Last lecture

Energy principle:

$$\boxed{\Delta E_{sys} + \Delta E_{surr} = 0}$$

E

Kinetic Energy:

$$\boxed{K = \frac{1}{2} m v^2}$$

Work:

$$\boxed{\Delta E = W = \vec{F} \cdot \Delta \vec{r}}$$

Force × time changes momentum

Force × distance changes energy

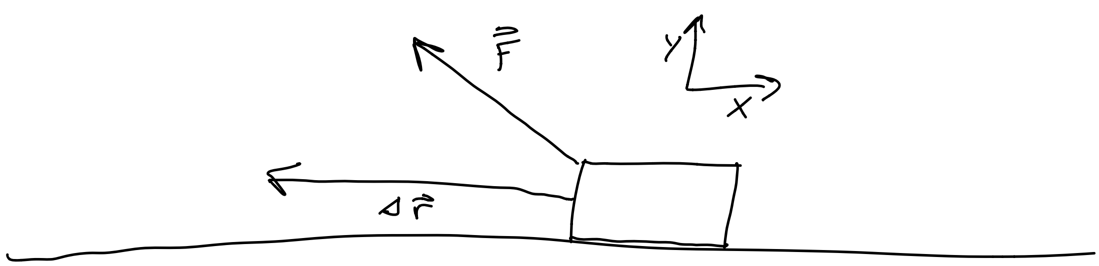
- Notes about dot product:
  - Is a scalar!
  - Is commutative ($\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$)

So: If a force $\vec{F}$ is being applied over a displacement of $\Delta\vec{r}$, then

$$\boxed{W = \vec{F}\cdot\Delta\vec{r}}$$

constant Force

Ex: How much work to pull a block w/ const force?

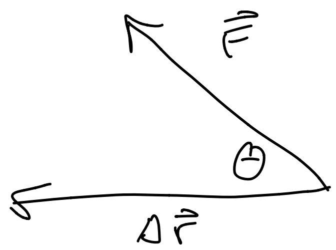


$$\vec{F} = -0.3\,\text{N}\,\hat{x} + 0.25\,\text{N}\,\hat{y} \qquad \left((-0.3, 0.25, 0)\right)$$

$$\Delta\vec{r} = -1.5\,\text{m}\,\hat{x}$$

$$W = \vec{F}\cdot\Delta\vec{r} = (-0.3\,\text{N})(-1.5\,\text{m}) + (0.25\,\text{N})(0)$$

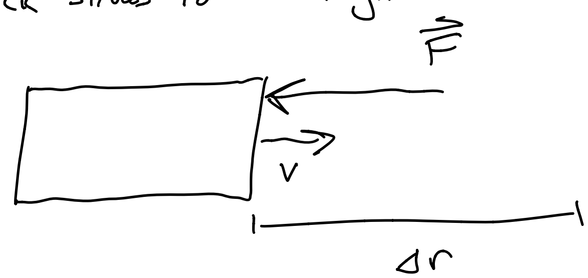
$$W = 0.45\ \text{J}$$



would get the same answer if we found θ then $W = |\vec{F}||\Delta\vec{r}|\cos\theta$

Note: Work can be negative

Ex: block slides to the right while I push to the left



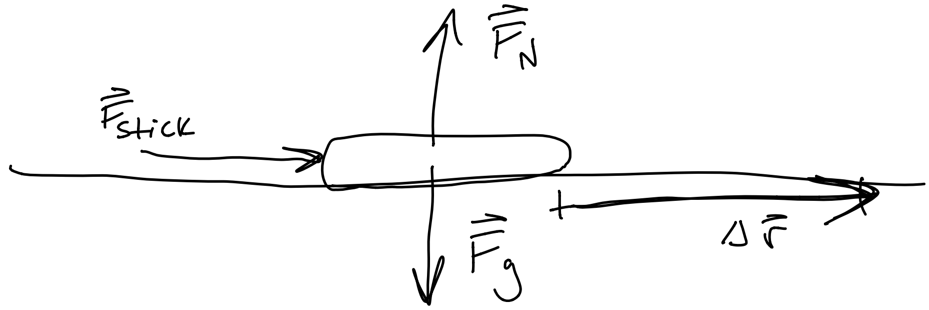
$$W = \vec{F}\cdot\Delta\vec{r} = |\vec{F}||\Delta\vec{r}|\cos(180°) = -|\vec{F}||\Delta\vec{r}|$$

