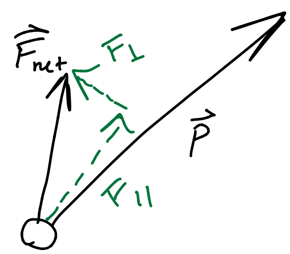
Last lecture

Basic idea

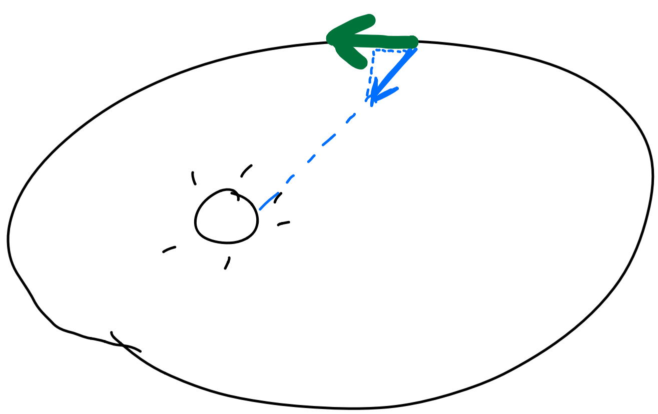
An object moving with momentum $\vec{p}$ expieriences a net force $\vec{F}_{net}$



- Some of $\vec{F}_{net}$ acts parallel to the object's momentum speeding it up (increasing $|\vec{p}|$)

- Some of $\vec{F}_{net}$ acts perpendicular to the momentum, changing its direction

Ex:



$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

Compare to:

$$\vec{F}_{net} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}_{net} = F_x \langle 1, 0 \rangle + F_y \langle 0, 1 \rangle$$

$$\vec{F}_{net} = F_x \hat{x} + F_y \hat{y}$$

$\boxed{\text{Convenient to break } F_{net} \text{ into } \parallel \text{ \& } \perp}$

Do the same with $\frac{d\vec{p}}{dt}$

$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}}{dt}_{\parallel} + \frac{d\vec{p}}{dt}_{\perp}$$

$$\frac{d\vec{p}}{dt_{\parallel}} = \frac{dp}{dt}\hat{p} = \vec{F}_{net,\parallel}$$

$$\frac{d\vec{p}}{dt_{\perp}} = p\frac{d\hat{p}}{dt} = \vec{F}_{net\perp}$$

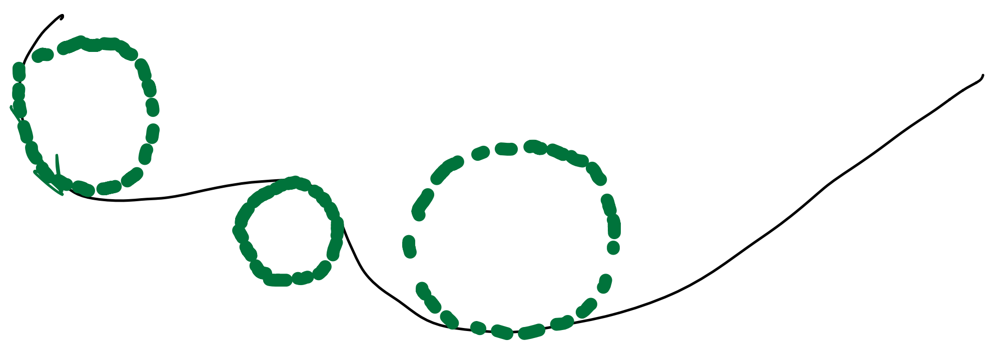
Why is this helpful?

- $p \frac{d\hat{p}}{dt}$ simplifies for circular motion

- circular motion is very common

2nd point first

- Even if curving motion isn't exactly circular, we can use circles to describe it



- At each instant along a curved path, your motion can be considered as a part of a circle

- It may be a different circle the next instant, but still a circle!

