Thomas Jefferson **Physics Olympiad**

A high school physics contest

This exam covers concepts related to astrophysics, specifically stars, and explores how physics can be used to analyze the properties of these celestial bodies.

TJ Physics Team <tjhsstphysicsteam@gmail.com>

Stars and Astrophysics

If you have any questions or concerns, please email TJ Physics Team.

Contents

Rules

Teams will have a total of four days to complete the TJPhO. All teams are required to submit their response by uploading a PDF to a Google Form. Each submitted page should also have the problem number. All other formatting decisions are delegated to the teams themselves, with no one style favored over another. Teams may either submit a handwritten or typed response. If teams choose to submit a handwritten response, all handwriting must be legible, and any writing that cannot be read by the graders will not receive credit. If teams choose to submit a typed response, we suggest that teams use LaTeX for typesetting. The easiest way we've found to typeset LaTeX is with Overleaf.

The winning teams will be decided by the number of points. If two teams have the same number of points, the higher-ranking one will be the one who submitted first.

Even if you are not able to fully solve a problem, please write up what you can. We want to see how you approached the problem, and partial credit may be awarded.

Collaboration

Students participating in the competition may only correspond with other members of their team. No other correspondence is allowed, including mentors, teachers, professors, and other students. Teams may use any computational resources they might find helpful, such as Wolfram Alpha/Mathematica, Matlab, Excel, or programming languages (C++, Java, Python, etc). Teams are not allowed to use online/print resources with the exception of language documentation for any problems that may require computation. Teams are under no circumstances allowed to post content on online forums asking questions related to the exam. We welcome teams to email us if there are any questions or concerns.

Submission

Teams must submit their solutions through the provided Google Form by the deadline 11:59 PM EST on June 10th. Late submissions will not be considered. Solutions should be written in English and submitted as PDF documents with the .pdf extension. Only one person per team, the person who registered through the Google Form, should send the final submission.

Awards

The top five teams will receive 3 one-year subscriptions to WolframAlpha Notebook Edition, a book from Impact Books, and a copy of a course on Lagrangian mechanics by

Physics with Elliot.

Sponsors

We are incredibly grateful to our sponsors, Dr. Adam Smith and Dr Abdel Hady Ebrahim, and Thomas Jefferson High School for Science and Technology for their support. We would also like thank our sponsors WolframAlpha, Impact Books, and Physics with Elliot for making this Olympiad possible.

Introduction

In the few hundred million years following the big bang, stars began to form as clouds of hydrogen collapsed due to gravity. In the cores of these stars, fusion processes began to produce increasingly heavy elements, while releasing light and heat into the rest of the universe. When stars reached the end of their life cycles, they shed stellar material in the form of planetary nebula or supernovae, which allowed future stars to form. Over billions of years, generations of stars formed and died out. In that process, planets formed in orbits around these stars, and in the case of the Earth, were able to support life. Throughout human history, people have looked up into a night sky filled with stars. During the last century, advances in science and technology have allowed humanity to gain an improved understanding of these celestial bodies. From our familiar sun, to extreme objects such as black holes where our understanding of physics break down, stars play a pivotal role in the universe.

The TJ Physics Team is proud to announce the sixth annual TJ Physics Olympiad (TJPhO)! We hope this contest will introduce you to the fascinating field of stars and astrophysics. To review concepts commonly used in astronomy to observe stars, we have included review problems covering topics such as stellar parallax, radiation laws, and nuclear fusion processes.

This contest covers a wide range in stellar physics including radiation pressure, white dwarf stars, binary star systems, hydrostatic equilibrium, and supernova explosions. We hope you have as much fun learning about stars as we had writing this contest.

Happy exploring!

TJ Physics Team Officers

1 Background Problems

Gravitation (40 pts.)

1 There always exists an attractive gravitational force between any two bodies with mass. *Newton's Law of Universal Gravitation* states that

$$\mathbf{F} = -\frac{Gm_1m_2}{|\mathbf{r}|^2}\,\hat{\mathbf{r}},$$

where $G = 6.67 \times 10^{-11}\ \mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{kg}^{-2}$ is the gravitational constant, m_1 and m_2 are the masses of the two bodies, and $\mathbf{r}$ is the displacement vector between the two bodies. The negative sign simply indicates that the gravitational force is always attractive.

