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# Transiting Exoplanets

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**Abstract**

This assignment dealt with a M6V star in a star-planet system that is in circular orbit. The values of the mass, radius and temperature of the star are known to be $0.102M_{\odot}$, $0.137R_{\odot}$ and 2800K. The period of the system is also known to be P = 1.14176084 days. The aim was to use the measured spectra of star at different timestamps compared to its spectral template and the transit data of a star as a function of time in order to produce the radial velocity curve and the light curve of the of the star-planet system. These sets of data were fitted using appropriate fitting functions and the fits were plotted as a function of time. Many characteristics of the system were able to be obtained from the various fitting parameters. The mass and the radius of the planet were found to be $10.41 \pm 0.3$ earth masses and $2.21 \pm 0.01$ earth radii respectively. From these the density of the planet was able to obtained, which was found to be $5300 \pm 400$ $kg/m^3$ or $0.96 \pm 0.07$ earth densities. The large size and mass of the exoplanet along with its average density being just slightly less than earths suggest this planet is a 'super-earth' with composition similar to that of our own planet. The temperature of the star in the system along with the semi-major axis were used to calculate the flux at the surface of the exoplanet in order to determine if it possessed the conditions for supporting life. This value of flux at the surface was found to be $14.2kW/m^2$, about 10.5 times that of the solar flux at earths surface. This would make the planet much too hot to have any liquid water present.

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# 1 Introduction

Exoplanets are planets that orbit stars other than our own sun. The most effective ways that have been created in order to discover exoplanets are the radial velocity and transit techniques. The first exoplanets were discovered in the 90s and just recently the milestone of 5000 discovered exoplanets was reached [1]. Each method has their own benefits and each can be used to calculate different characteristics of star-planet systems, and combining the data from the two methods allows further analysis of the system. The RV method is predominantly used to find the mass of the orbiting exoplanet which exerts a wobble on the star as it circles it. The transit method is predominantly used to figure out the size of the exoplanet along with the semi major axis and many more parameters. This assignment will use the transit and RV techniques through python to determine these characteristics.

# 2 Theory

## 2.1 Radial Velocity Technique

From the spectrum of a star the radial velocity can be determined from the Doppler shift. The measured spectrum can be compared to a spectral template where there are no relative velocity effects. By measuring the Doppler shifts of a number of spectra at different time intervals the radial velocity of the star can be plotted as function of time. This is known as the RV curve. In the case that the radial velocity of the star is not constant and instead oscillates about a certain centre of mass, the existence of an orbital companion, which can be a planet or even another star, can be inferred. The RV curve can thus allow the calculation of the mass of this companion given the knowledge of the mass of the star.

This project assumes circular motion and thus the radial velocity curve is sinusoidal. The function used to fit the RV curve in this assignment is a sine function of the form

$$K_\star \sin(bx + c) + d \tag{1}$$

where $K_\star$ is the amplitude of the RV curve and $d$ is the offset of the curve from a radial velocity of zero. The parameter $b$ is given by $2\pi/P$ where $P$ is the orbital period. Without the application of the transit technique as explained in section 2.2 the parameter $c$ can just be found as a fitting parameter of the RV curve. However utilising the parameters found using the transit method it can be found that

$$c = \left(\frac{2\pi}{P}\right) \times (T_0 + \Delta T_{start})$$

where $T_0$ is the central transit time as calculated using the transit technique and $\Delta T_{start}$ is the difference in time between the beginning of the transit data and the beginning of the RV data. This allows the fitting of just two parameters, $K_\star$ and $d$ in order to find a fit to the RV curve.

From the amplitude $K_\star$ of the radial velocity curve the minimum mass of the exoplanet, $M_p \sin i$ can be found. The amplitude $K_\star$ is given by

$$K_\star = \frac{M_p \sin i}{(M_\star + M_p)^{2/3}} \left(\frac{2\pi G}{P}\right)^{1/3} \tag{2}$$

where $M_\star$ and $M_p$ are the masses of the star and planet respectively, $P$ is the orbital period, $G$ is the gravitational constant and $i$ is the orbital inclination where $i = \pi/2$ corresponding to edge on. The mass of the star is known to be 0.102 $M_\odot$ and has a temperature of 2800 K.

