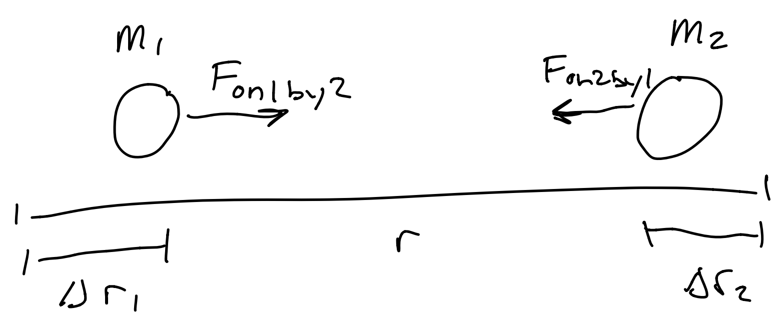
Ex: Two stars

If star 1 moves a distance Δr_1
star 2 moves Δr_2

What is ΔU?

System: Star 1 + star 2
Surr: None

$$\Delta U = -W_{int}$$



$$W_{int} = |\vec{F}_{1,2}||\Delta\vec{r}_1| + |\vec{F}_{2,1}||\Delta\vec{r}_2|$$

$$|\vec{F}_{1,2}| = |\vec{F}_{2,1}| \equiv F$$

$$W_{int} = F\Delta r_1 + F\Delta r_2$$

$$W_{int} = F(\Delta r_1 + \Delta r_2)$$

$$\Delta U = -F(\Delta r_1 + \Delta r_2)$$

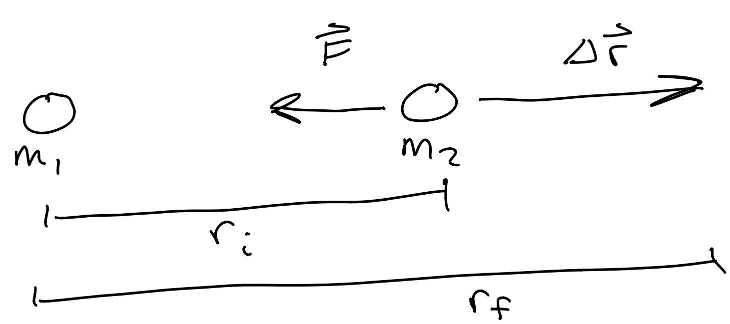
$$\Delta U = -F\Delta r$$

It's the same as one star moving the entire distance Δr

If Δr is large enough, then F will change and $F\Delta r$ is not valid

We need to integrate

Consider:



$\vec{F} \cdot \Delta\vec{r}$ is negative

$$W_{int} = -\int_{r_i}^{r_f} F \, dr$$

$$= -\int_{r_i}^{r_f} \frac{G m_1 m_2}{r^2} \, dr$$

$$W_{int} = G m_1 m_2 \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$\Delta U = -W = -G m_1 m_2 \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$\Delta U = U_f - U_i = -G m_1 m_2 \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

If the two objects start out very far apart, then F is 0 and so U_i is 0

$$r_i \rightarrow \infty, \; U(r_i) \rightarrow 0$$

$$U_f - 0 = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{\infty}\right)$$

$$U(r) = -\frac{Gm_1m_2}{r}$$

is the <u>total</u> potential energy at distance of sep r.

- increasing r increases U
- negative energy? Fine as long as $K \geq 0$

Another interesting result:

