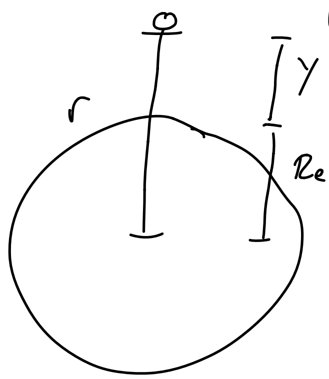
Earlier, we saw $\Delta U = mg\Delta y$

Knowing $U = \dfrac{-Gm_1m_2}{r}$, we can derive this



$$U = \frac{-Gm_em}{r}$$

$$r = R_e + y$$

$$U = -\frac{Gm_em}{R_e+y}$$

if $|y| << R_e$

$$\frac{1}{R_e+y} \approx \frac{R_e - y}{R_e^2}$$

$$U \cong -\frac{Gm_em}{R_e^2}\left(R_e - y\right)$$

$$\cong -\frac{Gm_em}{R_e} + \frac{Gm_e}{R_e^2}my$$

$$U \cong -\frac{Gm_em}{R_e} + mgy$$

$$\Delta U = U_f - U_i = \Delta mgy = mg\Delta y$$

$$\Delta U = mg\Delta y$$

$$U \neq mgy$$

$$U = -\frac{Gm_e m}{R_e} + mgy$$

This will always cancel when we find ΔE_{sys}, so we usually just don't write it

Ex: Find max height of a projectile

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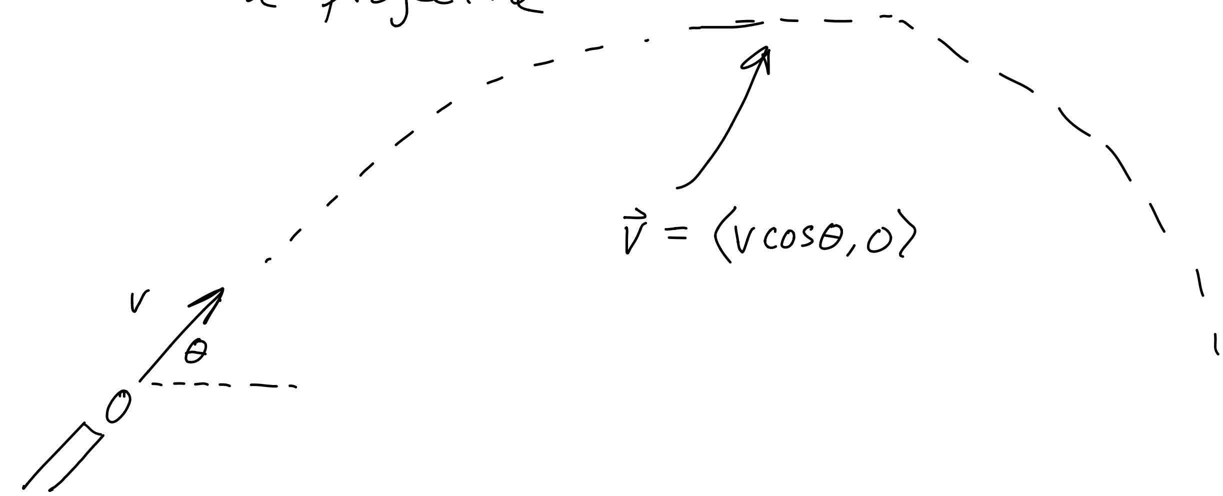


