Last Lecture

We finished by introducing a new principle:

$$\Delta \vec{p}_{total} = 0$$

If I release a ball from rest, gravity will accelerate it and its momentum goes from $0 \longrightarrow -0.44 \ \mathrm{kg}\frac{m}{s}$

$$\Delta \vec{p}_{ball} = \langle 0, -0.44, 0 \rangle \ \mathrm{kg}\frac{m}{s}$$

But $\Delta \vec{p}_{ball}$ is not $\Delta \vec{p}_{total}$

$$\Delta \vec{p}_{total} = \underbrace{\Delta \vec{p}_{ball}}_{\text{system}} + \underbrace{\Delta \vec{p}_{earth}}_{\text{surroundings}}$$

$$\Delta \vec{p}_{total} = 0 \ , \ \Delta \vec{p}_{bell} = -\Delta \vec{p}_{earth}$$

System vs surroundings

System: the object, or set of objects, we are interested in

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

$\vec{p}$ is the momentum of the system, $\vec{F}$ is the net force imparted on the system

Surroundings : anything that the system interacts with

$$\Delta \vec{p}_{sys} + \Delta \vec{p}_{surr} = 0$$

System = ball surr = Earth

$\Delta \vec{p}_{sys} = \langle 0, -.44, 0 \rangle$

$\Delta \vec{p}_{surr} = \langle 0, .44, 0 \rangle$

$\Delta \vec{p}_{sys} + \Delta \vec{p}_{sur} = 0$

We can choose the system!

System = ball + Earth

surr = None

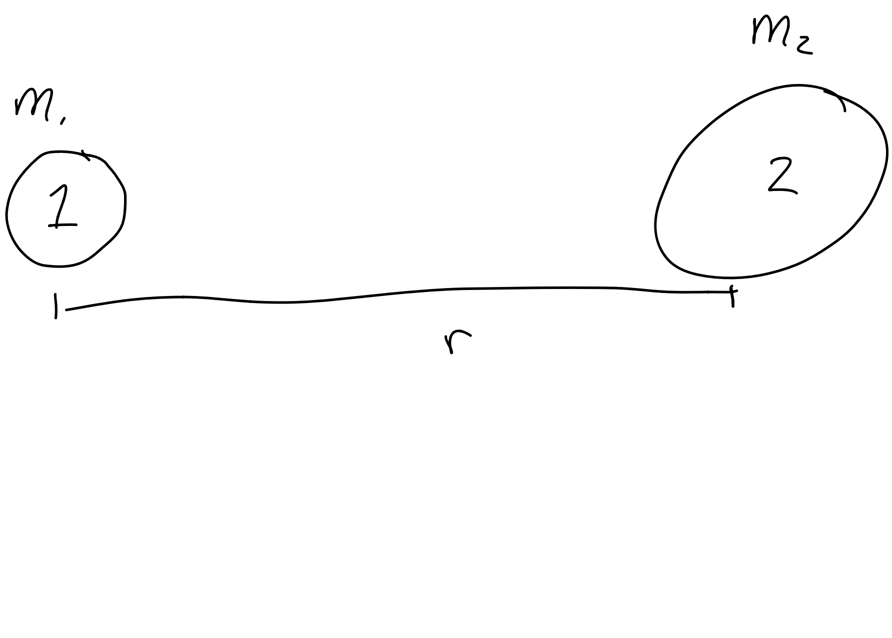
$\Delta \vec{p}_{sys} = \Delta \vec{p}_{ball} + \Delta \vec{p}_{earth}$

$= \langle 0, -.44, 0 \rangle + \langle 0, .44, 0 \rangle$

$= \vec{0}$

$0 + 0 = 0$

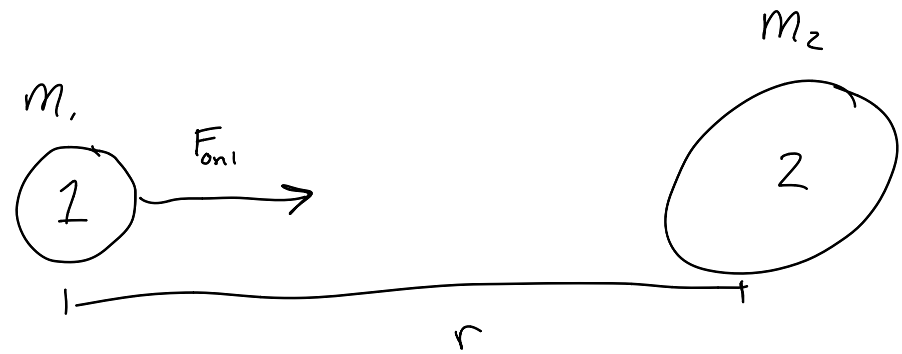
Ex: Two stars



Let system be star 1:

