWinX12, canWinX13	canWinO11, canWinO12, canWinO13
canWinX21, canWinX22, canWinX23	canWinO21, canWinO22, canWinO23
canWinX31, canWinX32, canWinX33	canWinO31, canWinO32, canWinO33
canWinX	canWinO

- forcedMove positions:

forcedMoveX11, forcedMoveX12, forcedMoveX13
 forcedMoveX21, forcedMoveX22, forcedMoveX23
 forcedMoveX31, forcedMoveX32, forcedMoveX33

Knowledge Base (KB):

- We need at least one move by X :

c1 : moveX11 \vee moveX12 \vee moveX13 \vee moveX21 \vee moveX22 \vee moveX23 \vee moveX31 \vee moveX32 \vee moveX33

- canWin moves for X 's and O 's:

Completing a row pattern:

c2 : $X11 \wedge X12 \wedge ?13 \implies \text{canWinX13}$	c5 : $O11 \wedge O12 \wedge ?13 \implies \text{canWinO13}$
c3 : $X21 \wedge X22 \wedge ?23 \implies \text{canWinX23}$	c6 : $O21 \wedge O22 \wedge ?23 \implies \text{canWinO23}$
c4 : $X31 \wedge X32 \wedge ?33 \implies \text{canWinX33}$	c7 : $O31 \wedge O32 \wedge ?33 \implies \text{canWinO33}$

Table 1: DPLL with NO Heuristics

Iteration No	QA1	QA2	QA3	QA4	QB1	QB2	QB3	QB4	QC1	QC2	QC3	QC4	QD1	QD2	QD3	QD4	Comments
1	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = T
2	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = F by C1
3	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	-	QA3 = F by C1
4	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	-	QA4 = F by C1
5	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	-	QB2 = F by R1
6	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	-	QB2 = F by D1
7	T	F	F	F	F	F	T	-	-	-	-	-	-	-	-	-	QB3 = T
8	T	F	F	F	F	F	T	F	-	-	-	-	-	-	-	-	QB4 = F by C2
9	T	F	F	F	F	F	T	F	F	-	-	-	-	-	-	-	QC1 = F by R1
10	T	F	F	F	F	F	T	F	F	F	-	-	-	-	-	-	QC2 = F by D18
11	T	F	F	F	F	F	T	F	F	F	F	-	-	-	-	-	QC3 = F by R3
12	T	F	F	F	F	F	T	F	F	F	F	-	-	-	-	-	BackTrack to Iter. 7
13	T	F	F	F	F	F	F	-	-	-	-	-	-	-	-	-	QB3 = F
14	T	F	F	F	F	F	F	T	-	-	-	-	-	-	-	-	QB4 = T
15	T	F	F	F	F	F	F	T	F	-	-	-	-	-	-	-	QC1 = F by R1
16	T	F	F	F	F	F	F	T	F	T	-	-	-	-	-	-	QC2 = T
17	T	F	F	F	F	F	F	T	F	T	F	-	-	-	-	-	QC3 = F by C3
18	T	F	F	F	F	F	F	T	F	T	F	F	-	-	-	-	QC4 = F by C3
19	T	F	F	F	F	F	F	T	F	T	F	F	F	-	-	-	QD1 = F by R1
20	T	F	F	F	F	F	F	T	F	T	F	F	F	F	-	-	QD2 = F by R2
21	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	-	QD3 = F by D20
22	T	F	F	F	F	F	F	T	F	T	F	F	F	F	F	F	QD4 BackTrack To Iter. 16
23	T	F	F	F	F	F	F	T	F	F	-	-	-	-	-	-	QC2 = F
24	T	F	F	F	F	F	F	T	F	T	F	-	-	-	-	-	QC3 = F by D26
25	T	F	F	F	F	F	F	T	F	T	F	F	-	-	-	-	BackTrack to Iter. 1
26	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = F
27	F	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = T
28	F	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	QA3 = F by C1
29	F	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	QA4 = F by C1
30	F	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	QB1 = F by D4
31	F	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	QB2 = F by R2
32	F	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	QB3 = F by D13
33	F	T	F	F	F	F	F	T	-	-	-	-	-	-	-	-	QB4 = T
34	F	T	F	F	F	F	F	T	T	-	-	-	-	-	-	-	QC1 = T
35	F	T	F	F	F	F	F	T	T	F	-	-	-	-	-	-	QC2 = F by C3
36	F	T	F	F	F	F	F	T	T	F	F	-	-	-	-	-	QC3 = F by C3
37	F	T	F	F	F	F	F	T	T	F	F	F	-	-	-	-	QC4 = F by C3
38	F	T	F	F	F	F	F	T	T	F	F	F	F	-	-	-	QD1 = F by R1
39	F	T	F	F	F	F	F	T	T	F	F	F	F	F	-	-	QD2 = F by D9
40	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	QD3 = T
41	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	QD4 = F by C4

