Problem Q3.3. *Solution.* We want to solve $Ax = b$, where $b = [0, 0, 1]^T$. We know that $A = LU$, so we have $LUx = b$, where we can set $Ux = y$. We now have the following matrix equation:

$$Ly = b \tag{8}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

to get the following system of equations:

$$y_1 = 0 \tag{10}$$

$$y_1 + y_2 = 0 \tag{11}$$

$$y_2 + 2y_3 = 1, \tag{12}$$

which yields $y = [0, 0, \frac{1}{2}]^T$. Now that we have y, we can find x using $Ux = y$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix} \tag{13}$$

where we get the following system of equations:

$$x_1 + x_2 = 0 \tag{14}$$

$$x_2 + x_3 = 0 \tag{15}$$

$$x_3 = \frac{1}{2} \tag{16}$$

So $x = [\frac{1}{2}, \frac{-1}{2}, \frac{1}{2}]^T$.