$$\Delta U = -W_{int} = -\int_i^f \vec{F} \cdot d\vec{r}$$

$$U(r) = U_i - \int_{r_i}^{r} \vec{F} \cdot d\vec{r}$$

if $\vec{F}$ + $d\vec{r}$ align, $\vec{F} \cdot d\vec{r} = F\,dr$

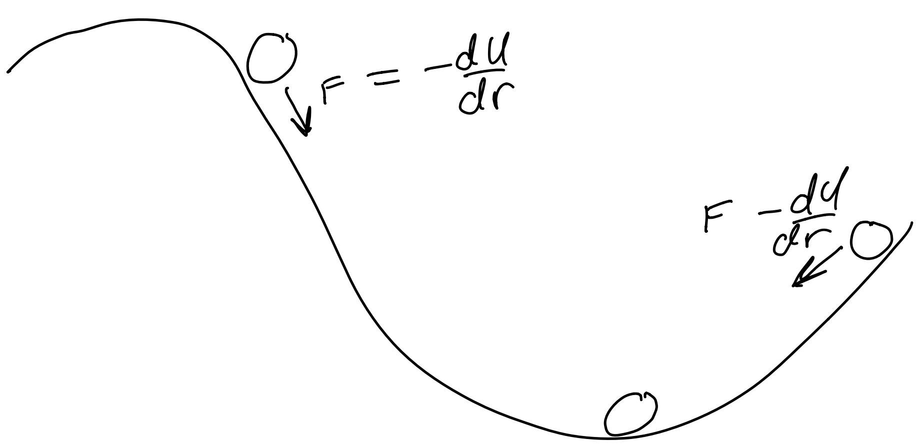
$$U(r) = U_i - \int_{r_i}^{r} F\,dr$$

$$\frac{dU}{dr} = \frac{dU_i}{dr} - \frac{d}{dr}\int_{r_i}^{r} F\,dr$$

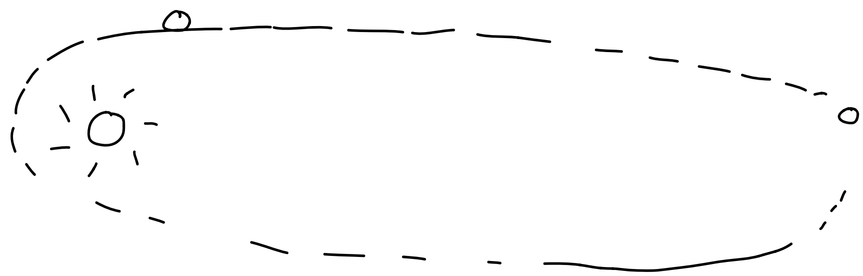
$$\frac{dU}{dr} = -F$$

$$\boxed{F = -\frac{dU}{dr}}$$

Force wants to minimize potential energy



Ex: Comet Neowise



Furthest point in orbit: $r_i = 300\,AU \; (4.5 \times 10^{13} m)$

$V_i = 5.7 \times 10^4 \frac{m}{s}$

Closest approach: $r_f = 2\,AU \; (3 \times 10^{11} m)$

$V_f = ?$

system: comet + sun

surr: None (Jupiter is only 0.1% mass of sun!)

mass of comet $= m_c = 2 \times 10^{13}$ kg

mass of sun $= m_s = 2 \times 10^{30}$ kg

$$\Delta E_{sys} = 0$$

$$E_f - E_i = 0$$

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m_c v_i^2 - G\frac{m_s m_c}{r_i} = \frac{1}{2} m v_f^2 - G\frac{m_s m_c}{r_f}$$

Know: m_c, m_s, v_i, r_i, r_f

Want: v_f

$$\frac{1}{2} m v_f^2 = G m_s m_c \left(\frac{1}{r_f} - \frac{1}{r_i}\right) + \frac{1}{2} m_c v_i^2$$

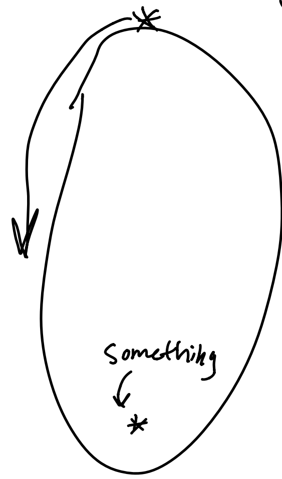
$$v_f = \sqrt{2 G m_s \left(\frac{1}{r_f} - \frac{1}{r_i}\right) + v_i^2}$$

$$v_f = \sqrt{(2)(6.7\times10^{-11})(2\times10^{30})\left(\frac{1}{3\times10^{11}} - \frac{1}{4.5\times10^{13}}\right) + \left(5.7\times10^{4}\right)^2}$$

$$\boxed{v_f = 6.2\times10^{4}\ \text{m/s}} \qquad (140{,}000\ \text{mph})$$

The curious case of S2

Years ago, astronomers discovered a star orbiting ... nothing!



when the star is 970 AU away (1.5×10^{14} m)

its speed is 4×10^6 m/s

when the star is 120 AU (1.8×10^{13})

speed is 8.3×10^6 m/s

The mass of S2 is $15 \times M_{sun} = 3 \times 10^{31}$ kg

what is mass of the unknown object?

