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# Supernovae Characteristics

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## ABSTRACT

This assignment was split up into two main parts. In the first part, the redshifts of 8 SNe Ia were calculated by comparing the observed wavelengths of the  $H\alpha$  line in the supernovae spectra with its rest wavelength, 6563Å. The SNe Ia light curves were fitted with polynomials in order to find each peak apparent B magnitude,  $B_{max}$ , and the difference in magnitude relative to this peak 15 days later, producing the light curve parameter  $\Delta m_{15}(B)$ . The absolute peak magnitudes  $M_{max}(B)$  of the supernovae were calculated using the distances associated with the calculated redshifts and the measured apparent peak magnitude. This was then plotted against the light curve parameter  $\Delta m_{15}(B)$  in order to establish a luminosity decline relationship. The slope of this plot was found to be 1.06, meaning that more luminous supernovae (lower absolute magnitudes) have smaller values of  $\Delta m_{15}(B)$ . The slope of this curve was used to correct the values of  $B_{max}$ . It was found that the  $\chi^2$  statistics were reduced from 2681.4 to 24.9 and the RMS was reduced from 0.47 to 0.26 for the corrected data. This correction was seen to improve the use of SNe Ia for measuring distances in the Universe. The second part of the assignment used the Joint Light curve Analysis SNe Ia sample in order to determine the type of Universal model consistent with the data. The analysed models included a mass-less model of the universe, a flat universe with no dark energy and the preferred model of the Universe. The  $\chi^2$  statistics were compared for each fit, with the preferred model of the Universe having a  $\chi^2$  value of about 598, compared to values of 862 and 2901 for the mass-less and dark-energy-less models respectively. Intuitively, the preferred model of the Universe was seen to be a statistically better fit than the other two models.

## 1. INTRODUCTION

This assignment is split up into two main parts. The first of these works with a data-set containing the spectra and light-curve data for a sample of 8 type Ia supernovae (SNe Ia). From the spectral features the redshifts will be calculated by comparing the observed wavelength of the  $H\alpha$  emission line in each spectrum to the rest wavelength of  $H\alpha$ , 6563Å. Next the apparent B-band magnitude of each supernova will be plotted as a function of time. Analysis of these light curves will attempt to verify the luminosity decline relationship for type Ia supernovae. Using this luminosity decline relationship the peak apparent magnitudes of the supernovae will be corrected in order to test if type Ia supernovae are standardisable candles.

In the second part of the assignment the Joint Light curve Analysis (JLA) supernovae sample will be used. The distance moduli of these supernovae will be calculated and compared to those predicted by a number of different cosmological models, including a mass-less model, the preferred model and a flat

model with no dark energy. The  $\chi^2$  statistics for each fit will be compared in order to deduce the most accurate model for the data in the sample.

## 2. THEORY

### 2.1 THE HUBBLE DIAGRAM

The Hubble diagram is used to relate the distance to a stellar object to its redshift. The original Hubble diagram from 1929 plotted the recessional velocity as a function of distance however the convention in modern times is to plot the distance on the y-axis and the velocity or redshift on the x-axis. Different ways of plotting the Hubble diagram exist such as plotting the distance modulus against the base 10 logarithm of the redshift  $z$ , or  $z \cdot c$  with  $c$  being the speed of light, which will be the type of Hubble diagram used for Section 4 of this assignment. Another way of plotting the Hubble diagram for type Ia supernovae is to plot the peak apparent magnitude of the supernova on the y-axis and the base 10 logarithm of  $z \cdot c$ , which will be used in Section 3 of this assignment. The paper by [1] also uses this method of plotting the Hubble diagram.

### 2.2 SNE IA AS COSMOLOGICAL PROBES

Supernovae are highly energetic, luminous stellar explosions. The supernovae that will be analysed in this assignment are type Ia supernovae; the explosions of white dwarf stars in binary systems. The light curves of SNe Ia are standardisable, meaning their light curves can be standardised so that the supernovae can be employed to measure distances in the Universe. In order to standardise the peak apparent magnitudes of type Ia supernovae the relationship between the absolute magnitude and the light curve parameter  $\Delta m_{15}(B)$ , the change in apparent magnitude in the B band measured at 15 days after the peak magnitude, is analysed. The findings in [1] give that more luminous SNe Ia have smaller values of this parameter and this relationship will be investigated further in this assignment. From the slope of this luminosity decline relation the apparent peak magnitudes of the supernovae are corrected so that they can be used as cosmological probes. This correction is given by

$$B_{corr} = B - \text{slope} \times (\Delta m_{15}(B) - 1.1) \quad (1)$$

where the 1.1 value is a scaling relative to a SN with this decline rate from [1].

### 2.3 COSMOLOGICAL MODELS OF THE UNIVERSE

The second part of this assignment will compare three different models of the Universe to the JLA SN sample in order to determine which type of universe the data is consistent with. The first of these is a mass-less universe with a Hubble constant of  $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . In this model the distance in Mpc is related to the redshift of an object and the Hubble constant by the following equation:

$$d = \frac{cz}{H_0} \left(1 + \frac{z}{2}\right) \quad (2)$$

The LambdaCDM function from the astropy.cosmology package can be used to create this model.

The second of these is the preferred model of the Universe with a Hubble constant of  $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , 30% dark matter and 70% dark energy. The third model is a flat universe assuming matter but no dark energy. The FlatLambdaCDM function from astropy.cosmology package can be used to create these two models of the Universe.

