ObsTools - Background

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Introduction - The CCD Equation

In order to create an exposure time calculator, we must consider the theoretical signal-to-noise ratio. The central equation is the so-called 'CCD Equation' (Howell 2011):

$$SNR = \frac{E\Delta t}{\sqrt{E\Delta t + n_{pix}(R\Delta t + D\Delta t + rn^2)}} \tag{1}$$

Where, E is the average target signal (in e/sec), n_{pix} is the number of pixels to consider, R is the sky background rate (in e/pix/sec), D is the dark current (in e/pix/sec), rn is the read-noise (in e/pix) and Δt is the exposure time (in seconds). In my calculations, I have used:

$$n_{pix} = \frac{2 \times FWHM}{pixel\, scale} \approx \frac{2 \times FWHM}{0.8} \tag{2}$$

Here, the FWHM refers to the seeing (i.e the stellar FWHM). The signal rate is given by:

$$E = F_{obj}\,(\pi r_{tel}^2)\, T\ q\, \Delta\lambda \tag{3}$$

Where, F_{obj} is the object flux density (photons/sec/cm^2/nm), q is the quantum efficiency of the detector, $\Delta\lambda$ is the bandpass and T is the transmission efficiency (i.e of atmosphere, telescope and camera). The flux density is related to the magnitude in filter λ as:

$$F_{obj} = \frac{f}{\lambda_{eff}} = 10^{0.4(Zpt_\lambda - m_\lambda)}/\lambda_{eff} \tag{4}$$

Here, λ_{eff} is the effective wavelength of the filter. We can see from these equations that there are multiple parameters needed to calculate the SNR. For the ObsTools dashboard, this required a combination of experimental fitting to data and approximations. Here I will list the steps undertaken and the models used.

The Camera and Filter Parameters

For the 50cm, we have an Apogee Alta U42 camera. The quantum efficiency as a function of wavelength was extracted from here: `https://neurophysics.ucsd.edu/Manuals/Apogee/ALTA%20U42%20Specifications.pdf`

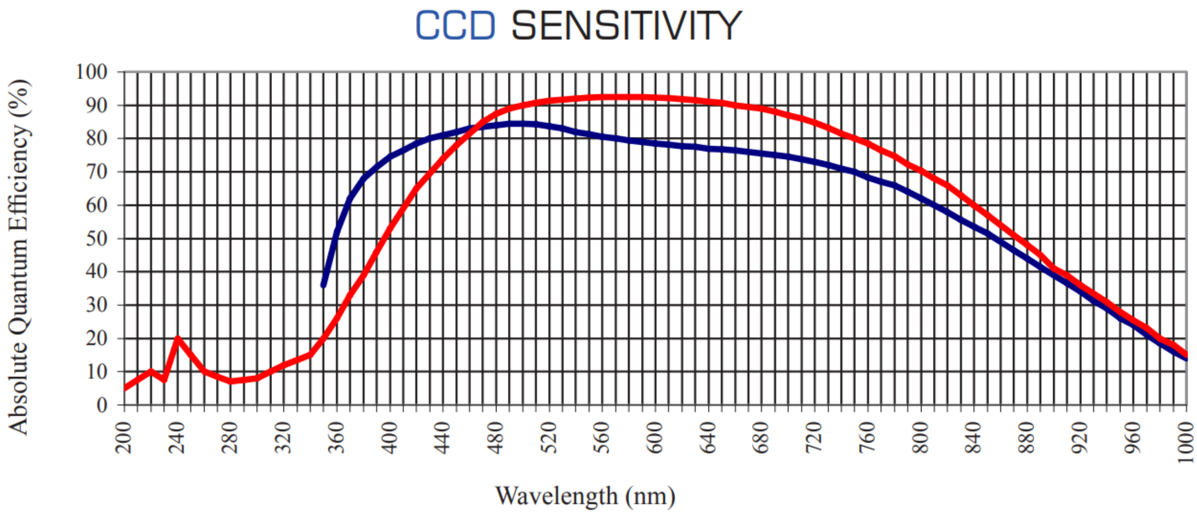

Figure 1: The absolute quantum efficiency as a function of wavelength for the Apogee Ulta 42 camera.

