Journal Report 5

9/30/19-10/7/19
Michael Huang
Computer Systems Research Lab
Period 1, White

Daily Log

Monday September 30

Began analyzing uneven amounts of 1s and -1s. Tried transferring an epsilon amount of 0s into 1s. Came up with an equation, but there were too many variables. Probably will not be fruitful.

Tuesday October 1

Came with a new approach. Assume there are x -1s, 1 0, and y 1s. Then evaluate different bounds based on this. Found that 100/316 was a lower bound when x+y was not between 1/18 and 4/3.

Thursday October 3

Finished the work from Tuesday with another construction. This finished the proof for all combinations of -1, 0 ,1.

Timeline

Date	Goal	Met
9/23	Find a lower bound for {-1,0,1}^{3} with varying amounts of 1s and -1s as well	Sorta. I found a conjecture for a lower bound which seems to be correct, however the dataset is too big for Mathematica to verify.
9/30	Find a way to make Mathematica work with weighted vertices in order to finalize the {-1,0,1}^{3} case with varying amounts of 1s and -1s	Yes, by specifying the exact number of vertices required by Mathematica. This shows 100/316 to be a minimum for symmetric cases.
10/7	Find an answer for asymmetric amounts of 1s and -1s.	Yes, the answer is intuitively the same as above, and I found a proof showing that.
10/14	Find the correlation coefficient between the scarcity of the graph and the maximum independent vertex set for different sets using $\{-1,0,1\}$	
10/21	Find an effective approximation algorithm for MIS	

Reflection

My work this week actually accelerated my timeline. I was slated to find a proof by next week, but I actually found it this week in looking for the answer. I began with a failed approach, but trying to turn an ϵ number of 0s into 1s to create my imbalance. This led to trying to prove the following inequality $\frac{a^3(2+\epsilon)^3+(1-\epsilon)^2(2a+\epsilon)}{(2a+1)^3-(1-\epsilon)^3} > \frac{a^3+2a}{(2a+1)^3-1}$. The combination of a and ϵ was very awkward in this equation and I couldn't make anything out of it.

I returned with a different approach, which turned out to be successful. I let there be $x-1$s, 1 0 and y1s, where x and y did not need to be integers(They were required to be positive). I looked to find constructions for this that would show a lower bound of $\frac{100}{316}$. I plugged in the construction $\{1,-1\}^3,(0,0,1),(0,0,-1)$. This gave us the expression

$$\frac{(x+y)^3+x+y}{(x+y+1)^3-1} > \frac{100}{316}$$

. Setting $(x+y)$ our variable, we get that this construction creates the lower bound for all (x+y) except when $\frac{1}{18} < (x+y) < \frac{4}{3}$.

I then plugged in the following construction

$$(0,0,1),(0,0,-1),(1,1,1),(-1,-1,-1),(0,1,1),(0,-1,-1),(1,0,1),(-1,0,-1)$$

. Analyzing this construction gave us the formula

$$\frac{x^3+y^3+x+y+(x+y)^2}{(x+y+1)^3-1} > \frac{100}{316}$$

We again wanted to work in $(x+y)$ so we used the inequality that among positive numbers, $x^3+y^3 \geq \frac{1}{4}(x+y)^3$. Now, plugging this in, we find that this construction, plugs in the holes with an equality case only at $(x+y)=\frac{4}{3}$ and $x=y$. Thus proof is complete, and not only that, but we have proved that there is only one equality case.