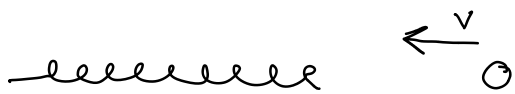
Question: A moving ball compresses a spring and temporarily comes to rest.





system: ball + spring

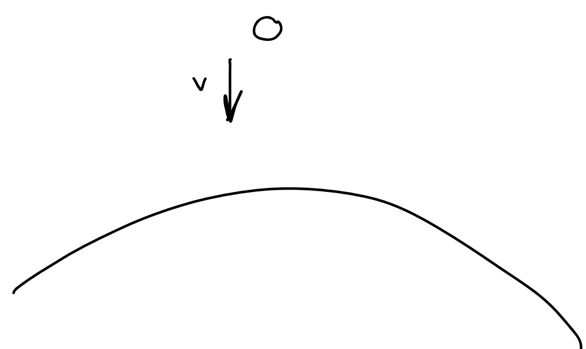
$\Delta E_{sys} = W_{surr} = 0$

$E_i = \frac{1}{2} m v^2$

$E_f = 0$

$\boxed{\Delta E_{sys} = -\frac{1}{2} m v^2}$ ?

Ex: Ball falling toward Earth




System: ball + Earth

Surr : None

$\Delta E_{sys} = 0$

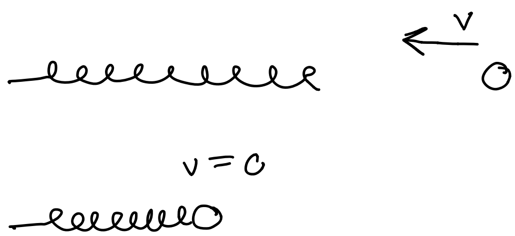
$\Delta E_{sys} = \underbrace{\Delta K_{ball}}_{>0} + \underbrace{\Delta K_{earth}}_{>0} > 0$ ?

- When the ball was stopped by the spring, where did its energy go?

- When the ball + Earth both gained speed, where did that energy come from?

# Another form of Energy: Potential Energy

Potential energy: energy associated w/ interactions in multi-particle systems




system: ball + spring

$\Delta E_{sys} = W_{surr} = 0$

$$E_i = \frac{1}{2}mv^2 + U_{i,\text{ball-sp}}$$

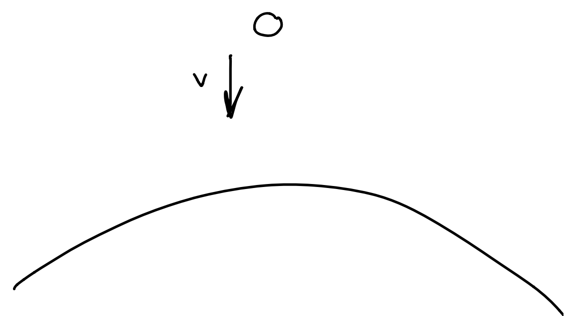
$$E_f = U_{f,\text{ball spring}}$$

$$\Delta E = \Delta U - \frac{1}{2}mv^2 = 0$$

$$\Delta U = \frac{1}{2}mv^2$$

The kinetic energy of the ball was transformed into potential energy "stored" in the spring

Ex: Ball falling toward Earth




System: ball + Earth

Surr: None

$$\Delta E_{sys} = 0$$

$$\Delta E_{sys} = \underbrace{\Delta K_{ball}}_{>0} + \underbrace{\Delta K_{earth}}_{>0} + \underbrace{\Delta U_{ball-earth}}_{?} = 0$$

$$\Delta U_{ball-earth} = -\left(\Delta K_{ball} + \Delta K_{earth}\right)$$

Potential energy stored in the gravitational field was converted into kinetic energy

Potential energy:

- "Stored" energy in multiparticle systems
- Has the "potential" to turn into kinetic energy
- Depends on relative position of interacting objects

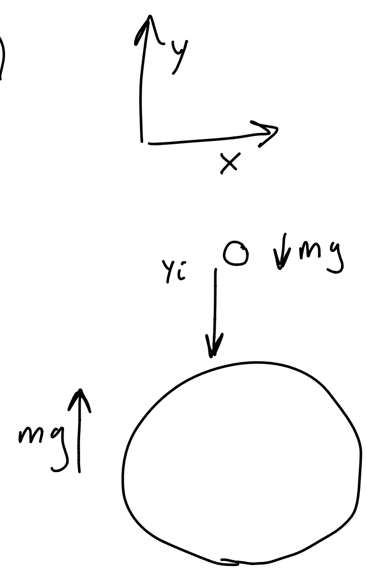
- A spring which is highly compressed will launch an object faster (it has more potential energy)

