Final Exam/Project/Thing Math 8, Winter 2020

Write your own high-quality solutions, complete with correct grammar and punctuation. Use whatever resources you want, but write all solutions *in your own words*. Understand, then write without copying. Prove all of your claims, to every last detail that you can muster. You are allowed to use any theorem from the course, and you can reference theorems by their name or by their theorem number.

1. Consider the set
$$(A \times X) \cup (B \times Y)$$
and the set
$$(A \cup B) \times (X \cup Y).$$
One of these is always subset of the other. Determine which inclusion is correct and prove that inclusion.

2. Prove: If $\mathcal{P}(A) = \mathcal{P}(B)$ then $A = B$.

3. Prove: If A has 3 elements, then there exist $x, y, z \in A$ such that $x \neq y$, $y \neq z$, and $x \neq z$.

4. Define $f : \mathbb{N} \to \mathbb{N}$ to be the unique function that satisfies $f(0) = 5$ and $f(n+1) = f(n) + f(n)$ for all $n \in \mathbb{N}$. Prove that $3 \leq f(n)$ for all $n \in \mathbb{N}$.