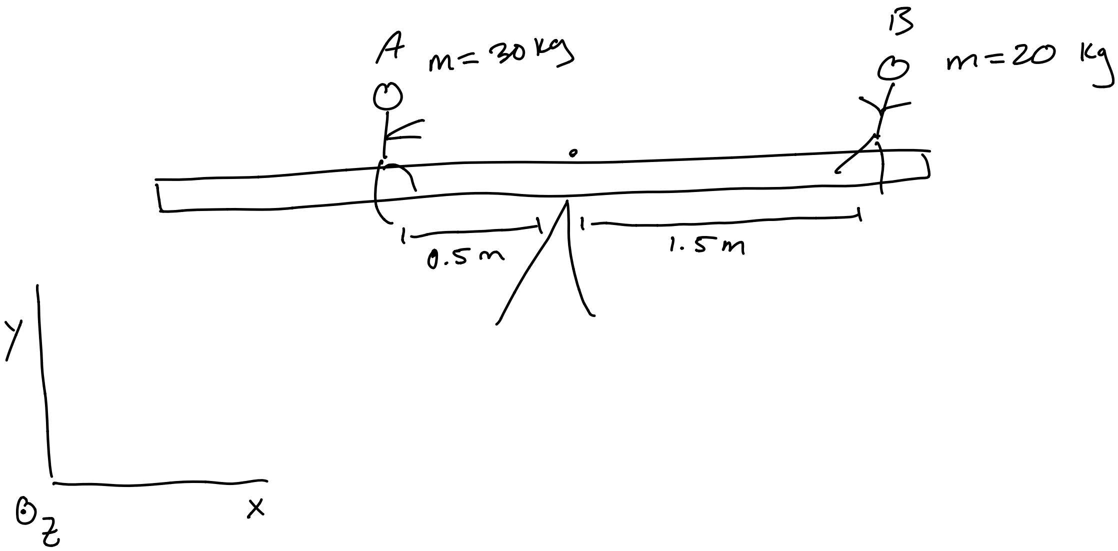
Torque: $\vec{\tau} = \vec{r} \times \vec{F}$

$$|\vec{\tau}| = |\vec{r}|\,|\vec{F}| \sin\theta$$

Classic example: the teeter totter (see-saw?)



$$\vec{\tau}_A = \vec{r}_A \times \vec{F}_A$$

$$\vec{r}_A = -0.5\,\hat{x}$$

$$\vec{F}_A = -(30\,\text{kg})(9.8\,\tfrac{m}{s^2})\,\hat{y}$$

$$\vec{\tau}_A = -(0.5m)\,\hat{x} \times (-294\,N)\,\hat{y}$$

$$\vec{\tau}_A = (147\,Nm)\,\hat{z}$$

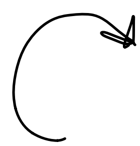
$$\vec{\tau}_B = (1.5\,m)\,\hat{x} \times (-196\,N)\,\hat{y}$$

$$\vec{\tau}_B = (-294\ Nm)\,\hat{z}$$

$$\vec{\tau} = \vec{\tau}_A + \vec{\tau}_B = (147\ Nm)\,\hat{z} - (294\ Nm)\,\hat{z}$$

$$\vec{\tau} = -147\ Nm\ \hat{z}$$

What direction?



Even though kid "B" weighs less, the teeter totter "teets" toward him, since he is sitting farther away

New fundamental principle

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

$$\Delta \vec{L}_{sys} + \Delta \vec{L}_{surr} = 0$$

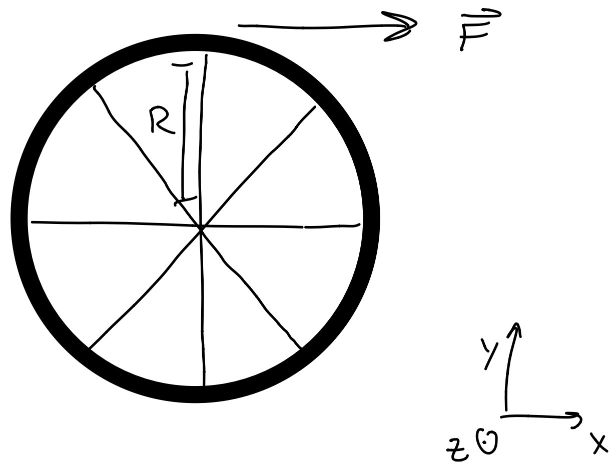
(conservation of angular momentum)