Since $\Delta E = W$, energy will decrease; block slows down

of course it does!

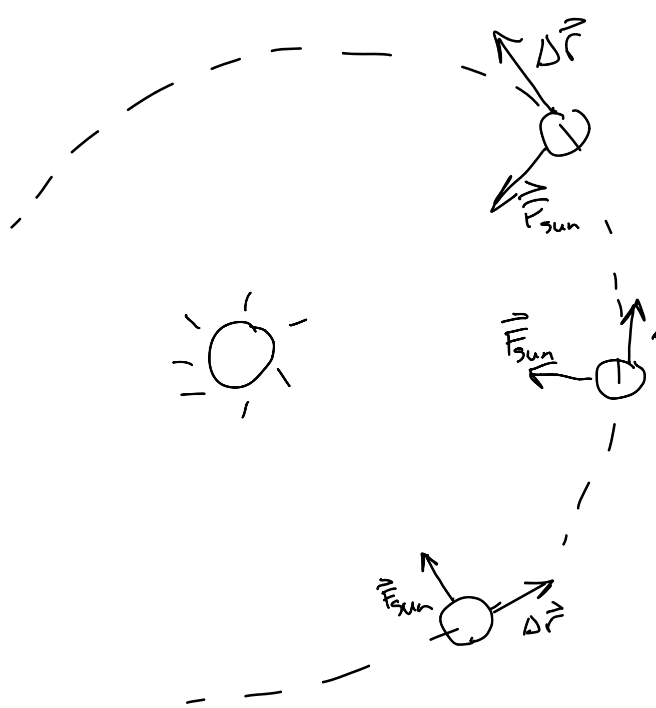
Not all forces do work

Ex: hockey puck being accelerated by a stick



$$W_{gravity} = \vec{F}_g \cdot \Delta\vec{r} = 0$$

Ex: Circular orbit



$$W_{sun} = \vec{F}_{sun} \cdot \Delta\vec{r} = 0$$

- The energy of the Earth does not change!
- The

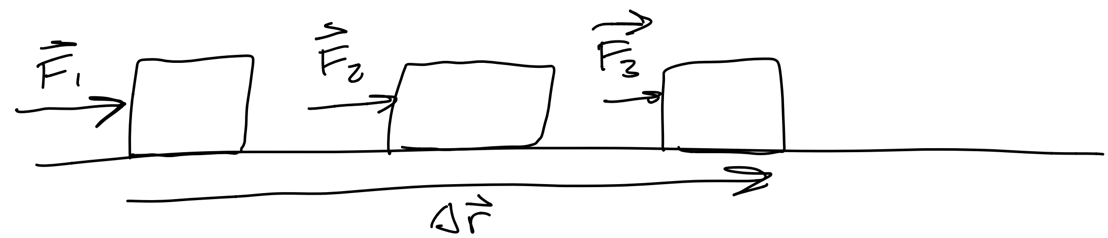
This confirms our result from Ch. 5: perpendicular forces do not change $|\vec{p}|$ only $\hat{p}$

