Practice Problems

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1 Introduction

The way to learn how to solve problems is to practice. Here are a few DP and network flow problems.

2 The Algorithm

First, and most importantly, read the problem. Make sure you understand it thoroughly, and note the limits. As you think about an algorithm, think about three things: run time, memory usage, and coding time. There *is* a working solution; if your solution doesn't quite work in time, keep looking! While an incorrect solution may get significant partial credit, you really rack up points when you get a full solution, since those last three cases might be worth a third or half of the problem.

3 The Problems

1. (Subset Sum) For many sets of consecutive integers from 1 through N ($1 \leq N \leq 39$), one can partition the set into two sets whose sums are identical.

   For example, if N=3, one can partition the set 1, 2, 3 in one way so that the sums of both subsets are identical:

   {3} and {1,2}

   This counts as a single partitioning (i.e., reversing the order counts as the same partitioning and thus does not increase the count of partitions).

   If N=7, there are four ways to partition the set {1, 2, 3, ... 7} so that each partition has the same sum:

   {1,6,7} and {2,3,4,5}, {2,5,7} and {1,3,4,6}, {3,4,7} and {1,2,5,6}, {1,2,4,7} and {3,5,6}

Given N, your program should print the number of ways a set containing the integers from 1 through N can be partitioned into two sets whose sums are identical. Print 0 if there are no such ways.

2. The cows are setting up umbrellas on the beach for shade. The beach can be represented as a 50×50 grid, with cows sitting in certain squares, and an umbrella covering two orthogonally adjacent squares. The cows want to shade as many of themselves as possible, but are firmly against waste and do not want to shade any squares that do not have cows. Find out the maximum number of cows that can be covered without covering any cow-less area.

3. Farmer John has a number of cows capable of milking themselves ($1 \leq N \leq 10000$). Whenever a cow enters the barn, she instantaneously hooks herself up to the milking machine and produces milk at a constant rate (potentially different for each cow) for the duration of her stay in the barn, and unhooks herself immediately before leaving the barn. Farmer John can only measure the aggregate output of all cows in the barn at any given time; given a set of K these measurements ($1 \leq K \leq 100$) and the times when each cow entered and left the barn, find the milk production rate of each cow.

4. (Political Tables) At a politics conference, there are ($4 \leq N \leq 1000$) participants. They are numbered $1, 2, 3, ..., N$ and have to be seated at $2 \leq M \leq 10$ tables. Due to technical constraints, they must be seated in contiguous blocks within each table, meaning that, for instance, $1, 2, 3$ may sit at one table, then $4, 5, 6, 7$ at the next, and so forth; but it can never happen that if A and B are at the same table and $A < B$, that there is a C with $A < C < B$ who isn't aat the same table. Each participant has a political view that can be characterized by an integer $-10 \leq \text{view} \leq 10$.

   The organizers would like to seat at least 2 people at each table, and to balance the political views at each table. Therefore, they want the absolute value of the sum of the political views at each table to be as small as possible. Therefore find the least possible value of the sum of the absolute values of the sums of the political views at each table.