2025 TJ PHYSICS OLYMPIAD

2025 TJ Physics Olympiad: Motion

- The contest lasts from **June 14th, 2025** 12 A.M EST to **June 16th, 2025** 11:59 P.M EST.
- All solutions must either be typeset in LaTeX/Typst (preferred), or clearly and legibly handwritten. Illegible handwriting will not receive any points (keep in mind cursive can be hard to read as well).
- Solutions should mention the concepts used (e.g. "by conservation of energy"), but please don't write a 5-paragraph essay.
- The usage of Generative AI is PROHIBITED (and will not be useful). Usage can result in disqualification.
- You MAY use online software including, but not limited to, Wolfram Alpha, Matlab, Python, or Julia, solely as a mathematical aid.
- All questions are equally weighted, but not necessarily the same level of difficulty. The subparts may not be equally weighted. Also, the problems may not be given in order of difficulty, so make sure to try all of them.
- Even if you are not able to fully solve a problem, please write up what you can. We want to see how you approached the problem, and partial credit may be awarded.
- Join our discord to ask questions
- If you need to contact us for clarifications, please email us at tjhsstphysicsteam@gmail.com
- Yes, this cover was designed to look like the cover of the $F_{\text{net}} = ma$ exam.

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

Sanchali Banerjee, Tiger Deng, Aarush Deshpande, Jason Hao, Sophia Hou, Kabeer Parmar, Ryan Singh, and Eric Xie

Introduction

From the earliest water wheels that harnessed river currents to the jet and rocket engines that propel us skyward, motion and energy has powered our machines and understanding of nature. In the atmosphere, heat differentials drive ocean currents, and in engines, chemical and thermal energy convert into mechanical work. Conservation laws, fluid dynamics, thermodynamic efficiency, and statistical behavior, govern how energy is stored, transferred, and transformed into motion.

TJ Physics Team is proud to announce the seventh annual TJ Physics Olympiad (TJPhO)! We hope that our problems deepen your appreciation for how motion and energy intertwine in every ”engine”, whether natural or engineered, and inspire you to apply these principles in your own problem-solving.

Happy exploring!

TJ Physics Team Officers

A Relativistic Rocket

A rocket is launching from Earth, with a constant velocity v, and ejects photons to produce thrust (cue the *Little Einsteins* theme song). Assume it is a hot day, and the index of refraction for light is given by $\eta = n_0(1 + ay)$, where $a = 1.5 \times 10^{-6}\,\mathrm{m}^{-1}$, and y is the distance from the surface of the earth. Ignore the curvature of the Earth. Unless stated otherwise, all quantities are measured in the ground frame.

A) Assuming the rocket launches at an angle θ from the normal, find an approximate expression for the distance $d(t)$ (where $t = 0$ corresponds to rocket launch) away from the rocket you must stand to be able to see the rocket when its height is $h \sim 1\,\mathrm{km}$ (see Fig 1). Neglect the height of the person, and the light emitted by the Sun. You may also assume the observer can only see a tiny bit above the horizontal, and that the height of the rocket is such that the light becomes close to parallel at the ground level.

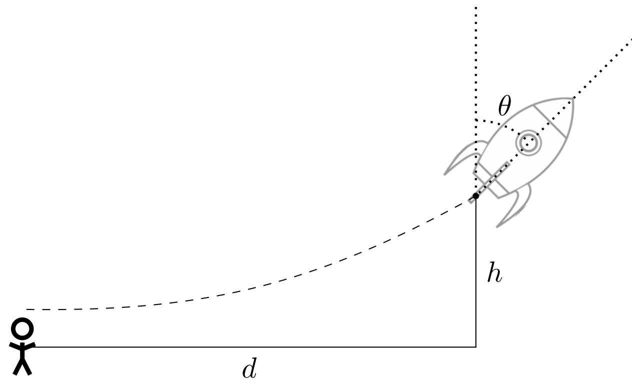

Figure 1

B) Repeat the previous problem, assuming the observer can look at any angle $\leq 90°$ from the horizontal. If your answer involves hyperbolic trig functions, you should correctly approximate your solution down to trigonometric functions for full credit.

C) Assume the observer maintains the distance d, and the rocket is at the same order of magnitude of h as in the previous part. If the rocket has a proper length L, what is the length of the rocket in the observers frame?

Solar Panels

Solar Panels are made out of photovoltaic cells, which are semiconductors. When photons hit the cells, they get absorbed. Then, they can excite electrons in the cell effectively only if their energies are greater than the band gap energy. If this is the case, the excited electrons jump from the valence band to the conduction band, where they can freely move through the semiconductor.

