

Advanced Graph Theory Problems

Videh Seksaria

TJHSST SCT

February 10, 2012

1 Problems

1. A revolution is occurring on FJ's farm. Bessie must travel through a perilous set of N bi-directional cow paths between K locations, each with a cow stationed upon it. If the cow is friendly, it will give her up to B bales of hay. If the cow is an enemy cow, it will take up to B bales of hay from her. Bessie wants to arrive at her destination with as many bales of hay as possible, what is her optimal route. What if there were multiple cows, each with a different starting and ending location? Note: Bessie can have negative bales at any given time.
2. FJ is bringing 4G connectivity to his farm. He has placed N signal towers at various locations, joined by K paths. He wishes to put a cable that connects every tower together for faster connections, how can he do so? What is FJ's cable company charged by multiplying each segment of cable's length, instead of adding them?
3. FJ has created a gigantic $N \times K$ chess board. Bessie and Bonnie are starting at $(0, 0)$ and (N, K) respectively. If both move like knights, what is the least number of moves for them to meet. A meeting is defined as Bessie and Bonnie being in rectilinearly neighboring squares. What is there are 3 cows?
4. FJ's barns are created in the form of a tree. In each of the N barns, there is 1 cow. FJ is planning a celebration for K of the N cows. He would like those cows to have to walk as little as possible. If a cow can only walk up from his/her barn, what is the closest barn every cow can arrive at?
5. Bessie is going on a journey! She is traveling across the border to visit Farmer Don. The paths to FD are consistent of N paths and K landmarks. She knows the scenery, and would like to spend quite a bit of time viewing it. So, she would like to take the Sth shortest path. How can she do so? What if she wanted the Sth longest path?

6. Bessie has given FJ a puzzle to solve. The puzzle is made of an $N \times N$ grid filled with the numbers 1 to N^2 , not necessarily sorted. Given a set of valid operations (Swap, rotate and sub - rotate), what is the best sequence of operations to return the puzzle to sorted order. What is different operations have different costs?
7. A virus is spreading around the N cities in FJ's farm. The C cows in each city have a different level of immunity, and thus a different death rate, N_i . FJ needs to save the cows of the world sets out on a dangerous trip to save as many as he can. When he arrives at a city, the virus is destroyed, and no more cows in that city will die. Assume deaths occur by the day, so that all the cows due to die that day die at the end of the day at the exact same time. Given that the cities are connected by K paths, and that a path can only be crossed once in any given direction, what is the least number of cows they will have to die? What if FJ cannot travel through a city that is 'clean' between two that are 'sick'. What if FJ and FD work together, each starting in the same city and splitting up?
8. Bessie has decided to run away. Given a map with Fs denoting farmland and Cs denoting cities, what is location of the city furthest from any farmland, given that $(0, 0)$ denotes the upper left - hand corner.
9. FJ is going on a world tour! Bessie wishes to meet him in a city made of $N \times M$ squares. FJ is located in the bottom right square, and Bessie has decided to go crowd surfing. Given a map of the city with a direction (N, S, E, W) denoting the direction the crowd will push Bessie in, an 'FJ' marking FJ, and an 'X' denoting an empty square, from how many outer squares can Bessie reach FJ?

The above problems are "Standard Problems" and have not been taken from any particular source. The surrounding problem statements are of the author's creation.