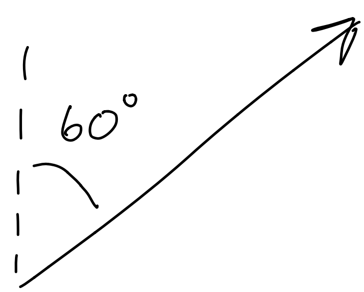
How to find xyz components of a vector

- Start w/ 2D

Ex: speed = 15 $\frac{m}{s}$

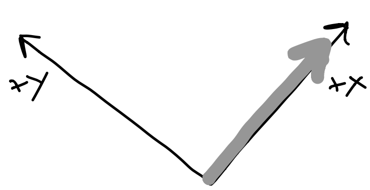


1) What coordinate system to use?

- Can use anything

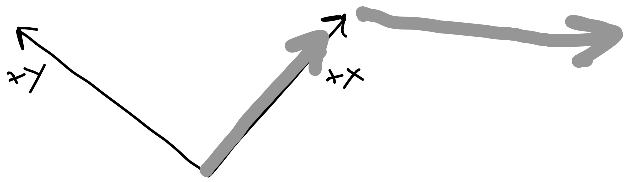
- typically, +x points to right
  +y points up

if I wanted to:



$\vec{v} = \langle 15, 0 \rangle \ \frac{m}{s}$

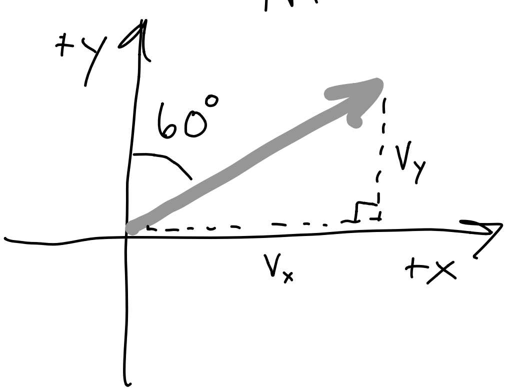
That's fine, but then every vector must be measured that way



Usually: $+x$ is horizontal & right

$+y$ is vertical & up

$|\vec{v}| = 15\,\frac{m}{s}$, want v_x, v_y



$\vec{v} = \langle v_x, v_y \rangle$

$\vec{v} = \langle v_x, v_y \rangle$

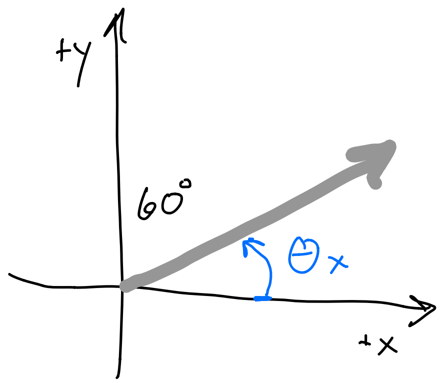
$\vec{v} = |\vec{v}|\,\hat{v} = \left(15\,\frac{m}{s}\right)\hat{v}$

What is $\hat{v}$? $\hat{v} = \langle \cos\theta_x, \cos\theta_y \rangle$

We're given an angle: 60° but is that θ_x?

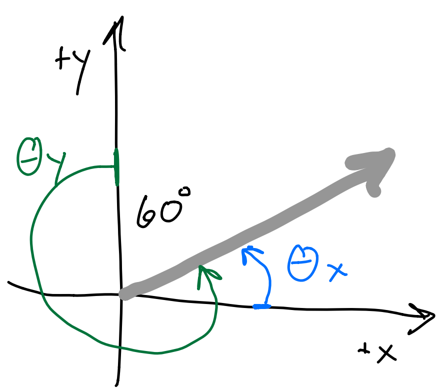
What is θ_x?