$\boxed{\text{Run program}}$

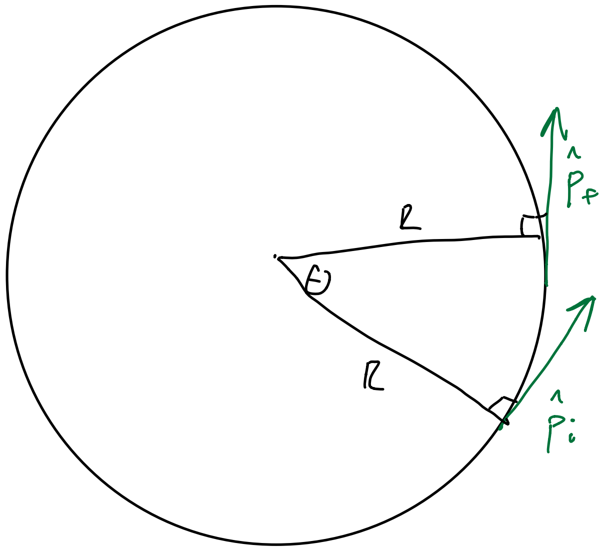
$\boxed{\text{Run program}}$

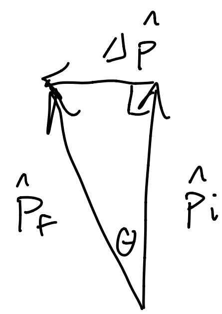
book calls this the "kissing circle"

instantaneous radius of curvature

Now get $\dfrac{d\hat{p}}{dt}$

For circular motion





$$\tan\theta = \frac{|\Delta\hat{p}|}{|\hat{p}_i|}$$

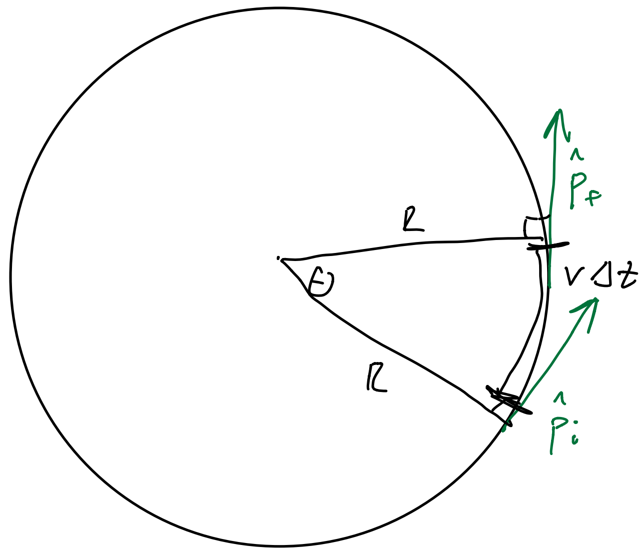
if θ is very small, $\tan\theta \longrightarrow \theta$

$$\theta = \frac{|\Delta\hat{p}|}{|\hat{p}_i|}$$

Defn of angle: $\frac{\text{arc length}}{\text{radius}}$

$$\theta = \frac{2\pi R}{R} = 2\pi$$

$$\theta = \frac{v\Delta t}{R}$$



$$\theta = \frac{v\Delta t}{R} = \frac{|\Delta \hat{p}|}{|\hat{p}_i|} \quad ; \quad |\hat{p}_i| = 1$$

$$\left|\frac{\Delta \hat{p}}{\Delta t}\right| = \frac{v}{R}$$

$$\lim_{\Delta t \to 0} \left|\frac{\Delta \hat{p}}{\Delta t}\right| = \left|\frac{d\hat{p}}{dt}\right| = \frac{v}{R}$$

Therefore: on a smooth circle

$$\left|\frac{d\vec{p}}{dt}_{\perp}\right| = p\left|\frac{d\hat{p}}{dt}\right| = p\,\frac{v}{R} = \frac{mv^2}{R}$$

