Problem Q2.1. *Solution.* Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$. Using Gram-Schmidt, we get

$$\begin{aligned}
q_1 &= \frac{w_1}{||w_1||} \\
&= \frac{[1,1,0]^T}{\sqrt{1^2+1^2}} \\
&= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\
q_2 &= \frac{w_2 - (q_1^T w_2) q_1}{||w_2 - (q_1^T w_2) q_1||} \\
&= \frac{[1,1,1]^T - \frac{1}{2}[1,1,0]^T}{||[1,1,1]^T - \frac{1}{2}[1,1,0]^T||} \\
&= \frac{[1/2, 1/2, 1]^T}{\sqrt{\frac{3}{2}}} \\
&= \frac{1}{\sqrt{\frac{3}{2}}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\
Q &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \\
R &= \begin{bmatrix} \sqrt{2} & 2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}
\end{aligned}$$

Thus, one of our QR factorizations is

$$QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ \frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}.$$

Another QR factorization can be gotten by changing the signs of the first column of Q and then changing the sign of the entries of the first row of R:

$$QR = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2/3} \\ -\frac{1}{\sqrt{2}} & \sqrt{2/3} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & -2/\sqrt{2} \\ 0 & \sqrt{3/2} \end{bmatrix}.$$