system: 1

surroundings: 2 (+ rest ~~of Universe~~)



$$\vec{F}_{net} = \left\langle \frac{Gm_1m_2}{r^2}, 0, 0 \right\rangle$$

$$\Delta \vec{p}_1 = \vec{F}_{net} \Delta t$$

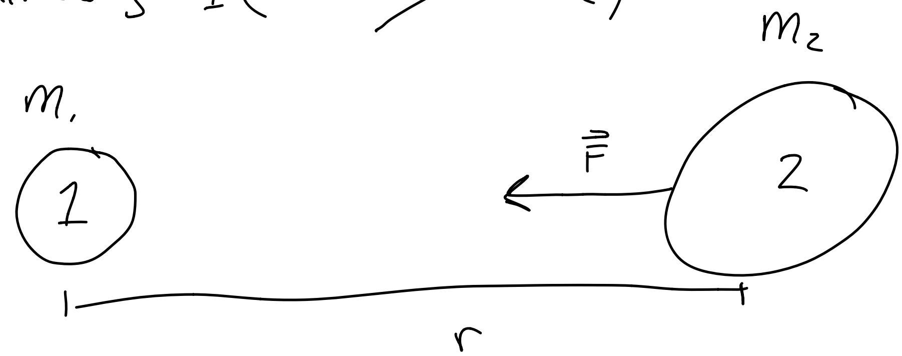
$$\Delta \vec{p}_{sys} = \left\langle \frac{Gm_1m_2}{r^2} \Delta t, 0, 0 \right\rangle$$

$$\Delta \vec{p}_{surr} = \left\langle -\frac{Gm_1m_2}{r^2} \Delta t, 0, 0 \right\rangle$$

Let system be star 2:

system: 2

surroundings: 1 (+ rest ~~of Universe~~)



$$\vec{F}_{net} = \left\langle -\frac{Gm_1m_2}{r^2}, 0, 0 \right\rangle$$

$$\Delta\vec{p}_2 = \vec{F}_{net}\,\Delta t$$

$$\Delta\vec{p}_{sys} = \left\langle -\frac{Gm_1m_2}{r^2}\Delta t, 0, 0 \right\rangle$$

$$\Delta\vec{p}_{surr} = \left\langle \frac{Gm_1m_2}{r^2}\Delta t, 0, 0 \right\rangle$$

Let system be 1 + 2

System: Stars 1 + 2

Surr: None

$$\vec{p}_{sys} = \vec{p}_1 + \vec{p}_2$$

$$\Delta\vec{p}_{sys} = \Delta\vec{p}_1 + \Delta\vec{p}_2$$

$$\Delta\vec{p}_1 = \vec{F}_{on1by2}\,\Delta t$$

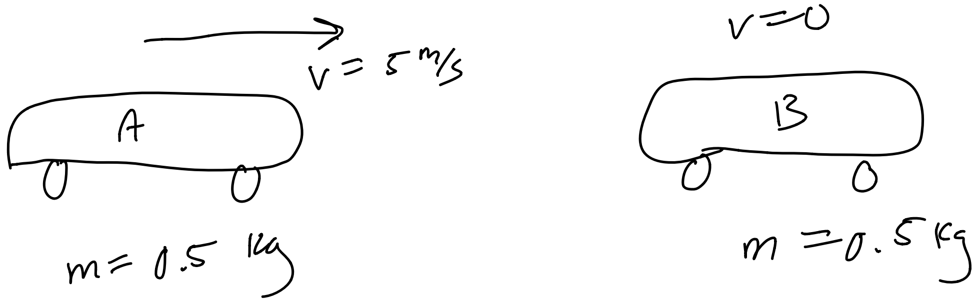
$$\Delta\vec{p}_2 = \vec{F}_{on2by1}\,\Delta t$$

