USING nD-LAPLACE TO TRAIN PRIVACY-PRESERVING CLUSTER ALGORITHMS ON DISTRIBUTED N-DIMENSIONAL DATA

by

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INTRODUCTION

1.1. RESEARCH QUESTIONS

Main question:
How can the nD-Laplace algorithm be applied in training privacy-preserving clustering algorithms on distributed n-dimensional data?

1. RQ1: How can 2D-Laplace be used to protect the data privacy of 2-dimensional data which is employed for training clustering algorithms?
2. RQ2: How can 3D-Laplace be extended to protect the data privacy of n-dimensional data which is employed for training clustering algorithms?
3. RQ3: What is the impact of different privacy budgets, dataset properties, and other clustering algorithms on the research conducted for research question 2?

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LITERATURE REVIEW

This chapter lays out the theoretical foundation of this work. To review the past literature, it is first necessary to gather the required knowledge for it.

2.1. DIFFERENTIAL PRIVACY

In practice, data is often sent to a central storage point. This requires trust, and because all data is collected in one place, the risk of private data leakage becomes very high. By applying differential privacy, noise can be added to the data to protect it. This is principle is illustrated in figure 2.1 with the following actors:

1. Trusted curator: The system that receives data from users. It is assumed in this setting that the system is trustworthy and that the data is securely stored.

2. Adversary: An adversary is someone who uses the data. This could be, for example, a data scientist who wants accurate results, or an attacker who wants to obtain as much data as possible.

3. The users are clients (for example, websites or mobile apps) who entrust their data to a central server.

With the introduction of differential privacy, the privacy of a user would be ensured (to a certain extent). This will be further explained in the next section.

Although differential privacy solves many problems (as mentioned earlier in the introduction), it remains difficult to calibrate the mechanism. There is an important trade-off between utility and privacy for the adversary, where a data scientist wants accurate data while the noise must be sufficient to prevent an attacker from obtaining too much information. For this reason, the following chapters will be devoted to outlining the mathematical background of differential privacy. We will examine which factors influence this calibration and whether other methods contribute to it. Afterward, we will also further explain other types of differential privacy (local and geo-indistinguishable) in the same way.

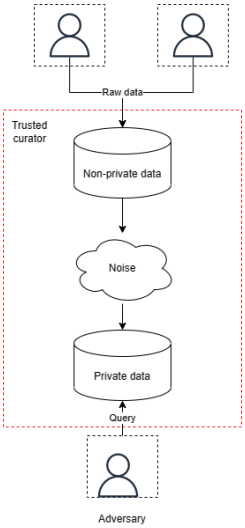

Figure 2.1: General approach for setting up (central) differential privacy.

2.1.1. DEFINITIONS

We examine the different notations and types of differential privacy we consider in this research.

ϵ-DIFFERENTIAL PRIVACY

Dwork et al formulated the notion of privacy as: Participating in a database should not significantly increase the risk of an individual's privacy being compromised Dwork [2006]. This is mathematically formulated in the same research with the name Differential Privacy (DP). Using the definition of privacy, it is formulated as the maximal possible change when adding or removing a single record [Dwork, 2006; Friedman and Schuster, 2010]. This is reflected using the formal mathematical formulation as formulated by dwork et al:

$$\Pr[K(D_1) \in S] \leq \exp(\epsilon) \times \Pr[K(D_2) \in S] \tag{2.1}$$

So, given a randomization function K, it gives ϵ-differential privacy if dataset D_1 and D_2 are differing at most one element [Dwork, 2006]. The ϵ determines the amount of noise (privacy budget) [Friedman and Schuster, 2010]. The lower the value of ϵ means, the higher the privacy guarantee. In this regard, it is important for a method that ensures differential privacy to take this into account. For this reason, a common way to determine the ϵ is to calculate the sensitivity. This value is calculated based on the impact of a function or query on the data. For example, if there is a method called sum for the summation of data points, the sensitivity of the method is 1. This is because removing one data point would greatly affect the outcome and ϵ-differential privacy could no longer be guaranteed. It is also mathematically defined by dwork et al:

$$\Delta f = \max_{D_1, D_2} \| f(D_1) - f(D_2) \|_1 \tag{2.2}$$

(ϵ, δ)-DIFFERENTIAL PRIVACY

The formal notion of differential privacy has only ϵ input. This formulation is really strict, but most methods relax this a little which is defined as (ϵ, δ).

$$\Pr[K(D_1) \in S] \leq \exp(\epsilon) \times \Pr[K(D_2) \in S] + \delta \tag{2.3}$$

This means the sensitivity (now denoted as delta δ) is used to loosen up the definition. The purpose of ϵ now is to calibrate the desired amount of privacy. To some extent, the delta represents the probability of the algorithm leaking information [Aitsam, 2021]. With ϵ-differential privacy, there would be no difference in the case of information leakage (delta = 0). However, with (ϵ, δ)-differential privacy, the information can leak up to the probability of delta.

ϵ-LOCAL DIFFERENTIAL PRIVACY

As the name suggests, Local Differential Privacy (LDP) is executed on the client-side instead of on the server, as was the case in figure 2.1. This is illustrated in figure 2.2. Local differential privacy was introduced to remove the "trusted" curator, preventing sensitive data leakage even if an attacker gains access to the dataset [Xiong et al., 2020]. The definition for LDP is the as for equation 2.3 with $\delta = 0$ being equal to equation 2.1.

Describe issues with local differential privacy

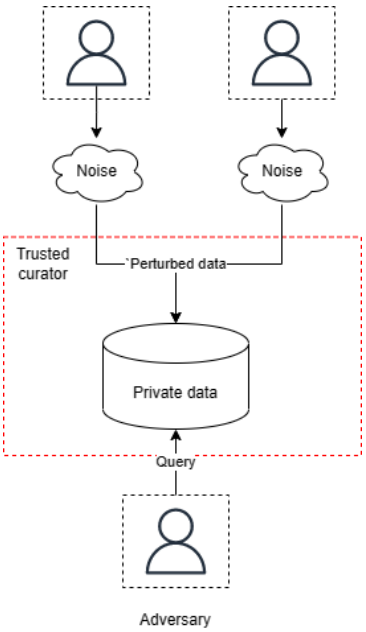

Figure 2.2: Local differential privacy, which moves the noise-adding step to the client-side.

ϵ-GEO-INDISTINGUISHABILITY

The last and for this study's most important type of differential privacy is Geo-indistinguishability (GI). This is a type of differential privacy that is specifically designed for location data.

2.1.2. MECHANISMS

In this section, we will explain the various mechanisms that are used to achieve differential privacy. The different mechanisms are not limited to the type of DP and some can be applied to multiple types.

RANDOMIZED RESPONSE MECHANISM

The random response method is a relatively simple method and was first applied in 1965 by Warner et al. It was originally used to mask the answers of individuals by randomly switching the answers with predictable randomness [Warner, 1965]. Therefore, the method is mainly used for categorical data. This method satisfies its own set of requirements for LDP [Xiong et al., 2020], which differs from the formal definition that was mentioned earlier

Since then, it is still one of the better-known methods, and larger organizations such as Google use it [Erlingsson et al., 2014]. They have named their extension RAPPOR and expanded it with bloom filters to be able to collect numerical data as well. It has been possible to ensure ϵ-differential privacy in this way, and it is also possible to preserve LDP [Xiong et al., 2020].

LAPLACE MECHANISM

The method that was originally proposed in the differential privacy paper by Dwork et al. is the Laplace algorithm [Dwork, 2006]. Therefore, the method works by configuring the ϵ and δ. A shorthand definition is provided by Rey et al [Xiong et al., 2020]:

$$M\left(f\left(x\right),\varepsilon\right)=f\left(x\right)+\left(Z_1,\ldots,Z_d\right) \tag{2.4}$$

The mechanism is based on the Laplace distribution with scale $\lambda f/\epsilon$, where λf is the same as in equation 2.2. Therefore, the Laplace mechanism is tightly linked to the definition of pure-dp and so it is ϵ-differential privacy [Dwork, 2006]. It is also suitable for preserving LDP [Xiong et al., 2020]. One disadvantage is that sensitivity is always required, and this parameter can sometimes be difficult to configure. Especially when there is no clear function and the entire dataset is perturbed. This can make it challenging to find the right balance between ensuring privacy and utility.

To this end, the sensitivity can be calculated in two forms: global and local. Global sensitivity is calculated over two different datasets and is part of the original definition of DP [Dwork, 2006]. It is called global because it is independent of the queried dataset. Usually, this is not the desired situation, since local sensitivity always has more context of the dataset in question [Nissim et al., 2007]. As a result, the trade-off for noise is much more precise, and the balance between utility and privacy is much better. This local sensitivity definition [Nissim et al., 2007].

$$LS(f,x) = \max_{x' \cdot d(x,x') \leq 1} |f(x) - f(x')| \tag{2.5}$$

A methodology to calculate the local sensitivity is by using the smooth sensitivity method, which was proposed by the same authors [Nissim et al., 2007]. This method aims at smoothing out the local sensitivity by focusing on reducing the noise. The goal is to smooth out the amount of noise, to reduce the risk of something being revealed in the data. Due to this, a disadvantage of this method is that it can be computationally expensive. Also, the introduction of this method makes Laplace preserve (ϵ, δ)-DP instead of pure ϵ-DP.

GAUSSIAN MECHANISM

Another mechanism that works comparably is the Gaussian mechanism. This mechanism makes use of the Gaussian distribution to add noise to the data.

2.2. CLUSTERING

Explain why these three algorithms

2.2.1. METHODS

In this chapter, a brief description is given of each clustering algorithm on how it works. The different parameters for the algorithm are also highlighted.

K-MEANS

The K-Means algorithm is based on the algorithm of Loyd et al. The starting point is determined randomly by choosing k number of centroids [Lloyd, 1982]. Each point is then assigned to a centroid based on the Euclidean distance. A new centroid is then determined based on the cluster average. This is repeated until the given number of iterations is reached or until the results are stable.

Choosing K: The most important parameter of the K-Means algorithm is the value of k. This value determines the number of clusters to consider and has a big influence on the results [Ahmed et al., 2020].

The first method is called an "elbow" plot [Kodinariya and Makwana, 2013]. This method can be used to determine the best k by applying the algorithm multiple times and estimating the best k.

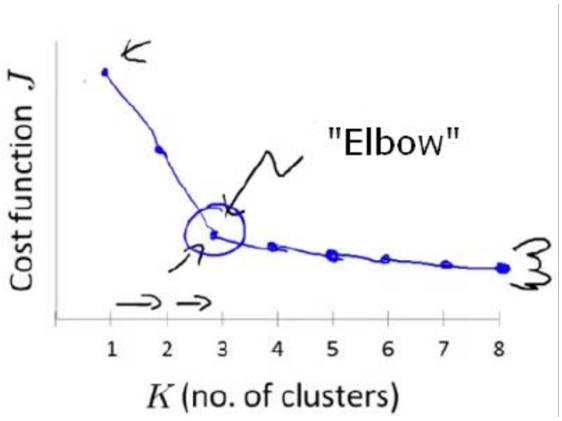

Figure 2.3: Illustration of determining k using the "elbow" method [Kodinariya and Makwana, 2013]

In this situation, a Silhouette plot can be used to determine the k. This method uses calculates the Silhouette coefficient for each cluster [SAPUTRA et al., 2020]. It takes into consideration the separation and cohesion of the cluster. The plot can then be made by plotting the silhouette coefficient on the y-axis and k on the x-axis [SAPUTRA et al., 2020]. Then the K with the highest coefficient can be selected based on that.
Another popular method is the Gap statistic method [Yuan and Yang, 2019]. It compares the total within-cluster variation for different values of k with their expected values under a null reference distribution of the data [Tibshirani et al., 2001]. A practical appliance of this method uses a line plot for comparing the k-value and gap value [Yuan and Yang, 2019]. Based on the visual change of the line, someone is be-able to select the best k.

All in all, there is no fixed method to choose a good k for K-Means. The elbow method is common in the existing literature and is very popular due to its simplicity. However, one

disadvantage is that it can be difficult to determine the "elbow" point, as it is not always present [Kodinariya and Makwana, 2013]. In that case, the silhouette method or gap statistic method can be chosen, with the algorithm for silhouette being the most obvious choice due to its simplicity.

