# Real-Time Schedulability Tests for Preemptive Multitasking

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**Abstract.** When developing multitasking real-time systems, schedulability tests are used to formally prove that a given task set will meet its deadlines. A wide range of such tests have appeared in the literature. This tutorial acts as a guide to the major tests available for preemptive multitasking applications.

## 1. Introduction

Safety-critical applications often involve several distinct activities, each of which has 'hard' inviolable timing constraints (Burns and Wellings 1990). Developing such systems so as to guarantee that no critical task ever misses a deadline is a significant intellectual challenge.

In the past programmers resorted to a rigid, pre-determined order for execution of (parts of) tasks, so that the times at which jobs will complete could be predicted in advance. Unfortunately this *cyclic executive* model results in programs that are hard to understand and maintain because the code for logically independent tasks is interleaved. Such methods are now being superseded by *preemptive scheduling* approaches (Locke 1992).

However, because preemptive scheduling results in an order for task execution that changes dynamically, it is difficult to know in advance whether each task will meet its deadlines or not. A formal tool to overcome this is the notion of a *schedulability test*. Given information such as the execution time of each task, the frequency with which it needs to run, and the particular scheduling policy used, these tests tell the programmer whether a given task set can meet its deadlines.

Over the years a bewildering variety of these tests have been proposed, for a wide range of scheduling policies. They vary considerably in their complexity and capabilities. This review summarises the most well-known tests currently available, using consistent notations and terminology, in order to provide a comparative catalogue of the tests available. The survey considers tests for *hard real-time* tasks, executing under a *preemptive* scheduling policy, in a *uniprocessor* environment.

The tests work by considering a worst-case scenario and checking that each task gets a sufficient allocation of shared resources in this situation; if so then it will also succeed under more favourable circumstances. Historically, there have been two distinct approaches. Tests based on the notion of *processor utilisation*, i.e., the percentage of processor time that may be occupied by tasks, developed first. Later work saw the emergence of tests based on *response times*, i.e., the exact duration by which tasks may be delayed. Examples of

both methods are considered here. Ultimately, however, the two approaches have been recognised as equivalent (Lehoczky 1990, p. 203; Audsley et al. 1995, p. 182), offering the same capabilities in different styles.

As far as possible the tests are grouped into broad categories based on their capabilities. (This grouping is inevitably somewhat arbitrary due to the varying mix of features supported.)

This report is *not* a self-contained tutorial on schedulability theory; complementary tutorials by Audsley et al. (1995) and Mercer (1992) give gentler introductions to the field.

Section 2 briefly reviews the overall computational model assumed by scheduling theory. Section 3 describes schedulability tests developed for sets of independent, non-communicating tasks. Section 4 describes tests for interacting, communicating task sets. Section 5 presents a tabular summary of the capabilities of each test reviewed. Precise citations are given throughout so that the reader may easily access further information on any of the tests presented, especially their proofs of correctness and examples of their use. A glossary of terminology is provided as an appendix, again accompanied by precise citations throughout. Underlined text denotes a cross reference to the glossary.

## 2.   Computational Model for Preemptive Multitasking

Hard real-time schedulability theory uses a simplified, abstract computational model to represent the behaviour of a preemptive, multitasking system (Audsley et al. 1993, §2).

The model assumes that the programmer wishes to implement a task set on a particular processor. Each task $i$ arrives infinitely often, each arrival separated from the last by at least $T_i$ time units. A periodic task arrives regularly with a separation of *exactly* $T_i$ time units. A sporadic task arrives irregularly with each arrival separated from its predecessor by *at least* $T_i$ time units. (Aperiodic tasks, which arrive irregularly with no minimum separation, are not usually considered because hard real-time guarantees cannot be made for them.)

At each arrival, task $i$ issues a notional invocation request for up to $C_i$ units of processor time, its worst case computation time. Each invocation of task $i$ must have this request satisfied before its deadline $D_i$ expires.

The scheduler services these requests according to a particular *scheduling policy*. Each task making a request is notionally placed in a ready queue (ISO 1994, §D.2.1) at which time the task is said to be released (Audsley et al. 1993, §2). The scheduler selects a task to run from the ready queue as per the scheduling policy it implements. Tasks of higher priority can preempt the running task $i$, resulting in a degree of interference $I_i$ to the progress of task $i$. Tasks stop being ready by suspending themselves (by executing a 'delaying' statement). Suspended tasks wait in a separate delay queue until they become ready again (Burns et al. 1995, p. 476).

Scheduling decisions are based on the priority of ready tasks. In *static-priority* scheduling there is a fixed base priority associated with each task, although the task may temporarily acquire a higher active priority at run time. Static-priority policies include rate monotonic and deadline monotonic scheduling. When discussing static-priority policies we assume there are $n$ tasks, with (usually) *unique* base priorities, ranked from highest-priority task 1 to lowest-priority task $n$.

$C_i$  Worst case <u>computation time</u> that may be required by an invocation of task $i$.

$T_i$  Lower bound between successive <u>arrivals</u> of task $i$. This is the period of a periodic task and the minimum separation of the triggering event for a sporadic task.

$D_i$  The <u>deadline</u> for each invocation of task $i$, measured from its arrival time (Tindell et al. 1994, p. 149).[a] Usually $D_i \leq T_i$ (Audsley and Burns 1990, p. 6; Tindell et al. 1994, p. 134). If no deadline is specified for a periodic task $i$ then $D_i = T_i$ is assumed.

[a] Audsley et al. (1993, p. 285) measure $D_i$ from *release* time.

*Figure 1.* Task characteristics specified by the programmer (Audsley et al. 1993, §3).

*Dynamic-priority* scheduling policies assign variable priorities at run time. Dynamic-priority policies include <u>earliest deadline first</u> and <u>least laxity</u> scheduling. Dynamic-priority policies can lead to better processor utilisation than static-priority ones, but are harder to implement (Manabe and Aoyagi 1995, p. 213), and thus less common.

All communication between tasks residing on the same processor is through controlled access to shared variables, with mutual exclusion guaranteed by semaphores or a similar *locking protocol*. This communication model supports timing predictability because the worst-case <u>blocking time</u> $B_i$ that task $i$ may experience while awaiting access to a shared variable can be determined *a priori*. Locking protocols include the <u>priority ceiling</u>, <u>ceiling locking</u>, <u>kernelised monitor</u> and <u>stack resource</u> protocols.

For simplicity the computational model usually assumes that only tasks consume time. The scheduler itself is treated as if it executes instantaneously. The overheads of context switching, interrupt handling, and shared resource locking are factored into the worst case computation time for each task invocation (Klein and Ralya 1990, p. 12; Leung and Whitehead 1982, p. 238; Audsley et al 1994).

## 3.   Independent Tasks

The tests in this section apply to a set of $n$ tasks, residing on the same processor, when the tasks are independent (i.e., they do not interact or otherwise communicate using shared resources and hence cannot block one another). Mathematical symbols used are defined in Figures 1 and 2.

### 3.1.   *Deadlines Equal Periods*

The following tests apply to sets of independent <u>periodic</u> tasks whose deadlines are equal to their periods.

> $B_i$  Worst case blocking time an invocation of task $i$ may experience due to shared resources being locked by lower-priority tasks.
>
> $I_i$  Worst case interference an invocation of task $i$ may experience due to preemptions by higher-priority tasks.
>
> $U_i$  Worst case processor utilisation percentage by task $i$ and tasks of higher priority.
>
> $J_i$  Worst case release jitter for an invocation of task $i$ due to scheduling delays.
>
> $R_i$  Worst case response time for an invocation of task $i$, measured from its *arrival* time.[a] Usually $R_i = J_i + C_i + B_i + I_i$. A schedulable task must have $R_i \leq D_i$.
>
> $r_i$  Worst case response time for an invocation of task $i$, measured from its *release* time (Audsley et al. 1993, p. 286). Usually $r_i = C_i + B_i + I_i$. When there is no release jitter $r_i = R_i$.
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> ---
> [a]Tindell et al. (1994, p. 150) use "$r_i$" where we use "$R_i$".

*Figure 2.* Task characteristics defined by the scheduling policy and locking protocol used (Audsley et al. 1993, §3).

*Rate Monotonic Scheduling*

In a seminal paper, Liu and Layland (1973) proved that any such task set is schedulable using rate monotonic scheduling if

$$\sum_{i=1}^{n} \frac{C_i}{T_i} \leq n(2^{\frac{1}{n}} - 1). \tag{1}$$

Expression $C_i/T_i$ is the percentage of time that task $i$ will occupy the processor at each arrival, in the worst case. The test thus states that the sum of these processor utilisation percentages, for all $n$ tasks, must not exceed the 'utilisation bound' (Liu and Layland 1973, §5; Baker 1991, p. 68). The utilisation bound expression $n(2^{1/n} - 1)$ converges to 69% for large values of $n$, so task sets passing this test may have low processor utilisation. This is a sufficient but not necessary test (Audsley et al. 1993, p. 284).

Lehoczky et al. (1989) then developed a more discerning test. Let

$$U_i(t) = \frac{\sum_{j=1}^{i} C_j \left\lceil \frac{t}{T_j} \right\rceil}{t},$$

be the worst-case processor utilisation needed by all tasks $j$ of priority greater than or equal to $i$ during an interval of duration $t$ (Lehoczky et al. 1989, p. 167). The amount of processor time required is calculated as the sum of each computation time $C_j$ multiplied by the number of periods $\lceil t/T_j \rceil$ started by task $j$ within $t$ time units.[1] This is divided by $t$ to

give the utilisation percentage. The test then states that the task set is schedulable if,

$$\text{for all tasks } i, \quad \min_{0 < t \leq T_i} U_i(t) \leq 1. \tag{2}$$

In other words, for each task $i$, at every moment of time $t$ within its period $T_i$, processor utilisation $U_i$ up until time $t$ must not exceed 100%. The equation defining $U_i$ assumes the worst-case scenario where all tasks arrive simultaneously, so it is only necessary to test that each task $i$ will meet its *first* deadline to know that it will *never* miss a deadline (Lehoczky et al. 1989, p. 167).

