Last time:

momentum, position, velocity of a system of many particles

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_N\vec{v}_N$$

$$\vec{v} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_N\vec{v}_N}{m_1 + m_2 + \dots + m_N}$$

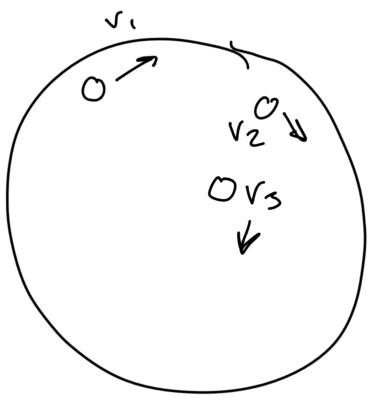
$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_N\vec{r}_N}{m_1 + m_2 + \dots + m_N}$$

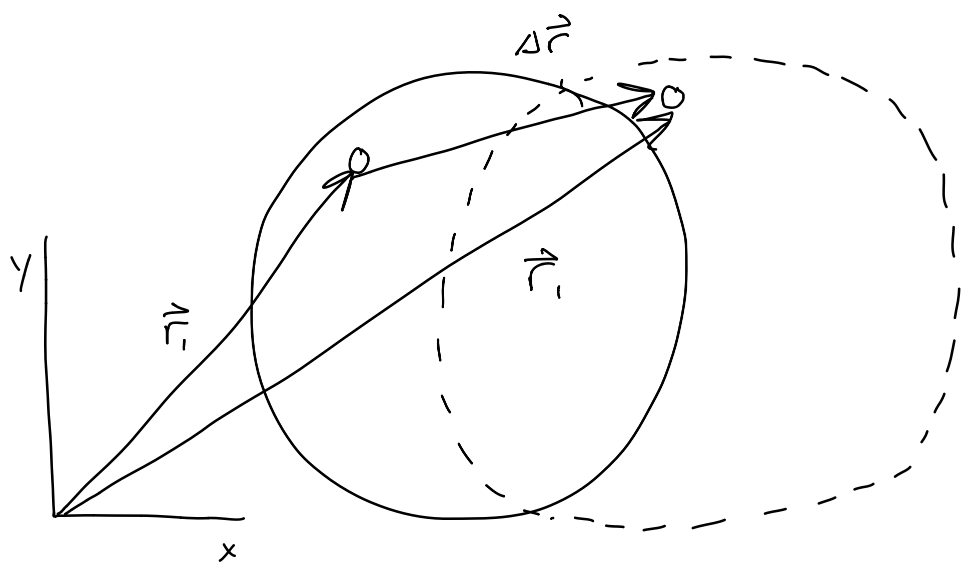
Conclusion: The system behaves like its own particle!

<u>Show phet sims</u>

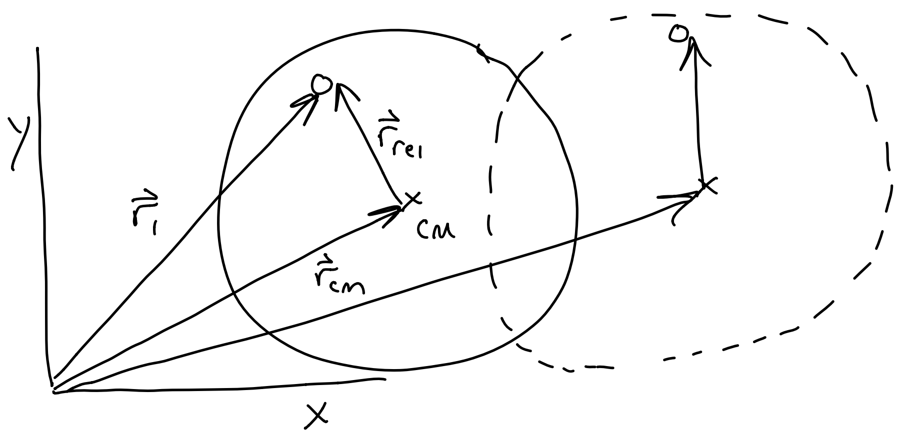
The energy of a multiparticle system

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots + \frac{1}{2} m_N v_N^2$$