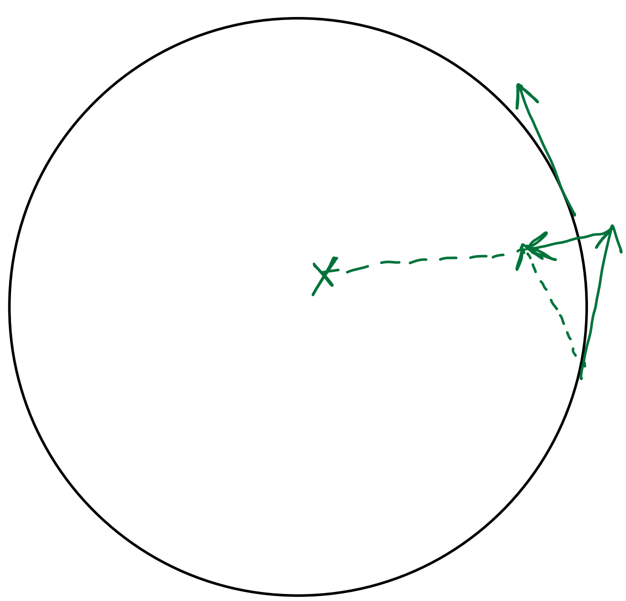
For circular motion:

$$|\vec{F}_{net,\perp}| = \frac{mv^2}{R}$$

often called the "centripetal" force

Latin $\longrightarrow$ center-seeking

For an object moving in a circle, its acceleration is toward the center



The centripetal force is real + causes a change in the objects momentum.

Ex: You are driving your car (1100 kg) 15 m/s (~35 mph) when you suddenly turn around a sharp curve. During this curve, the car's inst. radius of curvature is 20 m.

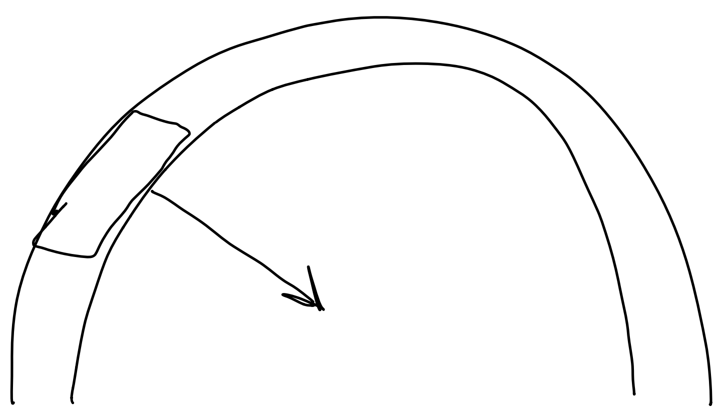
What is the force on the car?

$$\frac{d\vec{p}}{dt}_{\parallel} = \frac{dp}{dt}\hat{p} = 0 \quad \text{so} \quad \vec{F}_{\parallel} = 0$$

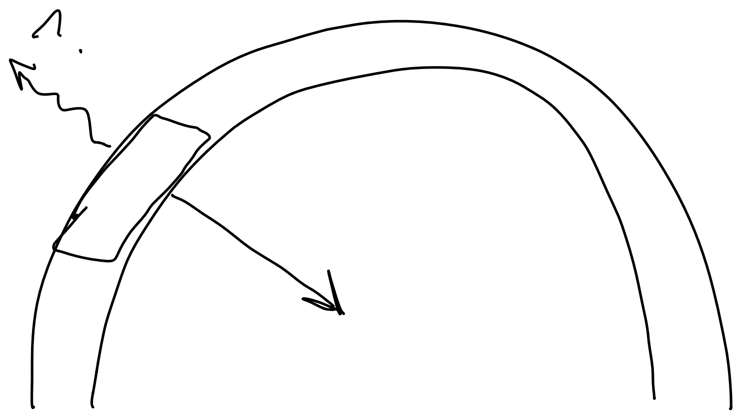
$$|\vec{F}_{net,\perp}| = \frac{mv^2}{R}$$

