

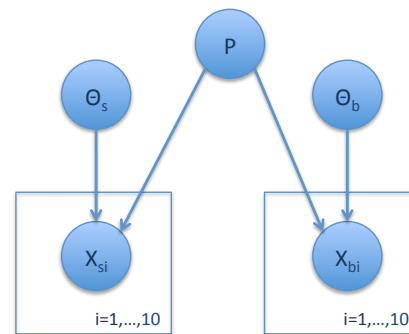
SYST/STAT 664: Homework Assignment 7

due April 9, 2018, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity. *Points may be deducted if you do not provide information on how you arrived at your answer. Please submit your R code either as a separate attachment on Blackboard or in your main submission.*

1. This problem concerns the HCB data from Assignment 6. For this problem, assume the observations are normally distributed with unknown depth-specific means Θ_s and Θ_b and common but unknown standard deviation Σ .

Surface	Bottom
3.74	5.44
4.61	6.88
4.00	5.37
4.67	5.44
4.87	5.03
5.12	6.48
4.52	3.89
5.29	5.85
5.74	6.85
5.48	7.16



Assume that experts have provided the following prior information based on previous studies.

- The unknown means Θ_s and Θ_b are independent and normally distributed with mean μ and standard deviation τ . The unknown precision $P = \Sigma^{-2}$ is independent of Θ_s and Θ_b and has a gamma distribution with shape α and scale β .
- Experts specified a 95% prior credible interval of [3, 9] for Θ_s and Θ_b . A good fit to this credible interval is obtained by setting the prior mean to $\mu=6$ and the prior standard deviation to $\tau=1.5$.
- A 95% prior credible interval of [0.75, 2.0] is given for the unknown standard deviation Σ . This translates to a credible interval of [0.25, 1.8] for $P = \Sigma^{-2}$. A good fit to this credible interval is obtained by setting the prior shape to $\alpha = 4.5$ and the prior scale to $\beta = 0.19$.

Find the following conditional distributions:

- The conditional distribution for Θ_s given Θ_b , P , and the observations.
 - The conditional distribution for Θ_b given Θ_s , P , and the observations.
 - The conditional distribution for P given Θ_s , Θ_b , and the observations.
2. Using the distributions you found in Part 1, draw 10,000 Gibbs samples of (Θ_s, Θ_b, P) . Estimate 95% credible intervals for Θ_s , Θ_b , $\Sigma=P^{-1/2}$, and $\Theta_b - \Theta_s$.
 3. Do a traceplot of $\Theta_b - \Theta_s$. Find the autocorrelation function of $\Theta_b - \Theta_s$ and the effective sample size for your Monte Carlo sample for $\Theta_b - \Theta_s$.
 4. Comment on your results. Compare with Assignment 6.