What is energy?

Energy gives an object the ability to exert a force

Ex: I lift this book what happens?

1. I exert an upward force which changes the momentum of the book & moves it

2. How am I able to exert this force?

   The muscles in my arm expand & contract to pull my arm

3. What gives my muscles the "power" to move in this way?

   The cells in my body have used chemical reactions to convert food into <u>energy</u>

- Energy gives my body the ability to move & exert force (calories)

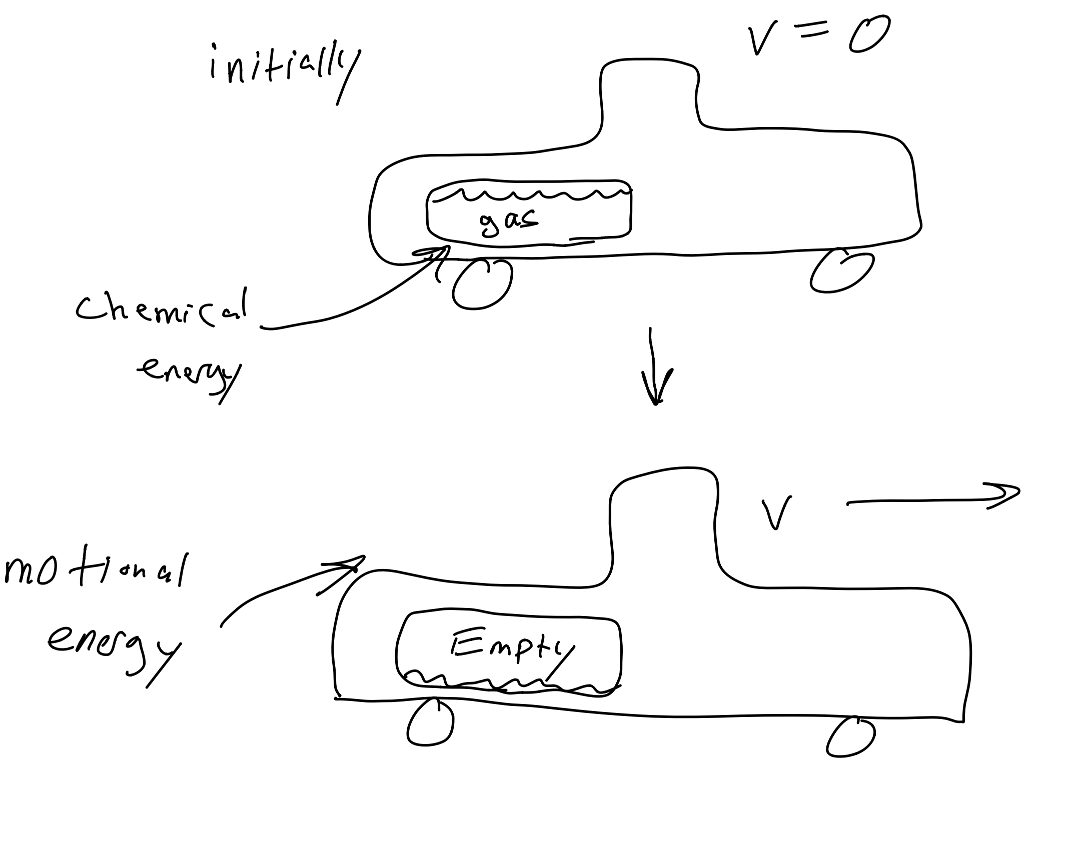
- Your car uses a chemical rxn to convert gasoline into energy, which it needs in order to force the wheels to turn

Etc. We have some intuition about energy; in this chapter we want to formalize it

- most important thing to know about energy
  - like momentum, it cannot be created or destroyed
    - It can move or change form, but the total amount is constant

The total amount of energy in the universe is constant

- We talk about cars "consuming energy" Really, it just transforms

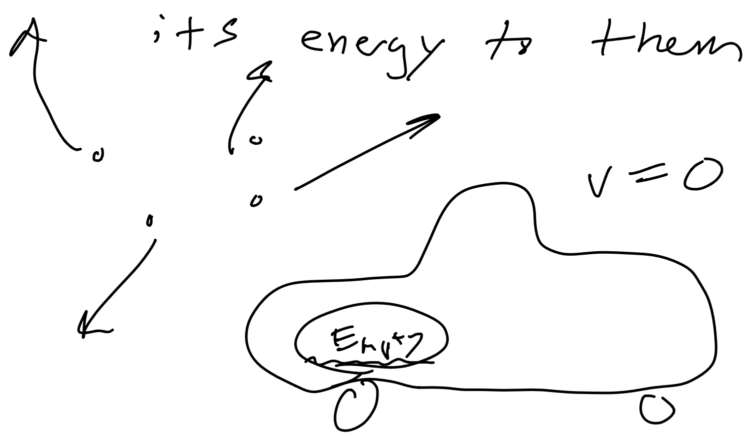


motional energy of moving car
with empty tank

=

chemical energy of full tank

- Car gradually slows down
because it collides w/ trillions
of air molecules & transfers
its energy to them



Fundamental Principle: Energy Conservation

$$\Delta E_{universe} = 0$$

$$\boxed{\Delta E_{system} + \Delta E_{surr} = 0}$$

If the energy of our system changed,
then so did the energy of surroundings

$$\Delta E_{sys} = \left(\text{Energy inputs from Surroundings}\right)$$

+ or −

Car (system) lost energy (slowed down)
so surroundings (Earth, air, road)
gained an equal amount
of energy

What is the "motional" energy of a car?

