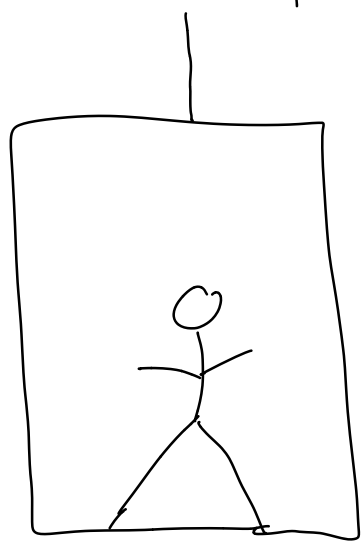
Changing momentum: simple example



$m = 75$ kg

(165 lbs)

Elevator starts at rest, & over the course of 2 s accelerates to 4 m/s (going down)

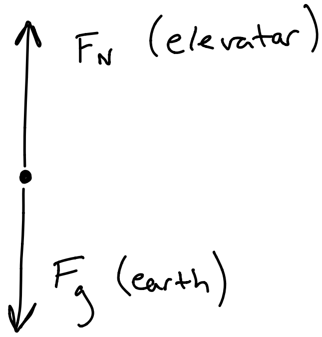
What is the passenger's apparent weight during this constant acceleration?

Apparent weight $\longrightarrow$ Normal Force

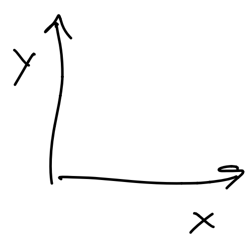
① Choose system

The person

(2)



(3)



(4) $\vec{F}_g = \langle 0, -F_g, 0 \rangle$

$\vec{F}_N = \langle 0, F_N, 0 \rangle$

$$\frac{dp_y}{dt} = F_N - F_g \quad \neq 0$$

accel is constant

$$\frac{dp_y}{dt} = \frac{d}{dt}(mv_y) = m\frac{d}{dt}v_y = ma_y$$

$$ma_y = F_N - F_g \Rightarrow \quad F_N = F_g + ma_y$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\langle 0, -4, 0 \rangle \text{ m/s} - \langle 0, 0, 0 \rangle}{2\text{ s}}$$

$$\vec{a} = \langle 0, -2, 0 \rangle \text{ m/s}^2$$

$$a_y = -2$$

$$F_N = F_g - 2m$$

$$F_N = mg - 2m$$

$$F_N = m(g - 2) = 585 \text{ N}$$

$$(132 \text{ lb})$$

Elevator accelerates away, F_N decreases
you feel lighter

In this example, only $|\vec{p}|$ was changing

In general, $\vec{p}$ can change in both mag + direction

- We will look at a special subset of problems: curving motion

Ex: Race car driver rounds a turn

Roller coaster does a loop

Tarzan swings on a vine

Goal: use momentum principle

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$ to calculate

forces expierienced by objects undergoing this motion

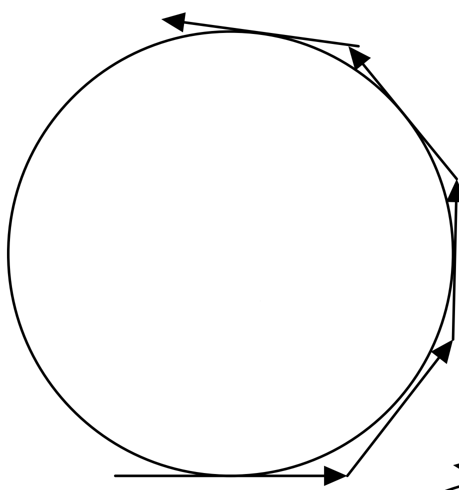
What does $\vec{F}_{net}$ look like for curving motion?

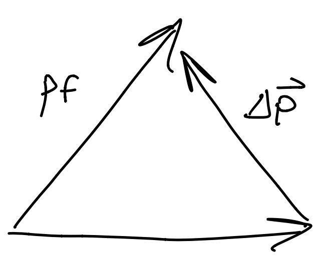
Ex: racecar driver

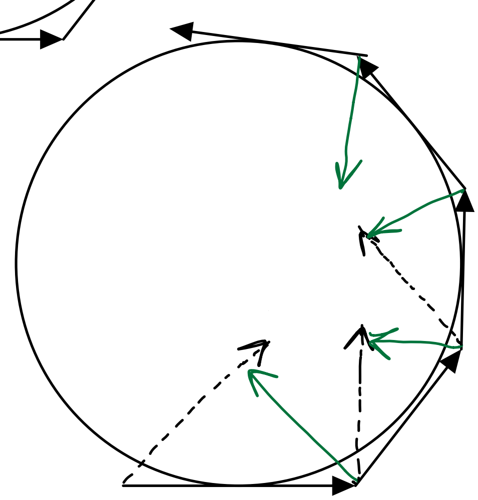
?