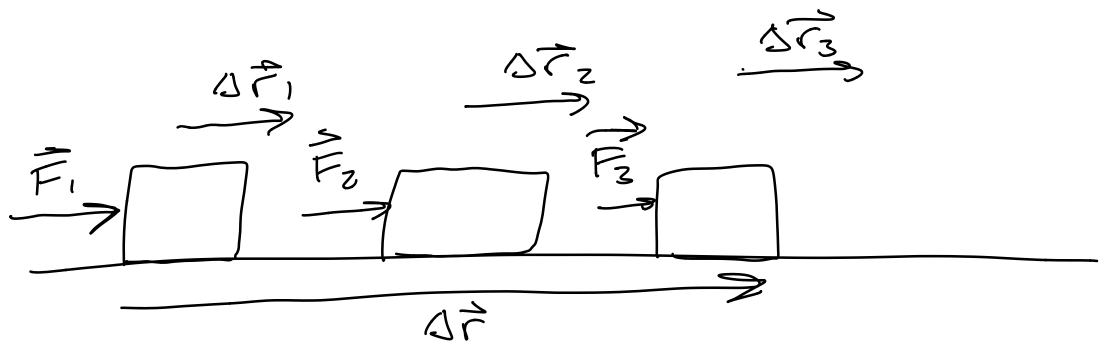
Our defn of W:

$W = \vec{F} \cdot \Delta\vec{r}$ is only valid if $\vec{F}$ is constant over $\Delta\vec{r}$

(like $\Delta\vec{p} = \vec{F}\Delta t$)

- If $\vec{F}$ is changing along $\Delta\vec{r}$





$$W = \vec{F}_1 \cdot \Delta\vec{r}_1 + \vec{F}_2 \cdot \Delta\vec{r}_2 + \vec{F}_3 \cdot \Delta\vec{r}_3$$

$$W = \sum_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot \Delta\vec{r}$$

$$W = \int_i^f \vec{F} \cdot d\vec{r}$$

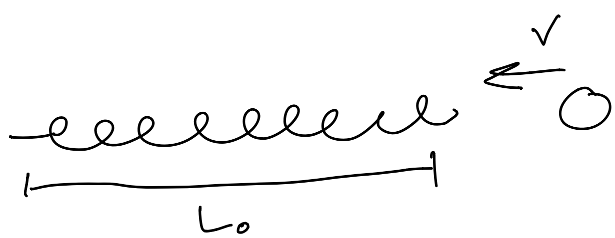
You haven't seen integrals like this yet... Don't worry

Ex: Work done by a spring

A ball is moving horizontally when it runs into a spring, compressing it by 20 cm

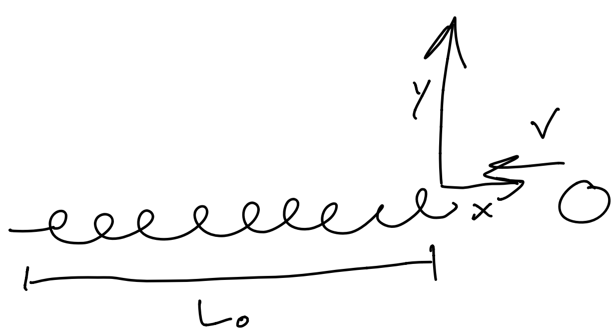
$K = 100\ N/m$

How much work does the spring do on the ball?



Q: Will W be positive or negative?

pick coordinate system so $x = 0$ @ L_0



$$\vec{F}_{spring} = K|s|\,\hat{x} = -Kx\,\hat{x}$$

$$\Delta\vec{r} = dx\,\hat{x}$$

$$W = \int_{x=0}^{x=-.2} -kx\hat{x} \cdot dx\hat{x} = \int_0^{-.2} -kx\,dx = -\frac{1}{2}kx^2 \Big|_0^{-.2}$$

$$= -\frac{1}{2}k(-.2)^2 = -2\ N \cdot m$$

The spring did $-2\ J$ of work on the ball (the ball lost energy + transferred it to the spring)

$$\Delta E_{sys} + \Delta E_{surr} = 0$$

$$\Delta E_{sys} = -\Delta E_{surr}$$

$$\Delta E_{sys} = W_{surr}$$

"The change in energy of the system is equal to the work done on the system by the surroundings"

