HW-1
Math 117

Siddharth Deshpande

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Sets (DeMorgan's Identities

1.

- The first law that we are going to prove is:

$$X - \bigcup_{F \in \mathcal{F}} F = \bigcap_{F \in \mathcal{F}} (X - F)$$

i)

$$\boxed{X - \bigcup_{F \in \mathcal{F}} F \subseteq \bigcap_{F \in \mathcal{F}} (X - F)}$$

- Let x be a variable such that, $x \in X - \bigcup\limits_{F \in \mathcal{F}} F$, then, we know that $x \in X$ and $x \notin \bigcup\limits_{F \in \mathcal{F}} F$ by definition.
- Therefore, because $x \notin \bigcup\limits_{F \in \mathcal{F}} F$, we know that $\forall F \in \mathcal{F},\ x \notin F$ by definition.
- Furthermore, because $x \in X$ and $x \notin F,\ \forall F \in \mathcal{F}$, we know that, $x \in X - F$, for all F in the set $\mathcal{F}$ by definition.
- Therefore, $\forall F \in \mathcal{F},\ x \in (X - F)$. Then, by definition $x \in \bigcap\limits_{F \in \mathcal{F}} (X - F)$.
- Therefore, now we know that, $X - \bigcup\limits_{F \in \mathcal{F}} F \subseteq \bigcap\limits_{F \in \mathcal{F}} (X - F)$

ii)

$$\boxed{X - \bigcup_{F \in \mathcal{F}} F \supseteq \bigcap_{F \in \mathcal{F}} (X - F)}$$

- Let c be a variable such that, $x \in \bigcap_{F \in \mathcal{F}} (X - F)$. Therefore, by definition we know that, $\forall F \in \mathcal{F},\ x \in (X - F)$.

- Then, $x \in X$ and $x \notin F$, $\forall F \in \mathcal{F}$ by definition. Then, because $x \notin F, \forall F \in \mathcal{F}$, we know that, $x \notin \bigcup_{F \in \mathcal{F}} F$.

- Since $x \in X$ and $x \notin \bigcup_{F \in \mathcal{F}} F$, we know by definition that, $x \in X - \bigcup_{F \in \mathcal{F}} F$.

- Therefore, by definition, $X - \bigcup_{F \in \mathcal{F}} F \supseteq \bigcap_{F \in \mathcal{F}} (X - F)$

- Because we have proved that the subset holds in both directions, we know that

$$\boxed{X - \bigcup_{F \in \mathcal{F}} F = \bigcap_{F \in \mathcal{F}} (X - F)}$$

2.

- Now we shall prove the second identity:

$$X - \bigcap_{F \in \mathcal{F}} F = \bigcup_{F \in \mathcal{F}} (X - F)$$

i)

$$\boxed{X - \bigcap_{F \in \mathcal{F}} F \subseteq \bigcup_{F \in \mathcal{F}} (X - F)}$$

- Let x be a variable such that, $x \in X - \bigcap_{F \in \mathcal{F}} F$. Then, by definition we have that, $x \in X$ and, $x \notin \bigcap_{F \in \mathcal{F}} F$.

- Because $x \notin \bigcap_{F \in \mathcal{F}} F$, we know that, $\exists F \in \mathcal{F} : x \notin F$. Without loss of generality, let us denote this element of $\mathcal{F}$ that does not contain x as F_1.

- Therefore, because $x \in X$ and, $x \notin F_1$, we know that, $x \in (X - F_1)$ by definition. But, $F_1 \in \mathcal{F}$, therefore, by definition we know that, $x \in \bigcup_{F \in \mathcal{F}} (X - F)$ as, $x \in (X - F_1)$ and $F_1 \in \mathcal{F}$.

- Therefore, by definition we have that, $X - \bigcap_{F \in \mathcal{F}} F \subseteq \bigcup_{F \in \mathcal{F}} (X - F)$

ii)

$$\boxed{X - \bigcap_{F \in \mathcal{F}} F \supseteq \bigcup_{F \in \mathcal{F}} (X - F)}$$

- Let x be a variable such that, $x \in \bigcup_{F \in \mathcal{F}} (X - F)$. Then, we know that $x \in X$ and we also know that, $\exists F \in \mathcal{F} : x \notin F$. Let us call this element of $\mathcal{F}$ which does not contain x as F_1.

- Therefore, because $x \notin F_1$ and $F_1 \in \mathcal{F}$ we have by definition that, $x \notin \bigcap_{F \in \mathcal{F}} F$.

- Finally, because $x \in X$ and, $x \notin \bigcap_{F \in \mathcal{F}} F$, by definition we know that, $x \in X - \bigcap_{F \in \mathcal{F}} F$.

- And therefore, by definition, $X - \bigcap_{F \in \mathcal{F}} F \supseteq \bigcup_{F \in \mathcal{F}} (X - F)$

- Because we have proved the subset holds in both directions we know that,

$$\boxed{X - \bigcap_{F \in \mathcal{F}} F = \bigcup_{F \in \mathcal{F}} (X - F)}$$