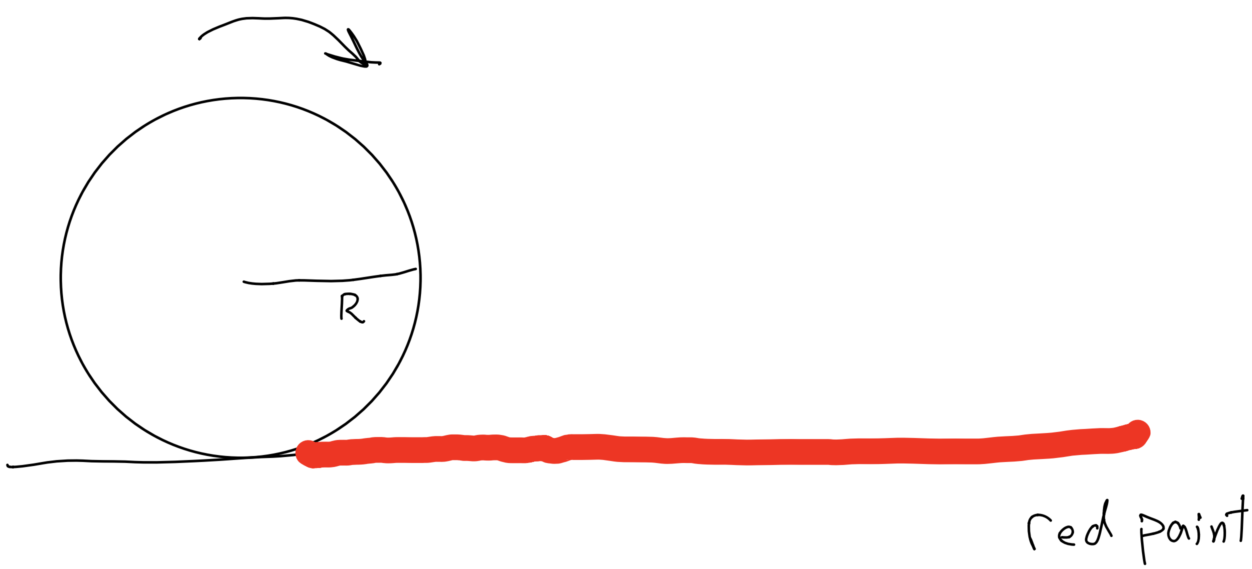
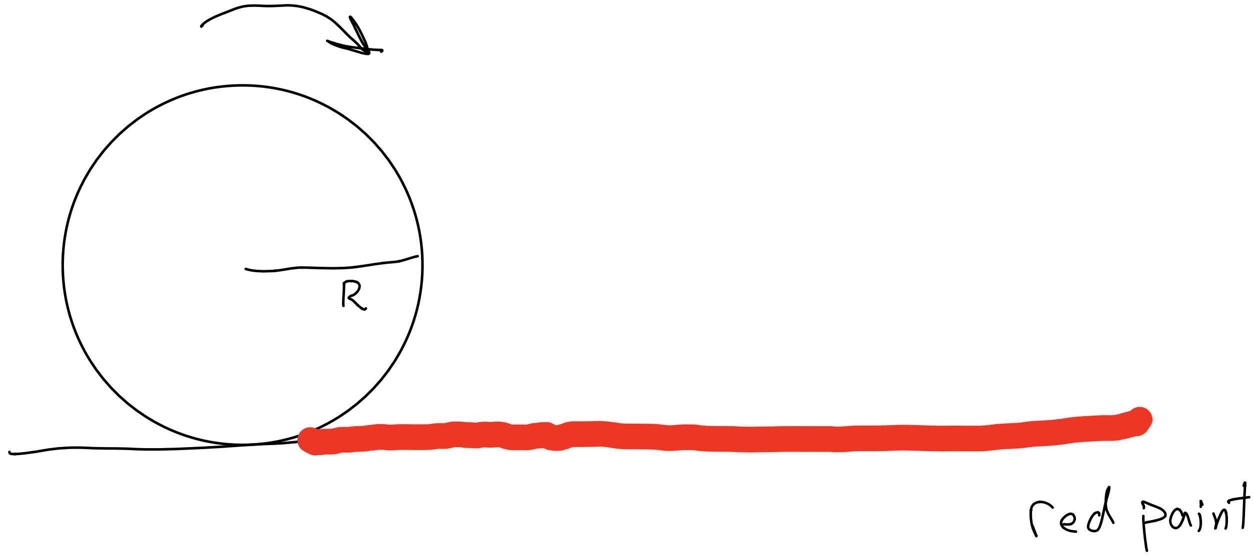
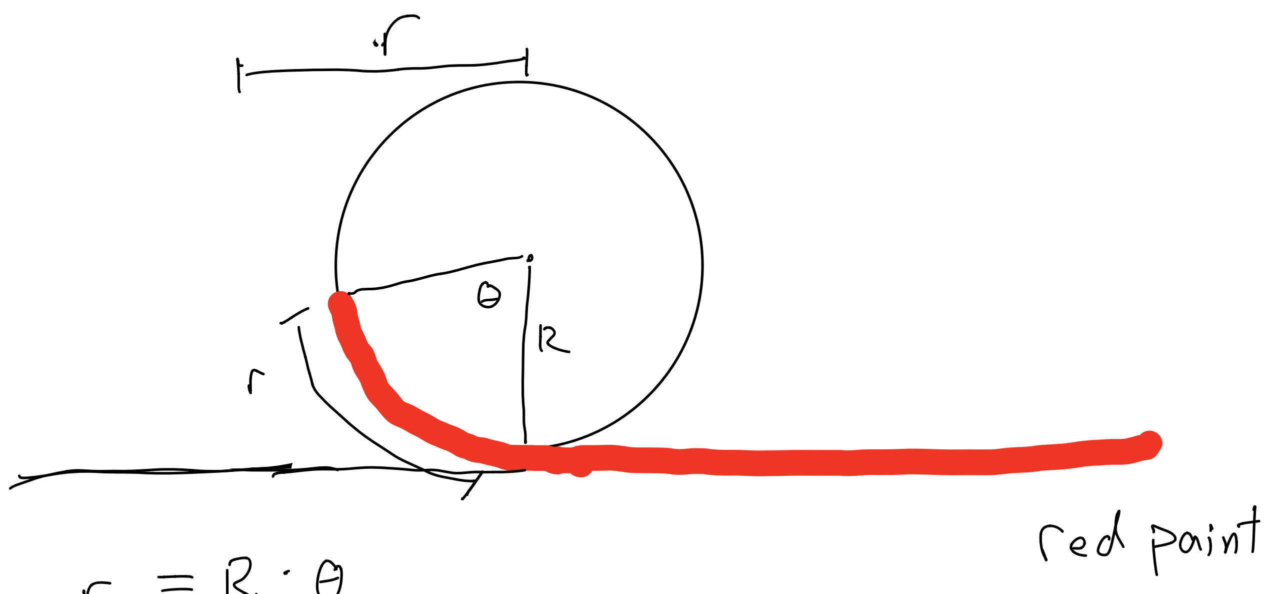
Common Special case:
rolling w/o slipping

