Exercice 1

· juste pour faire quelques conversions.

1. $\Delta t = 22$ s.
2. $d = 600\ \mu m = 0{,}6$ mm.

Exercice 2

1. a) $x = 1 \quad y = -1 \qquad \vec{OM} = \vec{e_x} - \vec{e_y}$

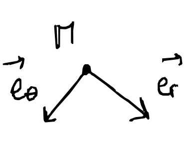
b) $x = \frac{5}{2} \quad y = 0 \qquad \vec{OM} = \frac{5}{2}\vec{e_x}$

c) $x = 2 \quad y = 1 \qquad \vec{OM} = 2\vec{e_x} + \vec{e_y}$

d) $x = -1 \quad y = 0 \qquad \vec{OM} = -\vec{e_x}$

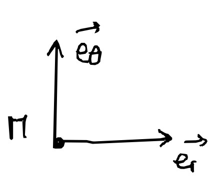
2. a) $r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ $\qquad \vec{OM} = \sqrt{2}\,\vec{e_r}$

$\theta = -\frac{\pi}{4}$



b) $r = \frac{5}{2} \qquad \vec{OM} = \frac{5}{2}\vec{e_r}$

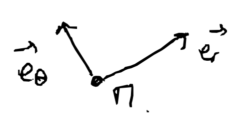
$\theta = 0$



c) $r = \sqrt{3} \qquad \vec{OM} = \sqrt{3}\,\vec{e_r}$

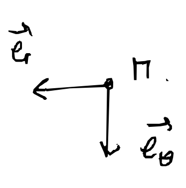
$\theta = \arctan\left(\frac{1}{2}\right)$

$\approx 26°$



d) $r = 1 \qquad \vec{OM} = \vec{e_r}$

$\theta = \pi$



3. a) $\vec{OM_i} = -\vec{e_x} - \vec{e_y}$

$\vec{OM_f} = 3\vec{e_x}$

donc $\Delta\vec{OM} = 4\vec{e_x} + \vec{e_y}$

b) $\vec{OM_i} = \vec{e_y}$

$\vec{OM_f} = -\vec{e_x}$

donc $\Delta\vec{OM} = -\vec{e_x} - \vec{e_y}$

$$\langle \vec{V}_{a)} \rangle = \frac{\Delta\vec{OM}}{\Delta t} = \frac{4}{\Delta t}\vec{e_x} + \frac{1}{\Delta t}\vec{e_y}$$

$$= 0{,}8\,(m.s^{-1})\,\vec{e_x} + 0{,}2\,(m.s^{-1})\,\vec{e_y}$$

$$\langle \vec{V}_{b)} \rangle = \frac{\Delta\vec{OM}}{\Delta t} = -\frac{1}{\Delta t}\vec{e_x} - \frac{1}{\Delta t}\vec{e_y}$$

$$= -0{,}2\,(m.s^{-1})\,\vec{e_x} - 0{,}2\,(m.s^{-1})\,\vec{e_y}$$

Exercice 3

1. $\frac{df}{dx} = 2x - \frac{1}{x^2} \qquad \frac{dg}{dt} = -A\omega \sin\omega t$

$$\frac{dh}{dy} = 2y e^{-ay} - ay^2 e^{-ay} = y(2-ay)e^{-ay}$$

2. $F(x) = x^3 + 2x + C \to -3$

$G(t) = \frac{1}{2}t^2 + \sin(t) + C \to \sin(1) - \frac{1}{2}$

$H(y) = y^2 e^{-ay} + C \to 0$

3. $\vec{OM}(t) = V_0 t\,\vec{e_x} + \left(y_0 + \frac{1}{2}a_0 t^2\right)\vec{e_y}$

$\vec{V}(t) = V_0\vec{e_x} + a_0 t\,\vec{e_y}$

$\vec{a}(t) = a_0\vec{e_y}$

$\vec{OM}(t) = A\cos(\omega t)\,\vec{e_x} + A\sin(\omega t)\,\vec{e_y}$

$\vec{V}(t) = -\omega A\sin(\omega t)\,\vec{e_x} + \omega A\cos(\omega t)\,\vec{e_y}$

$\vec{a}(t) = -\omega^2 A\cos(\omega t)\,\vec{e_x} - \omega^2 A\sin(\omega t)\,\vec{e_y}$

↳ on peut remarquer que $\vec{a}(t) = -\omega^2\vec{OM}(t)$

4. $\vec{v}(t) = -gt\,\vec{e_z} + C\vec{e_y} + C'\vec{e_x}$

$= V_0\vec{e_y} - gt\,\vec{e_z}$

$\vec{OM}(t) = C\vec{e_x} + \underset{+C'}{V_0 t\,\vec{e_y}} - \underset{+C''}{\frac{1}{2}gt^2\vec{e_z}}$

