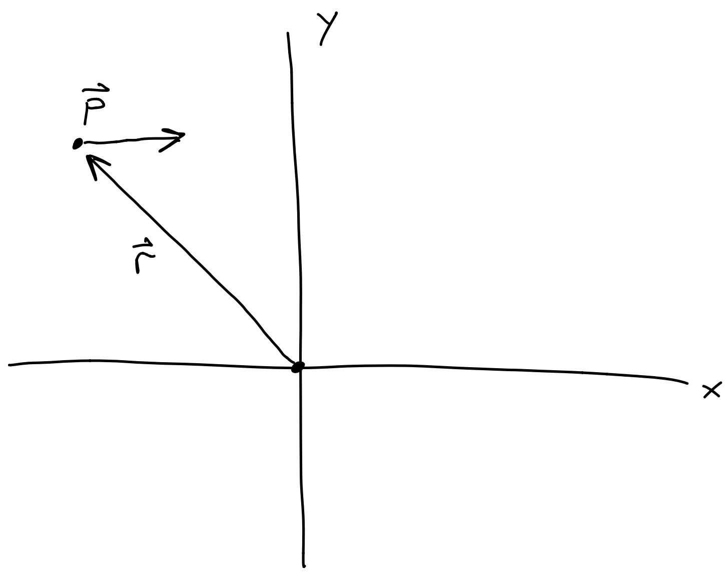
Last time:

$$\vec{L}_{orb} = \vec{r} \times \vec{p}$$

Ex:



$\vec{r} = \langle -4, 3, 0 \rangle$ m

$\vec{p} = \langle 10, 0, 0 \rangle \frac{kg\,m}{s}$

What is $\vec{L}$?

2 ways:

1)
$$\vec{L} = \vec{r} \times \vec{p} = \langle y p_z - z p_y ,\ z p_x - x p_z ,\ x p_y - y p_x \rangle$$

$$= \langle (3)(0) - (0)(0) ,\ (0)(10) - (-4)(0) ,\ (-4)(0) - (3)(10) \rangle$$

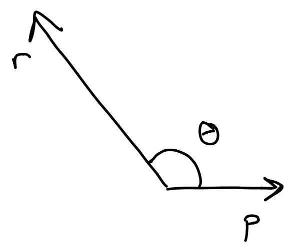
$$\vec{L}_{orb} = \langle 0, 0, -30 \rangle \frac{kg\,m^2}{s}$$

2) $|\vec{L}| = |\vec{r}||\vec{p}|\sin\theta$

$|\vec{r}| = \sqrt{3^2 + 4^2} = 5 \text{ m}$

$|\vec{p}| = 10 \frac{\text{kg m}}{\text{s}}$

$\sin\theta$



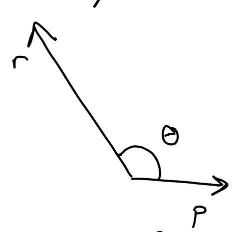
$\hat{r} = \langle -\frac{4}{5}, \frac{3}{5}, 0 \rangle = \langle \cos\theta, \sin\theta, 0 \rangle$

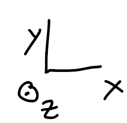
$\theta = \cos^{-1}\left(\frac{4}{5}\right) = 143.1^\circ$

$\sin\theta = \frac{3}{5}$

$|\vec{L}_{orb}| = (5\text{m})\left(10\frac{\text{kg m}}{\text{s}}\right)\left(\frac{3}{5}\right) = 30 \frac{\text{kg m}^2}{\text{s}}$