Making the assumption that $M_\star \gg M_p$ then the approximation $M_\star + M_p \approx M_\star$ can be made. Then rearranging 2 gives the value of the minimum mass of the planet in terms of $K_\star$, mass of the star and the orbital period.

$$M_p \sin i = K_\star M_\star^{2/3} \left(\frac{P}{2\pi G}\right)^{1/3} \tag{3}$$

## 2.2 Transit Technique

For this technique the relative flux of the star is observed as a function of time. When the planet passes in front of the star it blocks a fraction of the total light coming from the star for a certain period of time, causing the relative flux measured to drop for a certain period of time. Ignoring limb darkening, which is the decrease in apparent brightness of the star from disk centre to edge due to differences in the optical depth, this transit depth $\Delta F$ is given by

$$\Delta F = \left(\frac{R_p}{R_\star}\right) = \rho^2 \tag{4}$$

and from this equation the radius of the planet can be determined as the radius of the planet is known to be $0.137R_\odot$. $\rho$ is just a simple way to write the ratio of the planet radius to the stellar radius. A fit to this light curve outputs the fit parameters $T_0$ (central transit time), the period P, the semimajor axis $a$ of the system (in stellar radii), the ratio $\rho$, the orbital inclination of the system $i$ and the out of transit flux $f_{oot}$. The transit model works as follows.

The time $t$ is converted into phase $\phi$ by

$$\phi = \frac{2\pi}{P}(T - T_0) \tag{5}$$

Then the orbital coordinates in units of $R_\star$ are calculated using trigonometry which gives

$$x = \frac{a}{R_\star \sin(\phi)}, \quad y = \frac{a}{R_\star \cos(\phi)} \tag{6}$$

The function z(t) is given by

$$z(t) = \frac{a}{R_\star}\sqrt{\sin^2\phi + (\cos i \cos\phi)^2} \tag{7}$$

and making use of the definitions for the x and y coordinates in (6), z(t) can be rewritten as

$$z(t) = \sqrt{x^2 + (\cos iy)^2} \tag{8}$$

Then (8) is used to calculate the flux. Conditional statements are defined in the function for different ranges of z(t). These are as following

$$f(z,\rho) = \begin{cases} 1, & \text{if } z(t) > 1+\rho \\ 1 - 1/\pi(\rho^2\kappa_0 + \kappa_1 - \sqrt{\frac{4\rho^2 - (1+z^2-\rho^2)^2}{4}}, & \text{if } 1-\rho < z(t) \leq 1+\rho \\ 1-\rho^2, & \text{if } z(t) \leq 1-\rho \end{cases} \tag{9}$$

Equation (9) is then multiplied by the parameter $f_{oot}$, the out of transit flux, to scale the light curve.

So essentially the transit model takes in values for the time $t$ and the parameters $T_0$ (central transit time), the period P, the semimajor axis $a$ of the system (in stellar radii), the ratio $\rho$, the orbital inclination of the system $i$ and the out of transit flux $f_{oot}$ and outputs the flux of the system as a function of time. Exactly how this model is used to fit the transit data will be explained in section 4.2.

# 3 Plotting the Radial Velocity Curve

## 3.1 Method

The radial velocity data was downloaded into the file using numpy.load and the different components in the data-set were saved into separate arrays. The measured spectra, their respective timestamps along with the wavelengths, velocities and the spectral template made up these different components in the RV data-set. The number of spectra in the data-set was found, which was 40. One of the recorded spectra was plotted on top of the spectral template in order to make sure the data made sense.

Next the cross correlation method was used on each spectrum in order to probe the shift of each spectrum relative to the spectral template. For this the correlate function from the scipy.signal package was utilised. This shift has a distinct peak at a certain value of velocity for each spectrum so a Gaussian fitting function was used to find the mean shift at each timestamp using the curve_fit function from the scipy.optimize package. A loop over all the spectra is performed in order to find the mean radial velocity of each individual spectrum relative to the spectral template. These mean radial velocities are then plotted against the time stamps to see the plot of the radial velocity of the star as a function of time.

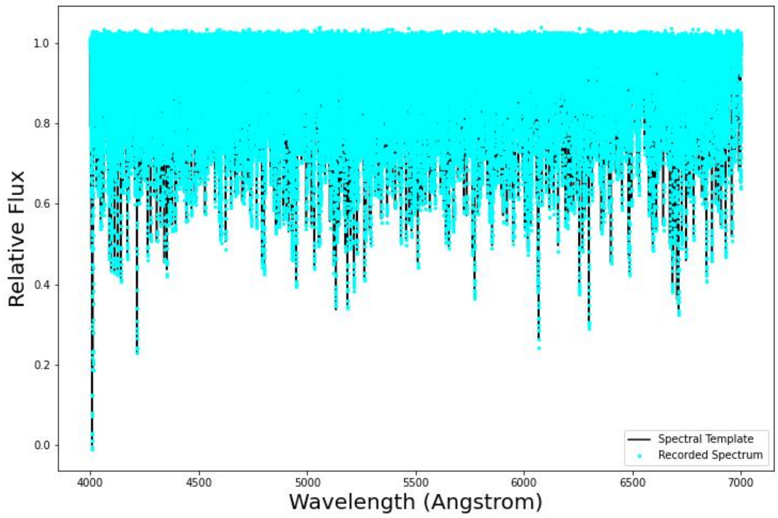


Figure 1: First recorded spectrum of the star in the data-set.