Gravity is a conservative force (work done is independent of path), meaning that it has an associated potential. The *gravitational potential energy* of a two-object system is given by

$$U = -\frac{Gm_1m_2}{r},$$

where r is the distance between the two objects.

Suppose an object of mass m orbits around another object of mass M. The *total energy* E is given by

$$E = -\frac{GMm}{2a}$$

for circular and elliptical orbits, where a is the semi-major axis of the orbit.

For all parts to this question, express your answer in terms of m, M, r, R, and any fundamental constants. Remember that you may use integral calculators as needed.

a) A spacecraft of mass m orbits around a stationary star of mass M and radius R such that $m \ll M$. The orbit is completely circular with a radius of r. Find the velocity of this spacecraft.

b) We say that the spacecraft "falls into the star" when it's distance from the center of star is less than R ($r < R$). What is the maximum angular momentum for this to occur at some point in the spacecraft's orbit, assuming the maximum distance from the star is still r?

c) If the spacecraft's initial orbit was circular with radius r, what instantaneous change in the velocity would it need to undergo to achieve the orbit from part b?

d) Find the period of the orbit in part b), where the period is the amount of time it takes the spacecraft to complete one orbit.

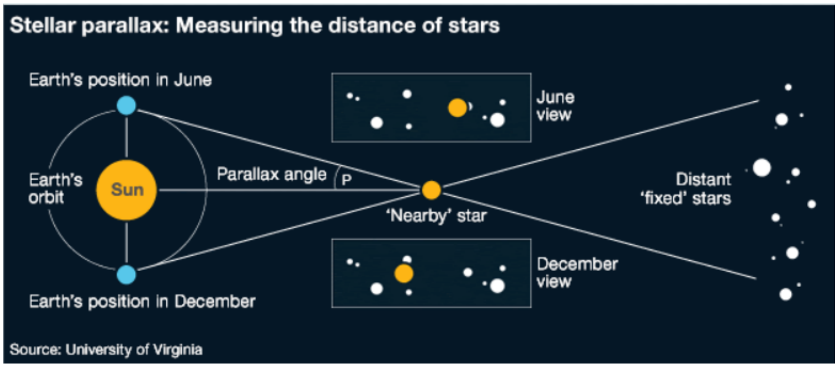
e) Now suppose that at time $t = 0$, an identical spacecraft located a distance r from the center of the star has no initial velocity. At what time t would the spacecraft reach the surface of the star?

Parallax and Intensity (30 pts.)

2 In the early 19th century, it was believed that stars reflected the sun's light instead of producing their own. For this to be possible, stars would have to be much closer to the sun than they actually are. In this problem we will investigate how the intensity of light changes with distance and how to calculate distances to nearby stars.

As the Earth revolves around the sun, the positions of nearby stars in the night sky shift slightly compared to very distant stars (which are considered to be completely stationary). Knowing the radius of the Earth's orbit around the sun, and assuming the orbit is perfectly circular, one can calculate the distances to nearby stars.

The stellar parallax angle is the angle representing the shift in the star's position. The maximum angular change in the star's position is twice the parallax.



The intensity of the star is defined as the rate of energy transfer per area:

$$I = \frac{P}{4\pi d^2}$$

where I is the intensity of the starlight, P is the power emitted by the star, and d is the distance from the light source.

a) Tau Ceti is a star with an observed parallax of 0.2738 arc seconds. An arcsecond is equal to $1/3600$ of a degree. The radius of the Earth's orbit is 1.496×10^{11} m. What is the distance of Tau Ceti from the Earth?

b) The intensity of the light from Tau Ceti is 1.176×10^{-9} W/m^2. Find the power outputted by that star.

c) The power outputted by the sun is on the order of 10^{26} W. The radius of Tau Ceti is 5.5×10^8 m. If Tau Ceti reflected all of the light hitting its surface, estimate to an order of magnitude the intensity of the reflected light reaching Earth from Tau Ceti.

The minimum intensity of light that humans can see is around 10^{-10} W/m^2. If stars only reflected the sun's light, they would appear far too dim for humans to see. The discovery of parallax provided the first evidence that stars produced their own light.

Radiation Laws (30 pts.)

