SYST 664 / CSI 674: Homework Assignment 6

due March 30, 2016, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity. *Points may be deducted if you do not provide information on how you arrived at your answer. Please submit your R code either as a separate attachment on Blackboard or in your main submission.*

1. Concentrations of the pollutants aldrin and hexachlorobenzene (HCB) in nanograms per liter were measured in ten surface water samples, ten mid-depth water samples, and ten bottom samples from the Wolf River in Tennessee. The samples were taken downstream from an abandoned dump site previously used by the pesticide industry. The full data set can be found at http://www.biostat.umn.edu/~lynn/iid/wolf.river.dat. For this problem, we consider only HCB measurements taken at the bottom and the surface. The question of interest is whether the distribution of HCB concentration depends on the depth at which the measurement was taken. The data for this problem are given below.

Surface	Bottom
3.74	5.44
4.61	6.88
4	5.37
4.67	5.44
4.87	5.03
5.12	6.48
4.52	3.89
5.29	5.85
5.74	6.85
5.48	7.16

We will assume the observations are normally distributed with unknown depth-specific means Θ_s and Θ_b and precisions P_s and P_b.

   a. Assume independent improper reference priors for the surface and bottom parameters:
   $$g(\Theta_s, \Theta_b, P_s, P_b) = g(\Theta_s, P_s)\, g(\Theta_b, P_b) \propto \rho_s^{-1} \rho_b^{-1}.$$
   Find the joint posterior distribution of $(\Theta_s, \Theta_b, P_s, P_b)$.
   b. Find 90% credible intervals for Θ_s, Θ_b, P_s, and P_b. Comment on your results.
   c. Use direct Monte Carlo to sample 10,000 observations from the joint posterior distribution of $(\Theta_s, \Theta_b, P_s, P_b)$. Use your Monte Carlo samples to estimate 90% posterior credible intervals for Θ_s, Θ_b, P_s, and P_b. Compare with the result of Part b.

2. Repeat Problem 1, but this time assume that equal standard deviations, $P_s = P_b = P$. Use the improper reference prior $g(\Theta_s, \Theta_b, P) \propto \rho^{-1}$ Compare your results with Problem 1. *Hint: First find the posterior distribution of* (Θ_s, P) *given the surface data. Then use the bottom data to update the distribution of* (Θ_b, P).

3. Do you think the assumption of normally distributed observations is reasonable? Why or why not? What about the assumption that the standard deviations at the two depths are equal?

4. Comment on the effect of measurement depth on the distribution of HCB concentration.