Begin with clicker questions

We know how to find $|\vec{L}|$

What is $\hat{L}$?

I'll give you the answer and then explain it!

If: $\vec{r} = \langle x, y, z \rangle$

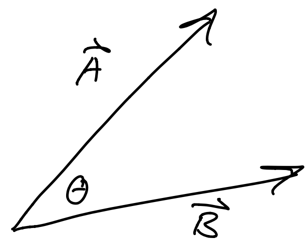
$\vec{p} = \langle p_x, p_y, p_z \rangle$

$$\vec{L} = \langle y p_z - z p_y, z p_x - x p_z, x p_y - y p_x \rangle$$

Where did this come from??

This is a new vector multiplication operation called the <u>cross product</u>

Given two vectors $\vec{A}$ + $\vec{B}$



$\vec{A} \times \vec{B}$:

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

Direction: perpendicular to the plane formed by $\vec{A}$ + $\vec{B}$

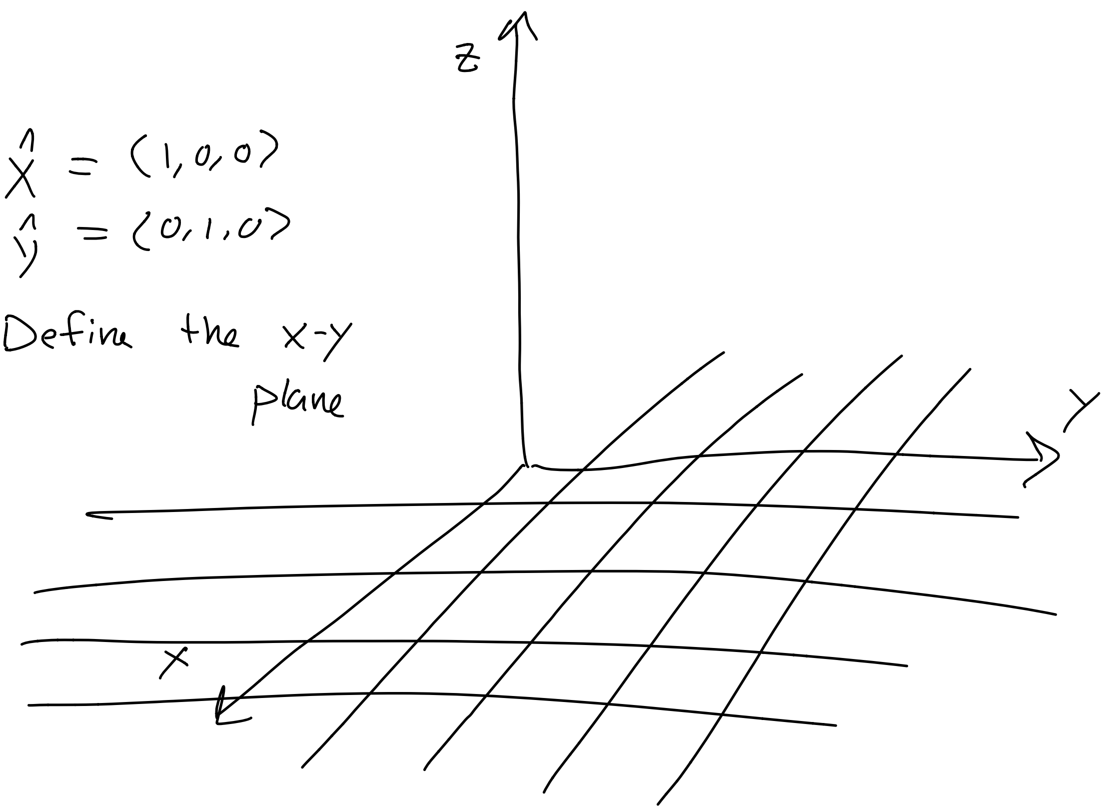
What does this mean?

