Conclusion:

$$K = K_{cm} + K_{rel}$$

translational

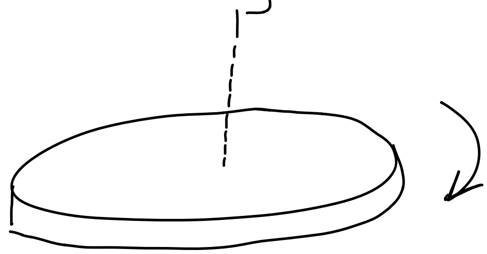
$$K = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

What is K_{rot}?

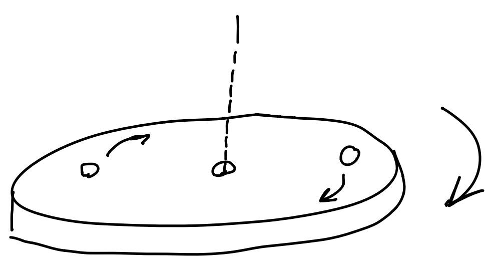
- Consider a rigid body (shape isn't changing)

  which is rotating about an axis

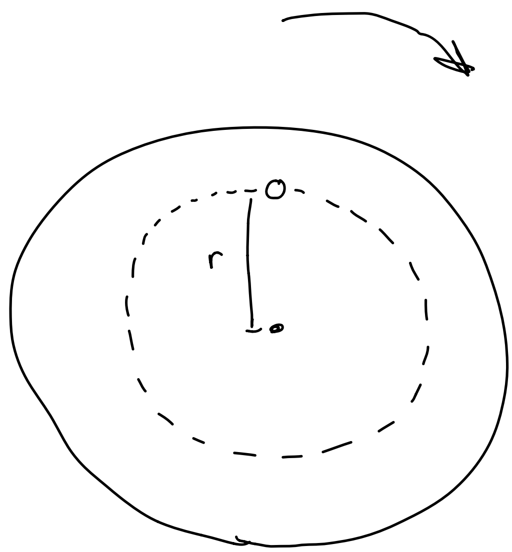


Axis of rotation:

- the straight line through all fixed points of a rotating object around which all other points rotate



View from top down $\quad f = \# \text{rotations}/\text{sec}$



This point travels a distance

$2\pi r$ per rotation

The point completes "f" rotations per second

So each second, it travels $2\pi r f$

so, $|\vec{v}| = 2\pi r f$

- This means that atoms far away from the axis of rotation are moving faster

Usually, we specify the speed of rotation as rad/sec, rather than rot/sec

ω = rad/sec

$$\omega = \frac{\text{rot}}{\text{sec}} \times \frac{\text{rad}}{\text{rot}} = 2\pi f$$

$$\omega = 2\pi f$$

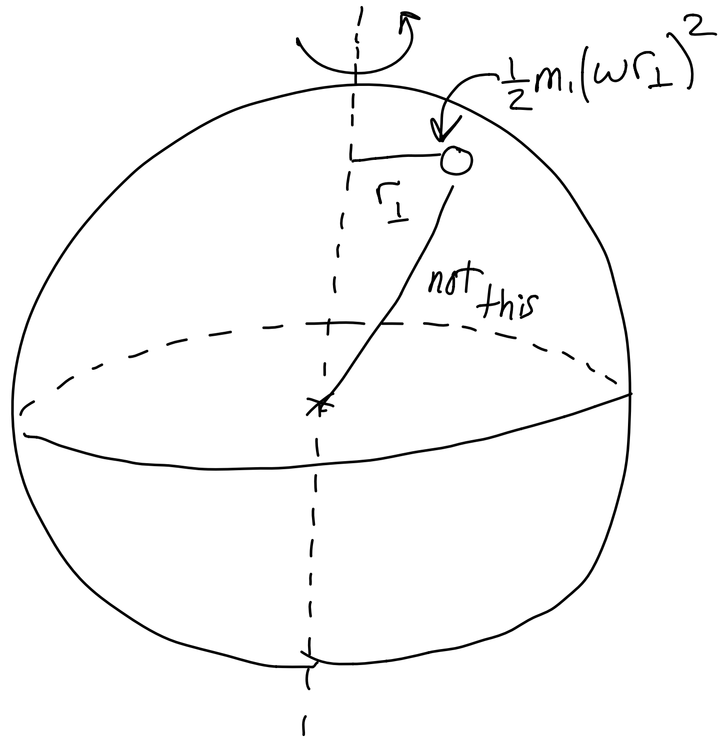
so $V_{rel} = \omega r$

$$K_{rot} = \frac{1}{2} m_1 v_{1,rel}^2 + \frac{1}{2} m_2 v_{2,rel}^2 + \frac{1}{2} m_3 v_{3,rel}^2 + \ldots$$

