Last class

Constant Force motion

$$\vec{F}_{net} = \text{const}$$

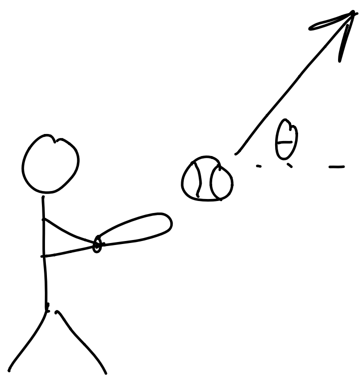
$$p_x(t) = p_{xi} + F_x t$$

$$V_x(t) = \frac{p_x(t)}{m} = V_{xi} + \frac{F_x}{m} t$$

$$X(t) = X_i + V_{xi} t + \frac{1}{2}\frac{F_x}{m} t^2$$

What about an object moving in 2D?

Ex $\downarrow$



Use momentum principle in x & y directions independently

<u>X:</u>

$$p_x(t) = p_{xi} + F_x t$$

$$v_x(t) = v_{xi} + \frac{F_x}{m} t$$

$$x(t) = x_i + v_{xi} t + \frac{1}{2} \frac{F_x}{m} t^2$$

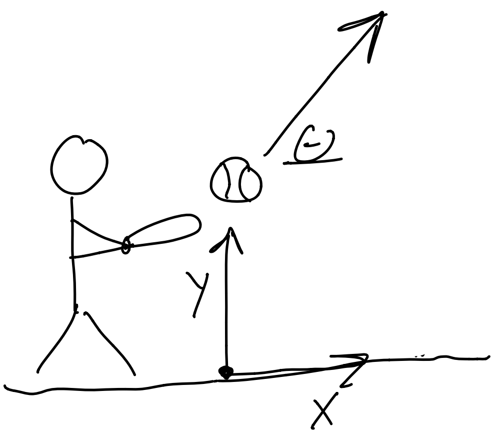
<u>Y:</u>

$$p_y(t) = p_{yi} + F_y t$$

$$v_y(t) = v_{yi} + \frac{F_y}{m} t$$

$$y(t) = y_i + v_{yi} t + \frac{1}{2} \frac{F_y}{m} t^2$$

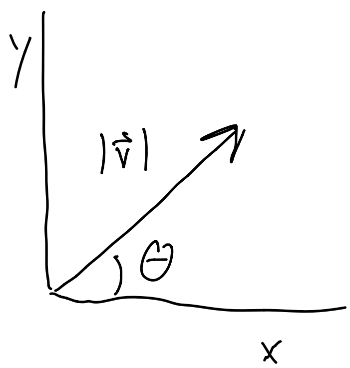
1) Pick a coordinate system & list initial quantities



$x_i = 0, \quad y_i = 0$ $\qquad \vec{r}_i = \langle 0, 0 \rangle$

p_{xi}, p_{yi}?

$$\vec{v} = |\vec{v}|\,\hat{v}$$



$$\hat{v} = \langle \cos\theta, \cos(\theta - \tfrac{\pi}{2}) \rangle$$

$$\hat{v} = \langle \cos\theta, \sin\theta \rangle$$

$$\vec{v} = |\vec{v}| \langle \cos\theta, \sin\theta \rangle$$

$$v_{x_i} = |\vec{v}|\cos\theta, \quad v_{y_i} = |\vec{v}|\sin\theta$$

$p_{xi} = m|\vec{v}|\cos\theta \, , \quad v_{yi} = m|\vec{v}|\sin\theta$

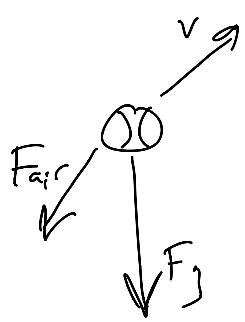
$x_i = 0 \qquad y_i = 0$

$p_{xi} = m|\vec{v}|\cos\theta \qquad p_{yi} = m|\vec{v}|\sin\theta$

2) Find $\vec{F}_{net}$ + Draw

What is the ball interacting with?

