In chapter 7, we briefly discussed new forms of energy that became important if the atoms inside your object interact w/ the surroundings

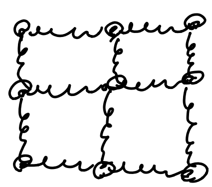
Solid block:

model: $\bullet^{m}$

single point of mass w/ no internal structure

$E: K + U_{sys}$

model:



$E = K + U_{sys} + E_{int}$ $\qquad E_{int} \longleftrightarrow \text{Temp}$

- more complicated model
- Don't need for simple problems like falling mass in a vaccuum
- Do need when considering friction, air resistance, collisions, etc

In chapter 9, we consider another case where we lose track of energy if we treat the system like a single particle

Ex: Throwing a frisbee

140 g

As I throw this frisbee, I do 30 J of work on it (chem energy in my body converted to kinetic energy of frisbee)

What is the resulting speed of the frisbee?

system: frisbee
surr: none

$\overset{m}{\bullet} \longrightarrow v$   treat frisbee like a point of mass

$$E_{sys} = 30\ J = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2}{0.14\ kg}(30\ J)} = 20.7\ m/s$$

When I test this, it is <u>wrong</u>

Ex:

Ball at top of ramp $\left(m = 535\,g \quad h = 25.5\,mm\right)$

$\Delta E_{sys} = 0$

$E_i = mgh$

$E_f = \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = mgh$

$v = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)(0.0255\,m)}$

$= 0.71$ m/s

What is happening?

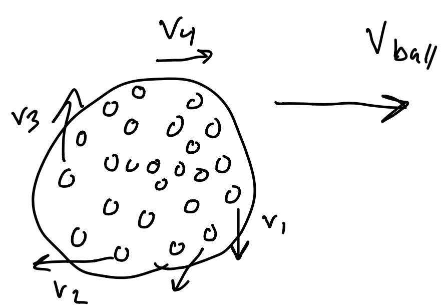
- We have not violated energy conservation
- It is not all friction/air resistance

  (we would see this even in a vaccuum + on ice)

  - The "missing" energy is not internal, the ball's temp does not change

→ The issue is that the ball + the frisbee are not "point masses"

