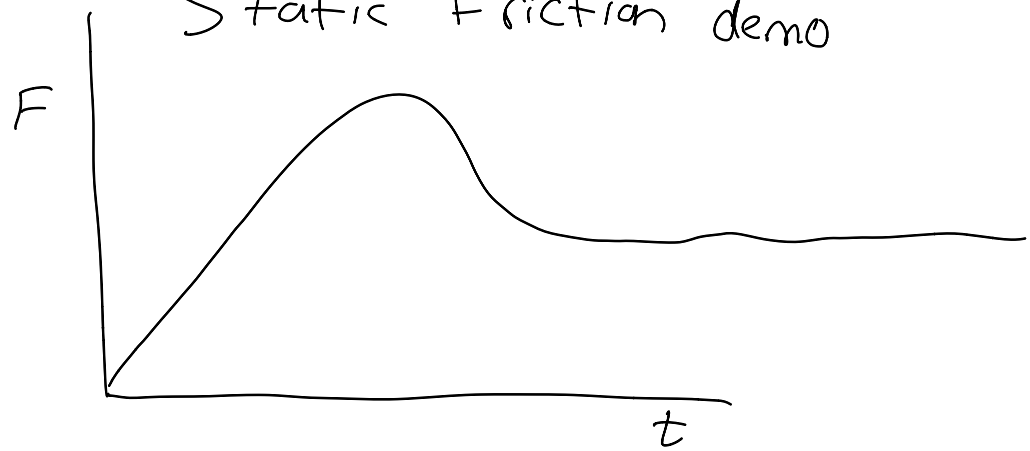
Ask: A function $f(x)$ so that $f''(x) = -f(x)$

Static Friction demo



I exert force on book, book compresses springs, springs push back on book, $F_{net} = 0$, book doesn't move.

As I increase the force, the springs compress more, until eventually the book breaks loose & begins to move. The force required to start moving:

$F_F = \mu_s F_N$; μ_s is coeff of static friction

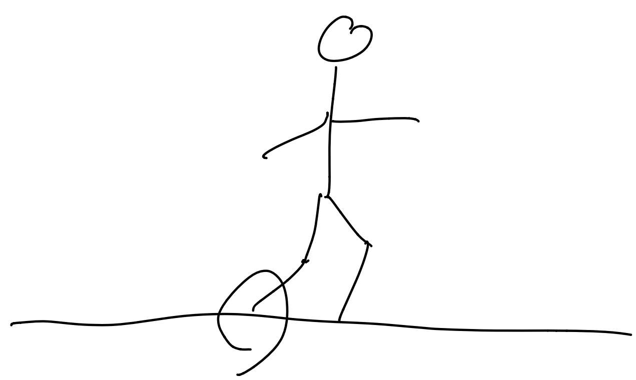
$F_{static} = \mu_s F_N$ ← Force needed to start moving

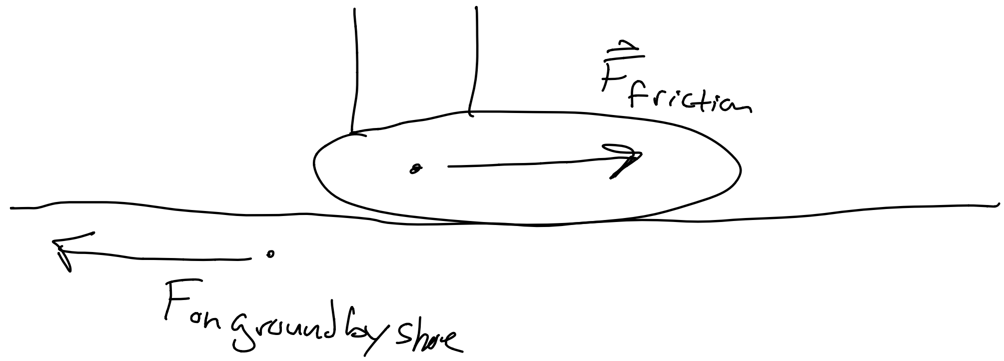
$F_{kinetic} = \mu_k F_N$ ← Force needed to keep moving