$$\vec{V}_1 = \frac{d\vec{r}_1}{dt}$$



$$\vec{r}_1 = \vec{r}_{cm} + \vec{r}_{rel}$$

position of m_1 = position of cm +
position of m_1
relative to cm

$$\vec{V}_1 = \frac{d\vec{r}_1}{dt} = \frac{d}{dt}\left(\vec{r}_{cm} + \vec{r}_{1,rel}\right)$$

$$\vec{V}_1 = \vec{V}_{cm} + \vec{V}_{1,rel}$$

$$K_1 = \frac{1}{2} m v_1^2$$

$$\vec{V}_1 = \langle V_x + V_{1x},\ V_y + V_{1y},\ V_z + V_{1z} \rangle$$

$$V_1^2 = (V_x + V_{1x})^2 + (V_y + V_{1y})^2 + (V_z + V_{1z})^2$$

$$v_1^2 = v_x^2 + v_{1x}^2 + 2v_x v_{1x}$$
$$+ v_y^2 + v_{1y}^2 + 2v_y v_{1y}$$
$$+ v_z^2 + v_{1z}^2 + 2v_z v_{1z}$$

$$v_1^2 = v_{cm}^2 + v_{rel}^2 + 2\vec{v}_{cm} \cdot \vec{v}_{rel}$$

$$K_1 = \frac{1}{2} m_1 \left(v_{cm}^2 + v_{rel,1}^2 + 2\vec{v}_{cm} \cdot \vec{v}_{rel,1} \right)$$

$$K_1 = \frac{1}{2} m_1 v_{cm}^2 + \frac{1}{2} m_1 v_{rel,1}^2 + m_1 \vec{v}_{cm} \cdot \vec{v}_{rel,1}$$

$$K_{tot} = K_1 + K_2 + \ldots + K_N$$

$$= \frac{1}{2} m_1 v_{cm}^2 + \frac{1}{2} m_1 v_{rel,1}^2 + m_1 \vec{v}_{cm} \cdot \vec{v}_{rel,1}$$

$$+ \frac{1}{2} m_2 v_{cm}^2 + \frac{1}{2} m_2 v_{rel,2}^2 + m_2 \vec{v}_{cm} \cdot \vec{v}_{rel,1}$$

$$+ \quad \ldots$$

$$= \frac{1}{2} (m_1 + m_2 + \ldots m_N) v_{cm}^2$$

$$+ \frac{1}{2} m_1 v_{rel,1}^2 + \frac{1}{2} m_2 v_{rel,2}^2 + \ldots \frac{1}{2} m_N v_{rel,N}^2$$

$$+ \vec{v}_{cm} \cdot \left(m_1 \vec{v}_{rel,1} + m_2 \vec{v}_{rel,2} + \ldots + m_N \vec{v}_{rel,N} \right)$$

$$= \frac{1}{2} M v_{cm}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i v_{rel,i}^2 + \vec{V}_{cm} \cdot \vec{p}_{rel}$$

$$\vec{p}_{rel} = \vec{p} - \vec{p}_{cm}$$

$$= (m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots m_N \vec{v}_N) - (m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots)$$

$$= 0$$

$$\boxed{K_{tot} = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} \sum_{i=0} m_i V_{rel,i}^2}$$

What do these terms mean?

$\frac{1}{2} m V_{cm}^2$ is the "normal" kinetic energy

If I throw the frisbee ($m = 140$ g)

at 20 m/s, then

$$\frac{1}{2} m V_{cm}^2 = \frac{1}{2}(0.14 \text{ kg})(20 \text{ m/s})^2 = 28 \text{ J}$$

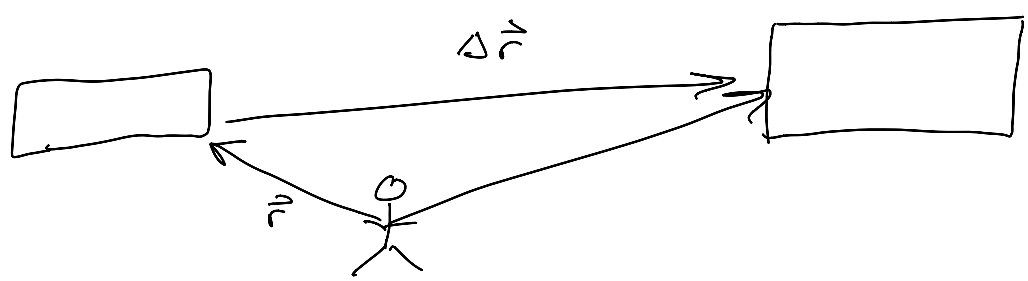
What is $\frac{1}{2} \sum_i m_i V_{rel,i}^2$???