Work done ON vs work done BY

- The object applying the force is doing the work
- The object expieriencing the force is being worked ON

The system is being worked ON, surr doing work

Ex: Ball & spring

System = spring

2 J of work done on spring by ball (surr)

System = ball

−2 J of work done on ball by spring

System = spring & ball

No work was done on the ball/spring system

If the ball was initially moving with speed $|\vec{v}| = 8$ m/s, what is its new speed? $m = 0.6$ kg

System: ball

$$\Delta E_{sys} = W_{surr}$$

$$E_f = E_i + (-2J)$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - (2J)$$

$$v_f = \sqrt{v_i^2 - \frac{2}{m}(2J)} = \sqrt{8^2 - \frac{2}{0.6}(2)} = \sqrt{57.33}$$

$$\boxed{v_F = 7.6\ \frac{m}{s}}$$

$(m = 0.2\ kg)$

Ex: A pitcher throws a baseball from rest to a speed of 36 m/s (~80 mph).

How much work did pitcher do on ball?

<u>Dont know $\vec{F}$ or $\Delta\vec{r}$</u>

sys: ball
surr: pitcher

$$\Delta E_{sys} = W_{surr}$$

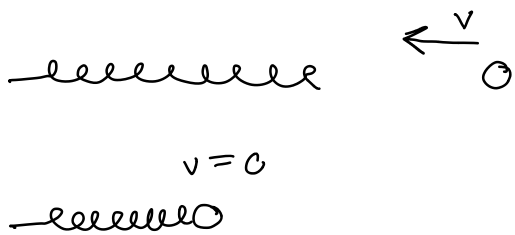
$$E_f - E_i = W_{surr}$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W_{surr}$$

$$\left(\frac{1}{2}\right)(0.2\,kg)\left((36\,\tfrac{m}{s})^2 - 0^2\right) = 129.6\ J$$

$$W_{surr} = 129.6\ J$$

Question: A moving ball compresses a spring and temporarily comes to rest.



system: ball + spring

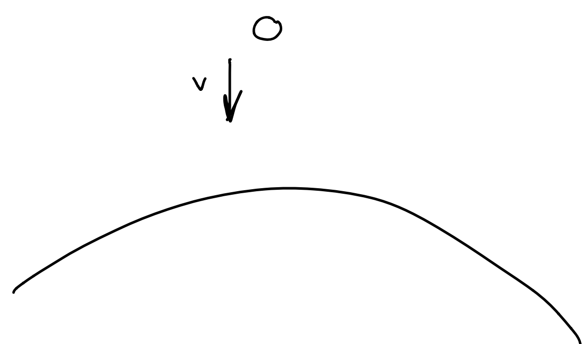
$\Delta E_{sys} = W_{surr} = 0$

$E_i = \frac{1}{2} m v^2$

$E_f = 0$

$\boxed{\Delta E_{sys} = -\frac{1}{2} m v^2}$?

Ex: Ball falling toward Earth



System: ball + Earth

Surr: None

$\Delta E_{sys} = 0$

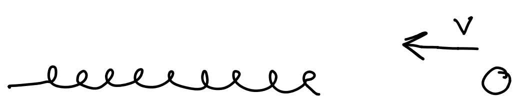
$\Delta E_{sys} = \underbrace{\Delta K_{ball}}_{>0} + \underbrace{\Delta K_{earth}}_{>0} > 0$?

- When the ball was stopped by the spring, where did its energy go?

- When the ball + Earth both gained speed, where did that energy come from?