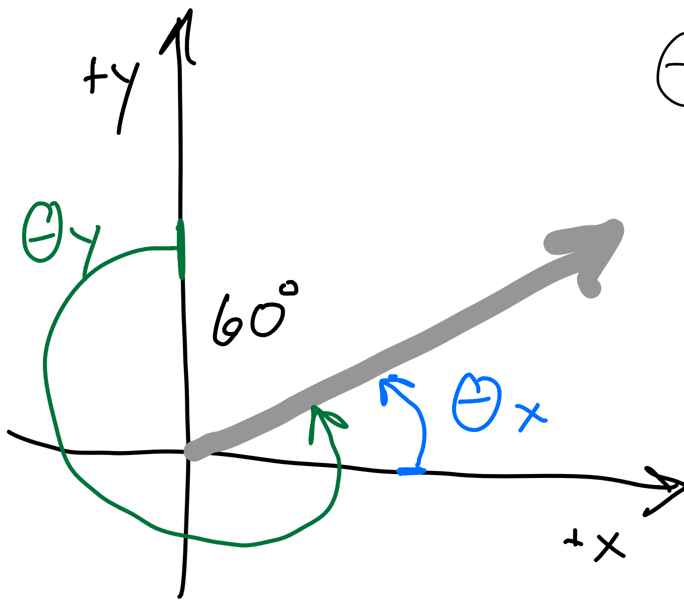
- The angle measured CCW from +x



What is θ_y?

- The angle measured CCW from +x





$$\theta_x + 60^\circ = 90^\circ$$

$$\theta_x = 30^\circ$$

$$\theta_y + 60^\circ = 360^\circ$$

$$\theta_y = 300^\circ$$

$$\hat{v} = \langle \cos(30^\circ), \cos(300^\circ) \rangle$$

$$\theta_y = \theta_x - 90^\circ$$

$$= \theta_x - \frac{\pi}{2}$$

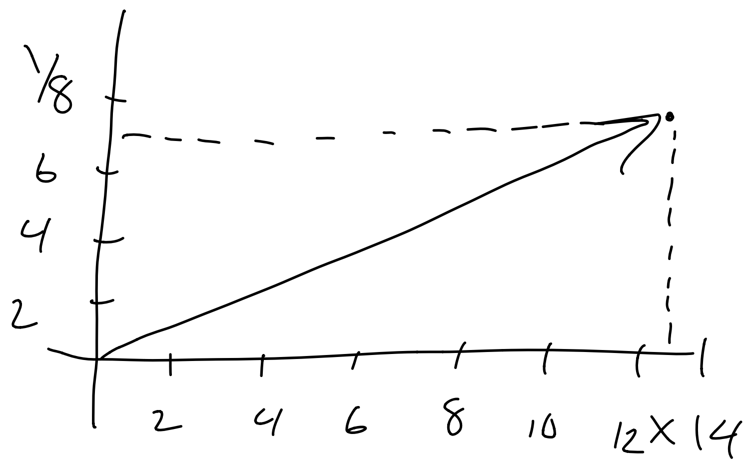
$$30^\circ - 90^\circ = -60^\circ + 360^\circ = 300^\circ$$

$$\hat{v} = \langle 0.866\ldots, 0.5 \rangle$$

$$\vec{v} = 15\,\frac{m}{s}\,\langle .866,\ .5\rangle$$

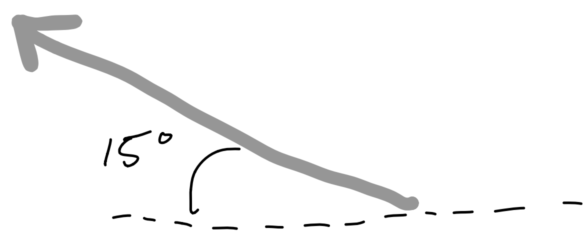
$$\vec{v} = \langle 13,\ 7.5\rangle\,\frac{m}{s}$$