In general, $\mu_s \gtrsim \mu_k$

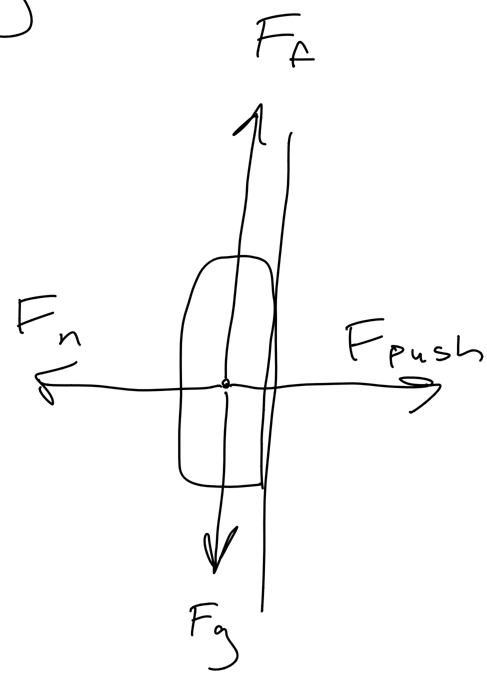
Pull really hard to get an object to start moving; once it starts you don't need to pull as hard

Static Friction is how we walk!





It's also how I can hold this book against the wall w/o it slipping



In order to keep it here,

I need $F_{net,y} = 0$

$$F_f + F_g = 0$$

$$\mu_s F_N - mg = 0$$

$$F_{net\,x} = 0$$

$$F_N = F_{push}$$

$$\mu_s F_{push} - mg = 0$$

$$F_{push} = \frac{mg}{\mu_s}$$

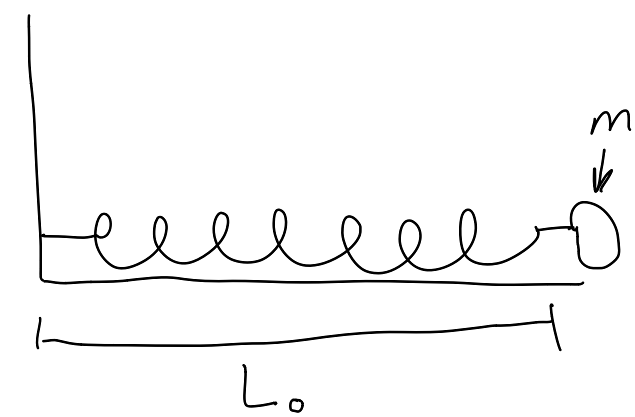
$m = 5\ kg$

$g = 9.8\ m/s^2$ $\quad \mu_s = 0.5$

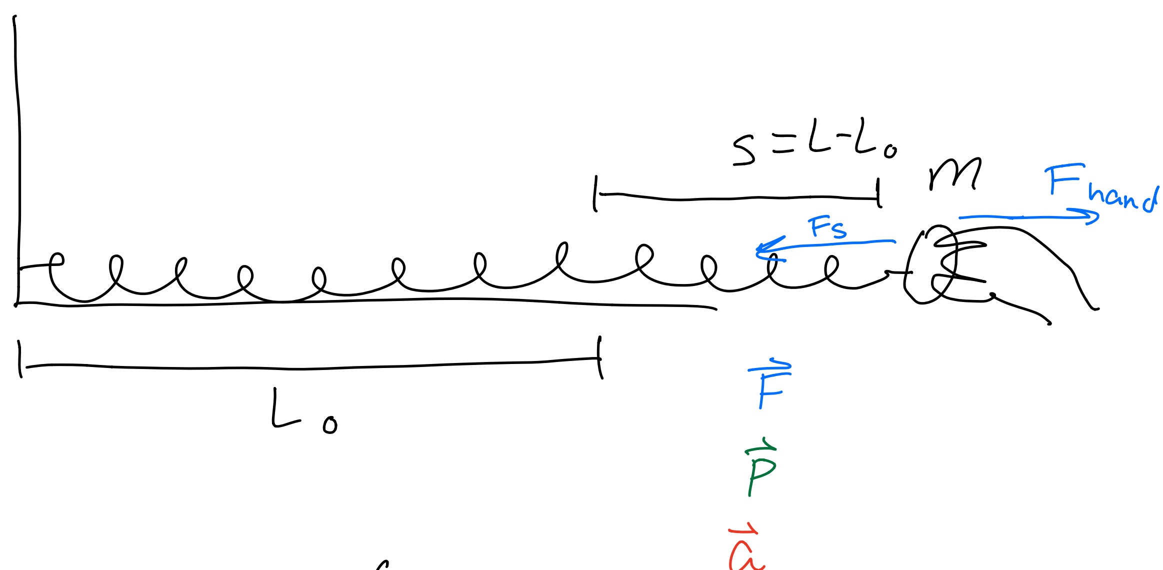
$F_{push} = 98\ N$

(22 lbs)