System: star + obj
surr: none

$\Delta E_{sys} = 0$

$E_i = E_f$

$$\frac{1}{2} m_* v_i^2 - \frac{G m_{obj} m_*}{r_i} = \frac{1}{2} m_* v_f^2 - \frac{G m_{obj} m_*}{r_f}$$

$$\frac{1}{2}\left(v_i^2 - v_f^2\right) = G m_{obj}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

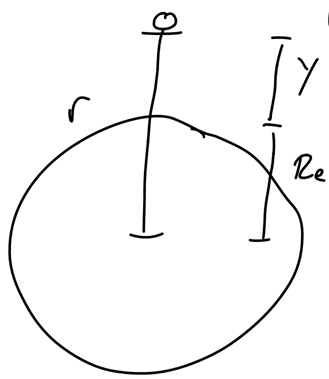
$$m_{obj} = \frac{\frac{1}{2}\left(v_i^2 - v_f^2\right)}{G\left(\frac{1}{r_i} - \frac{1}{r_f}\right)} = 8 \times 10^{36}\ \text{kg}$$

$\approx$ 4,000,000 × mass of sun

A black hole

Earlier, we saw $\Delta U = mg\Delta y$

Knowing $U = \frac{-Gm_1m_2}{r}$, we can derive this



$$U = -\frac{Gm_em}{r}$$

$$r = R_e + y$$

$$U = -\frac{Gm_em}{R_e+y}$$

if $|y| << R_e$

$$\frac{1}{R_e+y} \approx \frac{R_e - y}{R_e^2}$$

$$U \cong -\frac{Gm_em}{R_e^2}\left(R_e - y\right)$$

$$\cong -\frac{Gm_em}{R_e} + \frac{Gm_e}{R_e^2}my$$

$$U \cong -\frac{Gm_em}{R_e} + mgy$$

$$\Delta U = U_f - U_i = \Delta mgy = mg\Delta y$$

$$\Delta U = mg\Delta y$$

$$U \neq mgy$$

$$U = -\frac{Gm_e m}{R_e} + mgy$$

This will always cancel when we find ΔE_{sys}, so we usually just don't write it

Ex: Find max height of a projectile

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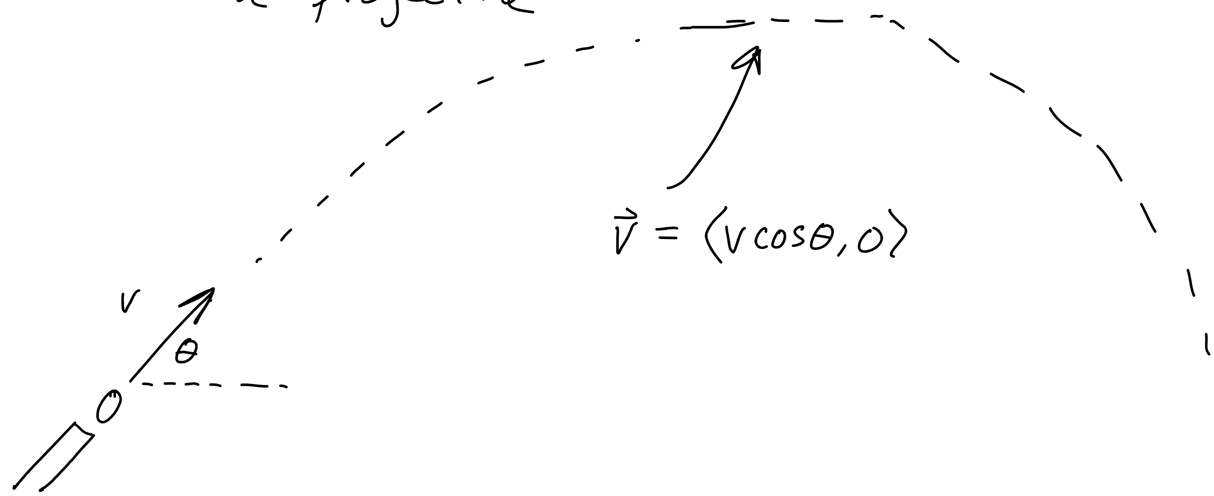


