Problem Q3.1. Solution.

$$A = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \cos\pi & \sin\pi \\ \sin\pi & -\cos\pi \end{bmatrix}$$
 (1)

$$= \begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}. \tag{2}$$

The first matrix on the right side of (1) is the rotation matrix when $\theta=\frac{\pi}{4}$, which is 45 degrees counterclockwise on the unit circle. The second matrix on the right side of (1) is the reflection matrix when $\theta=\pi$, which is the y-axis on the unit circle.