Affinity Propagation

Affinity Propogation (AP) is an algorithm that clusters data points by iteratively passing messages between them. Each point sends and receives messages about the attractiveness of other points as cluster centers (exemplars) and the suitability of itself as a center [Keller et al., 2021]. The method was introduced by Frey et al. and does not require any hyperparameters [Frey and Dueck, 2007]. Still, there are important properties that could potentially impact the clustering [Wang et al., 2007].

Choosing preference(p): Indicates the preference that a data point is selected as cluster center [Wang et al., 2007]. It highly influences the number of clusters, a high one would lead to more clusters and a small one to less [Moiane and Machado, 2018]. A good choice depending on the data is to set the p to the median of all data similarities [Wang et al., 2007]. But, the effectiveness of this could highly be influenced based on the dataset. To validate if the preference is correctly set, it is possible to analyze the silhouette coefficient [Moiane and Machado, 2018].

Choosing damping factor($lam \in [0,1]$): The damping factor is used to improve the stability (convergence) of the algorithm [Wang et al., 2007]. By default, this value is 0.5 and can be increased to 1 to reduce the impact of numerical oscillations. This can be applied manually, by re-running the algorithm and finding the optimal or just being increased. However, both approaches take a lot of time, especially for bigger datasets [Wang et al., 2007].

To conclude on this, damping is important if big datasets are considered. However, this research does not use large datasets or consider time complexity as a metric. The preference on the other hand could influence the results a lot. For this, the silhouette coefficient can be evaluated to choose the best option. In general, it should be sufficient to take the median.

DBSCAN

Density-based spatial clustering of applications with noise (DBSCAN) was introduced by [Ester et al.] and works by drawing a radius (neighborhood) around data points. It then groups all points within this radius as clusters. The main advantage is its ability to find arbitrarily shaped clusters and detect outliers [Liu et al., 2012]. To do this, the DBSCAN algorithm uses the inputs $minPts$, $radius(\epsilon)$ and a distance function [Schubert et al., 2017]. The ϵ is used to draw a neighborhood and the $minPts$ is used as a weight to evaluate which points should be inside the neighborhood. For the distance function, the Euclidean distance is used, to be consistent with the other algorithms.

Choosing radius(ϵ): The desired ϵ can be calculated using the K-NearestNeighbours algorithm [Ester et al.; Schubert et al., 2017]. The general approach for this is to choose a $K = 2 * N - 1$ (where N is the number of features) and plot the distance for each point. This can then be plotted using a k-dist plot and the best "elbow" can be chosen for deciding the ϵ (similar to choosing the k for K-means) [Elbatta and Ashour, 2013].

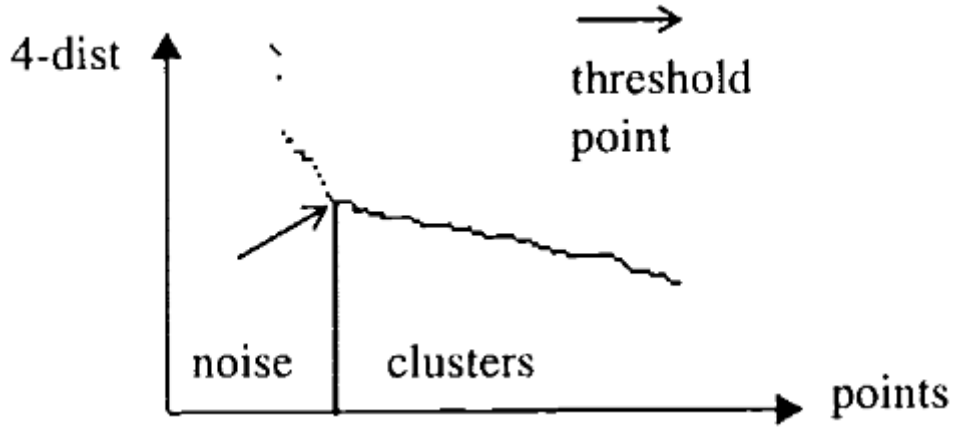

Figure 2.4: K-dist plot example for $minPts = 4$ based on a 2-dimensional dataset [Ester et al.]

Choosing minimum points ($minPts$): This hyperparameter is considered by a paper written by Sander et al. The work describes a way of calculating this parameter by doing two times the feature amount [Sander et al., 1998]. So, using this approach a dataset with two features will have an $minPts$ of four. This is confirmed by Schubert et al. to use the default $minPts = 4$ for a 2-dimensional dataset [Schubert et al., 2017].

In conclusion, the parameters of DBSCAN are a little harder to estimate than for AP. But, as with K-means for k there exists a selection method the ϵ. Although DBSCAN requires the most hyperparameters out of the three clustering algorithms, determining them is clearer than for K-means. There are clear methods for determining both the $minPts$ and ϵ.

2.2.2. Evaluation methods

Clustering comparison measures are important in cluster analysis for external validation by comparing clustering solutions to a "ground truth" clustering [Vinh et al.]. These external validity indices are a common way to assess the quality of unsupervised machine learning methods like clustering [Warrens and van der Hoef, 2022]. A method that could be used for this is the Rand Index [Rand, 1971]. It is a commonly applied method for comparing two different cluster algorithms [Wagner and Wagner]. An improvement of this method is adjusted for chance by considering the similarity of pairwise cluster comparisons [Vinh et al.]. Both the Rand Index (RI) and Adjusted Rand Index (ARI) [Hubert and Arabie, 1985] report a value between 0 and 1. Where 0 is for no-similarity and 1 for identical clusters. Alternatives for RI are the Fowles-Mallows Index and Mirkin Metric. However, these two methods have their disadvantages. Respectively, being sensitive to a few clusters and cluster sizes [Wagner and Wagner]. The ARI metric suffers from cluster size imbalance as well, so it only provides not a lot of information on smaller clusters [Warrens and van der Hoef, 2022]. Instead, they recommend using the cluster index metric that was proposed by Fränti et al. [Fränti et al., 2014].

Another popular group of methods is the information theoric-based measures [Vinh et al.]. This metric measures the information between centroids; the higher the value, the better [Vinh et al.]. Mutual Information (MI) is such metric, which calculates the probability of an element belonging to cluster C or C'. But, is not easy to interpret as it does not have a maximum value [Wagner and Wagner]. To this end, Normalized Mutual Information

(NMI) can be used to report a value between 0 and 1 using the geometric mean [Strehl and Ghosh, 2002]. The metric exists also in an adjusted version as Adjusted Mutual Information (AMI). This works in the same way as for the Adjusted Rank Index (ARI) and is mostly needed if the number of data items is small in comparison to the number of clusters [Vinh et al.].

Besides the external validity measurements for clustering, it is also possible to use internal validation methods. These metrics focus entirely on the intrinsic dataset properties, instead of relying on an external baseline cluster algorithm [Craenendonck and Blockeel]. Assessing two important concepts of clustering: compactness and separation [Hassani and Seidl, 2017]. Both studies, consider three different metrics and measure both concepts at the same time [Hassani and Seidl, 2017]:

1. Calinski-Harabasz Index (CHI) [Caliński and Harabasz, 1974] is used to measure the cluster variance (well-separated clusters) and low variance within the clusters (tightly coupled data). A high score indicates better clustering.

2. Silhouette Index [Rousseeuw, 1987] this metric is similar, by also measuring cohesion within clusters and separation of clusters. However, this metric uses the pairwise distance Hassani and Seidl [2017]. A score of -1 indicates incorrect clustering and +1 for dense clusters Rousseeuw [1987].

3. Davies-Bouldin [Davies and Bouldin, 1979] uses the average distance between centroids. A lower score indicates good clustering.

K-Means scores relatively high for CHI [Craenendonck and Blockeel; Hassani and Seidl, 2017] and SI [Craenendonck and Blockeel]. The same applies to DBSCAN, which scores relatively high on SI and DB due to the sensitivity of noise [Craenendonck and Blockeel].

Existing literature

Comparable studies with differential privacy use external validation [Sun et al., 2022; Xia et al., 2020]. Their experiment setup uses a so-called non-private cluster algorithm as external validation. This cluster algorithm is trained without the perturbed data and compared with the same clustering algorithm that is trained with perturbed data. Thus, the non-private variant functions as an external validation by providing the ground truth.

They compare the mutual information between a baseline cluster algorithm using AMI [Huang et al., 2021] or NMI [Sun et al., 2022; Xia et al., 2020]. Another study for evaluating DP with AP uses both ARI and AMI. In addition to mutual information and rand index scores, it is also not uncommon to calculate the error between the two cluster algorithm's centroids [Huang et al., 2021; Xia et al., 2020]. These two studies used Relative Error (RE) for this.

2.3. LITERATURE REVIEW

The related literature is divided into three parts:

1. Differential privacy methods
2. Cluster methods with (L)DP

Afterward, we provide a summary for both in the form of a table. This table includes components such as the type (LDP or DP) and whether there is a public code available.

2.3.1. DIFFERENTIAL PRIVACY METHODS

As was discussed in earlier sections, the Laplace method was the first one to establish DP [Dwork, 2006]. In the search for related literature, research has mainly focused on algorithms that can be used for general applications. This includes investigating whether the mechanisms can also be applied to n-dimensional data, as is the case with our mechanism.

The first paper is provided by Soria-Comas et al. and considers the distribution of the dataset for the provided noise [Soria-Comas and Domingo-Ferrer, 2013]. It does however not depend on the data and does not calibrate sensitivity. Their work claims the Laplace mechanism is not optimal for a univariate function and aims at improving it by introducing their Laplace-based mechanism. The method that is proposed performs slightly better than Laplace. For multivariate/multiple queries the Laplace distribution underperforms.

Quan et al. also proposed a new method as an extension of Dwork et al.'s Laplace algorithm [**?**]. They introduced the staircase mechanism for 1-dimensional noise, which was later extended to support multidimensional data [Geng et al., 2015].

The mechanism aims to improve optimally by adding the same level of privacy while adding less noise. It is represented as a staircase-shaped probability density function, hence the name Staircase Mechanism (SM). This mechanism accepts three configurable parameters, which are comparable to those of the Laplace mechanism. The authors' work can handle multidimensional numerical data and preserve the pure version of differential privacy.

Another paper introduces a new local differential privacy (LDP) mechanism for working with numerical data [Nguyên et al., 2016] Their primary focus is on estimating means and frequencies, with a particular emphasis on machine learning techniques such as Support Vector Machines (SVM) and linear regression using Empirical Risk Minimization. Initially, the authors analyze Duchi et al.'s method [Duchi et al., 2013] and highlight several shortcomings. To address these issues, they introduce Harmony as a mechanism for LDP perturbation. Additionally, they extend other work (Bassily and Smith's method) to introduce a method for perturbing multi-dimensional categorical attributes. The authors then create a hybrid model by combining Duchi et al.'s method and Bassily and Smith's method to handle respectively numeric and categorical data. This allows them to compare their method Harmony to the hybrid mechanism and measure the utility/accuracy differences.

Duchi et al. improved their method by proposing a formalization of the trade-off between statistical utility and (local) privacy, analyzing multiple types of estimation problems [Duchi et al., 2017]. Examples include mean, median, and density estimation. To achieve this, they use minimax, a technique for finding the worst-case probability distribution [Minimax]. Additionally, they focus on existing work and propose several optimization strategies for it.

This work extends the method proposed in the previous paragraph by Duchi et al. by adding support for bounded and multidimensional data [Wang et al., 2019]. The authors introduce the Piecewise Mechanism (PM) to handle both numeric and categorical data and the Hybrid Mechanism (HM) which combines PM and Duchi et al.'s method for 1-dimensional data. The effectiveness of their method is demonstrated using Support Vector Machines (SVM), linear regression, and mean estimation. PM and HM are compared to Laplace, SCDF , and Duchi et al.'s solutions. OUE is also used for comparison, but for categorical data only, as this is not supported by the other methods.

2.3.2. Cluster methods with (L)DP

This chapter examines the various studies that have been conducted on clustering in combination with differential privacy. Initially, we looked at the most fundamental papers in this field. Subsequently, the focus shifted toward researching well-known papers that have been published since 2020.