ω is the same for each particle

$$K_{rot} = \frac{1}{2} m_1 (\omega r_{\perp 1})^2 + \frac{1}{2} m_2 (\omega r_{\perp 2})^2 + \frac{1}{2} m_2 (\omega r_{\perp 3})^2 + \ldots$$

$r_\perp$ is the perpendicular distance to axis of rotation



$$K_{rot} = \frac{1}{2}m_1(\omega r_{\perp 1})^2 + \frac{1}{2}m_2(\omega r_{\perp 2})^2 + \frac{1}{2}m_2(\omega r_{\perp 3})^2 + \ldots$$

$$= \frac{1}{2}\left(m_1 r_{\perp_1}^2 + m_2 r_{\perp_2}^2 + m_3 r_{\perp_3}^2 + \ldots \right)\omega^2$$

$$I \equiv m_1 r_{\perp_1}^2 + m_2 r_{\perp_2}^2 + m_3 r_{\perp_3}^2 + \ldots$$

$$I = \sum_i m_i r_{\perp i}^2$$

I: moment of inertia

- Property of the material
- Units: $kg \cdot m^2$

$$K_{rot} = \frac{1}{2} I \omega^2$$

Compare to: $K_{trans} = \frac{1}{2} m v^2$

$$m \longrightarrow I$$

$$v \longrightarrow \omega$$

mass: resists change in v
(takes more energy to accelerate a car to 50 mph than a baseball)

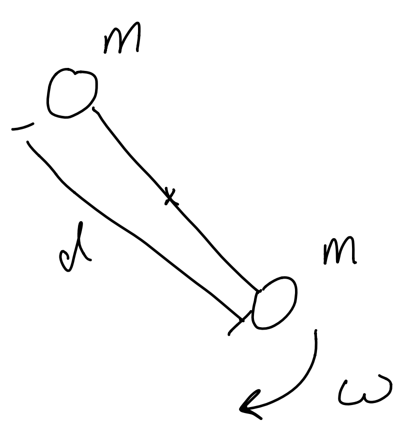
I : resists change in ω

Takes more energy to increase ω if :

- object is more massive
- mass is located further from axis of rotation

Ex :

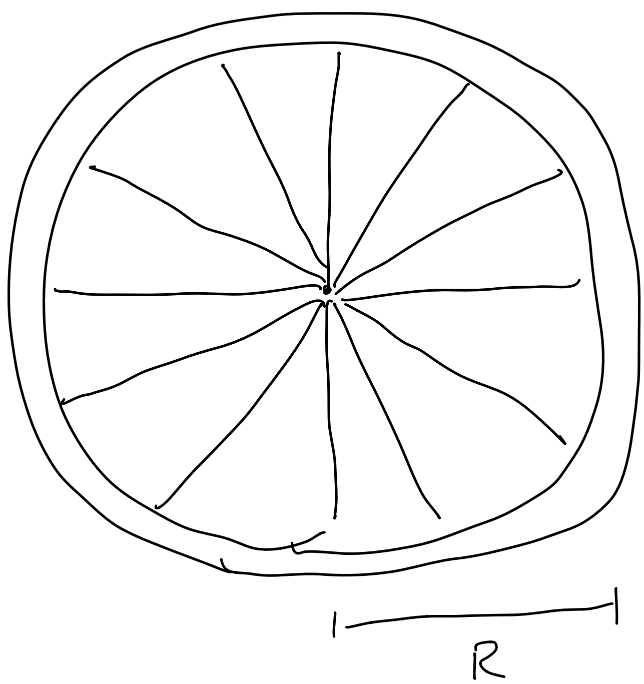
Dumbell (massless rod)



$$I = \sum_i m_i r_{\perp i}^2 = \left[m\frac{d}{2} + m\frac{d}{2} \right] = md$$

