What I want:

Given the initial conditions of the ball ($\vec{r}_i$, $\vec{p}_i$, m) and a constant Force $\vec{F}_{net}$, I can tell you

1) where the ball is
2) its momentum (+ velocity)

at <u>any</u> time just by plugging t into a formula

$$v = v(t)$$

$$x = x(t)$$

Consider:

Object starts with

$\vec{p}_i$, $\vec{r}_i$, + is subject to a constant force $\vec{F}_{net}$

First look at 1 dimension:

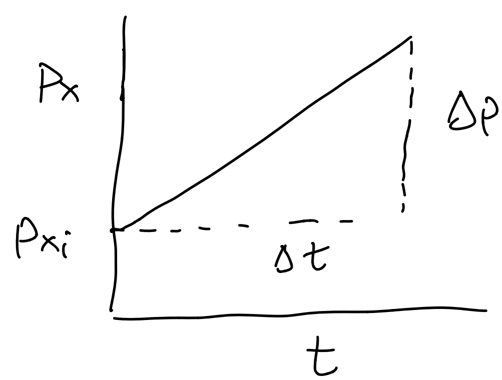
$$\vec{p}_i = p_{xi}$$

$$\vec{F}_{net} = F_x$$

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

$$\Delta p_x = F_x \Delta t$$

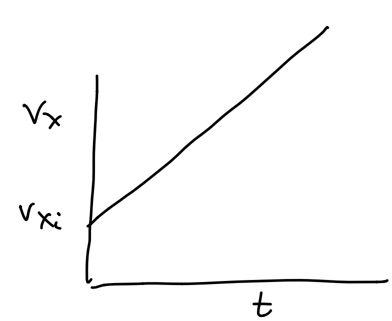
$$\frac{\Delta p_x}{\Delta t} = F_x = \text{const}$$



$$p_x(t) = p_{xi} + F_x t$$

$$p_x(t) = m v_x(t)$$

$$v_x(t) = v_{xi} + \frac{F_x}{m} t$$



How to go from $V_x(t)$ to $x(t)$

- Cant just say $x(t) = V_x(t)\, t$

  $V_x(t)$ is changing

- we know that:

$$V_{avg} = \frac{\Delta x}{\Delta t}$$

IF $\vec{a}$ is constant:

$$\boxed{V_{avg,x} = \frac{V_{xi} + V_{xf}}{2}}$$

$$V_{xf} = V_{xi} + \frac{F_x}{m} t$$

$$V_{avg\,x} = \frac{V_{xi} + V_{xi} + \frac{F_x}{m} t}{2}$$

$$V_{avg\,x} = V_{xi} + \frac{1}{2}\frac{F_x}{m} t$$

$$V_{avg\,x} = \frac{\Delta x}{t} = V_{xi} + \frac{1}{2}\frac{F_x}{m} t$$

$$\Delta x = V_{xi} t + \frac{1}{2}\frac{F_x}{m} t^2$$

$$x(t) = x_i + V_{xi} t + \frac{1}{2}\frac{F_x}{m} t^2$$

Kinematic Equations (constant F_x)

$p_x(t) = p_i + F_x t$ ← momentum prin

defn of momentum

$V_x(t) = V_i + \frac{F_x}{m} t$

$V_{avg\,x}(t) = V_{xi} + \frac{1}{2}\frac{F_x}{m} t$ $\quad V_{avg} = \frac{V_i + V_f}{2}$

$x(t) = X_i + V_{xi} t + \frac{1}{2}\frac{F_x}{m} t^2$ $\quad V_{avg} = \frac{\Delta x}{\Delta t}$

ONLY NEW THING

$$V_{avg} = \frac{V_i + V_f}{2}$$

Example: mass 1200 kg

A car driving at 30 $\frac{m}{s}$ spots a pedestrian and slams the brakes, applying a constant force of 9000 N.

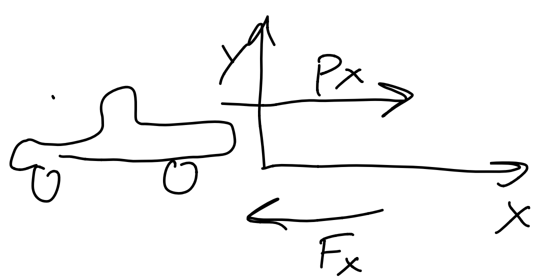
a) How long for car to stop?

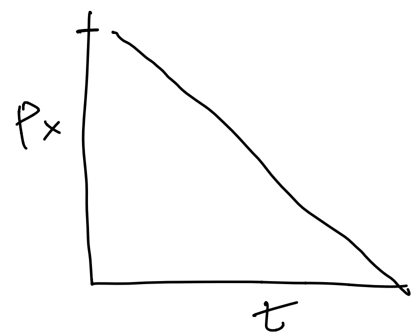
b) How much distance travelled during that time?