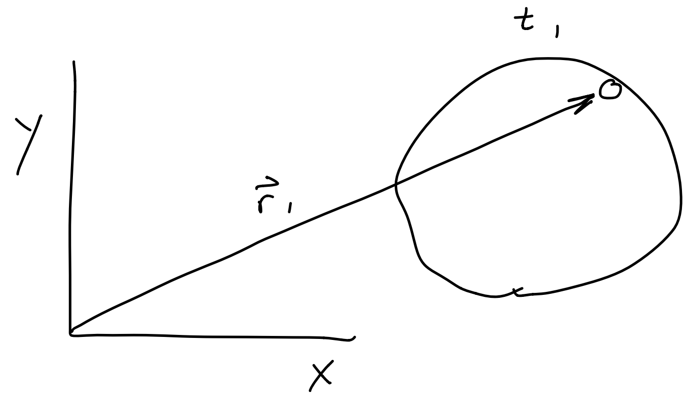
→ They are rotating

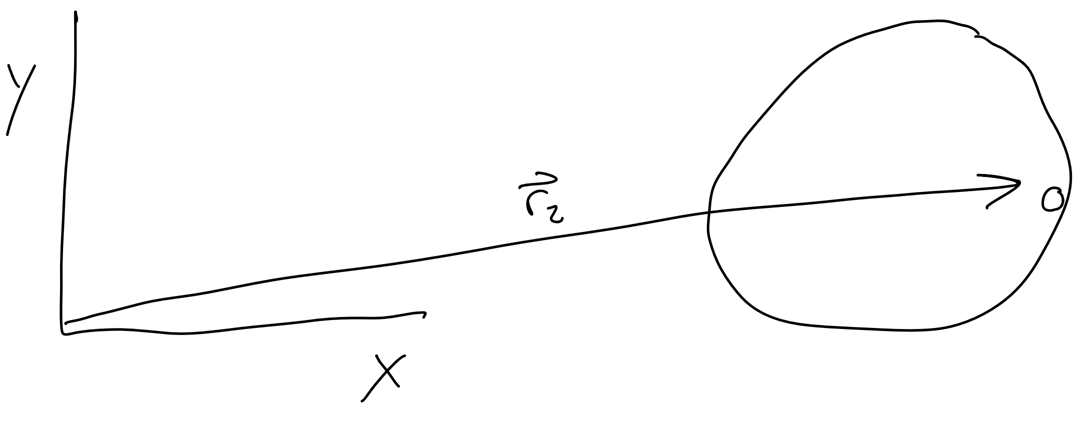


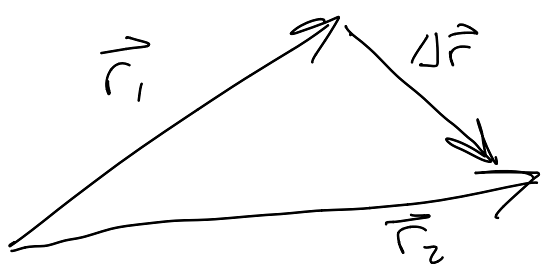

There is kinetic energy assoc with the speed of the ball's molecules as they rotate

In fact, we should clarify what we mean by "the speed of the ball"

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$



At any moment, molecules at different areas of the ball are moving w/ different speeds in different directions

So which one is "the speed" of the ball?

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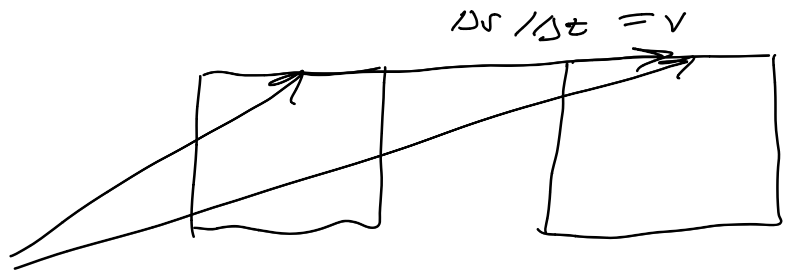
To answer this, we return to the concept of momentum

Momentum + velocity of multiparticle systems

Consider a system of 2 sliding blocks (friction-less)

$m_1 = 5 \text{ kg}$ $\qquad$ $m_2 = 2 \text{ kg}$

$v_1 = 2 \text{ m/s}$ $\qquad$ $v_2 = 3 \text{ m/s}$




system: box 1

$\vec{p}_1 = \langle 10\,\frac{kg\,m}{s}, 0, 0 \rangle$

$\vec{v}_1 = \langle 2\,\frac{m}{s}, 0, 0 \rangle$

system: box 2

$\vec{p}_2 = \langle 6\,\frac{kg\,m}{s}, 0, 0 \rangle$

$\vec{v}_2 = \langle 3\,m/s, 0, 0 \rangle$

system: box 1 + box 2

$\vec{p} = \langle 16\,\frac{kg\,m}{s}, 0, 0 \rangle$

$\vec{v} = ?$

$\vec{v} \stackrel{?}{=} \vec{v}_1 + \vec{v}_2 = \langle 5\,m/s, 0, 0 \rangle$

system moves faster than either object?

$$\vec{v} \stackrel{?}{=} \frac{\vec{v}_1 + \vec{v}_2}{2} = \langle 2.5 \tfrac{m}{s}, 0, 0 \rangle$$

Not quite right, either

To find $\vec{v}$, remember defn of $\vec{p}$

$$\vec{p} = m\vec{v}$$

$$\vec{p}_{sys} = \langle 16 \tfrac{kg\,m}{s}, 0, 0 \rangle$$

$$m_{sys} = 5\,kg + 2\,kg = 7\,kg$$

$$\vec{v}_{sys} = \vec{p}_{sys} / m_{sys} = \langle 2.29 \tfrac{m}{s}, 0, 0 \rangle$$