Although harder to check, schedulability test 2 may pass task sets that test 1 fails, and thus allows higher processor utilisation. Test 2 is both necessary and sufficient (Lehoczky et al. 1989, p. 168).

A variant, also defined by Lehoczky et al. (1989), shows that rather than checking *all* times $t$ it is possible to reduce the number of calculations required for test 2 by checking only certain *scheduling points*. These are all multiples of the periods of each higher-priority task. For some task $i$ let

$$P_i = \left\{ kT_j \ \middle| \ 1 \leq j \leq i, \ \ k = 1, \ldots, \left\lfloor \frac{T_i}{T_j} \right\rfloor \right\}$$

be the scheduling points that occur within its period $T_i$, i.e., each absolute time of the $k^{\text{th}}$ arrival of each higher or equal priority task $j$ (Lehoczky et al. 1989, p. 167). Task $j$ can arrive up to $\lfloor T_i / T_j \rfloor$ times in $T_i$ time units.[2] A set of independent periodic tasks using rate monotonic scheduling will then meet all its deadlines if (Lehoczky et al. 1989, p. 168),

$$\text{for all tasks } i, \quad \min_{t \in P_i} U_i(t) \leq 1, \tag{3}$$

where utilisation $U_i(t)$ is as defined for test 2 above.

Manabe and Aoyagi (1995, §4) recently suggested a further optimisation of test 3. It defines the significant scheduling points for task $i$ as only $T_i$, the largest multiple of $T_{i-1}$ less than or equal this point, the largest multiple of $T_{i-2}$ less than or equal this new point, and so on.

*Earliest Deadline First Scheduling*

Since the dynamic earliest deadline first scheduling policy is optimal for independent tasks, Liu and Layland (1973, p. 186) showed that any task set is schedulable under this policy if processor utilisation by all tasks does not exceed 100% (Giering III and Baker 1994, p. 55). A sufficient and necessary test (Liu and Layland 1973, p. 183) for independent tasks using earliest deadline first scheduling is thus trivial (Liu and Layland 1973, p. 184; Baker 1991, p. 69):

$$\sum_{i=1}^{n} \frac{C_i}{T_i} \leq 1. \tag{4}$$

### 3.2. *Deadlines Less than Periods and Sporadic Tasks*

The following tests were developed to relax the assumptions that all tasks are periodic and have deadlines equal to their periods. They allow sporadic tasks to be introduced to the task set, and deadlines to be less than or equal to task interarrival times.

*Deadline Monotonic Scheduling*

Audsley et al. (1991, §2) observed that an invocation of some task $i$, in a task set with static priorities allocated according to the deadline monotonic scheduling policy, will experience worst-case interference

$$I_i = \sum_{j=1}^{i-1} \left\lceil \frac{D_i}{T_j} \right\rceil C_j$$

from tasks with higher deadline-monotonic priorities. $I_i$ is the sum, for each higher-priority task $j$, of its computation time $C_j$ multiplied by the number of times $\lceil D_i/T_j \rceil$ it arrives before the deadline $D_i$ of task $i$.

The entire deadline-monotonic task set can be scheduled if,

$$\text{for all tasks } i, \quad \frac{C_i}{D_i} + \frac{I_i}{D_i} \le 1. \tag{5}$$

Since deadlines can be less than their periods, $D_i$ is used instead of $T_i$ to determine utilisation. The test checks that the processor utilisation $C_i/D_i$ by task $i$ before its deadline, plus the degree of interference $I_i/D_i$ it experiences before the deadline, does not exceed 100%. This test is suitable for sporadic as well as periodic tasks (no explicit sporadic server is needed to handle sporadic events).

The above interference measure is overly pessimistic, however, because it includes the entire worst-case computation time $C_j$ of each task invocation that *arrives* even though part of task $j$ may complete *after* deadline $D_i$ has passed. A more precise definition is (Audsley et al. 1991, §2)

$$I_i = \sum_{j=1}^{i-1} \left( \left\lfloor \frac{D_i}{T_j} \right\rfloor C_j + \min \left( C_j, D_i - \left\lfloor \frac{D_i}{T_j} \right\rfloor T_j \right) \right).$$

For each higher priority task $j$, the left-hand term defines the overhead of whole invocations of task $j$ completed before $D_i$ and the right-hand term defines whatever fraction of $C_j$ can be accommodated in the time remaining before deadline $D_i$.

Both variants of test 5 are sufficient but not necessary (Audsley et al 1991, §2) because they assume a worst-case interleaving of higher-priority tasks that may never actually occur. A necessary test for deadline monotonic scheduling must account for the actual interleaving of tasks; the tests presented in Section 4.3 below do this. (Audsley et al. (1991, §2) also present an algorithm that achieves such a result.)

Recently, Manabe and Aoyagi (1995, §5) proposed a test for deadline monotonic scheduling, based on the principles illustrated in test 3. Again the test works by checking processor

utilisation for task $i$, and higher-priority tasks, at all its schedulability points (Manabe and Aoyagi 1995, p. 217):

$$\text{for all tasks } i, \quad \min_{t \in Q_i} U_i(t) \leq 1. \tag{6}$$

Term $U_i(t)$ is defined as for test 2 above. The new feature is the choice of scheduling points,

$$Q_i = \{D_i\} \cup \left\{ kT_j \,\middle|\, 1 \leq j \leq i - 1, \quad k = 1, \ldots, \left\lfloor \frac{D_i}{T_j} \right\rfloor \right\}.$$

Here the significant scheduling points for an invocation of task $i$ are its deadline $D_i$, and all multiples $k$ of the period of higher priority tasks $j$ that can arrive within $D_i$ time units. (As they did for test 3, Manabe and Aoyagi (1995, Def. 6) also suggest an optimisation of this definition to further reduce the number of scheduling points that need to be evaluated.)

*Static-Priority Scheduling*

The following tests apply to any static assignment of base priorities to tasks, including that defined by the rate monotonic and deadline monotonic scheduling policies.

Recently, Park et al. (1996) proposed an approach to schedulability testing based on linear programming. Unlike the other tests we consider, it can be used when worst-case computation times are not yet known. For each task $i$, their minimum utilisation bound $M_i$ is defined as the result of applying the following procedure (Park et al. 1996, p. 59):

$$\text{Find the smallest} \quad M_i = \sum_{j=1}^{i} \frac{C_j}{T_j}$$

$$\text{subject to} \quad \sum_{j=1}^{i} C_j \left\lceil \frac{D_i}{T_j} \right\rceil = D_i,$$

where each $C_j$ must be greater than 0. In other words, we must find suitable computation times for all tasks $j$, of equal or higher priority than $i$, that minimise the processor utilisation bound for task $i$. The condition on the second line ensures that all task arrivals by some task $j$, before the deadline $D_i$ of task $i$, are completed at exactly the deadline. (The *minimum utilisation bound* is found when there is no processor idling before the deadline (Park et al. 1996, p. 59).) Note that this allows us to *discover* suitable computation times, in terms of the task deadlines and periods, rather than supplying them.

An entire task set is then schedulable if,

$$\text{for all tasks } i, \quad \sum_{j=1}^{n} \frac{C_j}{T_j} \leq M_i. \tag{7}$$

In other words, the overall processor utilisation for all $n$ tasks must be less than every individual task utilisation bound $M_i$ (Park et al. 1996, p. 60).

The following test, unlike its predecessors, is not based on the principle of processor utilisation bounds. Joseph and Pandya (1986) showed that a task set will meet all its

deadlines if,

$$\text{for all tasks } i, \quad R_i \leq D_i,$$
$$\text{where} \quad R_i = C_i + I_i$$
$$\text{and} \quad I_i = \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j. \tag{8}$$

In other words, for each task $i$, the test checks that its response time $R_i$ is always less than its deadline $D_i$. The worst case response time for an invocation of task $i$ is defined to be its worst case computation time $C_i$ plus its worst case interference $I_i$.

Interference term $I_i$ determines how much preemption task $i$ will experience, due to higher-priority tasks $j$, during the interval of time defined by $R_i$. For each task $j$ this interference is execution time $C_j$ multiplied by the number of arrivals $\lceil R_i/T_j \rceil$ that $j$ may have in $R_i$ time units. Thus interference during $R_i$ is defined in terms of the number of preemptions that may occur during the interval of time defined by $R_i$: the definition of $R_i$ is recursive! Joseph and Pandya observe that "equations of this form do not lend themselves easily to analytical solution" (Joseph and Pandya 1986, p. 391), but see Section 4.3 below.

## 4.  Communicating Tasks

The tests above assumed that tasks do not interact. The tests in this section allow for communication between tasks on the same processor, via access to shared variables, controlled using a particular locking protocol.

### 4.1.  Deadlines Equal Periods

The following tests apply to communicating periodic (or sporadic with a periodic server) tasks whose deadlines equal their period.