- grav
- air



$\vec{F} \propto -\vec{v}$

- Fair changes
- Can't use constant force equations
- Ignore (for now)

$\vec{F}_{net} = \vec{F}_g = \langle 0, -mg \rangle$

$F_x = 0 \, , \quad F_y = -mg$

$$P_x(t) = m|\vec{v}_i|\cos\theta + 0(t)$$

$$P_x(t) = m|\vec{v}_i|\cos\theta$$

$$x(t) = x_i + |\vec{v}_i|\cos\theta\, t + \cancel{\frac{1}{2}\frac{F_x}{m}t^2}$$

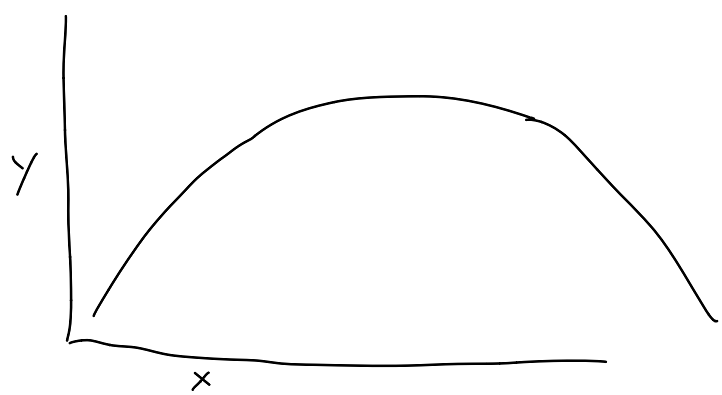
$$x(t) = x_i + |\vec{v}_i|\cos\theta\, t$$

$$P_y(t) = m|\vec{v}_i|\sin\theta - mgt$$

$$y(t) = y_i + |\vec{v}_i|\sin\theta\, t - \frac{1}{2}gt^2$$

$$y(t) = |\vec{v}_i|\sin\theta\, t - \frac{1}{2}gt^2$$

In the x direction: motion @ constant speed

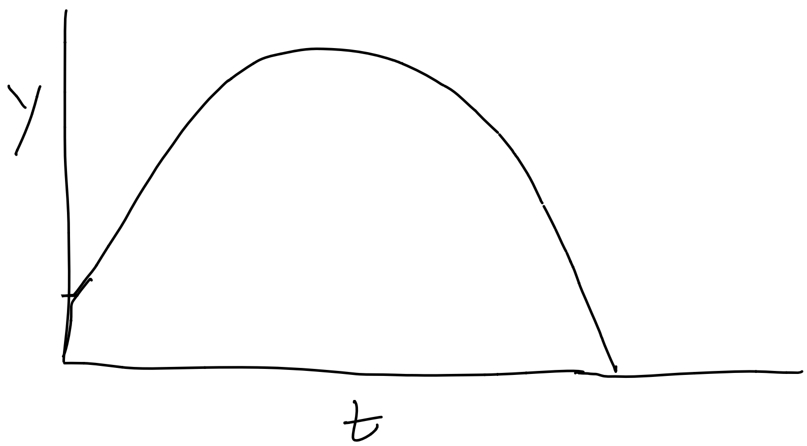


max height?

Horizontal range?

Time of flight?

Let's start with time of flight



when does $y = 0$?

$$y(t) = y_i + |\vec{v}_i| \sin\theta\, t - \frac{1}{2} g t^2$$

$$0 = |\vec{v}_i| \sin\theta\, t - \frac{1}{2} g t^2$$

$$t\left(|\vec{v}_i| \sin\theta - \frac{1}{2} g t\right) = 0$$

$t = 0$ is a solution (of course)

