DATA, MODELS & UNCERTAINTY IN THE NATURAL SCIENCES

Problem Set 4

In the frames posted as `http://geoweb.princeton.edu/people/simons/GOLFBALL/000000??.jpg`, you see a disembodied arm throwing a golf ball. The nominal frame rate is 25 fps.

Figure 1: A golf ball, 4 cm in diameter, on its flight through the old Simons living room.

1. Using MATLAB's `imread` and `ginput`, collect the data pairs (y, t) of the flying golf ball as best you can.
2. Fit an equation of the form $y(t) = m_1 + m_2\, t - (m_3/2)\, t^2$ to the data, where t is (relative) time and y is (relative) height. Write your own routine to calculate the "generalized inverse" for the **overdetermined** case. Expressing the problem as $\mathbf{y} = \mathbf{G} \cdot \mathbf{m}$, this amounts to finding the estimated model parameter vector as $\hat{\mathbf{m}} = (\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{y}$.

   What value do you find for the acceleration due to gravity?
3. Assign uncertainties to your height measurements, i.e. construct a covariance matrix for the data, $\mathbf{C}_y$. From there, calculate the covariance matrix of the model, $\mathbf{C}_m$, and discuss its features.
4. Redo the inversion by calculating $\hat{\mathbf{m}} = (\mathbf{G}^{\mathrm{T}} \cdot \mathbf{C}_y^{-1} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{C}_y^{-1} \cdot \mathbf{y}$ instead.
5. Perform a χ^2 test for the significance of your fits. Discuss your results.
6. Neglecting statistical correlations between the model parameters you determined, construct confidence intervals around your estimates. Discuss your results.

Note that you will find reading Chapters 1 and 2 in the book by *Aster, Borchers & Thurber* helpful.