$$= \frac{(1100\,\text{kg})(15\,\frac{m}{s})^2}{20}$$

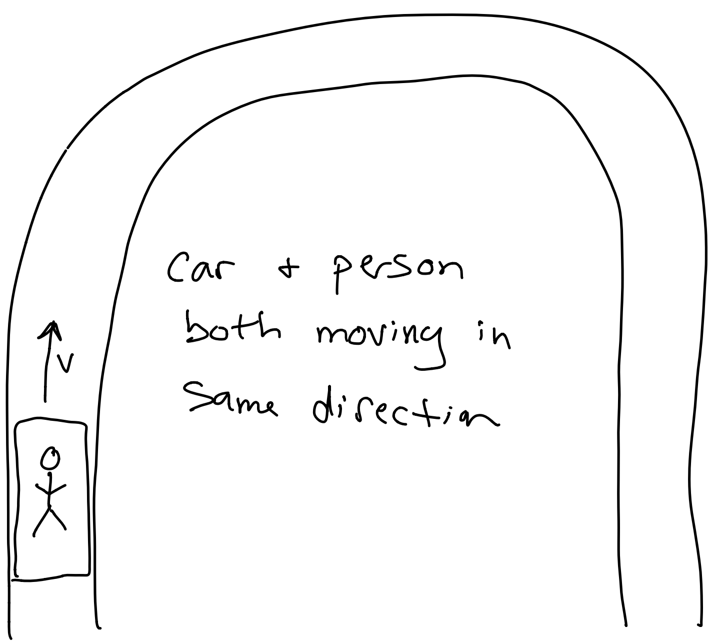
$$|\vec{F}_{net,\perp}| = 12{,}375\ \text{N}$$

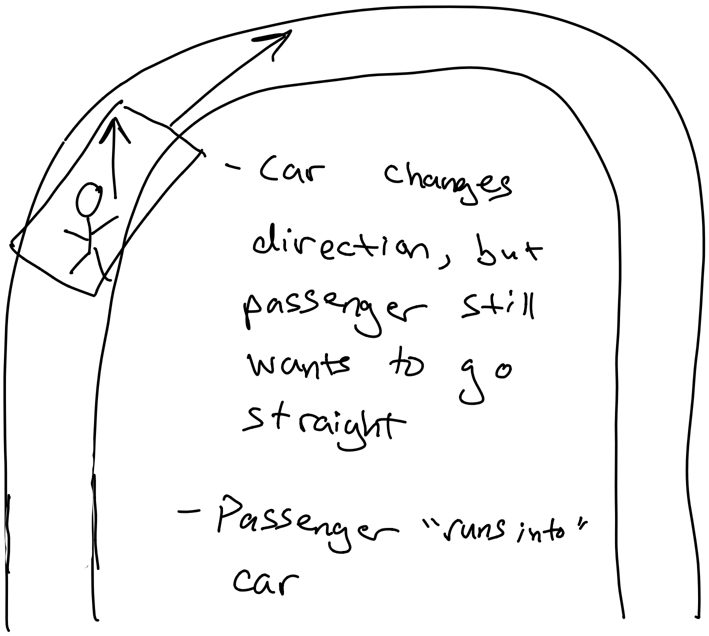


Wait, if we're in the car, don't we feel a force in the other direction?



- This is a sensation caused by the car moving away from us





- Car changes direction, but passenger still wants to go straight
- Passenger "runs into" car