*Rate Monotonic Scheduling*

Sha et al. (1987) showed that any such task set, when using the priority ceiling protocol, is schedulable if the following generalisation of test 1 holds (Baker 1991, p. 68):

$$\text{for all tasks } i, \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq i(2^{\frac{1}{i}} - 1). \tag{9}$$

Thus, for every task $i$, the sum of processor utilisation by equal or higher priority tasks $j$, plus processor utilisation lost due to blocking, must be less than the bound. Blocking figure $B_i$ is the worst-case computation time of the longest critical section of a task of *lower* priority than task $i$ (Chen and Lin 1990, p. 329; Baker 1991, p. 69). Expression $B_i/T_i$ is thus the worst case 'blocking utilisation' for task $i$: the task can be blocked at most once in period $T_i$ when the priority ceiling protocol is enforced.

An equivalent variant of test 9, applicable to the full set of $n$ tasks, is (Sha et al. 1989, p. 248)

$$\frac{C_1}{T_1} + \cdots + \frac{C_n}{T_n} + \max\left(\frac{B_1}{T_1}, \ldots, \frac{B_{n-1}}{T_{n-1}}\right) \leq n(2^{\frac{1}{n}} - 1). \tag{10}$$

Test 9 also applies when sporadic tasks are introduced using the sporadic server algorithm (Sprunt et al. 1989, p. 57).

A further generalisation of test 9 accounts for tasks whose priority does not obey rate monotonicity (Gomaa 1993, §11.4.8). Since the minimum arrival separation of a sporadic task, such as an interrupt server, may not necessarily be a measure of its true importance or required responsiveness, the test allows for the base priority of sporadic tasks to be higher than the value that would be allocated according to their interarrival times. (Under these circumstances tests 1 and 9 are not sufficient to guarantee schedulability (Gomaa 1993, p. 133).)

For some periodic task $i$ let $H_i$ be the set of tasks with higher priority than, and periods shorter than, task $i$. These tasks obey the usual rate monotonicity relationship. Let $S_i$ be those tasks with higher priority than, but periods *longer* than, task $i$. These tasks do *not* obey rate monotonic priority allocation. Preemption by such a task is treated in the test like blocking: like lower-priority tasks they can block task $i$ at most once during period $T_i$ because their period is longer than $T_i$. The schedulability test, assuming use of the priority ceiling protocol, is then

$$\text{for all tasks } i, \quad \sum_{j \in H_i} \frac{C_j}{T_j} + \frac{C_i}{T_i} + \frac{B_i}{T_i} + \sum_{k \in S_i} \frac{C_k}{T_i} \leq n(2^{\frac{1}{n}} - 1). \tag{11}$$

For each task $i$ the first term is the degree of interference $i$ can experience in its period due to tasks $j$ in $H_i$: each task $j$ may preempt an invocation of task $i$ several times because $T_j \leq T_i$. The second term is the processor utilisation of task $i$ itself. The third term is the degree of blocking $i$ may encounter from lower (rate monotonic) priority tasks. The fourth term is the preemption utilisation that $i$ can experience due to tasks $k$ in $S_i$: each task $k$ may preempt an invocation of task $i$ at most once because $T_k \geq T_i$, so $T_i$ is used instead of $T_k$ to calculate utilisation lost due to these tasks.

Again this test is not a necessary one and a generalisation of test 3 is also available that may pass task sets that test 11 fails. Sprunt et al. (1989, p. 57) use the following test in the situation where rate monotonic scheduling is used for periodic tasks except for a high-priority sporadic server task whose assigned priority may be higher than that warranted by its minimum interarrival time:

$$\text{for all tasks } i, \quad \min_{t \in P_i}\left(U_i(t) + \frac{B_i}{t}\right) \leq 1. \tag{12}$$

$P_i$ is defined as for test 3, and $U_i(t)$ as for test 2 above. At each scheduling point $t$ the first term defines utilisation due to periodic tasks with rate-monotonic priorities higher or equal to $i$, and the second is the percentage of blocking due to tasks with lower rate-monotonic priorities. However, when calculating the blocking time $B_i$, for a task $i$ that has a rate monotonic priority higher than the *rate monotonic* priority of a sporadic server, then the

execution time of that server must be counted as a possible source of 'blocking' (Sprunt et al. 1989, p. 58). Such high-priority servers may 'block' (in fact preempt!) task $i$ (once) during $T_i$.

*Earliest Deadline First Scheduling*

Test 4 can be trivially extended for systems using earliest deadline first scheduling and the kernelised monitor protocol (Chen and Lin 1990, p. 328):

$$\sum_{i=1}^{n} \frac{C_i + B}{T_i} \leq 1. \tag{13}$$

Here blocking time $B$ is the longest computation time that *any* task may spend in *any* critical section. If $B$ is large then task sets passing this test will have low utilisation.

  Chen and Lin (1990) then showed that a much tighter bound can be achieved when their dynamic version of the priority ceiling protocol is used instead of the kernelised monitor (Giering III and Baker 1994, p. 56; Baker 1991, p. 69):

$$\sum_{i=1}^{n} \left( \frac{C_i}{T_i} + \frac{B_i}{T_i} \right) \leq 1. \tag{14}$$

Blocking time in test 14 is again due to lower-priority tasks only, thus showing that this scheduling approach is more efficient than that assumed by test 13.

### 4.2.   Deadlines Less than Periods

The following test applies to communicating periodic tasks, and allows their deadlines to be less than or equal to their periods.

*Earliest Deadline First Scheduling*

Baker (1991) developed a tighter bound on utilisation than in test 14 by using the stack resource protocol (Baker 1990) to control semaphore locking, under earliest deadline first scheduling. A set of tasks, with higher priorities allocated to tasks with shorter (static!) deadlines $D_i$ (Baker 1991, §2.5.1), is then schedulable if (Baker 1991, p. 83),

$$\text{for all tasks } i, \quad \sum_{j=1}^{i} \frac{C_j}{D_j} + \frac{B_i}{D_i} \leq 1. \tag{15}$$

For each task $i$, $B_i$ is the execution time of the longest critical section in some task $k$ such that $D_i < D_k$. However even when task deadlines equal periods this test gives a better bound than test 14 (Baker 1991, p. 84) because the stack resource model offers better schedulability.

### 4.3. Deadlines Less than Periods and Sporadic Tasks

The following tests apply to communicating periodic, sporadic and, in later cases, sporadically periodic tasks, with deadlines that may be less than or equal to their interarrival times.

*Static-Priority Scheduling*

Audsley et al. (1993) define a general static-priority schedulability test for communicating tasks by extending test 8. It is suitable for rate monotonic and deadline monotonic priority allocations. The test applies to truly sporadic tasks as well as periodic ones.

A set of periodic and sporadic tasks is schedulable if,

$$\text{for all tasks } i, \quad R_i \leq D_i,$$
$$\text{where} \quad R_i = C_i + B_i + I_i$$
$$\text{and} \quad I_i = \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j. \tag{16}$$

As before, the test checks that the response time $R_i$ of task $i$ is always less than its deadline $D_i$. Assuming use of the priority ceiling protocol, blocking time $B_i$ equals the longest critical section of any lower priority task accessing a shared variable with a ceiling priority as great as the priority of task $i$ (Audsley et al. 1993, p. 286).

Again the definition of response time is recursive. Fortunately, however, Audsley et al. (1993, p. 287) show that it is possible to solve this equation iteratively. Let $R_i^x$ be the $x^{\text{th}}$ approximation to the value of $R_i$. Starting with $R_i^0 = 0$, equation

$$R_i^{x+1} = C_i + B_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i^x}{T_j} \right\rceil C_j$$

converges to $R_i$. Evaluation stops either when the equation has converged, i.e., $R_i^{x+1} = R_i^x$, or, because $R_i^{x+1} \geq R_i^x$ for any $x$, iteration can stop as soon as $R_i^{x+1} > D_i$ in which case the test has failed.

In cases where the scheduler implementation may introduce appreciable release jitter $J_i$ for task $i$, test 16 can be generalised to (Audsley et al. 1993, p. 288),[3]

$$\text{for all tasks } i, \quad R_i \leq D_i,$$
$$\text{where} \quad R_i = J_i + r_i$$
$$\text{and} \quad r_i = C_i + B_i + I_i$$
$$\text{and} \quad I_i = \sum_{j=1}^{i-1} \left\lceil \frac{r_i + J_j}{T_j} \right\rceil C_j. \tag{17}$$

Interval $r_i$ is used instead of $R_i$ to calculate interference because the task cannot be preempted before it is released. Also, whereas the worst-case scenario is usually a moment when all tasks arrive simultaneously, this is not the case when there may be release jitter. The worst

> $T_j$  Lower bound between successive arrivals of <u>sporadically periodic</u> task $j$'s outer 'period'.
>
> $t_j$  The inner period of sporadically periodic task $j$.
>
> $m_j$  The number of inner arrivals of sporadically periodic task $j$ per each outer arrival: $m_j t_j \leq T_j$.

*Figure 3.* Task characteristics specified for a sporadically periodic task (Audsley et al. 1993, p. 290).

case for some task $i$ in this situation is when it is *released* at the same moment that all higher-priority tasks $j$ are themselves released (Audsley et al. 1993, p. 288). The jitter value $J_j$ accounts for this by allowing a preemptive invocation of task $j$ to arrive *before* task $i$ is released. Again the recursive definition can be solved iteratively.

Audsley et al. (1993) then extend these tests to allow for <u>sporadically periodic</u> tasks. Figure 3 shows the symbols used in this case.

A sporadically-periodic task $j$ has sporadic 'outer' arrivals separated by $T_j$ but, having arrived, then rearrives repeatedly according to some 'inner' period $t_j$ a fixed number of times $m_j$ (Audsley et al. 1993, §5). This behaviour is found in "bursty" communications media, for instance. To use the inner period as the task's interarrival time would be unnecessarily pessimistic and lead to low processor utilisation, so a more complex analysis is desirable.