## 3.2 Analysis and Results

It is clearly seen in figure 2 that a Gaussian function is very suited to the peak fitting of the shift of each spectra with respect to the spectral template. As previously stated the fitting of a Gaussian to this peak gives the mean and standard deviation of the data. However the errors associated with the mean radial velocity are found to be much too large to be reflected as the actual error as they are of the same order of magnitude as the amplitude of the RV curve and if taken to be the true errors would render the fit to the data meaningless. Therefore another method must be used to calculate the errors in the RV data, which will be explained further in section 5.1 .

As seen in figure 3 there is a clear sinusoidal relationship between the mean value of the radial velocity of the system with time. The point which the RV curve oscillates about is offset from zero, which suggests the whole system is moving away at a certain velocity. The value of this velocity, along with the amplitude of the RV curve will be calculated in further sections.

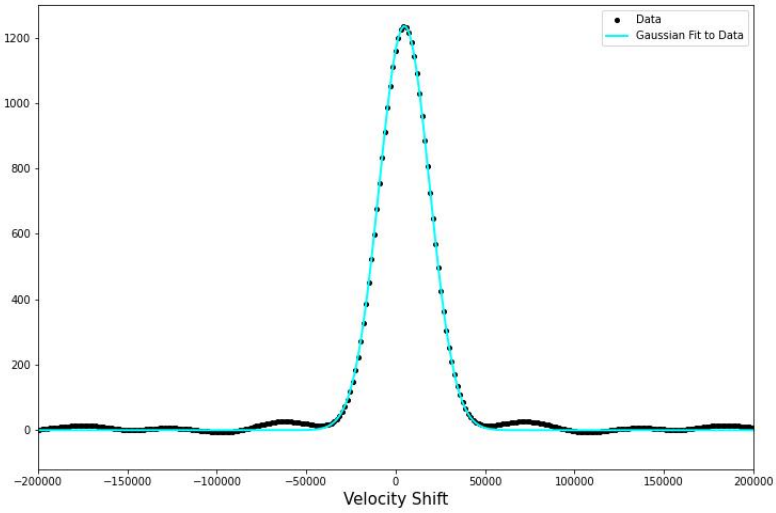


Figure 2: Shift in the first recorded spectrum of the star in the data-set compared to the spectral template. This plot has a peak at 4971.545 ± 53.400 m/s, which corresponds to the recorded radial velocity of the star in the system.

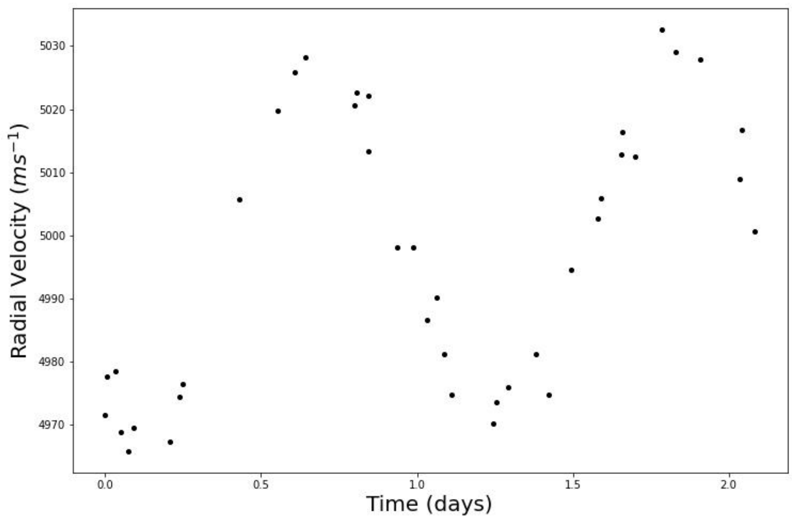


Figure 3: Plotted radial velocity of the star as a function of time. Each value of radial velocity is found by finding the mean value of the shift in the recorded spectrum of the star at a certain timestamp relative to the stellar template. This velocity directly corresponds to the radial velocity of the star.

