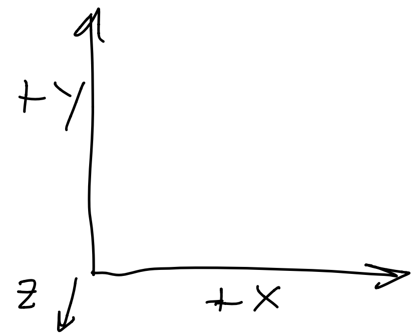
Outline:

- Expressing cartesian vectors
    - Notation, drawing
    - The origin
- The magnitude of vectors
- Unit vectors
- Vector operations:
    - Scalar mult
    - Addition/subtraction
        - Relative postion vector
- Unit vectors and angles

Vector from here to the clock?



$\vec{r} = \langle -2, 1, 2 \rangle$ m

<u>Drawing vectors</u>

- Arrow
- Head @ location of object
- Tail originates @ origin
- There is no single origin
    - position is relative

$\vec{r} = \langle -2, 1, 2 \rangle$ m

How far away is the clock?

3? 1? 2?

- Magnitude of $\vec{r}$

$$|\vec{r}| = \sqrt{(-2)^2 + 1^2 + 2^2}$$

$$|\vec{r}| = 3$$

is $|\vec{r}|$ a vector?

- it's a <u>scalar</u>

What if I want to find a vector to represent the halfway point? Same direction but only half of the distance?

$$\vec{r}_2 = \frac{1}{2}\vec{r} = \frac{1}{2}\langle -2, 1, 2 \rangle \text{m} = \langle -1, \tfrac{1}{2}, 1 \rangle \text{m}$$

$$|\vec{r}_2| = \sqrt{(-1)^2 + \left(\tfrac{1}{2}\right)^2 + 1^2}$$

$$|\vec{r}_2| = 1.5 \text{ m} \qquad \left(\tfrac{1}{2}|\vec{r}|\right)$$

- Scalars <u>scale</u>

- How to get a vector pointing in opposite direction of $\vec{r}$?

  $-1\,\vec{r}$

- Unit vectors

  magnitude is a scalar

  vector = mag × direction

  vector is $\vec{r}$, direction is $\hat{r}$ (r-hat)

$$\vec{r} = |\vec{r}|\,\hat{r} \implies \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = \langle -2, 1, 2 \rangle \qquad |\vec{r}| = 3$$

$\hat{r} = \frac{1}{3}\langle -2, 1, 2 \rangle = \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$

$|\hat{r}| = \sqrt{\left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$

$|\hat{r}| = 1$

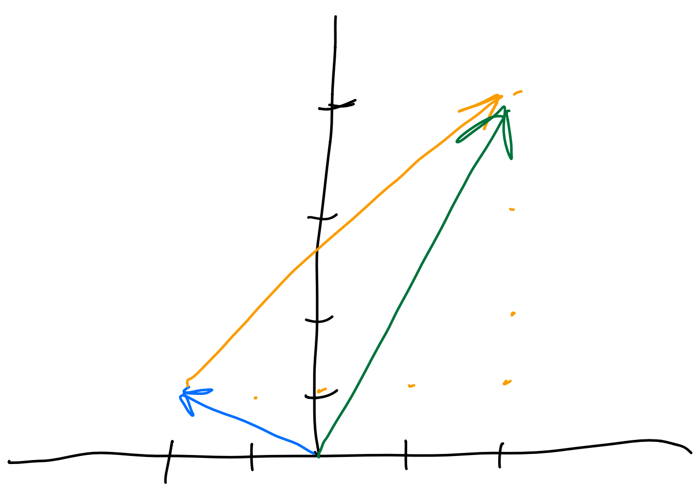
Addition & Sub

What if I start at

$\vec{r} = \langle -2, 1, 2 \rangle$ m

and move 4 m $\hat{x}$

3 m $\hat{y}$



Start at

$$\vec{r} = \langle -2, 1, 2 \rangle \text{ m}$$

move by

$$\vec{r}\,' = \langle 4, 3, 0 \rangle \text{ m}$$

then $\vec{r}_{new} = \vec{r} + \vec{r}\,'$

$$= \langle -2+4,\ 1+3,\ 2+0 \rangle \text{ m}$$

$$\vec{r}_{new} = \langle 2, 4, 2 \rangle \text{ m}$$

I started at a distance

$|\vec{r}| = 3$ m away

I moved from there a distance

$$|\vec{r}\,'| = \sqrt{4^2+3^2+0^2} = 5 \text{ m}$$

Am I now 8 m away?