Two vectors define a surface

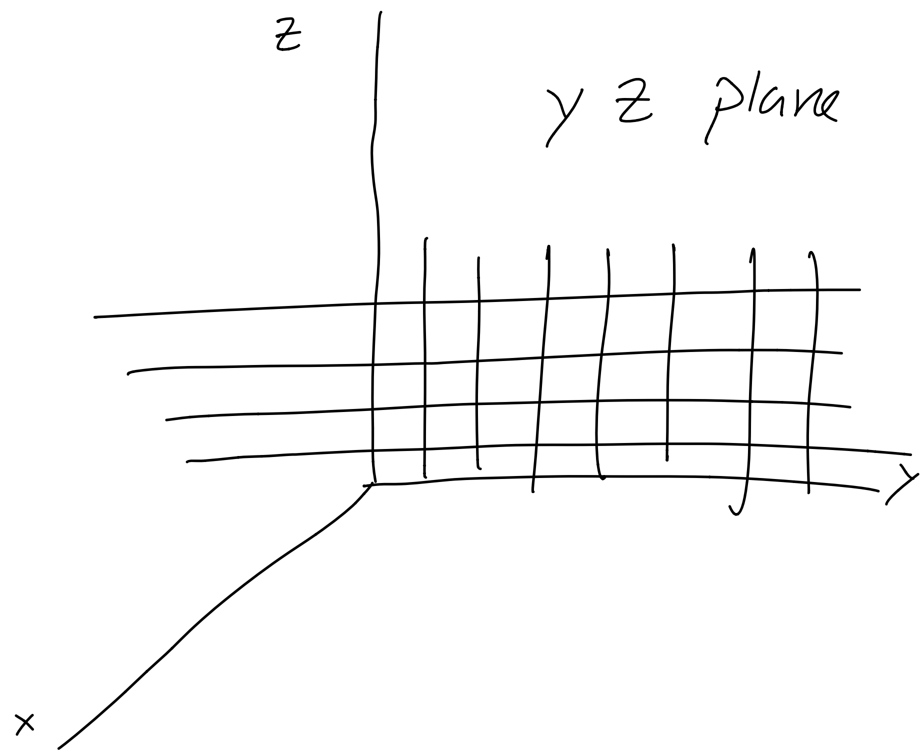
$\hat{x} = \langle 1, 0, 0 \rangle$

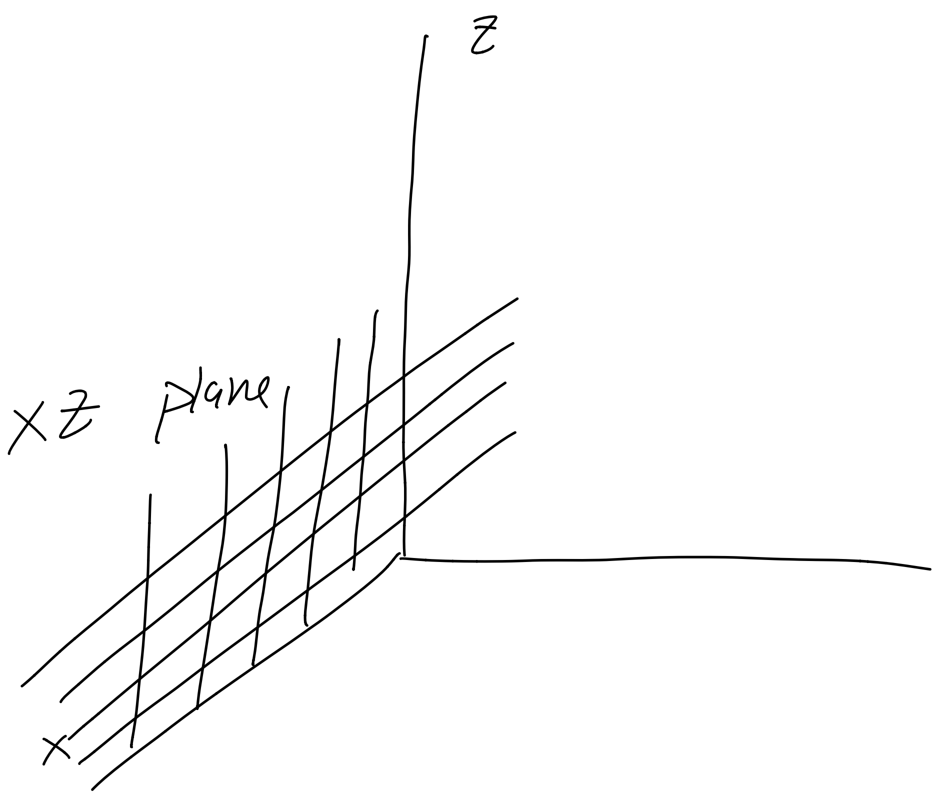
$\hat{y} = \langle 0, 1, 0 \rangle$

Define the x-y plane

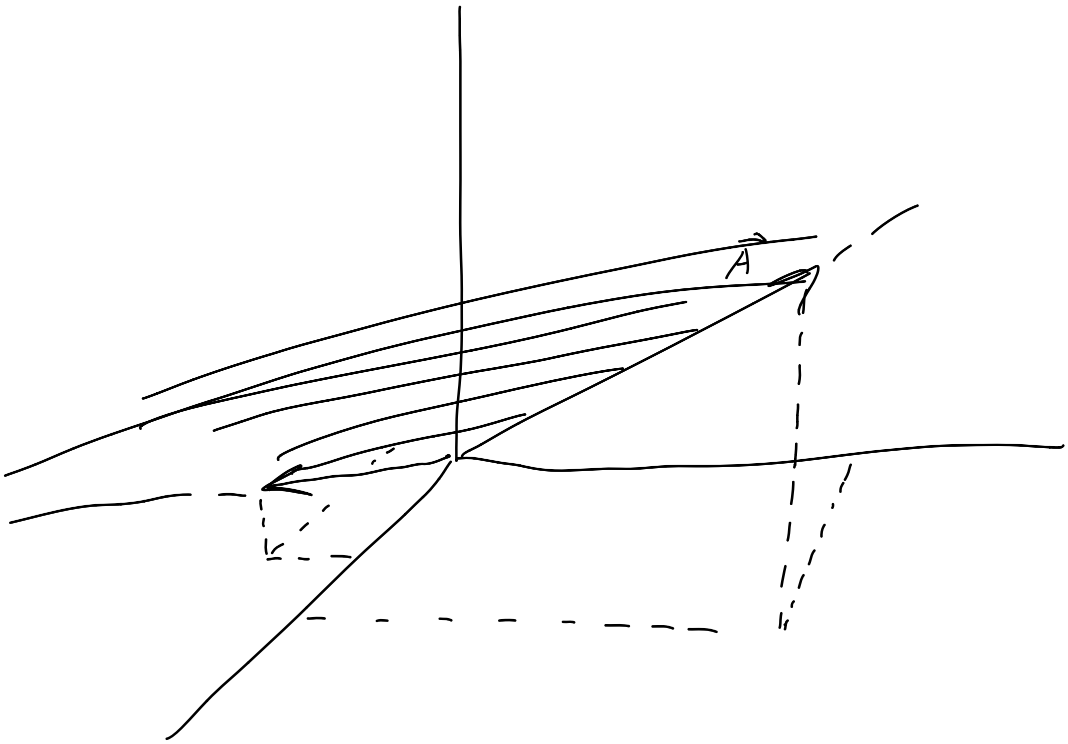