# 4 Fitting the Transit Flux

## 4.1 Method

The transit data was downloaded and the times and measured flux were stored in separate arrays. The period, $P$, was stored in a separate variable. Firstly the measured flux of the star as a function of time was plotted in order to see the light curve of the star.

Next a transit fitting function was defined in order to plot the shape of the light curve of the star. This transit function works as explained in section 2.2. The scipy.optimize.curve_fit was used to produce a fit to the transit data using this transit function, outputting the fit parameters $T_0$, $a$, $i$, $\rho$ and $f_{oot}$.

## 4.2 Analysis and Results

Figure 4 shows the recorded flux of the star as a function of time. It is evident that there is a steep drop in the recorded flux, which reflects the exoplanet passing in front of the star and blocking out a small amount of light. This data was then fitted as explained in 4.1 in order to evaluate the characteristics of the system. Using the curve_fit function in the scipy.optimize package takes the transit model, the transit data and initial guesses for the fit parameters and outputs the optimal parameters for the transit model. The initial guesses for the fit parameters have to be educated guesses in order to stop curve_fit from being stuck in a local optimum and not returning the desired fitting parameters

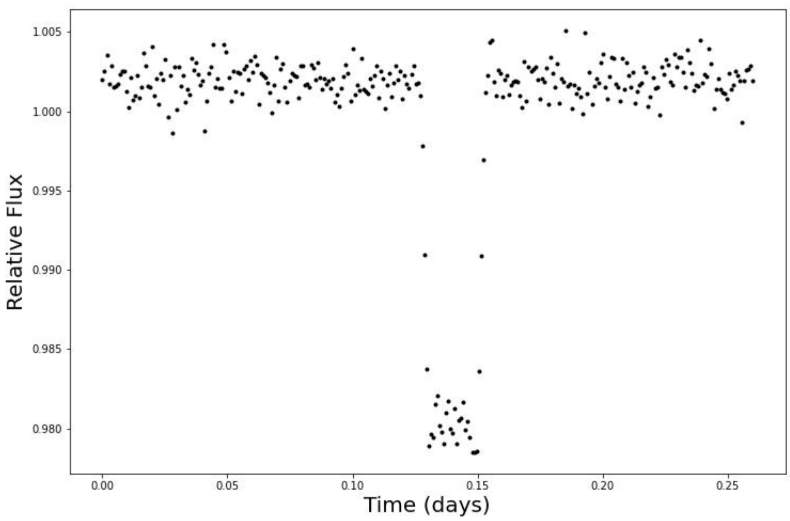


Figure 4: Plot of the recorded light curve of the star. The sudden drop in the recorded relative flux is due to the exoplanet passing in front of the star, leading to a reduction in the recorded flux for a certain period of time.

As seen in figure 5, the method of fitting the function to the transit data produced an excellent fit. From this the parameters of the fit and their respective errors were found using curve_fit, as seen in table 1. The central transit time parameter, $T_0$, will be useful in the next section when fitting the RV curve.

| Fit Parameter | Value | Error |
|---|---|---|
| $T_0$ | 0.14002 | $6 \times 10^{-5}$ |
| a | 15.6 | $6 \times 10^{-1}$ |
| $\rho$ | 0.1478 | $8 \times 10^{-4}$ |
| i | 1.58 | $10^{-2}$ |
| $f_{oot}$ | 1.00197 | $6 \times 10^{-5}$ |

Table 1: Parameters of the transit fit.

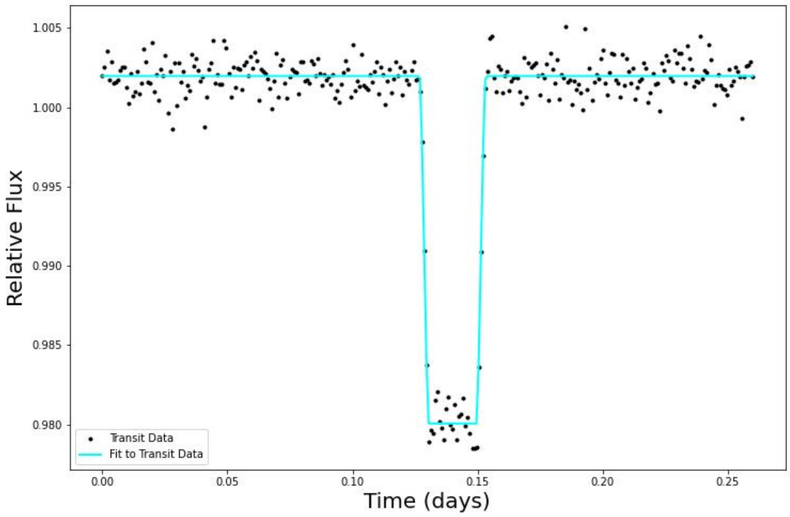


Figure 5: Fit of the transit Data

# 5 Fitting the Radial Velocity Curve

## 5.1 Method

This section involved fitting the RV data with a sinusoidal function. Firstly the times were 'folded' in order for the data to cover just one period. The fitting function used was a sine function of the form

