1. $$\left[\frac{\ln(n)}{n+1}\right]_{n=1}^{\infty} \qquad a_n = \frac{\ln(n)}{n+1}$$

$$a_1 = \frac{\ln(1)}{2} \qquad a_3 = \frac{\ln(3)}{4} \qquad a_5 = \frac{\ln(5)}{6}$$

$$a_2 = \frac{\ln(2)}{3} \qquad a_4 = \frac{\ln(4)}{5}$$

$$\lim_{n\to\infty} \frac{\ln(n)}{n+1} = 0 \text{ ; converges}$$

2. $$\left[\left(1-\frac{4}{n+2}\right)^n\right]_{n=1}^{\infty} \qquad a_n = \left(1-\frac{4}{n+2}\right)^n$$

$$a_1 = \left(1-\frac{4}{3}\right)^1 \qquad a_4 = \left(1-\frac{4}{6}\right)^4$$

$$a_2 = \left(1-\frac{4}{4}\right)^2 \qquad a_5 = \left(1-\frac{4}{7}\right)^5$$

$$a_3 = \left(1-\frac{4}{5}\right)^3$$

$$\lim_{n\to\infty}\left(1-\frac{4}{n+2}\right)^n :$$

$$\ln a_n = \ln\left(1-\frac{4}{n+2}\right)^n = n \ln\left(1-\frac{4}{n+2}\right)$$

$$\lim_{n\to\infty} \ln a_n = n \lim_{n\to\infty} \ln\left(1-\frac{4}{n+2}\right) = \infty \cdot 0$$

$$\lim_{n\to\infty} \frac{\ln\left(1-\frac{4}{n+2}\right)}{\frac{1}{n}} = \frac{0}{}$$

$$\lim_{n\to\infty} \frac{\left[\ln\left(1-\frac{4}{n+2}\right)\right]'}{\left[\frac{1}{n}\right]'} = \qquad -4\left(\frac{1}{n+2}\right)' =$$

$$-4 \cdot -1(n+2)^{-2}$$

$$\left[\ln\left(1-\frac{4}{n+2}\right)\right]' = \frac{\left(\frac{-4}{n+2}\right)}{1-\frac{4}{n+2}} = \frac{\frac{+4}{(n+2)^{2}}}{\frac{n+2-4}{(n+2)}} = \frac{4}{(n+2)^{\cancel{2}}} \cdot \frac{\cancel{(n+2)}}{n-2}$$

$$= \frac{+4}{(n+2)(n-2)}$$

$$\left[\frac{1}{n}\right]' = \frac{-1}{n^2}$$

$$\frac{\frac{+4}{(n+2)(n-2)}}{\frac{-1}{n^2}}$$

$$\frac{+4}{(n+2)(n-2)} \cdot \frac{n^2}{-1} = \lim_{n\to\infty}\left[\frac{-4n^2}{n^2-4}\right] = -4$$

$$\lim_{n\to\infty} \ln a_n = -4 \qquad \lim_{n\to\infty} = \boxed{e^{-4}}$$

③ $\left[\frac{(n+7)}{(n+8)}^{n}\right]_{n=1}^{\infty}$ $\qquad a_n = \left(\frac{n+7}{n+8}\right)^n$

$a_1 = \left(\frac{8}{9}\right)^1$ $\qquad a_4 = \left(\frac{11}{12}\right)^4$

$a_2 = \left(\frac{9}{10}\right)^2$ $\qquad a_5 = \left(\frac{12}{13}\right)^5$

$a_3 = \left(\frac{10}{11}\right)^3$

$$\lim_{n\to\infty}\left(\frac{n+7}{n+8}\right)^n$$

$$a_n = \left(\frac{n+7}{n+8}\right)^n \Rightarrow \ln a_n = \ln\left(\frac{n+7}{n+8}\right)^n$$

$$\ln a_n = n \ln\left(\frac{n+7}{n+8}\right) = \frac{\left[\ln\left(\frac{n+7}{n+8}\right)\right]'}{\left[\frac{1}{n}\right]'}$$

$$= \frac{\frac{\left[\frac{(n+7)}{(n+8)}\right]'}{\frac{n+7}{n+8}}}{\frac{-1}{n^2}} \qquad \frac{(n+8)-(n+7)}{(n+8)^2} = \frac{1}{(n+8)^2}$$

$$= \frac{1}{(n+8)^{\cancel{2}}} \cdot \frac{\cancel{n+8}}{(n+7)} \cdot \frac{n^2}{-1}$$

$$= \frac{-n^2}{n^2+7n+8n+56} = \frac{-n^2}{n^2+15n+56} = 1$$

$$\boxed{\begin{aligned} \ln a_n &= -1 \\ a_n &= e^{-1} \end{aligned}}$$

4 $b_n = (n)^{\frac{4.9}{n}}$

$\ln b_n = \ln (n)^{\frac{4.9}{n}}$

$$= \frac{4.9}{n} \ln(n) = \frac{[\ln(n)]'}{\left[\frac{n}{4.9}\right]'}$$

$$= \frac{\frac{1}{n}}{\frac{1}{4.9}} = \frac{1}{n} \cdot 4.9 = \frac{4.9}{n}$$

$$\ln b_n = \frac{4.9}{n}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} e^{\frac{4.9}{n}} = 1$$