Angular momentum cannot be created or destroyed; like momentum & energy

Ex: Rotating bicycle wheel (demo)

As I speed up this wheel, I apply a torque, causing it to spin

- The angular moment principle tells us how fast it will spin



$M = 5\ \mathrm{kg}$

$R = 0.3\ \mathrm{m}$

$|\vec{F}| = 50\ \mathrm{N}$

$|\vec{\tau}| = 50\ \mathrm{N} \cdot 0.3\ \mathrm{m} = 15\ \mathrm{N \cdot m}$

$\vec{\tau} = \langle 0, 0, -15 \rangle\ \mathrm{N \cdot m}$

$\frac{d\vec{L}}{dt} = \vec{\tau}$ Apply torque for 0.5 s

$\vec{L}_f = \vec{L}_i + \vec{\tau}\,\Delta t$ $\qquad \mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{m} \cdot \mathrm{s}$

$= 0 + \langle 0, 0, -15 \rangle \cdot 0.5\ \mathrm{N \cdot m \cdot s}$

$\vec{L}_f = \langle 0, 0, -7.5 \rangle\ \frac{\mathrm{kg\,m^2}}{\mathrm{s}}$

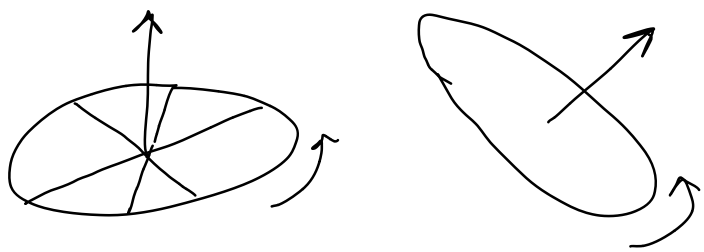
$\vec{L} = I\vec{\omega} = MR^2\vec{\omega}$ $\qquad \langle 0, 0, -7.5 \rangle\ \frac{\mathrm{kg\,m^2}}{\mathrm{s}} = (5\,\mathrm{kg})(0.3\,\mathrm{m})^2\,\vec{\omega}$

$\vec{\omega} = \langle 0, 0, -16.7 \rangle\ \frac{\mathrm{rad}}{\mathrm{sec}} \approx 2.7\ \frac{\mathrm{rot}}{\mathrm{sec}}$

- What direction is the angular momentum?

- Can I change the angular momentum without slowing the wheel?

