Journal Report 3
9/16/19-9/23/19
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Daily Log

Monday September 9

Began working on how to prove a lower bound of Maximum Independent Vertex Set for cases of $\{-1, 0, 1\}^3$ with varying amounts of 0s, 1s, and -1s.

Tuesday September 10

Realized that only ratio mattered. Set a fractional value for the number of 0s. Looked into why the number for -1,0,1 was 10/26 instead of 8/26.

Thursday September 12

Found another independent vertex set that works better for lower amounts of 0s. Mathematically calculated the min fraction based on the two. Happens on the set -1,-1,0,0,0,1,1. This would give 100/316 .316. Close to literature. Unsure if this is true, Mathematica cannot calculate.

Timeline

Date	Goal	Met
9/9	Have functional code and plug in numbers	Yes, I have code that works for about 124 distinct vectors by plugging things into Mathematica.
9/16	Find a lower bound for {-1,0,1}^{3} with varying amounts of 0s	Yes, I found that of the $6n^2 + 12n + 8$ terms (n being the number of 0s), I can always make $2n^2 + 4n + 2$
9/23	Find a lower bound for {-1,0,1}^{3} with varying amounts of 1s and -1s as well	Sorta. I found a conjecture for a lower bound which seems to be correct, however the dataset is too big for Mathematica to verify.
9/30	Find a way to make Mathematica work with weighted vertices in order to finalize the {-1,0,1}^{3} case with varying amounts of 1s and -1s	
10/7	Find an effective approximation algorithms and test their validity.	

Reflection

This week, I looked to build off some of my work from last week. Last week I proved the case for $\{-1,0,1\}^3$ for varying (integer) amounts of 0s. This week, I looked to also vary the amounts of 1s and -1s (maintaining symmetry). I quickly realized that this was the equivalent of having a fractional amount of 0s. When working with an integer amount of 0s, it was enough to try small cases and then take the limit going to infinite. Now, with a fractional number of 0s, we can't do that.

I examined what vectors made the set $\{-1,0,1\}^3$ have 10, instead of the predicted 8, and saw that they used all eight of elements of the set$\{-1,1\}^3$ as well as $\{0,0,\pm1\}$. I looked to replicate this, allowing my set to contain a 1s, b 0s, and a -1s. Applying this set, gives a total of $8a^3 + 2ab^2$. Equating this to the result from last week, $2a^3 + 4a^2b + 2ab^2$, gives us an equality case of $3a = 2b$. If we show that the equality case holds here, this will definitively prove the minimum for all sets symmetric basis sets of $\{-1,0,1\}$. It would provide a value of 100/316 which is approximately 0.316. This is better than what I have currently and only barely behind the literature's value of 0.313.

This week has made me change my goal for next week. I want to finish what I started this week and find a way for Mathematica to calculate this result. I'm going to try to implement weighted vertices so that Mathematica only sees 26 vertices instead of 316. I think I should be able to show that this set has a Maximum Independent Vertex Set of size 100 as analytically, I couldn't think of any other set that would be larger.

Once, I finish these sets, I will get back on track to calculating approximate Maxmimum Independent Vertex Sets. There's no guarantee that this will be effective as the error ratio may be much too high, but I believe that it is worth giving a shot to. I will research algorithms that have the best combination of time complexity and correctness.