

# Dynamic Programming (Silver)

Andre Kessler

October 9, 2009

Dynamic programming is a technique for dramatically reducing the runtimes of certain algorithms from exponential to polynomial. This is done by solving subproblems and storing the results. For people in silver, your main goal should be to honing your ability to recognize a problem as DP, finding the recursive formula for such a problem, coding the problem, and doing all of this quickly. Thus, this lecture will be about giving you practice doing so.

## 1 Problem 1: Max Sum Subwindow

Find the subwindow of maximum sum in  $A[1 \dots N]$ . What about the one of at least length  $k$ ? In a circular array?

### 1.1 Solution

## 2 Problem 2: Longest Increasing Subsequence

Find the longest increasing subsequence in the array  $A[1 \dots N]$ .

### 2.1 Solution

## 3 Problem 3: DAGs

Find the a) shortest and b) longest path(s) in a directed acyclic graph (DAG).

## 4 Problem 4: Bad Hair Day (USACO 06)

Farmer John's  $N$  cows are having a bad hair day! Since each cow is self-conscious about her messy hair style, FJ wants to count the number of other cows that can see the top of other cows' heads. Each cow has a height  $H$  and is standing in a line of cows all facing east. Thus, cow  $i$  can see the heads of the cows in front of her, as long as these cows are strictly shorter than cow  $i$ . Compute the number of cows that each cow can see. Try for  $O(N)$ .

### 4.1 Solution

## 5 Problem 5: Ant Counting (Jacob Steinhardt, 05)

Bessie was poking around the ant hill one day watching the ants march to and fro while gathering food. She realized that many of the ants were siblings, indistinguishable from one another. She also realized the sometimes only one ant would go for food, sometimes a few, and sometimes all of them. This made for a large number of different sets of ants! Being a bit mathematical, Bessie started wondering. Bessie noted that the hive has  $T$  ( $1 \leq T \leq 1,000$ ) families of ants which she labeled  $1 \dots T$  ( $A$  ants altogether). Each family had some number  $N_i$  ( $1 \leq N_i \leq 100$ ) of ants. How many groups of sizes  $S, S+1, \dots, B$  ( $1 \leq S \leq B \leq A$ ) can be formed (mod 1,000,000)?

### 5.1 Solution

## 6 Problem 6: Beetle on a Branch (BOI)

A beetle finds itself on a thin horizontal branch. There are  $N$  drops of dew on that same branch, each holding  $M$  units of water. Their beetle-based integer coordinates are  $x_1, x_2, \dots, x_N$ . The sun is shining brightly and evaporating one unit of water per minute. The beetle is so thirsty that it can drink a drop of dew in no time at all, and the beetle can crawl at one unit of length per minute. But will this pay off? That's what buzzes the beetle. Write a program which, given coordinates of dew drops, calculates the maximal amount of water that the beetle can possibly drink. Note that  $0 \leq N \leq 300$ ,  $1 \leq M \leq 1,000,000$ , and  $-10,000 \leq x_i \leq 10,000$ .

### 6.1 Solution