The approach is to extend the definition of $I_i$ from test 16 to account for preemptions by a higher-priority sporadically-periodic task $j$ (Audsley et al. 1993, p. 290):

$$
\text{for all tasks } i, \quad R_i \leq D_i,
$$
$$
\text{where} \quad R_i = C_i + B_i + I_i
$$
$$
\text{and} \quad I_i = \sum_{j=1}^{i-1} \left( \min \left( \left\lceil \frac{R_i - T_j \left\lfloor \frac{R_i}{T_j} \right\rfloor}{t_j} \right\rceil, m_j \right) + m_j \left\lfloor \frac{R_i}{T_j} \right\rfloor \right) C_j. \quad (18)
$$

As before, interference $I_i$, due to each higher priority task $j$, is determined by the number of arrivals of $j$ multiplied by its execution time $C_j$, in the interval $R_i$. The second major term in the summation determines the total number of arrivals occurring due to 'complete' outer arrivals in $R_i$, i.e., the number of outer arrivals $\lfloor R_i/T_j \rfloor$ times the number of inner arrivals $m_j$ per outer arrival. The first major term is the number of inner arrivals due to a remaining 'incomplete' outer arrival. This is found by subtracting the time consumed by complete outer arrivals $T_j \lfloor R_i/T_j \rfloor$ from $R_i$ and determining how many inner periods $t_j$ can occur in the remaining time, bounded by the maximum number of such arrivals $m_j$. (This new definition of $I_i$ is a generalisation of the previous one. For tasks in which $t_j = T_j$, i.e., standard periodic or sporadic tasks, it simplifies to the definition in test 16 (Audsley et al. 1993, p. 290).)

> $S_i$   Tick scheduling overheads encountered while an invocation of task $i$ is runnable.
>
> $T_{tic}$   Polling period of tick scheduler.
>
> $C_{int}$   Computation time required to service an interrupt (excluding the interrupt routine itself).
>
> $C_{ql}$   Computation time required to 'awaken' the first suspended task (i.e., move it from the delay queue to the ready queue).
>
> $C_{qs}$   Computation time required to awaken a subsequent suspended task (assumed to be significantly smaller than handling the first one).

*Figure 4.* Tick scheduler implementation characteristics (Burns et al., 1995).

A further generalisation of the interference definition in test 17 again allows for release jitter (Audsley et al. 1993, p. 291):[4]

$$\text{for all tasks } i, \quad R_i \leq D_i,$$
$$\text{where} \quad R_i = J_i + r_i$$
$$\text{and} \quad r_i = C_i + B_i + I_i$$
$$\text{and} \quad I_i = \sum_{j=1}^{i-1} \left( \min \left( \left\lceil \frac{J_j + r_i - T_j \left\lfloor \frac{J_j + r_i}{T_j} \right\rfloor}{t_j} \right\rceil, m_j \right) \right.$$
$$\left. + m_j \left\lfloor \frac{J_j + r_i}{T_j} \right\rfloor \right) C_j. \tag{19}$$

Other extensions to this method allow for more accurate characterisations of implementation overheads due to tick scheduling (Burns et al. 1995). Figure 4 shows the special symbols used in this case.

Although a tick scheduler can be modelled as a high-priority periodic process, this has been found to be overly pessimistic because the worst case execution of the scheduler, when all tasks need to be moved to the ready queue simultaneously, occurs rarely (Burns et al. 1995, p. 477; Burns and Wellings 1995, p. 715). The following test is more precise. It assumes that periodic tasks are awoken by a polling tick scheduler with period $T_{tic}$. Thus a periodic task $i$ will experience release jitter proportional to the phase difference between its own period $T_i$ and that of the scheduler (Burns et al. 1995, p. 479):

$$J_i = T_{tic} - gcd(T_{tic}, T_i), \quad \text{for periodic task } i.$$

For a sporadic task $i$ initiated by an external interrupt there is no release jitter (although the overheads of implementing the interrupt handler must be included in $C_i$). However a sporadic task whose arrival is signalled by another task may have to wait for up to $T_{tic}$ time

units to be alerted to the event by the scheduler:

$$J_i = T_{tic}, \quad \text{for 'polled' sporadic task } i.$$

Another factor to be considered is the delay caused to an invocation of task $i$ by occurrences of the tick scheduler (Burns et al. 1995, p. 478; Tindell et al. 1994, §6). Task $i$ runs for up to $r_i$ time units. The tick scheduler can be invoked

$$L_i = \left\lceil \frac{r_i}{T_{tic}} \right\rceil$$

times during this interval. We can also determine that the scheduler will be required to move some task $j$ from the delay to ready queues up to

$$K_i = \sum_{j=1}^{n} \left\lceil \frac{\left\lceil \frac{r_i}{T_{tic}} \right\rceil T_{tic}}{T_j} \right\rceil$$

times while $i$ is running. This is the interval $\lceil r_i/T_{tic} \rceil T_{tic}$ in which the tick scheduler may be invoked divided by the interarrival time $T_j$ of $j$. (This assumes each task $j$ suspends itself only at the end of each invocation.) The overall interference caused by the tick scheduler to an invocation of task $i$ is then

$$S_i = L_i C_{int} + \min(K_i, L_i) C_{ql} + \max(K_i - L_i, 0) C_{qs}.$$

The first term is the basic cost of handling $L_i$ interrupts. The second term is the overhead of moving 'first' tasks, if any. This is the (long) time $C_{ql}$ to move each one multiplied by the number to be shifted; this may be as low as $K_i$ times, but no more than $L_i$. The third term is the cost of moving any remaining task invocations; this is the (short) computation time $C_{qs}$ multiplied by the total number to be moved $K_i$ less the number of 'first' tasks $L_i$, or 0 if no such tasks remain.

The test is then (Burns et al. 1995, p. 479),

$$\begin{aligned}
\text{for all tasks } i, \quad & R_i \leq D_i, \\
\text{where} \quad & R_i = J_i + r_i \\
\text{and} \quad & r_i = C_i + B_i + I_i + S_i \\
\text{and} \quad & I_i = \sum_{j=1}^{i-1} \left\lceil \frac{\left\lceil \frac{r_i}{T_{tic}} \right\rceil T_{tic}}{T_j} \right\rceil C_j.
\end{aligned} \tag{20}$$

The response time now includes an explicit allowance $S_i$ for tick scheduling overheads. The interference term replaces the simple bound from test 17 with an exact bound expressed in terms of how many times the tick scheduler can invoke each higher-priority task $j$ during time interval $r_i$.

Burns and Wellings (1995) present a further variant of this approach in which the overheads of the initial and final context switch associated with each task invocation, and the overheads of implementing the interrupt handler associated with a sporadic task, are expressed explicitly, rather than being incorporated into each $C_i$. They also discuss the practicalities

> $w_i$  Level $i$ busy period, the longest interval during which tasks of priority equal
> or greater than $i$ are continuously executing.

*Figure 5.* Task characteristic defined for tasks with arbitrary deadlines (Tindell 1994, §7).

of dealing with these and other system timing characteristics in an Ada programming environment. In particular, they show how to factor application-specific characteristics into the schedulability test, such as tasks with worst-case computation times that vary according to a known pattern, and task sets that never encounter the worst-case situation in which they all arrive simultaneously.

### 4.4. Arbitrary Deadlines

The following tests apply to communicating periodic, sporadic and, in some cases, sporadically-periodic tasks, with arbitrary deadlines that may *exceed* their interarrival times. Special notation used in this situation is shown in Figure 5.

*Static-Priority Scheduling*

All of the tests below can be used for any static-priority scheduling policy, including rate and deadline-monotonic scheduling. Unique static priorities are assumed (Lehoczky 1990, p. 202).

Lehoczky (1990) showed that a processor-utilisation based schedulability test can be established in terms of a level $i$ busy period, i.e., the total time for which tasks of priority level $i$ or higher execute continuously. When deadlines may exceed periods there can be more more than one incomplete invocation of some task $i$ in existence at a time. The computational model adopted assumes that later invocations are delayed until all earlier ones have completed (Tindell 1994, §2).

Firstly, define

$$w_i(x, t) = \sum_{j=1}^{i-1} C_j \left\lceil \frac{t}{T_j} \right\rceil + xC_i$$

to be the duration of the level $i$ busy period for up to $x$ invocations of task $i$ and however many invocations of higher-priority tasks $j$ can occur in a window of duration $t$ time units. (Lower-priority tasks are ignored (Lehoczky 1990, p. 203), so blocking time is not considered by this test.) From this we can determine the processor utilisation by these tasks, in this window, as follows (Lehoczky 1990, p. 203):

$$U_i(x, t) = \min_{u \leq t} \frac{w_i(x, u)}{t}.$$

The minimum such utilisation value represents the time at which the $x^{\text{th}}$ invocation of task $i$ is completed (Lehoczky 1990, p. 203).

The maximum number of invocations of task $i$ that occur in a level $i$ busy period is then (Lehoczky 1990, p. 204)

$$N_i = \min\{x \mid U_i(x, xT_i) \leq 1\}.$$

This is the smallest number of invocations $x$ such that the level $i$ utilisation, within $x$ periods $T_i$, is less than 100%. That is, all $x$ invocations of task $i$ have completed within this interval.

The schedulability test is then,

$$\text{for all tasks } i, \quad \max_{x \leq N_i} U_i(x, (x-1)T_i + D_i) \leq 1. \tag{21}$$

For each task $i$, the test checks that the maximum processor utilisation for all $N_i$ invocations of task $i$ occurring in a level $i$ busy period is less then 100%, when the $x^{\text{th}}$ such invocation is required to complete before its deadline. The deadline, measured from the start of the busy period, for the $x^{\text{th}}$ invocation of task $i$ is $D_i$ plus the $x^{\text{th}}$ arrival time, which is $x-1$ times period $T_i$.