Intepretetation:

$$\vec{v}_{sys} = \frac{\vec{p}_{sys}}{m_{sys}} = \frac{\vec{p}_1 + \vec{p}_2}{m_1 + m_2} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$\vec{v}_{sys}$ is the average velocity, weighted by mass

Like your score on an exam:

P1: worth 10 pts, you score 40%

P2: worth 20 pts, you score 90%

$$\text{grade} = \frac{(10)(40) + (20)(90)}{10 + 20} = 73.3\ \%$$

points possible $\longrightarrow$ mass

score $\longrightarrow$ velocity

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This generalizes to as many particles as I want

system: $m_1$ @ $\vec{v}_1$

$m_2$ @ $\vec{v}_2$

$m_3$ @ $\vec{v}_3$

$\vdots$

$m_N$ @ $\vec{v}_N$

$$\vec{p}_{sys} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_N\vec{v}_N$$

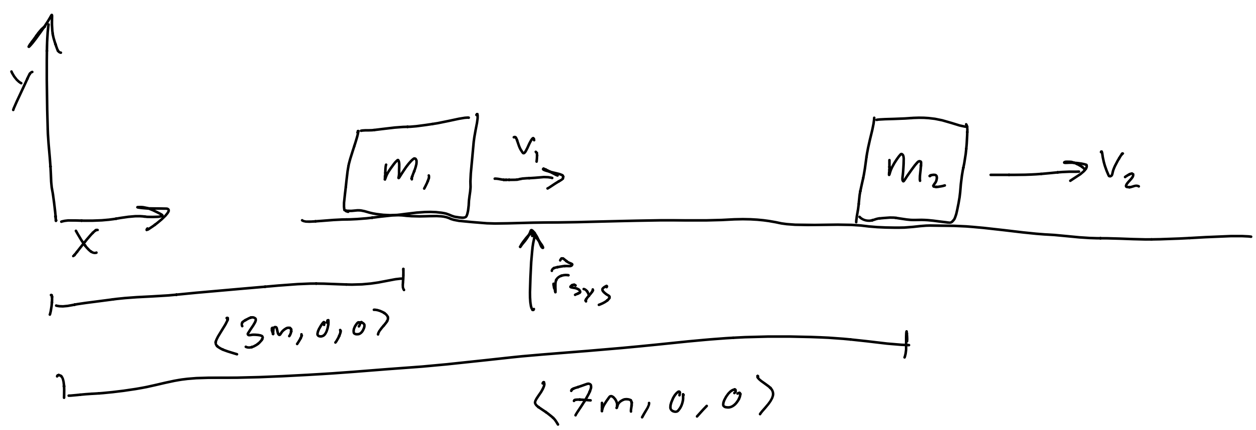
$$\vec{v}_{sys} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_N\vec{v}_N}{m_1 + m_2 + \ldots m_N}$$

"Where" is my system?

Each particle has its own position

Just like velocity, take the weighted average

$$\vec{r}_{sys} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots m_N \vec{r}_N}{m_1 + m_2 + \dots m_N}$$




$$\vec{r}_{sys} = \frac{(5\,kg)\langle 3m, 0, 0\rangle + (2\,kg)\langle 7m, 0, 0\rangle}{7\,kg}$$

$$\vec{r}_{sys} = \langle 4.1, 0, 0 \rangle m$$

A system of many particles has its own:

mass: $(m_1 + m_2 + \dots m_N)$

position: $\left(\vec{r}_{sys} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots m_N\vec{r}_N}{m_1 + m_2 + \dots m_N}\right)$

velocity: $\left(\vec{v}_{sys} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_N\vec{v}_N}{m_1 + m_2 + \dots + m_N}\right)$

Note that $\vec{v}_{sys} = \dfrac{d\vec{r}_{sys}}{dt}$

accel, net force, etc ...

A system of many particles itself behaves like a particle!

$\vec{r}_{sys}$ = Center of mass $(\vec{r}_{cm})$

$\vec{v}_{sys}$ = Velocity of center of mass $(\vec{v}_{cm})$

Show phet simulations...