system = ball + Earth

Surr = None

$$E_i = E_f$$

$$E = K_{ball} + \cancel{K_{earth}} + U_{ball-earth}$$

$y = 0$

$$E_i = \frac{1}{2}mv^2 - \frac{G m_e m}{R_e}$$

$$E_f = \frac{1}{2}m(v\cos\theta)^2 - \frac{G m_e m}{R_e} + mgy$$

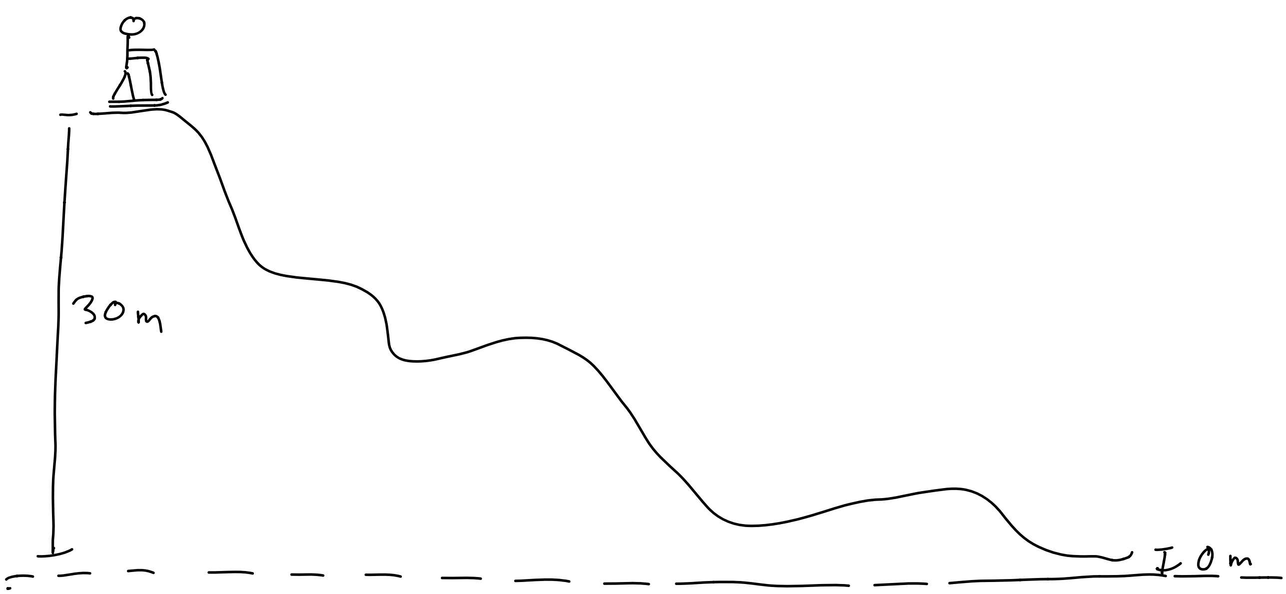
$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2\cos^2\theta + mgy$$

$$\frac{1}{2}v^2(1-\cos^2\theta) = gy$$

$$y = \frac{v^2\sin^2\theta}{2g}$$; same thing we found in Ch 2

The path doesn't matter

Ex: Skier on a hill



Ignore friction:

sys = skier + Earth

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$mgy_i = \frac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{2gy_i} = \sqrt{2(9.8)(30)}$$

$$\boxed{v_f = 24.2\ m/s}$$

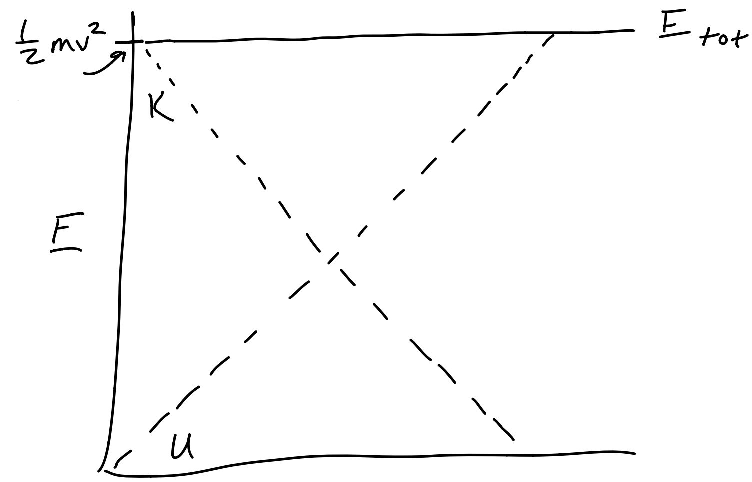
Ex: I throw a ball up in the air

How high will it go?