We call this the "Kinetic" energy

For a particle w/ mass m &
velocity $\vec{v}$

$v = |\vec{v}|$

$$\boxed{K = \frac{1}{2} m v^2}$$

Energy of motion

Relativity: $E_{total} = \gamma mc^2$

$c =$ speed of light $(3 \times 10^8 \ m/s)$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$E_{rest} = E(v=0) = mc^2$$

$$E_{rest} = mc^2$$

$$E_{kin} = \gamma mc^2 - mc^2$$

$$E_{kin} \simeq \frac{1}{2}mv^2, \quad v << c$$

$$\boxed{K = \frac{1}{2}mv^2}$$

So: If car moves @ $v = 30 \ m/s$
+ $m = 1100 \ kg$

$$K = \frac{1}{2}(1100 \ kg)(30 \ \tfrac{m}{s})^2$$

$$K = 495{,}000 \ kg \left(\frac{m}{s}\right)^2$$

$1 \text{ Joule (J)} = 1\, kg \left(\frac{m}{s}\right)^2$

$K = 495{,}000\ J$

$1 \text{ Calorie} = 4184\ J$

Ex: Pulling a 3kg block, + I increase the force so that the block's speed increases from 4 m/s to 5 m/s, what is the change in chemical energy in my body?

system: me + block

$$\Delta E_{(me + block)} = 0$$

$$E_i = E_{me,i} + \frac{1}{2}(3\,kg)(4\,m/s)^2$$

$$E_f = E_{me,f} + \frac{1}{2}(3\,kg)(5\,m/s)^2$$

$$\Delta E = \Delta E_{me} + 13.5\ J = 0$$

$$\Delta E_{me} = -13.5\ J$$

Chemical energy left my body + transformed into kinetic energy

We have said "Energy is the ability to exert force"

Not exactly, let's formalize

- Compare to momentum

momentum: ability to resist force
(greater momentum, more force required to change motion)

$$\Delta \vec{p} = \vec{F} \Delta t$$

$$\Delta E = ?$$

$$\Delta E = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$v_i = v$

$v_f = v + \Delta v$

$$\Delta E = \frac{1}{2} m (v + \Delta v)^2 - \frac{1}{2} m v^2$$

$$\frac{dE}{dv} = \lim_{\Delta v \to 0} \frac{\frac{1}{2} m (v+\Delta v)^2 - \frac{1}{2} m v^2}{\Delta v}$$

$$\Delta E \longrightarrow \frac{dE}{dv} (\Delta v)$$

$$\frac{dE}{dv} = \frac{d}{dv}\left(\frac{1}{2}mv^2\right)$$
$$= mv$$

$$\Delta E = mv\,\Delta v$$

$$\Delta v = \frac{\Delta p}{m}$$

$$\Delta E = mv\frac{\Delta p}{m} = v\,\Delta p = v\,F\,\Delta t$$

$$v\,\Delta t = \Delta x$$

$$\Delta E = F \cdot \Delta x$$

Change in energy = "work"

Force × time $\longrightarrow$ Δp

Force × displacement $\longrightarrow$ ΔE

Work = force × displacement

units?

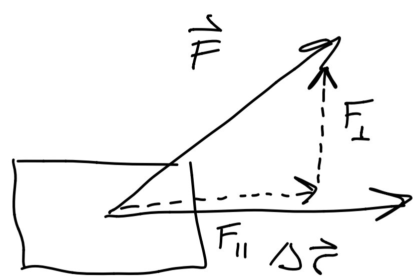
$$N \cdot m = kg\frac{m}{s^2}\,m = kg\left(\frac{m}{s}\right)^2 = J \checkmark$$

work = force × displacement

- But W is a scalar, $\vec{F}$ + $\Delta\vec{r}$ are both vectors

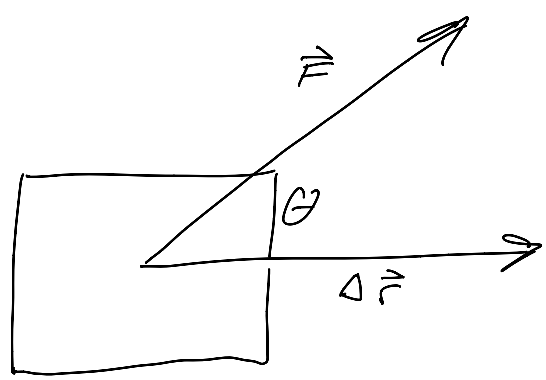
- How do we multiply $\vec{F}$ + $\Delta\vec{r}$ together?