Another form of Energy: Potential Energy

Potential energy: energy associated w/ interactions in multi-particle systems



system: ball + spring

$\Delta E_{sys} = W_{surr} = 0$

$E_i = \frac{1}{2}mv^2 + U_{i\,\text{ball-sp}}$

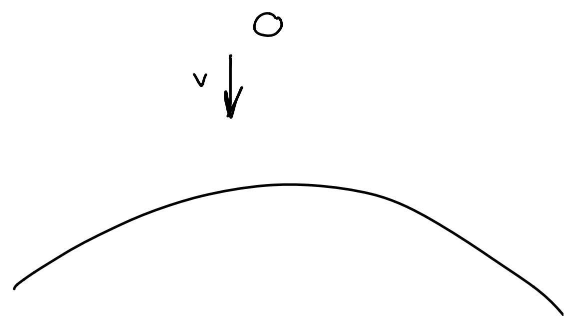
$E_f = U_{f\,\text{ball sp'ng}}$

$\Delta E = \Delta U - \frac{1}{2}mv^2 = 0$

$\Delta U = \frac{1}{2}mv^2$

The kinetic energy of the ball was transformed into potential energy "stored" in the spring

Ex: Ball falling toward Earth



System: ball + Earth

Surr: None

$$\Delta E_{sys} = 0$$

$$\Delta E_{sys} = \underset{>0}{\Delta K_{ball}} + \underset{>0}{\Delta K_{earth}} + \underset{?}{\Delta U_{ball-earth}} = 0$$

$$\Delta U_{ball-earth} = -\left(\Delta K_{ball} + \Delta K_{earth}\right)$$

Potential energy stored in the gravitational field was converted into kinetic energy

Potential energy:

- "Stored" energy in multiparticle systems
- Has the "potential" to turn into kinetic energy
- Depends on relative position of interacting objects

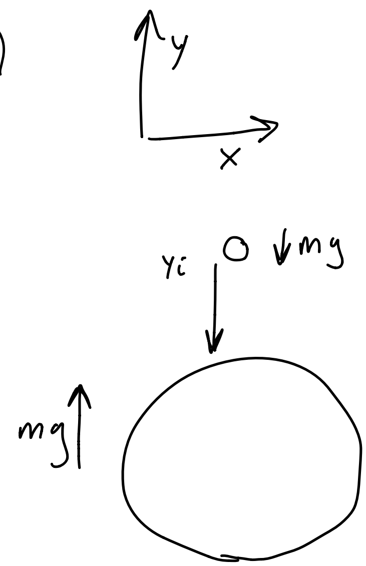
- A spring which is highly compressed will launch an object faster (it has more potential energy)

- A ball dropped from above the Earth is moving faster when it hits the surface if it was released higher

To calculate potential energy:

$$\Delta K_{earth} + \Delta K_{ball} + \Delta U_{ball-earth} = 0$$

$$\Delta U = -(\Delta K_{earth} + \Delta K_{ball})$$



Earth moves up a small distance

$$W_{on\,earth} = mg\,\Delta y_{earth} \approx 0$$

$$W_{on\,ball} = -mg\,\Delta y$$

$$\Delta U = -(\Delta K_{earth} + \Delta K_{ball})$$
$$= -(0 + -mg\Delta y)$$
$$\Delta U = mg\Delta y$$

Ex: 100 g ball falls from a height of 1 m

What is its speed when it lands?

Method 1: Use $y(t) = y_i - \frac{1}{2}gt^2 = 0$ to find Δt

$\Delta p = -mg\Delta t$

Method 2:

system = ball + Earth

Surr = None

$\Delta E_{sys} = 0$

$$\underbrace{\Delta K_{Earth}}_{\approx 0} + \Delta K_{ball} + \Delta U = 0$$

$$\Delta K_{ball} = -\Delta U$$

$$= -mg\Delta y$$

$$= -mg(y_f - y_i)$$

$$\Delta K_{ball} = mg(y_i - y_f)$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mg(y_i - y_f)$$

$$\frac{1}{2}mv_f^2 = mgy_i$$

$$v_f = \sqrt{2gy_i}$$

$$v_f = \sqrt{(2)(9.8\ m/s^2)(1m)}$$

$$\boxed{v_f = 4.43\ m/s}$$