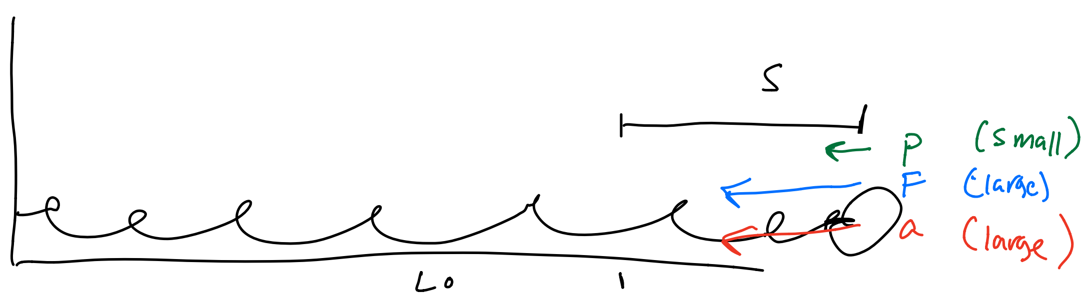
The spring revisited



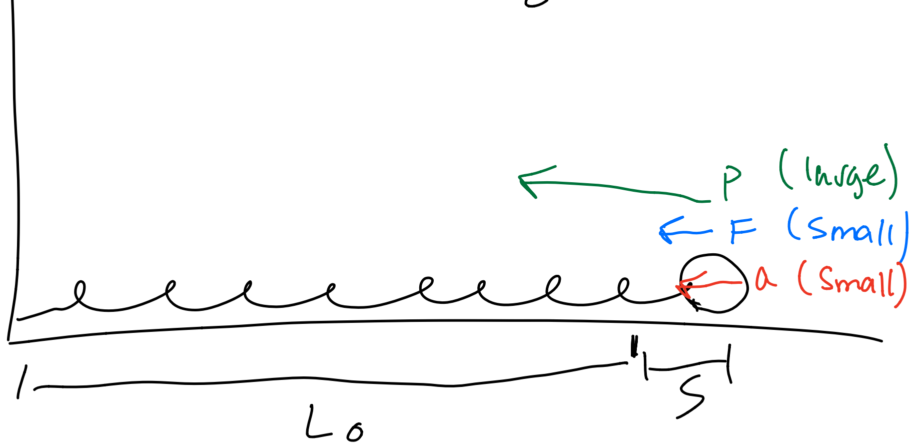
Stretch + Hold



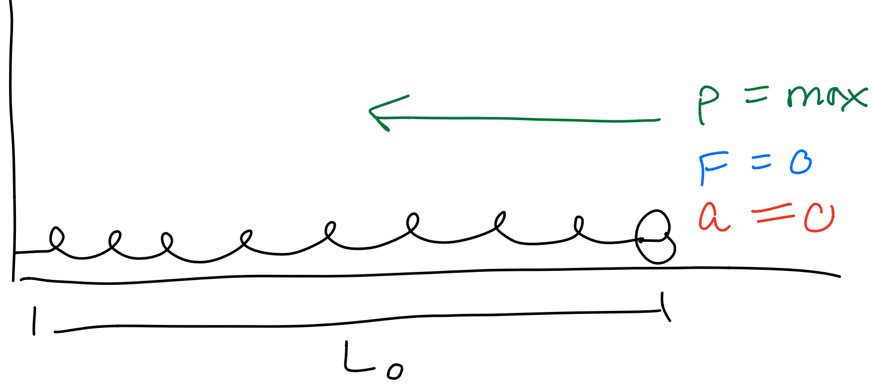
Release (Far from equilibrium)