An infinite sheet with $z = 0$ everywhere


z
y z plane
x
y

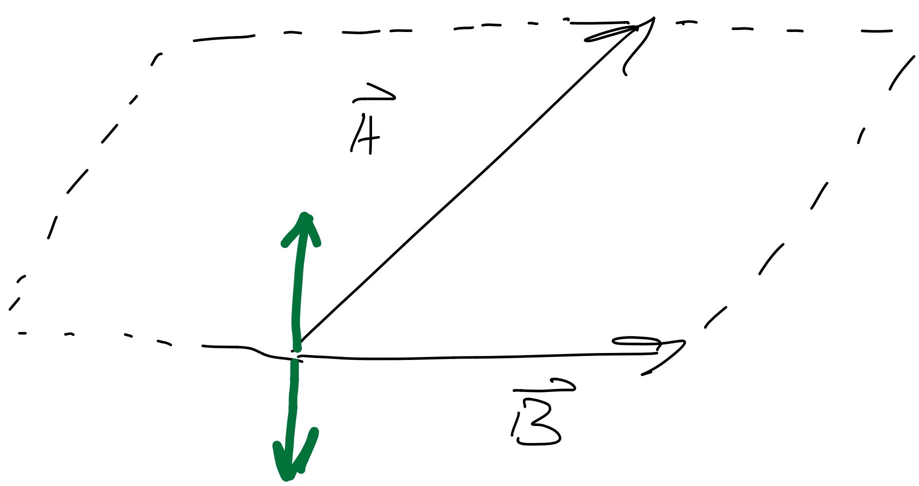

z
xz plane
x

Any two vectors form a plane, not just $\hat{x}, \hat{y}, \hat{z}$



Draw the vectors from the same origin & the imagine stretching a sheet from one to the other