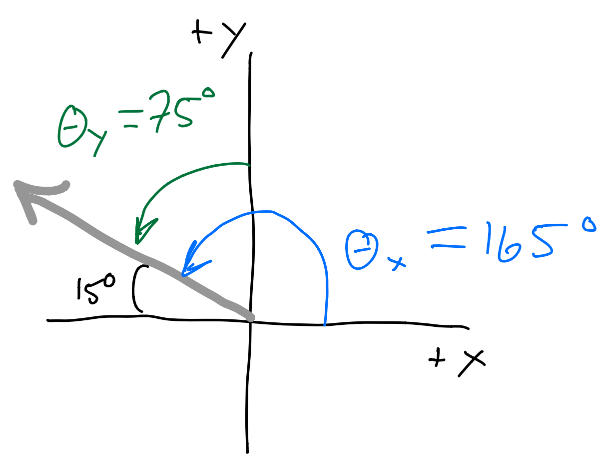
Check:



Ex: A ball travels to the left & makes an angle of $15°$ w/ horizontal $|\vec{p}| = 12 \frac{kg\,m}{s}$

What is $\vec{p}$?





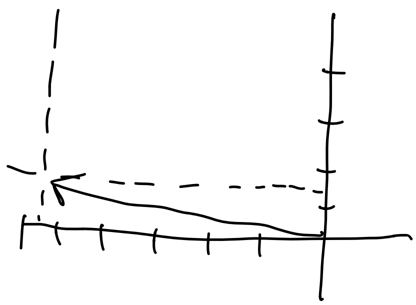
$$\theta_y = \theta_x - 90°$$

$\vec{p} = |\vec{p}|\,\hat{p}$, $|\vec{p}| = 12 \frac{kg\,m}{s}$ $\hat{p} = \langle \cos 165°, \cos 75°, 0 \rangle$

$$\hat{p} = \langle -0.97, 0.26, 0 \rangle$$

$$\vec{p} = \left(12 \frac{kg\,m}{s}\right) \langle -.97, .26, 0 \rangle$$

$$\vec{p} = \langle -11.6, 3.1, 0 \rangle \frac{kg\,m}{s}$$



In 3D, this becomes

$$\hat{r} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z \rangle$$

- I will never have you draw a 3D vector
- I will give you these angles directly

Now let's go the reverse direction

Given $\vec{v} = \langle v_x, v_y \rangle$, tell me

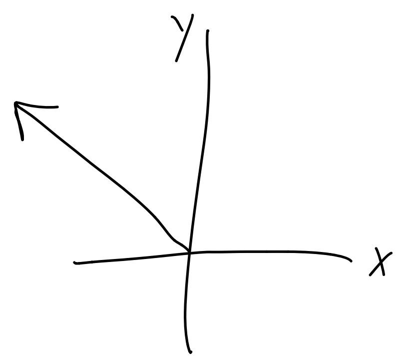
$|\vec{v}|$ and θ_x, θ_y

Example: $\vec{r} = \langle -15, 25 \rangle$ m

$|\vec{r}| = ?$

$\theta_x = ?$

$\theta_y = ?$



$$|\vec{r}| = \sqrt{(-15)^2 + (25)^2} = 29.15 \text{ m}$$

θ_x & θ_y ?

$$\vec{r} = |\vec{r}|\,\hat{r} \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{29.15}\langle -15, 25 \rangle$$

know know

$$\hat{r} = \langle -0.51, 0.86 \rangle$$

$$\hat{r} = \langle -0.51, 0.86 \rangle$$

AND

$$\hat{r} = \langle \cos\theta_x , \cos\theta_y \rangle$$

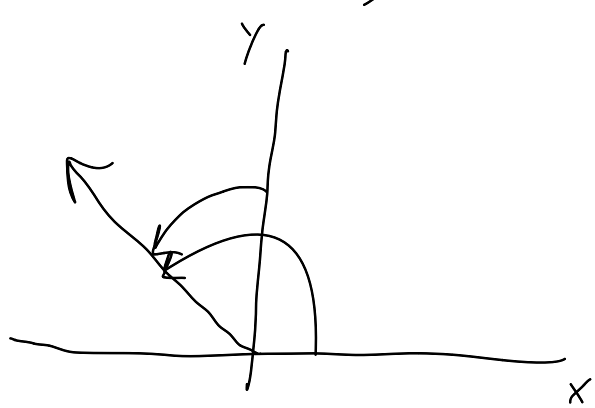
$$\langle -.51, .86 \rangle = \langle \cos\theta_x , \cos\theta_y \rangle$$

