

DP Problems

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1 Introduction

Since DP is so common, you really ought to know how to recognize and solve DP problems. For the following problems, find the recurrences necessary to solve the DP and make sure it would run in time.

2 The Problems

1. Find the N -th ($0 < N < 10,000$) Lucas number, where $F_0 = 1$, $F_1 = 2$, $F_n = F_{n-1} + F_{n-2}$.
2. You are given a triangle of integers with $1 \leq N \leq 100$ rows, where the K -th row contains K elements. A path down the triangle is set of numbers starting from the top and ending at the bottom such that each successive number is one row below and directly to one side or the other. Find the maximum sum of the numbers in one path.
3. In the above, consider two paths. Find the maximum sum of the integers in either of the paths. The paths may overlap, but numbers overlapped only count once.
4. You are given a set of C denominations of coins, ($1 \leq C \leq 20$), where each denomination is a positive integer less than 2^31 . Compute the number of ways to form a value of N , $0 \leq N \leq 100000$, with these denominations.
5. Given a set of denominations, as above, compute the highest value which is impossible to form.
6. You are given a function $f(N)$, which gets
 - $N = 1$: 1
 - N odd: $3N + 1$.
 - N even: $\frac{N}{2}$.

Given i, j , $1 \leq i \leq j \leq 100000$, find the maximum of the number of iterations before $f(N)$ becomes 1 for all numbers between i and j .