Kevin and Trees

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1 Kevin

Problem 1.1. Kevin the cow is on a tree with N vertices ($N \leq 10^5$). The distance between two pairs of vertices in a tree is defined as the number of edges in the unique simple path between them.

This is a general sketch of the methods we will use, to be accompanied by a lecture.

Solution. We start the solution of the problem by describing the general idea of the solution. We will first use Centroid Decomposition. Then we will use Tree DP, and we will finish by applying the Fast Fourier Transform. There are three sections, each devoted to part of it. □

2 Centroid Decomposition

The centroid of a tree (not the center of the tree, something totally unrelated) is a point on a tree such that all subtrees from this vertex have size at most $N/2$, where N is the number of vertices in the tree. The proof of existence of a centroid and algorithm for finding the centroid of a tree is left to the reader and will be covered if we have more time. Essentially, the algorithm to find the centroid is $O(N)$. We keep finding centroids until we get a tree of all centroids. In order to solve this problem, we will define the polynomial of a tree T to be $P(x) = \sum_{i<j} x^{d(i,j)}$, where i and j are pairs of all elements. We will rely heavily on the fact that $d(i, j)$ is bounded by the number of vertices in the tree.

3 Tree DP and FFT

Obviously, we want to compute $P(x)$ for the tree, and then we will be done. Now, how do we build the tree up? If we have $P(x)$ from all the subtrees, we just need to find these distances between the subtrees, in other words, we want to find the sum of the products of these polynomials between subtrees. But this is just asking for the second symmetric sum, so we may conclude ehre.