Table 2: DPLL with Pure Symbol and Unit Clause heuristics

Iteration	QA1	QA2	QA3	QA4	QB1	QB2	QB3	QB4	QC1	QC2	QC3	QC4	QD1	QD2	QD3	QD4	Comments
1	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Q1A = T
2	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QA2 = F by C1
3	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QA3 = F by C1
4	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QA4 = F by C1
5	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	-	Unit Clause : QB1 = F by R1
6	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	-	Unit Clause : QB2 = F by D1
7	T	F	F	F	F	F	-	-	F	-	-	-	-	-	-	-	Unit Clause : QC1 = F by R1
8	T	F	F	F	F	F	-	-	F	-	F	-	-	-	-	-	Unit Clause : QC3 = F by D2
9	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	-	Unit Clause : QD1 = F by R1
10	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	F	Unit Clause : QD4 = F by D3
11	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	F	Q3B = T
12	T	F	F	F	F	F	T	F	F	-	F	-	F	-	-	F	Unit Clause: QB4 = F by C2
13	T	F	F	F	F	F	T	F	F	F	F	-	F	-	-	F	Unit Clause: QC2 = F by D18
14	T	F	F	F	F	F	T	F	F	F	F	-	F	-	F	F	Unit Clause: QD3 = F by R3
15	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	F	Q4C = F Backtrack to Iter. 11
16	T	F	F	F	F	F	F	-	F	-	F	-	F	-	-	F	Q3B = F
17	T	F	F	F	F	F	F	T	F	-	F	-	F	-	-	F	Q4B = T
18	T	F	F	F	F	F	F	T	F	-	F	F	F	-	-	F	Unit Clause: QC4 = F by R4
19	T	F	F	F	F	F	F	T	F	-	F	F	F	F	-	F	Unit Clause: QD2 = F by D22
20	T	F	F	F	F	F	F	T	F	T	F	F	F	F	-	F	Unit Clause: QC2 = F by D18
21	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Q3D = T/F Backtrack to Iter. 1
22	F	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA1 = F
23	F	T	-	-	-	-	-	-	-	-	-	-	-	-	-	-	QA2 = T
24	F	T	F	-	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA3 = F by C1
25	F	T	F	F	-	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA4 = F by C1
26	F	T	F	F	F	-	-	-	-	-	-	-	-	-	-	-	Unit Clause: QA4 = F by D4
27	F	T	F	F	F	F	-	-	-	-	-	-	-	-	-	-	Unit Clause: QB2 = F by R2
28	F	T	F	F	F	F	F	-	-	-	-	-	-	-	-	-	Unit Clause: QB3 = F by D13
29	F	T	F	F	F	F	F	-	-	F	-	-	-	-	-	-	Unit Clause: QC2 = F by R2
30	F	T	F	F	F	F	F	-	-	F	-	F	-	-	-	-	Unit Clause: QC4 = F by D14
31	F	T	F	F	F	F	F	-	-	F	-	F	-	F	-	-	Unit Clause: QD2 = F by R2
32	F	T	F	F	F	F	F	T	-	F	-	F	-	F	-	-	Q4B = T
33	F	T	F	F	F	F	F	T	-	F	F	F	-	F	-	-	Unit Clause: QC3 = F by D26
34	F	T	F	F	F	F	F	T	-	F	F	F	-	F	-	F	Unit Clause: QD4 = F by D27
35	F	T	F	F	F	F	F	T	T	F	F	F	-	F	-	F	QC1 = T
36	F	T	F	F	F	F	F	T	T	F	F	F	F	F	-	F	Pure Symbol: QD1 = F
37	F	T	F	F	F	F	F	T	T	F	F	F	F	F	T	F	Pure Symbol: QD3 = T

c8 : $X11 \wedge ?12 \wedge X13 \implies \text{canWin}X12$
c9 : $X21 \wedge ?22 \wedge X23 \implies \text{canWin}X22$
c10 : $X31 \wedge ?32 \wedge X33 \implies \text{canWin}X32$

c11 : $O11 \wedge ?12 \wedge O13 \implies \text{canWin}O12$
c12 : $O21 \wedge ?22 \wedge O23 \implies \text{canWin}O22$
c13 : $O31 \wedge ?32 \wedge O33 \implies \text{canWin}O32$

c14 : $?11 \wedge X12 \wedge X13 \implies \text{canWin}X11$
c15 : $?21 \wedge X22 \wedge X23 \implies \text{canWin}X21$
c16 : $?31 \wedge X32 \wedge X33 \implies \text{canWin}X31$

c17 : $?11 \wedge O12 \wedge O13 \implies \text{canWin}O11$
c18 : $?21 \wedge O22 \wedge O23 \implies \text{canWin}O21$
c19 : $?31 \wedge O32 \wedge O33 \implies \text{canWin}O31$