Direction: $-\hat{z}$



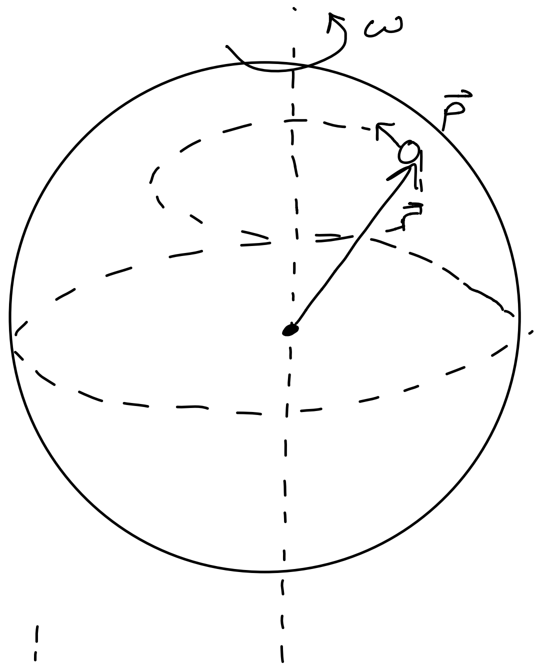


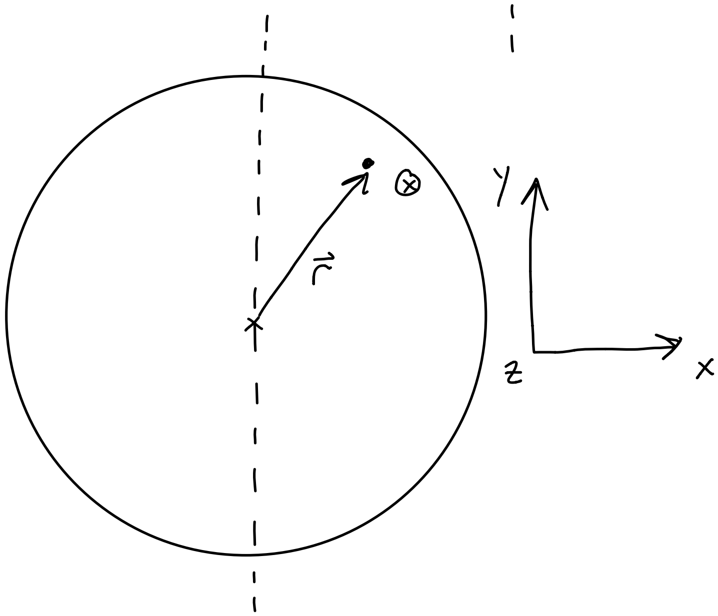
$\vec{L}_{orb} = \langle 0, 0, -30 \rangle \frac{\text{kg m}^2}{\text{s}}$

Ch 9: $K_{tot} = K_{trans} + K_{rel}$

Ch 11: $\vec{L} = \vec{L}_{orb} + \vec{L}_{rel}$

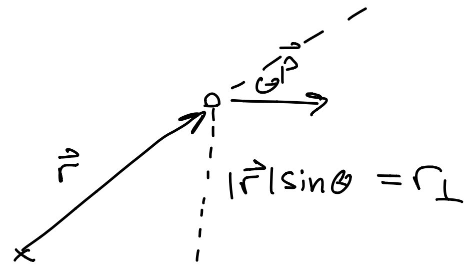
$\vec{L}_{rel}$: total angular momentum relative to COM





