1. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$

$a_1 = \frac{1}{3^1}$ $\quad$ $a_3 = \frac{1}{3^3}$

$a_2 = \frac{1}{3^2}$ $\quad$ $\boxed{a_n = \frac{1}{3^n}}$

3. (a) $1, 1, 1, 1$

$(n)^0$

(b) $1, -1, 1, -1, \ldots$

$(-1)^{n+1}$

(c) $\frac{3}{4}, \frac{15}{16}, \frac{63}{64}, \frac{255}{256}$

$$\frac{4^n - 1}{4^n}$$

(d) $0, \frac{1}{\sqrt{\pi}}, \frac{4}{\sqrt[3]{\pi}}, \frac{9}{\sqrt[4]{\pi}}$

$$\frac{(n-1)^2}{\pi^{\frac{1}{n}}}$$

5. $S = \sum_{n=1}^{\infty} a_n \qquad S_N = 3 - \frac{9}{N^2}$

(a) $\sum_{n=1}^{10} a_n$

$= 3 - \frac{9}{10^2}$

$$\overbrace{a_1 + a_2}^{S_2} + a_3$$
$$\underbrace{a_1 + a_2 + a_3}_{S_3}$$

$\sum_{n=4}^{16} a_n$

$= \left(3 - \frac{9}{16^2}\right) - \left(3 - \frac{9}{9}\right)$

$= 3 - \frac{9}{16^2} - 2$

(b) $a_3 = S_3 - S_2 =$

$3 - \frac{9}{3^2} - \left(3 - \frac{9}{2^2}\right)$

$= \cancel{3} - \frac{9}{9} - \cancel{3} + \frac{9}{4} = -1 + \frac{9}{4} = \frac{-4+9}{4} = \frac{5}{4}$

(c) $a$ ... $= S$

(c) $a_n = s_n - s_{n-1}$

$$\left(3 - \frac{9}{n^2}\right) - \left(3 - \frac{9}{(n-1)^2}\right)$$

$$= \cancel{3} - \frac{9}{n^2} \cancel{-3} + \frac{9}{(n-1)^2} = \boxed{\frac{9}{(n-1)^2} - \frac{9}{n^2}}$$

(d) $\sum_{n=1}^{\infty} a_n = \boxed{\lim_{N\to\infty} 3 - \frac{9}{N^2} = 3}$

(7) $a_1 = -10$

$a_2 = \frac{-10}{4}$

$r = \frac{-10}{4} \cdot \frac{1}{-10} = \frac{1}{4}$

$$\sum_{n=0}^{\infty} -10 \cdot \left(\frac{1}{4}\right)^{n-1} = \frac{a}{1-r} = \frac{-10}{1-\frac{1}{4}} = \frac{-10}{\frac{3}{4}}$$

$$= -10 \cdot \frac{4}{3} = \frac{-40}{3}$$

(9) (a) $\sum_{n=1}^{\infty} \frac{8^n}{7^n}$  $a = \frac{8^1}{7^1}$

$\boxed{|r| > 1 \Rightarrow \text{DIV}}$  $\frac{a}{1-r}$

$a = \frac{8}{7}$  $r = \frac{8}{7}$

$r = \frac{a_2}{a_1} = \frac{8^{\cancel{2}}}{7^{\cancel{2}}} \cdot \frac{\cancel{7}}{\cancel{8}} = \frac{8}{7}$

(b) $\sum_{n=2}^{\infty} \frac{1}{3^n}$

$a = \frac{1}{3^2} = \frac{1}{9}$

$r = \frac{a_3}{a_2} = \frac{1}{3^3} \cdot \frac{9}{1} = \frac{9}{27} = \frac{1}{3}$

$$\frac{\frac{1}{9}}{1-\frac{1}{3}} = \frac{1}{9} \cdot \frac{3}{2} \boxed{= \frac{1}{6}}$$

(c) $\sum_{n=0}^{\infty} \frac{3^n}{8^{2n+1}}$

$a = \frac{3^0}{8^1} = \frac{1}{8}$

$$\frac{a}{1-r} = \frac{\frac{1}{8}}{1-\frac{3}{64}} = \frac{1}{8} \cdot \frac{64}{61}$$

$$r = \frac{a_1}{a_0} = \frac{3}{8^3} \cdot 8 = \frac{3}{64} \qquad \boxed{= \frac{8}{61}}$$

(d) $\displaystyle\sum_{n=5}^{\infty} \frac{7^n}{8^n}$

$$a = \frac{7^5}{8^5} \qquad \frac{a}{1-r} = \frac{\frac{7^5}{8^5}}{1-\frac{7}{8}} = \frac{7^5}{8\cancel{8}^4} \cdot \frac{\cancel{8}}{1}$$

$$r = \frac{a_6}{a_5} = \frac{7^{\cancel{6}}}{8^{\cancel{6}}} \cdot \frac{8^{\cancel{5}}}{\cancel{7^5}} = \frac{7}{8}$$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{6^{n+4}}$

$$a = \frac{6}{6^5} = \frac{1}{6^4}$$

$$r = \frac{a_2}{a_1} = \frac{1}{6^4} \cdot \frac{6^4}{1} = 1 \Rightarrow \boxed{\text{Div}}$$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n + 3^n}{8^n} = \overset{(*)}{\sum_{n=1}^{\infty} \frac{7^n}{8^n}} \overset{(**)}{+} \sum_{n=1}^{\infty} \frac{3^n}{8^n}$

(*) $a = \frac{7}{8}$

$$r = \frac{a_2}{a_1} = \frac{\cancel{49}7}{8\cancel{64}} \cdot \frac{\cancel{8}}{\cancel{7}} = \frac{7}{8}$$

$$\frac{\frac{7}{8}}{1-\frac{7}{8}} = \frac{7}{\cancel{8}} \cdot \cancel{8} = 7$$

(**) $a = \frac{3}{8}$

$$r = \frac{a_2}{a_1} = \frac{\cancel{9}3}{\cancel{64}8} \cdot \frac{\cancel{8}}{\cancel{3}} = \frac{3}{8}$$

$$\frac{\frac{3}{8}}{1-\frac{3}{8}} = \frac{3}{8} \cdot \frac{8}{5} = \frac{3}{5}$$

$$\Rightarrow \sum \frac{7^n + 3^n}{8^n} = \boxed{7\tfrac{3}{5}}$$

$n=1$