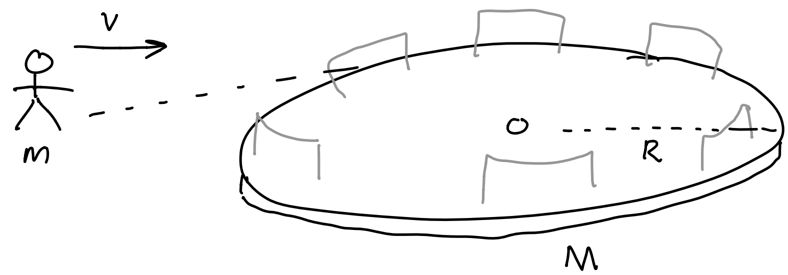
merry go round

Example:

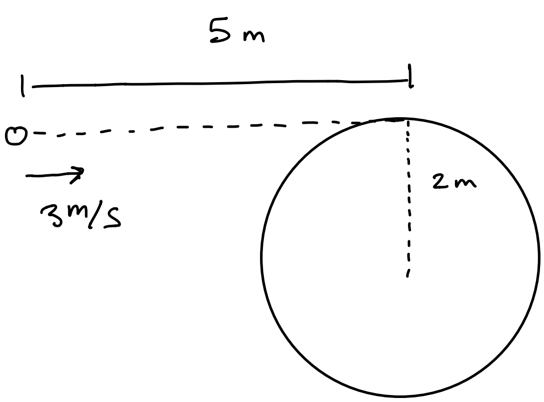


Kid runs + jumps on ride.

Will cause ride to spin.

How fast does it spin?

Conservation of angular momentum



$m = 40$ kg

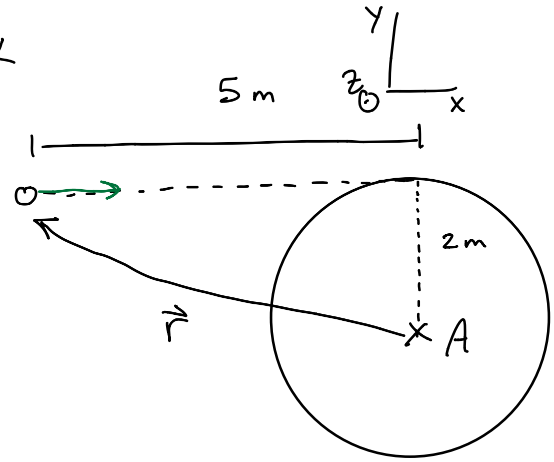
$M = 300$ kg

$I = \frac{1}{2} M R^2$

System: Kid + disk

$\vec{L} = \vec{L}_{kid} + \vec{L}_{disk}$

Calc $\vec{L}$ relative to center of disk



$$\vec{L}_i = \vec{L}_{kid}$$

$$= \vec{r} \times \vec{p}$$

$$\vec{r} = \langle -5, 2 \rangle \text{ m}$$

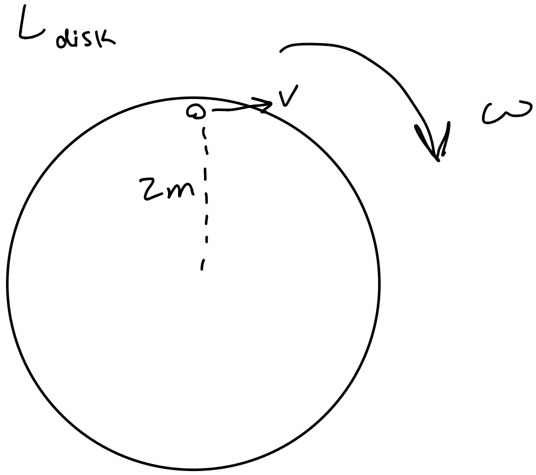
$$\vec{p} = \langle (40\text{kg})(3\tfrac{m}{s}), 0 \rangle = \langle 120, 0 \rangle \ \frac{kg\,m}{s^2}$$

$$\vec{r} \times \vec{p} = \langle yp_z - zp_y,\ zp_x - xp_z,\ xp_y - yp_x \rangle$$

$$= \langle 0,\ 0,\ (-5\text{m})(0) - (2\text{m})(120\ \tfrac{kg\,m}{s}) \rangle$$