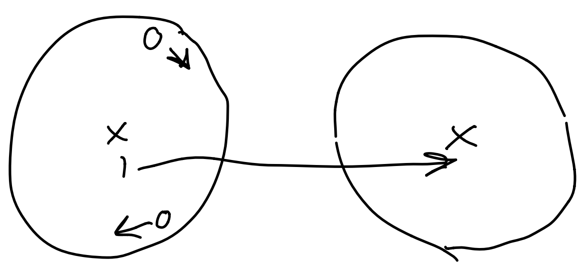
A rolling ball is no different

It is a collection of molecules, but

$$\vec{V}_{ball} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots m_N \vec{v}_N}{m_1 + m_2 + \ldots + m_N}$$

When we talk about "the velocity" or "the position"

of a rolling ball or a spinning frisbee, we mean that of its center of mass



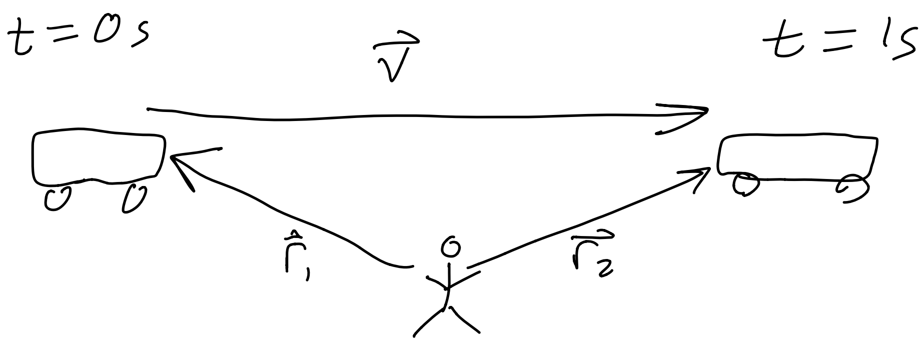
Energy of a multiparticle system:


system: $m_1 + m_2$

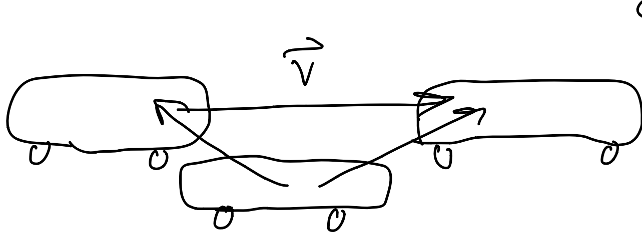
$K: \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

How I measure "v" depends on my own velocity!

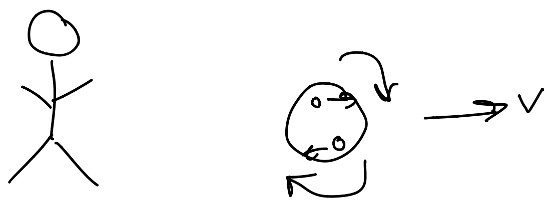
Ex: Car speeds past you




velocity relative to observer vs moving car



Why is this important?



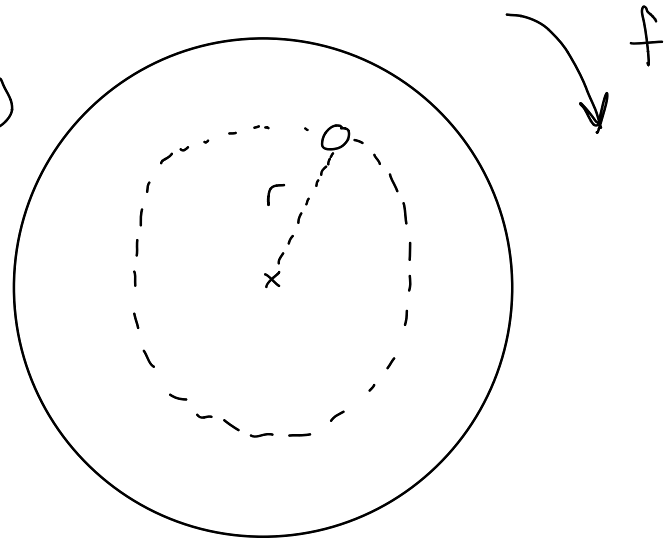

- The velocity of the CM is easy to measure
- The velocity of rotating molecules is hard

