

Network Flow

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Network flow is the problem of finding the maximum amount of “flow” that one can push through a directed graph from a source to a sink. This lecture will be about the Ford-Fulkerson algorithm because it’s efficient and pretty easy to understand.

1 The Algorithm

At each step in the algorithm, it tries to find a path from the source to the sink along which it can augment the flow. It then “pushes” the flow through the network, and repeats until no more augmenting paths can be found. The runtime is dependent on the method one uses to find the path in the first place. If the graph only contains edges of capacity one, a BFS can be used to find the path. If the capacities vary, Dijkstra’s algorithm is the method of choice. The runtime of the algorithm using BFS is $O(E^2V)$, although poorer schemes for selecting the path can make the time dependent on the max flow in the graph.

```
ford_fulkerson (G, s, t) {
    initialize flow array;
    while (augmenting path p exists) {
        capacity[p] = min(capacity[u][v] for edges (u, v) in p);
        for (each edge (u, v) in p) {
            flow[u][v] += capacity[p];
            flow[v][u] = -flow[u][v];
        }
    }
    return flow;
}
```

2 Problems

1. Farmer John has cooked F types of foods and prepared D types of drinks. Each of his N cows has decided whether she is willing to eat a particular food or drink a particular drink. If each dish or drink can only be consumed by one cow, find the maximum number of cows that can be fed both food and drink that conform to their wishes.
2. Farmer John has a set of gifts to give out to his cows, and knows how many gifts he can give each cow (different for each cow) before more gifts cease to have an effect. Help him distribute the gifts among the cows to maximize the total happiness of his herd.
3. Bessie wants to navigate her spaceship through a dangerous asteroid field in the shape of an $N \times N$ grid. The grid contains K asteroids, which are conveniently located at the lattice points of the grid. Bessie has a powerful weapon that can vaporize all the asteroids in any given row or column of with a single shot. Given the location of all the asteroids, find the minimum number of shots Bessie needs to fire to eliminate all of the asteroids.