$$K_\star \sin(bx + c) + d$$

where the phase shift c and the parameter b are known from the given period, the central transit time parameter from the transit fit and the difference between the starting times of the transit and radial velocity data as explained in section 2.1. Again curve_fit is used to fit for the parameters $K_\star$ and d for a sinusoidal fit of the radial velocity data.

As explained in section 3.2, the values found for the errors in the radial velocities from the Gaussian fit are much too large to be considered as the real errors. Hence another route was taken in order to produce the errors in the RV data. The radial velocity data was taken away from the values of the best fit at each timestamp in order to find the residuals of the fit and this was plotted against the timestamps as seen in figure 7. The standard deviation of the residuals was found and this value was set as the error in radial velocity for each data point, which was found to be about $\pm 3.76$ m/s. As there is now an error in the radial velocity data the fitting of the RV curve should be recalculated as curve_fit takes into account the errors in the y-data, or in this case the radial velocity data. Doing this as before recalculates the parameters $K_\star$ and d whilst taking into account the errors in the data. There is also a systemic velocity which is due to the fact that the entire system is moving away from us at a certain speed. This speed is given by the fitting parameter $d$ and hence this can be taken away from the array of radial velocities in order to find the true radial velocity of the system as a function of time. The value for the amplitude K stays unchanged from the previous calculation.

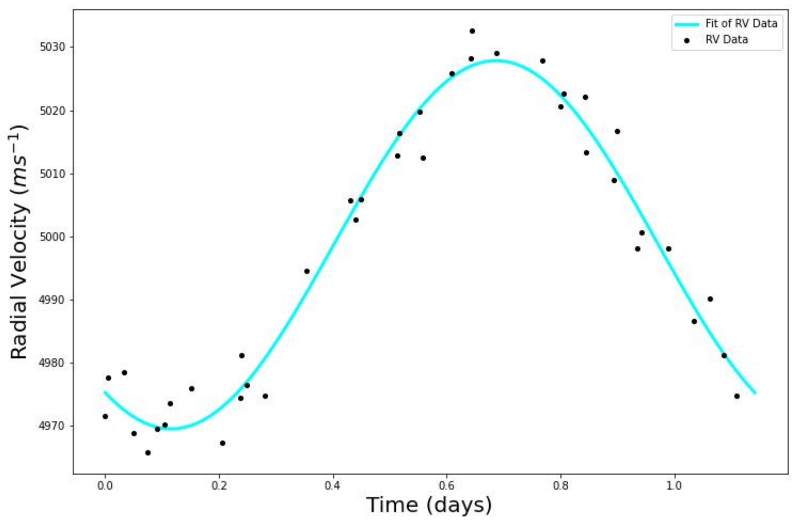


Figure 6: Fit to the raw radial velocity data without taking into account the spread of the data. The function used to fit the curve was a sine function (equation (1)) as laid out in section 2.1

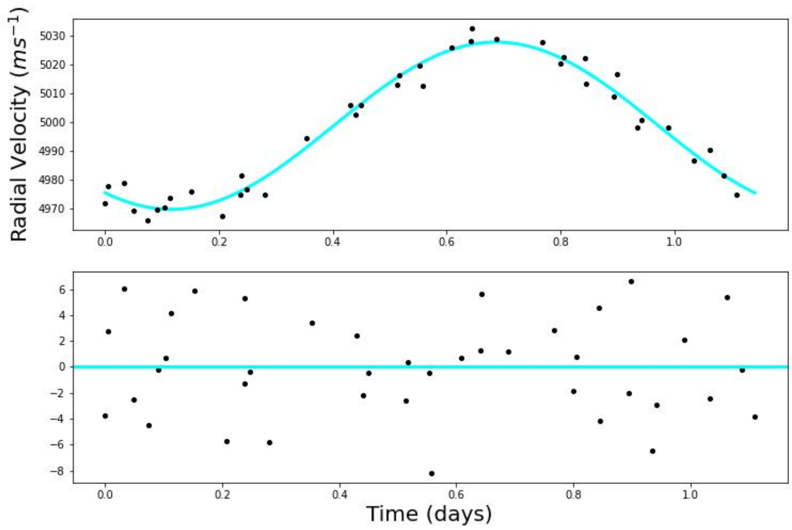


Figure 7: Residuals of the RV data plotted below the fitted RV curve. The residuals were found by subtracting the radial velocity data from the best fit. When these residuals were found the standard deviation of the data was found and set equal to the error in the radial velocity for each of the RV data points.

