

# SYST 664 / CSI 674: Homework Assignment 3

due February 12, 2018 at 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity.

1. The table below is taken from the Hoff text and shows the joint distribution of occupations taken from a 1983 study of social mobility.<sup>1</sup>

father's occupation	son's occupation				
	farm	operatives	craftsmen	sales	professional
farm	0.018	0.035	0.031	0.008	0.018
operatives	0.002	0.112	0.064	0.032	0.069
craftsmen	0.001	0.066	0.094	0.032	0.084
sales	0.001	0.018	0.019	0.010	0.051
professional	0.001	0.029	0.032	0.043	0.130

- Find the marginal distribution of fathers' occupations.
  - Find the marginal distribution of sons' occupations.
  - Find the conditional distribution of the son's occupation given that the father is a farmer.
  - Find the conditional distribution of the father's occupation given that the son is a farmer.
  - Comment on these results. What do they say about changes in farming in the population from which these data are drawn?
2. Times were recorded at which 41 vehicles passed a fixed point on the M1 motorway in Bedfordshire, England on March 23, 1985.<sup>2</sup> The times were subtracted to form 40 intervals between successive cars. These interarrival times, rounded to the nearest second, are:
- 12, 2, 6, 2, 19, 5, 34, 4, 1, 4, 8, 7, 1, 21, 6, 11, 8, 28, 6, 4, 5, 1, 18, 9, 5, 1, 21, 1, 1, 5, 3, 14, 5, 3, 4, 5, 1, 3, 16, 2
- A common model for interarrival times is a random sample from an exponential distribution. Do you think an exponential distribution provides a good model for the interarrival times? Justify your answer.
  - When interarrival times are randomly sampled from an exponential distribution, the counts of events per unit time are a random sample from a Poisson distribution. Using a time unit of 15 seconds, find the number of cars passing in each 15-second block of time after the initial car. (The initial car is used to bound the recording interval, so the total car count in your data set should be 40.) Do you think a Poisson distribution provides a good model for the count data? Justify your answer.
  - Assume that  $\Lambda$ , the rate parameter of the Poisson distribution (and the inverse of the mean of the exponential distribution), has a discrete uniform prior distribution on 20 equally spaced values between (0.2, 0.4, ..., 3.8, 4.0) cars per 15-second interval. Find the posterior mean, standard deviation, median and 95<sup>th</sup> percentile of  $\Lambda$  given the observations. Describe what your results mean in terms of traffic on this motorway.

<sup>1</sup> Logan, J.A. (1983) A Multivariate Model for Mobility Tables. *The American Journal of Sociology* 89(2): 324-349.

<sup>2</sup> These data were taken from Hand, et al., *A Handbook of Small Data Sets*, Chapman and Hall, 1994.