- A ball dropped from above the Earth is moving faster when it hits the surface if it was released higher

To calculate potential energy:

$$\Delta K_{earth} + \Delta K_{ball} + \Delta U_{ball-earth} = 0$$

$$\Delta U = -(\Delta K_{earth} + \Delta K_{ball})$$




Earth moves up a small distance

$$W_{on\,earth} = mg\,\Delta y_{earth} \approx 0$$

$$W_{on\,ball} = -mg\,\Delta y$$

$$\Delta U = -(\Delta K_{earth} + \Delta K_{ball})$$
$$= -(0 + -mg\Delta y)$$

$$\Delta U = mg\Delta y$$

Ex: 100 g ball falls from a height of 1 m

What is its speed when it lands?

Method 1: Use $y(t) = y_i - \frac{1}{2}gt^2 = 0$ to find $\Delta t$

$\Delta p = -mg\Delta t$

Method 2:

system = ball + Earth

Surr = None

$\Delta E_{sys} = 0$

$$\underbrace{\Delta K_{Earth}}_{\approx 0} + \Delta K_{ball} + \Delta U = 0$$

$$\Delta K_{ball} = -\Delta U$$

$$= -mg\Delta y$$

$$= -mg(y_f - y_i)$$

$$\Delta K_{ball} = mg(y_i - y_f)$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mg(y_i - y_f)$$

$$\frac{1}{2}mv_f^2 = mgy_i$$

$$v_f = \sqrt{2gy_i}$$

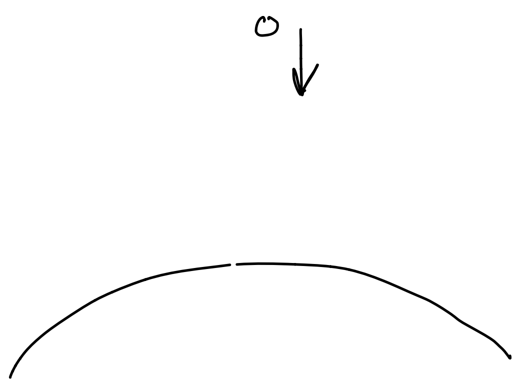
$$v_f = \sqrt{(2)(9.8\ m/s^2)(1m)}$$

$$\boxed{v_f = 4.43\ m/s}$$

Potential energy only exists between objects

- Earth doesn't have a potential energy
- The ball doesn't have potential energy
    - The Earth + ball system does

- Two ways of looking at the same thing:



system = ball

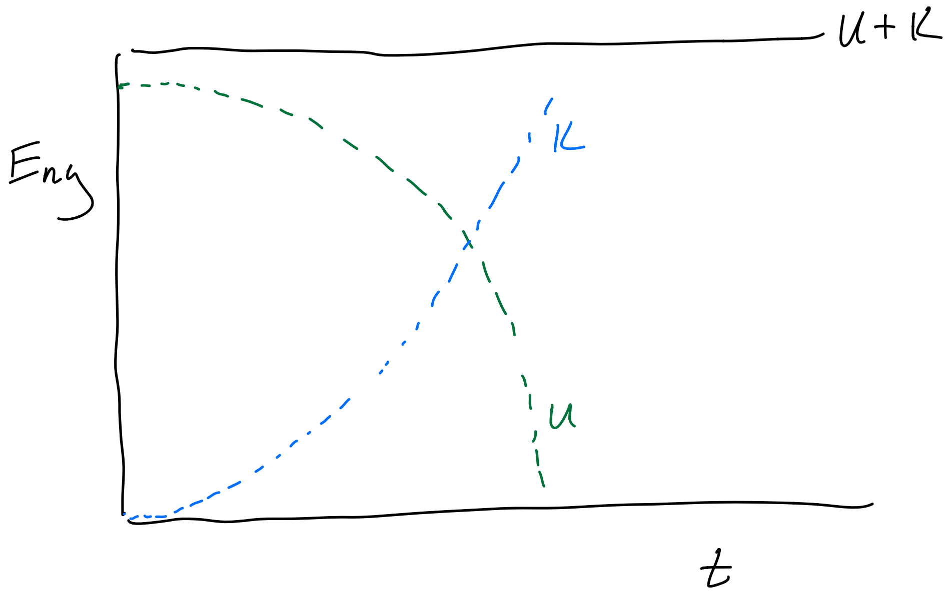
$$\Delta E_{ball} = W_{earth}$$

"The gravitational force of the Earth did work on the ball, increasing its speed" system = ball + Earth