- Yes, by tilting it



I am applying a torque

- This is why bikes are so steady when they are moving fast

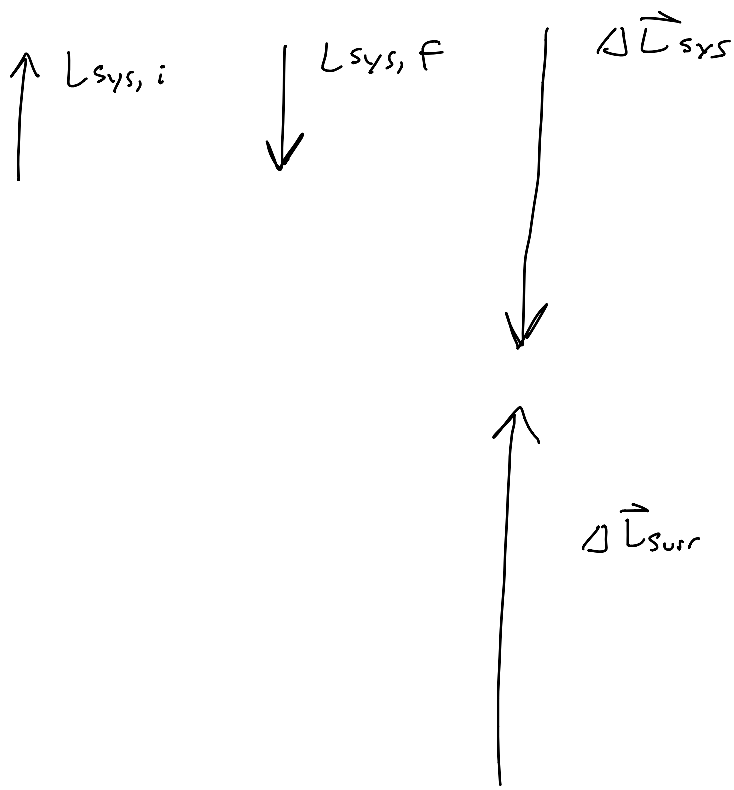
When I turned the wheel, $\vec{L}$ changed

Where did it go?

- I exert torque on wheel
- Wheel exerts torque on me
- My body transfers that torque to the Earth
- $\vec{L}$ of Earth changes

We can't detect a change in $\vec{L}$

Let me demonstrate a different way



3 main principles

1) $\frac{d\vec{p}}{dt} = \vec{F}_{net}$

$\Delta\vec{p}_{sys} + \Delta\vec{p}_{surr} = 0$

2) $\Delta E_{sys} + \Delta E_{surr} = 0$

3) $\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$

$\Delta\vec{L}_{sys} + \Delta\vec{L}_{surr} = 0$

Q: Can I change $\vec{\omega}$ without any torque?

Demo Yes, by changing I

$\vec{L}$ is conserved

$\vec{L}_i = I_i \vec{\omega}_i$

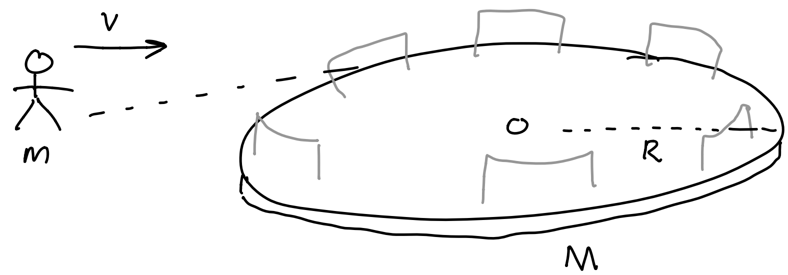
$\vec{L}_f = I_f \vec{\omega}_f$

$I_i \vec{\omega}_i = I_f \vec{\omega}_f$

$\vec{\omega}_f = \frac{I_i}{I_f} \vec{\omega}_i$

merry go round

Example:

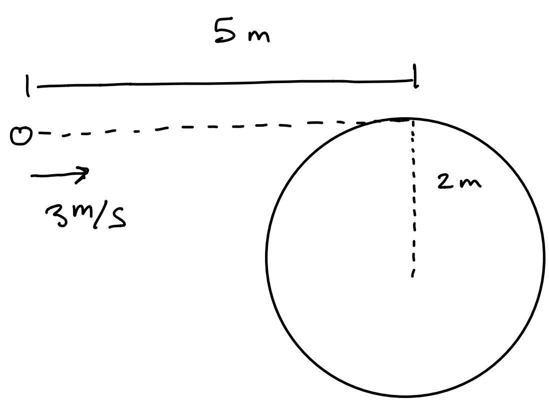


Kid runs + jumps on ride.

Will cause ride to spin.

How fast does it spin?

