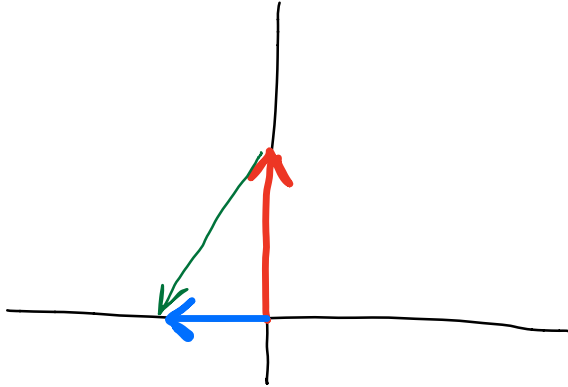


Q 1.4.c

$$\vec{r}_p = \langle 0, 3, -2 \rangle \text{ m}$$

$$\vec{r}_e = \langle -1, 0, -6 \rangle \text{ m}$$



$$\vec{r} = \vec{r}_e - \vec{r}_p$$

$$\vec{r} = \langle -1, 0, -6 \rangle \text{ m} - \langle 0, 3, -2 \rangle \text{ m}$$

$$\vec{r} = \langle -1, -3, -4 \rangle \text{ m}$$

Q1.4.e

$$\vec{r} = \langle 3, 5, -2 \rangle \text{ m}$$

$$\hat{r} = ?$$

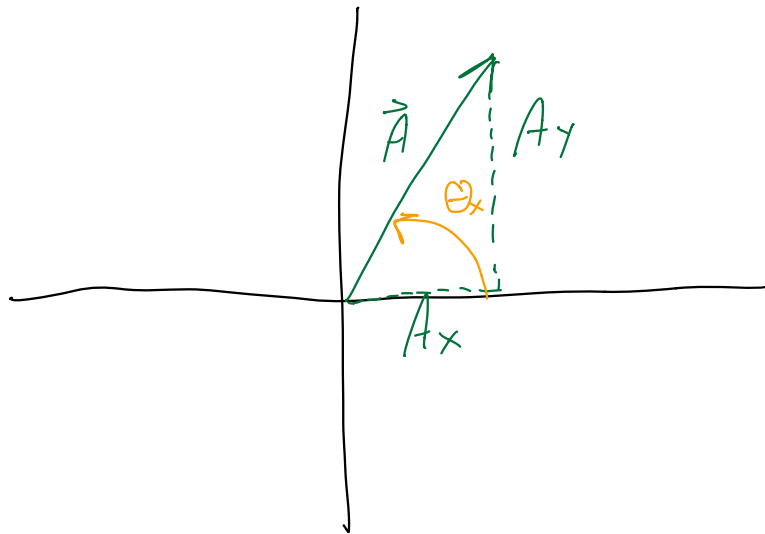
$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$|\vec{r}| = \sqrt{3^2 + 5^2 + (-2)^2} = 6.16$$

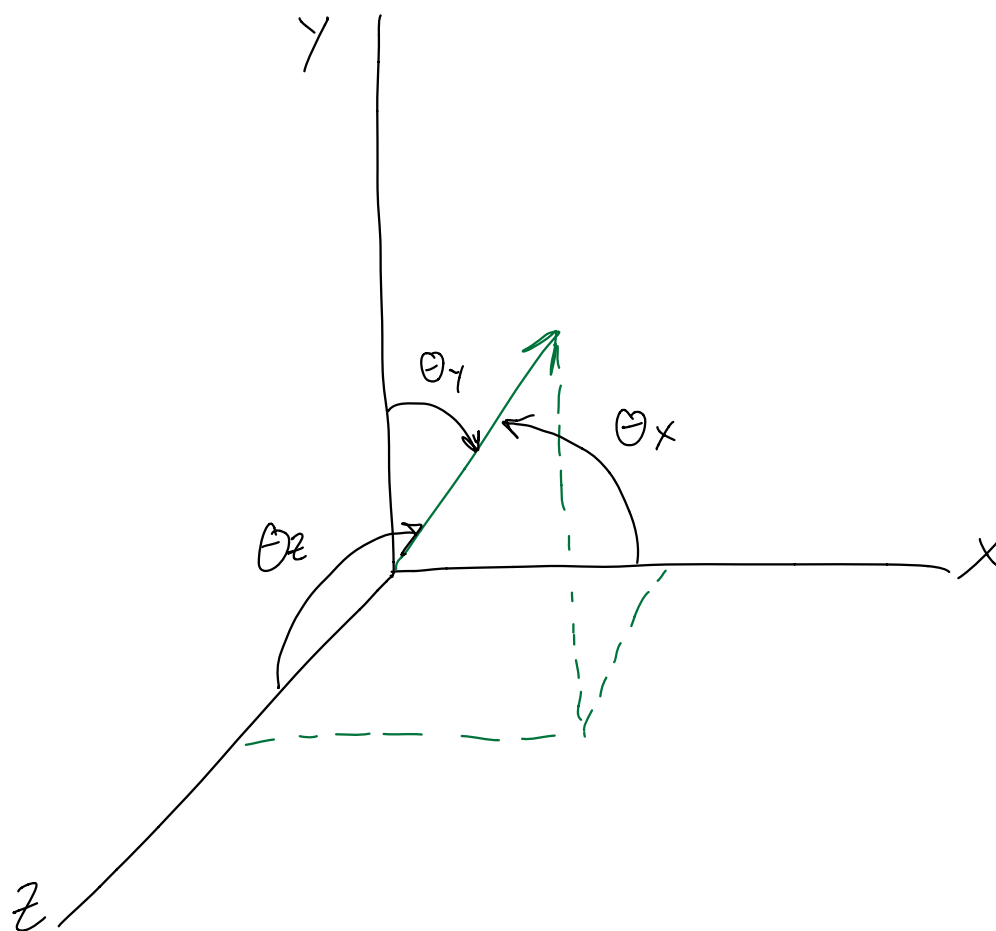
$$\hat{r} = \frac{1}{6.16} \langle 3, 5, -2 \rangle$$

