

**Problem Q2.** *Solution.* Suppose  $U$  is an orthogonal upper triangular matrix. Then  $U$  has orthonormal columns and also the rows of  $U$  are orthonormal. Particularly, this means that norms of both the rows and columns of  $U$  are 1. Additionally, the transpose of  $S$  is also its inverse. Since  $U$  is any orthogonal upper triangular matrix, it must be that it is a diagonal matrix whose main diagonal entries are either 1 or -1.

Now we prove the other direction: Suppose that  $U$  is a diagonal matrix whose main diagonal entries are either 1 or -1. From this we can gather that  $U$  is square and that the rows and columns of  $U$  are orthonormal. Hence,  $U$  is an orthogonal matrix whose entries are on the diagonal, thus  $U$  is upper triangular as well.