

**Problem Q2.1.** *Solution.* Let  $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ . Using Gram-Schmidt, we get

$$\begin{aligned}
 q_1 &= \frac{w_1}{\|w_1\|} \\
 &= \frac{[1, 1, 0]^T}{\sqrt{1^2 + 1^2}} \\
 &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\
 q_2 &= \frac{w_2 - (q_1^T w_2)q_1}{\|w_2 - (q_1^T w_2)q_1\|} \\
 &= \frac{[1, 1, 1]^T - \frac{1}{2}[1, 1, 0]^T}{\|[1, 1, 1]^T - \frac{1}{2}[1, 1, 0]^T\|} \\
 &= \frac{[1/2, 1/2, 1]^T}{\sqrt{\frac{3}{2}}} \\
 &= \frac{1}{\sqrt{\frac{3}{2}}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\
 Q &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \\
 R &= \begin{bmatrix} \sqrt{2} & 2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}
 \end{aligned}$$

Thus, one of our QR factorizations is

$$QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}.$$

Another QR factorization can be gotten by changing the signs of the first column of Q and then changing the sign of the entries of the first row of R:

$$QR = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2/3} \\ -\frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & -2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}.$$