system = ball + Earth

Surr = None

$$E_i = E_f$$

$$E = K_{ball} + \cancel{K_{earth}} + U_{ball-earth}$$

$y = 0$

$$E_i = \frac{1}{2}mv^2 - \frac{G m_e m}{R_e}$$

$$E_f = \frac{1}{2}m(v\cos\theta)^2 - \frac{G m_e m}{R_e} + mgy$$

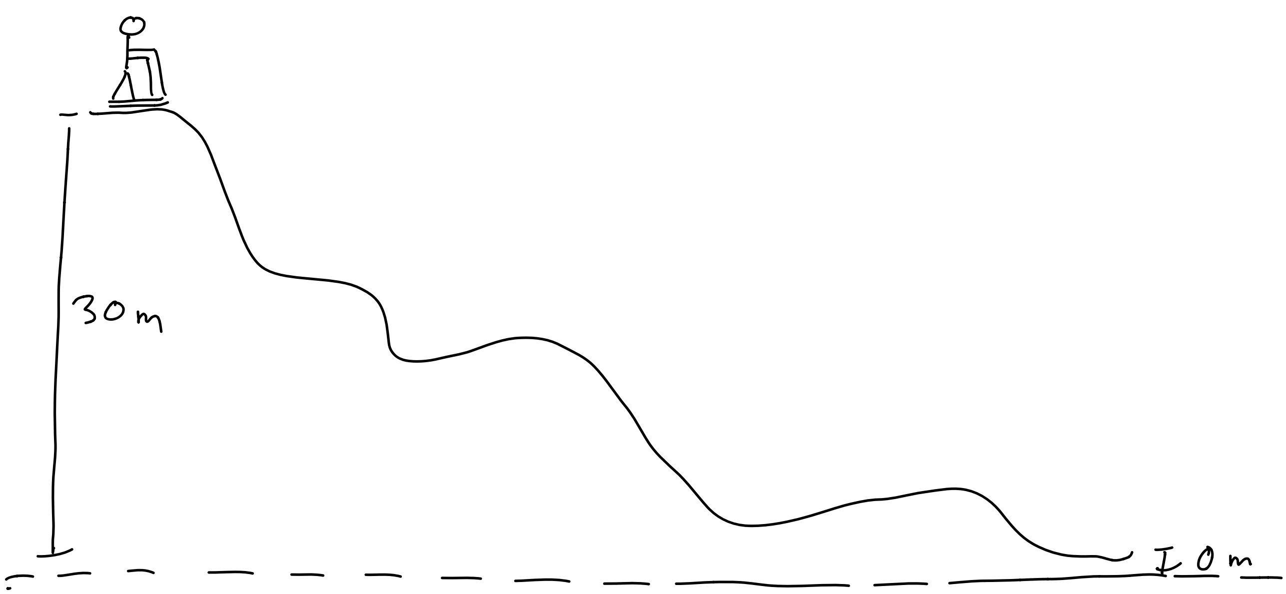
$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2\cos^2\theta + mgy$$

$\frac{1}{2}v^2(1-\cos^2\theta) = gy$

$y = \dfrac{v^2\sin^2\theta}{2g}$; same thing we found in Ch 2

The path doesn't matter

Ex: Skier on a hill



Ignore friction:

sys = skier + Earth

$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$

$mgy_i = \frac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{2gy_i} = \sqrt{2(9.8)(30)}$

$\boxed{v_f = 24.2\ m/s}$