$$E_i = E_f$$

$$\frac{1}{2}mv^2 = mgy$$

$$y = \frac{v^2}{2g}$$



Can I ever have $U > E$?

$$K + U = E$$

if $U > E$

$$K = E - U < 0$$

$$\frac{1}{2}mv^2 < 0 \quad !!!$$

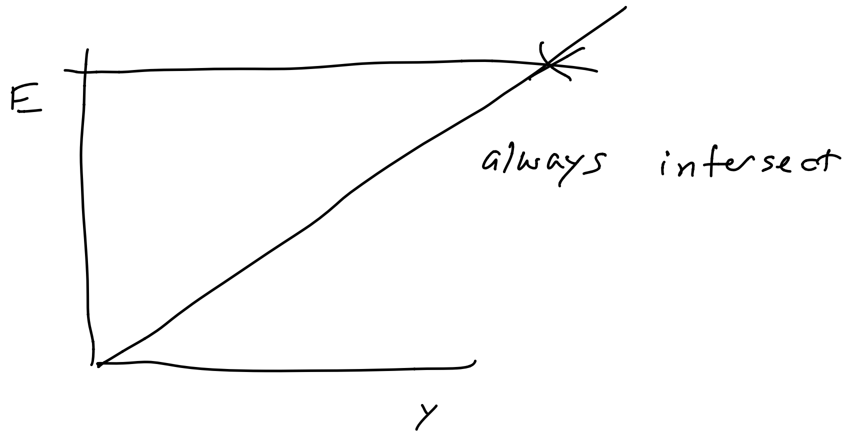
no!

Can $K > E$?

$$U = E - K < 0 \checkmark$$

Fine. U can be negative

No matter how hard I throw the ball, it always reaches a max height & then returns



- What if I throw it fast enough that $U = mgy$ no longer holds

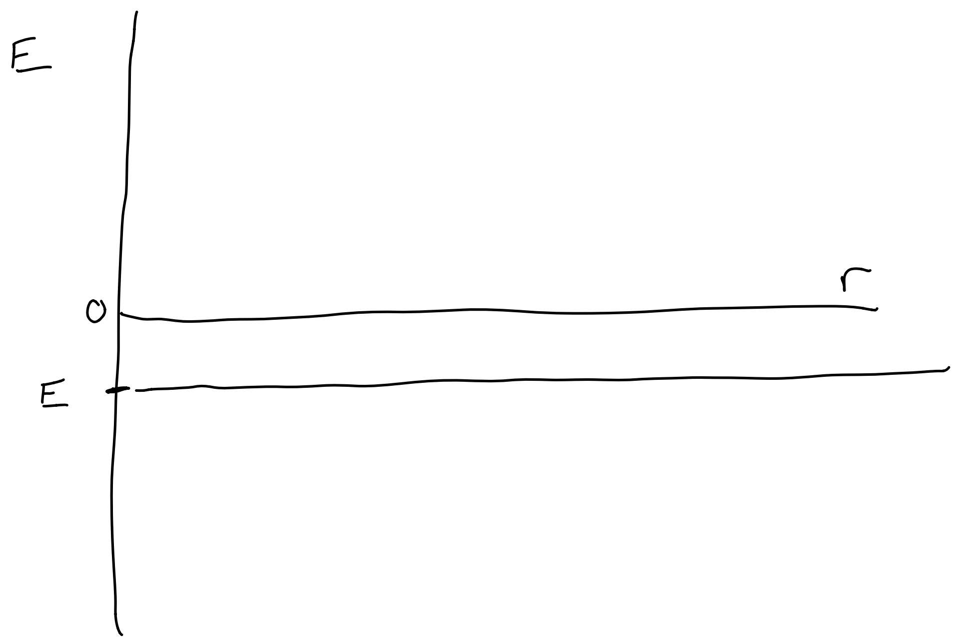
(For a spaceship)