## 5.2 Analysis and Results

The original fit of the phase-folded RV data was conducted without the use of the error in the data. When using the standard deviation in the residuals as explained before as the error in the data for fitting the curve, the fitting parameters are only changed by a minuscule amount. The fits of the phase folded RV curves with and without the use of errors in the data are seen in figures 8 and 6 respectively. The fitting parameters for the RV curve are seen in table 2. In order to find the radial velocity of the star itself, without taking into account the systemic velocity, the parameter $d$ is taken away from all the values of radial velocity. This true RV curve of the star can be seen in figure 9.

| Parameter | Value | Error |
|---|---|---|
| $K_\star$ (m/s) | -29.1 | 0.8 |
| d (m/s) | 4998.7 | 0.6 |

Table 2: Parameters of the most accurate RV fit in which the error in the radial velocities was taken to be the standard deviation of the radial velocity residuals.

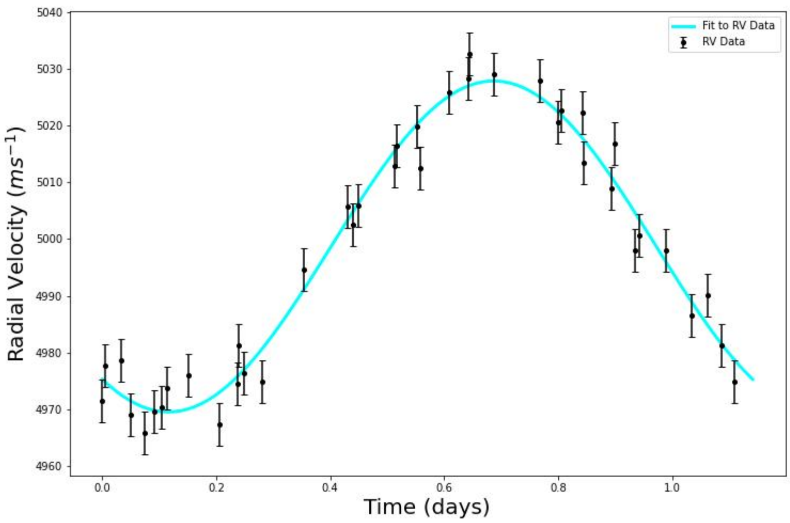


Figure 8: Fit to the raw radial velocity data taking into account the standard deviation of the residuals of the data. This slightly alters the fitting parameters but only minimally in this case.

# 6 System

The radius of the star can be found from (4) using the known values of the radius of the star and the fitting parameter from the transit fit, $\rho$. The radius of the planet is first found in terms of solar radii and then multiplied by the ratio of ratio of the solar radius to the earth radius in order to find the radius of the planet in earth radii. The radius of the exoplanet was found to be $2.21 \pm 0.01$ Earth Radii.

The mass of the star is found using the fact that the star has a mass of $0.102 M_\odot$ and $M_\odot = 1.99 \times 10^{30} kg$. Then from 3 the value of the minimum mass of the planet in kg can be found using the fitting parameter $K_\star$, the mass of the star and the given orbital period. However in order to take into account the error in the sine of the orbital inclination, both sides of (3) were multiplied by $\sin i$. This gave the equation

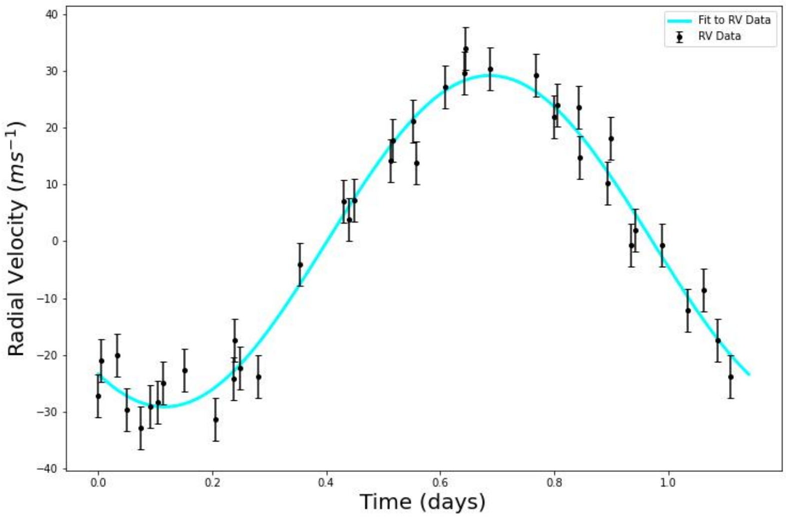


Figure 9: it to the RV curve of the star taking into account the systemic velocity of the star-planet system. The fitting parameter $d$ which corresponds to the systemic velocity is taken away from all radial velocities to produce the true radial velocity curve of the star.