$$\vec{L} = \vec{r} \times \vec{p}$$

$$|\vec{L}| = |\vec{r}|\,|\vec{p}| \sin\theta$$

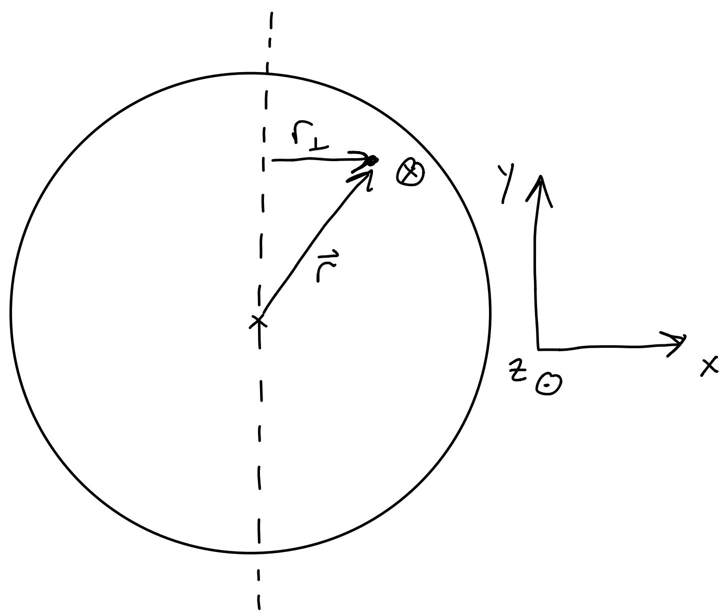


$$|\vec{L}| = \left(|\vec{r}| \sin\theta\right) |\vec{p}|$$

$$= r_\perp |\vec{p}|$$

$r_\perp$ = component of $\vec{r}$ perp to $\vec{p}$

= $\perp$ distance to axis of rotation



$$|\vec{L}| = r_\perp |\vec{p}| \qquad \text{Direction: } \hat{y}$$

$$|\vec{p}| = mv$$

$$v = \omega r_\perp$$

$$= m\omega r_\perp$$

$$|\vec{L}| = r_\perp (m\omega r_\perp) = m\omega r_\perp^2$$

$$|\vec{L}_{rel}| = m_1 r_{\perp 1}^2 \omega + m_2 r_{\perp 2}^2 \omega + m_3 r_{\perp 3}^2 \omega + \ldots$$

$$= \left(\sum_i m_i r_{\perp i}^2 \right) \omega$$

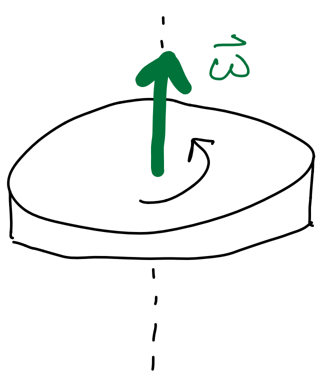
$$|\vec{L}_{rel}| = I \omega$$

Direction: Given by right hand rule

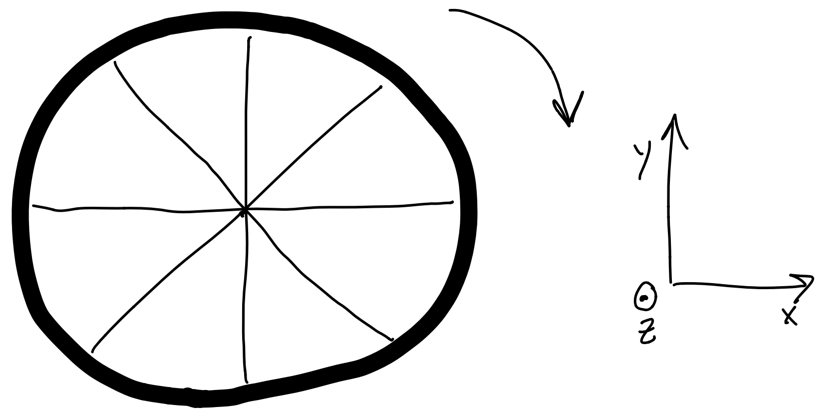
$$\vec{L}_{rel} = I \vec{\omega},$$

$$|\vec{\omega}| = \frac{\text{rad}}{\text{sec}} = 2\pi \cdot \frac{\#\text{rots}}{\text{sec}}$$

direction of $\vec{\omega}$: given by right hand rule



Bike tire:



$R = .3\,m$

$M = 2\,kg$

$I = MR^2$

$f = 5\ \frac{rot}{sec}$

$|\vec{\omega}| = 2\pi f = 31.4\ \frac{rad}{sec}$

$\vec{\omega} = -31.4\ \frac{rad}{s}\ \hat{z}$

$\vec{\omega} = \langle 0, 0, -31.4 \rangle\ \frac{rad}{s}$

$\vec{L}_{rel} = I\vec{\omega}$

$= MR^2 \langle 0, 0, -31.4 \rangle\ \frac{rad}{sec}$

$= (2kg)(0.3m)^2 \langle 0, 0, -31.4\ \frac{rad}{sec} \rangle$

$\vec{L}_{rel} = \langle 0, 0, -18.84 \rangle\ \frac{kg\ m^2}{s}$

$\vec{L} = I\vec{\omega}$

Linear momentum:

$\vec{p} = m\vec{v}$

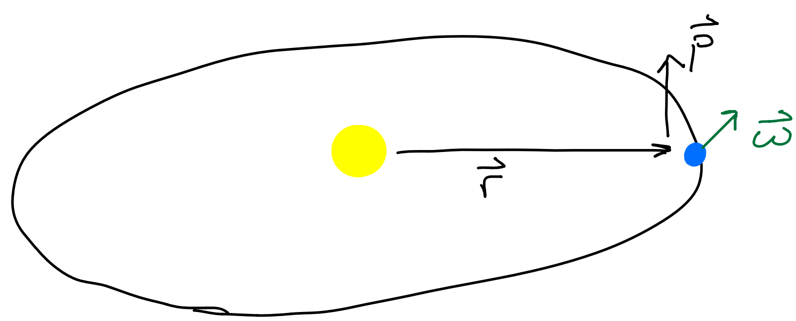
$m \longrightarrow I$

$\vec{v} \longrightarrow \vec{\omega}$

$\vec{p} \longrightarrow \vec{L}$

Total angular momentum: $\vec{L} = \vec{L}_{orb} + \vec{L}_{rel}$

Ex: $\vec{L}$ of Earth



$\vec{L} = \vec{r} \times \vec{p} + I_{earth}\vec{\omega}$

Momentum principle:

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

What is $\frac{d\vec{L}}{dt}$?

$$\vec{L} = \vec{r} \times \vec{p}$$

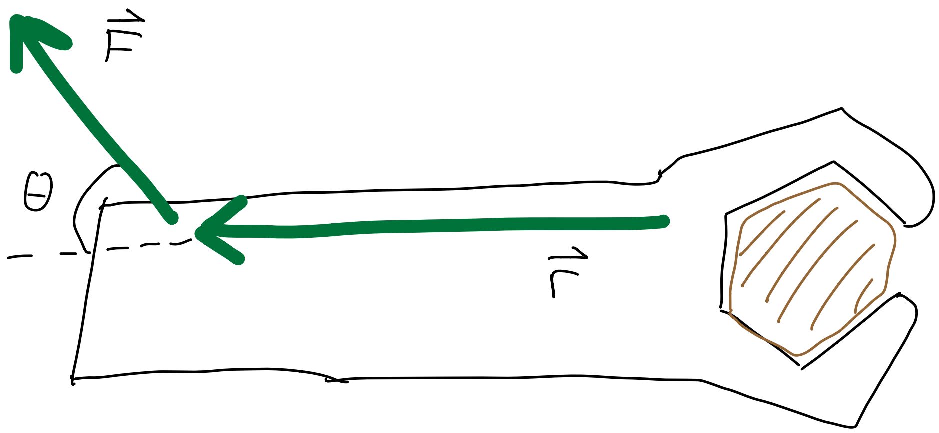
$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right) = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{r} \times \vec{F} + \vec{v} \times m\vec{v}$$

$$= \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau} \quad \text{(torque)}$$