(A) When photons excite electrons in a photovoltaic cell, they generate a photocurrent, producing electricity. In real photovoltaic cells, about 90% of the excited electrons can move freely in the conduction band.

i) Find the number of photons that hit the solar panel per unit area every second with energies high enough to excite electrons. Assume the sun is a blackbody with temperature T = 5800K and that the solar panel is a closed sphere surrounding the sun on the sun's surface. You can also assume the minimum wavelength of light that reaches the solar panel is $\lambda_{\text{min}} = 280$nm and a band gap energy of 1.1 eV.

ii) Find the magnitude of the current that is generated in a realistic (non-idealized) semiconductor.

(B) Photovoltaic cells themselves consist of two layers of semiconductor materials, called p-type and n-type semiconductors. Silicon, which is typically used in semiconductors, can be "doped" by replacing a small amount of the atoms, usually about 1 in 10^7, with those of another element. If silicon is "doped" with phosphorus, it creates an n-type semiconductor and an excess of electrons. However, if silicon is "doped" with aluminum, a p-type semiconductor is created, and an excess of "holes" (which represent the absence of electrons) is maintained. These p-n junctions in photovoltaic cells create diodes. So, we will model the solar cells as ideal diodes.

i) Assuming the cell is at a temperature of 300K, find the maximum theoretical voltage across the diode. Model the energy transfer between the sun and the solar panel as a thermal energy transfer.

ii) Find the maximum power this photovoltaic cell can generate in this situation, in terms of the power P emitted by the sun.

(In reality, most solar panels have an efficiency of around 30% of their maximum power production possible).

Light Bouncing

We have two fixed, perfectly reflective walls, with the distance between them being 2ℓ. A double-sided mirror with mass m (which is lined up with the walls) is placed in the center between the two walls. On each side of the mirror, light with energy E_0 is placed, which bounces back and forth between the mirror and the walls. The mirror is given a speed v_0 to the left. Assume $v_0 \ll c$, that v_0 is insufficient to reach the left wall, ℓ is much greater than the wavelength of the light, and that there is no energy dissipation. The mirror will oscillate between the two walls.

(A) Find the energy of the light to the left of the mirror when the mirror is closest to the left wall.

(B) Estimate the period of the oscillations. Assume that the maximum displacement $A \ll \ell$.

Fluids in Action

In this problem, we aim to demonstrate how one arrives at some common equations used for modeling fluids, used when designing hydropower. Note that this question will require knowledge of Vector Calculus. If you need to learn it, https://www.damtp.cam.ac.uk/user/tong/vc.html is excellent, and so is the first chapter of Griffiths Electrodynamics.

A) Just to get in the fluids' mindset, let's start with some problems that should be approachable without significant fluids' knowledge.

   i) Suppose we have a can of base area A and height H, and is filled with water of density ρ. A small hole of area $A_0 \ll A$ is then opened at the bottom of the can. At the same time, a piston pushes down on the top of the water with a constant force F. Find the height of the water in the can as a function of time.

   ii) Repeat the previous problem, without the $A_0 \ll A$ assumption.

B) Now, we shall derive some important equations in fluid dynamics! Note that we shall often work with a velocity field $\mathbf{v}(\mathbf{x}, t)$, and we define the operator $\mathrm{D}/\mathrm{D}t$ (called the material derivative) as

$$\frac{\mathrm{D}}{\mathrm{D}t} \coloneqq \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

In essence, it is a way of relating the change in a Lagrangian quantity, which is a property observed by a moving fluid particle, to an Eularian quantity at a fixed point in space.

   i) The rate at which mass is moving through a surface is given by

$$\frac{\mathrm{D}M}{\mathrm{D}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho \,\mathrm{d}V + \oint_{\partial V} \rho \mathbf{v} \cdot \mathrm{d}\mathbf{S}$$

   where ∂V denotes the boundary of V, and ρ is the density of the fluid. Show that

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{v}$$

   This is called the continuity equation, and it was first obtained by Euler in 1753. Note that while we derived it in the context of fluid dynamics, it can also be applied to other quantities, such as electric charge.

   ii) By applying Newtons Second Law $\mathbf{F} = \mathrm{D}\boldsymbol{\mathcal{P}}/\mathrm{D}t$ to a volume, show that

$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = \rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}\right) = f^*$$

   where f^* is the effective force density (force per unit volume). This is called Cauchy's equation, and in conjunction with the continuity equation it governs the behavior of *all continuous matter*! Hint: what are all the contributions to the change in momentum?