### 2.4 THE JOINT LIGHT CURVE ANALYSIS (JLA)

The JLA is a state of the art supernovae sample containing 740 type Ia supernovae with redshifts between 0.01 and 1.3. The light curve parameters for each supernova are also seen with mass estimates

of their host galaxy. There are new, more modern methods for parameterising the light curves of SNe Ia than the simple luminosity decline relationship that is used in the first part of this assignment, where the absolute peak magnitude  $M_{max}(B)$  is seen to increase with increasing  $\Delta m_{15}(B)$  values. The light-curve width is given by the quantity  $x_1$  which takes into account the rise and fall of the light curve. There is a colour term  $c$  which is approximately equal to the colour index,  $B - V$ , with more luminous SNe Ia being found to have bluer colour indices. There is a further correction to be made for the total mass of the host galaxy called  $\Delta_m$ . This correction increases the brightness of SNe Ia in galaxies where the stellar mass is greater than  $10^{10} M_\odot$  by this value. These light curve parameters allow the calculation of the distance modulus by the following equation

$$\mu = m_\star^B - M_B + \alpha x_1 - \beta c \quad (3)$$

with  $m_\star^B$  being the apparent magnitude in the B band and  $M_B$  is the absolute peak magnitude of a type Ia supernova that has  $x_1, c = 0$ .

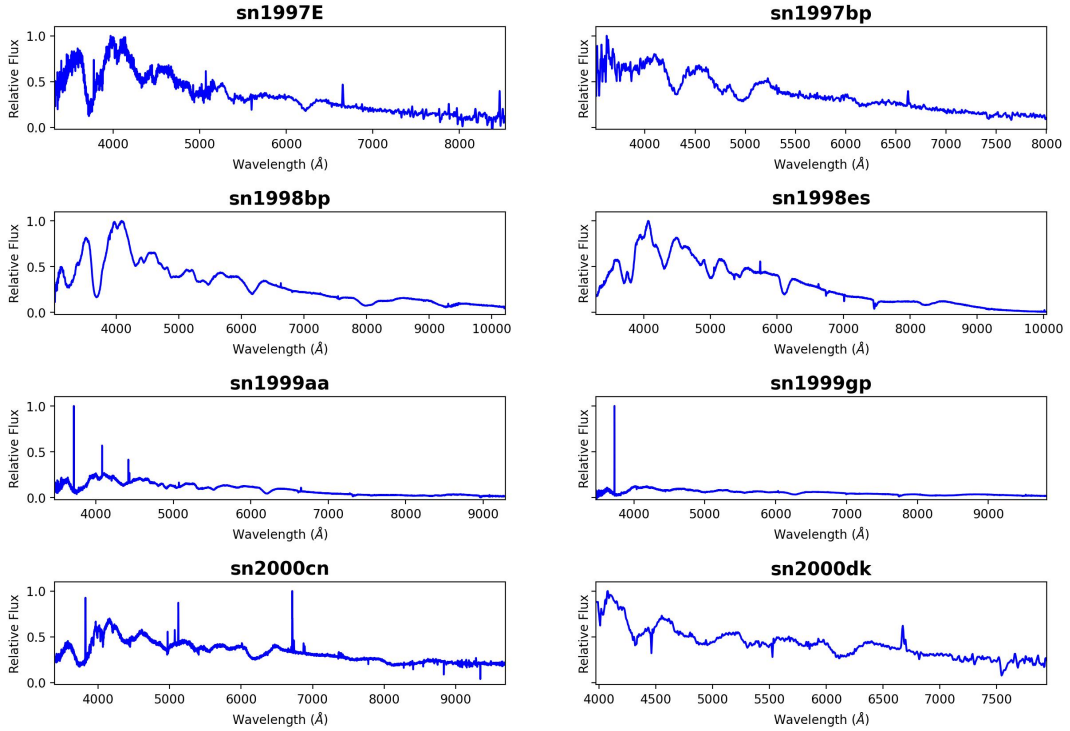


Figure 1: Plotted spectra of 8 SNe Ia in this sample. From the spectra the wavelengths of the  $H\alpha$  lines could be estimated and used in order to find the observed wavelength of the  $H\alpha$  line in each spectrum. The narrow  $H\alpha$  emission lines are features of the host galaxies of the supernovae and therefore are very useful for calculating the redshift of the galaxy and SN. The broad, deep Si II absorption is a feature of the SN ejecta and is not able to be used to calculate the redshift.

### 3. STANDARDISING SNE IA LIGHT CURVES

#### 3.1 MEASURING REDSHIFTS FROM SUPERNOVAE SPECTRA

In order to calculate the redshifts of each supernova the observed wavelength of the  $H\alpha$  emission line was calculated. Each type Ia supernova spectrum was plotted in order to identify the  $H\alpha$  line and give an estimate of its wavelength, as seen in Figure 1. A function was defined that fit a Gaussian to each  $H\alpha$  emission line over a specified range, given the initial guess of the wavelength of the line. The Python package used was the `scipy.optimize` package with the `curve_fit` function being used for the fitting of the

lines. The output of this code is the peak wavelength of each H $\alpha$  line and the associated errors, which were taken to be the standard deviation  $\sigma$ , along with plots of the fit to each H $\alpha$  line. These fitted H $\alpha$  lines can be seen in Figure 2. These wavelengths were then compared to the rest wavelength of H $\alpha$ , 6563Å, to produce the redshift of each supernova and the error associated with each of these.

In table 1 the observed wavelength of the H $\alpha$  line in each supernova spectrum along with the calculated redshifts are displayed. The redshift of each supernova was calculated from the H $\alpha$  line by the following formula:

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}} \quad (4)$$

In this table a further error was added to the redshift, corresponding to a radial velocity of 300 km/s or a redshift of 0.001 in order to take account of the systematic error associated with the data. Due to the small number of points in the H $\alpha$  emission line in the spectrum of sn1998bp the error in the wavelength was increased from  $1\sigma$  to  $2\sigma$ .

