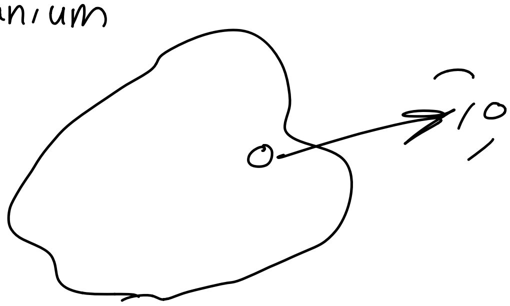
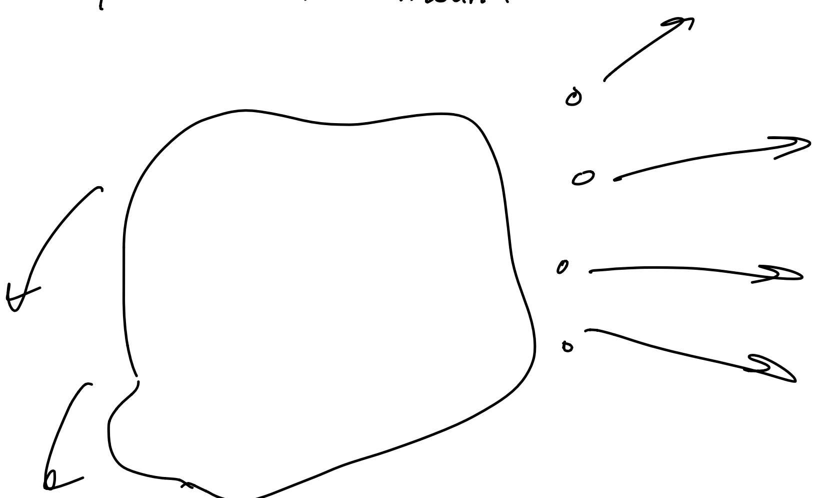
Suppose we have some amount of a certain radioactive element

Uranium

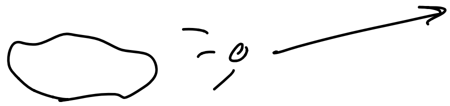


mass decreases

Depends on amount



if amount "N" is large, mass loss rate is also large $\frac{\Delta m}{\Delta t}$



if N is small, $\frac{dN}{dt}$ is also small

mass lost per second $\propto$ − mass

$$\frac{dN}{dt} \propto -N$$

Different rates for different materials

Uranium-238 : takes 4.5 billion years
to lose half of its mass

Carbon-14 takes ~ 5700 years

Some only take seconds
(half life)

$$\frac{dN}{dt} = -\frac{1}{\tau} N$$

τ is a time related to the half life
$T_{1/2} = \ln(2)\,\tau$

Consider an element with $\tau = 100\,s$
(radon-204, lead-190)

Let's say its lead

We start with 5 kg $\quad N(t=0) = 5\,kg$

$$\frac{dN}{dt} = \frac{-N}{100}$$

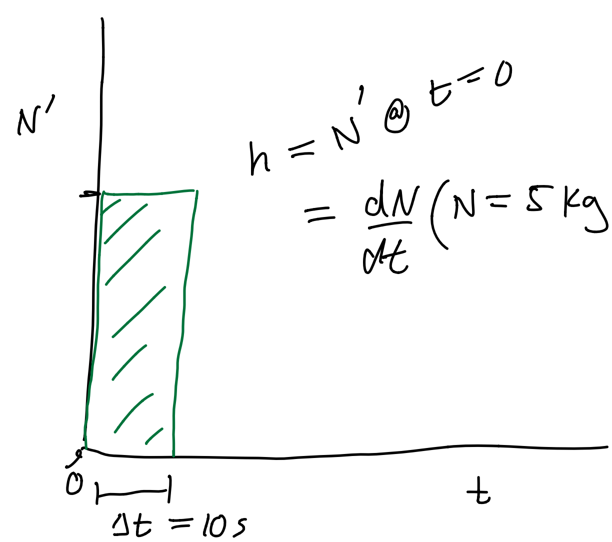
What is N after 30 s?