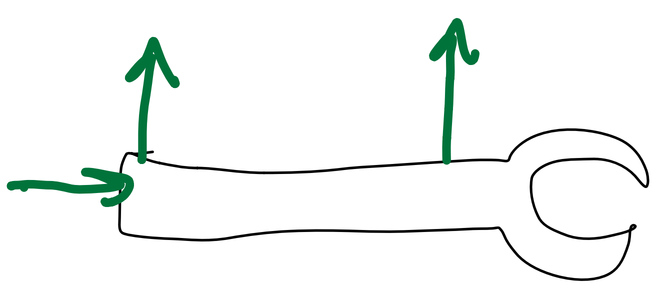
Torque: A "twisting" forc



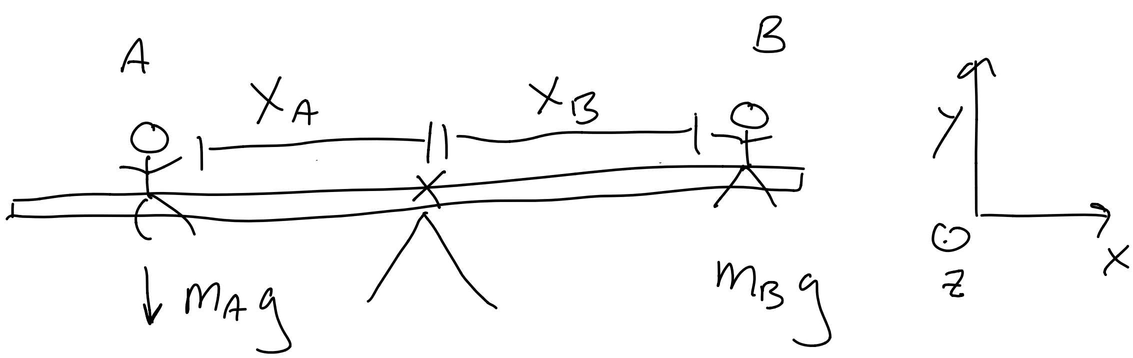
$$|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$$

Torque increases with:

- magnitude of applied force
- Distance away (lever arm)
- angle between $\vec{r}$ & $\vec{F}$



Ex:



$$\vec{\tau}_{net} = \vec{\tau}_A + \vec{\tau}_B$$

$$\vec{\tau}_A = \vec{r}_A \times \vec{F}_A$$

$$= \langle -x_A, 0, 0 \rangle \times \langle 0, -m_A g, 0 \rangle$$

$$\vec{\tau}_A = \langle 0, 0, m_A g x_A \rangle$$

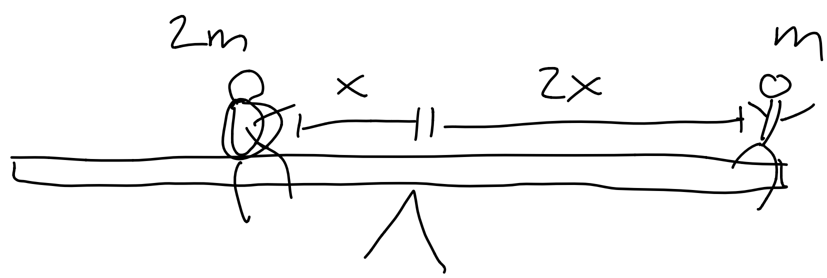
$$\vec{\tau}_B = \langle x_B, 0, 0 \rangle \times \langle 0, -m_B g, 0 \rangle$$

$$\vec{\tau}_B = \langle 0, 0, -m_B g x_B \rangle$$

$$\vec{\tau}_{net} = \langle 0, 0, m_A g x_A - m_B g x_B \rangle$$

$$\vec{\tau}_{net} = \langle 0, 0, m_A g x_A - m_B g x_B \rangle$$

If kid A is twice as heavy as kid B, kid B should sit twice as far away



Kid A : 30 kg , 2 m

Kid B : 40 kg , $x_B = ?$

$$m_A g x_A - m_B g x_B = 0$$

$$(30\text{ kg})(2\text{m}) = (40\text{kg})(x_B)$$

$$x_B = \frac{30}{40}(2\text{m}) = 1.5\text{ m}$$

If $m_A = 35\text{ kg}$, $m_B = 32\text{ kg}$

$x_A = 1.5\text{ m}$ $\quad x_B = 2.5\text{ m}$

$$\vec{\tau}_{net} = \langle 0, 0, m_A g x_A - m_B g x_B \rangle$$

$$= \langle 0, 0, (35\text{ kg})(9.8\tfrac{m}{s^2})(1.5\text{ m}) - (32\text{ kg})(9.8\tfrac{m}{s^2})(2.5\text{ m}) \rangle$$

$$= \langle 0, 0, -269.5 \rangle \text{ kg}\frac{m^2}{s^2}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \left(-269.5 \text{ kg}\frac{m^2}{s^2}\right)\hat{z}$$