$$M_p = \frac{K_\star M_\star^{2/3}}{\sin i}\left(\frac{P}{2\pi G}\right)^{1/3} \tag{10}$$

The mass of the planet in earth masses was found by dividing the mass of the planet in kg by the mass of the earth in kg. The error in $M_p$ was found using Gauss' error law. Now the value of $M_p$ could be multiplied by $\sin i$ once again in order to obtain the minimum mass of the planet. The error in the minimum mass of the planet was found by using Gauss' error law again. This gave a value for the minimum mass of the planet of $10.4 \pm 0.3$ Earth Masses. However as sin(i) is almost exactly equal to 1, when calculating for the actual mass of the exoplanet the value for the measured mass is unchanged within this number of significant figures.

From the mass and radius of the planet the density of the planet could be found. First of all the mass and radius and their respective errors were converted in SI units. The density was found by dividing the mass by the volume, $(4/3)\pi {R_p}^3$. The error in the density was again calculated using Gauss' error law. This density was found to be $5319.85 \pm 373.30\, kg\, m^{-3}$ or 0.96 $\pm$ 0.07 earth densities.

The parameter $a$ found from the transit fit is the semi-major axis of the system in units of the radius of the star. In order to find the value of the semi-major axis, the parameter $a$ is multiplied by the radius of the star. Changing the radius of the star into au gives the value of the semi-major axis in au, which was found to be 0.0100 $\pm$ 0.0004 au.

From the temperature of the star and the semi-major axis of the star-planet system, the stellar flux incident on the planet can be calculated. The temperature of the star is known to be 2800 K and the semi-major axis was calculated to be 0.0100 $\pm$ 0.0004 au. The flux detected at the surface of the planet can be calculated by the following equation

$$F_{detected} = \sigma T^4 \left(\frac{R_\star}{a}\right)^2 \tag{11}$$

where $a$ is the semi-major axis of the system and $\sigma$ is the Stefan-Boltzmann constant. This flux at the planet surface was found to be $14.2\, kW/m^2$, which is about 10.5 times the flux at earths surface.

This huge flux would make the planet far too hot to host liquid water.

It can be seen in figure 10 that the central transit time coincides with the zero point of radial velocity. This is the point at which the planet is exactly halfway through its path of crossing over the face of the star. At this point the radial velocity of the star (relative to the system) comes to zero as the planet and star at this point are moving in the transverse direction, with no velocity component in the radial direction. It specifically coincides with the zero point when the radial velocity of the star is changing from positive to negative, as the planet now begins to move away from us in the radial direction as it continues its orbit and passes behind the star, with the star moving towards us (negative radial velocity), orbiting the centre of mass. The difference in times is due to the fact that both data-sets were set to start at t = 0 however the actual data was recorded approximately 0.83 days apart.