$$= \langle 0.49, 0.81, -0.32 \rangle$$

Another way to describe unit  
vectors



$$A_x = \cos(\theta_x)$$



$$\hat{r} = \langle \cos \theta_x, \cos \theta_y, \cos \theta_z \rangle$$

## Example

A golf ball is hit off the tee w/ velocity

$$\vec{v} = \langle 80, 60, 0 \rangle \frac{m}{s}$$

$$\text{Speed: } |\vec{v}| = \sqrt{80^2 + 60^2} = 100 \frac{m}{s}$$

Direction:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{100} \langle 80, 60, 0 \rangle$$

$$\hat{v} = \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle$$

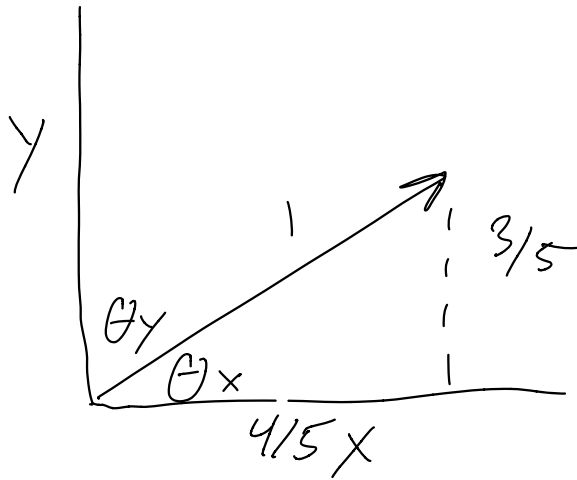
What is  $\theta_x$ ?  $\theta_y$ ?  $\theta_z$ ?

$$\hat{v} = \langle \cos \theta_x, \cos \theta_y, \cos \theta_z \rangle$$

$$\cos \theta_x = \frac{3}{5}$$

$$\theta_x = \cos^{-1}\left(\frac{4}{5}\right) = 0.64 = 37^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{3}{5}\right) = 0.93 = 53^\circ$$



What is velocity?

We know what speed is

- Velocity is the rate of change of position

If I start at some <sup>initial</sup> position  
called  $\vec{r}_i$  at some initial time  
 $t_i$ , and move to some  
separate position  $\vec{r}_f$  by time  
 $t_f$ , the

$$\vec{V}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Did this yesterday in lab!

A few notes:

- Time is a scalar, so  $t_f - t_i$   
is a scalar

-  $\vec{r}_f - \vec{r}_i$  is a vector

-  $\vec{V}_{avg}$  is a vector

Really 3 equations

$$\langle V_{av,x}, V_{av,y}, V_{av,z} \rangle =$$

$$\frac{\langle x_f, y_f, z_f \rangle - \langle x_i, y_i, z_i \rangle}{t_f - t_i}$$



$$V_{av, x} = \frac{x_f - x_i}{t_f - t_i}$$

$$V_{av, y} = \frac{y_f - y_i}{t_f - t_i}$$

. . . -

- We use final - initial  
so that  $\vec{V}_{av}$  points in  
the direction of motion

This concept of final - initial  
is so important that we have  
special notation for it

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta t = t_f - t_i$$

$\Delta$ : Greek letter "delta"  
Change

$$\vec{V}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\Delta \vec{r}}{\Delta t}$$

Ex: the bee in flight

$$\vec{r}_i = \langle 2, 4, 0 \rangle \text{ m} \quad \vec{r}_f = \langle 3, 3.5, 0 \rangle \text{ m}$$

$$t_i = 15 \text{ s}, t_f = 15.5 \text{ s}, \Delta t = 0.5 \text{ s}$$

$$\Delta \vec{r} = \langle 1, -.5, 0 \rangle, \quad \vec{V} = \frac{\Delta \vec{r}}{\Delta t} = \langle 2, -1, 0 \rangle \frac{\text{m}}{\text{s}}$$

Note:  $\vec{V}_{avg}$  is not avg speed

Indy 500?

Note: if I know the position and velocity of an object at a certain time, I can use them to predict the position of the object at some later time.

-Only works if velocity is constant!

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\Delta \vec{r} = \Delta t \vec{v}_{avg}$$

$$\vec{r}_f - \vec{r}_i = \Delta t \vec{v}_{avg}$$

$$\vec{r}_f = \vec{r}_i + \Delta t \vec{v}_{avg}$$

EX: A bird flies at constant velocity. At one moment, you see the bird at position

$$\vec{r}_i = \langle 5, 9, -3 \rangle \text{ m,}$$

flying w  $\vec{v}_{avg} = \langle 10, 0, 5 \rangle \frac{\text{m}}{\text{s}}$ .

What will the position be in 5 s?

$$\vec{r}_f = \vec{r}_i + \Delta t \vec{V}_{avg}$$

$$= \langle 5, 9, -3 \rangle_m + 5s \cdot \langle 10, 0, 5 \rangle \frac{m}{s}$$

$$= \langle 5, 9, -3 \rangle_m + \langle 50, 0, 25 \rangle_m$$

$$\boxed{\vec{r}_f = \langle 55, 9, 22 \rangle_m}$$

Q 1.7.b