$$p_{xi} = m v_{xi} = (1200\ kg)(30\ \tfrac{m}{s}) = 36000\ \tfrac{kg\,m}{s}$$

$$p_{xi} = 36000\ \tfrac{kg\,m}{s}$$

$F_{net} = ?$ $\quad F_{net} = -9000\ N$





$$p_x(t) = p_{xi} + F_x t$$

$$p_x(t) = 36000 - 9000t$$

car stops when $p_x(t) = 0$

$0 = 36000 - 9000t$

$9000t = 36000$

$t = 4s$

$x(t) = x_i + V_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$

$x_i = 0$

$t = 4$

$F_x = -9000$

$V_{xi} = \frac{p_{xi}}{m} = 30$

$x(4) = 0 + (30)(4) + \frac{1}{2}\left(\frac{-9000}{12000}\right)(4)^2$

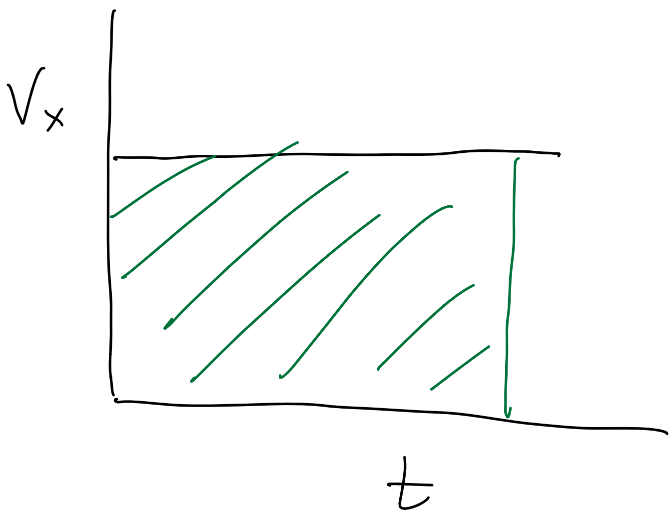
$= 120 + (-6)$

$x(4) = 114\ m$

$\Delta x = 114\ m$

Look graphically:

if F is 0:

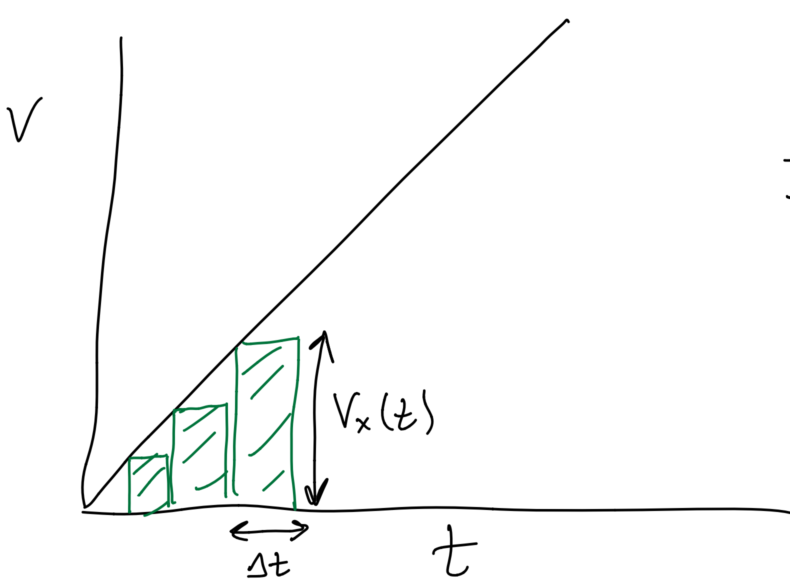


$V_x = V_i$

$\Delta x(t) = V_i \Delta t$

same as area under the curve

Now F_x is const



$V_x = V_i + \frac{F}{m} t$

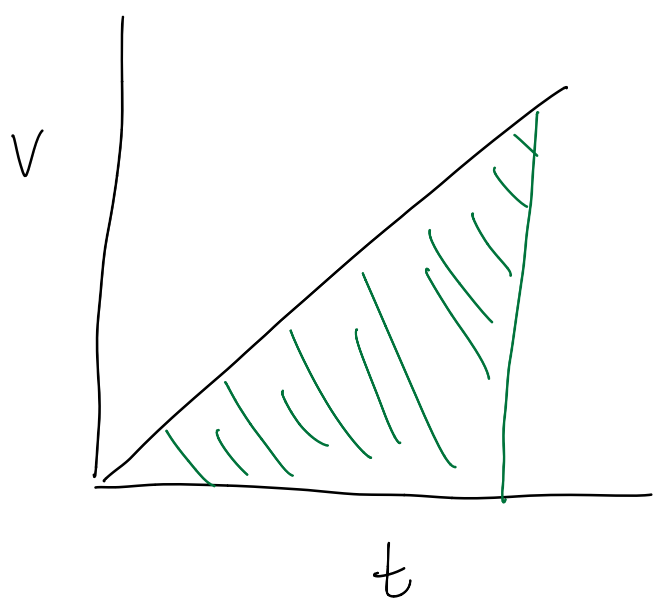
$\Delta x = ?$

If Δt is very small, treat V_x as const

$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \ldots$

Better approximation w/ smaller Δt

- What am I approximating? Area under the curve



$$\Delta x = \frac{1}{2} t (v_f - v_i)$$

$$= \frac{1}{2} t \left(v_i + \frac{F_x}{m} t - v_i\right)$$

$$\Delta x = \frac{1}{2} \frac{F_x}{m} t^2$$

In general: $\Delta x = \int_{t_i}^{t_f} v(t)\, dt$

By FTC:

$$\frac{dx}{dt} = v$$

inst velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$p_x(t) = p_i + F_x t$$

$$v_x(t) = v_i + \frac{F_x}{m} t$$

$$v_{x\,avg}(t) = v_{xi} + \frac{1}{2} \frac{F_x}{m} t$$

$$x(t) = x_i + v_{xi} t + \frac{1}{2} \frac{F_x}{m} t^2$$

$$\int_0^t v_x\, dt = v_i t + \frac{1}{2} \frac{F_x}{m} t^2 = \Delta x$$

$$x = x_i + v_{xi} t + \frac{1}{2} \frac{F_x}{m} t^2$$

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \Delta \vec{r} = \int_{t_i}^{t_f} \vec{v}\, dt$$