Let's proceed in steps of $\Delta t = 10\,s$

Given that $N(t=0) = 5\,kg$, what is $N(t=10s)$?

$$N(10) = N(0) + \int_0^{10} N'\,dt$$

How to integrate?
Use a rectangle?



$h = N'$ @ $t=0$

$$= \frac{dN}{dt}(N = 5\,kg) = \frac{-5\,kg}{100\,s}$$

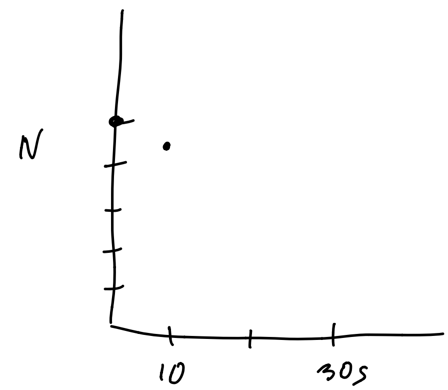
$$\int_0^{10} N'\,dt \approx \text{area of rectangle with height } \frac{N_o}{\tau} = \frac{-5}{100} = -.05$$

and width $\Delta t = 10\,s$

$$N(10s) \approx N(0) - (0.05\,\tfrac{kg}{s})(10s)$$

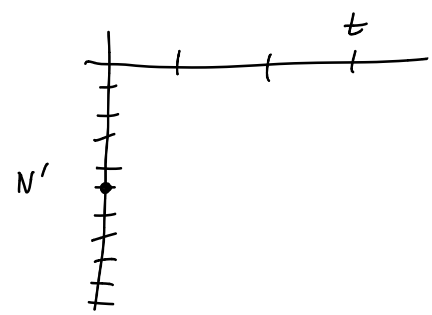
$$= 5\,kg - 0.5\,kg$$

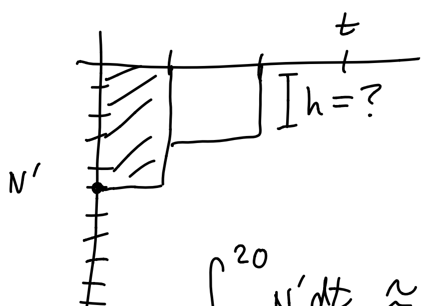
$$N(10s) = 4.5\,kg$$



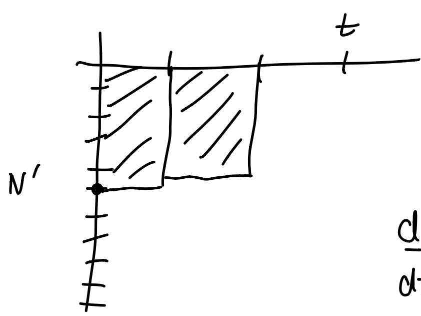
$N(20s) = ?$

$$N(20s) = N(10s) + \int_{10}^{20} N'\,dt$$





$\int_{10}^{20} N'\,dt \approx$ area of rectangle with width $10\,s$ + h ...

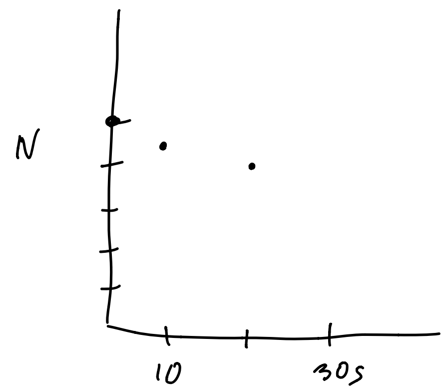


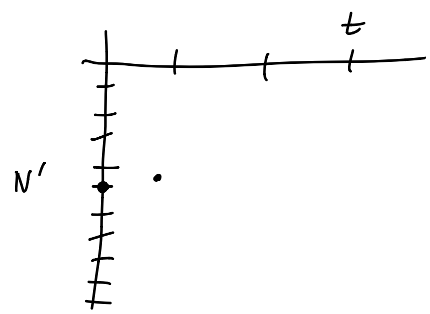
$$h = \frac{dN}{dt} \text{ @ } t = 10\,s$$

$$\frac{dN}{dt} = \frac{-N}{\tau} = \frac{-4.5\,kg}{100s} \Rightarrow h = -.045\,\frac{kg}{s}$$

