Main concepts so far:

velocity: $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

momentum: $\vec{p} = m\vec{v}$

momentum principle: $\Delta \vec{p} = \vec{F}_{net} \Delta t$

Book lists the so called "position update equation"

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

Not a new equation! Same thing as

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

The momentum update equation

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

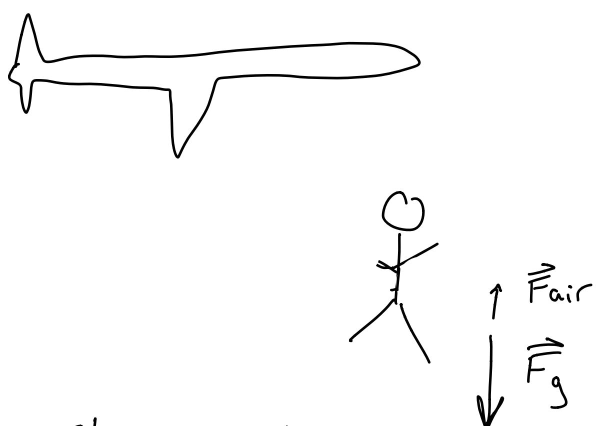
Same eqn as

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

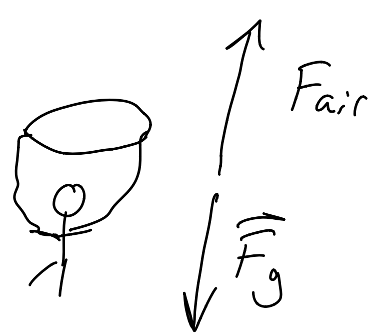
$\boxed{\text{QC 2.2.C}}$

Future momentum depends on 2 things: $\vec{p}_{now}$, $\vec{F}_{net} \Delta t$, now

Skydiver Example



initially, she accelerates downward



later, she opens her chute which increases air resistance so that the forces balance

Which best describes her motion from this point on?

a) Float w/o moving up or down

b) Continue falling at increasing speed

c) Fall at const speed

$\vec{F}_{net} = 0$, so $\Delta\vec{p} = 0$, so her momentum won't change. B<u>ut</u>, she already has downward momentum

$\vec{p}_{future} \rightarrow \vec{p}_{now}$

$\vec{p}_{future} \rightarrow \vec{F}_{net}\Delta t$

If an object has momentum, we can assume it interacted at some point in the past

- We don't need to know the entire history of an object, just $\vec{p}_{now}$

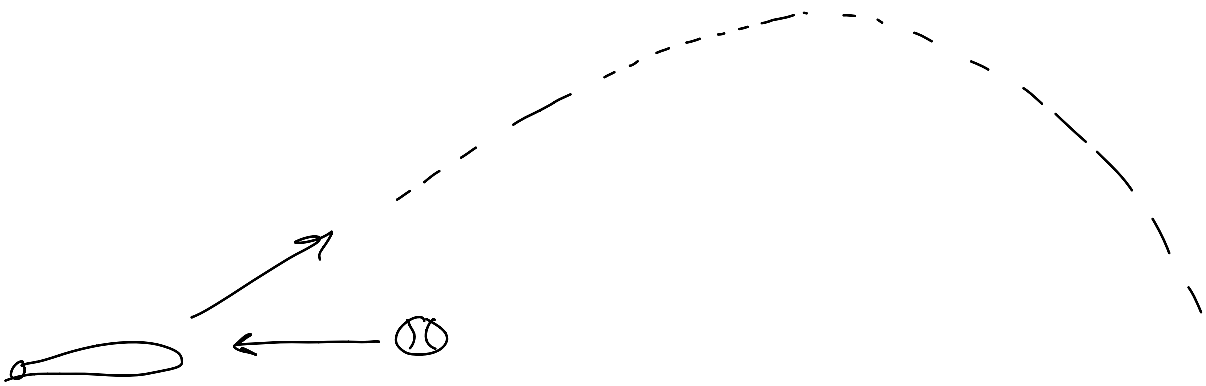
QC 2.4.b

2.4.c

Example:

Ball moving under influence of gravity

Gravity: $|\vec{F}| = mg$, $g = 9.81 \frac{m}{s^2}$

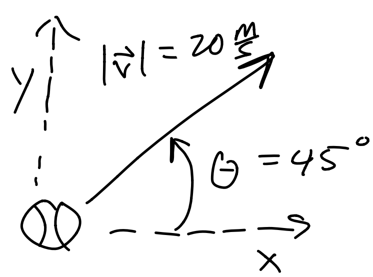


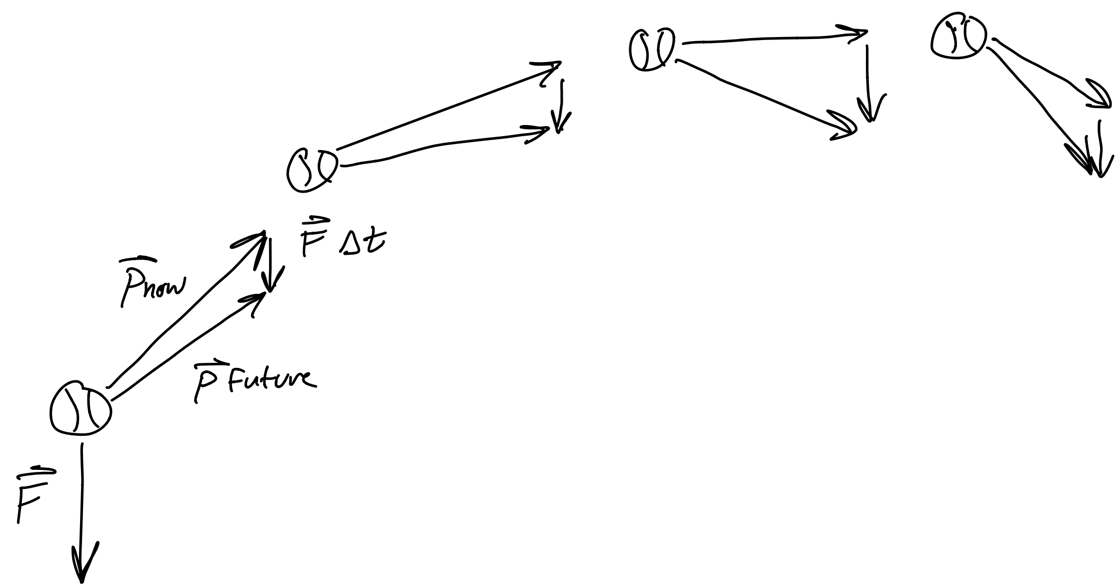
How?

1) Find $\vec{p}_{now}$
2) Find $\vec{F}_{net,now}$
3) $\vec{p}_{future} = \vec{p}_{now} + \vec{F}_{net,now} \Delta t$
4) $\vec{r}_{future} = \vec{r}_{now} + \vec{v}_{avg} \Delta t$, if Δt is small, $\vec{v}_{avg} = \frac{\vec{p}_{future}}{m}$

repeat

$m = 0.15 \text{ kg}$

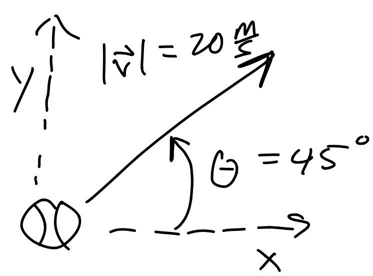




Start:

initial momentum?

$m = 0.15$ kg



$$\vec{v} = 20\,\frac{m}{s}\,\langle \cos\frac{\pi}{4},\ \cos^{-}\frac{\pi}{4},\ 0\rangle$$

$$= 20\,\frac{m}{s}\,\langle 0.71, 0.71, 0\rangle$$

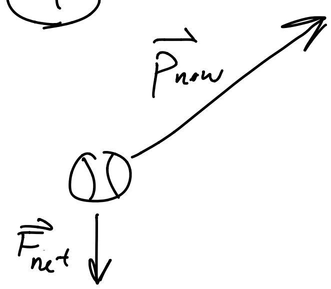
$$\vec{v} = \langle 14.1, 14.1, 0\rangle\ \frac{m}{s}$$

$$\vec{p}_{now} = m\vec{v} = 0.15 \text{ kg} \langle 14.1, 14.1, 0 \rangle \frac{m}{s}$$

$$\vec{p}_{now} = \langle 2.1, 2.1, 0 \rangle \frac{kg\, m}{s}$$

Let Δt be 0.5 s

1



$$|\vec{F}_g| = mg$$

direction is $-y$

any other forces?