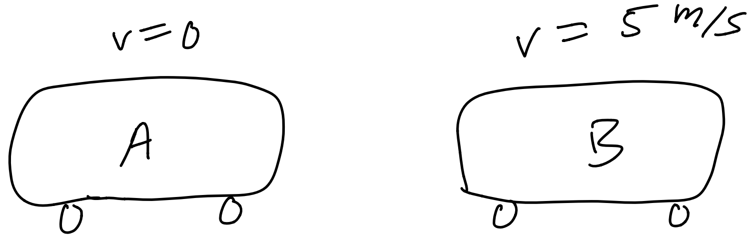
$$\Delta\vec{p}_{sys} = \vec{F}_{on1by2}\,\Delta t + \vec{F}_{on2by1}\,\Delta t$$

$$\vec{F}_{on1by2} = -\vec{F}_{on2by1}$$ Newton's 3rd

$$\Delta\vec{p}_{sys} = 0, \quad \Delta\vec{p}_{surr} = 0$$

Ex: Colliding carts





Let A be the system
B the surroundings

$$\Delta \vec{p}_A = \vec{p}_{Af} - \vec{p}_{Ai} = \langle 0,0,0 \rangle - \langle (0.5)(5), 0, 0 \rangle$$

$$\Delta \vec{p}_{sys} = \langle -2.5, 0, 0 \rangle \ \frac{kg\,m}{s}$$

Let B be the system

$$\Delta \vec{p}_B = \vec{p}_{Bf} - \vec{p}_{Bi}$$

$$= \langle (0.5)(5), 0, 0 \rangle - \langle 0, 0, 0 \rangle$$

$$\Delta \vec{p}_{sys} = \langle 2.5, 0, 0 \rangle \ \frac{kg\,m}{s}$$

A & B are
the System

$$\Delta \vec{p}_{sys} = \Delta \vec{p}_A + \Delta \vec{p}_B$$

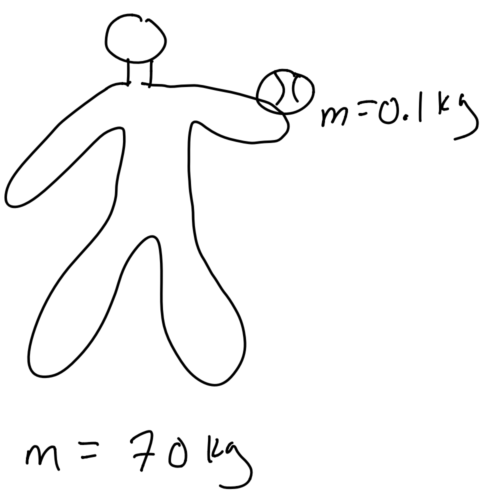
$$= \langle -2.5, 0, 0 \rangle + \langle 2.5, 0, 0 \rangle = 0$$

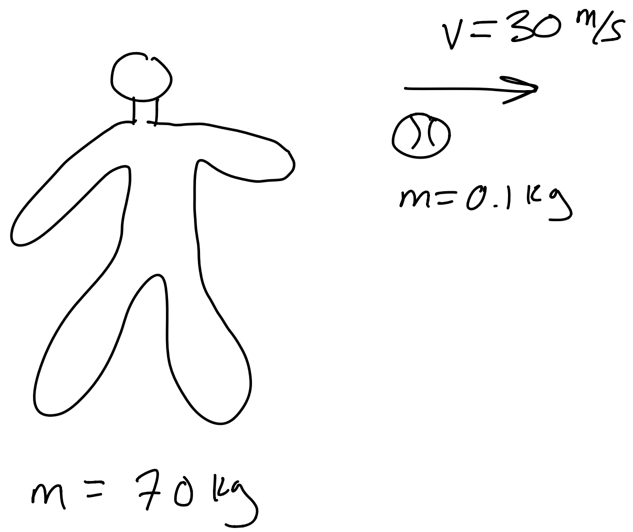
No external Force acts on the system

Why is this useful?

We can use it to make predictions

Ex: Astronauts playing catch





What happens to the astronaut?