$N(20s) \approx N(10s) + \left(-0.045 \frac{kg}{s}\right)(10\ s)$

$N(20s) = 4.5\ kg - .45\ kg = 4.05\ kg$



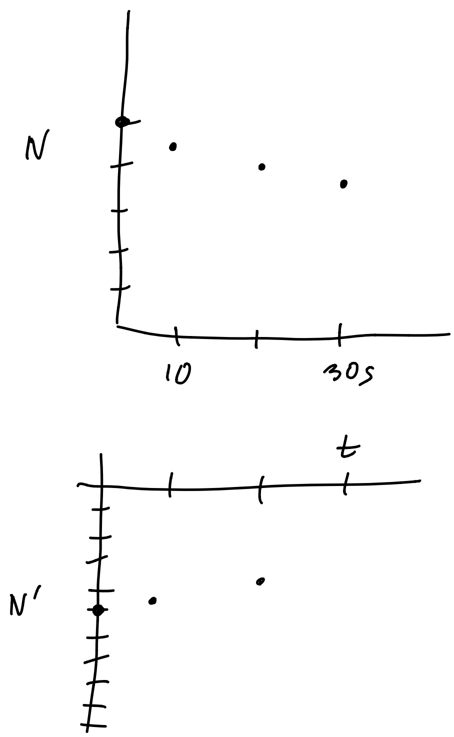


$N(30s)$?

$$N(30s) = N(20s) + \int_{20}^{30} N'\,dt$$

$$\approx N(20s) + \Delta t \cdot \frac{dN}{dt} \text{ @ } 20s$$

$$\approx 4.05\,kg + (10s)\left(-\frac{4.05\,kg}{100\,s}\right) = 3.65\ kg$$



After 30 s, there is ~ 3.65 kg left

Recap: What did we do?

Starting with N @ $t=0$, find N' @ $t=0$

Use N' at $t=0$ to estimate $N(t=10)$

Use $N(t=10)$ to estimate $N'(t=10)$

Use $N'(t=10)$ to estimate $N(t=20)$

$N(t=20) \longrightarrow N'(t=20)$

$N'(t=20) \longrightarrow N(t=30)$

How to code:

- Function for the derivative

  $deriv(y, x)$

- $x_i, x_F, \Delta t/n$

Start with $y_0 = y(x_i)$

$$y'(x_i) = deriv(y(x_i), x_i)$$

$$y_1 = y(x_i + \Delta x) = y_0 + y'(x_i)\Delta x$$

$$y_1' = y'(x_i + \Delta x) = deriv(y_1, x_i + \Delta x)$$

$$y_2 = y_1 + y_1' \Delta x$$

$$y_2' = deriv(y_2, x + 2\Delta x)$$

$$y_3 = y_2 + y_2' \Delta x$$

$$\vdots$$

1) calculate derivative using previous y, x
2) use derivative to find next y value
3) $x \mathrel{+}= \Delta x$
4) repeat

```
import numpy as np

def deriv(N,t,tau):
    return -N / tau

tau = 100
t0 = 0
tf = 10
dt = 1

N = 5
#t = 0
#while t <= tf:
    Nprime = deriv(N,t,tau)
    N = N + Nprime * dt
    t+=dt

for t in np.arange(t0,tf+dt,dt):
    Nprime = deriv(N,t,tau)
    N = N + Nprime * dt
print(N)
```