The data points to produce this plot were not available. This graph was sampled by eye at various discrete wavelengths and then linearly interpolated for estimation of quantum efficiency for each filter in the code (using the effective wavelength). Next, we needed the noise sources from the camera. These were found here: `https://neurophysics.ucsd.edu/Manuals/Apogee/Apogee%20Alta%20U42%20Camera%20Test%20Report.pdf`

Quantity	**Value**
Gain (e/count)	1.2
Dark Current (e/pix/sec)	0.14
Read Noise (e)	8.7

Table 1: Apogee Camera Parameters

For the filters, I referred to Bessell (2005) for the effective wavelength and bandpass for our filters.

Filter	λ_{eff} **(nm)**	$\Delta\lambda$ **(nm)**
B	436.1	89.0
V	544.8	84.0
i’	743.9	123.0
r’	612.2	115.0
g’	463.9	128.0

Table 2: Apogee Camera Parameters

Extinction Coefficients and Zeropoints

To find the zeropoint in each filter (Z_λ), we recall the definition of a calibrated magnitude:

$$m \approx m_{inst} - k_\lambda \chi + Z_\lambda = -2.5log_{10}(\frac{gain \times counts}{\Delta t}) - k_\lambda \chi + Z_\lambda \tag{5}$$

First, I needed to find the airmass extinction coefficients. To do this, we took a series of images of the standard star field SA107 in each filter at various airmass values. I then reduced the images and performed aperture photometry using my own photometry pipeline using Prose (). Then I performed a linear regression of instrumental magnitude as a function of airmass. This was done for multiple stars in the field, including SA107-484, SA107-619 and SA107-646.

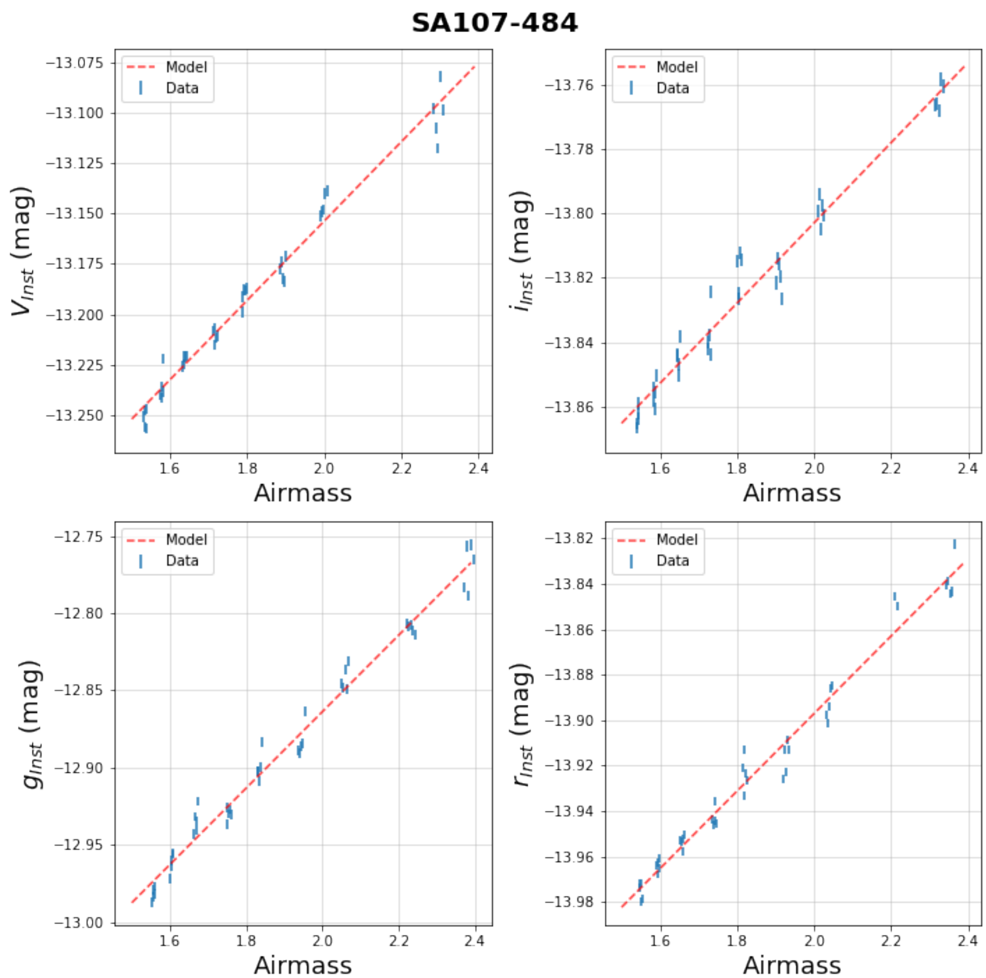

Figure 2: Instrumental magnitudes as a function of airmass for the Landolt standard SA107-484.