5 $c_n = \ln\left(\frac{2n-7}{9n+4}\right) \sim$ as $n \to \infty$ $\ln\left(\frac{2}{9}\right)$

$$\lim_{n\to\infty} c_n = \boxed{\ln\left(\frac{2}{9}\right)}$$

6 $\left[\frac{(-1)^{n+1}}{(n+1)^2}\right]_{n=1}^{\infty}$ $a_n = \frac{(-1)^{n+1}}{(n+1)^2}$

$a_1 = \frac{(-1)^2}{} = \frac{1}{}$ $a_4 = (-1)^5 = -1$

$$\frac{\quad}{2^2} \qquad 4 \qquad\qquad \frac{\quad}{5^2} \qquad \frac{\quad}{25}$$

$$a_2 = \frac{(-1)^3}{3^2} = \frac{-1}{9} \qquad a_5 = \frac{(-1)^6}{6^2} = \frac{1}{36}$$

$$a_3 = \frac{(-1)^4}{4^2} = \frac{1}{16}$$

$$\lim_{n\to\infty} \frac{(-1)^{n+1}}{(n+1)^2} = \frac{c}{(n+1)^2} = 0$$

7 $S_n = \frac{4^n}{(3n)!}$

$$\frac{\frac{4^{n+1}}{(3(n+1))!}}{\frac{4^n}{(3n)!}}$$

$$\Rightarrow \frac{4^{n+1}}{(3(n+1))!} \cdot \frac{(3n)!}{4^n} = \frac{4^{n+1}}{4^n} \cdot \frac{(3n)!}{(3n+3)!}$$

$$= 4^{\cancel{n}+1-\cancel{n}} \cdot \frac{\cancel{3n}}{(3n+3)(3n+2)(3n+1)(\cancel{3n})}$$

$$\boxed{= \frac{4}{(3n+3)(3n+2)(3n+1)}}$$

8 $\frac{n}{5+4n}$ Difference test:

$S_{n+1} - S_n$

$$\frac{n+1}{5+4(n+1)} - \frac{n}{5+4n}$$

$$= \frac{n+1}{5+4n+4} - \frac{n}{5+4n}$$

$$= \left(\frac{n+1}{4n+9}\right)\left(\frac{5+4n}{5+4n}\right) - \frac{n}{5+4n}\left(\frac{4n+9}{4n+9}\right)$$

$$= \frac{\cancel{5n} + \cancel{4n^2} + 5 + \cancel{4n} - \cancel{4n^2} - \cancel{9n}}{16n^2 + 56n + 45}$$

$$= \frac{5}{16n^2 + 56n + 45}$$

$$\frac{n}{5+4n}$$

9

$$a = \lim_{n \to \infty} a_n = \frac{1}{2}\left(a + \frac{2}{a}\right)$$

$$a = \frac{1}{2}a + \frac{1}{a}$$

$$\frac{a}{2} = \frac{1}{a}$$

$$\sqrt{a^2} = \sqrt{2}$$

$$a = \pm\sqrt{2}$$

10

1. $$a_n = \frac{n-4}{n+4}$$

$$a_{n+1} = \frac{n+1-4}{n+1+4} = \frac{n-3}{n+5}$$

$$\frac{(n-3)}{(n+5)} \cdot \frac{(n+4)}{(n-4)} \qquad \overset{@1}{\frac{-}{+} \cdot \frac{+}{-}} \qquad \overset{@100}{\frac{+}{+} \cdot \frac{+}{+}}$$

$$= \frac{n^2 + 4n - 3n - 12}{n^2 - 4n + 5n - 20} = \frac{n^2 + n - 12}{n^2 - n - 20}$$

$$a_n' = \frac{1(n+4) - 1(n-4)}{(n+4)^2}$$

$$= n + 4 - n + 4 = 8$$

$\overline{(n+4)^2}$

2. $a_n = \dfrac{\sqrt{n+4}}{9n+4} = \dfrac{(n+4)^{1/2}}{9n+4}$

$(a_n)' = \frac{1}{2}(n+4)^{-1/2}(9n+4)$

$a_{n+1} = \dfrac{\sqrt{n+1+4}}{9(n+1)+4} = \dfrac{\sqrt{n+5}}{9n+9+4} = \dfrac{\sqrt{n+5}}{9n+13}$

$\dfrac{9n+4}{9n+4}\left(\dfrac{\sqrt{n+5}}{9n+13}\right) - \left(\dfrac{\sqrt{n+4}}{9n+4}\right)\dfrac{9n+13}{9n+13} = \dfrac{(9n+4)\sqrt{n+5} - (9n+13)\sqrt{n+4}}{(9n+4)(9n+13)}$

3. $a_n = \dfrac{1}{4n+9} = (4n+9)^{-1}$

$(a_n)' = -1\,(4n+9)^{-2} \cdot 4$

$= \dfrac{-4}{(4n+9)^2}$

4. $a_n = \dfrac{\cos n}{4^n}$

$a_{n+1} = \dfrac{\cos(n+1)}{4^{n+1}}$

$\dfrac{a_{n+1}}{a_n} = \dfrac{\cos(n+1)}{4^{n+1}} \cdot \dfrac{4^n}{\cos(n)}$

$= \dfrac{\cos(n+1)}{\cos(n)} \cdot 4^{n-n-1}$

$= \dfrac{1}{}$

4