The work that was proposed by Nissim et al. aims to improve differential privacy methods, such as Laplace, which use sensitivity to compensate the noise for a function. In addition to compensating the function, they also consider the dataset itself. The algorithm for this is called "smooth sensitivity" and is used for instance-specific noise. To apply it, the authors introduce a method/framework to effectively calculate it. To demonstrate the effectiveness of the method, they use K-means, among other cluster algorithms. Their method requires the calculation of cluster distances, using Wasserstein distance instead of Euclidean distance.

Another study focuses on both interactive and non-interactive approaches for differential privacy in K-Means [Su et al., 2015]. The study builds upon the work done for DPLloyd, an interactive privacy extension of K-Means described by Blum et al. [Blum et al., 2005]. The DPLLoyd mechanism partitions an n-dimensional dataset into a grid and releases the count for each grid by adding Laplacian noise to each count. Another part of their research focuses on determining the width of the data cells. The grid estimation method used in their research is called the extended uniform grid approach (EUG), and the complete K-Means method is called EUGkM. The experiment consists of comparing against the DPLloyd mechanism, which performs better for an interactive setting. Therefore, they combine their algorithm for combining both aspects into a hybrid approach (EUGkM + DPLloyd approach) and show a better final performance.

The study of Nissim et al. researches the idea of finding the smallest possible radius in the Euclidean space R^d for a set of n points [Nissim and Stemmer, 2018]. They propose a new solution that uses locality-sensitive hashing (LSH) for differential privacy, which they use to find 1-cluster in the d-dimensional Euclidean space. This method works for differential privacy (LSH-GoodCenter), but they also extend this to the local model (LDP-GoodCenter). The algorithm to find this radius is used to count the points enclosed by the radius and Laplace noise is added to the count to preserve differential privacy. This mechanism is combined and applied to work with the K-Means algorithm (LDP-K-Mean). This method was later extended in a paper proposed by Kaplan et al. and introduces a similar LDP method [Kaplan and Stemmer, 2018]. They aim to reduce the number of interactions needed between the server and users to one, instead of the $O(k \log n)$ required for Nissam et al.'s solution [Nissim and Stemmer, 2018]. To increase the success probability, they use the same idea but extend it to have multiple centers instead of one large one. They call it

the LSH-Procedure and the algorithm Private-Centers is applied to generate centers to use with K-Means. Then, they apply the same method to the LDP method that was also originally proposed by Nissam et al. (LDP-GoodCenter). The most recent work by Stemmer et al. focuses on improving the work that was done by Kaplan et al. [Kaplan and Stemmer, 2018; Stemmer, 2021]. Because this work has a higher additive error, which means the noise that is added introduces a lot of error. To solve this, the authors aim at reducing this error by improving the original GoodCenter algorithm [Nissim and Stemmer, 2018]. Their extended method is called WeightedCenters and adds weights to candidate centers. So, in addition to calculating the centroids for each iteration, it also calculates the weights for each. In the final iteration, the weights are used to create the K-Means or K-Median clusters.

Sun et al. proposed a method for distributed clustering using local differential privacy (LDP) to preserve distance-based information. They claim to have the first non-interactive LDP algorithm for clustering [**?**]. This means, they are being able to perturb the data locally at once and sent it to the server to cluster with both K-Means and DBSCAN. They encode the client-side data into an anonymous hamming space using Bit Vector (BV) and modify the encoding to preserve Euclidean distance. As their method only is be-able to share distance information they were not able to use K-Means directly. To overcome this, they modified the algorithm and called it K-Cluster. Finally, the method is evaluated using Normalized Mutual Information (NMI) and Average Estimated Error (AEE).

PrivBV

Xia et al. noticed the shortcoming of Sun et al.'s work which is the need to share privacy-sensitive distance information [Xia et al., 2020]. Therefore, they propose a new interactive method for distributed K-means clustering using LDP. The method converts features to binary strings and uses a Random Response mechanism (RR) to perturb each feature into a feature vector. The privacy cost depends on the length of the bits of each feature transformation, meaning that a longer length yields more information at the cost of the privacy budget. In each iteration, the server-side calculates and sends K-means centroids to each user, who recalculates distances until the centroids become stable. The approach has the disadvantage of a high correlation between user data and the clusters. To solve this problem, the algorithm is improved by having the client-side send not only the user data but also a set of random zero strings. The server side then performs similar calculations to determine the true cluster. Huang et al. propose a private distributed K-means clustering algorithm for interval data that addresses a shortcoming in Xia et al.'s work by using Condensed Local Differential Privacy (CLDP) for small-scale values and LDP for large-scale values [Huang et al., 2021]. They preserve distance using a Square Wave (SW) mechanism and apply a classical K-Means algorithm on the server side to the perturbed data.

Another more recent mechanism that also builds around K-Means to preserve LDP, is called the LDPK mechanism [Yuan et al., 2021]. As K-Means works only with numerical data, they use K-prototypes for supporting mixed data types. The LDPK mechanism perturbs the user data first locally and interactively exchanges information with the server to complete the clustering process. The mechanism they use for perturbation is the Harmony algorithm, which was proposed earlier by [Nguyên et al., 2016]. To also support categorical data the S-Hist method is used, which was also introduced by Nguyen et al. But the author replaces this algorithm with OUE Wang et al. to improve accuracy. Due to the correlation between the cluster centroids and the real data, the server could still infer the correct information. Therefore, the authors also disturb the user's cluster information with an extra

extension to the LDPK method, called ELDPK. To this end, they perturb the clusters with the GRR (Generalized Random Response) algorithm. Their evaluation focuses on the privacy budget and the amount of data points. They show that if the amount of data points increases the clustering quality does as well.

The conclusion from the previous paragraphs is that most of the related work focuses on variants of the K-Means algorithm. Finally, two interesting studies focus on differential privacy and AP or DBSCAN. A study conducted by Cai et al. focuses on AP [Cai et al., 2020]. Their method involves adding Laplace noise to the responsibility matrix. For each sample data, a neighborhood is specified using a radius around the data point. This area is called the neighborhood density, and each sample point's preference value is adjusted according to its density value. Higher density yields a higher chance of belonging to a cluster center and then being ranked based on size. The perturbed responsibility matrix and densities are combined and used to run AP. Finally, they evaluated their method using the ARI, Fowlkes-Mallows Index (FMI), and AMI.

Another recent study focuses on differential privacy for DBSCAN [Bozdemir et al.]. The proposed solution involves clustering data between two or more parties using two servers. Secure two-party computation (S2PC) is used to achieve this. Using S2PC, both servers receive a random-looking secret share. To recover the original data, both servers would need to combine their shares using S2PC, which combines the data without the servers having access to the full value. The proposed protocol is named privacy-preserving DBSCAN (ppDBSCAN). The calculations in this study are based on squared Euclidean distance (SED) and are evaluated using different methods. To evaluate the performance of ppDBSCAN, the study compares its Adjusted Rand Index (ARI) to that of K-means.

2.3.3. Summary

Short summary

year	Name	Data type	Dataset	Code implementations	preserving	Type	Interactive	Methods	Privacy
2022	PrivBV: Distance-aware encoding for distributed...	-	Synthetic dataset	-	Local differential privacy	K-Means	Non interactive	-	(ϵ, δ)-LD
2021	Private distributed K-means clustering on inter...	-	??	-	Local differential privacy	K-Means	Interactive	-	-
2021	Locally Private k-Means Clustering	numerical	-	-	Local differential privacy	K-Means	Interactive	-	-
2021	Privacy-preserving mechanism for mixed data clu...	n-dimensional numeric & categorical	Adult dataset, US Census dataset	-	Local differential privacy	K-Prototypes	Interactive	LDPK and ELDPK	LDP
2021	Privacy-preserving Density-based Clustering	-	Deer dataset, Lsun dataset, S1	-	Differential privacy	DBSCAN	-	ppDBSCAN	-
2020	DP-AP: Differential Privacy-Preserving Affinity...	-	Iris dataset, Seeds dataset	-	Differential privacy	AffinityPropagation	-	DP-AP	-
2020	Distributed K-Means clustering guaranteeing loc...	n-dimensional numerical data	3D Road Network, CarGPS	-	Local differential privacy	K-Means	Interactive	LDPKmeans	-
2019	Distributed Clustering in the Anonymized Space...	n-dimensional numerical data	Aggregation dataset, Digit dataset, Pathbased d...	-	Local differential privacy	DBSCAN, K-Means	Non interactive	Distance Aware Bit Vector (DPBV)	-
2018	Clustering algorithms for the centralized and l...	n-dimensional numerical data	-	-	Local differential privacy	K-Means	Interactive	LDP-GOODCenter	(ϵ, δ)-LD
2018	Differentially private K-means with constant mu...	-	-	-	-	K-Means	Interactive	- LSH-Procedure - Private-Centers	(ϵ, δ)-DP
2015	Differentially Private k-Means Clustering	2 - 10-dimensional numerical data	Adult dataset, Gowalla dataset, Image dataset, ...	https://github.com/DongSuIBM/PrivKmeans	Differential privacy	K-Means	Both	EUGkM and hybrid EUGkM + DPLloyd	-
2011	Smooth sensitivity and sampling in private data...	n-dimensional numeric	-	-	Local differential privacy	K-Means	Non interactive	Smooth sensitivity for K-Means clustering	(ϵ, δ)-LD

Table 2.1: Summary table of the literature review for (L)DP clustering algorithms.

year	Name	Data type	Dataset	Code implementations	preserving	Type	Interactive	Methods
2019	Collecting and Analyzing Multidimensional Data ...	n-dimensional (PM), but HM is 1-dimensional and...	BR, MR	-https://github.com/forestneo/sunPytools/blob/m...	Local differential privacy	Differential privacy method	-	- Piecewise Mechanism (PM) - Hybrid Mecha
2017	Minimax optimal procedures for locally private ...	1-dimensional numerical data	-	https://github.com/forestneo/sunPytools/blob/ma...	Local differential privacy	Differential privacy method	-	-
2016	Collecting and analyzing data from smart device...	numerical, binary and categorical data. Domain ...	BR (4M records 12 categorical and 6 numeric), U...	-	Local differential privacy	Differential privacy method	-	Harmony
2015	The staircase mechanism in differential privacy	n-dimensional numerical data	-	https://github.com/IBM/differential-privacy-lib...	Differential privacy	Differential privacy method	-	Staircase mechanism (SM)
2013	Optimal data-independent noise for differential...	n-dimensional	-	-	Differential privacy	Differential privacy method	Non interactive	-

Table 2.2: Summary table of the literature review for (L)DP algorithms.

3

nD-Laplace

3.1. 2D-Laplace

The theory for this subject is inspired by the paper that was written by Andrés et al. [Andrés et al., 2012]. This notion of GI was introduced to solve the issue of privacy and location data. It offers an alternative approach for differential privacy by adding noise to the location locally before sending it to a location-based system (LBS) like Google maps. This section starts with an introduction to mathematics for the planar and polar Laplace algorithm. For each of the different subsections, we visualize and explain open challenges and theoretic for applying them for clustering.

Math symbols

θ Angle.

l Privacy level.

r Radius.

The other symbols can be found in section 2.1.

Geo-indistinguishability

As mentioned in the previous section, the GI method can be applied to preserve the privacy using a differential privacy method specific to spatial data. The formula to measure if an algorithm preserves ϵ-geo-indistinguishability can be expressed as [Andrés et al., 2012]:

$$K(x)(y) \leq e^{\epsilon * d(x,x')} K(x')(y) \tag{3.1}$$

Where K is a probability method reporting $x, x' \in X$ as $z \in Z$. The idea of this algorithm looks a lot like that of differential privacy using the La Place method; but includes distance. The intuition for this is that it displays the distinguishability level between two secret locations/points x and x' [Chatzikokolakis et al., 2015]. An extension of this is called d_x-privacy and is a more general notation of distance-aware differential privacy. Their definition for GI is, therefore, d_2-privacy, but is essentially the same as the proof provided for GI.