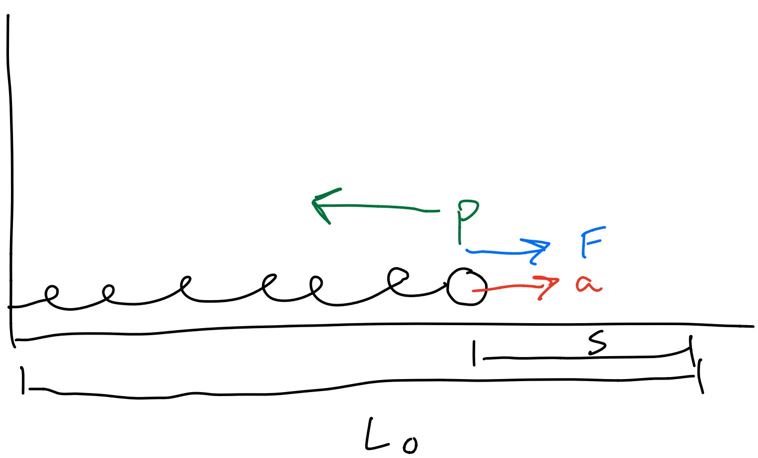


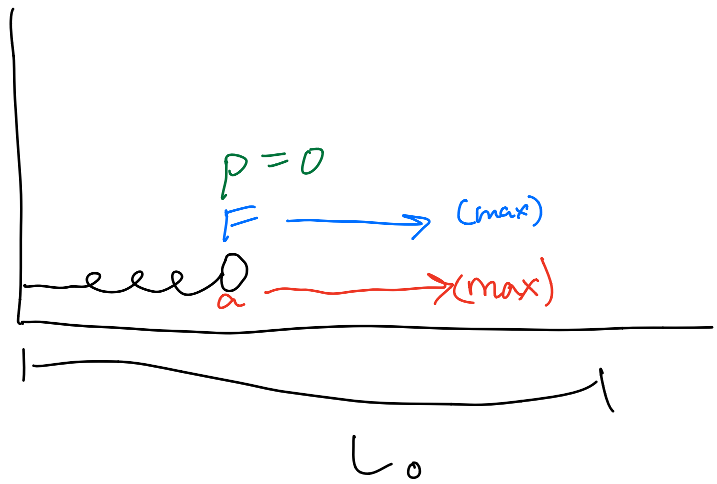
Close to eq.



@ equilibrium







Like projectile motion, I want exact equations

$x(t)$

$p(t)$

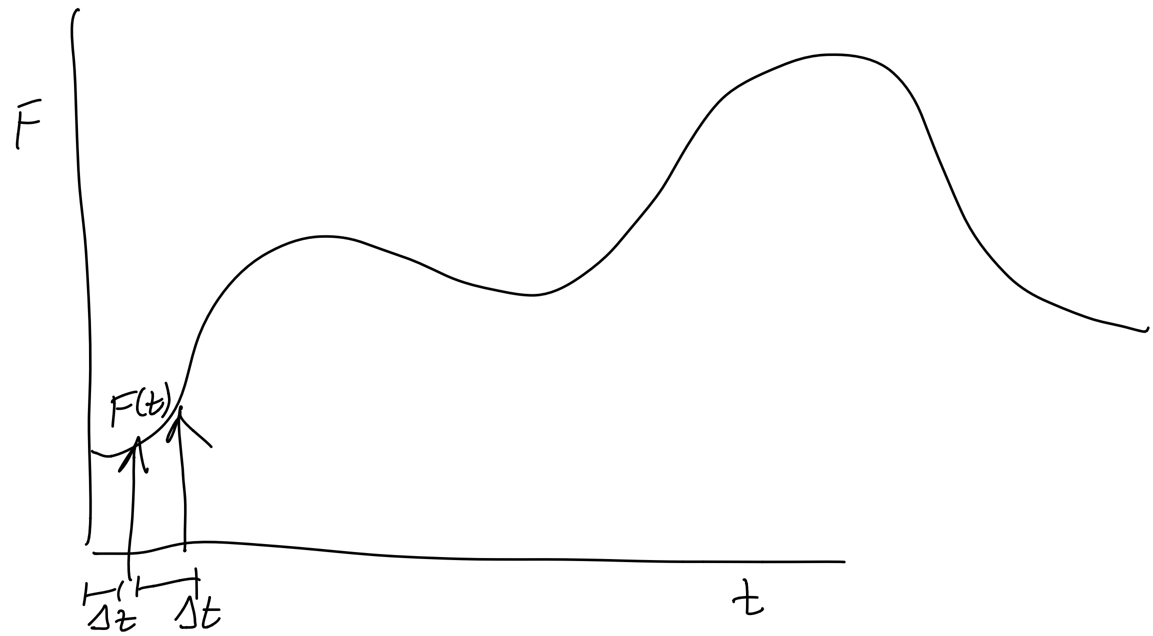
etc...