$$\vec{L}_i = \langle 0, 0, -240\ \frac{kg\,m^2}{s} \rangle$$

$$\vec{L}_f = \vec{L}_{kid,f} + \vec{L}_{disk}$$



Disk rotates with velocity $\vec{\omega}$

$$\vec{L}_{disk} = I\vec{\omega} = -I\omega\hat{z}$$

$$\vec{L}_{kid} = \vec{r} \times \vec{p}$$

$R=2$

$$\vec{r} = \langle 0, R \rangle m$$

$$\vec{p} = \langle mv, 0 \rangle$$

$$v = |\vec{\omega}| R$$

$$\vec{p} = \langle m\omega R, 0 \rangle$$

$$\vec{L}_{kid} = R\hat{y} \times m\omega R\hat{x} = -m\omega R^2\hat{z}$$

$$\vec{L}_f = -I\omega\hat{z} - m\omega R^2\hat{z}$$

$$= -\frac{1}{2}MR^2\omega\hat{z} - m\omega R^2\hat{z}$$

$$= -\left(\frac{1}{2}MR^2 + mR^2\right)\omega\hat{z}$$

$$= -\left(\frac{1}{2}M + m\right)R^2\omega$$

$$\vec{L}_i = \vec{L}_f$$

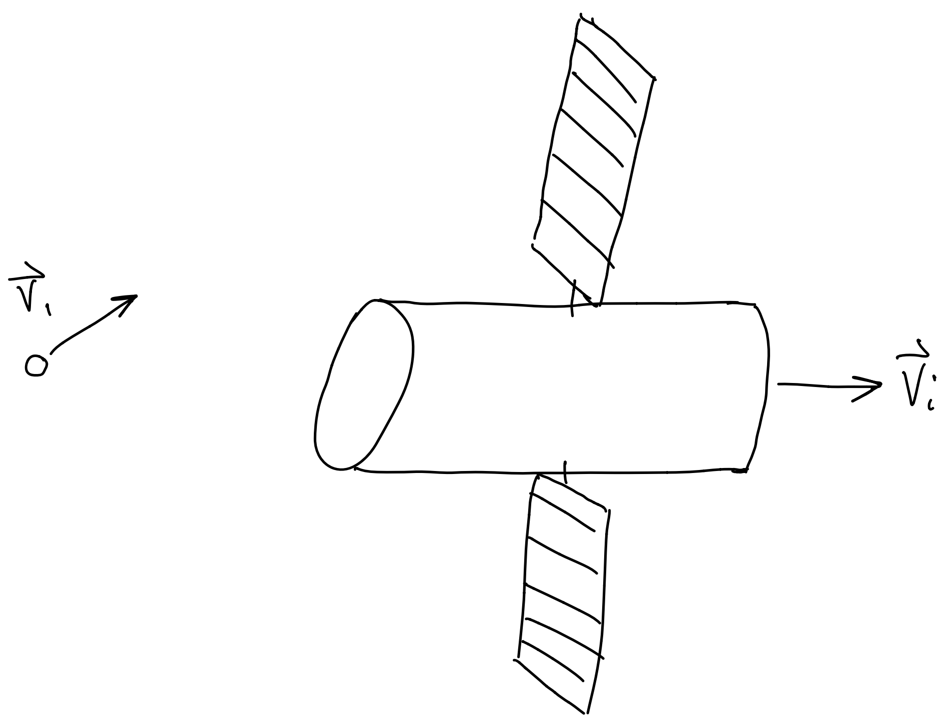
$$\left(-240\ \frac{kg\,m^2}{s}\right)\hat{z} = -\left(\frac{1}{2}M + m\right)R^2\omega$$

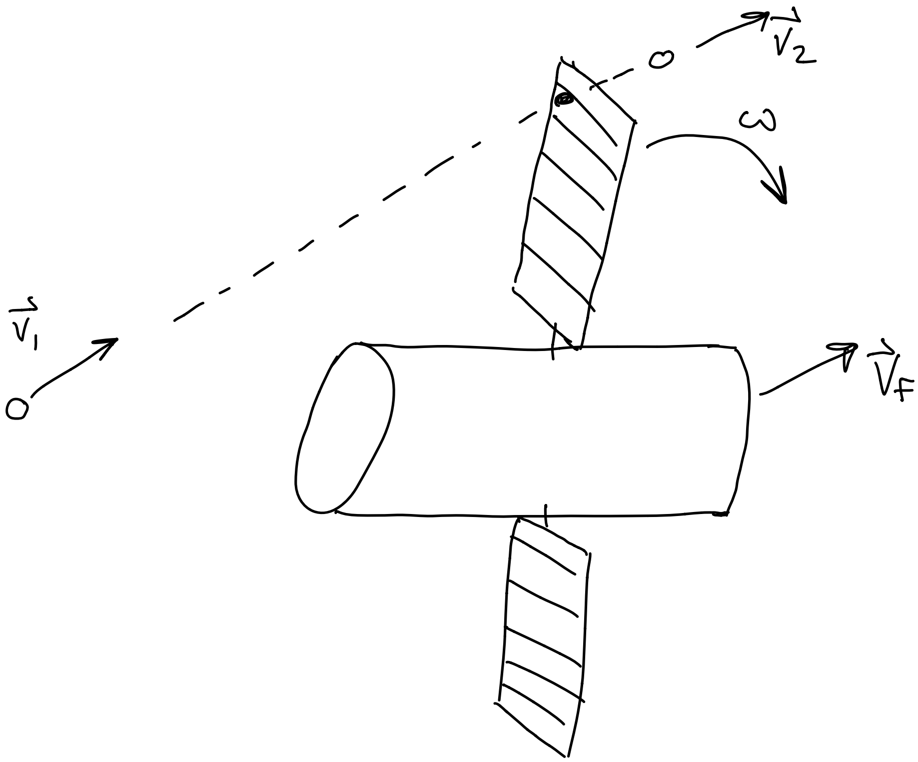
$$\omega = \frac{(240)}{(\frac{1}{2}M + m)R^2} = \frac{240}{(150+40)(4)} = 0.316 \ \frac{rad}{s}$$