If the ball does not slip when it makes contact with the ground, v + ω will be related

- silly thought experiment







$$r = R \cdot \theta$$

$$\frac{d}{dt} r = \frac{d}{dt}(R\theta)$$

$$\frac{dr}{dt} = v = R\frac{d}{dt}\theta = R\omega; \quad \omega = \frac{v}{R}$$

so

$$K = K_{trans} + K_{rot}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2$$

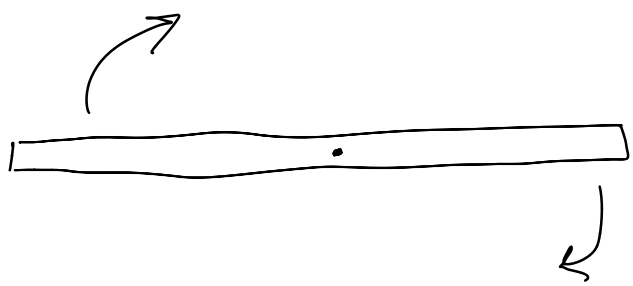
$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}}$$

One more example:

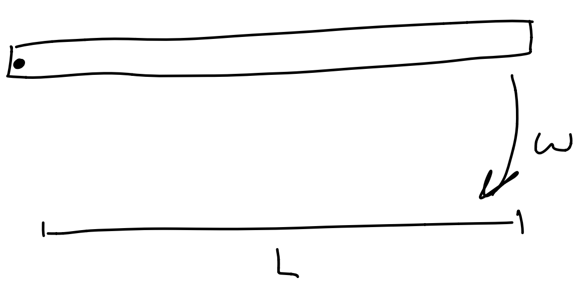
Kinetic energy of rotating thin rod

Haven't we already done this?