Tindell et al. (1994) then extended this result to define a response-time test that includes blocking times. Their schedulability test defines a worst-case response time $R_i$ in terms of a busy period window $w_i$ for a particular task invocation:

$$\text{for all tasks } i, \quad R_i \leq D_i, \tag{22}$$
$$\text{where} \quad R_i = \max_{q=0,1,2,\ldots} (w_i(q) - qT_i)$$
$$\text{and} \quad w_i(q) = (q+1)C_i + B_i + \sum_{j=1}^{i-1} \left\lceil \frac{w_i(q)}{T_j} \right\rceil C_j.$$

Term $w_i(q)$ represents the duration of the level $i$ busy period that includes the $(q+1)^{\text{th}}$ overlapping invocation of task $i$, i.e., covering the execution of this task invocation and $q$ preceding (incomplete) invocations. Thus the response time for any invocation of task $i$ is found by taking the window $w_i(q)$ that includes this invocation, and subtracting the part of the window $qT_i$ due to the $q$ previous invocations. The worst-case response time $R_i$ is then the maximum such value for any $q$. The test is bounded, however, because it can stop when a value of $q$ is reached such that $w_i(q) \leq (q+1)T_i$, i.e., when the $q^{\text{th}}$ occurrence of task $i$ completed *before* the next arrival. This marks the end of the busy period for task $i$ (at which time a lower priority task is free to run).

In calculating $w_i(q)$, the first term $(q+1)C_i$ is the execution time required by this invocation and its predecessors. The blocking term $B_i$ assumes that the priority ceiling protocol is used and equals the longest critical section executed by a task of lower priority than $i$ where the shared object has a priority ceiling at least as high as the priority of task $i$. While task $i$ is 'busy' a lower-priority task can block the *whole* set of incomplete invocations of $i$ only once (Tindell et al. 1994, pp. 138–9)! The final term defines the preemption time encountered by task $i$ due to higher-priority tasks $j$ in the interval $w_i(q)$, i.e., the worst-case number of arrivals $\lceil w_i(q)/T_j \rceil$ of task $j$ during $w_i(q)$, multiplied by its worst-case execution time $C_j$. As for test 16, the recursive equation can be solved iteratively (Tindell et al. 1994, p. 138).

Numerous variants of test 22 have been proposed. Release jitter can be accounted for with an extension analogous to that introduced in test 17 (Tindell and Clark 1994, p. 120; Tindell et al. 1994, §4):

$$\text{for all tasks } i, \quad R_i \leq D_i, \tag{23}$$

$$\text{where} \quad R_i = \max_{q=0,1,2,\ldots} (J_i + w_i(q) - qT_i)$$

$$\text{and} \quad w_i(q) = (q+1)C_i + B_i + \sum_{j=1}^{i-1} \left\lceil \frac{J_j + w_i(q)}{T_j} \right\rceil C_j.$$

Sporadically periodic tasks can also be incorporated (Tindell et al. 1994, §5). Here we need to know the number $P_i$ of complete, outer invocations of sporadically periodic task $i$ active at the beginning of the window of interest. For the $q^{\text{th}}$ successive, incomplete invocation of $i$ there can be

$$P_i = \left\lfloor \frac{q}{m_i} \right\rfloor$$

such arrivals. Similarly, the number $p_i$ of remaining incomplete inner invocations of $i$ can be determined. For the $q^{\text{th}}$ instance of $i$ this must be $q$ less the number of incomplete $i$ tasks due to whole outer arrivals, i.e.,

$$p_i = q - P_i m_i.$$

To account for higher-priority, preemptive, sporadically-periodic tasks $j$, we must allow for the number of arrivals they may have in the window of interest. Allowing for release jitter, let

$$F_j = \left\lceil \frac{J_j + w_i(q)}{T_j} \right\rceil - 1$$

be the number of complete outer periods of higher-priority task $j$ occurring in window $w_i(q)$, less the last one which can be treated as a simple periodic task (Tindell et al. 1994, p. 143). Similarly, let

$$f_j = \left\lceil \frac{J_j + w_i(q) - F_j T_j}{t_j} \right\rceil$$

be a bound on the number of inner invocations of $j$ in the remaining part of the window, i.e., the amount of time remaining once the outer invocations are removed, divided by the inner period $t_j$.

The test then becomes,

$$\text{for all tasks } i, \quad R_i \leq D_i,$$

$$\text{where} \quad R_i = \max_{q=0,1,2,\ldots} (J_i + w_i(q) - p_i t_i - P_i T_i)$$

$$\text{and} \quad w_i(q) = (P_i m_i + p_i + 1)C_i + B_i$$

$$+ \sum_{j=1}^{i-1} (\min(m_j, f_j) + F_j m_j)C_j. \tag{24}$$

$C_i^D$  The worst case computation time required by task $i$ to complete all externally observable events: $C_i^D \leq C_i^T$ and $C_i^D \leq R_i^D$.

$C_i^T$  The worst case computation time required by task $i$ to complete all activity: $C_i^T \leq R_i^T$ but $C_i^T$ may exceed $D_i$.

$R_i^D$  The worst case time by which task $i$ must complete all externally observable events: $R_i^D \leq R_i^T$ and $R_i^D \leq D_i$.

$R_i^T$  The worst case time by which task $i$ must complete all activity: $R_i^T$ may exceed $D_i$.

*Figure 6.* Task characteristics defined for tasks with deadlines prior to completion times (Burns et al. 1994).

In calculating $R_i$ we subtract all the previous, incomplete outer $F_i$ and inner $f_i$ arrivals of $i$ multiplied by their interarrival times $T_i$ and $t_i$, respectively. The recursive definition of $w_i(q)$ is (a) the number of outer $P_i m_i$ and inner $p_i$ invocations of $i$, plus the current invocation, multiplied by its execution time $C_i$, plus (b) the blocking time $B_i$, plus (c) the number of preemptions from higher-priority tasks $j$ multiplied by their worst case execution time $C_j$. The number of preemptions is determined by the number of arrivals of $j$ due to complete outer periods $F_j m_j$ and the least upper bound on the remaining number of inner arrivals $\min(m_j, f_j)$.

Another variant of test 22 allows for the case where each task must complete all externally observable activity before the deadline $D_i$, but internal activities can continue *after* the deadline has elapsed (Burns et al. 1994, §3). Task behaviour is divided into two parts, that which must be completed by the deadline, and 'local' actions that may be performed after the deadline. Special symbols used in this case are shown in Figure 6. Schedulability test 22 then becomes,

$$\text{for all tasks } i, \quad R_i^D \leq D_i,$$
$$\text{where} \quad R_i^D = \max_{q=0,1,2,\dots} (w_i(q) - qT_i)$$
$$\text{and} \quad w_i(q) = qC_i^T + C_i^D + B_i + \sum_{j=1}^{i-1} \left\lceil \frac{w_i(q)}{T_j} \right\rceil C_j^T. \tag{25}$$

The definition of window $w_i(q)$ requires the first $q$ occurrences of $i$ to have completed their entire computation time $C_i^T$, but it is sufficient for the $(q+1)^{\text{th}}$ occurrence to have completed only $C_i^D$ units of execution time before the deadline. (Interference due to higher-priority tasks $j$ still depends on their total execution times $C_j^T$. Whether $j$ is performing externally observable behaviour or not it still prevents task $i$ from progressing.)

Tindell and Clark (1994) define an extension of tests 22 and 20 to allow for tick scheduling overheads when deadlines exceed periods and the task set contains only sporadic and periodic tasks.

In this case the tick scheduler can be invoked

$$L_i = \left\lceil \frac{w_i(q)}{T_{tic}} \right\rceil$$

times during busy period $w_i(q)$. We can also determine that the scheduler will be required to move a task $j$ from the delay to dispatch queues up to

$$K_i = \sum_{j=1}^{i-1} \left\lceil \frac{J_j + w_i(q)}{T_j} \right\rceil$$

times while overlapping invocations of $i$ are runnable.[5] The overall interference $S_i$ caused by the tick scheduler to an invocation of task $i$ is calculated as for test 20. The test is then (Tindell and Clark 1994, p. 121),

for all tasks $i$,    $R_i \leq D_i$,

   where    $R_i = \max_{q=0,1,2,\ldots} (J_i + w_i(q) - qT_i)$

   and    $w_i(q) = (q+1)C_i + \sum_{j=1}^{i-1} \left\lceil \frac{J_j + w_i(q)}{T_j} \right\rceil C_j + S_i.$    (26)

(Blocking time has been ignored in this test.)

A further extension to test 24 then allows for sporadically periodic tasks and tick scheduling (Tindell et al. 1994, §6). The values for $P_i$, $p_i$, $F_j$, $f_j$ are calculated as for test 24. The worst case number of times tasks move from the pending to ready queues during $w_i(q)$ is (Tindell et al. 1994, p. 144)

$$K_i = \sum_{j=1}^{n} \min(m_j, f_j) + m_j F_j,$$

i.e., the arrivals $m_j F_j$ due to complete outer arrivals of $j$, plus the least upper bound on the possible inner arrivals in the remaining time $\min(m_j, f_j)$. Values of $L_i$ and $S_i$ are calculated as per test 20. The test is then,

for all tasks $i$,    $R_i \leq D_i$,

   where    $R_i = \max_{q=0,1,2,\ldots} (J_i + w_i(q) - p_i t_i - P_i T_i)$

   and    $w_i(q) = (P_i m_i + p_i + 1)C_i + B_i$

$$+ \sum_{j=1}^{i-1} \left( \min(m_j, f_j) + F_j m_j \right) C_j + S_i.$$    (27)

## 5.    Schedulability Tests Summary

The tables in this section summarise the major capabilities of the schedulability tests presented in this report.