$$\boxed{\omega = 0.316 \ \frac{rad}{sec}}$$

Another example:

A satellite in outer space





Basic idea:

Choose sytem to be rock + satellite

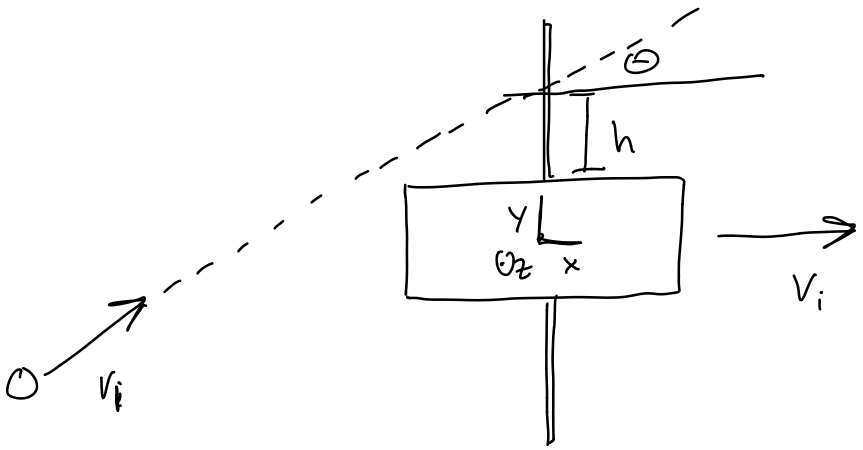
Then: $\vec{p}_i = \vec{p}_f$

$\vec{L}_i = \vec{L}_f$

m_1 = mass of rock
M = mass of satellite

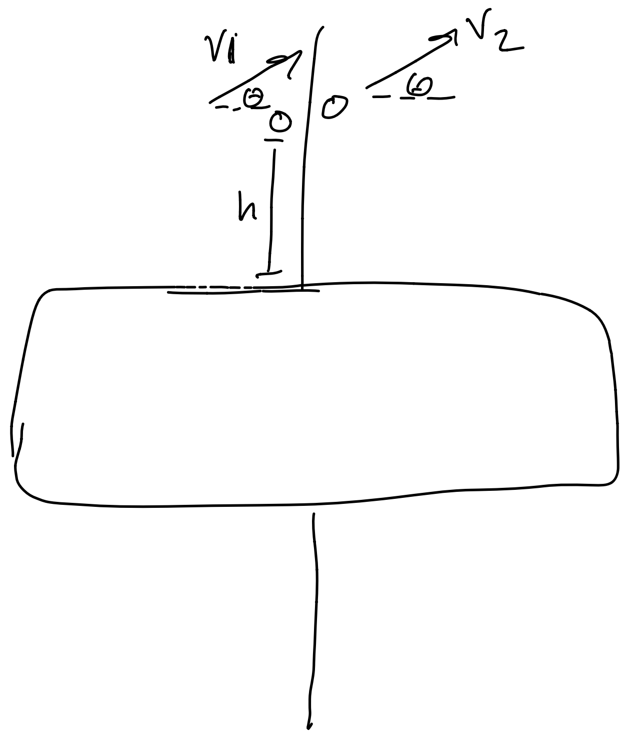
Also know R + L of satellite

Satellite moves in $\hat{x}$ direction with speed v



Calculate $\vec{p}$, $\vec{L}$ immediately before &

after collision



Momentum:

$$\vec{p}_i = \vec{p}_{sat,i} + \vec{p}_{meteor,i}$$

$$= M\vec{v}_i + m_1\vec{v}_1$$

$$\vec{p}_i = Mv_i\hat{x} + m_1 v_1 \left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)$$