$\Delta E_{sys} = 0$

$\Delta E_{sys} = \Delta K + mg\Delta y$

"Gravitational potential energy between the Earth + the ball was converted into kinetic energy, increasing the speed of the ball"

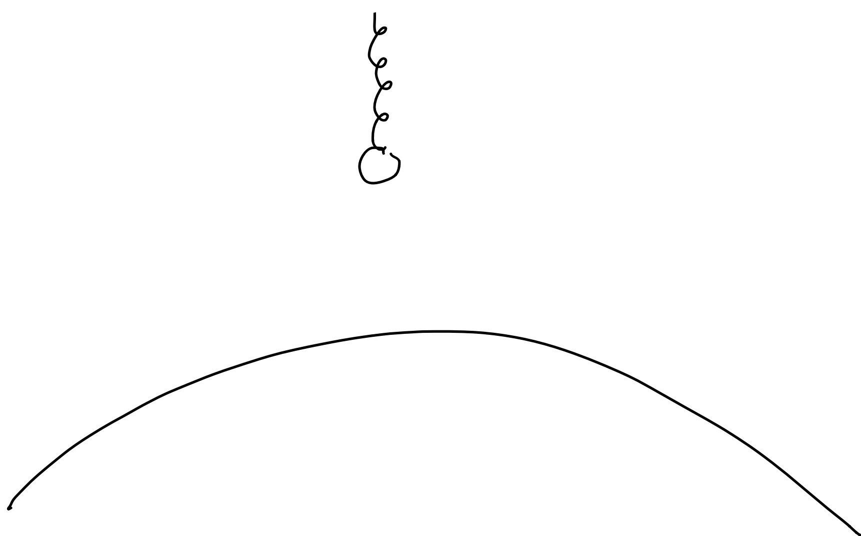


How do I know if I need to include potential energy?

Internal vs External Forces

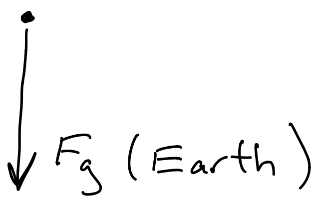
- Only forces exerted by objects in the surroundings can do work on the system
- We call these external forces
- Not the only possible forces

Ex:



system = ball + spring

External forces act <u>on</u> the system




Internal forces act within the system

$$F_{(\text{on ball by spring})} = ks$$

$$F_{(\text{on spring by ball})} = -ks$$

Internal Forces

- Exerted by & on objects within the system

$$\Delta E_{sys} = W_{surr}$$

$$\Delta E_{ball} = W_{\text{by spring}} + W_{\text{by Earth}}$$

$$\Delta E_{spring} = W_{\text{by ball}} + W_{\text{by Earth}}$$

$$\Delta E_{sys} = W_{(ball \to spring)} + W_{(spring \to ball)} + W_{(ball \to Earth)} + W_{(spring \to Earth)}$$

$$\Delta E_{sys} = W_{int} + W_{surr}$$

We want: $\Delta E_{sys} = W_{surr}$

$$\Delta E_{sys} - W_{int} = W_{surr}$$

$$\Delta U = -W_{int}$$

$$\Delta E_{sys} = \Delta K + \Delta U$$

$$\Delta E_{sys} = W_{surr}$$

$$\Delta K + \Delta U = W_{surr}$$

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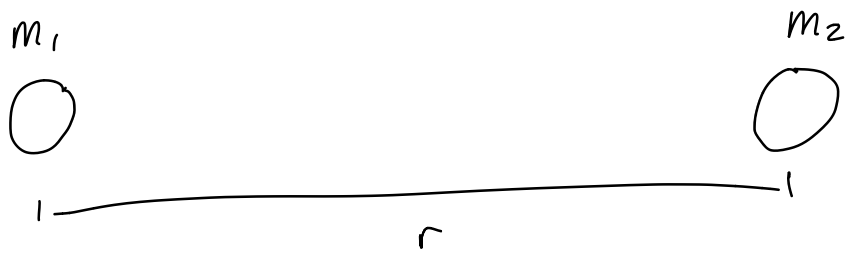
main point