no

$$\vec{F}_{net} = \langle 0, -9.8 \times 0.15, 0 \rangle \text{ N} = \langle 0, -1.47, 0 \rangle \text{ N}$$

$$\vec{p}_{future} = \vec{p}_{now} + \vec{F}_{net} \Delta t$$

$\frac{kg\, m}{s^2} \cdot s$

$$= \langle 2.1, 2.1, 0 \rangle \frac{kg\, m}{s} + 0.5 \langle 0, -1.47, 0 \rangle \text{ N} \cdot \text{s}$$

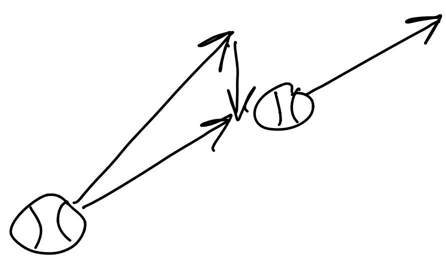
$$\vec{p}_{future} = \langle 2.1, 1.37, 0 \rangle \frac{kg\, m}{s}$$

$$\vec{v}_{avg} = \frac{\vec{p}_{future}}{m} = \langle 14.1, 9.13, 0 \rangle \frac{m}{s}$$

$$\vec{r}_{future} = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$= \langle 0, 2, 0 \rangle \, m + \langle 7.05, 4.57, 0 \rangle \, m$$

$$\vec{r}_{future} = \langle 7.05, 6.57, 0 \rangle \, m$$



② $\vec{p}_{now} = \langle 2.1, 1.37, 0 \rangle \, \frac{kg\, m}{s}$

$$\vec{F}_{net} = \langle 0, -1.47, 0 \rangle \, N$$

$$\vec{p}_{future} = \vec{p}_{now} + \vec{F}_{net}\Delta t$$

$$= \langle 2.1, 1.37, 0 \rangle \frac{kg\, m}{s} + \langle 0, -0.74, 0 \rangle \frac{kg\, m}{s}$$

$$\vec{p}_{future} = \langle 2.1, 0.63, 0 \rangle \frac{kg\, m}{s}$$

$$\vec{v}_{avg} = \frac{\vec{p}_{future}}{m} = \langle 14.1, 4.2, 0 \rangle \frac{m}{s}$$

$$\vec{r}_{future} = \vec{r}_{now} + \vec{v}_{avg}\Delta t$$

$$= \langle 7.05, 6.57, 0\rangle + 0.5\langle 14.1, 4.2, 0\rangle$$

$$= \langle 14.1, 8.67, 0\rangle \text{ m}$$

We can do better.

What I want:

Given the initial conditions of the ball $(\vec{r}_i, \vec{p}_i, m)$ and a constant Force $\vec{F}_{net}$, I can tell you

1) where the ball is
2) its momentum (+ velocity)

at <u>any</u> time just by plugging t into a formula

Consider:

Object starts with

$\vec{p}_i$, $\vec{r}_i$, + is subject to a constant force $\vec{F}_{net}$

First look at 1 dimension:

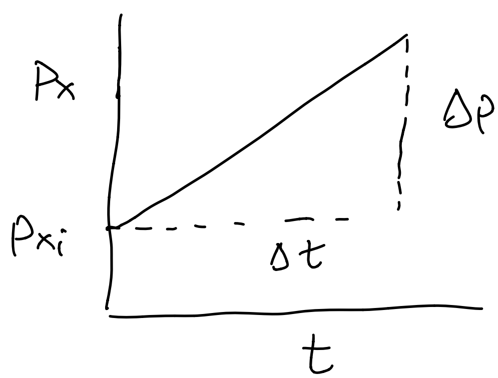
$$\vec{p}_i = p_{xi}$$

$$\vec{F}_{net} = F_x$$

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

$$\Delta p_x = F_x \Delta t$$

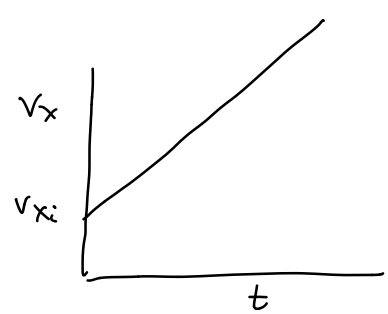
$$\frac{\Delta p_x}{\Delta t} = F_x = \text{const}$$