Start with momentum principle

$$\underbrace{\Delta p = F_{net}\,\Delta t}$$

This applies for time intervals Δt where F_{net} doesn't change

What if F_{net} does change?



$$p = p_i$$

$$p(t=\Delta t) = p_i + F(t=\Delta t)\Delta t$$

$$p(t=2\Delta t) = p(t=\Delta t) + F(t=2\Delta t)\Delta t$$

$$= p_i + F(t=\Delta t)\Delta t + F(t=2\Delta t)\Delta t$$

$$p(t=3\Delta t) = p(t=2\Delta t) + F(t=3\Delta t)\Delta t$$

$$p(t=3\Delta t) = p_i + F(t=\Delta t)\Delta t + F(t=2\Delta t)\Delta t$$

$$+ F(t=3\Delta t)$$

$$p(t=N\Delta t) = p_i + F(t=\Delta t) + F(t=2\Delta t) + \ldots + F(t=N\Delta t)$$

$$t = N\Delta t$$

$$p(t) = N\Delta t$$

$$p(N\Delta t) = p_i + \sum_{n=0}^{n=N} F(n\Delta t)\Delta t$$

Only an approximation

more accurate as $\Delta t \to 0$

$N \to \infty$

Exact

$$p(t) = p_i + \lim_{\substack{\Delta t \to 0 \\ N \to \infty}} \left(\sum_{n=0}^{n=N} F(n\Delta t)\Delta t \right)$$

$$\boxed{p(t) = p_i + \int_0^t F(t)\,dt}$$

Equivalent

$$\frac{dp}{dt} = \frac{d}{dt} p_i + \frac{d}{dt}\int_0^t F(t)\,dt$$

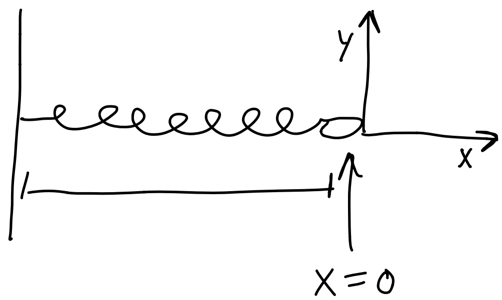
$$\frac{dp}{dt} = F$$

General momentum principle

$$\boxed{\begin{gathered}\frac{d\vec{p}}{dt} = \vec{F}_{net} \\ \Delta\vec{p} = \vec{p}_i + \int_0^t \vec{F}_{net}\,dt\end{gathered}}$$

$$\frac{d\vec{p}}{dt} = \left\langle \frac{dp_x}{dt}, \frac{dp_y}{dt}, \frac{dp_z}{dt} \right\rangle$$

Spring-mass system



$s = x$ $F_x = -kx$

$p_x = p_i + \int F_x \, dt$

Dont know $F(t)$

$$\frac{dp_x}{dt} = F_x$$

$$\frac{dp_x}{dt} = -kx$$

$$\frac{d}{dt}(mv_x) = -kx$$

$$m \frac{d}{dt}\left(\frac{d}{dt}x\right) = -kx$$

$$\frac{d^2}{dt^2}x = -\frac{k}{m}x$$

This is a differential equation

derivative of x also depends on x

Can't just integrate each side

have some function $f(t)$ so that $f''(t) = -f(t)$

What about $x(t) = \cos(t)$

$$\frac{d^2}{dt^2}\cos(t) = \frac{d}{dt}\left(-\sin(t)\right) = -\cos(t)$$

Close, but not quite:

