SYST/STAT 664: Homework Assignment 9

due April 23, 2018

Homework is due at class time on the date indicated. You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity.

1. This assignment uses the reaction time data from Assignment 8, taken from the Gelman et al. reference and found at this url:

    http://www.stat.columbia.edu/~gelman/book/data/schiz.asc

   As with Assignment 8, we will consider only the non-schizophrenic subjects (rows 1 to 11). A plate model is shown at right.

   - As with Assignment 8, assume the logarithms of the response times are independent and identically distributed normal random variables with person-specific mean θ_s ($s = 1,\ldots 11$).
   - Following Gelman, et al., assume all the observations have the same standard deviation σ^2.
   - The 11 means θ_s, $s=1, \ldots, 11$, are independent and identically distributed normal random variables with mean μ and standard deviation μ.
   - The unknown mean has a normal distribution with mean 5.52 and standard deviation 0.22. This reflects a prior 95% credible interval of [162, 385] ms for the population average reaction time, which is consistent with the literature on reaction times.
   - The inverse variance $1/\tau^2$ has a gamma distribution with shape ½ and scale 50. This reflects weak prior information focused on a value of 25 for $1/\tau^2$, or 0.2 for τ.
   - The inverse variance $1/\sigma^2$ has a gamma distribution with shape ½ and scale 50. This reflects weak prior information focused on a value of 25 for $1/\sigma^2$, or 0.2 for σ.
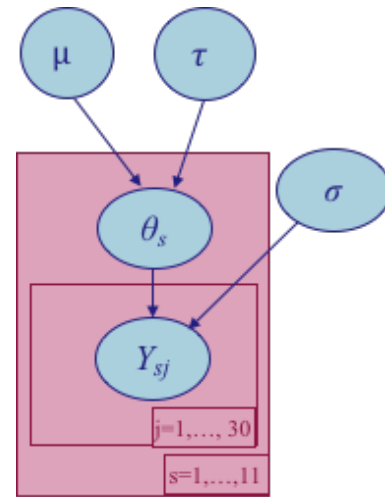
   Note that this model is similar to the math test scores example in Unit 7. Using the formulas in the Unit 7 notes as a model, find the posterior distribution for each of the unknown parameters given the other parameters.

2. Use Gibbs sampling to draw 5000 samples from the posterior distribution of the parameters μ, τ, σ, and θ_s, $s=1, \ldots, 11$. Find posterior credible intervals for each of these parameters.

3. Discuss your results. Compare your intervals for θ_s, $s=1, \ldots, 11$ with the results from Assignment 8. Discuss the impact of the assumption that all eleven standard deviations for the log reaction times are equal.