Completing a column pattern:

c20 : $X11 \wedge X21 \wedge ?31 \implies \text{canWin}X31$
c21 : $X12 \wedge X22 \wedge ?23 \implies \text{canWin}X23$
c22 : $X13 \wedge X23 \wedge ?33 \implies \text{canWin}X33$

c23 : $O11 \wedge O21 \wedge ?31 \implies \text{canWin}O31$
c24 : $O12 \wedge O22 \wedge ?23 \implies \text{canWin}O23$
c25 : $O13 \wedge O23 \wedge ?33 \implies \text{canWin}O33$

c26 : $X11 \wedge ?21 \wedge X31 \implies \text{canWin}X21$
c27 : $X12 \wedge ?22 \wedge X23 \implies \text{canWin}X22$
c28 : $X13 \wedge ?23 \wedge X33 \implies \text{canWin}X23$

c29 : $O11 \wedge ?21 \wedge O31 \implies \text{canWin}O21$
c30 : $O12 \wedge ?22 \wedge O23 \implies \text{canWin}O22$
c31 : $O13 \wedge ?23 \wedge O33 \implies \text{canWin}O23$

c32 : $?11 \wedge X21 \wedge X31 \implies \text{canWin}X11$
c33 : $?12 \wedge X22 \wedge X23 \implies \text{canWin}X12$
c34 : $?13 \wedge X23 \wedge X33 \implies \text{canWin}X13$

c35 : $?11 \wedge O21 \wedge O31 \implies \text{canWin}O11$
c36 : $?12 \wedge O22 \wedge O23 \implies \text{canWin}O12$
c37 : $?13 \wedge O23 \wedge O33 \implies \text{canWin}O13$

Completing a diagonal pattern:

c38 : $X11 \wedge X22 \wedge ?33 \implies \text{canWin}X33$
c39 : $X13 \wedge X22 \wedge ?31 \implies \text{canWin}X31$

c40 : $O11 \wedge O22 \wedge ?33 \implies \text{canWin}O33$
c41 : $O13 \wedge O22 \wedge ?31 \implies \text{canWin}O31$

c42 : $X11 \wedge ?22 \wedge X33 \implies \text{canWin}X22$
c43 : $X13 \wedge ?22 \wedge X31 \implies \text{canWin}X22$

c44 : $O11 \wedge ?22 \wedge O33 \implies \text{canWin}O22$
c45 : $O13 \wedge ?22 \wedge O31 \implies \text{canWin}O22$

c46 : $?11 \wedge X22 \wedge X33 \implies \text{canWin}X11$
c47 : $?13 \wedge X22 \wedge X31 \implies \text{canWin}X13$

c48 : $?11 \wedge X22 \wedge X33 \implies \text{canWin}O11$
c49 : $?13 \wedge X22 \wedge X31 \implies \text{canWin}O13$

c50 : $\text{canWin}X11 \vee \text{canWin}X12 \vee \text{canWin}X13$
 $\vee \text{canWin}X21 \vee \text{canWin}X22 \vee \text{canWin}X23$
 $\vee \text{canWin}X31 \vee \text{canWin}X32 \vee \text{canWin}X33 \implies \text{canWin}X$

c51 : $\text{canWin}O11 \vee \text{canWin}O12 \vee \text{canWin}O13$
 $\vee \text{canWin}O21 \vee \text{canWin}O22 \vee \text{canWin}O23$
 $\vee \text{canWin}O31 \vee \text{canWin}O32 \vee \text{canWin}O33 \implies \text{canWin}O$

- Conditions for winning moves by X :

c52 : canWinX11 \implies moveX11 **c55** : canWinX21 \implies moveX21 **c58** : canWinX31 \implies moveX31
c53 : canWinX12 \implies moveX12 **c56** : canWinX22 \implies moveX22 **c59** : canWinX32 \implies moveX32
c54 : canWinX13 \implies moveX13 **c57** : canWinX23 \implies moveX23 **c60** : canWinX33 \implies moveX33

- Conditions for forcedMoves by X :

c61 : canWinO11 \implies forcedMoveX11 **c64** : canWinO21 \implies forcedMoveX21
c62 : canWinO12 \implies forcedMoveX12 **c65** : canWinO22 \implies forcedMoveX22
c63 : canWinO13 \implies forcedMoveX13 **c66** : canWinO23 \implies forcedMoveX23

c67 : canWinO31 \implies forcedMoveX31
c68 : canWinO32 \implies forcedMoveX32
c69 : canWinO33 \implies forcedMoveX33

c70 : \neg canWinX \wedge forcedMoveX11 \implies moveX11 **c73** : \neg canWinX \wedge forcedMoveX21 \implies moveX21
c71 : \neg canWinX \wedge forcedMoveX12 \implies moveX12 **c74** : \neg canWinX \wedge forcedMoveX22 \implies moveX22
c72 : \neg canWinX \wedge forcedMoveX13 \implies moveX13 **c75** : \neg canWinX \wedge forcedMoveX23 \implies moveX23

c76 : \neg canWinX \wedge forcedMoveX31 \implies moveX31
c77 : \neg canWinX \wedge forcedMoveX32 \implies moveX32
c78 : \neg canWinX \wedge forcedMoveX33 \implies moveX33