3.1.1. Planar and polar Laplace

The idea of planar Laplace is to generate an area around $x0 \in X$ according to the multivariate Laplace distribution. The mechanism of planar Laplace is a modification of the Laplace algorithm to support distance [Andrés et al., 2012]. This distance method $dist(x, x')$ is defined as the Euclidean distance between two points or sets. Recalling the definition of

Laplace, this method $|x - x\cdot|$ is replaced by the distance metric. Hence, the definition of the Probability Density Function (pdf) by Andrés et al. is:

$$\frac{\epsilon^2}{2 * \pi} e(-\epsilon d(x_0, x)) \tag{3.2}$$

Which is the likelihood a generated point $z \in Z$ is close to $x0$. The method works for Cartesian coordinates but was modified to support polar coordinates by including θ. So each point is reflected as (r, θ) and can be modified by using a slight modification to work for polar Laplace. A point $z \in Z$ where $z = (r, \theta)$ is randomly generated using two separate methods for calculating r and θ.

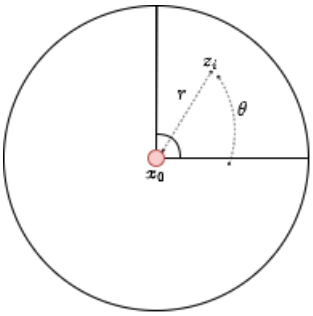

Figure 3.1: Representation of the generated $z = r\theta$ and original point $x0$.

Calculating r: This variable is described as $dist(x_0, z)$ and can be randomly drawn by inverting the CDF (**Link**) for the Laplace distribution:

$$C_\epsilon{}^{-1}(p) = -\frac{1}{\epsilon}(W\_1(\frac{p-1}{e}) + 1) \tag{3.3}$$

For this equation, W_1 is a Lambert W function with -1 branch. The Lambert w function, also called the product logarithm is defined as $W(x)e^{W(x)} = x$ [Lehtonen, 2016]. The purpose of the Lambert w function is to invert the CDF of the Laplace distribution to generate random noise for one of the coordinates (r) using the random value of p.

Calculating θ: The other coordinate (θ) is defined as a random number $[0, 2\pi]$.

To visualize these methods it is necessary to convert the polar coordinates for $z = (r, \theta)$ back to a plane (x, y). This is described as step 4 of the planar Laplace algorithm [Andrés et al., 2012] and visualized using figure **??**.

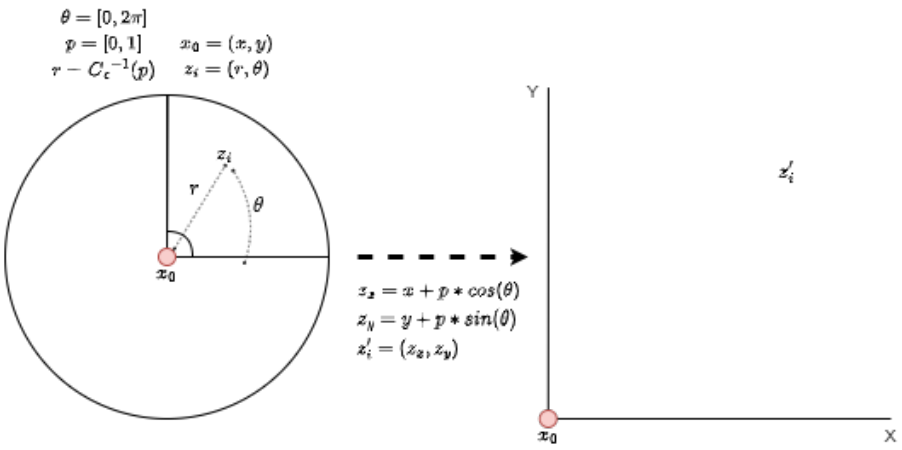

Figure 3.2: Representation of converting the perturbed point $z = (r, \theta)$ to a point z_x, z_y

3.1.2. TRUNCATION

Because we have a finite space, it can be possible the perturbed points are off-graph (outside the given domain). The solution was described in step 5 of the Laplacian mechanism for 2D space. This explains the idea of remapping to the closest admissible location in set A. For which $A \subset \mathbb{R}$, where A is the set of admissible locations [Andrés et al., 2012]. This is also described by chatzikokolakis et al, who also describes a method to do it. When a perturbed point z is located at the sea or in water, it is easily distinguishable as a fake location. They introduce a method to check this and efficiently remap to a nearby location.

Describe the method

Analyze other methods

3.1.3. OPTIMIZING FOR CLUSTERING

The decision of the parameters for the algorithm is straightforward as it depends on the ϵ. This constant is calculated by defining the radius r and the desired level of privacy l and ϵ is calculated using l/r. The l is a predefined constant $l \in R^+$ but usually will be below 10. For geographical data, the r can be configured by using meters as a unit of measure. Therefore, $r = 200$ corresponds to a radius of 200m around point x_0. So, regarding clustering, it is a challenge to define a reasonable radius.
The ϵ can be considered the inverse unit of r [Andrés et al., 2012]. A radius can be defined per-use case based on how crowded a place is [Chatzikokolakis et al., 2015].

Give the algorithm

A drawn area as shown in **??** can be expressed as a perturbation area P_{area} [Yan et al., 2022]. This metric was formulated as:

$$P_{area} = \left\{ center = x_0, radius = \frac{1}{N} \times \sum_{i=1}^{N} r_i \right\} \tag{3.4}$$

The method loops through each perturbed point r on center x_0 (recall **??**) and calculates the Euclidean distance for an n amount of perturbation points. Although the method does not contribute to the Laplace algorithm, it is useful for visualization purposes.

3.2. 3D-Laplace

The previous chapter focused on the effects of 2-dimensional noise on geographical data. This approach has recently been extended to support 3-dimensional data, which benefits indoor navigation [Min et al., 2022]. The method is similar to the 2D approach but includes the azimuth angle ψ, in addition to the polar angle θ and radius r.

3.2.1. Geo-indistinguishability

To establish the same privacy guarantees for 3-dimensional data as for 2-dimensional data, the original equation 3.1 is extended [Min et al., 2022].

$$K(x_1)(z) \leq e^{\epsilon * d_3(x_1,x_2)} K(x_2)(z) \tag{3.5}$$

Where x_1 and x_2 are two real data points in the same dataset X. The more x_1 and x_2 are similar, the more the perturbed location distributions $K(x_1)(z)$ and $K(x_2)(z)$ need to be similar.

3.2.2. Spherical Laplace

The implementation of Min et al. projects the dimensions onto a sphere instead of a circle [Min et al., 2022]. This sphere is a unit sphere calculated with a radius of 1. Based on this sphere the polar angle θ and azimuth angle ψ are randomly calculated.
Calculating θ and ψ: Both are drawn from the unit sphere using the following equations:

$$\theta = \frac{1}{\pi} \tag{3.6}$$

$$\psi = \frac{1}{2\pi} \tag{3.7}$$

The tuple $U = (\theta, \psi)$ is randomly selected based on the uniform distribution of the unit sphere [Min et al., 2022].
Calculating r: The radius r (distance from the center) is calculated using the following equation:

$$r = \frac{1}{2}\epsilon^3 * r^2 * e^{-\epsilon * r} \tag{3.8}$$

Where the gamma scale is the same as for 2D-Laplace, but with a shape of 3 instead of 2. The noise is added to the original location x to obtain the perturbed location z: $z = x + U * r$. A clear example of the noise that is generated by this method is shown in figure 3.3. Finally, we convert this to the Cartesian coordinate system to obtain the final location z:

$$\begin{aligned} z_x &= r * \sin(\theta) * \sin(\psi) \\ z_y &= r * \sin(\theta) * \cos(\psi) \\ z_z &= r * \cos(\theta) \end{aligned}$$

This is also visualized in figure 3.4.

3.2.3. Truncation

Todo: Add truncation to 3D-Laplace

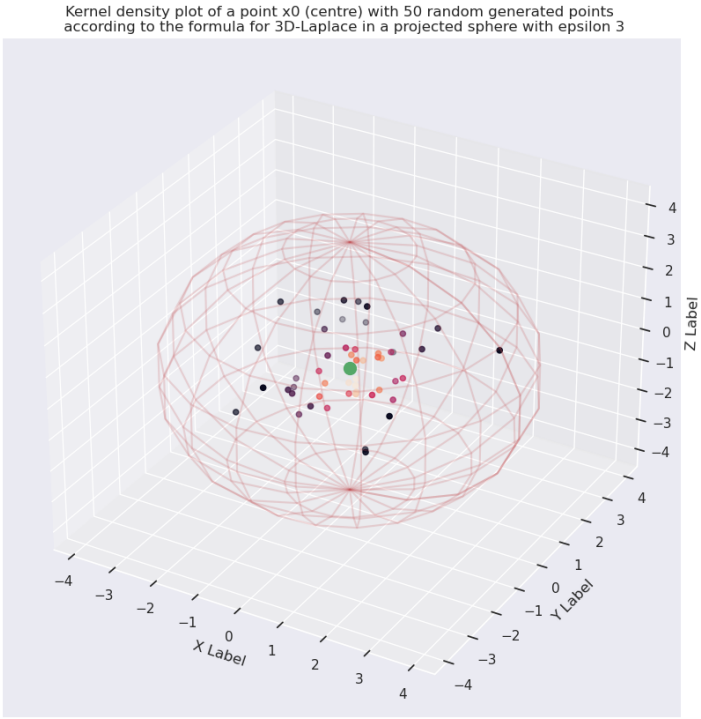

Figure 3.3: 50 random noise samples generated around point x_0 (green dot) using the 3D-Laplace noise method [Min et al., 2022] plotted on a sphere.

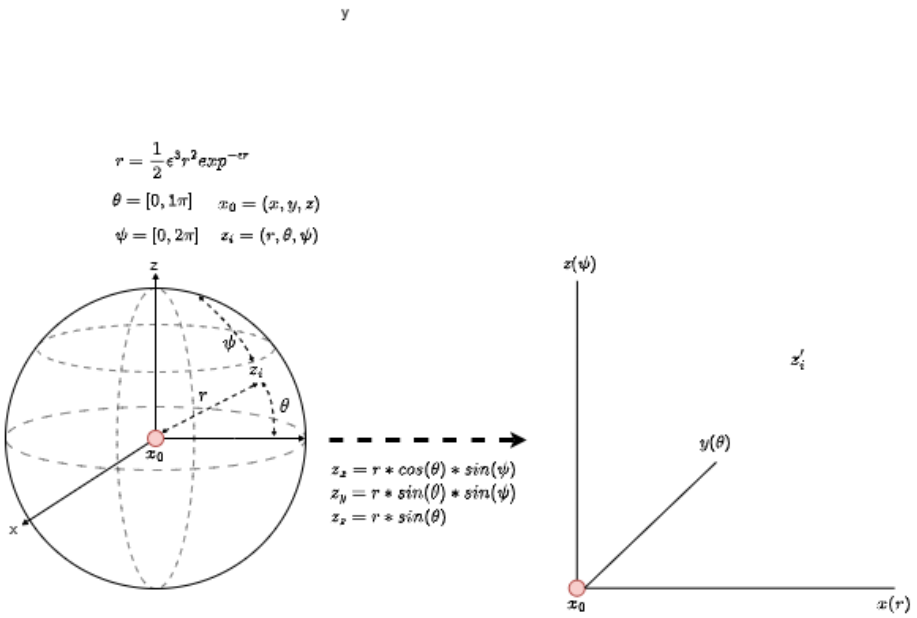

Figure 3.4: 3D-Laplace noise distribution according to the method proposed by Min et al. [Min et al., 2022]

3.3. nD-Laplace

As mentioned in the previous chapter, the paper that was introduced by Min et al. is be-able to handle 3-dimensional data. A small recap: a point (r, θ, ψ) gives us the spherical coordinates of a given 3-dimensional sphere. An important property for this is the fact that each of these coordinates can be generated separately [Andrés et al., 2012; Min et al., 2022]. The r gives us the radius or distance from (θ, ψ) to the center of the sphere [1]. So, instead of having just these two coordinates, we are be-able to extend this to n-dimensions by considering an n-hypersphere [Fernandes et al., 2019; Min et al., 2022]. To this end, besides points θ and ψ we also consider $\theta \in S^n$, where S is a unit hypersphere.