3 Stars can be approximated as ideal blackbodies, which means that they follow Wein's Displacement Law and the Stefan-Boltzmann law.

All ideal blackbodies radiate some radiation in all wavelengths, but the vast amjority of this radiation is close to a certain wavelength. The peak wavelength of the light is the one with the highest intensity. Wein's Displacement law relates the temperature of a star and its peak wavelength by:

$$\lambda T = b$$

where b is a constant equal to 2.898×10^{-3} m $\cdot$ K.

The Stefan-Boltzmann law is a statement on the power radiated by an ideal blackbody:

$$P = \sigma A T^4$$

where σ is a constant equal to $5.67 \times 10^{-8} \dfrac{\text{W}}{\text{K}^4 \cdot \text{m}^2}$ (Watts per kelvin to the fourth per meters squared).

a) Arcturus is a red-orange star 36.7 light years from earth (one light year is 9.46×10^{15} m) with peak wavelength of 676 nm and intensity of light of 4.33×10^{-8} W/m^2. Calculate the radius of the star.

Stars like Arcturus are known as red giants due to their large size. At the end of the sun's life, its outer layers will expand to past the orbits of Mercury, Venus, and possibly Earth. Even larger are red supergiants. One example of such a star is Betelgeuce, with a radius of 617 million kilometers. If placed at the center of our solar system, Betelgeuce would engulf all four terrestrial planets, and its surface would nearly engulf Jupiter.

b) The sun has a power output of 3.85×10^{26} W and is 1.51×10^{11} m away from Earth. Neglecting the greenhouse effect, if the Earth did not reflect light, what would be its equilibrium temperature? You may assume that the far side of the Earth has the same temperature as the near side.

c) In reality, the Earth has an albedo of 0.3, which means that 30% of the light reaching the Earth is immediately reflected. Find the equilibrium temperature taking into account effects from the albedo.

The Earth remains at a habitable temperature thanks to the greenhouse gases in the atmosphere, which absorb and re-emit radiation that was reflected and radiated from the surface. Through this "trapping of heat" the Earth's equilibrium temperature is significantly higher than it otherwise would be.

Nuclear Fusion (40 pts.)

4 Stars must generate vast amounts of power to emit as radiation. This energy is supplied inside the stellar core, in which temperatures are so high that nucleii can undergo nuclear fusion. During this process, two smaller nuclei are smashed together to form one larger nuclei, which has a mass slightly less than the sums of the two original masses. The rest energy of a particle is $E_0 = mc^2$, so this change in mass is converted to energy that is released by this reaction.

Nuclear fusion involves subatomic particles, which primarily include leptons and baryons. The leptons relevant to nuclear fusion in stars are electrons (denoted as e^-), electron neutrinos (ν_e), and their anti-leptons ($\overline{e^-}$, $\overline{\nu_e}$). The leptons have a lepton number of 1 and the anti-leptons have a lepton number of -1. Electrons have a charge of -1, neutrinos have a charge of 0, and their anti-particles have the opposite charge.

Baryons include protons, which have $+1$ charge, and neutrons, which have no charge. Protons and neutrons have baryon numbers of 1 while their antiparticles have baryon numbers of -1.

In a reaction involving these subatomic particles, charge, lepton numbers, and baryon number must be conserved.

A nuclear reaction can release energy in the kinetic energy of these particles, or in the form of radiation (denoted as γ) carried away by massless particles called photons.

a) The proton-proton chain (also known as the pp chain) is one of the reactions in which protons undergo nuclear fusion, eventually resulting in alpha particles (helium nuclei). The pp-III chain is one mechanism by which this could occur. Although it does not contribute to a large portion of the sun's energy output, the pp-III chain produces highly energetic neutrinos which scientists can more easily detect. The pp-III branch consists of the reactions with missing particles A_1, A_2, A_3, and A_4 (A_3 and A_4 are interchangeable):

$$^3_2\mathrm{He} + ^4_2\mathrm{He} \longrightarrow ^7_4\mathrm{Be} + A_1$$

$$ {}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \longrightarrow A_2 + \gamma $$

$$ {}^{8}_{5}\mathrm{B} \longrightarrow {}^{8}_{4}\mathrm{Be} + A_3 + A_4 $$

What are the particles/nuclei represented by A_1, A_2, A_3, and A_4?