Name	H $\alpha$ (Å)	Redshift
sn1997E	6653 $\pm$ 5	0.014 $\pm$ 0.002
sn1997bp	6621 $\pm$ 4	0.009 $\pm$ 0.002
sn1998bp	6629 $\pm$ 1	0.010 $\pm$ 0.001
sn1998es	6623 $\pm$ 3	0.009 $\pm$ 0.001
sn1999aa	6649 $\pm$ 2	0.013 $\pm$ 0.001
sn1999gp	6736 $\pm$ 3	0.026 $\pm$ 0.001
sn2000cn	6715 $\pm$ 2	0.023 $\pm$ 0.001
sn2000dk	6673 $\pm$ 6	0.017 $\pm$ 0.002

Table 1: Table containing the observed wavelength of the H $\alpha$  line and the redshift for each spectrum, with the associated errors. The redshifts are calculated from the observed wavelength and the rest wavelength of H $\alpha$ , using equation (4).

### 3.2 LIGHTCURVE PROPERTIES

A data-set containing light curve data for these SN Ia was used for this section. In the given data-set there existed no light curve data for sn1999gp and in its place there was light curve data for sn1999dq. Therefore for this part of the assignment the redshift and its associated error for sn1999gp was removed and replaced with the redshift of sn1999dq, which was given to be  $0.01468 \pm 0.00002$  [2] and this new dataset was used to analyse further sections.

The aim was to plot the light curve of each SNe Ia using the apparent B-band magnitude in the data-set and produce a fit of these light curves in order to calculate the light curve parameter  $\Delta m_{15}(B)$ , the change in the apparent B-band magnitude 15 days after the peak. Some B-band magnitudes in the

Name	Peak B Mag	$\Delta m_{15}(B)$
sn1997E	15.63 $\pm$ 0.03	1.3 $\pm$ 0.2
sn1997bp	14.09 $\pm$ 0.01	1.2 $\pm$ 0.1
sn1998bp	15.62 $\pm$ 0.02	1.9 $\pm$ 0.3
sn1998es	13.96 $\pm$ 0.05	0.9 $\pm$ 0.2
sn1999aa	14.89 $\pm$ 0.07	0.9 $\pm$ 0.2
sn1999dq	14.85 $\pm$ 0.04	1.0 $\pm$ 0.1
sn2000cn	16.78 $\pm$ 0.05	1.6 $\pm$ 0.3
sn2000dk	15.63 $\pm$ 0.02	1.7 $\pm$ 0.1

Table 2: Table displaying the peak apparent magnitude and light-curve parameter  $\Delta m_{15}(B)$  for each supernova. These values were be used to analyse the luminosity decline relation.

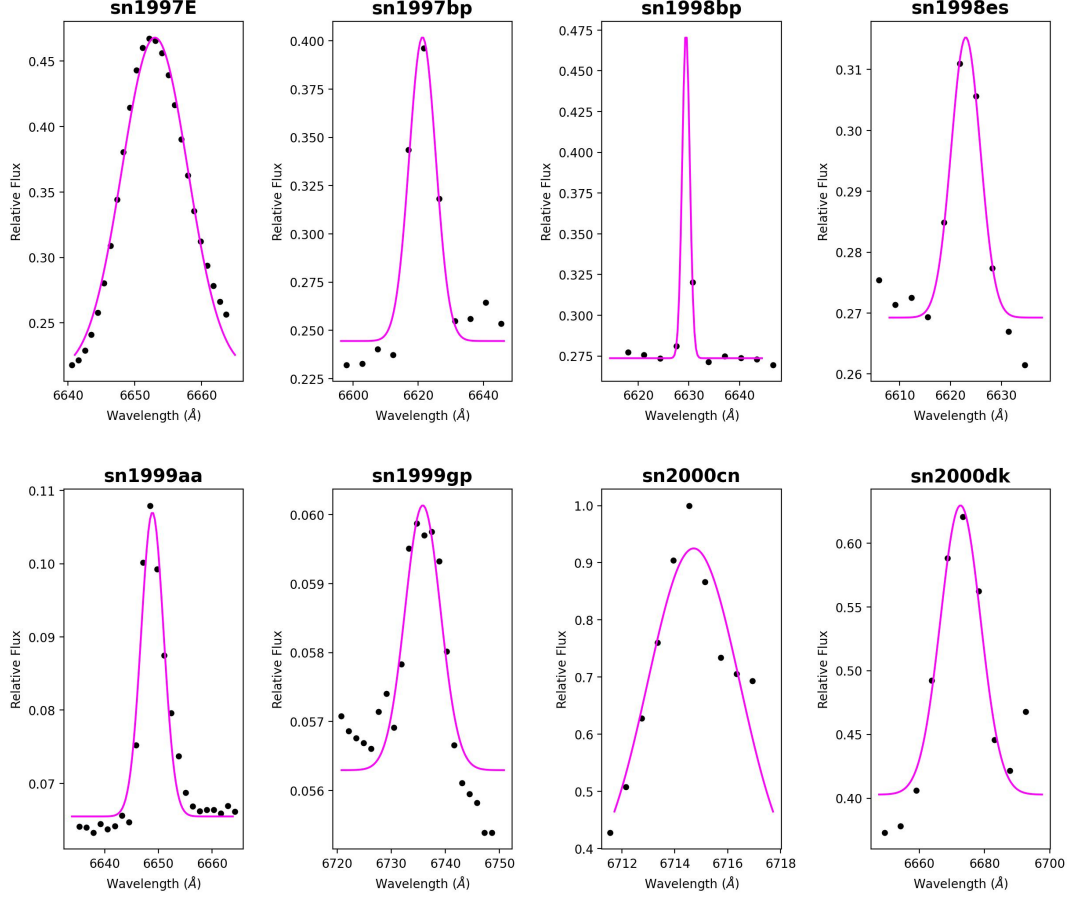


Figure 2: Gaussian fits of the  $H\alpha$  line in each spectrum. These sharp emission line is a feature of the host galaxy of the supernova. From these fits the wavelength of the  $H\alpha$  line were found and used to calculate the redshift of the galaxy (and hence the supernova) by comparing the observed wavelength with the rest wavelength of  $H\alpha$ ,  $6563\text{\AA}$ .