Note, accurate measurements of the coefficient for the B filter was not possible with the current data. This will need to be re-measured in the future, but we will use the value from Siding Spring Observatory for now. Taking a weighted average of all the other results, we get the extinction coefficients listed in the table below.

Filter	**Airmass Coefficient (k_λ)**
B*	0.309 ± 0.029 (*from SSO)
V	0.196 ± 0.004
i’	0.125 ± 0.003
r’	0.170 ± 0.003
g’	0.244 ± 0.005

Table 3: Airmass Extinction Coefficients for GHO

Now to find the zeropoints, we took images of the globular cluster NGC4590 in each filter at 5, 10, 20, 40 and 80 seconds. I then reduced these images and performed aperture photometry using my pipeline. I then queried synthetic photometry from the GAIA DR3 catalogue and matched the sky coordinates with a tolerance of 1” for each catalogue using TopCat. Then, for each exposure time, I calculated the zeropoint by rearranging Equation 5. Approximately 150 stars were used to find an average zeropoint across the field in each exposure. The final values were then calculated using a weighted average of each exposure time. The results are shown in Figure 3 and values are summarised in the table below. Again, we note the poor quality of the data for the B filter.

Filter	**Zeropoints (Z_λ)**
B	20.76 ± 0.06
V	22.03 ± 0.06
i’	21.83 ± 0.04
r’	22.07 ± 0.07
g’	22.22 ± 0.04