$$|\vec{v}_i| \sin\theta - \frac{1}{2} g t = 0$$

$$\boxed{t = \frac{2|\vec{v}_i| \sin\theta}{g}}$$

Ex: IF, $|\vec{v}_i| = 20 \frac{m}{s}$, $\theta = 45°$

$t = 2.89\ s$

Now that we know t, we can find horizontal distance

$$x(t) = x_i + |\vec{v}_i| \cos\theta\ t$$

$$= |\vec{v}_i| \cos(\theta) \left(\frac{2|\vec{v}_i| \sin\theta}{g} \right)$$

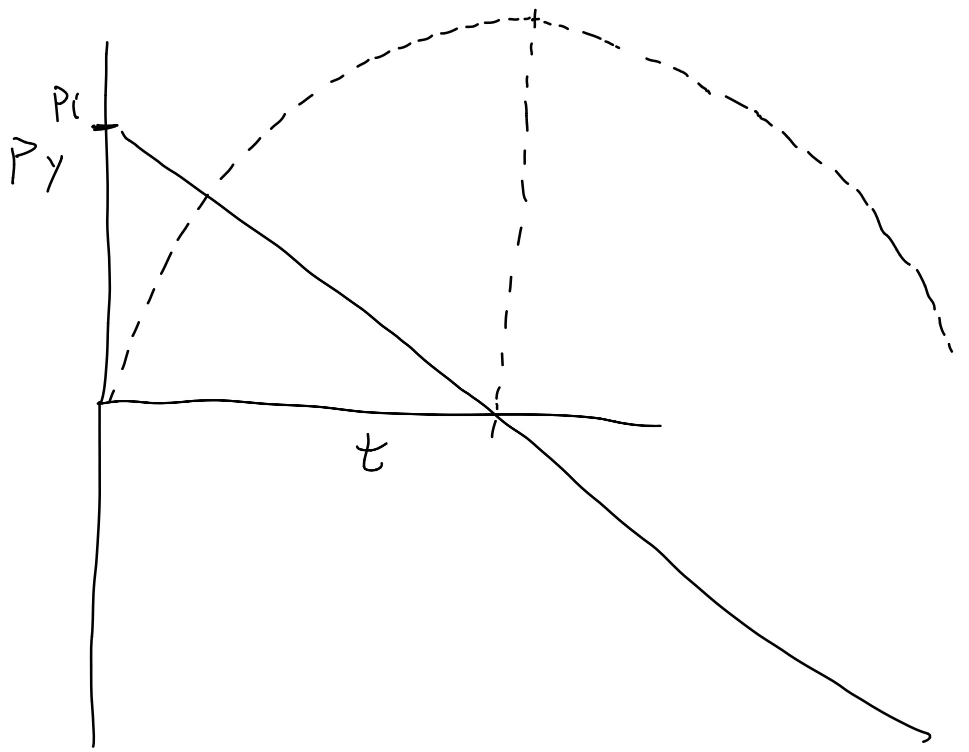
$$= \frac{2|\vec{v}_i|^2 \sin\theta \cos\theta}{g} \qquad \frac{1}{2}\sin(2\theta)$$

$$x_{max} = \frac{|\vec{v}_i|^2 \sin(2\theta)}{g}$$

Max height?

$$y(t) = y_i + |\vec{v}_i| \sin\theta \, t - \frac{1}{2} g t^2$$

t is not time of flight



Find t such that $p_y = 0$

$$p_y(t) = m|\vec{v}_i|\sin\theta - mgt$$

$$p_y(t) = m|\vec{v}_i|\sin\theta - mgt$$

$$0 = m|\vec{v}_i|\sin\theta - mgt$$

$$t = \frac{|\vec{V}_i|\sin\theta}{g}$$

- time to reach max height
- Note: $\frac{1}{2}$ total time of flight

$$y_{max} = y\left(\frac{|\vec{v}_i|\sin\theta}{g}\right)$$

$$y(t) = y_i + |\vec{v}_i|\sin\theta\, t - \frac{1}{2}gt^2$$

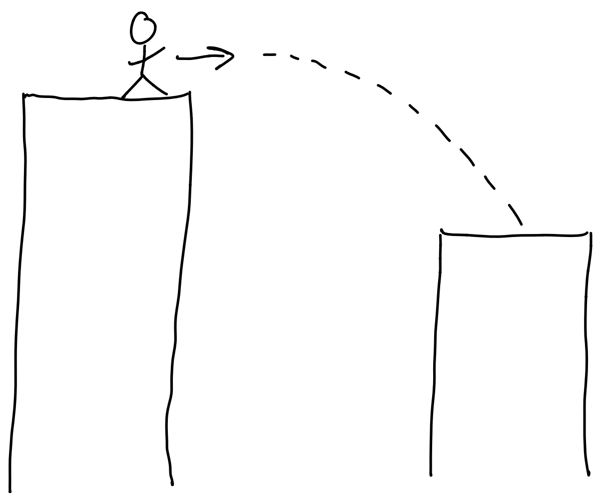
$$y\left(\frac{|\vec{v}_i|\sin\theta}{g}\right) = |\vec{v}_i|(\sin\theta)\left(\frac{|\vec{v}_i|\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{|\vec{v}_i|\sin\theta}{g}\right)^2$$

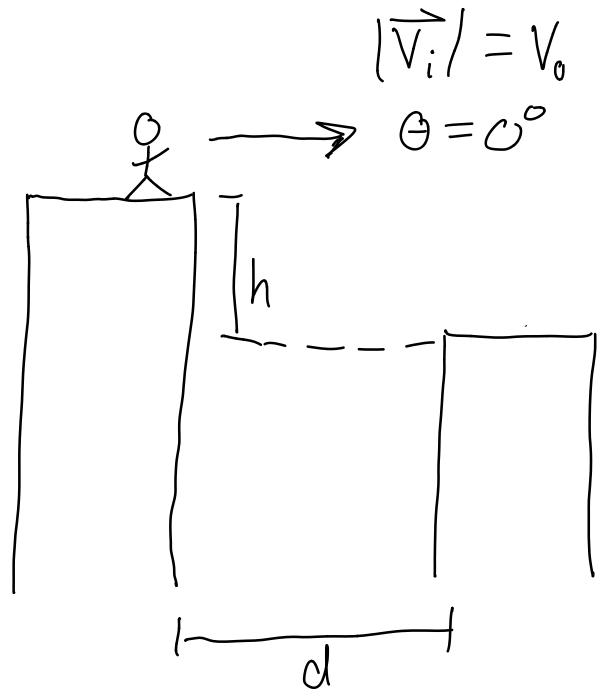
$$y_{max} = \frac{|\vec{v}_i|^2\sin^2\theta}{g} - \frac{1}{2}\frac{|\vec{v}_i|^2\sin^2\theta}{g}$$

