① A. $a_n = (6+6n)^{-7}$

$$\lim_{n\to\infty} n^k (6+6n)^{-7} = \frac{n^k}{(6+6n)^7}, \qquad k=?$$

$$\boxed{k=7} \Rightarrow \frac{n^7}{(6+6n)^7}$$

as $n \to \infty$, $\frac{n^7}{(6+6n)^7} \approx \frac{n^7}{6n^7} = \frac{1}{6}$

B. $a_n = \frac{7}{n^2+n}$

$$\lim_{n\to\infty} n^k \cdot \frac{7}{n^2+n} \approx \frac{7n^k}{n^2}, \quad \boxed{k=2} \Rightarrow \frac{7\cancel{n^2}}{\cancel{n^2}}$$

C. $a_n = \frac{4n^2+7n+4}{8n^7+2n+6}$

$$\lim_{n\to\infty} n^k \cdot \frac{4n^2+7n+4}{8n^7+2n+6} \approx \frac{n^k \cdot 4n^2}{8n^7},$$

$$\boxed{k=5} \Rightarrow \frac{4n^{2+5}}{8n^7} = \frac{1}{2}$$

D. $a_n = \left(\frac{4n^2+7n+6}{8n^7+2n+6\sqrt{n}}\right)$

Same principle as C. $\boxed{k=5}$

② $A_n = \frac{1}{n^7}$

$B_n = \frac{1}{n^5}$

$C_n = \frac{1}{n}$

1. $\sum_{n=1}^{\infty} \frac{8n^2+3n^6}{5n^7+9n^3-5} \leftarrow a_n$

$a_n \approx \frac{3}{5n}$ $\qquad \Sigma\, b_n \quad p=1 \Rightarrow \text{Div}$

$L = \lim_{n\to\infty} \frac{\frac{3}{5n}}{\frac{1}{n}} = \frac{3}{5\cancel{n}} \cdot \cancel{n} = \frac{3}{5}$

2. $\sum_{n=1}^{\infty} \frac{5n^5+n^7}{935n^{12}+9n^5+8}$

$a_n \approx \frac{n^7}{935n^{12}} = \frac{1}{935n^5}$

$b_n = \frac{1}{n^5}$

$L = \lim_{n\to\infty} \frac{1}{935\cancel{n^5}} \cdot \cancel{n^5}$

③ $\sum_{n=6}^{\infty} \frac{5n^3-9n^2+6}{3+3n^4}$ $\qquad 5n^3 \approx 5n^3-9n^2+6$

$3n^4 \approx 3+3n^4$

(a) $a_n \approx \frac{5n^3}{3n^4} = \frac{5}{3n}$ as $n\to\infty$ $\qquad b = \frac{1}{n}$

$b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{5n^3-9n^2+6}{3+3n^4} \cdot \frac{n}{1} = \boxed{\frac{5n^4-9n^3+6n}{3+3n^4}}$

(b) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{5n^4}{3n^4} = \boxed{\frac{5}{3}}$

(c) $\Sigma\, b_n = \Sigma \frac{1}{n}$, $p=1$, $\boxed{\text{div}}$

④ Div

5 1. $\sum_{n=2}^{\infty} \frac{4}{n^{10}-4}$, $b_n = \frac{1}{n^{10}}$

$\sum \frac{1}{n^{10}}$, $p = 10 \Rightarrow$ conv

2. $\sum_{n=1}^{\infty} \frac{9+10^n}{6+4^n}$ ~~$a = \frac{9+10}{6+4} = \frac{19}{10}$~~

$r = \frac{a_2}{a_1} = \frac{9+10^2}{6+4^2} \cdot \frac{10}{19}$

~~$S_N = \frac{a}{1-r}(1-r^{N+1})$~~ $= \frac{109}{22} \cdot \frac{10}{19}$

$= \frac{1090}{418} > 1 \Rightarrow$ Div

3. $\sum_{n=1}^{\infty} \frac{\ln(n)}{10n}$? Div

4. $\sum_{n=1}^{\infty} \frac{1}{7+\sqrt[7]{n^6}}$ $L = \lim_{n\to\infty} \frac{1}{7+\sqrt[7]{n^6}} \cdot n^{6/7} = 1$

$b_n = \frac{1}{n^{6/7}}$ $\sum \frac{1}{n^{6/7}}$, $p = \frac{6}{7} < 1 \Rightarrow$ Div

$0 < 1 < \infty$

5. $\sum_{n=1}^{\infty} \frac{4}{n(n+7)} = \sum_{n=1}^{\infty} \frac{4}{n^2+7n}$

$b_n = \frac{1}{n^2}$ $L = \lim_{n\to\infty} \frac{4}{n^2+7n} \cdot n^2 = \frac{4\cancel{n^2}}{\cancel{n^2}+\cancel{7n}} = 4$