data-set were given as 99.999, which is meant to signify that a measurement was not taken on these days. Therefore these days were ignored for the analysis of the data. Furthermore a correction was made to the time values to account for time dilation, which stretches the light curve by a factor of  $(1 + z)$ , therefore the time values for each supernova were divided by  $(1 + z)$  where  $z$  is the redshift of that supernova. A function was defined that fit a polynomial to each light curve, given the degree of the polynomial needed for each fit and the range of data over which to fit for each light curve. The output of this function was the peak magnitude and  $\Delta m_{15}(B)$  parameter for each light-curve and their respective errors, along with plots of the individual light curves and their fits.

Figure 3 shows the apparent B-band magnitude of each supernova plotted as a function of time, in units of days after the peak magnitude in the B-band. Also seen in these plots are the fits to the data, which were used in conjunction with the Python uncertainties package in order to find the peak magnitude and the light curve parameter  $\Delta m_{15}(B)$  along with the associated errors with these two values. These can be seen in table 2.

### 3.3 LIGHT CURVE STANDARDISATION

Firstly the distance to each supernova was calculated from the red-shifts using a cosmological model of a flat universe with cold dark matter, a Hubble constant of  $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $\Omega_M = 0.3$ . To create a cosmological model of the Universe for these conditions the FlatLambdaCDM function from

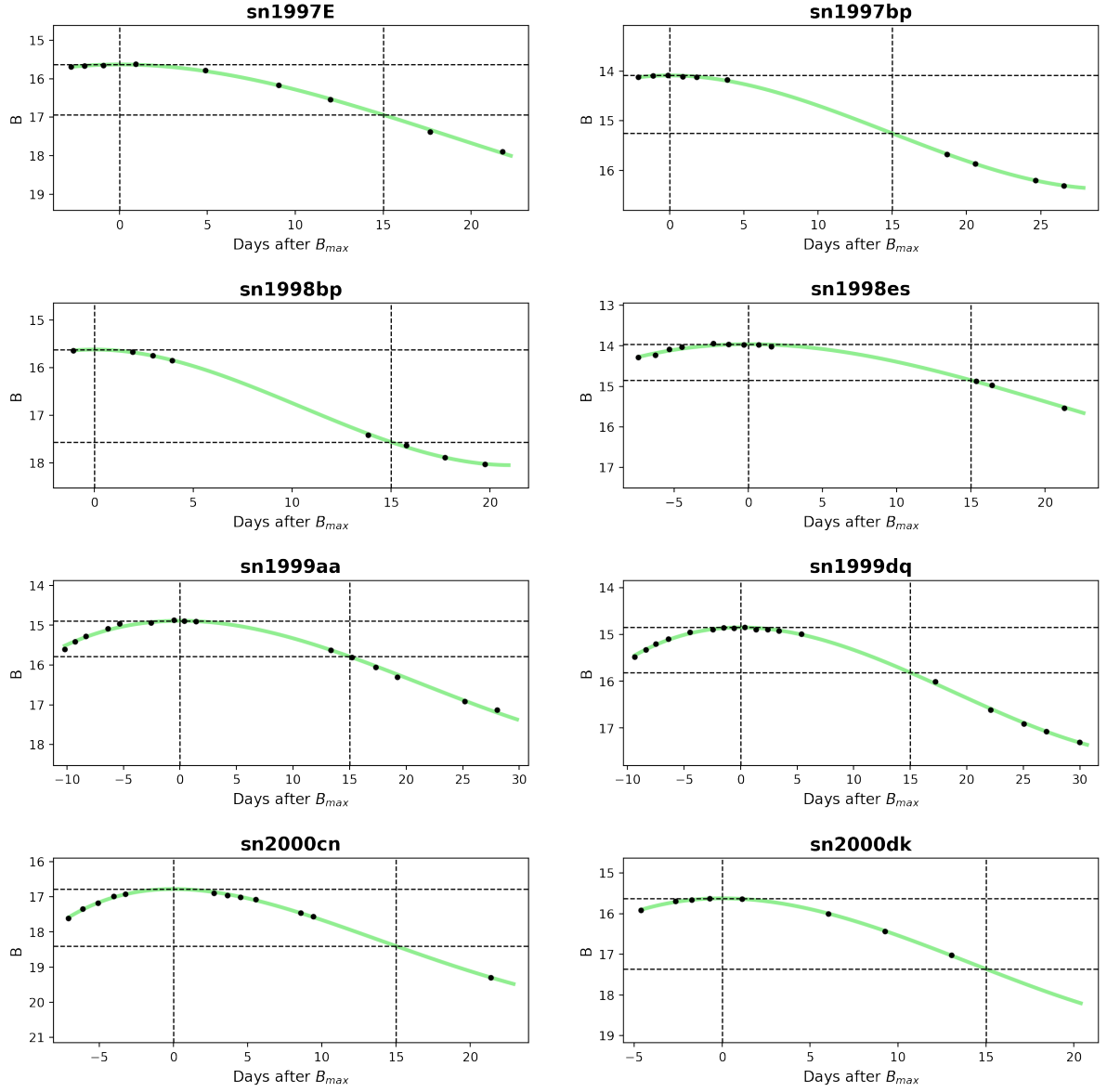


Figure 3: Light curves of the 8 SNe Ia in this sample. The apparent magnitude in the B band is plotted against time in days after the peak magnitude (making a correction for time dilation). These light curves are fitted with a polynomial in order to calculate the light curve parameter  $\Delta m_{15}(B)$  which gives the change in magnitude between the peak magnitude and the magnitude at 15 days after the peak. This parameter is used along with the redshift calculated in section 3.1 to identify a luminosity decline relation, which will be used to correct and standardise the peak magnitudes of the supernovae. The peak magnitude  $B_{max}$  and the lightcurve parameter  $\Delta m_{15}(B)$  for each supernova can be seen in table 2 along with the associated errors.