The first step to generate the noise is first to select the r again. This method is almost the same as for 3-dimensional . But, instead of applying a scale of 3, the scale will be n for the number of dimensions in the data [Fernandes et al., 2019]: link to formula

$$\gamma(n, 1/\epsilon) \tag{3.9}$$

For the other dimensions, we consider a vector $U = (\theta_1, \theta_2, \theta_n)$ which is uniformly selected based on a unit n-hypersphere S^n [Fernandes et al., 2019]. For this, we consider the work that was proposed by Marsaglia et al. for 4-sphere that can be used for selecting points from an n-hypersphere [Marsaglia, 1972]:

$$(2x_1(1-S)^{\frac{1}{2}}, x_2[(1-S)]^{\frac{1}{2}}, 1-2S) \tag{3.10}$$

Where S is the surface of the sphere and $x_1, x_2, \ldots, x_n$ are random variables generated using the Gaussian distribution [2]. For clarity, this is projected using the following algorithm:

Formula

3.3.1. Cartesian coordinates

As with the 2/3D-Laplace, the spherical coordinates need to be converted to Cartesian to be-able to cluster. It is comparable to the way we did it in the previous chapters, however as there are several more angles the equation is repeated and slightly different:

$$\begin{aligned} x_1 &= r * cos(\theta_1) \\ x_2 &= r * sin(\theta_1) * cos(\theta_2) \\ x_n &= r * sin(\theta_1) \ldots sin(\theta_{n-2}) * cos(\theta_{n-1}) \\ x_n &= r * sin(\theta_{n-1}) * sin(\theta_{n-2}) * sin(\theta_{n-1}) \end{aligned}$$

If we combine sections 1 and 2 of this chapter, we are being able to give a good overview of the solution using a similar image as for the 2D and 3D variants.

3.3.2. Curse of dimensionality

The algorithm shows some interesting behavior. If we continue adding dimensions, we notice the noise is shrinking proportionally. To understand this behavior, we first have to examine the formula for a hypersphere's volume.

$$S_n = \frac{2\pi^{n/2}}{\gamma(\frac{1}{2}n)} \tag{3.11}$$

Where γ is the gamma distribution that is determined based on the number of dimensions n [3]. As the amount of dimensions increases, the most volume is located on the hypersphere surface. The decreasing amount of volume is illustrated using this figure: When we convert the points to Cartesian coordinates, some will be located at the center (e.g., 0.5), while others will be close to the surface (e.g., 0.0). However, as the number of dimensions increases, the majority will be close to the surface (e.g., 0.99).

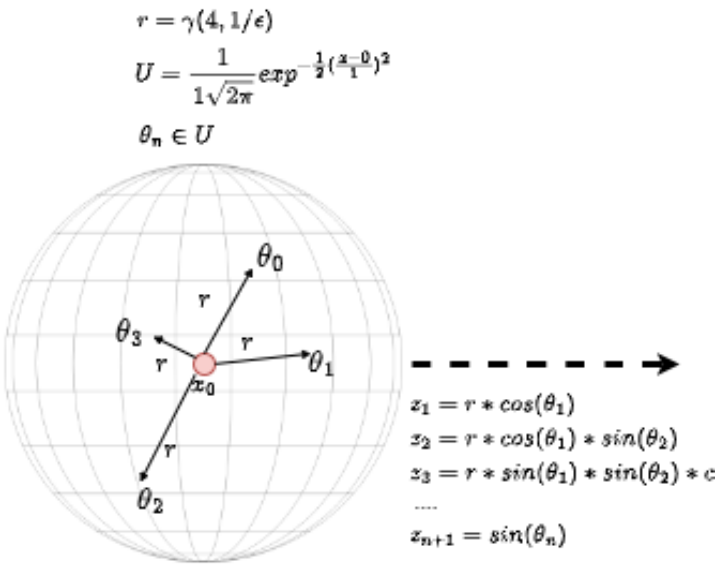

Figure 3.5: Overview of the nD-Laplace mechanism

This is a known challenge in machine learning , as well as for the nD-Laplace mechanism. Solutions for this can be using Principal Component Analysis (PCA) [Gorban et al., 2020]. This technique is used to reduce dimensionality by reducing the number of features while preserving the information. It uses the correlation between the features to contain that information.

specify

3.3.3. TRUNCATION

[1]https://mathworld.wolfram.com/SphericalCoordinates.html
[2]https://mathworld.wolfram.com/HyperspherePointPicking.html
[3]https://mathworld.wolfram.com/Hypersphere.html

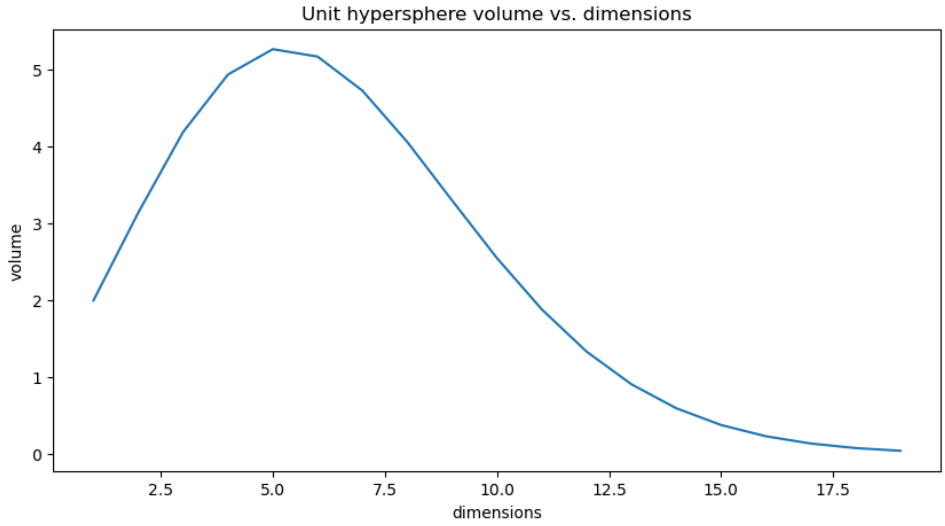

Figure 3.6: Illustration of the decreasing volume while increasing the number of dimensions

4

ATTACKS ON PRIVACY

This chapter focuses on the prevalent attacks that machine learning algorithms face. Towards the end, we will assess their impact on differential privacy and discuss evaluation methods.

4.1. MEMBERSHIP INFERENCE ATTACKS

An attack model that plays a big role in machine learning is a membership inference attack (MIA). With this attack, an adversary attempts to infer the training data $x \in X$ (member) from a given data point $z \in Z$ (non-member). The attack happens exclusively on supervised learning models, which either predict labels or probabilities. Most attacks on models trained on a centralized dataset occur during the inference phase, where the trained model is used to make predictions. [Rigaki and Garcia, 2021]. This is also why we are primarily interested in this phase, as we are not using a distributed learning model.

MIA depends on the knowledge of the attacker (adversarial knowledge), which can be divided into white-box and black-box approaches [Hu et al., 2022].

1. **White-box**: The attacker has all the data that is needed. Including target model parameters, the training dataset and even the architecture [Hu et al., 2022].

2. **Black-box**: The attacker has a limited amount of information, like training data distribution and the trained model [Hu et al., 2022].

The most well-known member inference attack is training shadow models [Rigaki and Garcia, 2021]. In this attack, an attacker trains multiple models. These models do not necessarily have to be the same as the original model, and the focus is mainly on the data input/output. It is a black-box attack, but often the attacker also needs knowledge of the data distribution to create a good shadow dataset [Rigaki and Garcia, 2021].

One of the earlier works that used this attack was Shokri et al. [Shokri et al., 2017]. An attacker trains multiple models (shadow model), with as goal to overfit the original modal. This idea is based on the fact that the model gives higher scores to the data on which it was trained (overfitting). Using this approach, attackers can retrieve the training data (member data) from the model by injecting a large amount of fake data (non-membership data).

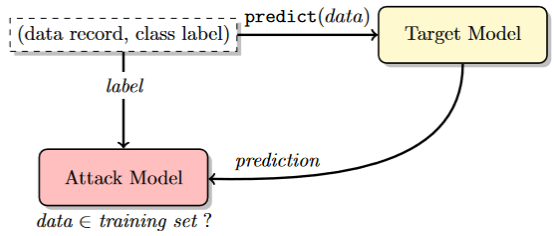

Figure 4.1: Black-box MIA attack on a machine learning model [Shokri et al., 2017]

Another approach to a black-box attack was introduced by Peng et al. and only considers that the attacker has access to the already trained model. They rescale the probabilities first using temperature scaling, to compensate for models that are overconfident

[Peng et al.]. So instead of having a probability between two classes with for example 99% against 1% it will be more evenly distributed based on the training data. They then proceed in clustering the probabilities into two clusters using K-Means and label the higher confidence scores as members.

The above attacks do rely on the model to also provide the confidence or probabilities of the predictions. This is often not the case for the practical appliance of a model, and therefore Choquette-Choo et al. introduced a label-only attack. While the existing models exploit the probability output for MIA, they solely rely on labels [Choquette-Choo et al., 2021]. For this, they make use of the "HopSkipJump" attack; a so-called decision-based attack [Chen et al., 2020]. Choquette-Choo et al. consider a more semi-black-box approach, for which the attacker still requires access to a subset of the original training data and the trained model. Another paper that also uses "HopSkipJump" requires only the trained model and achieves higher accuracy by using an approach with random data [Li and Zhang, 2021].

Access to only the output of the model is a typical characteristic of black-box attacks. If the attacker also knows architecture, for example, it is referred to as a white-box attack. Another take on this is prediction and confidence-based MIA which are both proposed by [Yeom et al., 2018]. They assume that an attacker knows the standard error and has access to the perturbation dataset. The algorithm is then able to extract the truth label by minimizing the loss.

4.2. Reconstruction attack

This attack is a threat, especially to differential privacy. The attack is primarily focused on reconstructing the data rather than focusing on machine learning.

Do research for this attack

4.3. Model inversion attack

Do research for this attack

4.4. Attack evaluation

In this section, we compile a list of attacks and evaluate which attacks are best suited for adoption in this research. We also assess whether differential privacy provides protection for each attack and discuss how this can be measured.

4.4.1. Member inference attacks

Most current research for MIA is evaluated for neural networks [Rigaki and Garcia, 2021]. Just a small percentage evaluates this attack for supervised learning, with the majority using classification with decision trees. For these attacks, most studies have used a black-box approach [Rigaki and Garcia, 2021]. This is not surprising, as these attacks have a high success rate and pose a greater risk of being exploited.

The introduction of differential privacy reduces the impact of a member inference attack [Hu et al., 2022; Rigaki and Garcia, 2021]. This is because the input to the model is perturbed. While it is still possible to retrieve the training data, the leaked privacy is significantly reduced. The leaked privacy can be measured using a metric called "adversarial advantage." This metric describes the percentage of privacy that is compromised in the event of a member inference attack [Yeom et al., 2018]. This is calculated by subtracting the False Positive Rate from the True Positive Rate. The TPR represents the number of correctly predicted member data (training data) and the FPR represents the number of correctly predicted non-member data.