$$y_{max} = \frac{|\vec{v}_i|^2\sin^2\theta}{2g}$$

if $|\vec{v}_i| = 20 \frac{m}{s}$, $\theta = 45°$

$y_{max} \approx 10.2$ m





- Will they make it to the other building?

Procedure

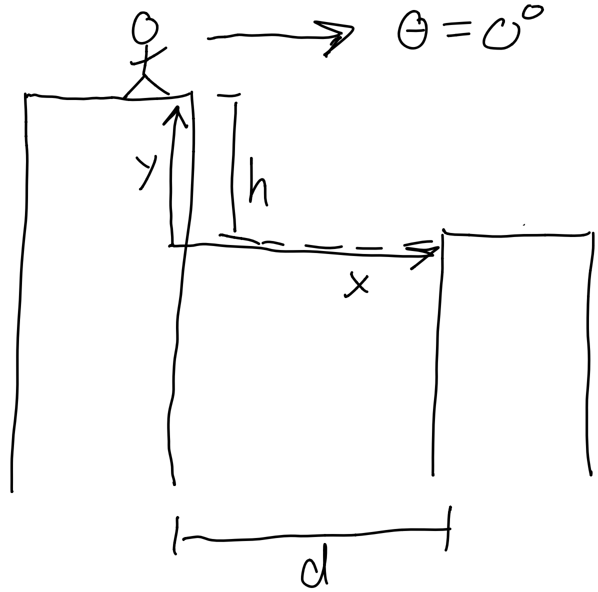
1) Find time for them to move $\Delta y = h$

2) Find Δx for that time

Let's call the height of the lower building $y = 0$

$|\vec{V_i}| = V_i$

$\theta = 0^\circ$



$X_i = 0$

$y_i = h$

$\vec{F}_{net} = \langle 0, -mg \rangle$

$$y(t) = y_i + |\vec{V_i}| \sin\theta \, t - \frac{1}{2} g t^2$$

$$= h + V_i \sin(0) t - \frac{1}{2} g t^2$$

$$y(t) = h - \frac{1}{2} g t^2$$

$0 = h - \frac{1}{2} g t^2$, $t = \pm\sqrt{\frac{2h}{g}}$, we want the t

$$t_{land} = \sqrt{\frac{2h}{g}}$$

$$x(t) = x_i + |\vec{v}_i| \cos\theta \, t$$

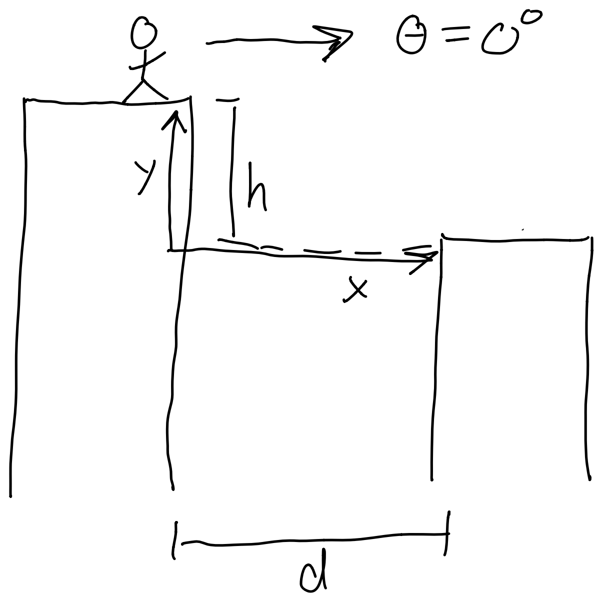
$$= v_i \cos(0) \, t$$

$$x(t) = v_i t$$

$$x(t_{land}) = (v_i)\left(\sqrt{\frac{2h}{g}}\right)$$

$$|\vec{v}_i| = v_i$$

$$\theta = 0^\circ$$



Say: $h = 10\ m$

$d = 15\ m$

$V_i = 12\ \frac{m}{s}$

$$X(t_{land}) = (12)\sqrt{\frac{2(10)}{9.8}} \approx 17.1\ m$$