$$\cos\theta_x = -.51$$

$$\cos\theta_y = .86$$

$$\theta_x = \cos^{-1}(-.51) = 121°$$

$$\theta_y = \cos^{-1}(.86) = 31° = \theta_x - 90°$$

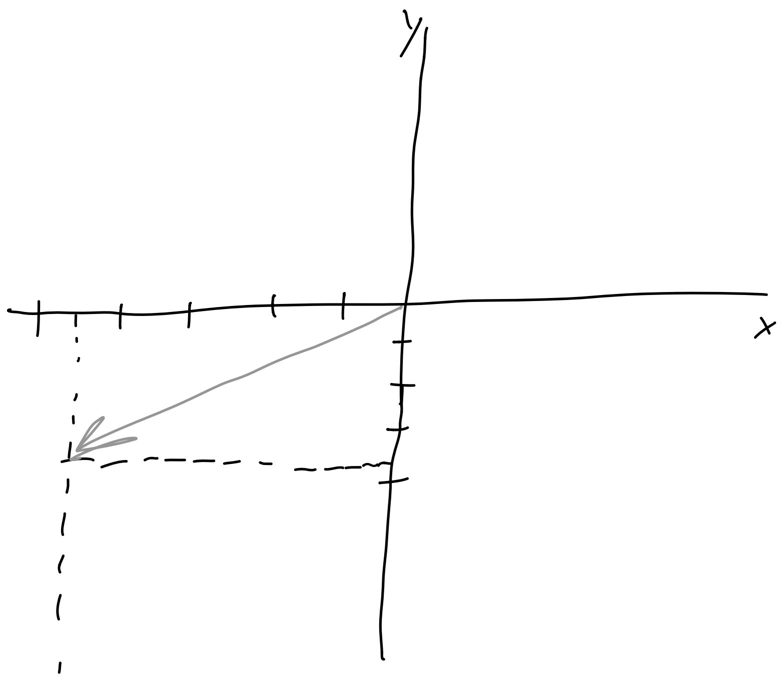


CAVEAT!

$\vec{F} = \langle -9600, -7400 \rangle$ N

$|\vec{F}| = ?$

$\theta_x, \theta_y = ?$



$|\vec{F}| = 12{,}121.1$ N

$$\hat{F} = \frac{\vec{F}}{|\vec{F}|} = \frac{1}{12121.1} \langle -9600, -7400 \rangle$$

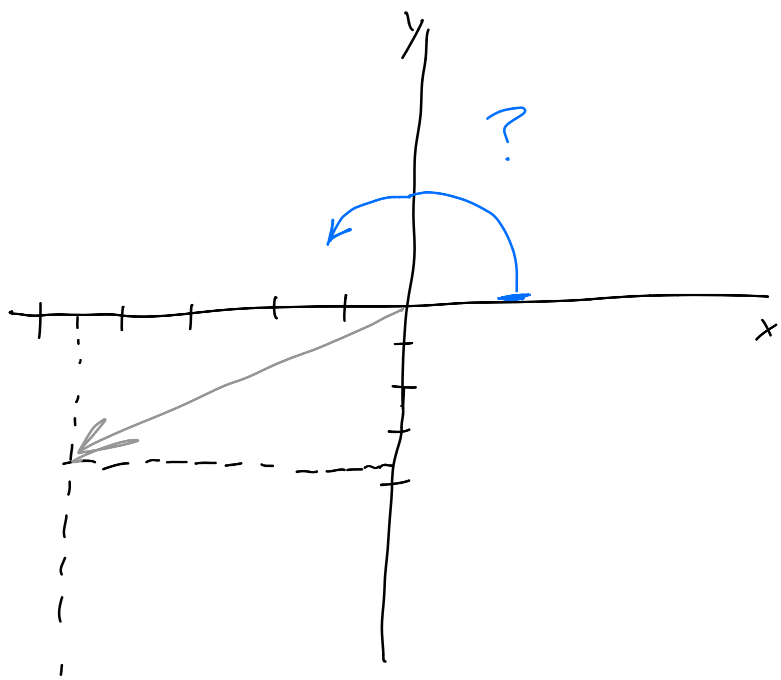
$$\hat{F} = \langle -0.79, -0.61 \rangle$$