$0 < 4 < \infty$

$\sum \frac{1}{n^2}$, $p = 2 > 1 \Rightarrow \boxed{\text{conv}}$

6 (a) $\sum_{k=1}^{\infty} \frac{k^8}{k^9+5}$ $b = \frac{1}{k}$ $p = 1 \Rightarrow$ Div

$L = \lim_{n\to\infty} \frac{k^8}{k^9} \cdot k = 1$

$\frac{1}{2} - \frac{18}{2} = \frac{-17}{2}$

(b) $\sum_{k=1}^{\infty} \frac{k^{\frac{1}{2}}}{k^9+5}$ $k^{1/2-9} = \frac{1}{k^{17/2}}$

$p > 1 \Rightarrow$ conv

(7) $\frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \dots$ $a = \frac{1}{8}$

$r = \frac{\frac{1}{11}}{\frac{1}{8}} = \frac{1}{11} \cdot 8 = \frac{8}{11} < 1 \Rightarrow$ conv

converges to $\frac{\frac{1}{8}}{1 - \frac{8}{11}} = \frac{\frac{1}{8}}{\frac{3}{11}} = \frac{1}{8} \cdot \frac{11}{3} = \frac{11}{24}$

$\sum_{n=0}^{\infty} \frac{1}{8+3n}$ $b = \frac{1}{n}$

$L = \lim_{n\to\infty} \frac{1}{8+3n} \cdot n = \frac{1}{3}$

(8) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+7}}$ $b_n = \frac{1}{n^2}$ $p=2>1$ Conv

$a_n = \frac{n}{n^{6/2}} = \frac{n}{n^3} = \frac{1}{n^2}$

(9) 1. $\sum_{n=1}^{\infty} \frac{8n^8 - n^4 + 7\sqrt{n}}{7n^{10} - n^3 + 6}$

as $n\to\infty$ $a_n \approx \frac{8n^8}{7n^{10}} = \frac{8}{7n^2}$

$b_n = \frac{1}{n^2}$

$L = \lim_{n\to\infty} \frac{8}{7\cancel{n^2}} \cdot \cancel{n^2} = \frac{8}{7}$

$0 < \frac{8}{7} < \infty$

$\sum \frac{1}{n^2}$; $p = 2 > 1 \Rightarrow \boxed{\text{conv}}$

2. $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^3}$ $0 \le \overset{a_n}{\frac{\cos^2 n}{n^{5/2}}} \le \overset{b_n}{\frac{1}{n^{5/2}}}$

$a_n \approx \frac{\sqrt{n}}{n^3} = n^{1/2-3} = n^{-\frac{5}{2}}$

$0 \le a_n \le b_n$

$b_n = \frac{1}{n^{5/2}}$

$\sum \frac{1}{n^{5/2}} \Rightarrow p = \frac{5}{2} > 1 \Rightarrow$ conv

3. $\sum_{n=1}^{\infty} \frac{(\ln(n))^6}{n+8} \Rightarrow$ DIV because $(\ln(n))^6 \gg n$

4. $\sum_{n=1}^{\infty} \frac{\cos(n)\sqrt{n}}{7n+7}$

$a_n \approx \frac{n^{1/2}}{7n} = \frac{1}{7} n^{1/2-1} = \frac{1}{7\sqrt{n}}$

$b_n = \frac{1}{\sqrt{n}}$ $\qquad 0 \le a_n \le b_n$

$-1 \le \cos n \le 1 \Rightarrow 0 \le \frac{\cos n}{\sqrt{n}} \le \frac{1}{\sqrt{n}}$

$\sum \frac{1}{n^{1/2}}$; $p = \frac{1}{2} < 1 \Rightarrow$ DIV $\Rightarrow$ inconclusive

5. $\sum_{n=1}^{\infty} \frac{7n^3}{n^4+7}$

$a_n \approx \frac{7n^3}{n^4} = \frac{7}{n}$

$b_n = \frac{1}{n}$

$L = \lim_{n\to\infty} \frac{7}{n} \cdot n = 7$

$\sum \frac{1}{n}$; $p = 1 \Rightarrow \boxed{\text{DIV}}$

(10) $\sum_{n=1}^{\infty} \frac{9}{5+6^n}$ $\qquad a = \frac{9}{11}$ $\qquad r = \frac{a_2}{a_1} = \frac{\cancel{9}}{5+36} \cdot \frac{11}{\cancel{9}}$

$= \frac{11}{41}$

$\frac{a_3}{a_2} = \frac{\cancel{9}}{5+216} \cdot \frac{41}{\cancel{9}} = \frac{41}{221}$