In conclusion, the attacks that use member inference attacks are all based on supervised machine learning. However, in this study, we use cluster algorithms. Therefore, a semi-supervised approach can be used, as illustrated in this figure:

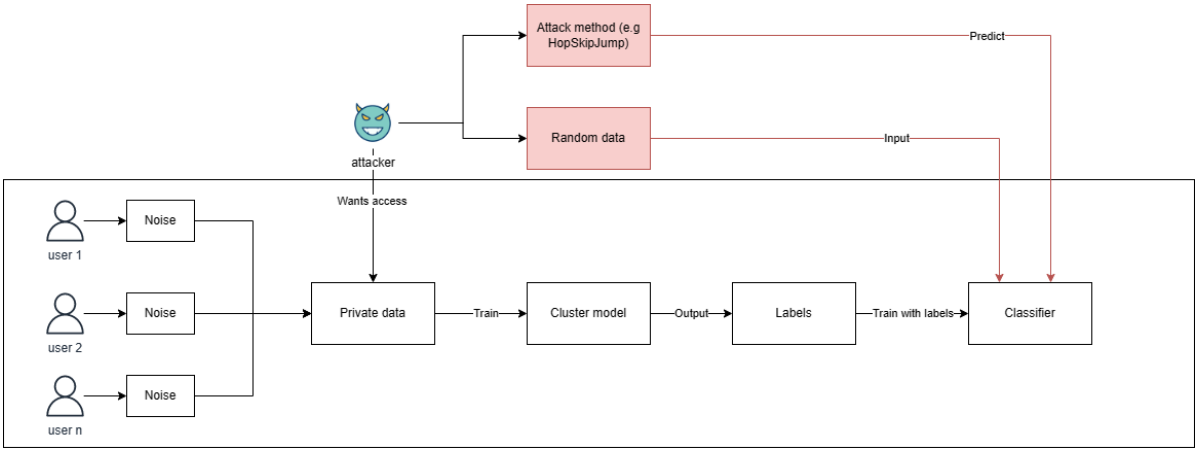

Figure 4.2: Semi-supervised black-box approach to execute a member inference attack.

Reconstruction attacks:

Evaluate after we did research for reconstruction attacks

Model inversion attacks:

Evaluate after we did research for model inversion attacks

5
METHODOLOGY

To gain insights into the proposed methods for researching the appliance of (ND)-Laplace for cluster algorithms we conducted experiments. The experiment results are used to evaluate our method against other literature. In this chapter, we explain:

1. Datasets
2. Environmental setup.
3. For each research question: Description of the different experiments.
4. For each research question: Results.

5.1. DATASETS

For this research, we will use a synthetic dataset for all three research questions. Research question 3 uses a "real-world" dataset to properly assess the different dataset properties that are the subject of this research question.

Dataset	Records	Centers	Dimensions	Standard deviation	Research
1	50	4	2	0.60	RQ 1
2	50	4	3	0.60	RQ 2
3	50	4	5	0.60	RQ 2

5.2. ENVIRONMENTAL SETUP

For running the experiments we make use of 16GB ram memory and i7-10750H 2.6Ghz processor. The experiments are run using a Docker container which runs a pre-configured distribution of Linux Alpine. It includes a pre-installed Anaconda environment for python [1,2]. We run the container using the dev-container feature for visual-studio code [3]. This allows us to create a reproducible experiment environment.

[1] https://github.com/devcontainers/images/tree/main/src/anaconda
[2] tag: mcr.microsoft.com/devcontainers/anaconda:0-3
[3] https://code.visualstudio.com/docs/devcontainers/containers

5.2.1. Libraries & code versions

We use python version 3.9.13 with Jupyter notebook for creating a reproducible experimental environment. The packages for python are:

1. Scikit-learn: 1.0.*
2. Yellow-brick: 1.5
3. Numpy: 1.24.*
4. Pandas: 1.4.*
5. Seaborn: 0.11.*
6. Mathplotlib: 3.5.*

5.3. Methods

This section explains what methods/ algorithms we used and how we evaluate them.

5.3.1. Clustering methods

For the three different algorithms: K-Means, AP and DBSCAN we analyzed the most important decisions regarding parameter selection. In this section, we give a short list and explanation of the different parameters we used throughout the experiments. For all three Scikit-learn was used, and for each of them we also provide the underlying formula.

K-Means

$$\sum_{i=0}^{n} \min_{\mu_j \in C} (||x_i - \mu_j||^2) \tag{5.1}$$

Parameter	Description	Value	Dataset
K-value	Calculated based on an "elbow" plot.	4 (see figure 7)	Dataset 1
K-value	TODO	??	Dataset 2
K-value	TODO	??	Dataset 3

Table 5.1: K-Means hyperparameters for dataset 1 - 3

Affinity Propagation

As specified in section 2.2.1, the clustering algorithm has two types of similarity. The responsibility is calculated by the following formula:

$$r(i,k) \leftarrow s(i,k) - max[a(i,k') + s(i,k') \forall k' \neq k] \tag{5.2}$$

Then the availability is given using this formula:

$$a(i,k) \leftarrow min[0, r(k,k) + \sum_{i' \ s.t. \ i' \notin \{i,k\}} r(i',k)] \tag{5.3}$$

And iteratively calculated while considering the damping factor.

Parameter	Description	Value	Dataset
Preference	We decided to use the median similarity as described in section 2.2.1	TODO	dataset 1
Preference	""	TODO	dataset 2
Preference	""	TODO	dataset 3
Damping factor	Default value as specified in section 2.2.1	0.5	dataset 1
Damping factor	""	0.5	dataset 2
Damping factor	""	0.5	dataset 3

Table 5.2: Affinity Propagation hyperparameters for datasets 1 - 3

DBSCAN

Parameter	Description	Value	Dataset
Epsilon	Decided using the k-distance plot	0.9 (see figure 8)	Dataset 1
Epsilon	""	TODO	Dataset 2
Epsilon	""	TODO	Dataset 3
Minimum points	Decided using the formula $minPts = n * 2$, where n is the number of features (2.2.1)	4	Dataset 1
Minimum points	""	6	Dataset 2
Minimum points	""	10	Dataset 3

Table 5.3: DBSCAN hyperparameters for datasets 1 - 3

5.3.2. Evaluation

With differential privacy, it is a trade-off of utility versus privacy. Therefore, for the evaluation of the 2D/3D-Laplace algorithms, we compare both criteria to achieve a consensus between utility and privacy. To reduce the measurement bias of results we executed them 10 times for multiple privacy budgets and report the average for each [Huang et al., 2021].

1. All experiments are performed 10 times and the average is reported.
2. All experiments are performed per privacy budget (epsilon). We have a fixed list for this: $0.05, 0.1, 0.5, 1, 2, 3, 5, 7, 9$.

Utility

Based on section 2.2.2, we can conclude that the corresponding literature mainly evaluates one clustering algorithm and not multiple ones. Furthermore, it can be concluded that if we only want to measure the coherence of the clusters, we can use an internal validation method. If we want a concrete measurement compared to the non-private version, an external validation method can be used. Both measurements are important to evaluate, so we use both external and internal validation.

External validation: We will use both ARI and AMI different strengths we evaluate both. For the validation we want to validate how much utility we lose using our method for a given privacy budget (ϵ). Therefore, we evaluate smaller sets of data to measure this for local perturbation. To compensate for this, the adjusted version is used for both Rand Index and Mutual Information. The implementation for these metrics is provided by the Scikit-learn package. With the underlying formulas:

$$AMI(U,V) = \frac{MI(U,V) - E(MI(U,V))}{avg(H(U),H(V)) - E(MI(U,V))} \tag{5.4}$$

Adjusted Mutual Information formula [Hubert and Arabie, 1985; Vinh et al.]

$$RI = \frac{a+b}{C_2^n} \tag{5.5}$$

$$ARI = \frac{RI - E(RI)}{max(RI) - E(RI)} \tag{5.6}$$

(Adjusted) Rand Index formula [Hubert and Arabie, 1985; Rand, 1971]

Internal validation: To evaluate the cluster algorithms, we use CHI and silhouette score. The expectation is that both will give a similar result, but they measure the cluster coherence in different ways.

Include formulas

PRIVACY

For this reason, we evaluate the privacy provided by the differential privacy algorithm by calculating the average (Euclidean) distance difference in comparison to the non-private data. Then, we evaluate the privacy method by simulating a membership inference attack and calculating the adversary advantage.

Privacy distance: We calculate the Euclidean average difference between non-perturbed data and the perturbed data. This is measured for each epsilon.

Membership inference attack (adversary advantage): We used the MIA that was proposed by Shokri et al. with the implementation that was provided by Adversarial Robustness Toolkit (ART) [noa, 2023]. An earlier study also explored this attack with as goal to evaluate differential privacy. Similar to this attack, we train a classifier with perturbed data and evaluate it using non-perturbed data (test-data / shadow-data) [Zhao et al., 2020]. In this way, we can demonstrate privacy compared to if we had not applied differential privacy. For the classifier, we use a RandomForestClassifier, as this classifier is not yet applied for shadow model attacks [Rigaki and Garcia, 2021]. We consider a semi-surpervised setup, so we generate the labels while using the K-Means algorithm. The 50 as the number of data samples is not enough to measure MIA. This is modified to 2000 samples.

Finally, we evaluate the adversary advantage (percentage) using $TPR - FPR$.

1. TPR: The amount the attacker inferred the member data correctly (the private training data).

2. FPR: The amount the attacker inferred the non-member data correctly (the non-private test data).

A visual setup of the experiment is given in figure: 5.1

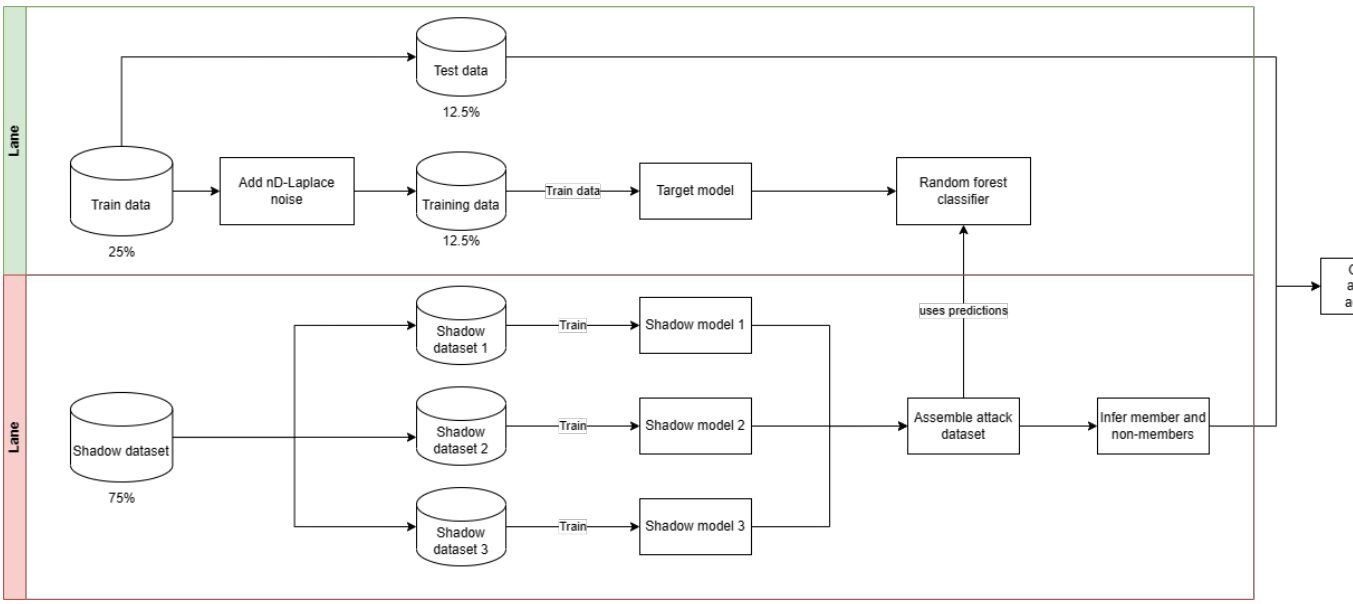

Figure 5.1: Member inference attack using shadow models. The green swim lane illustrates the normal setup and the red swim lane projects the adversary steps.

5.3.3. SCALING

Because we use a distance metric, we need to apply some data standardization. For this purpose, we use standard scaling provided by the Scikit-learn package [4]. This is only for clustering, so it is applied after all the perturbation algorithms.

5.3.4. RESEARCH QUESTION 1

TRUNCATION:

We explained the theory for truncation earlier in paragraph 3.1.2. The methods proposed work correctly for a geographic map where other (historic) locations for remapping are available.