b) (2 point) Positron electron annihilation accounts for some of the released energy. The equation for this process is:

$$ \mathrm{e}^{+} + \mathrm{e}^{-} \longrightarrow 2\gamma $$

The mass of electrons and positrons are 9.11×10^{-31} kg. What is the energy released in this process?

c) (2 point) ${}^{8}_{4}\mathrm{Be}$ quickly decays into two two alpha particles:

$$ {}^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He} $$

The atomic masses of ${}^{4}_{2}\mathrm{He}$, ${}^{3}_{2}\mathrm{He}$, and ${}^{1}_{1}\mathrm{H}$ are 4.0015 amu, 3.0160 amu, and 1.0073 amu, respectively. One amu is equal to 1.6605×10^{-27} kg. What is the energy released from one nucleus of Helium-3, an alpha particle, and a proton undergoing this process?

d) Neutrinos can be detected in large observatories, where the neutrinos excite a charge lepton in a large pool of water. The charged lepton then moves faster than the speed of light in the medium, which produces Cherenkov radiation. By analyzing the intensity and angle of the Cherenkov light, scientists can deduce the speed of the particle in the medium. A neutrino has a velocity related by $\dfrac{v}{c} = 1 - 3.64 \times 10^{-5}$. The neutrino mass is $0.120\ \mathrm{eV}/c^2$. What is the total energy of the neutrino?

The pp-III and pp-IV cycles can produce neutrinos of significantly higher energies compared to other proton-proton chains, but they are far more common in stars more massive and hotter than the sun. These two processes do not contribute significantly to the sun's energy output.

e) The pp-I chain has an energy output of 26.73 MeV but loses 0.59 MeV of that energy for every ${}^{4}_{2}\mathrm{He}$ produced, and it accounts for 83.3% of the alpha particles produced. The pp-II chain has an energy output of 19.8 MeV and energy loss of 0.81 MeV, and it accounts for 16.7% of the alpha particles produced. Assuming that the neutrino does not interact significantly with other particles in the interior of a star, what is the energy contribution to the luminosity of the star for every alpha particle produced?

f) The electron scattering cross section of a neutrino can be thought of as the surface area that an electron occupies. This is given as

$$ \sigma \approx 10^{-48} \left(\frac{E_\nu}{m_e c^2} \right)^2 \ \mathrm{m}^2. $$

The number density of electrons in the sun is on the order of 10^{-30} m^3 and the rest energy of an electron is on the order of 1 MeV. Estimate to an order of magnitude the mean free path of a neutrino with energy of 1 MeV in a medium with the same density as the sun? Is the assumption we made in the previous part valid?

2 Applied Problems

Solar Sails (30 pts.)

5 Light from the sun exerts radiation pressure on conductors due to the photoelectric effect. One concept for spacecraft design that would eliminate the need for propellant would use the radiation pressure from the sun to increase the energy of the probe, allowing it to visit other planets in the solar system. In this problem we investigate the possibility of using solar sails to propel spacecraft.

a) Find the force due to radiation pressure on an ideal sail of area A, distance r from a star with power output P, and facing the sun, assuming that the photoelectric effect occurs in this material and that all light is reflected from the sail. Express your answer in terms of A, r, P, and/or any fundamental constants.

b) A solar sail of mass m and area A orbits a star of mass M in a completely circular orbit of radius r_1. The sail uses radiation pressure to propel itself to a distance of r_2. What is the change in kinetic energy of the sail in terms of P, m, M, A, r_1, r_2, and/or any fundamental constants?

c) Now suppose the sail is not perfectly reflecting and that the energy reflected by the sail E_r and radiant energy that reaches the sail E_i always follows $E_r = nE_i$. If the sail reaches a certain temperature T_m, it will burn up. Find the minimum value of r_1 so that the sail is not at risk of burning up (the equilibrium temperature remains lower than T_m). Express your answer in terms of P, n, m, M, A, r_2, and/or any fundamental constants.

d) A solar sail of mass m and area A is directly facing the sun. At $t = 0$, the sail is in a circular orbit with no radial velocity. Assume the intensity of light I hitting the sail is constant and that $\Delta r \ll r_0$, where Δr is the change in radius in time t. What is the radial velocity v_r in terms of A, m, I, t, and/or any fundamental constants.

e) Now find v_r from the previous problem, assuming that the star has power P and that the sail begins at radius r_0 from the star. Assume that $\Delta r \ll r_0$. You may not assume that I is constant. Express your answer in terms of r_0, P, t, m, A, and/or any fundamental constants.