I need $\frac{d^2}{dt^2}x = -\frac{k}{m}x$

Try $x(t) = \cos\left(\sqrt{\frac{k}{m}}\,t\right)$

$$\frac{d^2}{dt^2}\cos\left(\sqrt{\tfrac{k}{m}}\,t\right) = \frac{d}{dt}\left(-\sqrt{\tfrac{k}{m}}\sin\left(\sqrt{\tfrac{k}{m}}\,t\right)\right) = -\sqrt{\tfrac{k}{m}}\sqrt{\tfrac{k}{m}}\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)$$

$$= -\frac{k}{m}\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)$$

$\sqrt{\frac{k}{m}} = \omega$ = angular frequency

f = # of oscillations / sec

$\omega = 2\pi f$

$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$x(t) = \cos(\omega t)$; $\omega = \sqrt{k/m}$

$\max(\cos) = 1$

$x(t) = A\cos(\omega t)$

A is a constant determined by where the mass is @ $t = 0$

Ex: If I pull the spring 10 cm from eq

$x(0) = A\cos(0) = A = 0.1$

$x(t) = 0.1\cos(\omega t)$

Alternative approach to constant force motion

Start:

$$\frac{dp_x}{dt} = F_x$$

$$p_x = p_i + \int_0^t F_x \, dt$$

$$= p_i + F_x \int_0^t dt$$

$$p_x = p_i + F_x t$$

$$v_x = \frac{p_x}{m} = v_i + \frac{F_x}{m} t$$

$$v_x = \frac{dx}{dt}$$

$$\frac{dx}{dt} = v_i + \frac{F_x}{m} t$$

$$dx = v_i dt + \frac{F_x}{m} t dt$$

$$\Delta x = x(t) - x_i = \int_0^t \left(v_i dt + \frac{F_x}{m} t dt \right)$$

$$= v_i t + \frac{1}{2} \frac{F_x}{m} t^2$$

$$x(t) = x_i + v_i t + \frac{1}{2} \frac{F_x}{m} t^2$$

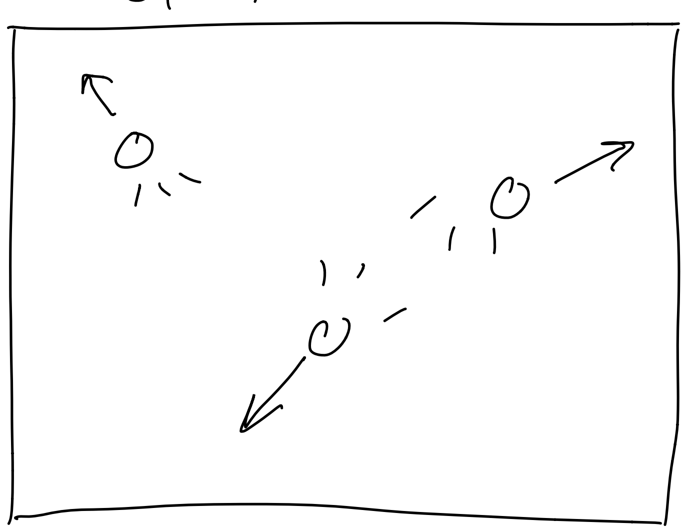
Think about the air in this room

What is air?

A bunch of atoms, mostly nitrogen & oxygen, moving around randomly

At room temp, $v \sim 500$ m/s
(1100 mph)

Not like solids



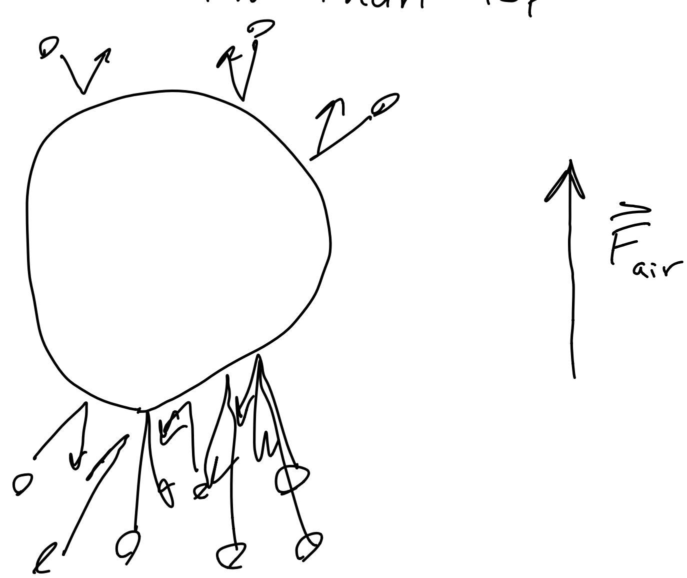
On average, $\vec{v} = 0$ (no preferred direction)