However, it is difficult to apply this to data clustering. The number of data points is not known beforehand, so we may remap to a location that is too far away. This way we lose important distance information, which hurts the clustering. Also, the truncation threshold is so clear (the points are outside the known 2D domain), that we do not have to rely on historical data for remapping. Our algorithm can be much simpler by re-calculating the noise until it will be within the domain:

Algorithm 1 Truncation algorithm ($T(\min, \max, x_0, z)$) for clustering with planar Laplace

Ensure: z
  $x_1, y_1 \leftarrow x_{min}$
  $x_2, y_2 \leftarrow x_{max}$
  $z_x, z_y \leftarrow z$
  if $x_1 < z_x < x_2$ and $y_1 < z_y < y_2$ **then**
    return z
  else
    $x, y \leftarrow x_0$
    $z_2 \leftarrow LP(\epsilon, x, y)$ ▷ See formula 3.3.
    return $T(x_{min}, x_{max}, x_0, z_2)$ ▷ Rerun recursively
  end if

This algorithm uses x_{min} and x_{max} to re-calculate the points within the domain using respectively the minimum X/Y and maximum X/Y. An example of this is visualized:

[4] https://scikit-learn.org/stable/modules/preprocessing.html

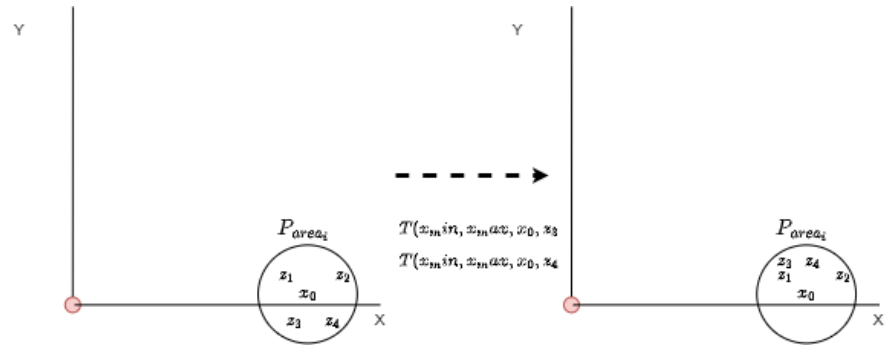

Figure 5.2: Representation of the remapping algorithm for clustering for points z_3 and z_4

ALGORITHM
The full algorithm for the perturbation:

Algorithm 2 Full algorithm for perturbing cluster data based on planar/2D-Laplace [Andrés et al., 2012]

Require: $x \in X$ ▷ 2D array of points
Require: $l \in R^+$
Ensure: $z \in Z$ ▷ 2D array of perturbed points
  $r = \frac{\sigma}{2}$ ▷ formula 4.1
  $\epsilon = \frac{l}{r}$ ▷ Calculating privacy budget [Andrés et al., 2012]
  $x_{min} \leftarrow min(X)$
  $x_{max} \leftarrow max(X)$
  $Z \leftarrow []$
  for $point_i \in X$ **do**
    $\theta \leftarrow [0, \pi 2]$ ▷ Random noise for θ
    $p \leftarrow [0, 1]$
    $z_i \leftarrow {C_\epsilon}^{-1}(p)$ ▷ formula 3.2
    $z_i \leftarrow T(x_{min}, x_{max}, point_i, z_i)$ ▷ algorithm 1.
    $x_{perturbed} \leftarrow point_{i_x} + (z_{i_x} * \cos(\theta))$ ▷ add noise to x-coordinate
    $y_{perturbed} \leftarrow point_{i_y} + (z_{i_y} * \sin(\theta))$ ▷ add noise to y-coordinate
    append $x_{perturbed}, y_{perturbed}$ to Z
  end for
  return Z

5.3.5. RESEARCH QUESTION 2

Starts after RQ1

5.3.6. RESEARCH QUESTION 3

Starts after RQ2

5.4. Results

5.4.1. Research question 1

For research question 1 the results are 2-dimensional plotted using a line diagram.

Utility

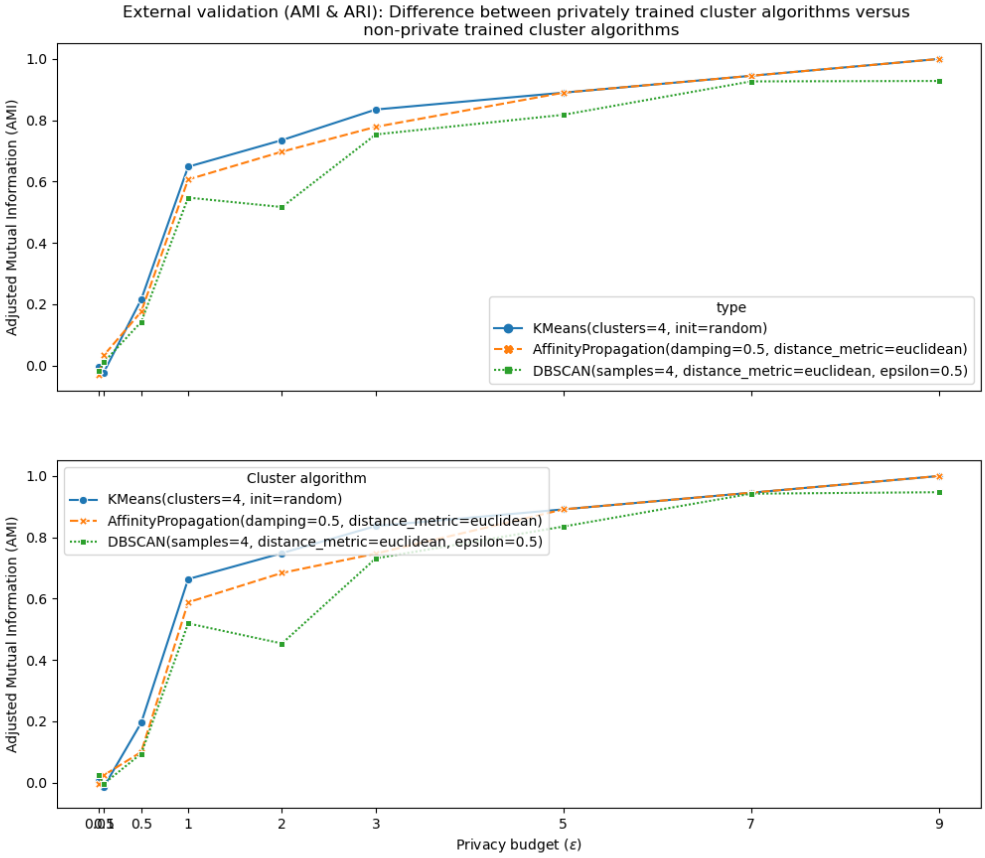

Figure 5.3: ARI and AMI evaluation for cluster algorithms 2D-Laplace for a dataset with shape (50, 2)

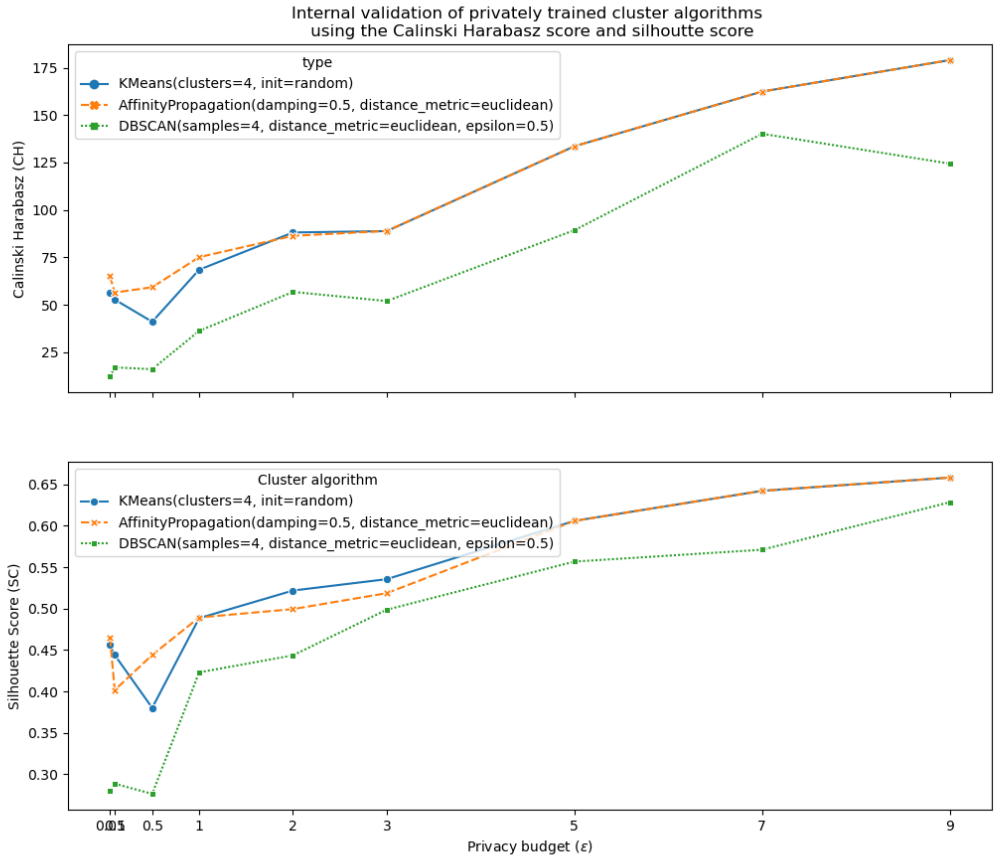

Figure 5.4: SH and CH for cluster algorithms trained with 2D-Laplace for a dataset with shape (50, 2)

PRIVACY

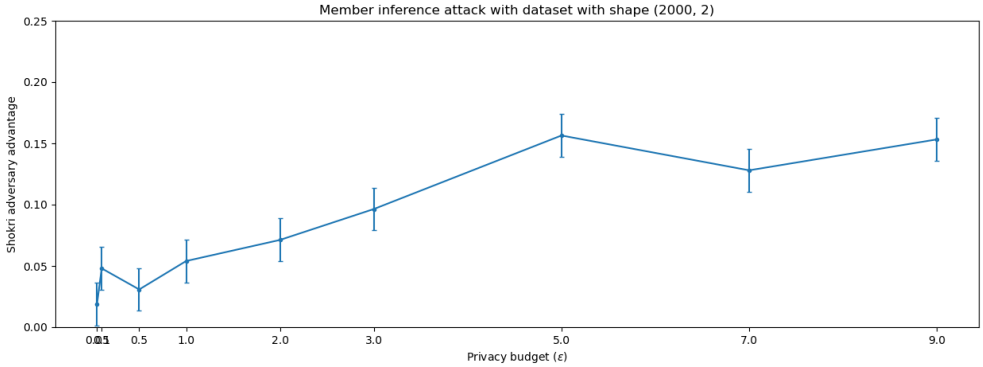

Figure 5.5: Shokri et al. attack mechanism using 3 shadow models and Yeom et al's adversary advantage ($TPR - FPR$) calculated per epsilon. (The error bar illustrates the fluctuation in the results using the standard deviation).

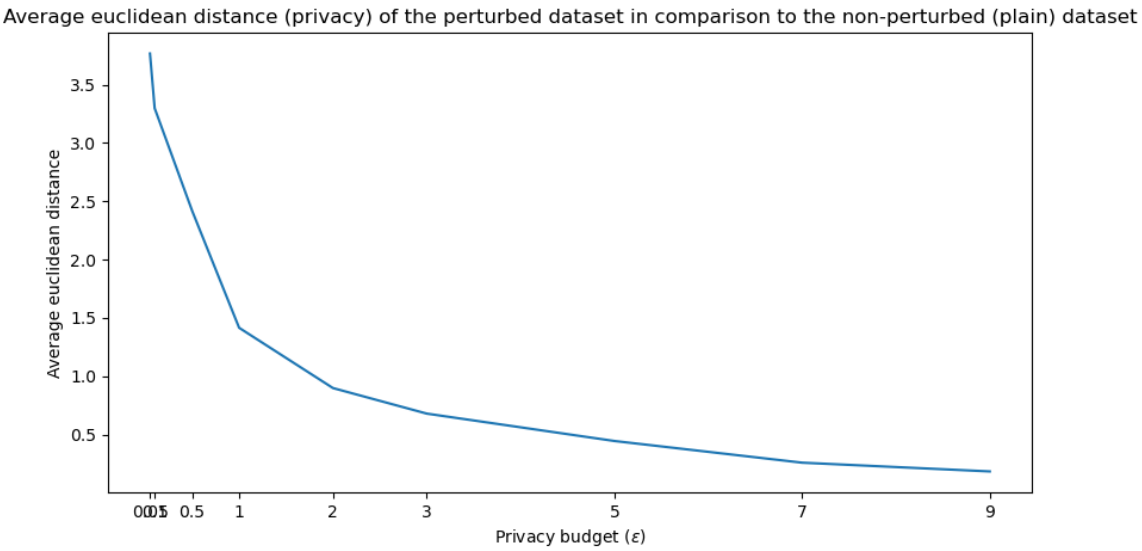

Figure 5.6: Average (euclidean) distance protection provided per epsilon.