the `astropy.cosmology` package was used. From these distances and the known peak magnitudes in the B-band, the absolute peak magnitude  $M_{max}(B)$  of each supernova in the B-band was calculated. This absolute magnitude was then plotted against the light curve parameter  $\Delta m_{15}(B)$  for each supernova in order to establish a luminosity decline relation. A linear fit to the data was performed, again using `curve_fit`, and the slope of this fit was used to correct the apparent peak magnitudes of the supernovae. Two Hubble diagrams were plotted; one for the uncorrected case and one for the corrected case, in which the peak B-band magnitude was plotted against  $\log_{10}(c \cdot z)$ , with  $c$  in km/s.

In Figure 4 a plot of  $M_{max}(B)$  against  $\Delta m_{15}(B)$  can be seen. Although this data does not seem to decline very steadily there is an observable decline in luminosity (increase in absolute magnitude) as the light curve parameter  $\Delta m_{15}(B)$  increases. A reason for this variation from linearity may be due to the fact that the  $H\alpha$  line was solely used in Section 3.1 to deduce the redshift of each SN and this may have caused an error in calculating the absolute peak magnitudes. Furthermore there is also a suggestion by [3] to reject events with  $\Delta m_{15}(B) > 1.5$ , and as multiple data points in this sample have values of this parameter greater than 1.5, this may be another source of error. Making an assumption that the data is linear, a fit of this plot was produced, with a slope of 1.06. This slope was used to make the correction to the apparent B-band magnitude using equation (1). The corrected apparent magnitude of each SN is seen in table 3.

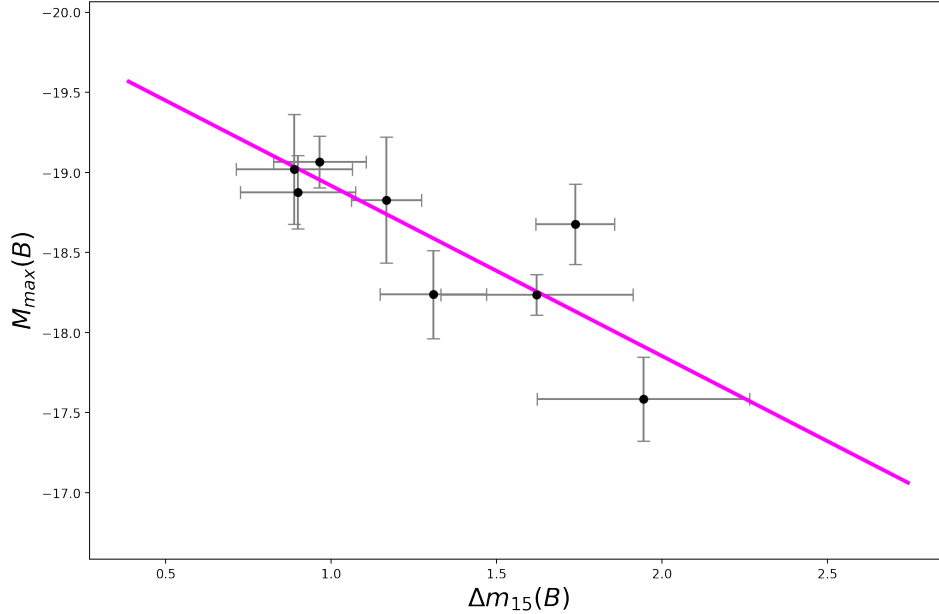


Figure 4: The luminosity decline relation between the peak absolute magnitude of each supernova in the B band and the light curve parameter  $\Delta m_{15}(B)$ . It can be seen from the plot that there is a declining relationship between the two parameters, thus indicating that more luminous SNe Ia have smaller  $\Delta m_{15}(B)$  values. The large error bars seen in the plot are due to some light curves having very few points to fit around their peaks and this leading to larger errors in the fitting parameters of the polynomial fits.

In Figure 5 two Hubble diagrams are seen where both the uncorrected and corrected peak B magnitudes are plotted against  $\log_{10}(c \cdot z)$ . These two Hubble diagrams were fitted with a linear fit and the residuals of each data set were found. Upon taking the standard deviation of each set of residuals the uncorrected fit residuals were found to have a RMS of 0.47 whereas the corrected residuals were found to have a RMS of 0.26, a significant reduction. Furthermore the  $\chi^2$  is reduced from 2681.4 to 24.9 for the corrected data. This reduction in spread in the corrected data shows that the correction to the peak magnitudes gives a more linear Hubble diagram and therefore improves the use of SNe Ia for distance measurements.