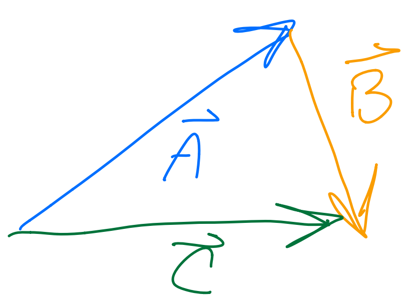
No! $|\vec{r}_{new}| = \sqrt{2^2+4^2+2^2} = 4.9m$

if I have two vectors,

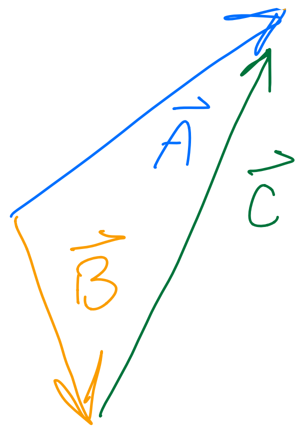
$\vec{A}$ + $\vec{B}$,

$$|\vec{A}+\vec{B}| \neq |\vec{A}| + |\vec{B}|$$

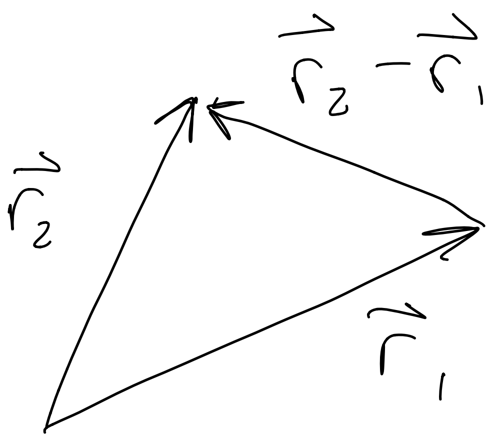
$$\vec{C} = \vec{A} + \vec{B}$$



$$\vec{C} = \vec{A} - \vec{B}$$

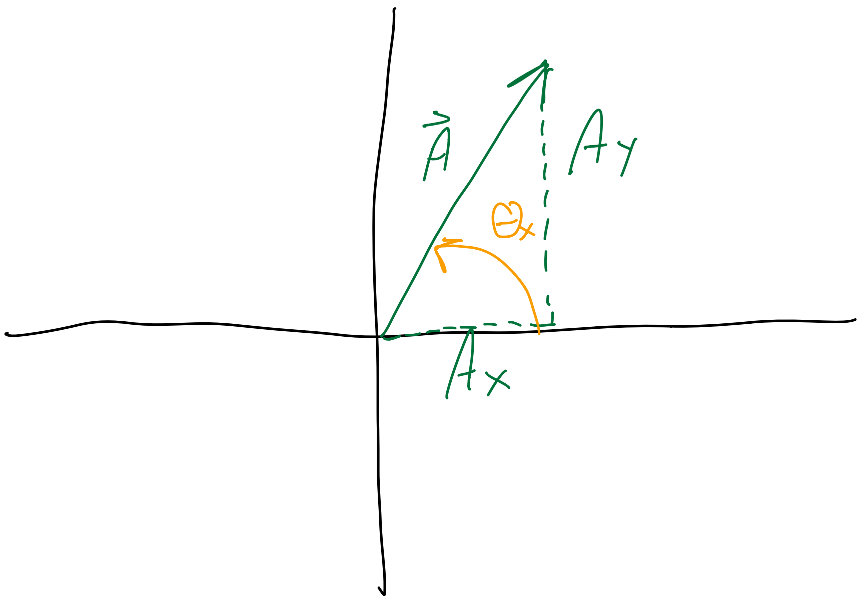


Relative position

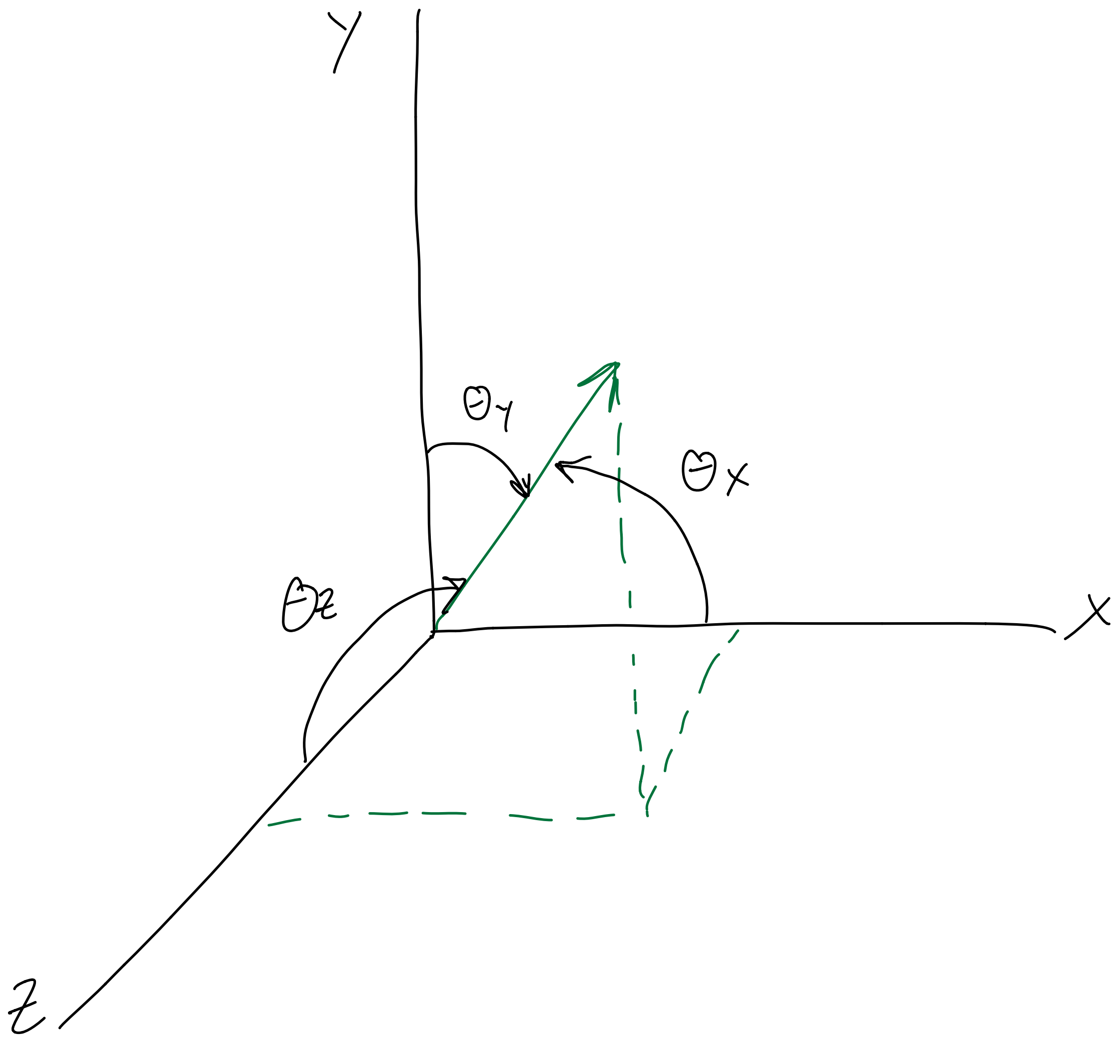


$\vec{r}_{21}$ = location of 2 relative to 1