Now imagine we're riding inside the ball at the center of mass

We just see a rotating ball

We can easily calculate the velocity now

Ball is rotating @ freq $f$



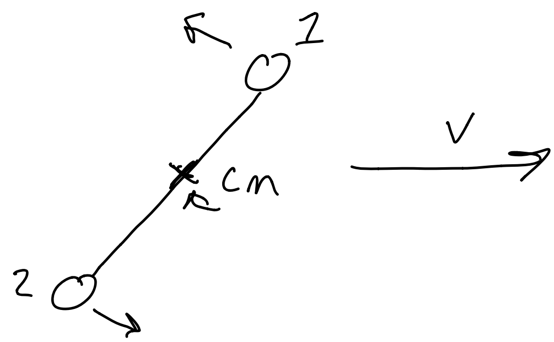

distance per rotation $= 2\pi r$

rotations per sec $= f$

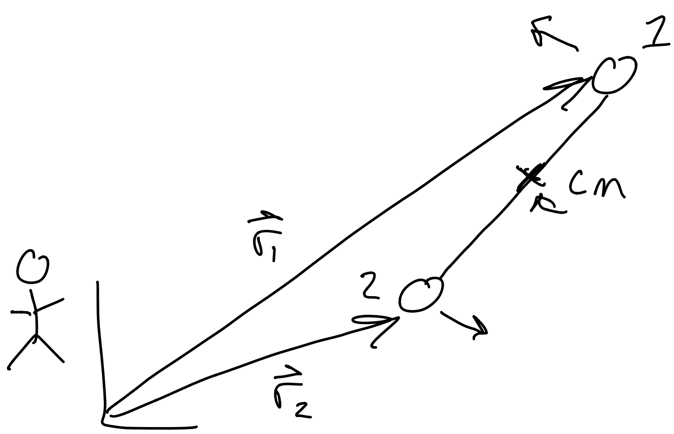
$$v = 2\pi r f = \left| \frac{d\vec{r}_{rel}}{dt} \right|$$

Conclusion: It's easy to describe the speed of a rotating molecule <u>relative to the center of mass</u>

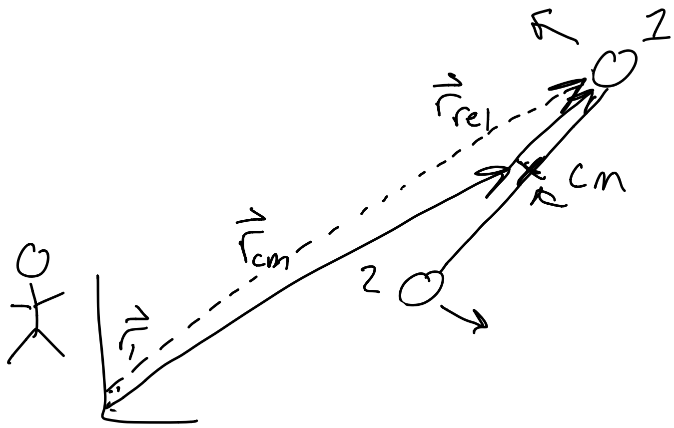
Consider a rotating dumbell




$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_1 v_2^2$$




$$|\vec{v}_1| = \frac{d\vec{r}_1}{dt}$$




$$\vec{r}_1 = \vec{r}_{cm} + \vec{r}_{rel}$$

position of $m_1$ = position of cm +
position of $m_1$
relative to cm

$$K_1 = \frac{1}{2} m_1 v_1^2$$

$$\vec{v}_1 = \frac{d}{dt}\left(\vec{r}_{cm} + \vec{r}_{rel}\right)$$

$$= \vec{v}_{cm} + \vec{v}_{rel}$$