$$U = \frac{-GM_e m}{r}$$

$$E_{initial} = K_i + U_i$$

$$= \frac{1}{2} m v_i^2 - \frac{GM_e m}{R_e}$$

$$E_i = \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}$$



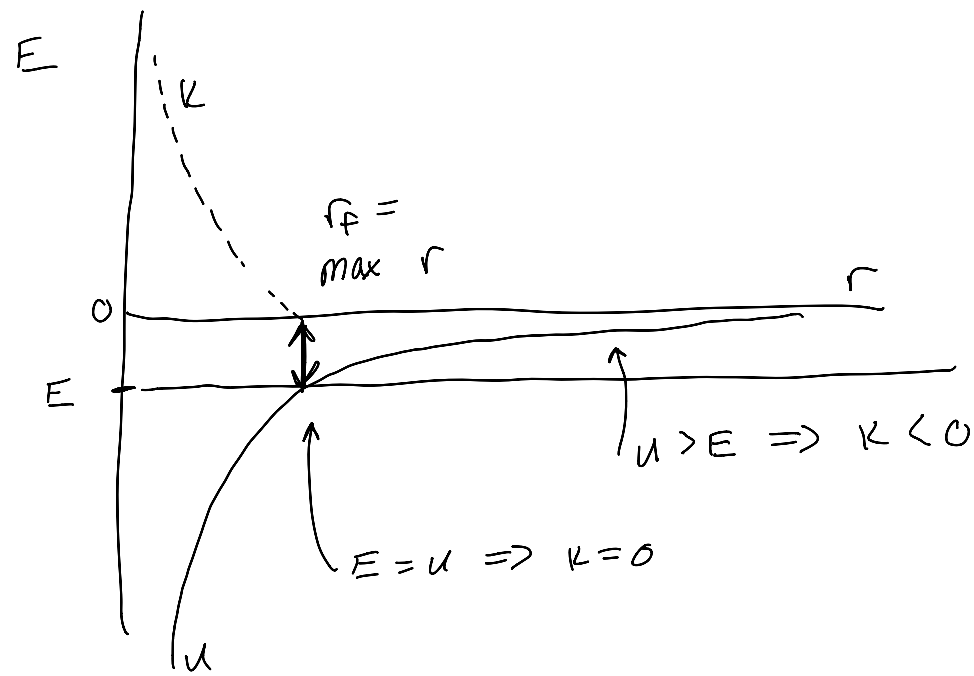
Shape of $-\frac{Gm_e m}{r}$?