   iii) For fluids without any viscosity, the only forces at play are gravity and pressure. Under this assumption, find f^* and substitute it into Cauchy's equation. This is called *Euler's Equation for ideal fluids*.

   iv) A fluid is in steady flow when the flow doesn't change with time. Show that for an incompressible fluid with steady flow and constant density ρ_0, the quantity

$$H \coloneqq \frac{1}{2}v^2 + \phi + \frac{p}{\rho_0}$$

where ϕ is a kind of effective potential, is conserved along streamlines. This is called Bernolli's theorem. Show that this is a generalization of the typical Bernoulli's equation

$$P_0 + \frac{1}{2}\rho v^2 + \rho g h = \text{const}$$

C) Having introduced some common ideas from fluid mechanics, we now turn to a familiar topic: waves! Waves controlled by pressure and gravity are called *gravity waves*, and those driven by pressure and surface tension are called *capillary waves*. In an interface between two fluids of similar density, such as a saline layer of the sea overlayed with a brackish layer, *internal gravity waves* may also arise.

Unfortunately, the more unusual types of waves originating from the non-linear aspects (the advective term) of fluid mechanics, such as the run-up of waves on a beach or a sonic boom, require a much more challenging amount of mathematics. Instead, we shall focus on applications involving small-amplitude surface waves.

i) As a fun thought, consider what would happen if you lifted a mound of water with height a and width λ, and it let splash back down in a characteristic time τ into a sea of depth $d \ll \lambda$. Since the sea is shallow, the ground will force all of the water to move away horizontally with a speed v. Estimate v up to a dimensionless coefficient.

ii) Small amplitude, inviscid gravity waves in incompressible water obeys

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{g} - \frac{1}{\rho_0}\nabla p, \qquad \nabla \cdot \mathbf{v} = 0 \tag{1}$$

In other words, the time derivative of the velocity is a gradient field, implying that the velocity takes the form of a gradient field plus a time independent field (such as one containing vorticity). However, for waves, we don't really care about the time independent field so we shall claim

$$\mathbf{v} = \nabla\psi$$

which, due to the incompressibility condition, must satisfy Laplace's equation $\nabla^2\psi = 0$. Find the general form solution for p in terms of ψ and other given variables in Equation 1.

iii) Assuming the water is surrounded by air (or a vacuum) above the surface, and that the amplitude of waves a very small ($a \ll d, \lambda$ to be precise), come up with two boundary conditions for the general form solution derived above.

Hint: it may be helpful to consider the water as a surface $z = h$ for some h.

Further analysis of shallow and deep water waves consists mostly of assuming that ψ takes the form of an elementary harmonic wave, and applying the above boundary conditions to derive a result. As such, we decided not to include it in the olympiad, but it's very interesting and we highly recommend reading about it!

For example, if you're curious about capillary waves, the effect of surface tension on deep water waves is to increase the gravitational acceleration to

$$g' = g_0 + \frac{\gamma k^2}{\rho_0}$$

where γ is the surface tension, and k would be determined by a boundary condition.

Hurricanes

A) Suppose that in a hurricane, the ocean has a constant temperature 30 °C and the ocean is in thermal equilibrium with the air directly above it. The air temperature in the upper troposphere is maintained at -55 °C from conduction with the upper atmosphere. If in a small time dt, the ocean rate of heat transfer from the ocean to the atmosphere is $\frac{dQ}{dt}$ and the rate of change of the kinetic energy of the air is $\frac{dK}{dt} = \alpha \frac{dQ}{dt}$, calculate numerically the maximum possible value of α.

B) In a rotating reference frame with angular velocity $\vec{\Omega}$, the coriolis force on a particle with mass m, velocity $\vec{v}$, and displacement $\vec{r}$ from the axis of rotation is $\vec{F}_{cor} = -2m\vec{\Omega} \times \vec{v}$, and this causes hurricanes to rotate counter-clockwise in the northern hemisphere and clockwise in the southern hemisphere. For a small volume of air moving directly east in a hurricane at 30° latitude in the northern hemisphere, let v_0 be the required velocity of the air particles with respect to the Earth for the component of Coriolis acceleration parallel to the ground to be comparable to the gravitational acceleration. You may assume that the air is close to the Earth's surface. Find numerically v_0 to one significant figure. The radius of the earth is 6.37×10^6 m.

C) Now we will consider a more formal derivation of the coriolis force in the general case for a hurricane. Again, consider a hurricane at a latitude of 30° in the northern hemisphere, but now with a wind speed much less than that found in the previous problem ($v \ll v_0$). Consider a model of the hurricane as a flat circle with radius much smaller than the Earth's radius, and in polar coordinates for any location on the circumference of this hurricane, let θ be the angle counterclockwise from North. Find the function $F_{\|}(\theta)$ in terms of Ω as defined in the previous problem, v, and/or any fundamental constants (where $F_{\|}$ is the component of the Coriolis force in the radial out direction and parallel to the ground). You may assume that the air in the hurricane always maintains the same height above the ground.