Name	$B_{max}$	$\Delta m_{15}(B)$	d [Mpc]	$M_{max}(B)$	$B_{corr}$
sn1997E	$15.63 \pm 0.03$	$1.3 \pm 0.2$	$59 \pm 8$	$-18.2 \pm 0.3$	$15.4 \pm 0.2$
sn1997bp	$14.09 \pm 0.01$	$1.2 \pm 0.1$	$38 \pm 7$	$-18.8 \pm 0.4$	$14.0 \pm 0.1$
sn1998bp	$15.62 \pm 0.02$	$1.9 \pm 0.3$	$44 \pm 5$	$-17.6 \pm 0.3$	$14.5 \pm 0.4$
sn1998es	$13.96 \pm 0.05$	$0.9 \pm 0.2$	$39 \pm 6$	$-19.0 \pm 0.3$	$14.3 \pm 0.2$
sn1999aa	$14.89 \pm 0.07$	$0.9 \pm 0.2$	$57 \pm 6$	$-18.9 \pm 0.2$	$15.2 \pm 0.2$
sn1999dq	$14.85 \pm 0.04$	$1.0 \pm 0.1$	$61 \pm 4$	$-19.1 \pm 0.2$	$15.0 \pm 0.2$
sn2000cn	$16.78 \pm 0.05$	$1.6 \pm 0.3$	$101 \pm 5$	$-18.2 \pm 0.1$	$16.1 \pm 0.4$
sn2000dk	$15.63 \pm 0.02$	$1.7 \pm 0.1$	$72 \pm 8$	$-18.7 \pm 0.3$	$14.8 \pm 0.2$

Table 3: Table displaying relevant data for each supernova in the data set. The values of  $B_{max}$  and  $\Delta m_{15}(B)$  are found from the fit of the supernova light curves. The distance to the supernova is found from the redshift of the supernova as found in section 3.1, assuming a flat universe with cold dark matter using the astropy.cosmology function FlatLambdaCDM. The absolute magnitude  $M_{max}(B)$  is found from the distance modulus formula, employing the previously calculated distance to the supernova. The corrected peak magnitude in the B band is calculated from equation (1) and allows the supernovae to be used as standardisable candles.

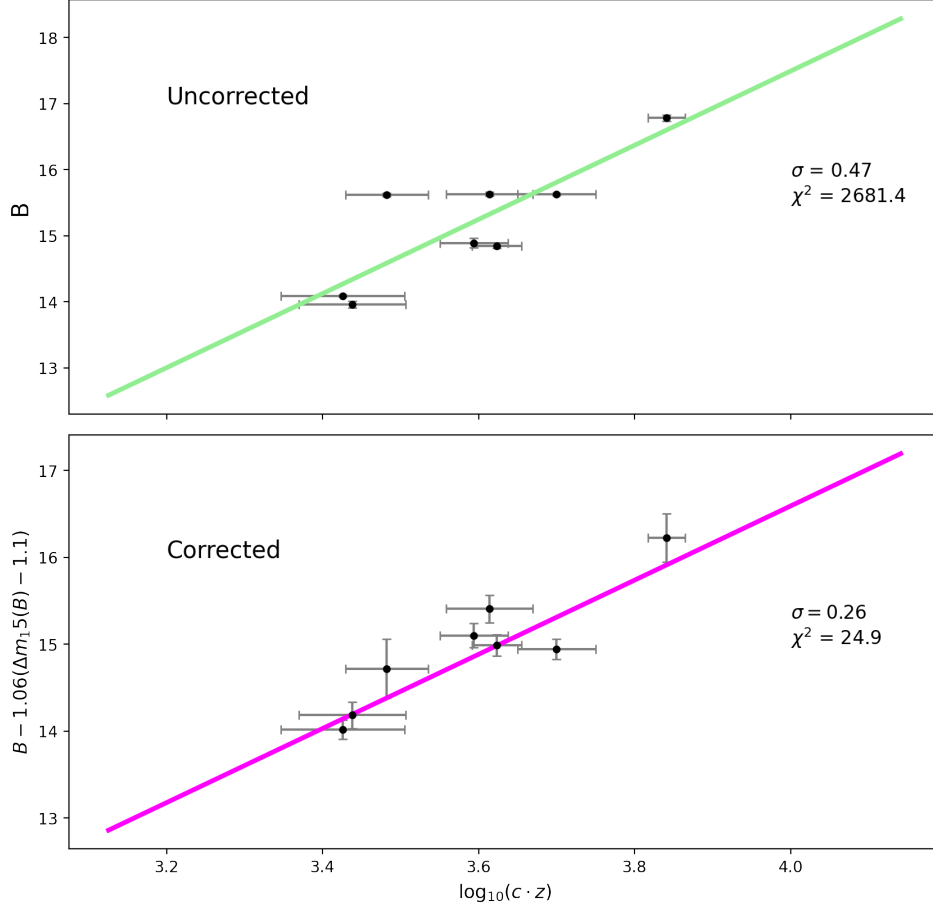


Figure 5: Hubble diagrams where the peak magnitude in the B band is plotted against  $\log_{10}(c \cdot z)$  for both the uncorrected and corrected cases for the supernovae sample. The corrected B magnitude improves the linearity of the data as seen in the reduction in the spread of the data (from the  $\sigma$  and  $\chi^2$  statistics of each plot).



#### 4. ANALYSING THE JLA SAMPLE

For this section the JLA SNe Ia sample was used and three different universal models as described in Section 2.3 were compared to the data in order to find the best fitting model. The paper by [4] gives the best fitting values of the constants for equation (3) to be used for this section, with  $\alpha = 0.141 \pm 0.006$ ,  $\beta = 3.101 \pm 0.075$ ,  $M_B = -19.5 \pm 0.02$  and  $\Delta_m = -0.07 \pm 0.02$ .