Shape of $-\frac{1}{r}$

1) near $r = 0$, $-\frac{1}{r} \rightarrow -\infty$

2) always negative, always increasing w/r

3) as $r \rightarrow \infty$, $-\frac{1}{r} \rightarrow 0$



- Object will travel until it reaches

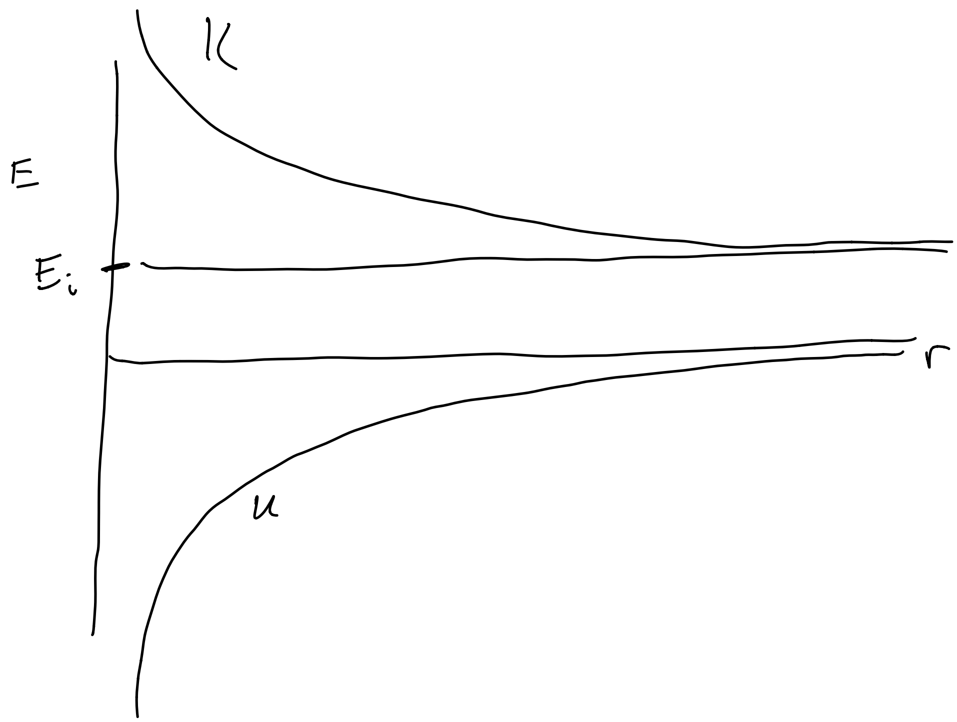
r_f where $E = \dfrac{-G m_e m}{r_f}$

Then it can travel no further

What if the object has a higher v_i?

$$E = E_i = \frac{1}{2} m v_i^2 - \frac{G m_e m}{R_e}$$

In particular, what if $\frac{1}{2} m v_i^2 > \dfrac{G m_e m}{R_e}$



-> U is always negative (or 0)

-> There is no point where $U \geq E$

Object keeps moving away forever!

→ Unbound state

- Object loses some K due to increasing U, but eventually escapes

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 - \frac{Gm_e m}{R_e} = \frac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{v_i^2 - \frac{Gm_e}{R_e}}$$

So, if $E < 0$, object is bound, + returns to Earth

if $E > 0$, object is unbound

- minimum speed necessary to escape given by $E = 0$

$$E = 0 = K_i + U_i$$

$$\frac{1}{2}mv_{esc}^2 - \frac{Gm_e m}{R_e} = 0$$

$$v_{esc} = \sqrt{\frac{2Gm_e}{R_e}} = \sqrt{\frac{(2)(6.7\times10^{-11})(6\times10^{24})}{6.4\times10^6}}$$

$$v_{esc} = 1.12\times10^4 \frac{m}{s} \quad (\approx 25{,}000 \text{ mph})$$

Final notes on potential energy

- Depends only on separation distance between objects (Not their individual positions)

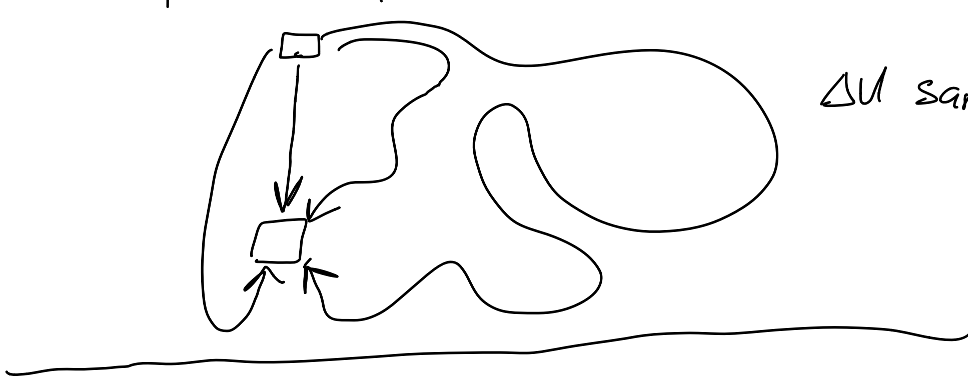
- if $r \to \infty$, $U(r) \to 0$

- For an attractive force (gravity) U increases with r

Opposite for a repulsive force

Can't have a "friction" potential

U is "path independent"



ΔU same