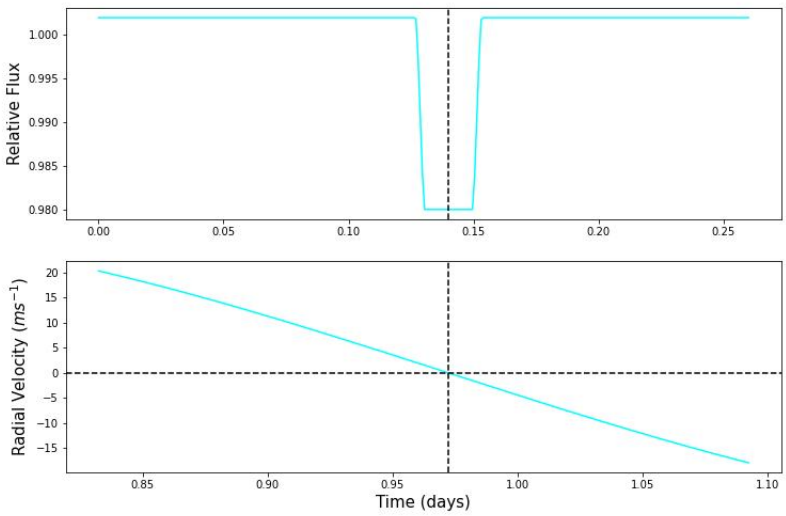


Figure 10: Relationship between the RV curve and the transit curve. It can be seen that the central transit time coincides with the zero point of radial velocity. This is the point at which the planet is exactly halfway through its path of crossing over the face of the star. At this point the radial velocity of the star (relative to the system) comes to zero as the planet and star at this point are moving in the transverse direction, with no velocity component in the radial direction. It specifically coincides with the zero point when the radial velocity of the star is changing from positive to negative, as the planet now begins to move away from us in the radial direction as it continues its orbit and passes behind the star, with the star moving towards us (negative radial velocity), orbiting the centre of mass. The difference in times is due to the fact that both data-sets were set to start at t = 0 however the actual data was recorded approximately 0.83 days apart.

# 7 Discussion

This project used recorded spectral data and recorded flux from a star as a function of time in order to plot the radial velocity of the star and the relative flux of the star as functions of time. These plots were then fitted using appropriate fitting functions in order to give the fit parameters, which could be utilised to find out the characteristics of the system.

Firstly, from the fit of the transit data, the parameter $\rho$ that gave the ratio of the radius of the planet to the radius of the star could be used to find the planets radius as the stellar radius was given. This resulted in a calculation for the radius of the planet of 2.21 $\pm$ 0.01 earth radii. The parameter giving the orbital inclination of the system could be used in order to calculate the mass of the planet.

Next the minimum mass of the planet was calculated, making use of equation (3) and the amplitude of the RV curve found by fitting the RV data. The orbital period and mass of the star are known so the calculation for this minimum mass is quite simple, giving a minimum mass of 10.405 $\pm$ 0.293 earth masses. The knowledge of the orbital inclination from the transit fit could be utilised in order to find the actual mass of the planet. As $\sin i = 0.99992$ the actual mass of the planet is nearly identical to the minimum mass measured of the planet, at 10.406 $\pm$ 0.294 earth masses.

From the fitting parameter $a$, the semi-major axis of the system in terms of the radius of the star, was used to find the semi-major axis of the star-planet system in astronomical units. This was calculated to be 0.0100 $\pm$ 0.0004 au. With the known mass and radius of the system, the density of the exoplanet could be deduced by the mass divided by the volume. This gave a value for the density of 0.96 $\pm$ 0.07 earth densities, or $5300 \pm 400\,kg/m^3$.

From the temperature of the star and the semi-major axis of the star-planet system, the stellar flux at the surface of the planet was calculated. This was found to be $14.2\,kW/m^2$, which is about 10.5 times the flux at earths surface. This huge flux would make the planet far too hot to host liquid water.

The number of exoplanets confirmed by NASA recently exceeded 5000 [1]. Of these 30% were gas giants, 35% are Neptune like, 31% are super earths and only 4% are earth size or smaller terrestrial planets. The large radius and mass and close orbit of the exoplanet in this system are common characteristics of the exoplanets we have detected thus far, and the density and size suggest it is a super-earth exoplanet. These are not necessarily general characteristics of exoplanets but more-so that the techniques used to detect them are more sensitive to larger, more massive planets that are in closer orbit with their companion star [2]. More massive planets have a greater effect on the radial velocity as it rotates the star and planets with larger radii and closer orbits have larger effects on the light curves of stars as they pass in front of them. The density of this planet suggests that it has a composition similar the that of earths, making it a rocky terrestrial planet. Most of the exoplanets that have been detected before now are Jovian planets with gaseous compositions which are lower in density than rocky planets but tend to be much greater in size.

# References


[1] Pat Brennan. Cosmic milestone: Nasa confirms 5,000 exoplanets. https://www.nasa.gov/feature/jpl/cosmic-milestone-nasa-confirms-5000-exoplanets, 2022. [Online; accessed 22 March 2022].

[2] Portia Wolf. Extrasolar planets. https://lasp.colorado.edu/outerplanets/exoplanets.php, 2022. [Online; accessed 22 March 2022].


# Appendices

## A1 Error Propagation

Many of the errors calculated in this assignment used Gauss' error law. For example the error in the mass was calculated using a function that took in values of $K_\star$, $\Delta K_\star$, $\sin i$ and $\Delta \sin i$ and outputted the error in mass. This equation this function used was as follows.

$$\frac{\Delta m}{m} = \sqrt{\left(\frac{\Delta K_\star}{K_\star}\right) + \left(\frac{\Delta \sin i}{\sin i}\right)}$$

This type of error in functions of multiplied or divided errors were also used to find the error in density.