$\Delta E_{sys} = \Delta K + \Delta U$, $\quad \Delta U = -W_{int}$

total work done by forces internal to the system

Ex: Two stars




If star 1 moves a distance $\Delta r_1$

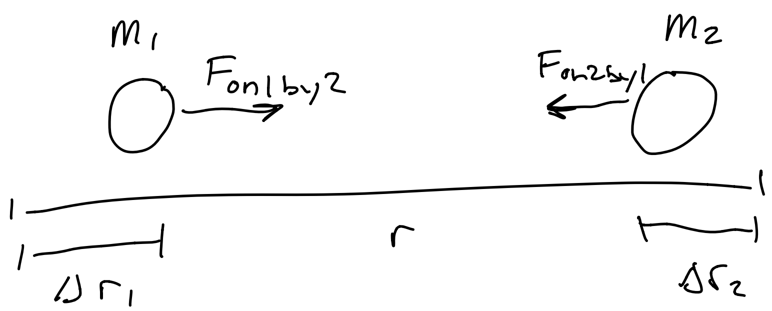
star 2 moves $\Delta r_2$

What is $\Delta U$?

System: Star 1 + star 2

Surr: None

$$\Delta U = -W_{int}$$




$$W_{int} = |\vec{F}_{1,2}||\Delta \vec{r}_1| + |\vec{F}_{2,1}||\Delta \vec{r}_2|$$

$$|\vec{F}_{1,2}| = |\vec{F}_{2,1}| \equiv F$$

$$W_{int} = F\Delta r_1 + F\Delta r_2$$

$$W_{int} = F(\Delta r_1 + \Delta r_2)$$

$$\Delta U = -F(\Delta r_1 + \Delta r_2)$$

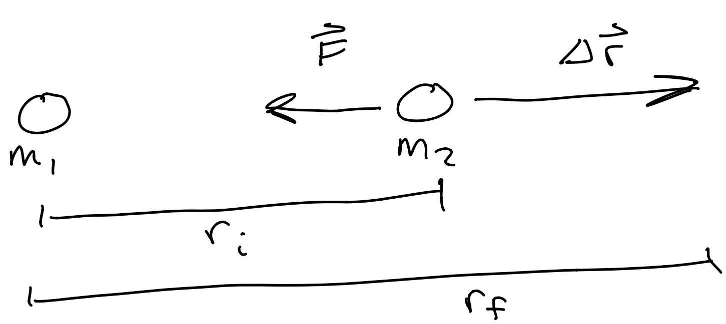
$$\Delta U = -F\Delta r$$

It's the same as one star moving the entire distance $\Delta r$

If $\Delta r$ is large enough, then $F$ will change and $F\Delta r$ is not valid

We need to integrate

Consider:



$\vec{F} \cdot \Delta\vec{r}$ is negative

$$W_{int} = -\int_{r_i}^{r_f} F\,dr$$

$$= -\int_{r_i}^{r_f} \frac{Gm_1m_2}{r^2}\,dr$$

$$W_{int} = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\Delta U = -W = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\Delta U = U_f - U_i = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

If the two objects start out very far apart, then $F$ is $0$ and so $U_i$ is $0$

$$r_i \to \infty,\ U(r_i) \to 0$$

$$U_f - 0 = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{\infty}\right)$$

$$\boxed{U(r) = -\frac{Gm_1m_2}{r}}$$

is the <u>total</u> potential energy at distance of sep $r$.

Another interesting result:

$$\Delta U = -W_{int} = -\int_i^f \vec{F} \cdot d\vec{r}$$

$$U(r) = U_i - \int_{r_i}^{r} \vec{F} \cdot d\vec{r}$$

if $\vec{F}$ & $d\vec{r}$ align, $\vec{F} \cdot d\vec{r} = F\,dr$

$$U(r) = U_i - \int_{r_i}^{r} F\,dr$$

$$\frac{dU}{dr} = \frac{dU_i}{dr} - \frac{d}{dr}\int_{r_i}^{r} F\,dr$$

$$\frac{dU}{dr} = -F$$

$$\boxed{F = -\frac{dU}{dr}}$$

Force wants to minimize potential energy