Table 4: Zeropoints for GHO 50cm filters

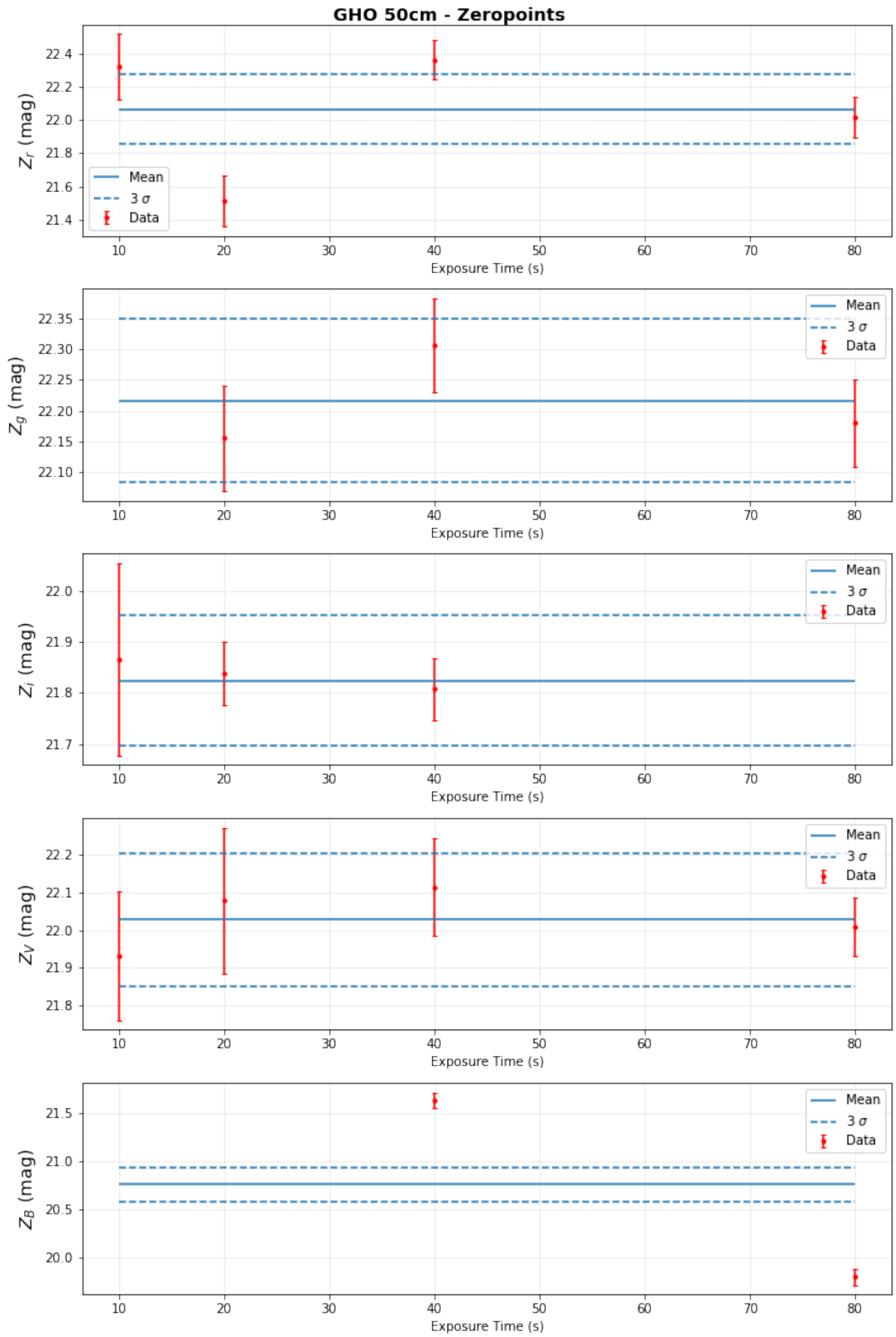

Figure 3: Zeropoints as a function of exposure time. The solid line shows the weighted average from all exposure times. The dashed lines show the 3 σ error.

Sky Background - Moon Model

To find the sky background rate, one would need to consider the light from the moon, airglow and zodiacal light. Whilst complex models exist to predict this, we do not currently require the accuracy needed to justify exploring this in detail. Instead, I have implemented a simple model based upon the moon phase, using measurements of zenith sky brightness during the full moon at GHO and sky brightness measurements from Siding Springs Observatory and SDSS. For our measurements, I calibrated images in each filter at approximately zenith with tracking turned off. I then took the median count value of each image as purely from the background, divided by exposure time and then converted to electrons using the known gain. I eventually hope to include angular separation of target and moon into the model also. The values listed in the following table are linearly interpolated to find an approximate sky brightness for a given moon phase. Note, we assume the values are symmetric around 180 degrees (i.e the value for 90 degrees and 270 degrees are equivalent.)

Moon Phase	R_g **(e/pix/sec)**	R_r **(e/pix/sec)**	R_i **(e/pix/sec)**
0^o (New Moon)	2	4	5
180^o (Full Moon)	15	9	11

Table 5: Sky Background Rate for Sloan filters on 50cm

Moon Phase	R_B **(e/pix/sec)**	R_V **(e/pix/sec)**
0^o (New Moon)	0.1	1
90^o (First Quarter)	0.2	2
180^o (Full Moon)	4	16.5

Table 6: Sky Background Rate for Bessell filters on 50cm

Transmission Efficiency

The final piece of the puzzle is to measure the transmission efficiency for the 50 cm. This requires all other parameters to have been measured, as it is essentially a free fitting parameter to get the calculations to match experimental SNR measurements. Consider:

$$\delta m = -2.5 \log_{10}(1 + \frac{1}{SNR}) \tag{6}$$

This implies that:

$$SNR = |\frac{1}{10^{-0.4\delta m} - 1}| \tag{7}$$

I once again used the photometry of the Landolt standard SA107-484 to measure the transmission efficiency. Using the magnitude errors from the photometry, I used Equation 7 to convert to SNR. I then took the average SNR of all the measurements. I then fit T by looping over all values between 0 and 1 (in steps of 0.00001) and minimising the function:

$$f(T) = |SNR_{measured} - SNR_{calculated}| \tag{8}$$

The final result was T = 0.0021 (i.e an efficiency of 0.21 %). Whilst this seems small, it is likely due to the efficiency of the camera/filter being absorbed into the quantum efficiency value.

References

Bessell, M. S. (2005). "Standard Photometric Systems". *Annual Review of Astronomy and Astrophysics*, 43(1)

Howell, S. B. (2011). "Handbook of CCD astronomy". *Cambridge University Press.*

Garcia L. J, et al. (2022)."PROSE: a PYTHON framework for modular astronomical images processing", *Monthly Notices of the Royal Astronomical Society*, 509(4)