Table 1 shows the types of tasks that the tests can deal with. The column labelled 'sporadic' is for truly sporadic tasks with no explicit task server. Presumably *any* of

*Table 1.* Types of tasks modelled.

| Test | periodic | sporadic | sporadic with server | sporadically periodic |
|------|----------|----------|----------------------|-----------------------|
| 1  | ✓ |   |   |   |
| 2  | ✓ |   |   |   |
| 3  | ✓ |   |   |   |
| 4  | ✓ |   |   |   |
| 5  | ✓ | ✓ |   |   |
| 6  | ✓ | ✓ |   |   |
| 7  | ✓ |   | ✓ |   |
| 8  | ✓ | ✓ |   |   |
| 9  | ✓ |   |   |   |
| 10 | ✓ |   |   |   |
| 11 | ✓ | ✓ | ✓ |   |
| 12 | ✓ |   | ✓ |   |
| 13 | ✓ |   |   |   |
| 14 | ✓ |   |   |   |
| 15 | ✓ |   |   |   |
| 16 | ✓ | ✓ |   |   |
| 17 | ✓ | ✓ |   |   |
| 18 | ✓ | ✓ |   | ✓ |
| 19 | ✓ | ✓ |   | ✓ |
| 20 | ✓ | ✓ |   |   |
| 21 | ✓ | ✓ |   |   |
| 22 | ✓ | ✓ |   |   |
| 23 | ✓ | ✓ |   |   |
| 24 | ✓ | ✓ |   | ✓ |
| 25 | ✓ | ✓ |   |   |
| 26 | ✓ | ✓ |   |   |
| 27 | ✓ | ✓ |   | ✓ |

*Table 2.* Scheduling policies modelled.

| | Scheduling policy | | | | |
|---|---|---|---|---|---|
| | Static priorities | | | Dynamic priorities | |
| Test | rate monotonic | deadline monotonic | any static allocation | earliest deadline first | least laxity |
| 1 | ✓ | | | | |
| 2 | ✓ | | | | |
| 3 | ✓ | | | | |
| 4 | | | | ✓ | |
| 5 | | ✓ | | | |
| 6 | | ✓ | | | |
| 7 | ✓ | ✓ | ✓ | | |
| 8 | ✓ | ✓ | ✓ | | |
| 9 | ✓ | | | | |
| 10 | ✓ | | | | |
| 11 | ✓ | | | | |
| 12 | ✓ | | | | |
| 13 | | | | ✓ | |
| 14 | | | | ✓ | |
| 15 | | | | ✓ | |
| 16 | ✓ | ✓ | ✓ | | |
| 17 | ✓ | ✓ | ✓ | | |
| 18 | ✓ | ✓ | ✓ | | |
| 19 | ✓ | ✓ | ✓ | | |
| 20 | ✓ | ✓ | ✓ | | |
| 21 | ✓ | ✓ | ✓ | | |
| 22 | ✓ | ✓ | ✓ | | |
| 23 | ✓ | ✓ | ✓ | | |
| 24 | ✓ | ✓ | ✓ | | |
| 25 | ✓ | ✓ | ✓ | | |
| 26 | ✓ | ✓ | ✓ | | |
| 27 | ✓ | ✓ | ✓ | | |

*Table 3.* Locking protocols modelled.

| | Locking protocol | | | | |
|---|---|---|---|---|---|
| Test | priority ceiling | ceiling locking | dynamic priority ceiling | kernelised monitor | stack resource |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | √ | √ | | | |
| 10 | √ | √ | | | |
| 11 | √ | √ | | | |
| 12 | √ | √ | | | |
| 13 | | | | √ | |
| 14 | | | √ | | |
| 15 | | | | | √ |
| 16 | √ | √ | | | |
| 17 | √ | √ | | | |
| 18 | √ | √ | | | |
| 19 | √ | √ | | | |
| 20 | √ | √ | | | |
| 21 | | | | | |
| 22 | √ | √ | | | |
| 23 | √ | √ | | | |
| 24 | √ | √ | | | |
| 25 | √ | √ | | | |
| 26 | | | | | |
| 27 | √ | √ | | | |

*Table 4.* Relationship of deadlines to interarrival times.

| Test | Deadlines vs. interarrival times | | |
|:---:|:---:|:---:|:---:|
| | deadlines equal periods | deadlines less than periods | deadlines greater than periods |
| 1 | √ | | |
| 2 | √ | | |
| 3 | √ | | |
| 4 | √ | | |
| 5 | √ | √ | |
| 6 | √ | √ | |
| 7 | √ | √ | |
| 8 | √ | √ | |
| 9 | √ | | |
| 10 | √ | | |
| 11 | √ | | |
| 12 | √ | | |
| 13 | √ | | |
| 14 | √ | | |
| 15 | √ | √ | |
| 16 | √ | √ | |
| 17 | √ | √ | |
| 18 | √ | √ | |
| 19 | √ | √ | |
| 20 | √ | √ | |
| 21 | √ | √ | √ |
| 22 | √ | √ | √ |
| 23 | √ | √ | √ |
| 24 | √ | √ | √ |
| 25 | √ | √ | √ |
| 26 | √ | √ | √ |
| 27 | √ | √ | √ |

the tests would allow a sufficiently-determined programmer to implement a sporadic task server as a periodic task, but only tests 11 and 12 make *explicit* allowance for the possibility. Park et al. (1996, p. 61) briefly mention that test 7 is suitable for use with a sporadic server. Gomaa (1993, §11.4.8), when discussing test 11, refers to sporadic tasks initiated by external interrupts, so we infer that the test is suitable for general sporadic tasks. Lehoczky (1990) and Manabe and Aoyagi (1995), when discussing tests 21 and 6, respectively, refer to periodic tasks only, but we see no impediment in applying these tests to sporadic tasks.

Not shown in table 1 are aperiodic tasks handled by the priority exchange or deferrable server algorithms (Sprunt et al. 1989) because (ideally!) any soft real-time tasks absorb idle time only and thus do not feature in these schedulability tests. (In fact both algorithms may introduce some degree of interference, and the priority exchange algorithm even makes the task set unstable!)

Table 2 shows the scheduling policies assumed by the tests. Tests that allow any static priority allocation have been marked as supporting rate and deadline monotonic scheduling too because such a capability can be used to construct a rate or deadline monotonic priority allocation if desired. None of the tests in our survey assumed least laxity scheduling.

Table 3 shows the shared variable locking protocols supported. Those tests with no check marks do not consider task communication at all. In a worst case scenario the behaviour of the priority ceiling and ceiling locking protocols is identical, so those tests designed for the priority ceiling protocol have been shown as suitable for both.

Burns et al. (1994) do not state which locking protocol is anticipated for test 25 but, given the test's lineage, it is safe to assume suitability for priority ceiling locking. Although Tindell and Clark (1994) do not include a blocking term in test 26, there seems to be no impediment to easily making such an allowance. Similarly, test 21 could be easily extended to consider blocking times. Park et al. (1996, p. 62) claim that test 7 can be easily extended with a blocking term for the priority ceiling protocol but, since this is not demonstrated in their paper, we have not marked this in the table.

Table 4 shows the allowed relationships of deadlines to interarrival times. Tests that allow deadlines less than or equal to periods can, of course, always handle the case of deadlines equal to periods, and tests that allow arbitrary deadlines can handle all three situations.


## 6.  Conclusions


Schedulability tests have progressed in sophistication (and complexity!) considerably in the last few years. As the tests have moved from academia to industrial applications, they have been forced to increase their scope. The concept of sporadically periodic tasks, and the addition of tick scheduling overheads into some of the later tests, were both in response to the need for more accuracy in particular practical applications. Other application-specific characteristics have been used to customise tests (Burns and Wellings 1995) and variations have been suggested to remove the simplifying assumption that context-switching overheads are accounted for in task execution times (Sha et al. 1991, §3). Another area of investigation is the addition of offsets to task release times so that the worst-case scenario in which all tasks want to be released simultaneously never occurs. The analysis required for this is

surprisingly complex, and an exact schedulability test has proven to be computationally infeasible (Tindell 1994, p. 11)!

Indeed, as the tests have become more and more complex it is natural to ask if they can be automated. In fact tools to do so are already appearing (Audsley et al. 1995, p. 192). For instance, the STRESS scheduling simulator developed by the University of York (Audsley et al. 1994) has a number of in-built feasibility tests. The SEW toolset, developed at Carnegie Mellon University (Strosnider 1995), is even more powerful, with an emphasis on checking schedulability of large-scale, multimedia systems.

Furthermore, some authors have directly addressed the computational issues involved in evaluating the tests. Audsley et al. (1991) presented an algorithm for implementing a sufficient and necessary test for deadline-monotonic scheduling. Concerned by the difficulty of assessing schedulability for dynamic scheduling policies, Ripoll et al. (1996, §4) recently proposed a new, efficient testing algorithm that works by simulating earliest deadline first scheduling. The tests developed by Manabe and Aoyagi (1995) were motivated by a desire to reduce computational complexity.

