

## DATA, MODELS & UNCERTAINTY IN THE NATURAL SCIENCES

## **Problem Set 4**

In the frames posted as http://geoweb.princeton.edu/people/simons/GOLFBALL/000000??.jpg, you see a disembodied arm throwing a golf ball. The nominal frame rate is 25 fps.

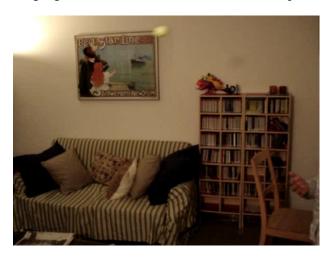


Figure 1: A golf ball, 4 cm in diameter, on its flight through the old Simons living room.

- 1. Using MATLAB's imread and ginput, collect the data pairs (y,t) of the flying golf ball as best you can.
- 2. Fit an equation of the form  $y(t) = m_1 + m_2 t (m_3/2) t^2$  to the data, where t is (relative) time and y is (relative) height. Write your own routine to calculate the "generalized inverse" for the **overdetermined** case. Expressing the problem as  $\mathbf{y} = \mathbf{G} \cdot \mathbf{m}$ , this amounts to finding the estimated model parameter vector as  $\hat{\mathbf{m}} = (\mathbf{G}^T \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^T \cdot \mathbf{y}$ .

What value do you find for the acceleration due to gravity?

- 3. Assign uncertainties to your height measurements, i.e. construct a covariance matrix for the data,  $C_y$ . From there, calculate the covariance matrix of the model,  $C_m$ , and discuss its features.
- 4. Redo the inversion by calculating  $\hat{\mathbf{m}} = (\mathbf{G}^{\mathrm{T}} \cdot \mathbf{C}_{y}^{-1} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{C}_{y}^{-1} \cdot \mathbf{y}$  instead.
- 5. Perform a  $\chi^2$  test for the significance of your fits. Discuss your results.
- 6. Neglecting statistical correlations between the model parameters you determined, construct confidence intervals around your estimates. Discuss your results.

Note that you will find reading Chapters 1 and 2 in the book by Aster, Borchers & Thurber helpful.

1/1 October 23, 2020