Although solar sails are a promising technology with the potential of cutting costs for interplanetary exploration by light-weight, unmanned probes, the acceleration of these sails remains much lower than that of spaceships using conventional propulsion methods. For that reason, solar-powered spacecraft would be too slow for most purposes.

Hydrostatic Equilibrium (60 pts.)

6 Gravitational forces inside stars are balanced primarily by pressure gradients. Consider a star of mass M, radius R, and uniform mass density.

a) Considering the forces on a thin spherical shell at distance r from the star's center and thickness dr, determine how pressure inside a star varies according to distance.

b) Use your result from part a to determine the pressure at the center of the star.

c) Suppose a disturbance increases the pressure at the star's center by ΔP. Find the new gravitational field inside the star at a distance r from the star's center.

d) Consider the initial star, before the disturbance from part c. Assuming the gas in the star is ideal, derive an estimate of how the temperature inside the star varies according to the distance to the star's center.

The hydrostatic equation derived in part a can also be used to analyze the atmosphere of stars. Consider a star whose atmosphere consists of an isothermal ideal gas of temperature T, where the gas molecules have mass m.

e) Suppose the air pressure at the star's surface is P_0. Find the air pressure at a height h above the star's surface.

In reality, most star atmospheres are not constant in temperature. Hotter air tends to rise in the atmosphere and push colder air to the star's surface, a process known as convection. Suppose these convective processes are adiabatic, where the gas molecules have adiabatic constant γ. Also, assume that these processes happen very slowly.

f) Suppose the temperature at the star's surface is T_0. Find the air temperature at a height h above the star's surface.

g) Our sun's atmosphere is composed primarily of diatomic hydrogen gas. Given that the surface temperature of the sun is approximately 5500 K, use your result from part f to estimate the air temperature at a height $100,000$ kilometers above the sun's surface.

White Dwarves (60 pts.)

7 When stars run out of hydrogen and helium to fuse, they undergo core collapse due to gravity. In lower-mass stars like the sun, electron degeneracy pressure builds up, which stops the collapse. Electron degeneracy pressure occurs due to Pauli's Exclusion Principle, which states that no two electrons can occupy the same states. Eventually, the star sheds its outer layers and only the core remains, which is known as a white dwarf.

Electron degeneracy pressure must be strong enough to keep the white dwarf "star" in stable equilibrium. Letting the system be the electrons in the white dwarf, the total energy consists of the gravitational binding energy and the kinetic energy of the electrons.

a) The gravitational binding energy is the negative of the energy required to separate each particle to a distance very far away from every other particle. What is the gravitational binding energy of a white dwarf with radius R and mass M, in terms of R, M, and/or any fundamental constants?

At the high densities inside white dwarf stars, electrons become degenerate, which means they occupy their lowest possible states. The wave number of an electron is $k = \frac{2\pi}{\lambda}$, where λ is the de Broglie wavelength. The number of quantum states below a certain wave number k is given by

$$N = \frac{2V}{(2\pi)^3} \cdot \left(\frac{4}{3}\pi k^3\right),$$

where V is volume.

For parts b and c, express your answer in terms of N, V, m, m_p, dp, and/or any fundamental constants.

b) In a system of degenerate electrons with volume V and number of electrons N, find the number of electron states with momenta between p and $p + dp$.

c) The kinetic energy of a particle of mass m is related to the magnitude of its momentum p by the formula $K = p^2/2m$. What is the total energy of a system of degenerate electrons with volume V and number of electrons N?

Stars can be approximated as being at constant entropy. The principle of minimum energy is a result of the Second Law of Thermodynamics. It states that a closed system (one that can transfer energy but not matter) at constant entropy will experience equilibrium at the state of lowest total energy.

d) The sun has a mass of 1.99×10^{30} kg and a radius of 6.96×10^{8} m. Find the equilibrium radius of a white dwarf star with mass M in terms of R_s, M_s, and M, where R_s and M_s are the radius and mass of the sun.

e) As the mass of a white dwarf increases, so does the velocity of electrons. If the velocity of electrons is comparable to the speed of light, find the equation of total energy in terms of M, R, m, m_p (mass of proton) assuming that for all electrons $p \gg mc$.

f) For some mass M_c, there is no equilibrium radius for the white dwarf. Find this critical value M_c.