Another way to describe unit vectors



$$A_x = \cos(\theta_x)$$



$$\hat{r} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z \rangle$$

Example

A golf ball is hit of the tee w/ velocity

$$\vec{v} = \langle 80, 60, 0 \rangle \frac{m}{s}$$

speed: $|\vec{v}| = \sqrt{80^2+60^2} = 100 \frac{m}{s}$

Direction:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{100} \langle 80, 60, 0 \rangle$$

$$\hat{v} = \langle \tfrac{4}{5}, \tfrac{3}{5}, 0 \rangle$$

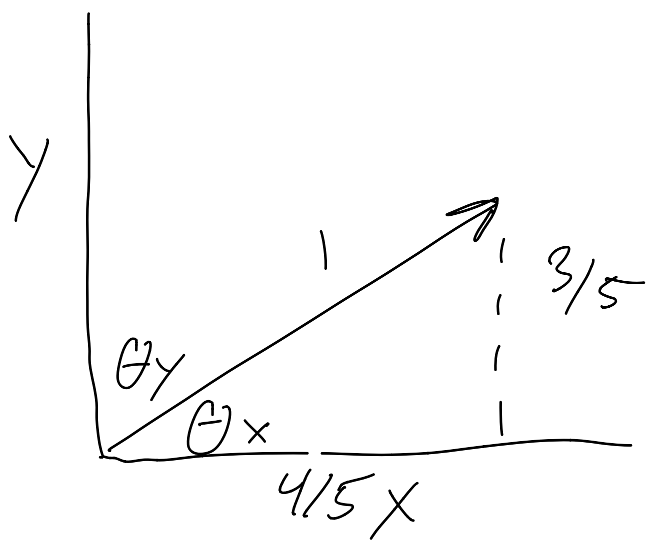
What is θ_x? θ_y? θ_z?