$\langle -0.79, -0.61 \rangle = \langle \cos\theta_x, \cos\theta_y \rangle$

$\theta_x = \cos^{-1}(-0.79) = 142.4^\circ$

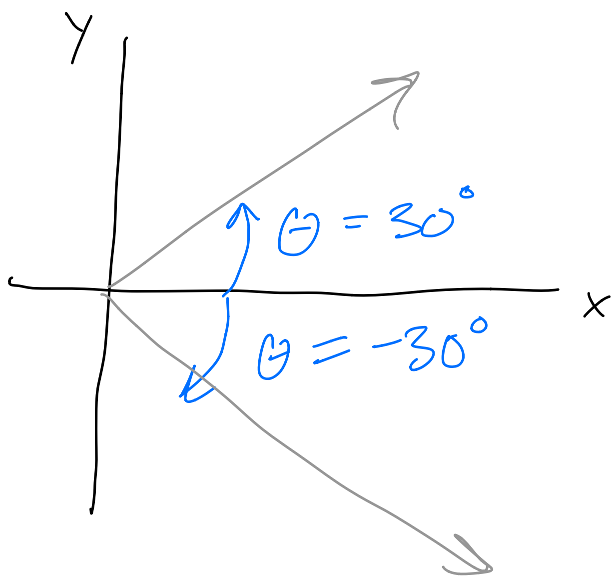
142° is in quad II

$\vec{F}$ is in III



Recall: $\cos(\theta) = \cos(-\theta)$

$\cos(\theta) = \cos(-\theta)$



cos is the same ccw as cw

$\cos^{-1}(-0.79) = 142.4°$

means $\pm 142.4°$

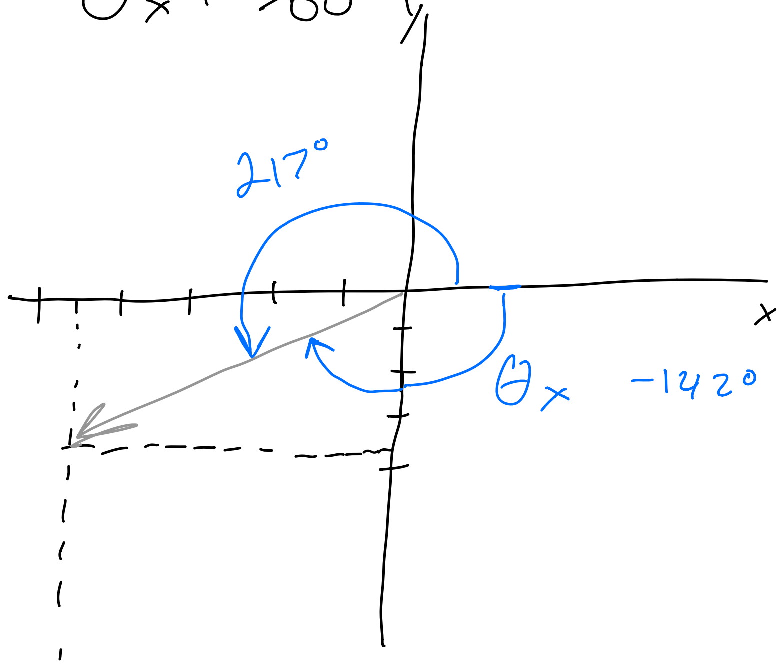
+ : ccw
− : cw

IF in quad 3 or 4, take −

We are in quad 3

so $\theta_x = -142.4°$

$\theta_x + 360° = 217.6°$



$\Theta_y = \theta_x - 90°$

$= 217.6° - 90°$

$\Theta_y = 127.6°$