Conservation of angular momentum

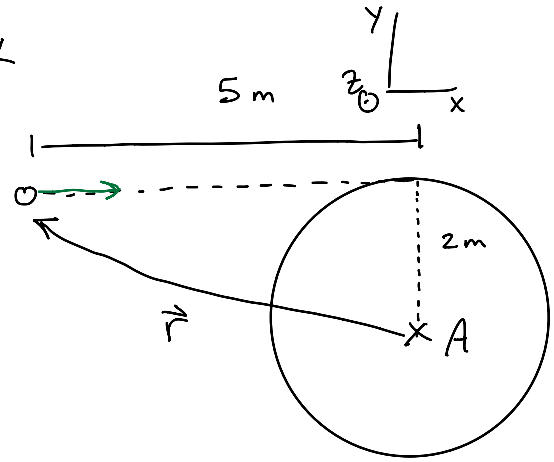


$m = 40$ kg
$M = 300$ kg
$I = \frac{1}{2} M R^2$

System: Kid + disk

$\vec{L} = \vec{L}_{kid} + \vec{L}_{disk}$

Calc $\vec{L}$ relative to center of disk



$\vec{L}_i = \vec{L}_{kid}$

$= \vec{r} \times \vec{p}$

$\vec{r} = \langle -5, 2 \rangle$ m

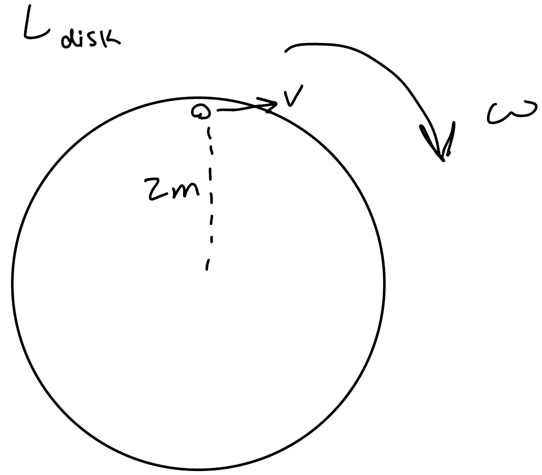
$\vec{p} = \langle (40\text{kg})(3\tfrac{m}{s}), 0 \rangle = \langle 120, 0 \rangle \ \text{kg}\tfrac{m}{s^2}$

$\vec{r} \times \vec{p} = \langle y p_z - z p_y , z p_x - x p_z , x p_y - y p_x \rangle$

$= \langle 0 , 0 , (-5\text{m})(0) - (2\text{m})(120 \tfrac{\text{kg}\,m}{s}) \rangle$

$\vec{L}_i = \langle 0, 0, -240 \ \text{kg}\tfrac{m^2}{s} \rangle$

$\vec{L}_f = \vec{L}_{kid,f} + \vec{L}_{disk}$



Disk rotates with velocity $\vec{\omega}$

$$\vec{L}_{disk} = I\vec{\omega} = -I\omega\hat{z}$$

$$\vec{L}_{kid} = \vec{r} \times \vec{p}$$

$R = 2$

$$\vec{r} = \langle 0, R \rangle m$$

$$\vec{p} = \langle mv, 0 \rangle$$

$$v = |\vec{\omega}| R$$

$$\vec{p} = \langle m\omega R, 0 \rangle$$

$$\vec{L}_{kid} = R\hat{y} \times m\omega R\hat{x} = -m\omega R^2\hat{z}$$

$$\vec{L}_f = -I\omega\hat{z} - m\omega R^2\hat{z}$$

$$= -\frac{1}{2}MR^2\omega\hat{z} - m\omega R^2\hat{z}$$

$$= -\left(\frac{1}{2}MR^2 + mR^2\right)\omega\hat{z}$$

$$= -\left(\frac{1}{2}M + m\right)R^2\omega$$

$$\vec{L}_i = \vec{L}_f$$

$$\left(-240\,\frac{kg\,m^2}{s}\right)\hat{z} = -\left(\frac{1}{2}M + m\right)R^2\omega$$

$$\omega = \frac{(240)}{(\frac{1}{2}M+m)R^2} = \frac{240}{(150+40)(4)} = 0.316 \ \frac{rad}{s}$$

$$\boxed{\omega = 0.316 \ \frac{rad}{sec}}$$