$$p_x(t) = p_{xi} + F_x t$$

$$p_x(t) = m v_x(t)$$

$$v_x(t) = v_{xi} + \frac{F_x}{m} t$$



We cant just say that

$$X(t) = X_i + V_x \cdot t$$

Because V_x is always changing

- instead

$V_x(t)$ is instantaneous velocity

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$V_x = \frac{dx}{dt}$$

$$dx = V_x\,dt$$

$$dx = \left(V_{xi} + \frac{F_x}{m}t\right)dt$$

$$\int_{x_i}^{x} dx = \int_0^t \left(V_{xi} + \frac{F_x}{m}t\right)dt$$

$$X - X_i = V_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$$

$$x(t) = x_i + V_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$$

Another way:

$$v_{avg,x} = \frac{v_{xi} + v_{xf}}{2} = \frac{v_{xi} + v_{xi} + \frac{F_x}{m}t}{2}$$

$$v_{avg,x} = \frac{2v_{xi} + \frac{F_x}{m}t}{2}$$

$$v_{avg,x} = v_{xi} + \frac{1}{2}\frac{F_x}{m}t$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$v_{avg,x} = v_{xi} + \frac{1}{2}\frac{F_x}{m}t = \frac{x - x_i}{t}$$

$$x(t) = x_i + v_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$$

Kinematic Equations (constant F_x)

$$p_x(t) = p_i + F_x t$$

$$v_x(t) = \frac{p_i}{m} + \frac{F_x}{m}t$$

$$v_{avg,x}(t) = v_{xi} + \frac{1}{2}\frac{F_x}{m}t$$

$$x(t) = x_i + v_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$$

Example: mass 1200 kg

A car driving at 30 $\frac{m}{s}$ spots a pedestrian and slams the brakes, applying a constant force of 9000 N.

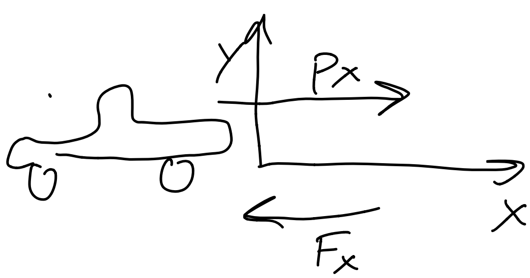
a) How long for car to stop?

b) How much distance travelled during that time?

$$p_{xi} = m v_{xi} = (1200\,\text{kg})\left(30\,\frac{m}{s}\right) = 36000\,\frac{kg\,m}{s}$$

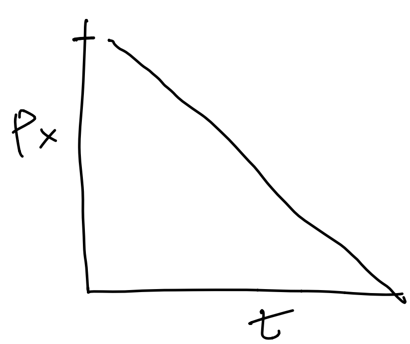
$$p_{xi} = 36000\,\frac{kg\,m}{s}$$

$F_{net} = ?$ $\quad F_{net} = -9000\,N$



$$p_x(t) = p_{xi} + F_x t$$

$$p_x(t) = 36000 - 9000t$$



Car stops when $p_x(t) = 0$

$$0 = 36000 - 9000t$$

$$9000t = 36000$$

$$t = 4\,s$$

$$x(t) = x_i + v_{xi}t + \frac{1}{2}\frac{F_x}{m}t^2$$

$x_i = 0$

$t = 4$

$F_x = -9000$

$$v_{xi} = \frac{p_{xi}}{m} = 30$$

$$x(4) = 0 + (30)(4) + \frac{1}{2}\left(\frac{-9000}{12000}\right)(4)^2$$

$$= 120 + (-6)$$

$$x(4) = 114\ m$$

$$\Delta x = 114\,m$$