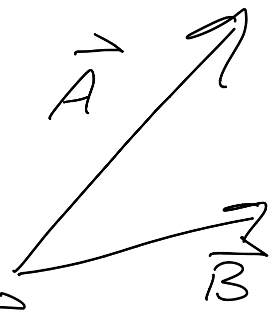
So: vectors $\vec{A}$ + $\vec{B}$ form a surface, the direction of $\vec{A} \times \vec{B}$ is directly away from the surface (perpendicular)



Two possibilities up or down

Solution: Right hand rule

- Lay hand along $\vec{A}$
- Curl fingers in direction of $\vec{B}$
- Thumb is direction of $\vec{A} \times \vec{B}$



$\vec{A} \times \vec{B}$ is into the page

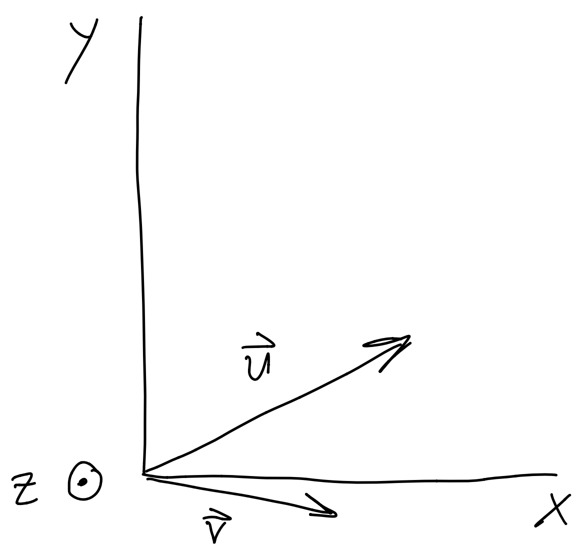
$\vec{A} \times \vec{B}$ is out of the page

Ex:

$$\vec{u} = 5\hat{x} + 3\hat{y}$$

$$\vec{v} = 4\hat{x} - 2\hat{y}$$

What is $\vec{u} \times \vec{v}$



Direction? $(-\hat{z})$

side note:

$\odot$ = "out of page"

$\otimes$ = "into page"

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$$

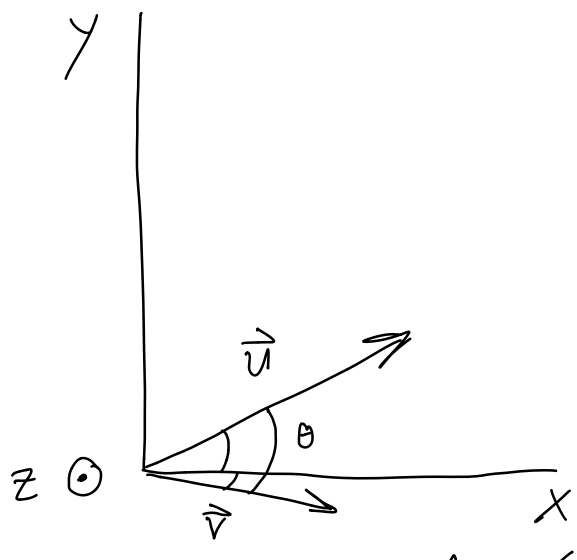
$$\vec{u} = 5\hat{x} + 3\hat{y}$$

$$\vec{v} = 4\hat{x} - 2\hat{y}$$

$$|\vec{u}| = \sqrt{5^2 + 3^2} = 5.8$$

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 4.5$$

What is θ?