A more realistic calculation of M_c accounts for the change in density inside stars and yields $M_c = 1.4M_s$. Once the mass of white dwarfs exceed this critical value, core collapse resumes, which may lead to a supernova and the formation of a neutron star (see Problem 9).

Binary Star Systems (50 pts.)

8 Many of the stars in the universe are not single stars, but instead a pair of two stars that orbit each other. According to some source, 85% of stars are part of binary star systems.

Detecting binary stars is made difficult, and in some cases impossible, due to diffraction and aberration effects. While aberration effects can be reduced and nearly eliminated, diffraction effects are impossible to remove. The angular separation required for two objects to be barely resolvable is proportional to the first power of the wavelength of light.

a) Estimate to one order of magnitude the minimum angular separation between two stars for them to be resolvable by a telescope one meter in diameter.

b) The minimum distance between two sources of yellow light (575 nm) located 1 kilometer away from a 10 cm wide telescope for them to be barely resolvable by Rayleigh's criterion is 7.02 mm. Kepler-16 is a binary star system 90 light years from earth that primarily emits light with a wavelength of 600 nm. Calculate the minimum distance between two stars for them to be resolvable by a 1 meter wide telescope.

The separation between the two stars in the Kepler-16 system is 3.29×10^{10}, which is less than the minimum distance for the two stars to appear distinguishable from one another. This is one case in which binary star systems cannot be reliably detected by direct observation.

Another method of detecting binary star systems is through observing Doppler shifts in the spectral lines of the stars.

c) Astronomers observe that wavelengths of a pair of spectral lines from a possible binary system oscillate with amplitude $\Delta\lambda$, where the two wavelengths oscillate $180°$ out of phase. The average wavelength of both spectral lines is λ_0. Assume that the stars pass directly in front of each other when observed from the Earth and that both move in circular orbits. Find the velocity of the two stars in terms of λ_0, $\Delta\lambda$, and/or any fundamental constants.

d) If the period of the oscillations in the previous part is T, what is the mass of each of the two stars? Express your answer in terms of λ_0, $\Delta\lambda$, T, and/or any fundamental constants.

e) Binary star systems can also be detected using the transit method, in which one star passes directly in front of or behind the other star, changing the total brightness. This method is particularly useful when one star is much more luminous (the power output is much greater) than the other.

   Suppose a star system of a main sequence star and a white dwarf orbit around each other in circular orbits. The sum of the orbital radii is d. Every interval of time T, a primary transit, in which the white dwarf passes in front of the main sequence star, occurs. This causes the intensity of light from the star system to decrease by ΔL. Let the intensity of light when there isn't a transit be L_0, and the mass densities of the main sequence star and white dwarf to be ρ_1 and ρ_2. During the secondary transit, when the white dwarf passes behind the main sequence star, no change in brightness is observed. What is the mass of the larger star in terms of d, L_0, ΔL, ρ_1, ρ_2, T, and any fundamental constants?

Neutron Stars and Supernovae (60 pts.)

9 This problem is divided into three parts. Each part can be solved independently. You may find the following information useful: the mass of the sun is 1.99×10^{30} kg, and the gravitational constant is $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}$.

When a white dwarf exceeds the Chandrasekhar limit, collapse due to gravity resumes. This can occur in binary star systems, in which a white dwarf star gains mass from it's stellar neighbor, or in massive stars, where the core becomes to massive to maintain stable equilibrium. As the core collapses, temperature increases, which allows for the fusion of increasingly heavy nuclei. This stops the core collapse temporarily until the material for fusion is used up.