$= V_0 t\,\vec{e_y} + \left(h - \frac{1}{2}gt^2\right)\vec{e_z}$

$z = 0$ pour $t = \sqrt{\frac{2h}{g}}$

Exercice 4

1. $\vec{OM} = \left[x_0 + \tau V_0\left(1 - e^{-t/\tau}\right)\right]\vec{e_x} + a_0\tau^2\left(1 - e^{-t/\tau} - \frac{t}{\tau}\right)\vec{e_y}$

2. $\vec{V} = V_0 e^{-t/\tau}\,\vec{e_x} + \left(-a_0\tau + a_0\tau e^{-t/\tau}\right)\vec{e_y}$

$\vec{a} = -\frac{V_0}{\tau}e^{-t/\tau}\,\vec{e_x} - a_0 e^{-t/\tau}\,\vec{e_y}$

$= -\frac{\vec{V}}{\tau} - a_0\vec{e_y}$

3. $\frac{x - x_0}{\tau V_0} = 1 - e^{-t/\tau}$

$1 - \frac{x - x_0}{\tau V_0} = e^{-t/\tau}$

$t = -\tau \ln\left(1 - \frac{x - x_0}{\tau V_0}\right)$

$$y(x) = a_0\tau^2\left(1 - \left(1 - \frac{x-x_0}{\tau V_0}\right) + \ln\left(1 - \frac{x-x_0}{\tau V_0}\right)\right)$$

$$= a_0\tau^2\left(\frac{x-x_0}{\tau V_0} + \ln\left(1 - \frac{x-x_0}{\tau V_0}\right)\right)$$

$$y(x=0) = a_0\tau^2\left(\frac{-x_0}{\tau V_0} + \ln\left(1 + \frac{x_0}{\tau V_0}\right)\right)$$

AN: $y(x=0) = -145$ m.

Exercice 5

1. $\vec{OM}(t) = x_0\cos(\omega t)\,\vec{e_x} + \left(y_0 + \frac{\ell}{2}\left(\cos(2\omega t) - 1\right)\right)\vec{e_y}$

2. $\vec{V} = -x_0\omega\sin(\omega t)\,\vec{e_x} - \ell\omega\sin(2\omega t)\,\vec{e_y}$

3. vitesse nulle pour $\sin(\omega t) = 0$ ET $\sin(2\omega t) = 0$.

donc $t = \frac{\pi}{\omega} \times n$ → entier naturel.

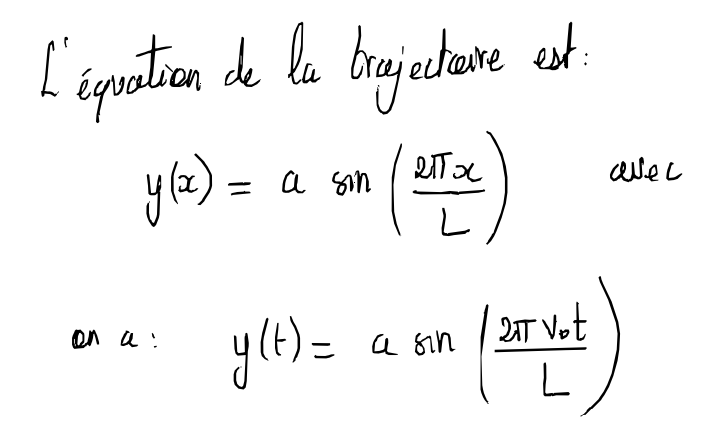
$\vec{OM}(t_n) = x_0(-1)^{n+1}\,\vec{e_x} + y_0\vec{e_y}$

donc $\pm x_0\,\vec{e_x} + y_0\vec{e_y}$

4. $\vec{a} = -x_0\omega^2\cos(\omega t)\,\vec{e_x} - 2\ell\omega^2\cos(2\omega t)\,\vec{e_y}$

Exercice 6

1.



$x = V_0 t \qquad V_0 = \frac{6L}{t_f} = 100\ m.s^{-1}$ ($6L$ → 1200 m ; t_f → 12 s)

2. L'équation de la trajectoire est :

$y(x) = a\sin\left(\frac{2\pi x}{L}\right)$ avec a à déterminer

on a : $y(t) = a\sin\left(\frac{2\pi V_0 t}{L}\right)$

$$\frac{dy}{dt} = \frac{2\pi a V_0}{L}\cos\left(\frac{2\pi V_0}{L}t\right)$$

$$\frac{d^2y}{dt^2} = -\left(\frac{2\pi a V_0}{L}\right)^2 \sin\left(\frac{2\pi V_0 t}{L}\right)$$

→ $\frac{d^2x}{dt^2} = 0$ par ailleurs.

donc $a_{max} = \left(\frac{2\pi a V_0}{L}\right)^2$

il faut donc $a < 10g\left(\frac{L}{2\pi V_0}\right)^2$

$= 9{,}9$ m.

↳ c'est très serré !