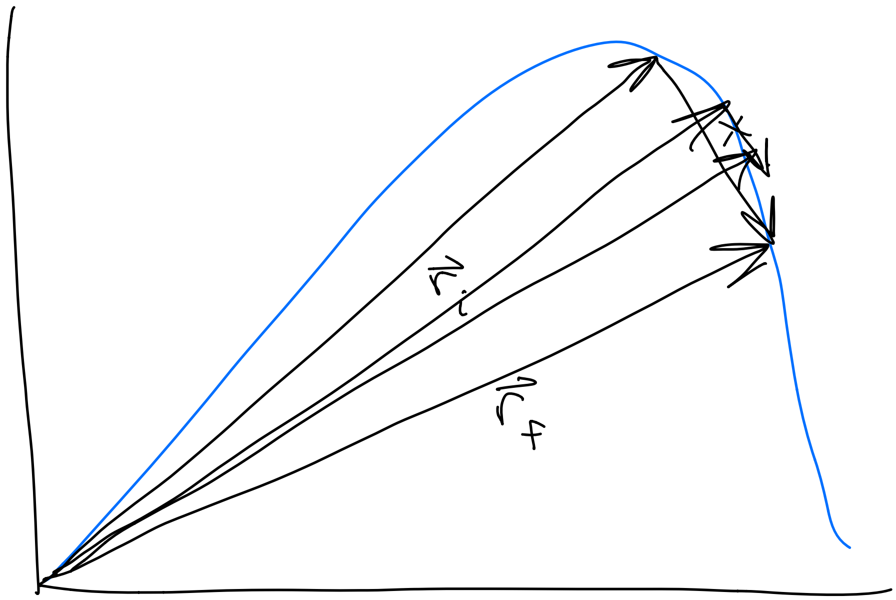


Instantaneous velocity

$$\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

make $\Delta t \rightarrow 0$

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\langle x, y, z\rangle = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$$

$$= \langle v_x, v_y, v_z \rangle$$

velocity is the rate of change of position

if position is changing over time, we have velocity.

what if velocity is changing?

acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

Car goes from 0 - 60 mph in 2.3 s

0 - 27 m/s in 2.3 s

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{2.3\ s} = \frac{\langle 27,0,0\rangle - \langle 0,0,0\rangle\ \frac{m}{s}}{2.3\ \ s}$$

$$\vec{a}_{avg} = \langle 11.7, 0, 0\rangle \frac{m}{s^2} \quad (m/s \text{ per } s)$$

Recap:

- Matter takes up space & contains mass
- matter moves at constant velocity, unless it's interacting w/ other matter
- velocity: $\vec{v} = \frac{d\vec{r}}{dt}$

Demo: catching a golf ball vs a bowling ball

- They are moving at ~ the same velocity
  - more interaction is required to change velocity of bowling ball

For any object moving at velocity $\vec{v}$, how strong of an interaction is needed to change its velocity?

- Answer: Depends on mass

  more mass

Momentum:

$$\vec{p} = m\vec{v}$$

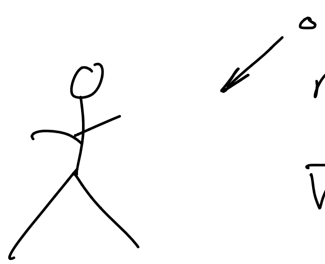
$$\vec{p} = m\vec{v}$$

scala vector

units? $kg \cdot \frac{m}{s}$

The greater an object's <u>momentum</u>, the more it resists change in it's motion

- Consider the above situation

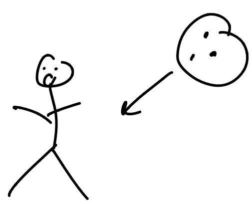


$m = 50\,g = 0.05\,kg$

$\vec{v} = \langle 1, -2, 0 \rangle \frac{m}{s}$

$$\vec{p} = m\vec{v} = 0.05\,kg \cdot \langle 1, -2, 0 \rangle \frac{m}{s}$$

$$\vec{p} = \langle 0.05, -0.1, 0 \rangle\, kg \frac{m}{s}$$



$m = 5\,kg$

$\vec{v} = \langle 1, -2, 0 \rangle \frac{m}{s}$

$$\vec{p} = \langle 5, -10, 0 \rangle\, kg \frac{m}{s}$$

- We see this idea everywhere
  - trains, semis, cars, & bikes stopping

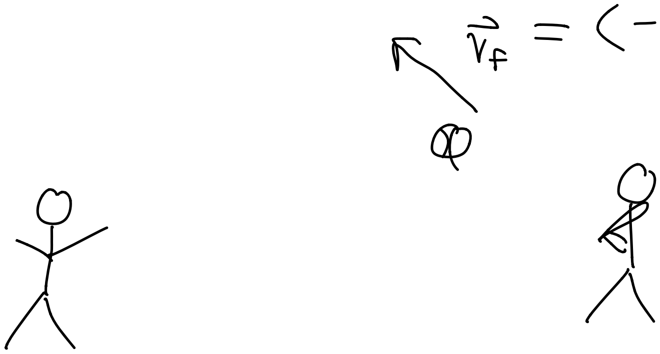
- Chg of $\vec{p}$ is impt concept

What is $\Delta\vec{p}$?

$m = 0.2\ \text{kg}$

$\vec{v}_i = \langle 30, 0, 0 \rangle \frac{m}{s}$

$\vec{v}_f = \langle -38, 14, 0 \rangle \frac{m}{s}$



$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\vec{p}_i = m\vec{v}_i = 0.2\ \text{kg}\ \langle 30, 0, 0 \rangle \frac{m}{s}$$

$$\vec{p}_i = \langle 6, 0, 0 \rangle\ \text{kg}\ \frac{m}{s}$$

$$\vec{p}_f = m\vec{v}_f = 0.2\ \text{kg}\ \langle -38, 14, 0 \rangle \frac{m}{s}$$

$$\vec{p}_f = \langle -7.2, 2.8, 0 \rangle\ \text{kg}\ \frac{m}{s}$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\begin{array}{rl} & \langle -7.2,\ 2.8,\ 0\rangle \\ - & \langle 6,\ 0,\ 0\rangle \\ \hline \end{array}$$

$$\Delta\vec{p} = \langle -13.2,\ 2.8,\ 0\rangle \ \mathrm{kg}\,\frac{m}{s}$$

$$|\Delta\vec{p}| = \sqrt{(-13.2)^2 + 2.8^2} = 13.5 \ \mathrm{kg}\,\frac{m}{s}$$

If you know $\vec{p}$ and m, you know $\vec{v}$

Ex: A hockey ($m = 0.1$ kg) slides on the ice with $\vec{p} = \langle 2, 0, -4\rangle \ \mathrm{kg}\,\frac{m}{s}$.

What will the position of the puck be in 3 s?
relative to current position

$$\vec{p} = m\vec{v}$$

$$\vec{v} = \frac{\vec{p}}{m} = \frac{1}{0.1\ \mathrm{kg}}\langle 2, 0, -4\rangle \mathrm{kg}\frac{m}{s} = \langle 20, 0, -40\rangle \frac{m}{s}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad \Delta\vec{r} = \vec{v}_{avg}\Delta t = \langle 20, 0, -40\rangle \frac{m}{s}(3\,s)$$

$$\Delta\vec{r} = \langle 60, 0, -120\rangle\ m$$