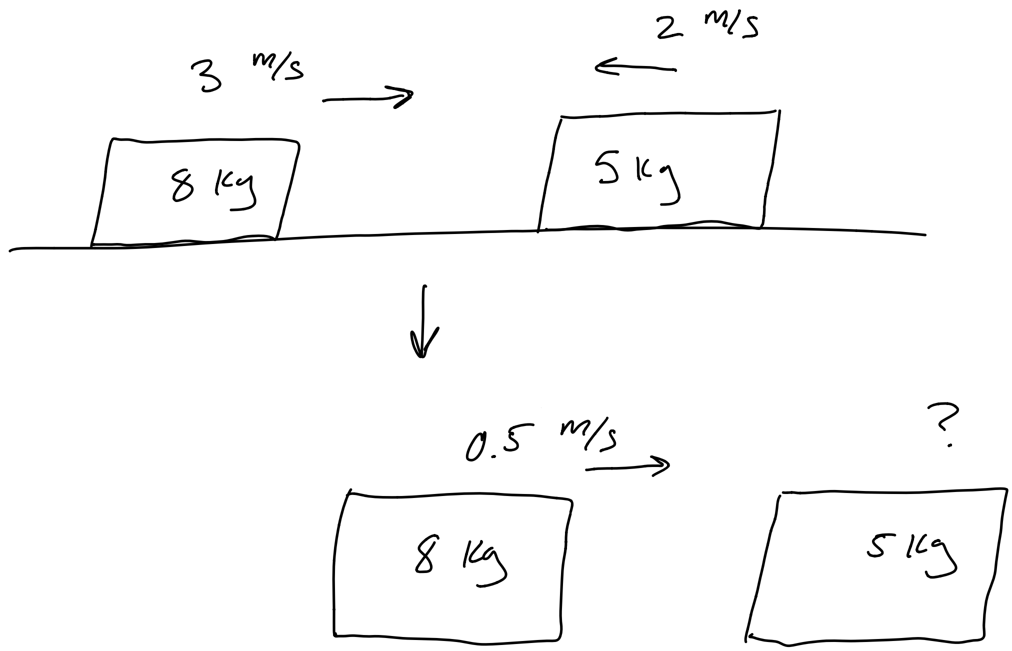
Collisions

Dealt with collisions back in ch 2



system: both blocks

surr: –

$\Delta \vec{p}_{sys} = 0$

$\vec{p}_i = \vec{p}_f$

$(8\,kg)(3\,m/s) - (5\,kg)(2\,m/s) = (8\,kg)(0.5\,\frac{m}{s}) + (5\,kg)(V_f)$

$\boxed{V_f = +2\,m/s}$

Q: Can I predict <u>both</u> final velocities, before the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

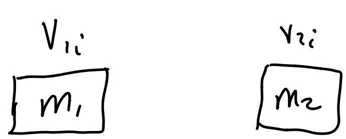
Given m_1, m_2, v_{1i}, v_{2i}: can I find v_{1f} & v_{2f}?

- Not using the above equation (1 eqn, 2 unknowns)

But we have learned more since Chapter 2

- Momentum is not the only thing conserved

Energy



1) $\Delta p_{sys} = 0$

2) $\Delta E_{sys} = 0$

Energy conservation + momentum conservation determine the final velocities

1) $p_i = p_f$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

2) $E_i = E_f$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

2 Eqns + 2 unknown

solve

Not so fast!

In our previous example:

$$E_i : \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$$

$$= \frac{1}{2}(8\,kg)(3\,\tfrac{m}{s})^2 + \frac{1}{2}(5\,kg)(2\,\tfrac{m}{s})^2$$

$$E_i = 46\ J$$

$$E_f = \frac{1}{2}(8\,kg)(0.5\,\tfrac{m}{s})^2 + \frac{1}{2}(5\,kg)(2\,\tfrac{m}{s})^2$$

$$E_f = 11\ J$$

???

$$\Delta E = -35\ J\ ?$$

Where did the energy go?