D) Suppose a hurricane has an eye of radius r_1 and an outer radius of r_2. The air in the hurricane at sea level has a constant density ρ, and the angular velocity of the Earth's spin is Ω. If the difference between pressure at distance r_1 from the hurricane's center and pressure at a distance r_2 is $-\Delta p$, where $\Delta p > 0$, find the wind speed at sea level at the outer edge of the hurricane v_0 in terms of ρ, r_1, r_2, Ω, Δp, and/or any fundamental constants. You may neglect the viscosity and any drag forces in the air, and assume that a single parcel of air always has a small radial velocity v_r such that $v_r \ll v_0$.

E) For a hurricane with inner radius $r_1 = 5$ km, outer radius $r_2 = 800$ km, and height $H = 10$ km, consider the function $v(r)$ for $5\,\text{km} < r < 800\,\text{km}$ (the wind speed as a function of distance from the hurricane's center). The absolute value of the pressure difference between that of $r = r_1$ and $r = r_2$ is $\Delta p = 3000$ Pa. This function can be written as $v(r) = Ar^{\alpha}$.

   i) If $v(r)$ and r are both in SI units, find numerically α and A, and express A to three significant digits with correct units.

   ii) Find the maximum wind speed for $5\,\text{km} < r < 800\,\text{km}$. The density of air at sea level is given as $\rho = 1.225\ \text{kg/m}^3$

Jittery Atoms

A small cylinder in a model heat engine contains N identical, non-interacting atoms that can occupy a discrete set of energy levels. As the piston moves, the gas exchanges heat with a high-temperature reservoir at T_H during the "intake" stroke and with a cold reservoir at T_C during the "exhaust" stroke. Under the assumption that between strokes, the gas reaches thermal equilibrium, the macroscopic state of the gas at any instant is specified by the set of occupation numbers,

$$n_1, n_2, \ldots n_m, \quad \sum_{j=1}^{m} n_j = N, \quad \sum_{j=1}^{m} \varepsilon_j n_j = E,$$

where ε_j is the j-th single-atom energy level. Boltzmann argued that the entropy of the gas is

$$S = k_B \ln \Omega$$

where Ω is the number of distinct microstates corresponding to the given occupation numbers:

$$\Omega = \frac{N!}{n_1! n_2! \cdots n_m!}$$

Since S is maximized (for fixed total energy E and total atom number N), the most likely occupation numbers $\{n_j\}$ satisfy

$$n_j = A e^{-\beta \varepsilon_j} \quad \text{(canonical distribution)},$$

where $\beta = 1/(k_B T)$ and A is a normalization constant (related to the chemical potential).

In this problem, you will derive the Boltzmann distribution and explain why n_j takes the form $Ae^{-\beta\varepsilon_j}$, and then apply these results to a concrete system.

A) **Derivation of the Boltzmann Distribution**

i) Determine how to maximize $\ln \Omega$ under $\sum_j n_j = N$ and $\sum_j \varepsilon_j n_j = E$ by using Lagrange Multipliers and/or taking a derivative with respect to n_j and showing that

$$\frac{\partial}{\partial n_j}[-\ln(n_j!) + \alpha n_j + \lambda \varepsilon_j n_j] = 0.$$

ii) Determine why $n_j = Ae^{-\lambda \varepsilon_j}$, and find a simplified approximation for entropy.

B) **Application: Four-Level Gas Molecule**

The working gas has four energy levels, $\varepsilon_1 = 0, \quad \varepsilon_2 = \varepsilon, \quad \varepsilon_3 = 3\varepsilon, \quad \varepsilon_4 = 5\varepsilon.$

i) Determine the entropy associated with this system if there are 0.8×10^{10} atoms in the first state, 0.5×10^{10} in the second state, 0.4×10^{10} and 0.3×10^{10} in the fourth.

ii) What is the maximum entropy possible for this system of four states if the total energy is not fixed?

iii) Determine the occupation numbers of each of the states if the total energy is fixed at $3 \times 10^{10}\varepsilon$. Give your answers in terms of $x = e^{-\beta\varepsilon}$.

iv) Determine the temperature of the system under the assumptions of part 3). Give your answer as a decimal multiple of $\frac{\epsilon}{k_B}$.

v) What is the maximum entropy possible for this system under the assumptions in part 3)?