$\vec{v}_{rel}$ is the velocity relative to the center of mass

Relative velocity

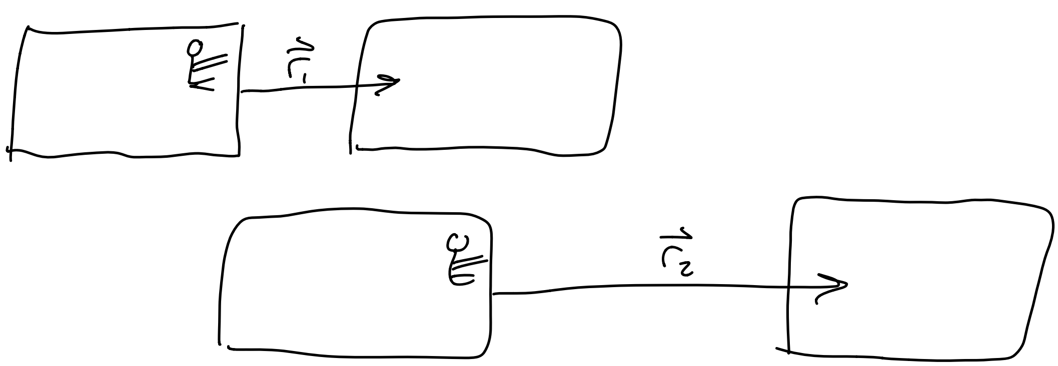
If a car speeds past you while you stand on the side of the road:

1)



Car moved a lot in a small time, you measure a large $\vec{v}$

2)



you measure $\vec{v}$ to be small

Who is right? Both!

What is $\vec{v}_{rel}$?

The velocity we see if we are traveling inside the object at the same speed as the center of mass

- Same thing as if the ball is rotating in place

Conclusion:

$$K = K_{cm} + K_{rel}$$

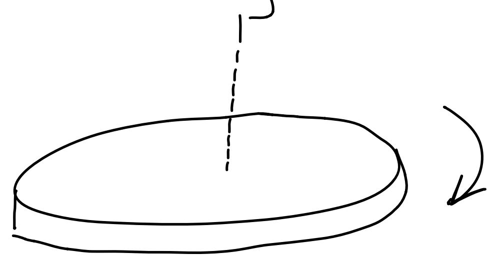
translational

$$K = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

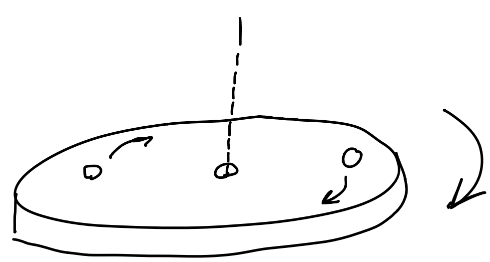
What is K_{rot}?

- Consider a rigid body (shape isn't changing) which is rotating about an axis

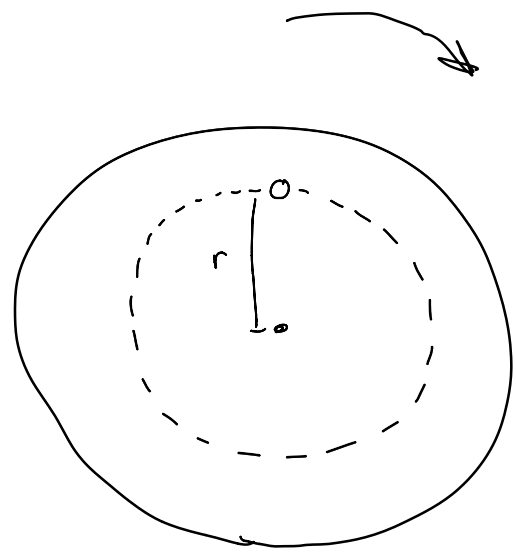


Axis of rotation:

- the straight line through all fixed points of a rotating object around which all other points rotate