We looked at a rod rotating around its center of mass



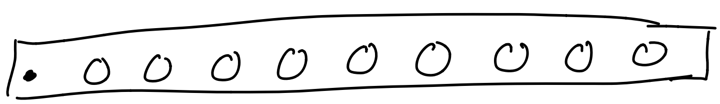
What if its rotating around the end?



$$K = \underbrace{K_{trans}}_{\frac{1}{2}mv_{cm}^2} + \underbrace{K_{rel}}_{\sum \frac{1}{2}mv_{rel}^2}$$

But the center of mass is rotating, so $K_{trans} \neq 0$ + we don't know v_{rel}

$$K = \sum \frac{1}{2}mv^2, \quad v = \omega r_\perp$$



$$K = \frac{1}{2}\left[m_1(\omega x_1)^2 + m_2(\omega x_2)^2 + \ldots\right]\omega^2$$

$$= \frac{1}{2}\left[m_1 x_1^2 + m_2 x_2^2 + \ldots\right]\omega^2$$

$$m_1 = m_2 = m_3 = \frac{M}{L}\Delta x$$

$$K = \frac{1}{2}\frac{M}{L}\left[x_1^2 \Delta x + x_2^2 \Delta x + \ldots\right]\omega^2$$

$$K \longrightarrow \frac{1}{2}\frac{M}{L}\omega^2 \int_{x_i}^{x_f} x^2 dx$$

$$K = \frac{1}{2}\frac{M}{L}\omega^2 \int_{x_i}^{x_f} x^2 dx$$

when rod rotates about the center:

$$x_i = -\frac{L}{2}, \quad x_f = \frac{L}{2}$$

Now:

$$x_i = 0, \quad x_f = L$$

$$K = \frac{1}{2}\left(\frac{M}{L}\int_0^L x^2 dx\right)\omega^2$$

$$K = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$$

Behaves like an object rotating with $I = \frac{1}{3}ML^2$

Note: $$\frac{M}{L}\int_0^L x^2 dx = \underbrace{\frac{M}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 dx}_{I_{cm}} + \underbrace{M\left(\frac{L}{2}\right)^2}_{M \cdot r_{cm}^2}$$

r_{cm} = dist from cm to rotate axis

Parallel axis theorem

Scenario:

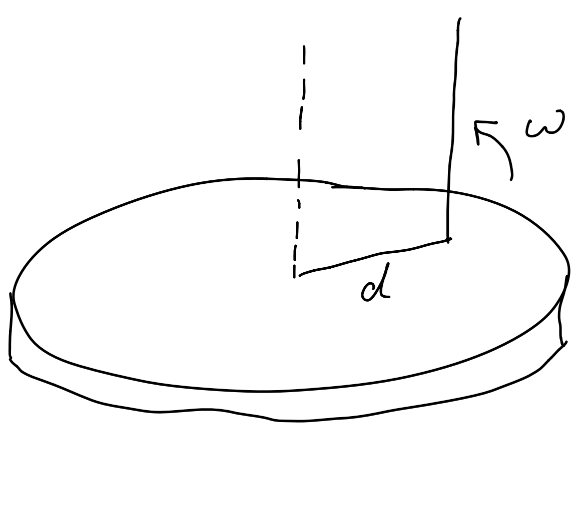
For any object rotating about an axis that isn't its center of mass:

$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M r_{cm}^2 \omega^2$$

K of object rotating around cm

K of rotating cm

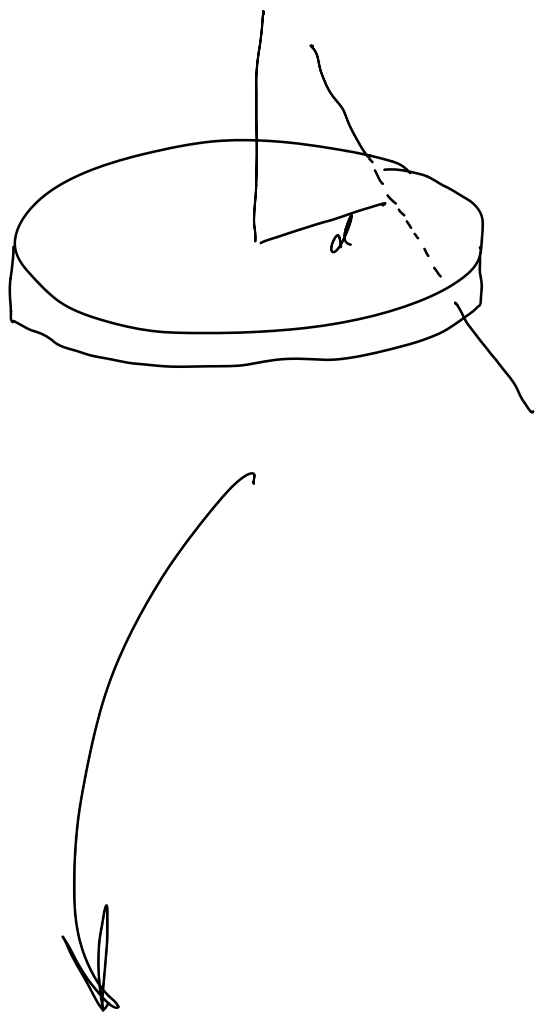
Called the parallel axis theorem



$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M r_{cm}^2 \omega^2$$

$$K = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\omega^2 + \frac{1}{2} M d^2 \omega^2$$

Only applies if axes are parallel



We have seen that there is kinetic energy
assoc with rotating objects

How does the motion of a rotating object
change if I apply a force to it?