We have considered only preemptive scheduling here, but there is currently a renewed interest in *non*-preemptive scheduling policies (Burns and Wellings 1996). Although such policies usually have lower processor utilisation than preemptive ones, they are cheaper to implement because mutually-exclusive access to shared variables is guaranteed automatically, without the need for a locking protocol. Also their conceptually simpler run-time behaviour makes this approach attractive in safety-critical applications (Bate et al. 1996).

Consideration of *soft* real-time tasks is another active area of research, aiming to allow non-critical tasks to profitably absorb any processor time left unused by those tasks with hard timing constraints (Davis 1994).

An especially dramatic development in recent years has been the impact scheduling theory has had on programming language design. Numerous features in the new Ada 95 standard (ISO 1994) have been included specifically to make it possible to write Ada programs amenable to real-time schedulability testing (Stoyenko and Baker 1994).

Finally, we note that although we have limited this survey to uniprocessor systems, schedulability techniques for multiprocessor and distributed systems are beginning to emerge (Sha and Sathaye 1995; Tindell and Clark 1994; Audsley et al. 1993, §4.1; Joseph and Pandya 1986, §8; Shin and Ramanathan 1994, p. 9). Unfortunately multiprocessor scheduling is not simply a matter of extending the uniprocessor methods. For instance, optimal scheduling policies for uniprocessor systems are not necessarily optimal for multiprocessor systems (Audsley et al. 1995, §8.2) and deciding schedulability for multiprocessor systems can be NP-hard (Leung and Whitehead 1982).

**A Glossary of Preemptive Scheduling Terminology**

The following terminology has been used when discussing scheduling principles in this article. Underlined text denotes a cross reference.

**Active (or effective) priority** (ISO 1994, §D.1; Audsley et al. 1993, §2) In static-priority scheduling, a variable priority value associated with each task, usually at least as great

as the base priority of the task. Priority inheritance may temporarily raise the active priority of a task above its base priority.

**Aperiodic task** (Sprunt et al. 1989, p. 28; Audsley and Burns 1990, §2.1) Some job that arrives randomly, typically in response to an external triggering event. Many such arrivals may occur in rapid succession. Hard real-time deadlines are impossible to enforce for aperiodic tasks: they may have soft 'performance' goals only. Also see sporadic task.

**Arrival** (Audsley et al. 1993, §2) The time at which a task invocation wishes to start running. This is the start of each period for a periodic task and the occurrence of the triggering event for an aperiodic or sporadic task.

**Base priority** (Audsley et al. 1993, §2) In static-priority scheduling, a fixed priority permanently associated with each task.

**Blocking** (Sha et al. 1990, p. 1177; Audsley and Burns 1990, p. 6) A task is blocked if it is ready to run but cannot because some shared variable it requires is locked by a task of *lower* priority (cf. preemption).

**(Worst case) blocking time** (Audsley et al. 1993, p. 285) An upper bound on the duration for which a task may be prevented from executing by a low priority task that has locked a shared resource needed by the higher-priority task. The actual worst-case blocking time is a property of the particular scheduling policy and locking protocol used, as this determines how many times each task invocation may be blocked. Typically, however, the value will be some multiple of the worst-case computation time spent by lower-priority tasks executing code in a critical region (Sha et al. 1990, p. 1177), or protected object (ISO 1994, §17.8.1). Actual computation times for these activities can be determined experimentally (Kenny and Lin 1991) or, more reliably, through formal analysis of high-level language (Chapman et al. 1994; Shaw 1989; Puschner and Koza 1989), or assembler (Lim et al. 1995), code.

**Busy period** (Lehoczky 1990, p. 202; Tindell and Clark 1994, p. 137) When task deadlines may exceed their interarrival times, the longest contiguous interval of time during which there are one or more unsatisfied invocation requests for a particular task.

**(Priority) ceiling locking (or emulation) protocol** (Stoyenko and Baker 1994, p. 104; ISO 1994, §D.1, D.3) A special case of the stack resource protocol (Stoyenko and Baker 1994, p. 104) for static-priority scheduling. It modifies priority inheritance by (a) statically allocating each shared variable a *ceiling value* defined as the maximum base priority of all tasks that may use it, and (b) making each task that locks a shared variable inherit the ceiling value from that variable as its active priority. This effectively prevents any task invocation from starting to run until all the shared variables it may want to use are free (ISO 1994, §D.2.1). Ceiling locking may cause tasks to be blocked in situations where they are not blocked by the priority ceiling protocol. However, in the worst case the same degree of blocking is experienced by both protocols and, since schedulability tests assume worst-case scenarios, any schedulability test applicable to the priority ceiling protocol also applies to the ceiling locking protocol. Ceiling locking is simpler to implement than the priority ceiling protocol.

**(Worst case) computation time** (Audsley and Burns 1990, §5.1) An upper bound on the amount of processor time required by a task for any invocation. Schedulability tests

usually assume that scheduling overheads are included in the computation time for each task (Gomaa 1993, p. 124): each invocation includes one context switch to the task itself, and one switch back to the task it preempted (Burns and Wellings 1995, p. 714; Burns et al. 1995, p. 477); a sporadic task initiated by an external interrupt must incorporate the interrupt handling overhead (Burns et al. 1995, p. 479); regular interrupts due to a tick scheduler must also be considered (Burns et al. 1995). Actual computation times can be determined experimentally (Kenny and Lin 1991) or, more reliably, through formal analysis of high-level language (Chapman et al. 1994; Shaw 1989; Puschner and Koza 1989), or assembler (Lim et al. 1995), code.

**Deadline** (Tindell et al. 1994, p. 149) A fixed time by which a task invocation must have completed its computation, measured relative to its arrival time. (Some authorities measure the deadline from release time (Audsley and Burns 1990, §2.1; Audsley et al. 1993, p. 255).) If no specific deadline is provided for a periodic task it is assumed to equal the period.

**Deadline monotonic (or inverse-deadline) scheduling** (Leung and Whitehead 1982, p. 240; Lehoczky 1990, p. 202; Audsley et al. 1991) A static-priority preemptive scheduling policy in which tasks with shorter (relative) deadlines have higher priority.

**Deferrable server algorithm** (Sprunt et al. 1989, §2.2) A method of servicing aperiodic tasks when using rate monotonic scheduling. It (a) implements a high-priority periodic task to handle aperiodic activities, (b) allows this task to suspend itself if it is activated when there are no outstanding aperiodic requests, while preserving its allotted computation time until such a request arrives, and (c) replenishes the computation time for the server task at the start of each period. The algorithm is simple and stable for the periodic tasks (Audsley and Burns 1990, p. 12) but allows lower utilisation than the priority exchange algorithm (Sprunt et al. 1989, p. 35).

**Delay (or pending) queue** (Burns et al. 1995, p. 476) A notional queue in which tasks that have been suspended wait, usually ordered by the time at which they become ready again. A task suspends itself by executing a 'delay' statement. This removes it from the ready queue and places it in the delay queue. It is removed from the delay queue and placed back in the ready queue by the scheduler, typically following a clock-driven interrupt. Also see tick scheduling.

**Earliest deadline first (or deadline driven) scheduling** (Liu and Layland 1973, §7; Audsley and Burns 1990, §3.2.1) A dynamic-priority preemptive scheduling policy in which the task with the earliest (absolute) deadline from the current moment has the highest priority.

**Feasible task set** (Audsley and Burns 1990, §2.2) A task set for which a schedule exists.

**Interference** (Audsley et al. 1993, §3) The degree of time for which a task invocation may be preempted.

**Invocation (or job) request** (Audsley et al. 1993, §2) A nominal event marking the arrival of a task. This 'event' involves the task requesting computational resources from the scheduler.

**Jitter** (Giering III and Baker 1994, p. 55; Locke 1992, p. 49) Variability in the actual separation between invocations of a periodic task from its intended period due to its ability to be scheduled anywhere between its arrival time and deadline. Jitter can be

reduced by shortening the task deadline, but this may make the task set infeasible (Giering III and Baker 1994, p. 55). Also see release jitter.

**Kernelised monitor protocol** (Chen and Lin 1990, §2.2) A shared variable access method for use with earliest deadline first scheduling. It assumes all tasks in critical sections are not preemptible. It can lead to low processor utilisation if there are critical sections with long execution times.

**Least laxity scheduling** (Audsley and Burns 1990, §3.2.2; Jones et al. 1996, §5) A dynamic-priority preemptive scheduling policy in which high priority is given to task invocations with the smallest difference between their upcoming absolute deadline and the remaining computation time they require.

**Necessary test** (Audsley et al. 1993, p. 284) A schedulability test that fails only infeasible task sets (cf. sufficient test).

**Optimal scheduling policy** (Audsley and Burns 1990, p. 6; Manabe and Aoyagi 1995, p. 213) A scheduling policy which can produce a schedule for any feasible task set. Among static-priority policies rate monotonic scheduling is optimal for independent periodic tasks with deadlines equal to their periods (Liu and Layland 1973, p. 178; Giering III and Baker 1994, p. 55) and deadline monotonic scheduling is optimal for periodic tasks with deadlines less than their period (Leung and Whitehead 1982, §2; Audsley et al. 1993, p. 284; Tindell and Clark 1994, p. 134; Burns and Wellings 1995, p. 708). Among dynamic-priority policies earliest deadline first is optimal for independent periodic tasks with deadlines equal periods (Liu and Layland 1973, p. 186; Giering III and Baker 1994, p. 55).

**Period transformation** (Sha and Goodenough 1990, p. 56; Audsley and Burns 1990, §3.3.1) Since the period of a task is not necessarily a measure of its relative importance, period transformation techniques seek to make a task set suitable for rate monotonic scheduling by breaking highly important tasks with long worst-case computation times into smaller, more frequently scheduled pieces.