Eventually, iron builds up in the stellar core. The fusion of iron is endothermic, so as other elements are used up in fusion processes, nuclear fusion is unable to support the weight of the star. Core collapse resumes again. In some stars, due to processes that are not yet completely understood, the core rebounds, leading to a supernova and the formation of a neutron star. In other stars, the core continues to collapse into a black hole. In this problem, we investigate the first scenario.

a) During core collapse, as temperature increases, protons can undergo spontaneous electron capture, forming neutrons.

   i Normally, free neutrons decay according to the equation

$$\mathrm{n} \longrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}.$$

   Neglecting the masses of the electron and antineutrino (which are negligible), and using $m_p = 1.67262 \times 10^{-27}$ kg, and a neutron, $m_n = 1.67493 \times 10^{-27}$ kg, find the energy E_0 released in this reaction.

   ii Using quantum statistics (also seen in Problem 7), the maximum momentum for a group of N degenerate electrons is

$$p_{\max} = \frac{(3\pi^2)^{1/3} h}{2\pi} \left(\frac{N}{V}\right)^{1/3}.$$

   The electrons are highly relativistic, so you may assume that $p \gg mc$, where m is the electron rest mass, p is the electron momentum, and c is the speed of light. The mass of an electron is 9.11×10^{-31} kg. At what electron number density n is the maximum energy of the electrons equal to the energy E_0?

   iii Using your answer for part aii), and assuming there is an electron for 2 nucleons (proton or neutron), find to one significant figure the mass density for the condition in the previous part.

During core collapse, energy is used up in the photodisintegration of iron nuclei into alpha particles and alpha particles into protons and neutrons. Once the mass density exceeds the value found in aiii), neutron decay is no longer spontaneous. Instead, the reverse reaction

$$\mathrm{p} + \mathrm{e}^- \longrightarrow \mathrm{n} + \nu_e$$

becomes possible and dominates in the star. As a result, the majority of protons in the core are converted to neutrons through electron capture.

This collapsed stellar core is now a neutron star. Neutron degeneracy pressure, which works in a similar way as electron degeneracy, is then able to stop the core collapse for stars below the Oppenhimer-Volkoff limit (estimated at between 2 and 3 solar masses).

b) The time scale of the core collapse depends only on the gravitational constant G and the density of the star. Assume the chandrasekhar mass is 1.4 solar masses and the radius of a white dwarf is on the order of 1000 km. Estimate to an order of magnitude the time scale of the core collapse.

c) While a neutron star is being formed, the star blasts its outer layers in a extremely powerful explosion known as a supernova. These explosions are the most energetic events known in the universe. Although the exact mechanisms of supernova explosions are not well known, one can derive the energy budget of a collapsing star.

  i The energy released in a supernova explosion is the change in gravitational binding energy. The radius of a neutron star is around 20 km and the radius of a white dwarf star near the chandrasekhar limit is around 10000 km. The chandrasekhar mass is around 1.4 solar masses. Find to one significant figure this change in gravitational binding energy.

  ii Around 10% of this energy powers the conversion of protons and electrons to neutrons. Some energy is used to unbind the stellar envelope, which is the stellar material not included in the neutron star, from the core. Assume that the binding energy of the stellar envelope to itself is on the same order of the initial binding energy of the star. What proportion of the total released energy found in part ci) is converted to unbinding the stellar envelope?

  iii Some energy is converted to kinetic energy of the stellar envelope. By observations, the speed of the ejecta is around 10000 km/s. For a star with total mass of 10 solar masses, find to one significant figure the percentage of the star's total energy (part ci) that is converted into kinetic energy of the stellar envelope.

You should have found that the formation of neutrons, unbinding of stellar envelope, and kinetic energy of the stellar envelope requires only a small portion of the energy released by the core collapse. The energy carried away by radiation also use only a small amount of the available energy. It is believed that neutrinos carry away the remaining energy.

While we know that energy must be transferred from the core to the stellar envelope, research is still ongoing to determine how this process occurs (see further resources section).

3 Further Resources

For those interested in further resources, here is an enumeration of articles and web pages regarding current equilibria applications and news, as well as resources for learning about equilibria. Click on the hyperlinks for more information.

- Pulsars
- Application of Parallax in Measuring Distances in the Solar System
- Heat Transfer from Radiation in the Sun
- Analysis on Radiation Pressure Effects on Spacecraft
- Cherenkov Effect (Used in Neutrino Detection)
- Composition of the Sun
- Stellar Life Cycle
- End of the Sun's Life Cycle
- Helium Flash
- Black Holes
- Supernova