$$\vec{p}_i = \left(Mv_i + m_1 v_1 \cos\theta\right)\hat{x} + m_1 v_1 \sin\theta\,\hat{y}$$

$$\vec{p}_f = M\vec{v}_f + m_1\vec{v}_2$$

$$\vec{p}_f = M\vec{v}_f + m_1 v_2 \left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)$$

$$\vec{p}_i = \vec{p}_f$$

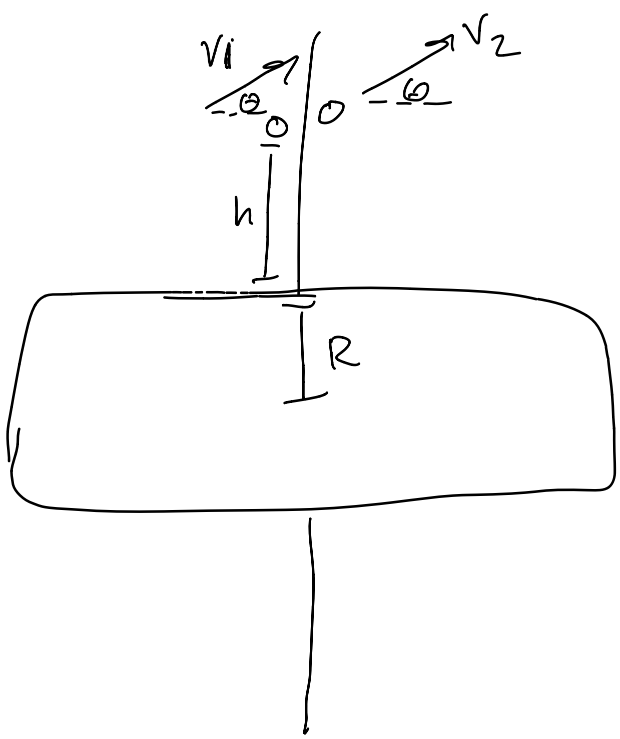
$$\left(Mv_i + m_1 v_1 \cos\theta\right)\hat{x} + m_1 v_1 \sin\theta\,\hat{y} = M\vec{v}_f + m_1 v_2 \left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)$$

$$M\vec{v}_f = \left(Mv_i + m_1 v_1 \cos\theta - m_1 v_2 \cos\theta\right)\hat{x} + \left(m_1 v_1 \sin\theta - m_1 v_2 \sin\theta\right)\hat{y}$$

$$v_{f,x} = v_i + \frac{m_1 (v_1 - v_2)\cos\theta}{M}$$

$$v_{f,y} = \frac{m_1}{M}(v_1 - v_2)\sin\theta$$

Angular momentum:



$$\vec{L}_i = \vec{L}_{meteror} = \vec{r}_i \times \vec{P}_i$$

$$\vec{r}_i = (R+h)\hat{y}$$

$$\vec{P}_i = m_1 \vec{v}_1 = m_1 v_1 (\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$$

$$\vec{r} \times \vec{P} = (R+h)\,\hat{y} \times (m_1 v_1 \cos\theta\,\hat{x} + m_1 v_1 \sin\theta\,\hat{y})$$

$$= (R+h) m_1 v_1 \cos\theta\,(\hat{y} \times \hat{x}) + 0$$

$$= -(R+h) m_1 v_1 \cos\theta\,\hat{z}$$

$$\vec{L}_f = \vec{L}_{meteor,f} + \vec{L}_{sat}$$

$$\vec{L}_{sat} = I\vec{\omega} \ , \quad I = I_{cylinder} = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$$

$$\vec{L}_{metero} = \vec{r} \times \vec{p}_f$$

$$\vec{r} = (R+h)\,\hat{y}$$

$$\vec{p}_f = m_1 v_2 (\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$$

$$\vec{r} \times \vec{p}_f = -(R+h) m_1 v_2 \cos\theta\,\hat{z}$$

$$\vec{L}_f = I\vec{\omega} - (R+h) m_1 v_2 \cos\theta\,\hat{z}$$

$$\vec{L}_i = \vec{L}_f$$

$$-(R+h) m_1 v_1 \cos\theta\,\hat{z} = I\vec{\omega} - (R+h) m_1 v_2 \cos\theta\,\hat{z}$$

$$I\vec{\omega} = (R+h) m_1 (v_2 - v_1) \cos\theta\,\hat{z}$$

$$\vec{\omega} = \frac{(R+h) m_1 (v_2 - v_1)\cos\theta}{\frac{1}{12}ML^2 + \frac{1}{4}MR^2}\,\hat{z}$$