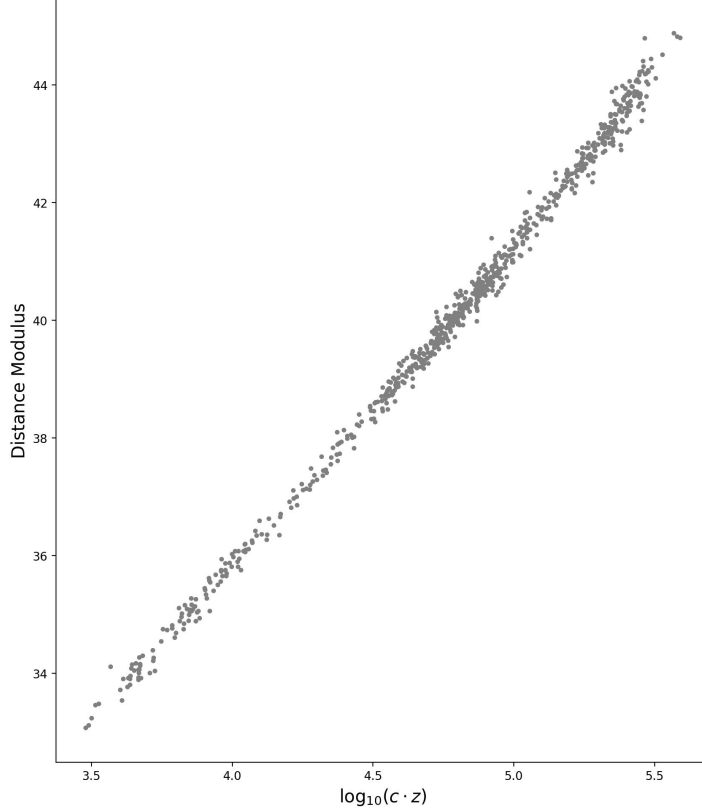


Figure 6: Hubble diagram where the distance moduli of the SNe Ia in the JLA sample are plotted against  $\log_{10}(c \cdot z)$ , where  $c$  is the speed of light in km/s. There is an increasing trend visible in the data, with redshift seen to increase with distance. The exact nature of this decrease will be explored when fitting the numerous cosmological models to the data later in this section.

##### 4.1 PLOTTING A HUBBLE DIAGRAM OF THE JLA SAMPLE

The JLA data was contained in a .csv file which was read into the Python script using pandas and the various columns were extracted from the table. The distance modulus of each supernova was calculated using equation (3). A function was defined that took light curve parameters of  $m_B$ ,  $x_1$  and  $c$  as inputs. The function uses equation (3), adding the correction  $\Delta_m$  to  $M_B$  where the stellar mass of the host galaxy of the supernova  $M_{stellar} > 10^{10} M_{\odot}$  or  $\log_{10}(M_{stellar}) > 10$ , as  $M_{stellar}$  is given in units of solar masses in the JLA data file. The function gives out the value of the distance modulus and its error for each supernova in the JLA file.

Figure 6 shows the distance moduli of the supernovae in the JLA data set plotted against the  $\log_{10}(c \cdot z)$  where  $c$  is the speed of light and  $z$  is the redshift of each supernova. It can be seen that the log of the redshift increases as the distance modulus increases, however the next section will deal with the model most consistent with the type of increase seen in this plot.

#### 4.2 COMPARING THE JLA DATA WITH DIFFERENT COSMOLOGICAL MODELS

The conditions for three different types of universes were created using the `astropy.cosmology` package. A mass-less universe was created with the `LambdaCDM` function, with a Hubble constant of 70 km/s/Mpc. The `FlatLambdaCDM` function was used to create conditions for both the preferred model, with the same Hubble constant and  $\Omega_M = 0.3$  and a flat universe model with no dark energy. The distance moduli associated with the supernovae redshifts were calculated for each model universe and plotted with the supernovae data in order to find the best fit to the data.

Figure 7 shows the distance moduli of the type Ia supernovae in the JLA data set plotted against the  $\log_{10}(c \cdot z)$  where  $c$  is the speed of light and  $z$  is the redshift of each supernova. Also seen in this plot are the theoretical values for the three different types of universe discussed above. It can be seen that for small redshifts all three models are approximately equal, however at larger and larger redshifts the mass-less model and the model without dark energy begin to deviate from the data. The residuals of the fits were found by taking the data away from each fit and dividing by the distance modulus errors of each data point. The  $\chi^2$  and RMS scatter statistics can be seen in the plot. Both the  $\chi^2$  and  $\sigma$  is lowest for the preferred model of the Universe, which is to be expected. The significance of this is that this model predicts the existence of dark energy, which opposes the gravitational force and is causing the universe to speed up in its expansion, rather than slow down.

