Problem Q5.1. *Solution.* Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$. To find an orthonormal basis for the columns space of A, all we do is use the Gram-Schmidt Procedure. Let $v_1 = (1, 0, -1)$ and $v_2 = (2, 1, 2)$. We find the first vector e_1 in our orthonormal basis:

$$\begin{aligned} e_1 &= \frac{(1,0,-1)}{||(1,0,-1)||} \\ &= \frac{(1,0,-1)}{\sqrt{1^2+0^2+(-1)^2}} \\ &= \frac{(1,0,-1)}{\sqrt{2}}. \end{aligned}$$

Now we find e_2:

$$\begin{aligned} e_2 &= \frac{(2,1,2) - \langle (2,1,2), (\frac{(1,0,-1)}{\sqrt{2}}) \rangle \frac{(1,0,-1)}{\sqrt{2}}}{||(2,1,2) - \langle (2,1,2), (\frac{(1,0,-1)}{\sqrt{2}}) \rangle \frac{(1,0,-1)}{\sqrt{2}}||} \\ &= \frac{(2,1,2) - \frac{1}{2}(0)(1,0,-1)}{||(2,1,2) - \frac{1}{2}(0)(1,0,-1)||} \\ &= \frac{(2,1,2)}{\sqrt{2^2+1^2+2^2}} \\ &= \frac{(2,1,2)}{3}. \end{aligned}$$

So an orthonormal basis of the column space of A is $e_1, e_2 = \frac{(1,0,-1)}{\sqrt{2}}, \frac{(2,1,2)}{3}$.