- For "ordinary" systems, we have learned
we can describe this relationship two ways:

(1) $$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$, momentum principle

(2) $$\Delta E = \vec{F}_{net} \cdot \Delta\vec{r}$$, energy principle

Two complementary ways of analyzing a system

(1) a net force changes a system's momentum
(2) a net force exerted over some distance
changes a systems energy

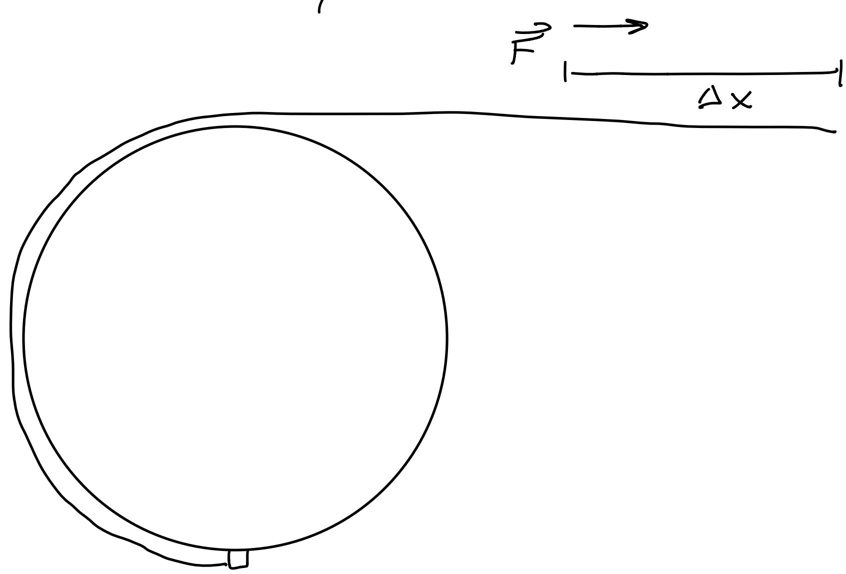
Conservation

$$\Delta \vec{p}_{sys} + \Delta \vec{p}_{sur} = \vec{0}$$

$$\Delta E_{sys} + \Delta E_{surr} = 0$$

What about a rotating object?

- We know you can do work to change its energy

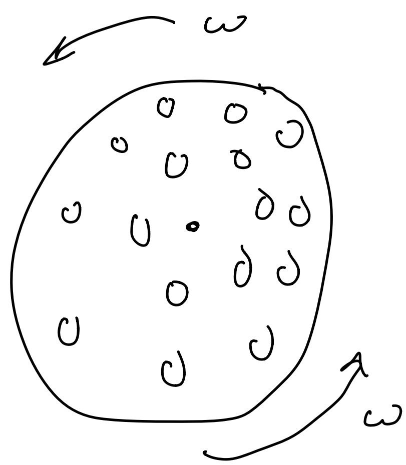


$$\Delta E = F \Delta x$$

$$\frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = F \Delta x$$

What about momentum?

What is the momentum of a rotating body?



$$\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_N \vec{v}_N$$

$$= \vec{p}_{cm}$$

If a ball is rotating but not moving,

$$\vec{p}_{cm} = \vec{0}$$

It doesn't matter what ω is,

$$\vec{p} = \vec{0}$$

This contradicts our intuition

What is momentum?

- The thing that changes when we apply a force

- Momentum resists changes in $\vec{v}$

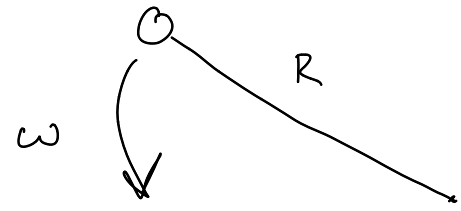
- What resists changes in ω?

A new quantity: angular momentum

- Quantifies a rotating object's resistance to changes in ω

$\vec{L}$

Consider a point of mass rotating at the end of a massless string



we know:

$$K = \frac{1}{2} m v^2$$

$$v = \omega R$$

Takes more energy to change ω if:

- m is large
- v is large
- R is large

Change in motion is resisted by m, v, + R

$$|\vec{L}| = m |\vec{v}| |\vec{r}| \sin\theta$$

Translational / Orbital Angular momentum

$$|\vec{L}_{trans}| = |\vec{p}||\vec{r}| \sin\theta$$

An object's resistance to change in ω increases with its momentum & its distance away from the axis of rotation