Is $\vec{F}_{net}$ 0 here?

constant speed, $\vec{F}_{net} = \vec{0}$







This makes sense.

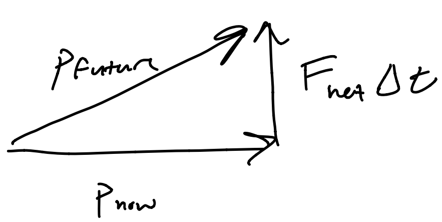
To change an object's direction, we must apply a force in a different direction than it is already moving

Consider:

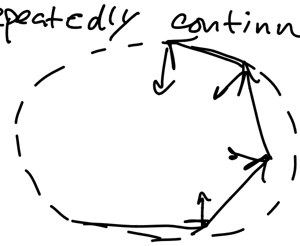
A soccer ball rolling by with momentum $\vec{p}$

What will its momentum be if I kick it in the perpendicular direction?

$$\vec{p} = \vec{p}_{now} + \vec{F}_{net} \Delta t$$



If I (somehow) repeatedly continue this



circular motion @ const speed

Conclusion: A force perpendicular to object's momentum changes it's direction

a continual force perpendicular to the object's momentum results in a circle

The larger the force, the smaller the circle

We can divide the net force into two parts:

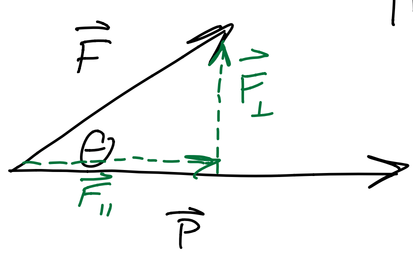
$\vec{F}_{\parallel}$, which increases the speed

$\vec{F}_{\perp}$, which changes the direction

Then:

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

Ex:



$$|\vec{F}_{\parallel}| = |\vec{F}|\cos\theta$$

$$\hat{F}_{\parallel} = \hat{p}$$

$$\vec{F}_{\parallel} = |\vec{F}|\cos\theta \, \hat{p}$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel}$$

OK so $\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$

We can write $\frac{d\vec{p}}{dt}$ the same way

$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}}{dt}_{\parallel} + \frac{d\vec{p}}{dt}_{\perp}$$

Mathematically:

$$\vec{p} = p\,\hat{p}$$

$|\vec{p}|$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(p\,\hat{p})$$

- both pieces are time dependent
- $F_{\perp}$ will change $\hat{p}$, $F_{\parallel}$ will change p
- Use product rule

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(p\,\hat{p}) = \hat{p}\frac{dp}{dt} + p\frac{d\hat{p}}{dt}$$

$$\frac{d\vec{p}}{dt} = \frac{dp}{dt}\hat{p} + \frac{d\hat{p}}{dt}p$$

Change in magnitude same direction as p ($F_{\parallel}$)

Change in direction, const magnitude ($F_{\perp}$)

Conclusion:

$$\frac{d\vec{p}}{dt}_{\parallel} = \frac{dp}{dt}\hat{p} = \vec{F}_{net,\parallel}$$

$$\frac{d\bar{p}}{dt}_{\perp} = p\frac{d\hat{p}}{dt} = \vec{F}_{net,\perp}$$

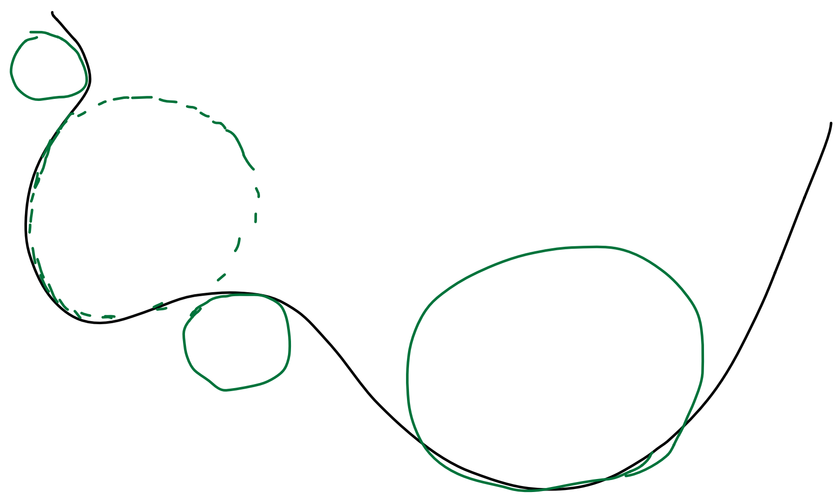
Why is this helpful?