$\theta = \theta_{xv} + \theta_{xu}$, $\hat{u} = \langle \cos\theta_{xu}, \cos\theta_{yu} \rangle$

$\hat{v} = \langle \cos\theta_{xv}, \cos\theta_{yv} \rangle$

To find θ, find $\hat{u}$ then use to find θ_{xu}

$\hat{v}$ θ_{xv}

$$\theta = \theta_{xv} + \theta_{xu}$$

An easier way:

Recall:

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z = |\vec{u}||\vec{v}| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$$

$$\vec{u} = 5\hat{x} + 3\hat{y}$$

$$\vec{v} = 4\hat{x} - 2\hat{y}$$

$$\vec{u} \cdot \vec{v} = 5 \cdot 4 + 3 \cdot (-2) = 14$$

$$\theta = \cos^{-1}\left(\frac{14}{5.8 \cdot 4.5}\right) = 57.6^\circ$$

$$|\vec{u} \times \vec{v}| = (5.8)(4.5)\sin 57.6^\circ$$

$$= 21.9$$

$$\vec{u} \times \vec{v} = \langle 0, 0, -21.9 \rangle = -21.9\,\hat{z}$$

$$(\vec{v} \times \vec{u}) = 21.9\,\hat{z}$$

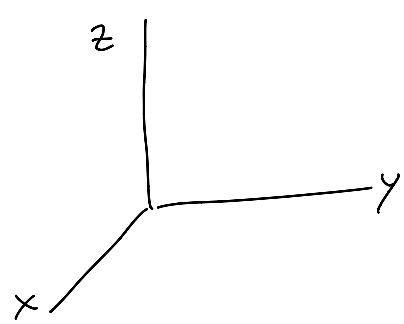
In general:

What is $\hat{x} \times \hat{y}$?

$$\langle 1,0,0 \rangle \times \langle 0,1,0 \rangle$$

$$|\hat{x} \times \hat{y}| = 1 \cdot 1 \cdot \sin 90^\circ$$

Dir: $\hat{z}$



$$\hat{x} \times \hat{y} = \hat{z}$$

$\hat{x} \times \hat{y} = \hat{z}$

$\hat{x} \times \hat{x} = ?$ $\qquad 1 \cdot 1 \cdot \sin 0$

$= 0$

$\hat{y} \times \hat{x} = ?$

$= -\hat{z}$

$\hat{x} \times \hat{z} = -\hat{y}$

$\hat{z} \times \hat{x} = \hat{y}$

- The cross product of any two of $(\hat{x}, \hat{y}, \hat{z})$ gives $\pm$ the third

- Cyclical

$$\begin{tikzcd} & \hat{x} \arrow[dr, bend left] & \\ \hat{z} \arrow[ur, bend left] & & \hat{y} \arrow[ll, bend left] \end{tikzcd}$$

$\hat{y} \times \hat{z}$?

$\hat{x} \times \hat{z}$?

$$\vec{A} = \langle A_x , A_y \rangle$$
$$\vec{B} = \langle B_x , B_y \rangle$$

$$\vec{A} \times \vec{B} = (A_x \hat{x} + A_y \hat{y}) \times (B_x \hat{x} + B_y \hat{y})$$

$$= A_x \hat{x} \times B_x \hat{x} + A_x \hat{x} + B_y \hat{y}$$
$$+ A_y \hat{y} \times B_x \hat{x} + A_y \hat{y} \times B_y \hat{y}$$

$$= A_x B_x (\hat{x} \times \hat{x}) + A_x B_y (\hat{x} \times \hat{y})$$
$$+ A_y B_x (\hat{y} \times \hat{x}) + A_y B_y (\hat{y} \times \hat{y})$$

$$= 0 + A_x B_y \hat{z} - A_y B_x \hat{z} + 0$$

$$= (A_x B_y - A_y B_x) \hat{z}$$

$$\vec{A} = \langle A_x, A_y, A_z \rangle$$

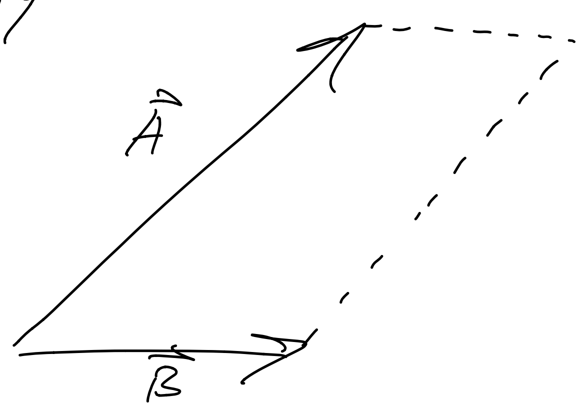
$$\vec{B} = \langle B_x, B_x, B_z \rangle$$

$$\vec{A} \times \vec{B} =$$

$$\langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

Geometry:



Area of parallelogram $= |\vec{A} \times \vec{B}|$