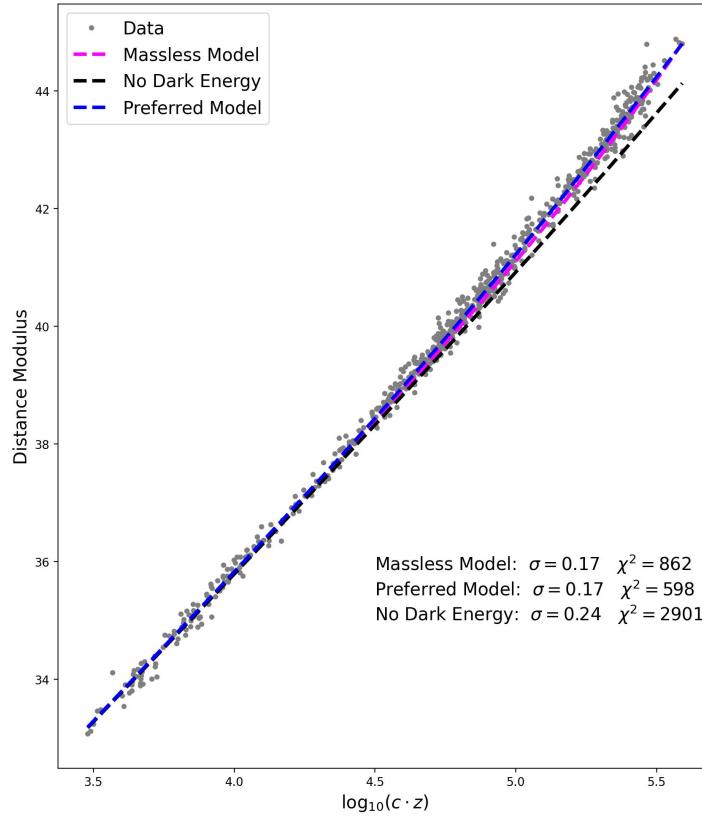


Figure 7: Numerous model fits to the JLA supernovae data. The three cosmological models that are seen plotted are a mass-less universe, a flat universe with cold dark matter (CDM) and no dark energy and the preferred model of the Universe: a flat model with CDM, a Hubble constant of approximately 70 km/s/Mpc, 30% dark energy ( $\Omega_M \approx 0.3$ ) and 70% dark energy ( $\Omega_\Lambda \approx 0.7$ ). It can be seen from the  $\chi^2$  statistics that the preferred model is the best fit to the data.

## DISCUSSION

This assignment used two different data-sets containing spectral and light curve data for two different samples of SNe Ia. The first of these contained the spectra and light curve data of 8 SNe Ia. The spectra were used to find the redshift of each supernova using the  $H\alpha$  line and calculate the peak apparent magnitude in the B band by fitting the light curve of each supernova. These fits allowed the calculation of the light curve parameter  $\Delta m_{15}(B)$  which gives the change in apparent magnitude of the supernova between the peak and 15 days after the peak. These values along with the absolute magnitudes of the supernovae, calculated from the apparent magnitudes and redshifts using the distance modulus, were used to produce a luminosity decline relation, with a slope of approximately 1.06. This found that more luminous supernovae, with larger absolute magnitudes, had smaller values of  $\Delta m_{15}(B)$ . This luminosity decline slope was used to make a correction to the peak apparent magnitudes of the supernovae in order to standardise them to use as standard candles: for measuring distances in the Universe. The correction to the apparent peak magnitudes was seen to reduce the  $\chi^2$  from 2681.4 to 24.9 and RMS scatter of the data was reduced from 0.47 to 0.26. This clearly made the peak magnitudes of the supernovae more suitable for distance measurements in the Universe.

The second data set was a state of the art sample of 740 SNe Ia called the Joint Light curve Analysis sample, which contained light curve information for each supernova in the data set, as well as some other values such as the stellar mass of the host galaxy of each supernova and the associated redshifts. From the data given for the JLA sample the distance moduli of each supernova was found using a modern correction to the distance modulus from [4]. These distance moduli were then plotted as a function of the base 10 logarithm of the corresponding redshifts multiplied by the speed of light in km/s. The relationship between these two values shows an observable increasing trend. This data was then compared to a number of cosmological models, with the  $\chi^2$  and RMS scatter statistics suggesting the preferred model of a flat universe, with cold dark matter and values of 30% dark matter, 70% dark energy and assuming a Hubble constant of approximately  $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , is the best fit to the data from the JLA sample. This model suggests the expansion rate of the Universe is not decelerating as one with an understanding of the gravitational force would first expect, but rather it is accelerating and suggests there is something in great abundance in our Universe that opposes the gravitational force, namely the mysterious and poorly understood dark energy.

## REFERENCES

- [1] Mario Hamuy, MM Phillips, Jose Maza, Nicholas B Suntzeff, RA Schommer, and R Aviles. A hubble diagram of distant type ia supernovae, 1995.
- [2] Bert W Rust, Dianne P O’leary, Katharine M Mullen, et al. Modelling type 1a supernova light curves. *Exponential Data Fitting and Its Applications*, pages 169–186, 2010.
- [3] A Saha, Allan Sandage, GA Tammann, Lukas Labhardt, FD Macchetto, and N Panagia. Cepheid calibration of the peak brightness of type ia supernovae. ix. sn 1989b in ngc 3627. *The Astrophysical Journal*, 522(2):802, 1999.
- [4] Daniel Moshe Scolnic, DO Jones, A Rest, YC Pan, R Chornock, RJ Foley, ME Huber, R Kessler, Gautham Narayan, AG Riess, et al. The complete light-curve sample of spectroscopically confirmed sne ia from pan-starrs1 and cosmological constraints from the combined pantheon sample. *The Astrophysical Journal*, 859(2):101, 2018.

## APPENDICES

### A1. ERROR PROPAGATION

Many of the errors calculated in this assignment used Gauss' error law. For example the error the absolute magnitudes for 3.3 were calculated using the following expression, derived from Gauss' error law:

$$\Delta M_{max}(B) = \Delta(B - 5 \log_{10}(d [\text{Mpc}]/10)) \quad (5)$$

$$= \sqrt{(\Delta B)^2 + \left( \frac{5}{\ln(10)} \frac{\Delta d}{d} \right)^2} \quad (6)$$

This Gaussian propagation of errors was also used for calculating errors in values such as the corrected distance modulus  $\mu$  in 4.

### A2. UNCERTAINTIES PACKAGE

The Python uncertainties package was utilised in Section 3.2 to find errors in the values of  $B_{max}$  and  $\Delta m_{15}(B)$ . The reason for using this package for this section was that the uncertainties package was found to work well with polynomials and the Gaussian error propagation of high order polynomials is a very long and tedious task. The ability to use this package within a loop was very useful and made the calculations of errors much simpler for this section.

The package was also used for the calculations of errors such as the error in the corrected peak B-magnitude. Performing mathematical operations on the unumpy arrays automatically propagates the error which was very useful.