- $p\frac{d\hat{p}}{dt}$ simplifies for circular motion

- circular motion is very common

Circular motion

- Not all motion is exactly circular, but most curving motion is instantaneously circular



"Kissing circle"

instantaneous radius of curvature

Run program

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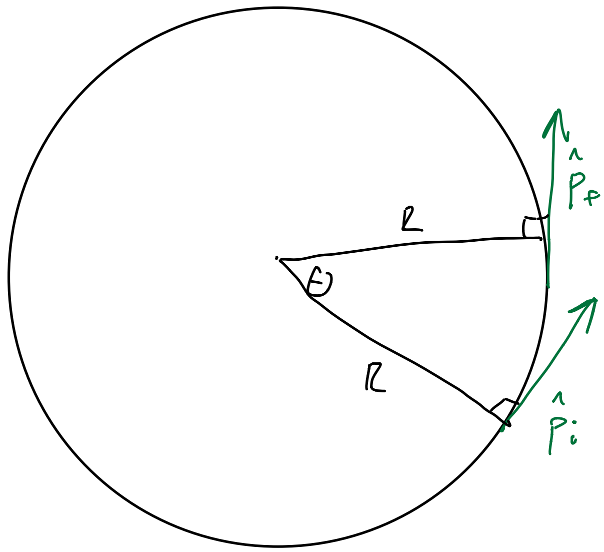
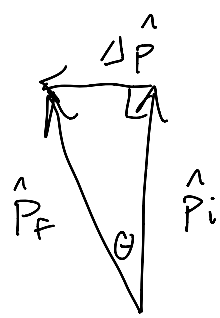
along a circle, what is $\frac{d\vec{p}}{dt}_\perp$?

We need $\frac{d\hat{p}}{dt}$





$$\tan\theta = \frac{|\Delta \hat{p}|}{|\hat{p}_i|}$$

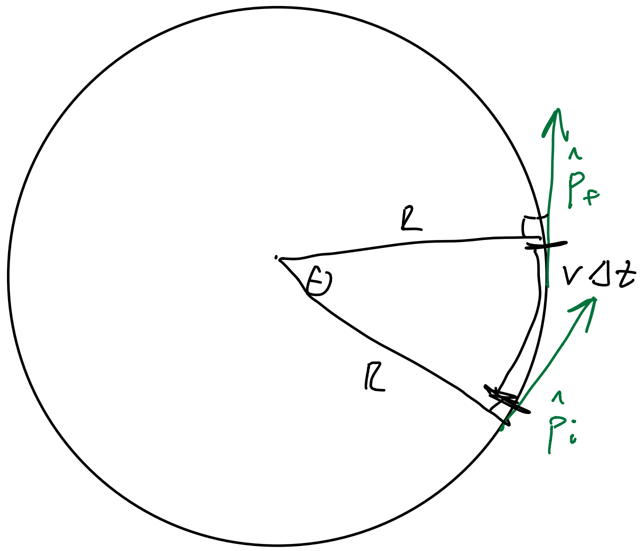
if θ is very small, $\tan\theta \longrightarrow \theta$

$$\theta = \frac{|\Delta \hat{p}|}{|\hat{p}_i|}$$

Defn of angle: $\dfrac{\text{arc length}}{\text{radius}}$

$$\theta = \frac{2\pi R}{R} = 2\pi$$

$$\theta = \frac{v\Delta t}{R}$$



$$\theta = \frac{v\Delta t}{R} = \frac{|\Delta \hat{p}|}{|\hat{p}_i|} \; ; \quad |\hat{p}_i| = 1$$

$$\left| \frac{\Delta \hat{p}}{\Delta t} \right| = \frac{v}{R}$$

$$\lim_{\Delta t \to 0} \left| \frac{\Delta \hat{p}}{\Delta t} \right| = \left| \frac{d\hat{p}}{dt} \right| = \frac{v}{R}$$

Therefore: on a smooth circle

$$\left| \frac{d\vec{p}}{dt}_{\perp} \right| = p \left| \frac{d\hat{p}}{dt} \right| = p \frac{v}{R} = \frac{mv^2}{R}$$