Energy is conserved

Kinetic Energy ... not necessarily

$$\Delta E = \Delta K + \Delta E_{int} = 0$$

$$= -35\ J + \Delta E_{int} = 0$$

$$\Delta E_{int} = 35\ J$$

(Temperature of the objects increased)

$$E_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 + E_{int,i}$$

$$E_f = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 + E_{int,f}$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 + E_{int,i} = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 + E_{int,f}$$

$E_{int,f}$ is always $\geq$ $E_{int,i}$

Kinetic energy can be converted into thermal energy, but the reverse never happens (2nd Law of Thermo)

- On the other hand, there are limits on <u>how much</u> kinetic energy can transform into thermal energy

Cant have $\frac{K_f}{K_i} = 0$ (violates momentum principle)

The smallest $\frac{K_f}{K_i}$ can possibly be:

$$K_f = \frac{m_1^2 v_{1i}^2 + m_2^2 v_{2i}^2}{(m_1+m_2)(m_1 v_{1i}^2 + m_2 v_{2i}^2)} K_i$$

This corresponds to

$$v_{1f} = v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

(objects stick together)

If $K_i = K_f$ $(E_{int,i} = E_{int,f})$

the collision is called elastic

If $K_f < K_i$, the collision is inelastic

If $K_f = \frac{m_1^2 v_{1i}^2 + m_2^2 v_{2i}^2}{(m_1+m_2)(m_1 v_{1i}^2 + m_2 v_{2i}^2)} K_i$

The collision is <u>maximally inelastic</u>

Most collisions are somewhere in between

We will consider 2 cases

1) Elastic

$$K_i = K_f$$

$$E_{int,i} = E_{int,f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 + \cancel{E_{int,i}} = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \cancel{E_{int,f}}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Know: m_1 m_2
v_{1i} v_{2i}

2 eqns, 2 unknowns : can be solved

First, solve from perspective of observer moving with velocity V_{2i}

$$V_{1i} \rightarrow V_{1i} - V_{2i} = V_{1i}'$$

$$V_{2i} \rightarrow V_{2i} - V_{2i} = 0$$

$$V_{1f} \rightarrow V_{1f} - V_{2i} = V_{1f}'$$

$$V_{2f} \rightarrow V_{2f} - V_{2i} = V_{2f}'$$

1) $m_1 V_{1i}' = m_1 V_{1f}' + m_2 V_{2f}'$

2) $\frac{1}{2} m_1 V_{1i}'^2 = \frac{1}{2} m_1 V_{1f}'^2 + \frac{1}{2} m_2 V_{2f}'^2$

1) $\rightarrow m_1 (V_{1i}' - V_{1f}') = m_2 V_{2f}'$

2) $\rightarrow m_1 (V_{1i}'^2 - V_{1f}'^2) = m_2 V_{2f}'^2$

$$= m_1 (V_{1i}' - V_{1f}')(V_{1i}' + V_{1f}') = m_2 V_{2f}'^2$$

$\div$ (2) by $m_1 (V_{1i}' - V_{1f}')$

2) $\rightarrow \dfrac{m_1 (V_{1i}' - V_{1f}')(V_{1i}' + V_{1f}')}{m_1 (V_{1i}' - V_{1f}')} = \dfrac{m_2 V_{2f}'^2}{m_2 V_{2f}'}$

2) $\rightarrow V_{1i}' + V_{1f}' = V_{2f}'$

1) $m_1\left(v_{1i}' - v_{1f}'\right) = m_2 v_{2f}' = m_2\left(v_{1i}' + v_{1f}'\right)$

$$(m_1+m_2)v_{1f}' = (m_1 - m_2)v_{1i}'$$

$$\boxed{v_{1f}' = \frac{m_1 - m_2}{m_1+m_2} v_{1i}'}$$

$$v_{2f}' = v_{1i}' + \frac{m_1 - m_2}{m_1+m_2} v_{1i}'$$

$$\boxed{v_{2f}' = \frac{2m_1}{m_1+m_2} v_{1i}'}$$

As seen by observer moving at v_{2i}

1) $\rightarrow$ $v_{1f} - v_{2i} = \left(\frac{m_1 - m_2}{m_1+m_2}\right)\left(v_{1i} - v_{2i}\right)$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1+m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i}$$