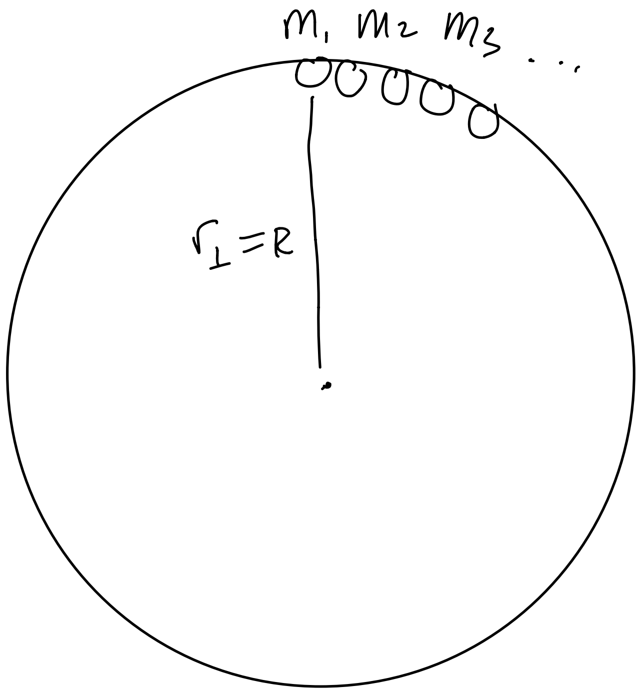
$$K_{rot} = \frac{1}{2} m d \omega^2$$

Ex: Energy required to rotate a bicycle wheel



Assume: spokes are (essentially) massless

Then



$$I = \sum_i m_i r_{\perp i}^2$$

$$= (m_1 R^2 + m_2 R^2 + m_3 R^2 + \dots)$$

$$= (m_1 + m_2 + m_3 + \dots) R^2$$

$$= M R^2$$

$$I = M R^2$$

$$K_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} M R^2 \omega^2$$

$R = 0.3 \ m$

$M = 2 \ Kg$

$\omega = 2\pi \times f$

$\quad = 2\pi \times 5$

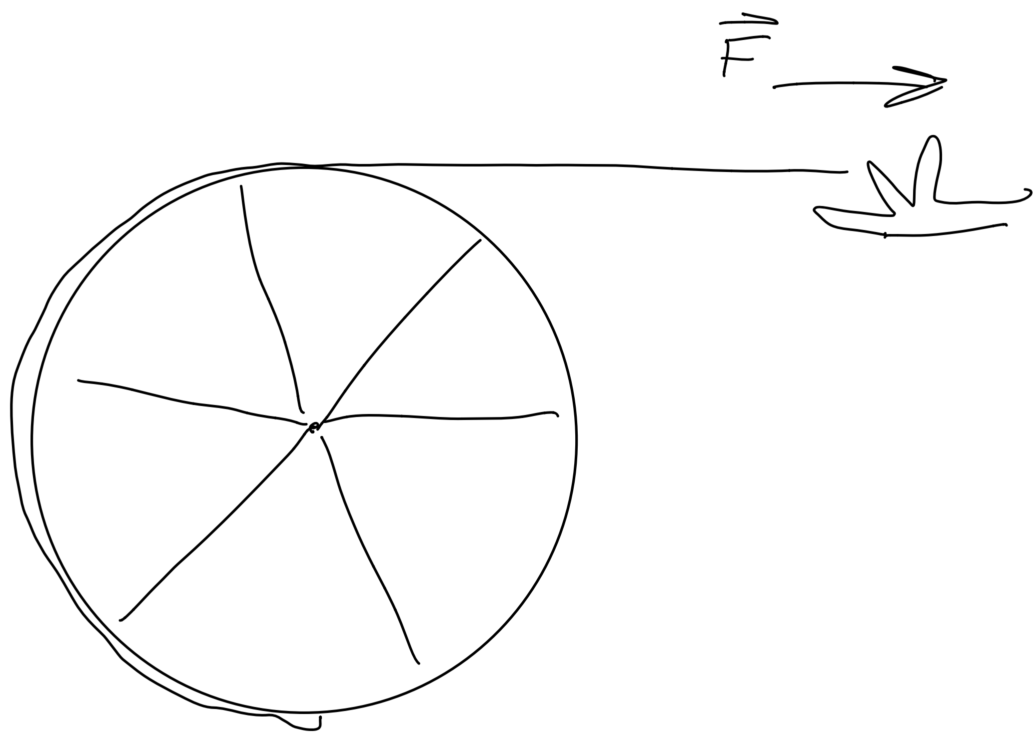
$\omega = 31.4 \ \frac{rad}{sec}$

$$K_{rot} = \frac{1}{2} (2kg)(0.3m)^2 \left(31.4 \ \frac{rad}{sec}\right)^2$$

$K_{rot} = 89 \ J$

The energy principle still applies!

$$\Delta E_{sys} = W_{surr}$$



initial: $f = 30$ rpm

final: $f = 120$ rpm

How much work did you do?

$\Delta E_{sys} = W_{surr}$

$E_f - E_i = W_{surr}$

$\frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W$

$I = MR^2$

$\frac{1}{2} MR^2 (\omega_f^2 - \omega_i^2) = W$

$\omega_f = 2\pi \left(\frac{rot}{sec} \right)$

$f = rpm \ \frac{rot}{min} \times \frac{1 \ min}{60 \ s}$

$\omega_i = 2\pi \left(\frac{30}{60} \right)$

$\omega_f = 2\pi \left(\frac{120}{60} \right)$

$$W = \frac{1}{2} M R^2 \left[(2\pi \cdot 2)^2 - \left(2\pi \cdot \frac{1}{2}\right)^2 \right]$$

$R = 0.3\ m$

$M = 2\ Kg$

$W = 13.3\ J$