SYST 664 / CSI 674: Homework Assignment 5

due February 26, 2018, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity. *Points may be deducted if you do not provide information on how you arrived at your answer. Please submit your R code either as a separate attachment on Blackboard or in your main submission.*

1. In previous years, students in this course collected data on people's preferences in the two Allais gambles from Assignment 2. For this problem, we will assume that responses are independent and identically distributed, and the probability is π that a person chooses both B in the first gamble and C in the second gamble.
   a. Assume that the prior distribution for π is Beta(1, 3). Find the prior mean and standard deviation for π. Find a 95% symmetric tail area credible interval for the prior probability that a person would choose B and C. Do you think this is a reasonable prior distribution to use for this problem? Why or why not?
   b. In 2009, 19 out of 47 respondents chose B and C. Find the posterior distribution for the probability that a person in this population would choose B and C. Find the posterior mean and standard deviation, and a 95% symmetric tail area credible interval for the posterior probability that a person in this population would choose B and C. Do a triplot.
   c. In 2011 another 47 responses were collected. This time, 20 out of 47 people said they preferred B and C. Assume that the Spring 2011 respondents have the same distribution of responses as the Spring 2009 respondents. Repeat part b, using the posterior distribution from the 2009 sample as your prior distribution.
   d. Comment on your results.

2. This problem concerns the automobile data from Assignments 3 and 4.
   a. As in Assignment 4, assume that counts of cars per 15-second interval are independent and identically distributed Poisson random variables with unknown mean Λ. Assume a uniform prior distribution for Λ. (As for Assignment 4, you can approximate this prior distribution by using a Gamma distribution with shape 1 and scale 10,000.) The car counts for the first 10 time intervals are 2, 2, 1, 1, 0, 4, 3, 0, 2, 1. Find the posterior distribution for Λ conditional on the first 10 observations. Plot the posterior density function for Λ conditional on the first 10 observations. Find a 90% 2-sided posterior credible interval for Λ.
   b. Using the posterior distribution from Part a, find the predictive distribution for the total number of cars in the next 11 time periods. What type of distribution is it? What are the parameters? Plot the probability mass function for the predictive distribution.
   c. Find the posterior predictive probability that between 10 and 30 cars will pass by in the next 11 time periods.
   d. If the number of cars passing during a single time period has a Poisson distribution with parameter Λ, then the number of cars passing during 11 time periods has a Poisson distribution with parameter 11Λ. Use the mean of the posterior distribution as a point estimate of Λ, and use this point estimate to find a Poisson distribution for predicting the number of cars passing during the next 11 periods. With this distribution, what is the probability that between 10 and 30 cars pass?
   e. Compare the distributions for Parts c and d. Discuss.