**Periodic task** (Audsley and Burns 1990, §2.1) Some job that arrives at fixed intervals (cf. aperiodic task). A periodic task is usually characterised by its period, computation time, deadline and shared variable access (Audsley et al. 1993, §2) requirements.

**Preemption** (Audsley et al. 1993, §2) A task is preempted if it is ready to run but cannot because a task of *higher* priority is running (cf. blocking).

**Priority** (Burns and Wellings 1991, p. 19) An allocation of (usually unique) numbers to tasks, used by a scheduler to determine which ready task should run. Tasks with higher priorities run in preference to those with lower priorities.

**Priority ceiling protocol** (Sha and Goodenough 1990, p. 57; Sha et al. 1990, §IV; Audsley and Burns 1990, §4.4) A method of preventing priority inversion and bounding blocking times, suitable for use with static-priority scheduling policies. It extends priority inheritance by (a) statically allocating each shared variable a *ceiling* value defined as the maximum base priority of all tasks that may use it, and (b) allowing a task to lock a variable only if its active priority is higher than the ceiling value of *any* variable currently locked by any other task (Sha et al. 1990, p. 1178). In the worst case with this protocol each task invocation is blocked at most once (cf. priority inheritance protocol), although it can be 'blocked' even if it does not access any shared variables (Sha and

Goodenough 1990, p. 58)! A high-priority task may start running even when some of the resources it will need are locked by lower-priority tasks (Sha and Goodenough 1990, pp. 56–7) (cf. ceiling locking protocol). The protocol also has the beneficial side-effect of preventing deadlocks. A dynamic version of the priority ceiling protocol is available for use with earliest deadline first scheduling, where the ceiling value for each variable changes according to upcoming deadlines (Chen and Lin 1990, pp. 332–3) (cf. stack resource protocol). Also see ceiling locking protocol.

**Priority exchange algorithm** (Sprunt et al. 1989, §2.2) A method of servicing aperiodic tasks when using rate monotonic scheduling. It (a) uses a periodic server task to handle aperiodic requests, (b) allows the server to suspend itself if there are no outstanding aperiodic requests by swapping its priority with the highest-priority periodic task, and (c) replenishes the available computation time (and priority) of the server at the start of its period. The algorithm has greater responsiveness than a simple periodic polling server but makes the periodic task set unstable (Audsley and Burns 1990 p. 11) (cf. deferrable server algorithm).

**Priority inheritance protocol** (Sha et al. 1990, §III; Chen and Lin 1990, p. 328) A method of preventing priority inversion. This is done by temporarily raising the active priority of a task locking a shared variable to the maximum base priority of any task blocked on that variable. In the worst case with this protocol a task is blocked at most once for *each* shared variable it attempts to access (cf. priority ceiling protocol).

**Priority inversion** (Sha and Goodenough 1990, p. 57; Sha et al. 1990, §II; Audsley and Burns 1990, §4.1) The situation where (a) a task of low priority is blocking one of higher priority and (b) a task of medium priority then preempts the low-priority one, thus further blocking the high priority task and 'inverting' the desired relationship about which task runs next. Priority inversion can be prevented by guaranteeing that tasks never attempt to access the same variable simultaneously (Audsley and Burns 1990, §4.2) or by using a priority inheritance protocol. More generally, 'priority inversion' can also refer to situations where the scheduler is undertaking actions on behalf of low-priority tasks while preempting a high-priority task (Burns et al. 1995, §II).

**Processor utilisation** (Chen and Lin 1990, p. 327; Manabe and Aoyagi 1995, p. 213) The overall percentage of time that a task (or task set) may require access to the processor. It is defined as the ratio of computation time divided by interarrival time.

**Rate monotonic scheduling** (Liu and Layland 1973, p. 178; Sha et al. 1991) A static-priority preemptive scheduling policy in which tasks with shorter periods have higher priorities. Also see period transformation.

**Ready (or run or dispatch) queue** (Burns et al. 1995, p. 476; Audsley et al. 1993, §2; ISO 1994, §D.2.1) A notional queue of task invocations that are ready to run given sufficient resources, usually ordered by their priorities.

**Release jitter** (Audsley et al. 1993, §4; Tindell et al. 1994, p. 134) The difference between the arrival time of a task invocation and its release time. This gap may be due to the scheduler taking some time to recognise task arrival (Audsley et al. 1993, p. 287) or due to a task awaiting input from the environment before it can start (Audsley et al. 1993, §4.1).

**Release time** (Audsley et al. 1993, §2) The time at which the scheduler acknowledges that a task invocation that has arrived is ready to run, typically by placing it in the ready queue. (The task does not necessarily begin running at this time, however.)

**(Worst case) response time** (Tindell et al. 1994, p. 150) The time at which a task invocation completes all activity, measured relative to its arrival time. For a task to be schedulable its worst case response time must not exceed its deadline. (A relaxation of this constraint is to require only that the task completes all *observable* activity by the deadline (Burns et al. 1994).)

**Shared resources** (ISO 1994, §D.2.1; Audsley and Burns 1990, p. 6) Any resource needed by more than one task. Typically this includes all shared variables and the processor. The processor is a preemptible resource because, once allocated to a task, it may temporarily be allocated to another. Access to shared variables is made non-preemptible in uniprocessor scheduling by protecting them with semaphores or monitors: once allocated to a task a variable cannot be allocated to another until unlocked (although the task accessing the variable may itself be preempted by a higher-priority task that does not use the variable).

**Sporadic server algorithm** (Sprunt et al. 1989, §3.1; Sha and Goodenough 1990, pp. 56, 60) A method of handling sporadic tasks using a preemptive server. For rate monotonic scheduling with sporadic task deadlines at least as great as their interarrival time it is the same as the deferrable server algorithm except that the available computation time is replenished after the interarrival time has elapsed from the last time a request was serviced, rather than periodically (Sprunt et al. 1989, p. 37). When sporadic task deadlines are less than their interarrival time the algorithm allows the base priorities for sporadic servers to be greater than their rate monotonic value (Sprunt et al. 1989, §4).

**Sporadic task** (Sprunt et al. 1989, p. 28; Audsley and Burns 1990, §2.1) A special case of aperiodic tasks where arrivals have a known minimum separation. This makes it possible to enforce hard deadlines. A sporadic task is characterised by this minimum separation and its computation time, deadline and shared variable access (Audsley et al. 1993, §2) requirements. For the purposes of worst case analysis sporadic tasks are usually treated like periodic tasks with a period equal to the minimum separation (Gomaa 1993, §11.4.6).

**Sporadically periodic task** (Tindell et al. 1994, p. 134; Audsley et al. 1993, §5) A combination of both periodic and sporadic task behaviour in which the task initially arrives sporadically but then rearrives periodically a fixed number of times. A sporadically periodic task is characterised by its computation time, 'outer' minimum arrival separation, 'inner' period, deadline and shared variable access requirements.

**Stability** (Sha and Goodenough 1990, p. 56; Gomaa 1993, p. 124) A scheduling policy is stable if, under conditions of transient overload, it is the lowest priority tasks that miss their deadlines. Rate monotonic scheduling is stable (Gomaa 1993, p. 124) whereas earliest deadline first is not (Audsley and Burns 1990, p. 11).

**Stack resource protocol** (Baker 1990, §3.2; Baker 1991) A shared variable access protocol, suitable for both dynamic and static scheduling policies. It (a) associates a static *preemption level* with each task, (b) maintains a dynamic *current ceiling* for each shared variable, (c) whenever a variable is accessed, raises the ceiling of the variable to be at

least as great as the preemption level of *any* task that may use the variable, and (d) allows a task invocation to start running only when its preemption level exceeds the current ceiling of *every* shared variable. This guarantees that tasks begin only when all the resources they may need are available.

**Sufficient test** (Audsley et al. 1993 p. 284) A schedulability test that passes only <u>feasible</u> <u>task sets</u> (cf. necessary test). It may *fail* some schedulable task sets, however.

**Suspension** (Audsley and Burns 1990, p. 6) A suspended task is one that has made itself temporarily inactive by executing a 'delay' statement. Suspended tasks reside in the <u>delay queue</u>.

**Task set** (Audsley et al. 1993, p. 285) A set of scheduling requirements typically consisting of several <u>periodic</u> and/or <u>sporadic task</u> requirements.

**Tick (or timer-driven) scheduling** (Burns et al. 1995, §II; Tindell et al. 1994, §6) A polling method of implementing 'delay' statements. A tick scheduler executes periodically, controlled by a regular clock interrupt. When activated it moves tasks from the <u>delay queue</u> to the <u>ready queue</u> if their delay interval has passed.

**Transient overload** (Audsley and Burns 1990, §3.3; Sha and Goodenough 1990, p. 56) The situation where one or more task invocations miss their deadlines. This may be due either to the programmer underestimating the worst-case <u>computation time</u> requirement for a task or <u>sporadic task arrivals</u> occurring at greater than their specified frequency.

## Notes

1. For some number $x$, $\lceil x \rceil$ is the smallest integer greater than or equal to $x$.

2. For some number $x$, $\lfloor x \rfloor$ is the largest integer less than or equal to $x$.

3. We use a different definition of $D_i$ than Audsley et al. (1993, p. 285), starting from *arrival* instead of release time.

4. We use a different definition of $D_i$ than Audsley et al. (1993, p. 285), starting from *arrival* instead of release time.

5. The formula for $K_i$ used here by Tindell and Clark (1994, p. 121), sums over higher-priority tasks, rather than *all* tasks. This differs from all other definitions of this form (Tindell et al. 1994, p. 144; Burns and Wellings 1995, p. 717; Burns et al. 1995, p. 478).

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