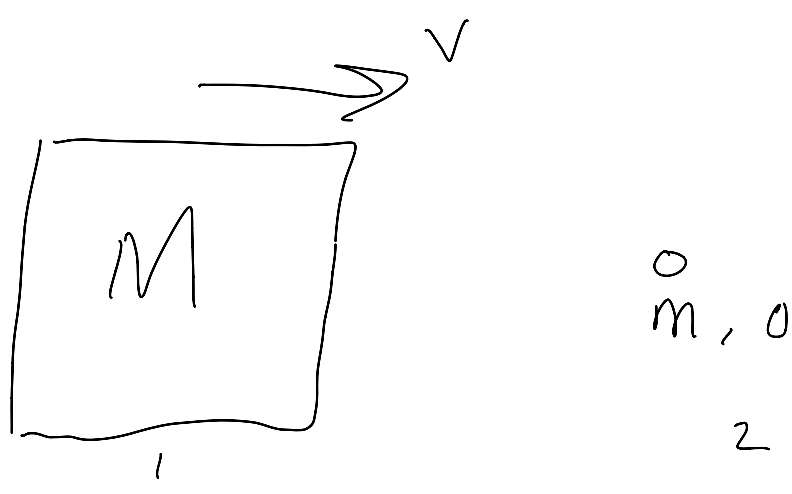
2) $\rightarrow$ $v_{2f} - v_{2i} = \left(\frac{2m_1}{m_1+m_2}\right)\left(v_{1i} - v_{2i}\right)$

$$v_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1i} - \left(\frac{m_1 - m_2}{m_1+m_2}\right)v_{2i}$$

$$\boxed{\begin{aligned} v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \\ v_{2f} &= \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{2i} \end{aligned}}$$

Special Case

Large object moving toward stationary, tiny object



$m << M \quad (m_2 << m_1)$

$$m_2 \to 0$$

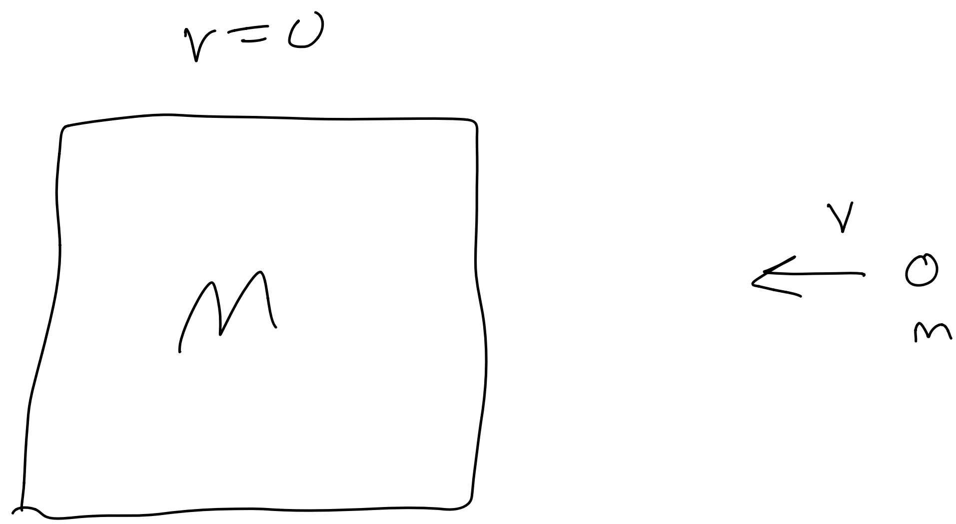
$$v_{1f} = \left(\frac{m_1 - \cancel{m_2}}{m_1 + \cancel{m_2}}\right) v_{1i} + \left(\frac{2\cancel{m_2}}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + \cancel{m_2}}\right) v_{1i} - \cancel{\left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{2i}}$$

$$v_{1f} \approx v_{1i} = v$$

$$v_{2f} \approx 2v_{1i} = 2v$$

Exactly the same:



$V_{1i} = 0$

$m_2 << m_1$

$$V_{1f} = \left(\frac{m_1 - \cancel{m_2}}{m_1 + \cancel{m_2}}\right) V_{1i} + \cancel{\left(\frac{2m_2}{m_1 + m_2}\right) V_{2i}}$$

$$V_{2f} = \cancel{\left(\frac{2m_1}{m_1 + m_2}\right) V_{1i}} - \left(\frac{m_1 - \cancel{m_2}}{m_1 + \cancel{m_2}}\right) V_{2i}$$

$V_{2f} \approx -V_{2i} = -v$ (bouncing)