Project 2

Realistic Projectile Motion

Overview

You will write a program to calculate the trajectory of a massive projectile in the presence of a gravitational force and variable air density. The user will provide the launch speed v_i and launch angle θ; your program will do several things:

1. Plot the trajectory y vs x
2. Print the max height and horizontal distance traveled
3. Print the total time of flight

You can assume that the projectile is being launched from (and lands on) the surface of the planet ($y_i = 0$).

Details

The derivative equations

While in flight, the forces acting on the projectile are: the gravitational force $-mg\hat{y}$, and the air resistance force $-\frac{1}{2}C\rho(y)Av^2\hat{v}$. The air density ρ is a function of y: $\rho(y) = \rho_0 e^{-\frac{y}{y_0}}$. The net force is therefore:

$$F_x = -\frac{1}{2}CA\rho_0 e^{-\frac{y}{y_0}} v v_x$$
$$F_y = -mg - \frac{1}{2}CA\rho_0 e^{-\frac{y}{y_0}} v v_y$$
$$v = \sqrt{v_x^2 + v_y^2}$$

These forces result in the following differential equations:

$$\frac{dv_x}{dt} = -\frac{1}{2m}CA\rho_0 e^{-\frac{y}{y_0}} v v_x$$
$$\frac{dv_y}{dt} = -g - \frac{1}{2m}CA\rho_0 e^{-\frac{y}{y_0}} v v_y$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dy}{dt} = v_y$$

You will solve these equations to plot the trajectory of the projectile. Note that, if the initial velocity is v_i, and the launch angle is θ, then the initial velocity in the x direction is $v_{x,i} = v_i \cos\theta$, and in the y direction: $v_{y,i} = v_i \sin\theta$

Inputting the program parameters

Note that these equations depend on the following constants:

- m: The mass of the projectile
- g: The gravitational acceleration, equal to 9.8 m/s^2 on Earth.
- C: the *drag coefficient*, a dimensionless constant used to modify air resistance
- A: the cross-sectional area of the projectile
- ρ_0: the atmospheric density at sea level

- y_0: the *scale height* of the atmosphere (at a height y, the atmospheric density is $1/e \approx 0.36$ of its sea-level value ρ_0)

It would be tedious to ask the user to specify these every single time the program is run. However, we want these values to be able to change without editing the source code. Our solution will be to read them from a text file. All of these constants will be read from a separate configuration file which specifies these parameters with the following format: (download the file here)

```
#This is a config file used for the program "projectile.py"

#Mass of projectile (kg)
MASS : 5

#Gravitational acceleration at surface of planet, m/s2
GRAV_ACCEL : 9.81

#Drag coeff, dimensionless
DRAG_COEFF : 1

#Cross-sectional area, m2
AREA : 0.05

#Atmospheric density at sea level (kg/m3)
DENSITY : 1.2

#Scale height of atmosphere
SCALE_HT : 8.5e3
```

Your program should be able to read this file in order to determined the value of m, g, C, A, ρ_0, and y_0. Your program should also be able to skip comments in the file (ignore lines that begin with the `#` character)

Choosing Δt

We must choose Δt so that $\frac{dv}{dt}$ is approximately flat over the interval. Since $\frac{dv}{dt}$ only changes with v (g is constant), we must choose a time interval Δt over which changes in velocity are small.

A good way to estimate this is to assume that the initial value of $\frac{dv}{dt}$ is constant, so that $t_0 \approx v_i / \frac{dv_i}{dt}$ is the time required for this constant force to bring the velocity all the way down to zero. Do this separately in each direction:

$$t_{0,x} \approx \frac{v_{x,i}}{\frac{1}{2m} C A \rho_0 v_i^2}$$
$$t_{0,y} \approx \frac{v_{y,i}}{\frac{1}{2m} C A \rho_0 v_i^2 + g}$$

Let t_0 be the minimum of $(t_{0,x}, t_{0,y})$, then choose Δt so that $\Delta t << t_0$ (say, $\Delta t \approx \frac{t_0}{100}$)

Requirements

Your program should follow the style and readability guidelines detailed here