SYST 664 / CSI 674: Homework Assignment 2

due February 5, 2018, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity.

1. In an experiment, subjects were given the choice between two gambles:

   Gambe 1:

A:	\$2500 with probability 0.33 \$2400 with probability 0.66 \$0 with probability 0.01	B:	\$2400 with certainty

   Suppose that a person is an expected utility maximizer. Set the utility scale so that u(\$0) = 0 and u(\$2500) = 1. Denote u(\$2400) by x. For what values of x would a person choose Option A? For what values would a person choose Option B?

   Gamble 2:

C:	\$2500 with probability 0.33 \$0 with probability 0.67	D:	\$2400 with probability 0.34 \$0 with probability 0.66

   For what values of x would a person choose Option C? For what values would a person choose Option D?

   This problem is a version of the famous *Allais paradox*, named after the prominent critic of subjective expected utility theory who first presented it. Kahneman and Tversky[1] found that 82% of subjects preferred B over A, and 83% preferred C over D. Explain why no expected utility maximizer would prefer *both* B in Gamble 1 and C in Gamble 2. (*A utility maximizer might prefer B in Gamble 1. A different utility maximizer might prefer C in Gamble 2. But the same utility maximizer would not prefer both B in Gamble 1 and C in Gamble 2.*) Discuss these results. Why do you think many people prefer B in Gamble 1 and C in Gamble 2? Do you think this is reasonable even if it does not conform to expected utility theory?

2. In a study of tumor incidence in rats, let $X_1, \ldots, X_n$ be a sequence of random variables indicating whether the i^{th} rat has a tumor, for $i = 1, \ldots, n$. Suppose the X_i are independent and identically distributed with $\Pr(X_i = 1|\theta) = \theta$. Assume that the tumor probability θ has 20 possible values, 0.025, 0.075, …, 0.975, and that all these values are equally likely *a priori*.

   a. Suppose researchers found tumors in 2 out of 13 rats. Find the posterior probability distribution for θ given the data from this study. Find the posterior mean and posterior standard deviation of θ.

   b. A follow-on study was carried out under similar conditions, and found 1 tumor in 18 rats. Assume that the second study has the same tumor probability θ as the first. Using the posterior distribution from the first study as the prior distribution for the second study, find the posterior probability distribution for θ given the data from both studies. Find the posterior mean and posterior standard deviation of θ.

   c. Show that the posterior distribution is the same if we start with a uniform prior distribution and use Bayes rule to update from a single sample of 3 tumors in 31 rats.

[1] Kahneman, D., P. Slovic, et al. (1982). *Judgment under Uncertainty: Heuristics and Biases*. Cambridge University Press.