A Powerful Piston

A vertical piston of mass m and cross-sectional area A slides without friction in a cylindrical chamber filled with n moles of an ideal diatomic gas. The piston moves without friction and is free to oscillate vertically. The entire system is thermally insulated, so any motion of the piston results in adiabatic compression or expansion of the gas. At equilibrium, the piston is at a height h_0 above the base of the cylinder and the pressure inside the chamber exactly balances the weight of the piston and atmospheric pressure P_0.

(A) Determine the equilibrium pressure P of the gas and use it to express the equilibrium temperature T_0 of the gas in terms of the given variables.

(B) Now suppose the piston is displaced upward by a small amount x and then released. Find the angular frequency of the oscillation of the piston.

(C) Suppose the piston is now released from rest at a height $h = h_0 + \Delta h$, where $\Delta h \ll h_0$. Assuming the system behaves as a harmonic oscillator, derive an expression for the total mechanical energy of the system in terms of the variables above, and identify the physical origin of this energy in the thermodynamic context.

Some Random Problems

We just had some other ideas that we didn't want to go to waste. They don't fully fit in with the theme, but we think they're fun!

A) n identical bricks of uniform density and length L are stacked, one above the other, near the edge of a table.

   i) What is the maximum length the farthest edge of the top brick can protrude over the edge of the table? (Hint: it is related to the nth harmonic number).

   ii) The table then starts to rotate with an angular velocity ω. Let coefficient of friction between the bricks be μ, and assume the table is made of bricks, and has a radius R. Find the minimum μ such that the maximum distance is the same as that in part (a).

B) Suppose there is a hemispherical sprinkler with negligible (as in not affecting the range of the water), but nonzero radius r, centered at the origin. It sprays water from its surface uniformly, i.e. the height (volume of water per surface area on the sprinkler is height) of water shot from any point on its surface is the same at H_0. The water shot from a point is shot along the line from the center of the hemisphere to said point, at a speed v.

   Find a formula in terms of x (with x ranging from 0 to the maximum range) for the distribution of the height of the water deposited at any point along the positive x-axis. You should approximate to the first degree when applicable. (Hint: Integrating your formula from $x = 0$ to $x = \frac{v^2}{g}$ should give you $\frac{\pi H_0 r}{2}$)

   Wording clarification: Note that the sprinkler deposits a volume of water over an area of ground, so each point on the ground receives a "height" of water. The height here has nothing to do with the trajectory of the water, and instead is a measure of how "wet" the ground is, like how weather stations report "We have had 1 inch of rain today".

Another Random Problem

Here's another problem which doesn't fully fit into the theme, but we think is quite interesting.

In this problem, we shall derive the shape of an axially symmetric pending droplet (think of a cartoon raindrop). Note that due to the axial symmetry, we shall work with cylindrical coordinates, with the origin at the bottom of the droplet.

If you're curious, we can create a droplet of this kind by taking for example, a pipette with a rubber bulb allowing us to vary the pressure. When the bulb is squeezed, a droplet emerges and will fall once it becomes large enough.

i) First, find the critical radius R_c, which is defined as the radius for which the hydrostatic pressure across a spherical drop with surface tension γ is equal to the pressure excess due to surface tension.

As an aside, for the effect of gravity to be negligible in a spherical droplet, its radius must be $a \ll R_c$.

ii) Show that given the arc length s of the droplet surface as viewed in the rz plane, and the angle of elevation θ of $\mathrm{d}s$, we must have

$$\frac{\mathrm{d}r}{\mathrm{d}s} = \cos\theta, \qquad \frac{\mathrm{d}z}{\mathrm{d}s} = \sin\theta$$

iii) Find the two radii of curvature of the raindrop R_1 and R_2, where R_1 the radius of the circle in the rz plane.

Hint: the second center of curvature lies on the z axis

iv) Young-Laplace's equation states that

$$\Delta p = -\gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Note that a contribution to the pressure discontinuity is always positive on the side of the surface containing the center of curvature, otherwise negative. Find an expression for $\mathrm{d}\theta/\mathrm{d}s$ in terms of θ, r, z, R_c, and R_0, where $R_0 = R_1 = R_2$ at $r = z = \theta = 0$

We now have a set of differential equations that can be solved numerically for the shape of the droplet, given a value of R_0! If you'd like to explore some more, you can run some simulations with different R_0 values. If $R_0 > R_{01} = 0.778...$, the radial distance grows monotonically with s, but if it's less than R_{01} it's shaped like an old-fashioned bottle with multiple waists. However, note that these simulations won't earn you any points on TJPhO :P