Answer: Not all of the force does work, just the component parallel to $\Delta\vec{r}$



only $F_\parallel$ does work

$$W = F_\parallel |\Delta\vec{r}|$$



$$W = |\vec{F}| \cos\theta \, |\Delta\vec{r}|$$

$W = |\vec{F}| \cos\theta \, |\Delta \vec{r}|$

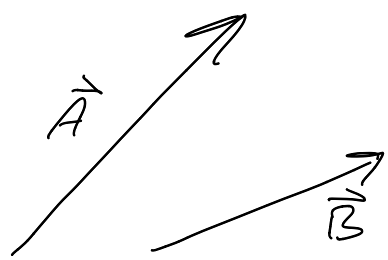
This is an example of a more general vector operation

- We know how to: add/sub
  mult by scalars

How can we multiply two vectors?

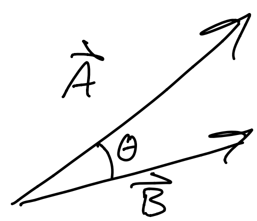
$\vec{A}$ times $\vec{B}$

One way: the dot product

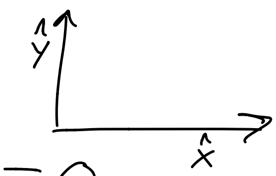


$\vec{A} = \langle a_x, a_y, a_z \rangle$
$\vec{B} = \langle b_x, b_y, b_z \rangle$

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta}$$



What is $\hat{x} \cdot \hat{y}$?



$$= |\hat{x}||\hat{y}| \cos(90^\circ) = 0$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\hat{x} \cdot \hat{x} = \cos(0) = 1$$

$$\hat{z} \cdot \hat{y} = \hat{z} \cdot \hat{x} = 0$$

$$\vec{A} \cdot \vec{B} = \langle a_x, a_y \rangle \cdot \langle b_x, b_y \rangle$$

$$= (a_x \hat{x} + a_y \hat{y}) \cdot (b_x \hat{x} + b_y \hat{y})$$

$$= a_x \hat{x} \cdot b_x \hat{x} + a_x \hat{x} \cdot b_y \hat{y}$$

$$+ a_y \hat{y} \cdot b_x \hat{x} + a_y \hat{y} \cdot b_y \hat{y}$$

$$= a_x b_x + a_y b_y$$

$$\vec{A} = \langle a_x, a_y, a_z \rangle, \quad \vec{B} = (b_x, b_y, b_z)$$

$$\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Take the component of $\vec{A}$ which is $\parallel$ to $\vec{B}$, & multiply it by $|\vec{B}|$

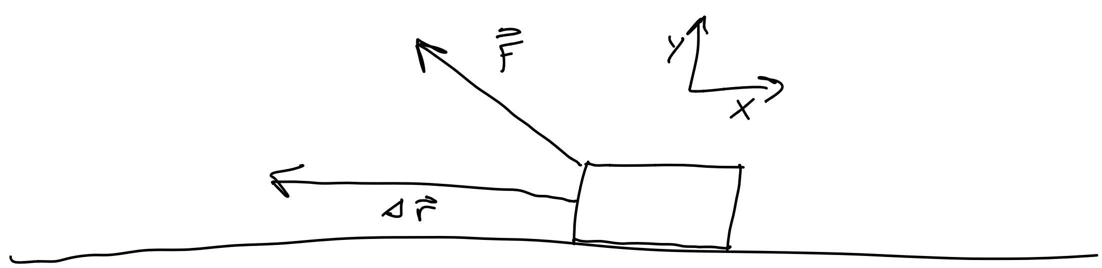
- Notes about dot product:
    - Is a scalar!
    - Is commutative ($\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$)

So: If a force $\vec{F}$ is being applied over a displacement of $\Delta\vec{r}$, then

$$\boxed{W = \vec{F} \cdot \Delta\vec{r}}$$

constant Force

Ex: How much work to pull a block w/ const force?

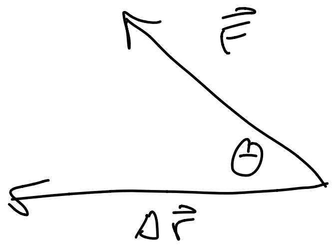


$$\vec{F} = -0.3^{N}\hat{x} + 0.25^{N}\hat{y} \qquad ((-0.3, 0.25, 0))$$

$$\Delta\vec{r} = -1.5^{m}\hat{x}$$

$$W = \vec{F} \cdot \Delta\vec{r} = (-0.3N)(-1.5m) + (0.25N)(0)$$

$$W = 0.45 \text{ J}$$



would get the same answer if we found

Θ then $W = |\vec{F}||\Delta\vec{r}|\cos\Theta$