$$\hat{v} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z \rangle$$

$\cos\theta_x = \frac{3}{5}$

$\theta_x = \cos^{-1}\left(\frac{4}{5}\right) = 0.64 = 37^\circ$

$\theta_y = \cos^{-1}\left(\frac{3}{5}\right) = 0.93 = 53^\circ$



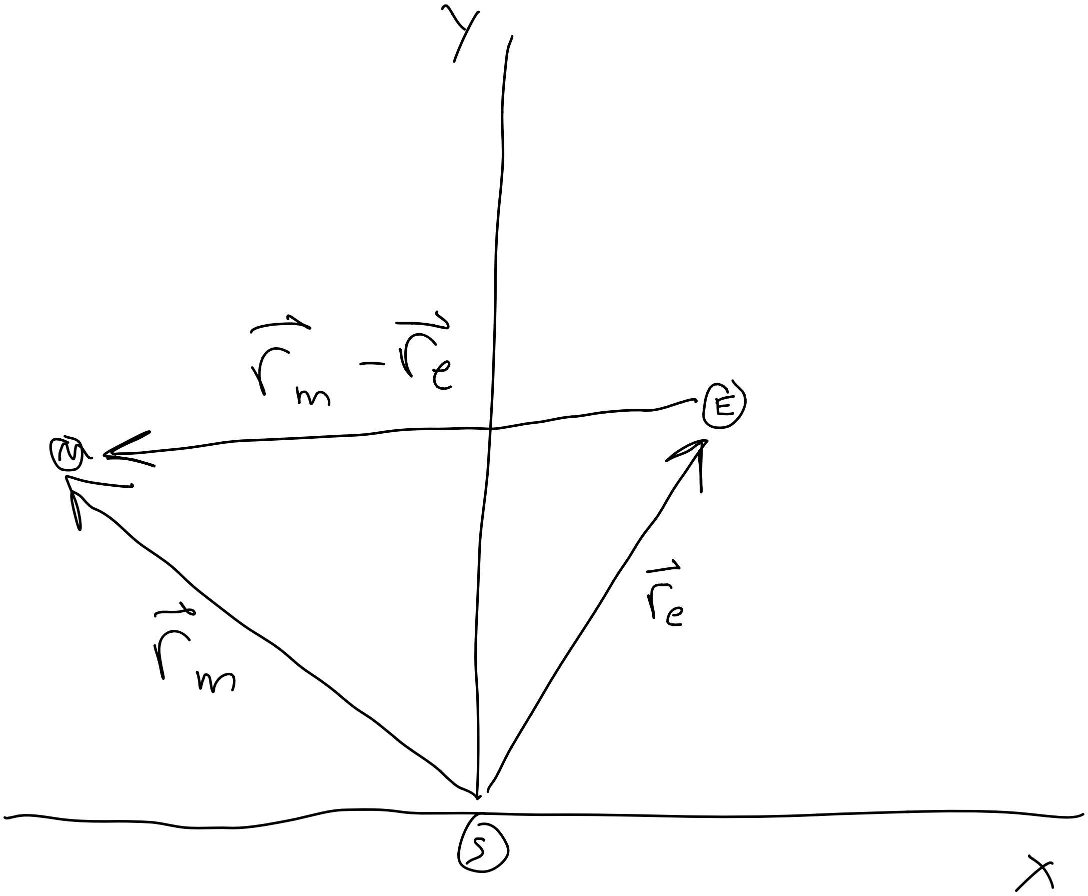
Ex: Relative to the sun the earth is at position

$$\langle 7.5\times 10^{10}, 13\times 10^{10}, 0\rangle \text{ m}$$

and mars is at position

$$\langle -20\times 10^{10}, 12\times 10^{10}, 0\rangle \text{ m}$$

What is the position of mars, relative to earth?



$$\vec{r}_m - \vec{r}_e = \langle -20\times10^{10}, 12\times10^{10}, 0\rangle m$$

$$- \langle 7.5\times10^{10}, 13\times10^{10}, 0\rangle m$$

$$\overline{\langle -27.5\times10^{10}, -1\times10^{10}, 0\rangle m}$$