5.4.2. RESEARCH QUESTION 2

5.4.3. RESEARCH QUESTION 3

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GLOSSARY

Adjusted Mutual Information Comparable with Adjusted Rand Index this algorithm is modified to account to chance. This means it accounts for a higher MI for a higher amount of clusters between two cluster algorithms. Therefore, the calculations are strongly influenced by that of Adjusted Rand Index [**?**]. . 10, 16

Adjusted Rand Index The Rand Index is improved and adjusted for chance [**?**]. This algorithm takes also into consideration the number of clusters and can be used to also compare different cluster algorithms [**?**].. v, 10, 16

Average Estimation Error This is the difference between an estimated value and the real value.. 16

Bit Vector List or array to store several bits.. 16

Calinski-Harabasz Index This is a way to measure the similarity of clusters [**?**]. It tells how well the clusters are separated from each other and how well the points are grouped.. 10, 16

Mutual Information This metric can be used to explain the amount of information about a random variable if compared to another random variable. Therefore, it can also be used to compare two cluster similarities.. v, 9, 16

Normalized Mutual Information The normalized version is a scaled version of Mutual Information to always be a value between 0 (no correlation) and 1 (perfect correlation). This version of Mutual Information is not adjusted and therefore highly influenced by cluster amount [**?**]. So it suffers the same issue as with Mutual Information.. 10, 16

Rand Index Compares the similarity between two clusters by comparing all pairs. It can therefore be used to measure the performance between two clustering algorithms [**?**]. . 16

ACRONYMS

AEE Average Estimated Error. 16, *Glossary:* Average Estimation Error

AMI Adjusted Mutual Information. 10, 14, 16, 30, *Glossary:* Adjusted Mutual Information

AP Affinity Propogation. 8–10, 14, 16, 29

ARI Adjusted Rank Index. 10, 14, 16, 30, *Glossary:* Adjusted Rand Index

BIRCH Balanced Iterative Reducing and Clustering using Hierarchies. 16

BV Bit Vector. 16

CHI Calinski-Harabasz Index. 10, 16, 31, *Glossary:* Calinski-Harabasz Index

DBSCAN Density-based spatial clustering of applications with noise. 8, 9, 14, 16, 29

DP Differential Privacy. 4–6, 10, 11, 16

DPC Density Peaks Clustering. 16

GI Geo-indistinguishability. 5, 16

LDP Local Differential Privacy. 4–6, 16

MI Mutual Information. 9, 16, *Glossary:* Mutual Information

NMI Normalized Mutual Information. 9, 10, 16, *Glossary:* Normalized Mutual Information

MATH SYMBOLS

θ Angle. 16

l Privacy level. 16

r Radius. 16

MATH SYMBOLS

$K(x)(Z)$ Randomization method for $x \in X$ and output $z \in Z$.. 16

$Pr(K(x_i) \in (Z))$ Probability of reporting $x \in X$ for $z \in Z$. 16

X Set of locations for a user. (R^2). 16

Z For every $x \in X$ a perturbed location $z \in Z$ is reported.. 16

ϵ The privacy budget ϵ determines the amount of noise that is added.. 16

HYPERPARAMETERS

.1. K-MEANS

For selecting the appropriate amount of clusters, we used an "elbow" plot in combination with the silhouette score.

1. Dataset 1: 4 clusters (see figure: 7)
2. Dataset 2: TODO
3. Dataset 3: TODO

.2. DBSCAN

For the selection of the appropriate epsilon, we used the k-distance plot.

1. Dataset 1: 0.9 epsilon (see figure: 8)
2. Dataset 2: TODO
3. Dataset 3: TODO

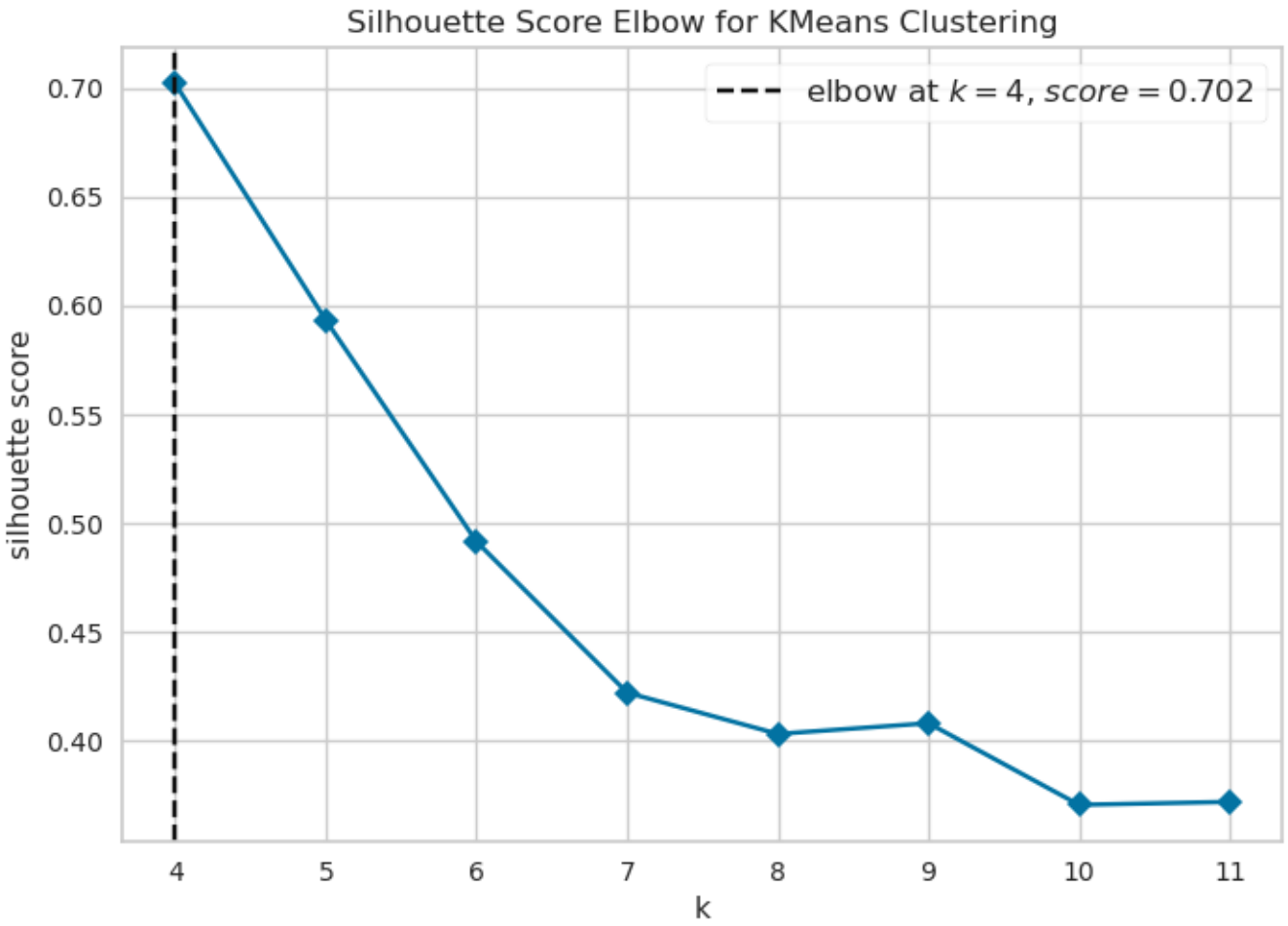

Figure 7: Selecting the k for K-Means for dataset 1 using the "elbow plot" using section 2.2.1

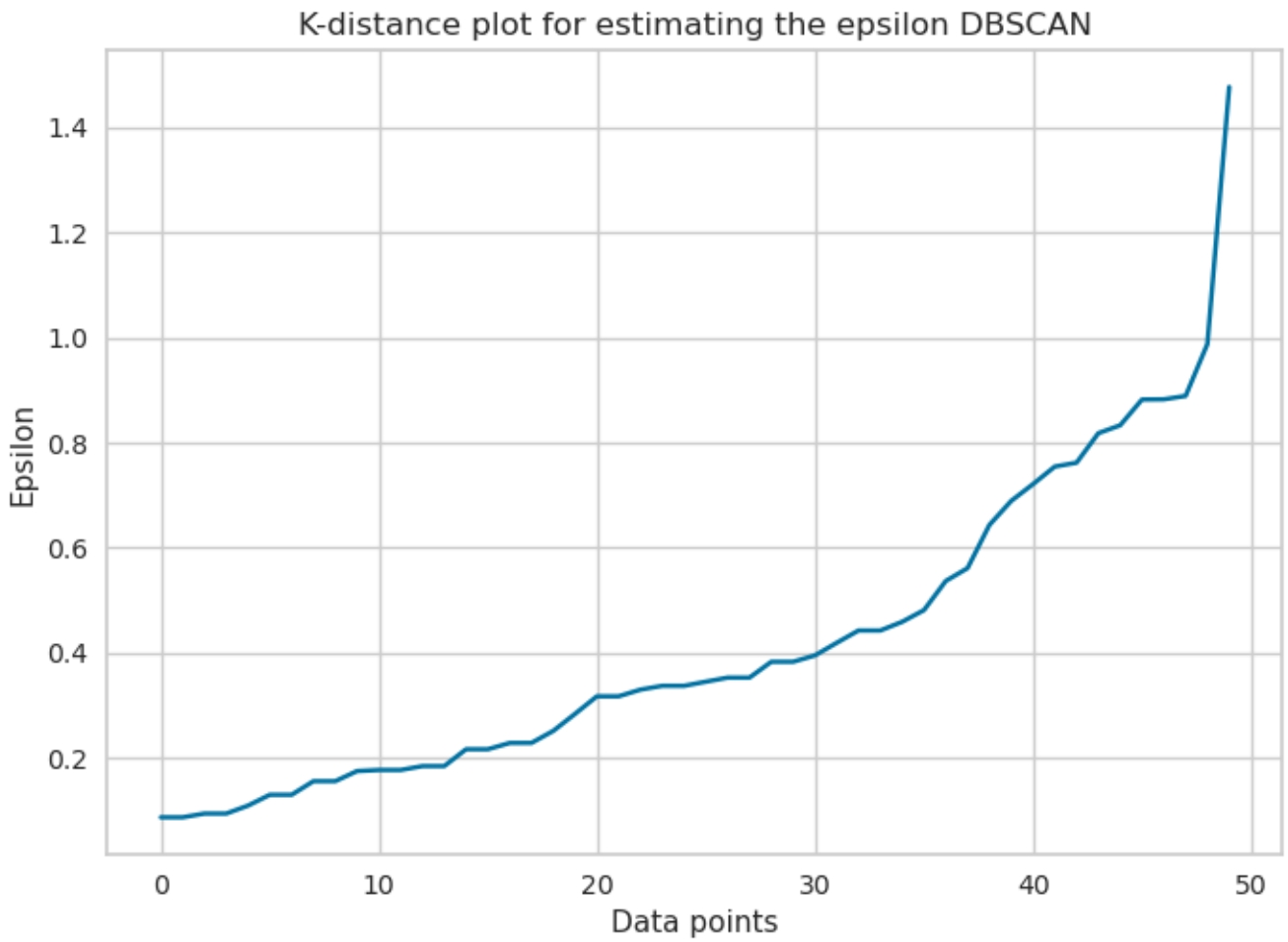

Figure 8: Selecting the ϵ for DBSCAN for dataset 1 using the "k-distance plot" using section 2.2.1