- Air molecules are constantly bombarding you from every direction

- Each time they collide, they exert a force on you (very small, since mass of a molecule is very small (2×10^{-23} g))

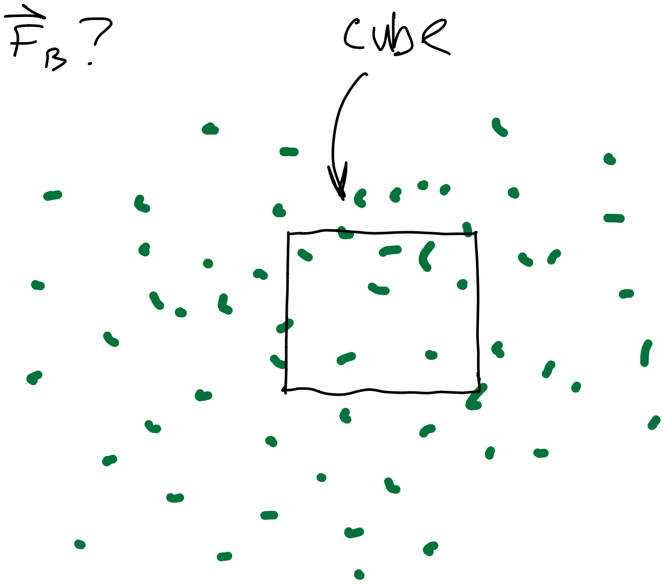
- But air gets less dense w/ altitude (due to gravity)

- more collisions on bottom than top



This is the buoyant force $\vec{F}_B$

What is $\vec{F}_B$?



molecules are always entering & exiting the box, but on average, the air density in the box is the same as everywhere around it

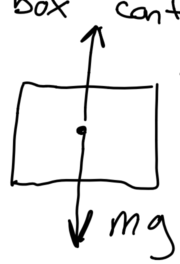
- the box is not moving ($p = 0$)

If it were moving, we would feel a rush of air

This means that $\vec{F}_{net} = 0$ on the box

What are the forces?

The box contains air, which has mass

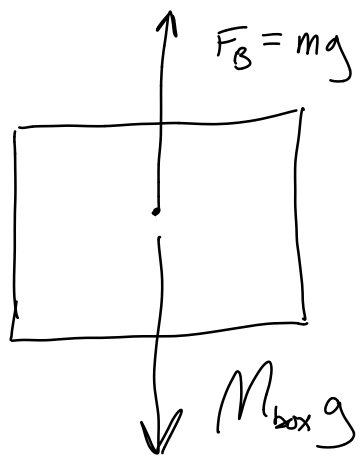


m is the total mass of air in the box

$$F_B - mg = 0 \Rightarrow F_B = mg$$

- Now, remove the box of air & replace it with an actual box filled with something (mass $= M_{box}$)

sum forces:



$$|\vec{F}_{net}| = mg - M_{box}g$$

Upward force equal to the weight of the air displaced by the object

if box has a volume V, the

$$m = (\text{air density}\left[\frac{kg}{m^3}\right] \times V\,[m^3]$$

$$m = \rho_{air} V$$

$$F_B = \rho_{air} V g$$

$$F_{net} = \rho_{air} V g - Mg$$

$$F_{net} = (\rho_{air} - \rho_{box}) V g$$

$$F_{net} = (\rho_{air} - \rho_{obj}) V_{obj}\, g$$

Object is:

- less dense than air?
  float upward
- as dense as air?
  not move
- denser than air?
  sink

Same principle applies to water:

$$F_B = \rho_{water} V_{obj}\, g$$

$$F_{net} = (\rho_{water} - \rho_{obj}) V_{obj}\, g$$

This is why an 800 million lb aircraft carrier floats

Why you sink in water when you exhale all air