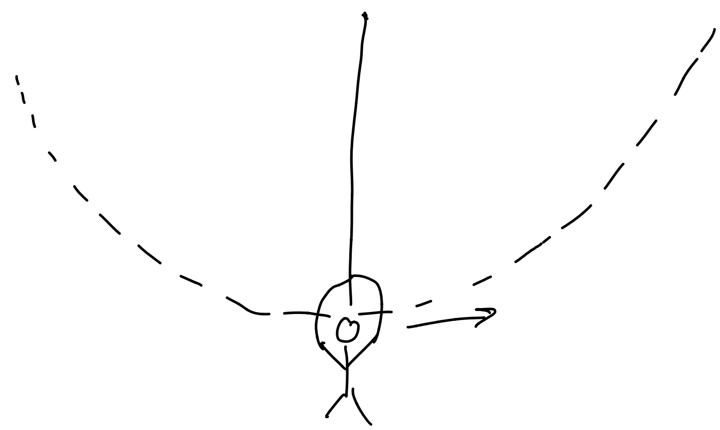
- Same reason that you lurch forward when the car suddenly stops

- This sensation of an outward force is sometimes given a name: centrifugal force
  However, it is not a real force, it is only a sensation! Do not put it on a FBD!

Ex: Tarzan + the vine

Tarzan ($m = 90$ kg) wants to use an 8 m vine to swing across a river.

1) he tests it by hanging on it
2) it breaks midway through, when his speed is 12 m/s



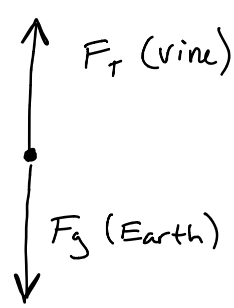
Why did this happen?

When he is hanging at rest, $F_T = mg = (90)(9.8) = 882$ N

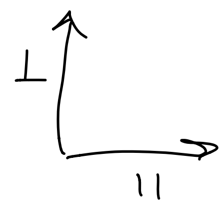
When he is swinging?

1. System: Tarzan
   Surroundings: vine, Earth

2. FBD



(3)



(4)

$$F_{net,\perp} = F_T - F_g$$

$$F_T - mg = \frac{dp}{dt}_\perp$$

$$F_T - mg = \frac{m v^2}{R}$$

$$F_T = mg + \frac{mv^2}{R} = 2502 \text{ N}$$

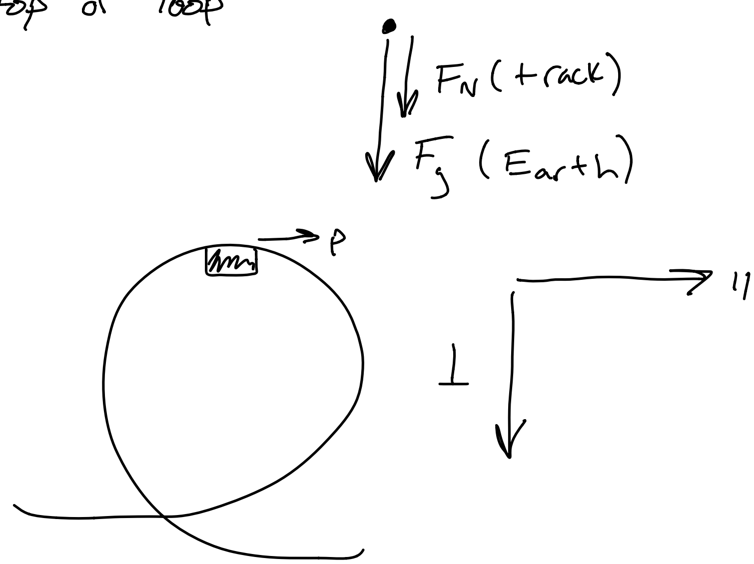
Ex: Roller coaster

A coaster cart ($m = 500$ kg) enters a circular loop of radius 30 m. What speed does it need in order not to fall?

1. Sys: roller coaster
   Surr: Earth track

2. @ top of loop

3.



$$F_{net,\perp} = F_g + F_N$$

$$\frac{dp}{dt}_\perp = \frac{mv^2}{R}$$

$$\frac{mv^2}{R} = mg + F_N$$

$$v = \sqrt{gR + \frac{R}{m}F_N}$$

V_{min} @ $F_N = 0$

$$V_{min} = \sqrt{gR}$$

$$V_{min} = 17.1 \text{ m/s} \quad (38 \text{ mph})$$