Newton's 3rd Law: Astronaut exerts force on ball, ball exerts force on astronaut

System:

astronaut + ball

Surr: None

$$\vec{p}_i = \vec{p}_{ast,i} + \vec{p}_{ball,i} = \langle 0, 0, 0 \rangle \ \frac{kg\,m}{s}$$

$$\vec{p}_f = \vec{p}_{ast,f} + \vec{p}_{ball,f}$$

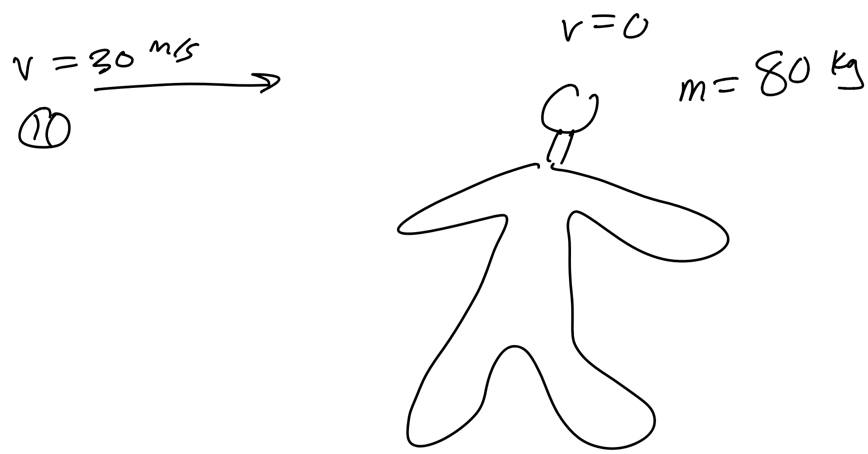
$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0$$

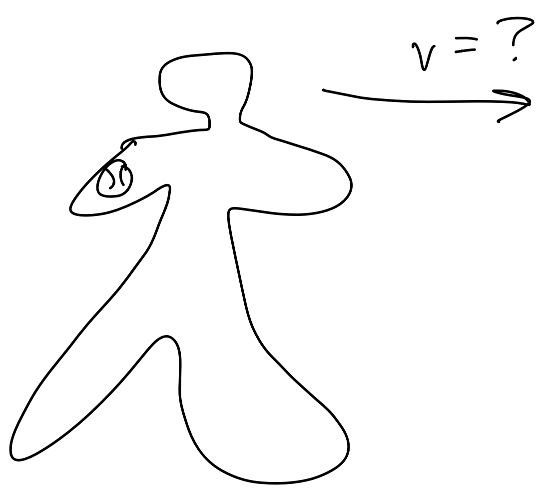
$$\vec{p}_{ast,f} + \langle (0.1)(30), 0, 0 \rangle - \langle 0, 0, 0 \rangle = \langle 0, 0, 0 \rangle$$

$$\vec{p}_{ast,f} = \langle -3, 0, 0 \rangle \ \frac{kg\ m}{s}$$

$$\vec{v} = \frac{\vec{p}}{m} = \langle -0.043, 0, 0 \rangle \ m/s$$

The other astronaut catches the ball





System:

astro + ball

$\vec{p}_i = \vec{p}_{ball,i} + \vec{p}_{ast,i} = \langle (0.1)(30,0,0) \rangle + \langle 0,0,0 \rangle$

$\vec{p}_i = \langle 3,0,0 \rangle \frac{kg\,m}{s}$

$\vec{p}_f = \vec{p}_{f\,ball} + \vec{p}_{f\,astro}$

$\vec{v}_{f\,ball} = \vec{v}_{f\,astro} = \vec{v}_f$

$\vec{p}_f = m_{ball}\vec{v}_{f\,ball} + m_{ast}\vec{v}_{f\,ast}$

$\vec{p}_f = (m_{ball} + m_{ast})\vec{v}_f$

$\Delta\vec{p} = 0 = \vec{p}_f - \vec{p}_i$

$= (m_{ball} + m_{ast})\vec{v}_f - \langle 3,0,0 \rangle = 0$

$80.1\,\vec{v}_f = \langle 3,0,0 \rangle$

$\vec{v}_f = \langle 0.037,0,0 \rangle$ m/s

1) Pick a system

2) Write $\vec{p}_i$, $\vec{p}_f$

3) $\Delta \vec{p} = 0$, solve for unknowns