View from top down $\quad f = \# \text{rotations}/\text{sec}$



This point travels a distance

$2\pi r$ per rotation

The point completes "f" rotations per second

So each second, it travels $2\pi r f$

so, $|\vec{v}| = 2\pi r f$

- This means that atoms far away from the axis of rotation are moving faster

Usually, we specify the speed of rotation as rad/sec, rather than rot/sec

$$\omega = \text{rad}/\text{sec}$$

$$\omega = \frac{\text{rot}}{\text{sec}} \times \frac{\text{rad}}{\text{rot}} = 2\pi f$$

$$\omega = 2\pi f$$

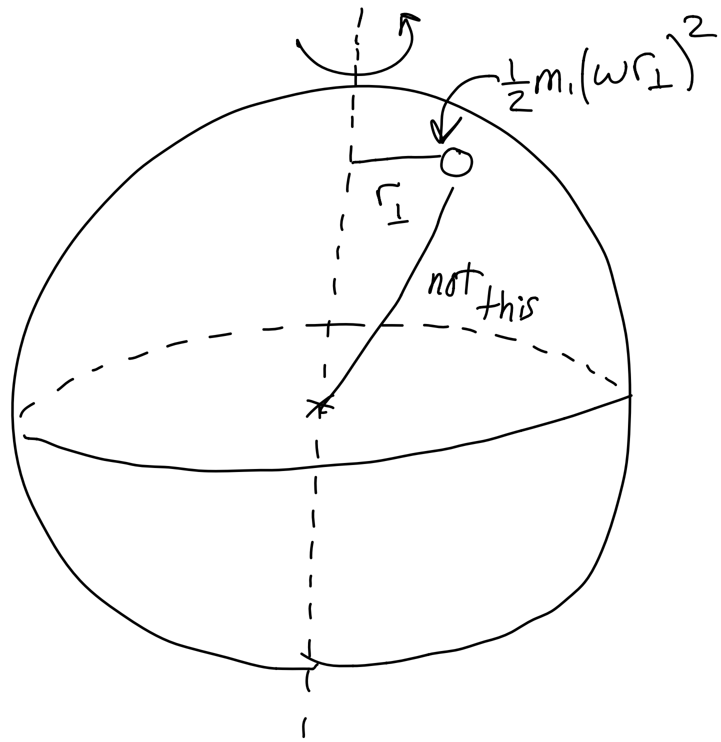
so $V_{rel} = \omega r$

$$K_{rot} = \frac{1}{2} m_1 v_{1,rel}^2 + \frac{1}{2} m_2 v_{2,rel}^2 + \frac{1}{2} m_3 v_{3,rel}^2 + \ldots$$

ω is the same for each particle

$$K_{rot} = \frac{1}{2} m_1 (\omega r_{\perp 1})^2 + \frac{1}{2} m_2 (\omega r_{\perp 2})^2 + \frac{1}{2} m_2 (\omega r_{\perp 3})^2 + \ldots$$

$r_\perp$ is the perpendicular distance to axis of rotation



$$K_{rot} = \frac{1}{2}m_1(\omega r_{\perp 1})^2 + \frac{1}{2}m_2(\omega r_{\perp 2})^2 + \frac{1}{2}m_2(\omega r_{\perp 3})^2 + \ldots$$

$$= \left(\frac{1}{2}m_1 r_{\perp_1}^2 + \frac{1}{2}m_2 r_{\perp 2}^2 + \frac{1}{2}m_3 r_{\perp 3}^2 + \ldots\right)\omega^2$$

$$I \equiv \frac{1}{2}m_1 r_{\perp_1}^2 + \frac{1}{2}m_2 r_{\perp 2}^2 + \frac{1}{2}m_3 r_{\perp 3}^2 + \ldots$$

$$I = \frac{1}{2}\sum_i m_i r_{\perp i}^2$$

I: moment of inertia

- Property of the material
- Units: $kg \cdot m^2$

$$K_{rot} = \frac{1}{2} I \omega^2$$

Compare to: $K_{trans} = \frac{1}{2} m v^2$

$$m \longrightarrow I$$

$$v \longrightarrow \omega$$

mass: resists change in v
(takes more energy to accelerate a car to 50 mph than a baseball)

I : resists change in ω

Takes more energy to increase ω if:

- object is more massive
- mass is located further from axis of rotation